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TRANSMISSION LINE MODEL ON LUMPED ELEMENTS

Bachelor's thesis

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PhD

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LAINESUHI MUDEL DISKREETEMENTIDEL

Bakalaureusetöö

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Tallinn 2020

Author's declaration of originality

I hereby certify that I am the sole author of this thesis. All the used materials, references to the literature and the work of others have been referred to. This thesis has not been presented for examination anywhere else.

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18 May 2020

Abstract

The aim of the work is to develop a laboratory work for a transmission line model. The main stages of the work are the calculation, simulation and fabrication of a physical model of transmission line on discrete elements. Validation of calculation and simulation results on a physical model and preparation of a structure for the laboratory manual.

This thesis is written in English and is 52 pages long, including 6 chapters, 31 figures and 8 tables.

Annotatsioon

LAINJUHI MUDEL DISKREETELEMENDEL

Töö eesmärgiks on lainejuhi mudeli laboritöö väljatöötamine. Töö peamiseks etappideks on diskreetelementidel lainejuhi mudeli arvutamine, simuleerimine ja füüsilise mudeli valmistamine. Arvutus – ja simulatsioonitulemuste valideerimine füüsilisel maketil ning laborijuhendi kondikava koostamine.

Lõputöö on kirjutatud inglise keeles ning sisaldab teksti 52 leheküljel, 6 peatükki, 31 joonist, 8 tabelit.

List of abbreviations and terms

EMF	<i>Electromotive force</i>
TP	<i>Test point</i>
SWR	<i>Standing wave ratio</i>

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Introduction

The transmission line is one of the main topics in electrical engineering. A transmission line is a device that helps transfer electrical energy from one point to another with a minimum of loss and distortion. For example, from a signal generator to an antenna. In the process of energy transfer, many physical phenomena arise, which we will consider in this thesis.

The main goal of the thesis is to create a laboratory work with which the student could study the transmission line.

The work consisted of theoretical basis. There we will examine what types of transmissions are, consider the losses in the transmission lines, as well important parameters of transmission line. One of the most important parts is the simulation in SPICE. The simulation will show how correctly we made the calculations and selected the values of the elements.

The main part is the creation of a board for laboratory work. Based on the results in the previous parts, we will design a PCB and recreate it in reality. Calculate the cost of such a PCB and test it. Based on the data, we will create graphs that show us how correctly we completed the work and analyze them.

And in the end, we'll create a manual for laboratory work. It will be used in TalTech university. Students will be able to learn the transmission line and prove their knowledge by practice.

1 Theoretical basis

1.1 Electrical reactance

In electrical engineering, the opposition of a circuit element to the flow of current due to that element's inductance or capacitance is reactance.

Reactance is used to compute amplitude and phase changes of sinusoidal alternating current going through a circuit element. It is denoted by the symbol X . As frequency goes up, inductive reactance also increases and capacitive reactance decreases. Reactance changes the phase so that the current through the element is shifted by a quarter of a cycle relative to the voltage applied across the element

Important parameter of transmission line is its capacity per unit length. Structurally, it consists of two or more conductive plates separated by a layer of material having dielectric properties. Due to this separation, direct current cannot pass through the capacitor, and alternating current is capable, but with a deviation from the original value. Due to recharging the plates, the current through the capacitor is ahead of the voltage by 90 degrees.

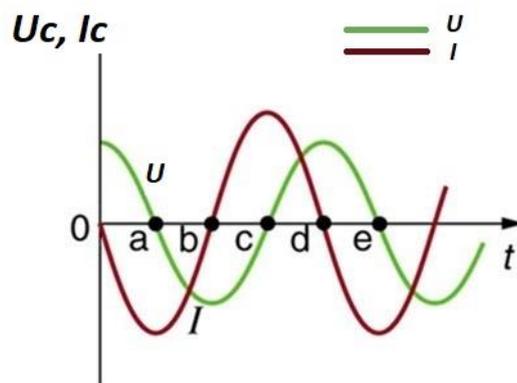


Figure 1. Graph of current and voltage across the capacitor as functions of time. [1]

As a result, the voltage drop will limit the current in the circuit across the capacitor, which creates reactive resistance to AC, inversely proportional to the rate of change of current (frequency) and capacitance of the capacitor.

Capacitive Reactance equation:

$$X_C = \frac{j}{\omega C} = -\frac{1}{j2\pi f C} \quad (1.1)$$

Where X_C capacitive reactance is in Ohms (Ω), f is frequency of voltage/current in Hertz (Hz) and C is Capacitance of capacitor in Farads (F). Constant j is imaginary unit, complex number whose square is -1 or $j = \sqrt{-1}$. ω is angular frequency, measures angular displacement per unit time, which equals $\omega = 2\pi f$.

Inductors and inductive reactance. When alternating current flows in the inductor, the magnetic field creates electromotive force (EMF) in its turns, which trying prevents the current from changing.

With increasing current, the EMF is negative and prevents the current from growing, while decreasing it is positive and prevents it from decreasing, thus resisting the change in current throughout the entire period.

As a result of the created counteraction, the voltage suppresses the EMF, equal in amplitude and opposite in sign, is formed at the terminals of the inductance coil in antiphase. When a sinusoidal voltage is applied to an inductor, the voltage leads the current by one-fourth of a cycle, or by a 90° phase.

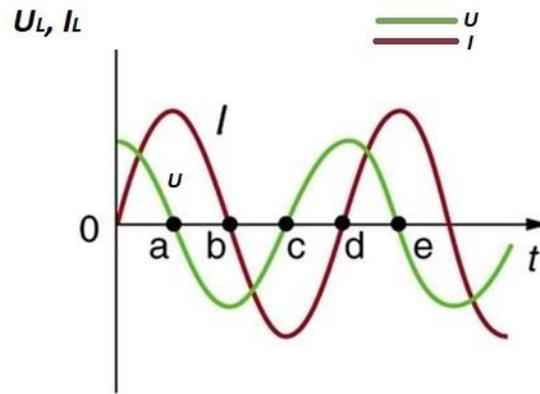


Figure 2. Graph of current and voltage across the inductor as functions of time. [1]

By overcoming the inductance forces, the current on the coil lags the voltage by 90 degrees.

Inductive Reactance equation:

$$X_L = j\omega L = j2\pi fL. \quad (1.2)$$

Where X_L is inductive reactance in Ohms (Ω), f is frequency of voltage/current in Hertz (Hz), L is inductance in Henryes (H) and j is imaginary unit.

Resistance. When a sinusoidal voltage is applied to an ideal resistor, the voltage is exactly in phase with the current, in other words, they have a 0° phase angle between them.

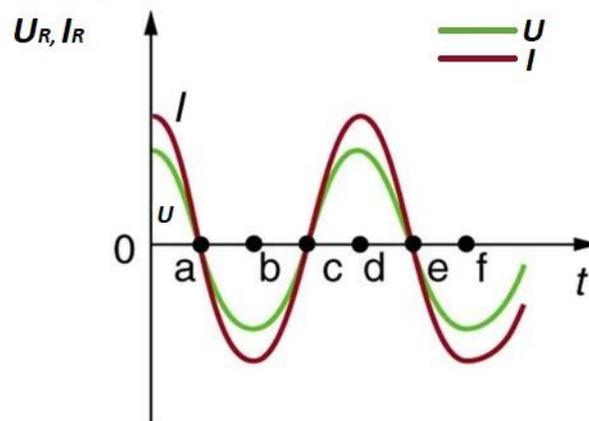


Figure 3. Graph of current and voltage across the resistor as functions of time, showing them to be exactly in phase. [1]

1.2 Transmission lines

Transmission line is a connector, which transmits energy from one point to another. The study of transmission line theory is helpful in the effective usage of power and equipment.

There are basically four types of transmission lines:

- Two-wire parallel transmission lines,
- Coaxial lines,
- Strip type substrate transmission lines,
- Waveguides.

While transmitting or while receiving, the energy transfer has to be done effectively, without the wastage of power. To achieve this, there are certain important parameters which has to be considered. [2]

Transmission-line parameters R' , L' , G' , and C' for three types of lines.

Parameter	Coaxial	Two-Wire	Parallel-Plate	Unit
R'	$\frac{R_s}{2\pi} \left(\frac{1}{a} + \frac{1}{b} \right)$	$\frac{2R_s}{\pi d}$	$\frac{2R_s}{w}$	Ω/m
L'	$\frac{\mu}{2\pi} \ln(b/a)$	$\frac{\mu}{\pi} \ln \left[(D/d) + \sqrt{(D/d)^2 - 1} \right]$	$\frac{\mu h}{w}$	H/m
G'	$\frac{2\pi\sigma}{\ln(b/a)}$	$\frac{\pi\sigma}{\ln \left[(D/d) + \sqrt{(D/d)^2 - 1} \right]}$	$\frac{\sigma w}{h}$	S/m
C'	$\frac{2\pi\epsilon}{\ln(b/a)}$	$\frac{\pi\epsilon}{\ln \left[(D/d) + \sqrt{(D/d)^2 - 1} \right]}$	$\frac{\epsilon w}{h}$	F/m

Notes: (1) R_s is the surface resistance of the conductors. (2) μ , ϵ , and σ pertain to the insulating material between the conductors. (3) $R_s = \sqrt{\pi f \mu_c / \sigma_c}$. (4) μ_c and σ_c pertain to the conductors. (5) If $(D/d)^2 \gg 1$, then $\ln \left[(D/d) + \sqrt{(D/d)^2 - 1} \right] \simeq \ln(2D/d)$.

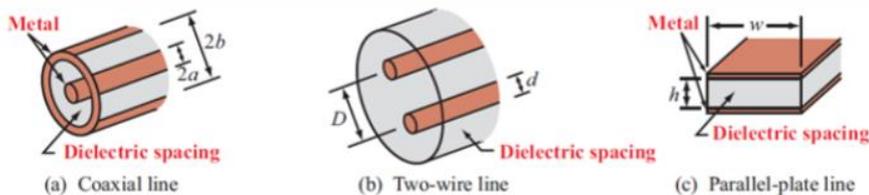


Figure 4. Important parameters of different types of transmission line. [3]

Electromagnetic waves can travel along waveguides using a number of different modes.

The transmission lines of electromagnetic energy at microwave frequencies are the so-called waveguides in the form of metal pipes with various cross-sectional configurations.

TE mode: characterized by the absence of a longitudinal component of the electric field ($E_z=0$). One of the features of such waves is that the ratio of the transverse components of the electric and magnetic fields [4] does not depend on the coordinates

TM mode: waves are characterized by the absence of a longitudinal component of the magnetic field ($H_z = 0$).

TEM mode: The Transverse electromagnetic wave cannot be propagated within a waveguide, but is included for completeness. The mode is commonly used within coaxial and open wire feeders. The TEM wave is characterized by the fact that both the electric vector (\mathbf{E}) and the magnetic vector (\mathbf{H}) are perpendicular to the direction of propagation. [5]

1.2.1 Characteristic impedance

Characteristic impedance Z_0 is defined as ratio of voltage and current of wave traveling in one direction. This is entirely different from leakage resistance of the dielectric separating the two conductors, and the metallic resistance of the wires themselves. Characteristic impedance is purely a function of the capacitance and inductance distributed along the line's length, and would exist even if the dielectric were perfect (infinite parallel resistance) and the wires superconducting (zero series resistance). [6]

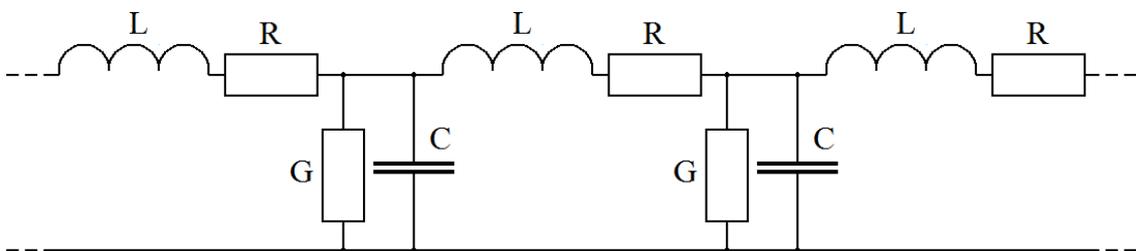


Figure 5. Equivalent circuit of transmission line.

The general expression for the characteristic impedance of a transmission line is:

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}, \quad (1.3)$$

Where R is the resistance of transmission line per unit length, considering the two conductors to be in series, L is the inductance of transmission line per unit length, G is the conductance of transmission line of the dielectric per unit length and C is the capacitance per unit length.

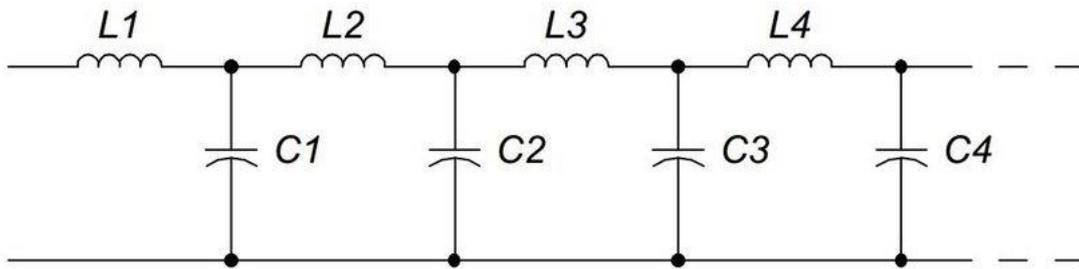


Figure 6. Equivalent circuit of lossless transmission line model. [7]

The real line always has losses. However, in some cases it is convenient to consider the line as ideal, i.e., without losses. A lossless line is a line in which there is no energy dissipation, which occurs when the primary parameters are $R = 0$ and $G = 0$.

$$Z_0 = \sqrt{\frac{L}{C}}. \quad (1.4)$$

1.2.2 Impedance

In DC circuits, the active resistance plays an important role. As for the sinusoidal AC circuits, there is no need to do just active resistance. Indeed, if in DC circuits capacitances and inductances are noticeable only during transients, then in AC circuits these components manifest themselves much more significantly. Therefore, for an adequate calculation of AC circuits, the term "electrical impedance" is introduced - Z .

As the three vector voltages are out-of-phase with each other, X_L , X_C and R must also be "out-of-phase" with each other with the relationship between R , X_L and X_C being the vector sum of these three components. This will give us the RLC circuits overall

impedance, Z . These circuit impedance's can be drawn and represented by an Impedance Triangle [8] :

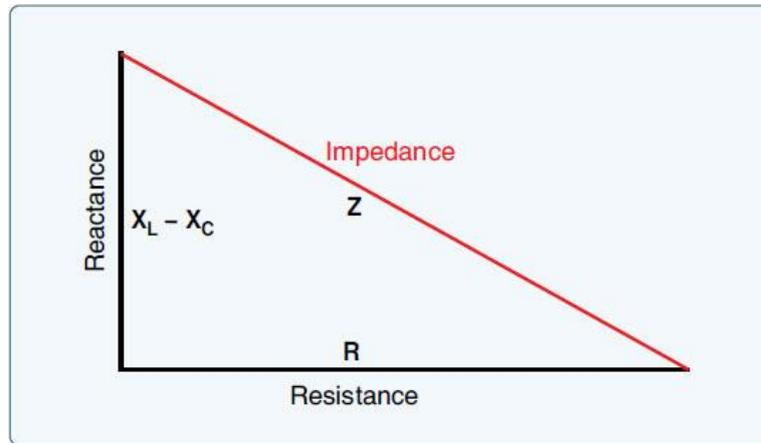


Figure 7. The Impedance Triangle for a Series RLC Circuit. [9]

Equation of impedance:

$$Z = R + jX. \quad (1.5)$$

In accordance with the Euler formula, we write the exponential form of the complex impedance:

$$Z = |Z|e^{j\varphi} \quad (1.6)$$

1.2.3 Resonance

Resonance occurs when frequency of exiting signal equals with self-oscillating frequency of LC-circuit. This happens at frequency when inductive and capacitive reactance having equal module.

$$|X_C| = |-X_L| \quad (1.7)$$

The resonance mechanism consists in the fact that the magnetic field of the inductance generates an electric current charging the capacitor, and the discharge of the capacitor creates a magnetic field in the inductance - a process that is repeated many times, by analogy with a mechanical pendulum.

Assuming that at the moment of resonance the inductive and capacitive components of the impedance are equal, the resonant frequency can be found from the expression known as Thompson's equation:

$$f_r = \frac{1}{2\pi\sqrt{LC}} \quad (1.8)$$

Where f_r is resonant frequency of the LC circuit in Hertz, (Hz).

1.2.4 Reflection coefficient

Voltage reflection coefficient - the ratio of the complex voltage amplitude of the reflected V^- wave to the complex voltage amplitude of the incident wave V^+ in a given section of the transmission line. This is typically represented with a Γ and can be written as: [10]

$$\Gamma = \frac{V^-}{V^+}, \quad (1.9)$$

which is a function of the load impedance Z_L and the source impedance Z_0 .

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}. \quad (1.10)$$

If the impedance of the device and the transmission line do not match with each other, then the energy gets reflected. Greater reflection coefficient means that more energy gets reflected and less forwarded.

1.2.5 Standing Wave Ratio (SWR)

The standing wave ratio characterizes the degree of matching of the antenna and the transmission line. In practice, always part of the transmitted energy is reflected and returned to the transmitter. The reflected energy degrades the operation of the transmitter and may damage it.

A standing wave is formed in the waveguide as a result of the addition of the incident wave and reflected from the load (antenna) at the end of the waveguide. The standing wave mode is achieved under the condition of maximum reflection of the signal at the end of the transmitter-antenna line. As a result of this addition, static maxima and minima

of the field strength are formed in the waveguide, i.e. the addition of the power of the reflected and incident signal forms an uneven distribution of field strength along the entire length of the waveguide (cable).

VSWR stands for Voltage Standing Wave Ratio, and is also referred to as Standing Wave Ratio (SWR). VSWR is a function of the reflection coefficient, which describes the power reflected from the antenna. If the reflection coefficient is given by Γ then the VSWR is defined by the following formula: [11]

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}. \quad (1.11)$$

Exist special VSWR cases:

1. Short circuit, if $Z_L = 0$, then $\Gamma = -1$,
2. Open circuit, if $Z_L = \infty$, then $\Gamma = 1$,
3. Matched load, if $Z_L = Z_0$, then $\Gamma = 0$.

In the first two cases $VSWR = \infty$, in third $VSWR = 1$.

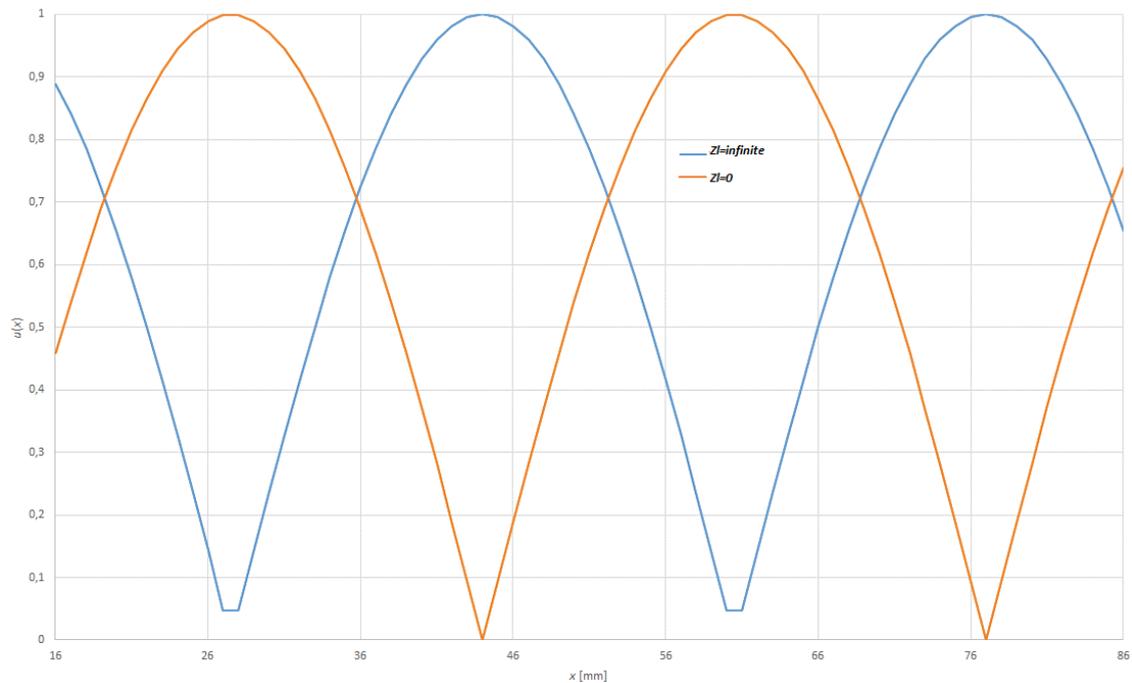


Figure 8. Standing wave on shorted and open ended lines.

1.2.6 Wavelength and wavenumber

The wavelength is the shortest spatial distance between points, which oscillate in one phase. Denote the wavelength, most often the letter λ .

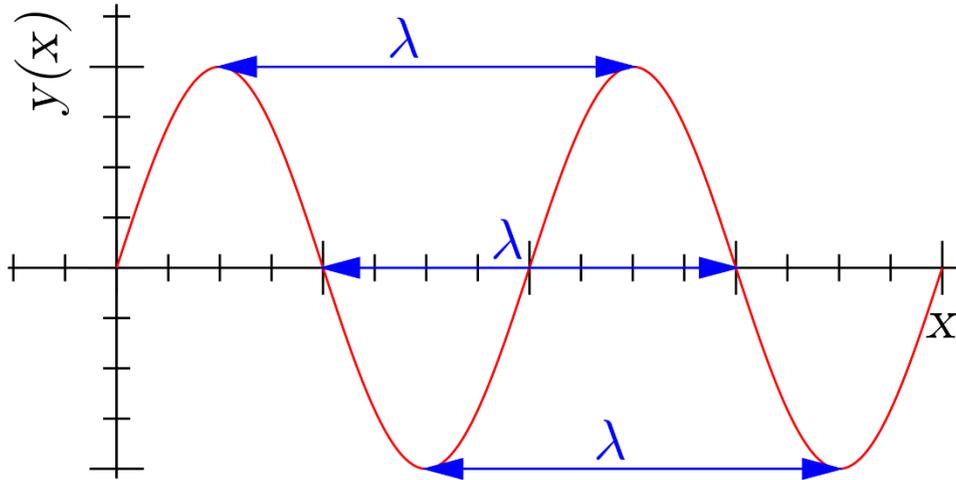


Figure 9. Illustration of concept of wavelength

The wavelength is equal to the quotient of the wave propagation velocity v and the wave frequency f :

$$\lambda = \frac{v}{f}. \quad (1.12)$$

Wavenumber is denoted by lower case k , and is a measure of phase per given length, for a traveling wave:

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{v}. \quad (1.13)$$

1.2.7 Propagation constant

As it propagates along the line, electromagnetic energy decreases. This is due to line loss.

Main losses in transmission line:

1. Losses in the metal parts of the cable. Wires and metal parts near the wires are heated due to current, thermal losses occur. As the frequency and resistance of the wires increase, these losses also increase.
2. Only in the static case is the current density in a wire homogeneously distributed over the cross-section area. However, if the currents vary with time the current

density is maximal at the conductor surface and it falls towards the centre of the wire. As the current flows mainly at the surface of the conductor, this effect is known as the skin effect. [12]

Skin depth refers to the point at which the current density reaches approximately 37% of its value at the surface of the conductor. Calculating skin depth requires the frequency of the AC signal and the resistivity and relative permeability of the conductive material. [13] Skin depth equation:

$$\delta = \sqrt{\frac{\rho}{\pi \cdot f \cdot \mu_r \cdot \mu_o}}, \quad (1.14)$$

Where δ is skin depth, ρ – resistivity of the conductive material, μ_r – relative permeability and μ_o – permeability of free space ($4\pi \cdot 10^{-7}$).

3. The current penetration depth is determined according to the expression:
4. Losses in cable insulation. These losses are due to imperfection of the insulating material and losses due to dielectric polarization.
5. Radiation and induction losses are similar in that both are caused by the fields surrounding the conductors. Induction losses occur when the electromagnetic field about a conductor cuts through any nearby metallic object and a current is induced in that object. As a result, power is dissipated in the object and is lost. [14]

Radiation losses occur because some magnetic lines of force about a conductor do not return to the conductor when the cycle alternates. These lines of force are projected into space as radiation, and these results in power losses. That is, power is supplied by the source, but is not available to the load. [14]

The propagation coefficient, which is a complex quantity, takes into account these losses and can be calculated by the equation:

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \quad (1.15)$$

Where γ is complex propagation constant, $\alpha = \frac{1}{2} \left(\frac{R}{Z_0} + GZ_0 \right)$ is attenuation constant in nepers/unit length and $\beta \approx \omega\sqrt{LC}$ - phase constant in radians/unit length.

If the transmission line is lossless, then resistance and conductance in the propagation constant are zero, $\alpha=0$. The equation for propagation constant is neatly simplified [15]:

$$\gamma = j\beta. \quad (1.16)$$

Where β and k are often treated as the same thing. Indeed, for TEM transmission lines the phase constant and wavenumber are equal $\beta = k$. [16]

1.2.8 Velocity factor

Electromagnetic energy propagates along a transmission line at a certain speed. The transmission speed depends on the line parameters and signal frequency. The phase velocity can be found from the expression:

$$v_p = \frac{\omega}{\beta}, \quad (1.17)$$

and the β , which, in turn, depends on the primary parameters of the line. If the α determines the propagation length and the quality of the connection, then the β determines the propagation speed of the signal energy in the line.

In the high-frequency range, for which

$$\beta = \frac{\omega}{\sqrt{LC}}, \quad (1.18)$$

the propagation speed of electromagnetic energy no longer depends on the frequency and is determined only by the transmission line parameters

$$v_p = \frac{1}{\sqrt{LC}}, \quad (1.19)$$

Where v_p is the velocity of propagation in m/s, C is the distributed capacitance per unit length in pF, L is the distributed inductance per unit length in μH .

The Velocity factor of a cable is the speed at which an electrical signal can propagate through the cable in comparison to the speed of light:

$$VF = \frac{v}{c}, \quad (1.20)$$

Where VF is velocity factor, v is the velocity of propagation in m/s, c is the speed of light in vacuum in m/s.

2 Calculations

Phase shift between two points on actual transmission line are caused by finite propagation velocity. In our model we use LC circuit in order to introduce phase shift on much smaller physical scale. For example to get 40° at 1 MHz we should need transmission line 22.2 meters long with $VF = 2/3$.

Another thing that we want to emulate is standing wave – when real transmission line is not properly terminated/loaded then reflected wave travels along it back towards source. This forward and reflected wave add up in line causing standing wave to appear. If our model is not properly terminated then same effect appears and amplitude of oscillations is strongly dependent on test point where it is measured.

We want to build model of transmission line based on lumped elements. Reasons being that this model can be used with cheaper measurement instruments working at low frequencies and also this model can be used to demonstrate that this kind of model is valid for transmission line. We will use twelve LC segments:

- Less work to solder things together
- Less measurements to be made during labwork
- Less components needed, meaning also lower costs.

2.1 Single segment

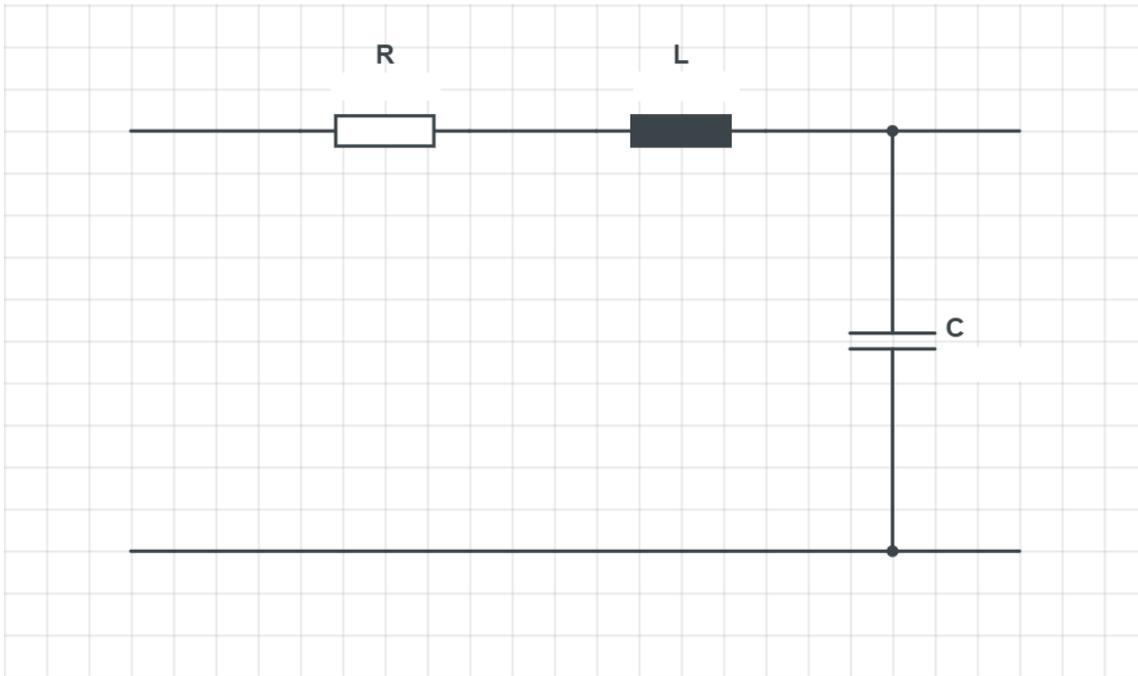


Figure 10. Equivalent circuit of single segment of model.

The length of transmission line that we want to simulate is 1.5λ . This means that the phase shift at each point should be $\frac{1.5 \cdot 360^\circ}{12} = 45^\circ$. To start calculating the line segment, we need to select the inductor and capacitor. The characteristic impedance of the lossless transmission line was equal (1.4) to the root of the ratio L ja C and this ratio must be 50Ω . Although our circuit is not lossless, in the case of small losses it could still be the case that the capacitance should be 2500 times less than the inductance. For our circuit, I chose the inductor $L = 5.3 \mu\text{H}$. Having the characteristic impedance and inductor, we can calculate the capacitor:

$$C = \frac{L}{Z_0^2} = \frac{5.6 \cdot 10^{-6} \text{H}}{50\Omega^2} = \frac{5.6 \cdot 10^{-6} \text{H}}{2500\Omega^2} \approx 2.2 \cdot 10^{-9} \text{F}, \quad (2.1)$$

Because our model differs from the real transmission line in that it behaves as a low pass filter, we should work below the resonance frequency to avoid amplitude loss. Resonance frequency can be found via Thompson's equation (1.8):

$$f_r = \frac{1}{2 \cdot \pi \cdot \sqrt{5.6 \cdot 10^{-6} \text{H} \cdot 2.2 \cdot 10^{-9} \text{F}}} \approx 1.4 \cdot 10^6 \text{ Hz} = 1.4 \text{ MHz}.$$

To calculate the reactance of the inductor and capacitor we need an operating frequency. Since our model differs from a real transmission line in that it acts as a low-pass filter, we must work below the resonant frequency to avoid loss of amplitude.

In other words, if we chose f as the operating frequency, then the resonance of the oscillatory circuit (Thompson's equation (1.8)) should be slightly higher than this. The initial approximation can be about 1.5 times. I choose a frequency of 1 MHz.

With frequency, we can calculate inductive reactance of inductor (1.2):

$$X_L = 2 \cdot \pi \cdot 1 \cdot 10^6 \text{ Hz} \cdot 5.6 \cdot 10^{-6} \text{ H} \approx j35.2 \Omega$$

and Capacitive Reactance of capacitor (1.1):

$$X_C = \frac{1}{2 \cdot \pi \cdot 1 \text{ MHz} \cdot 2.2 \cdot 10^{-9} \text{ F}} \approx -j72.3 \Omega.$$

We can find the effective series resistance R_L of inductor in the circuit using equation Q factor of inductor. Q factor of our inductor is 30.

$$Q_L = \frac{\omega L}{R_L} = \frac{2\pi f L}{R_L}, \tag{2.2}$$

$$R_L = \frac{2\pi f L}{Q} = \frac{2\pi \cdot 1 \cdot 10^6 \text{ Hz} \cdot 5.6 \cdot 10^{-6} \text{ H}}{30} = 1.2 \Omega.$$

Frequency response of circuit is ratio of output voltage to input voltage in case of this input voltage being sinusoidal. In fact, the frequency response of a system is simply its transfer function as evaluated by substituting $s = j\omega$. s is used for two reasons. First to make the notation simple and compact, and second, to get all the math into a standard form. s has both amplitude and phase associated with it.

Replacing $j\omega$ with s , we get inductive reactance of line $X_L = sL$ and capacitive reactance of line $X_C = \frac{1}{sC}$.

In order for the frequency response function to be found, the replacement takes place:

$$\begin{aligned} \frac{U_o}{U_i} = H(s) &= \frac{X_C}{R + X_L + X_C} = \frac{\frac{1}{sC}}{R + sL + \frac{1}{sC}} = \frac{\frac{1}{sC}}{\frac{sRC + s^2LC + 1}{sC}} \\ &= \frac{1}{sRC + s^2LC + 1} \end{aligned} \quad (2.3)$$

and because $s^2 = (j\omega)^2 = -\omega^2$:

$H(j\omega)$ is the amplitude response:

$$H(j\omega) = \frac{1}{1 - \omega^2LC + j\omega RC}. \quad (2.4)$$

φ is the phase response:

$$\varphi \angle H(j\omega) = -\arctan\left(\frac{\omega RC}{1 - \omega^2LC}\right). \quad (2.5)$$

We substitute the values of elements and calculate phase response:

$$\begin{aligned} \varphi &= -\arctan\left(\frac{\omega RC}{1 - \omega^2LC}\right) \\ &= -\arctan\left(\frac{2 \cdot \pi \cdot f \cdot R \cdot C}{1 - (2 \cdot \pi \cdot f)^2 L \cdot C}\right) = \\ &= -\arctan\left(\frac{2 \cdot \pi \cdot 1 \cdot 10^6 \text{ Hz} \cdot 1.2 \Omega \cdot 2.2 \cdot 10^{-9} \text{ F}}{1 - (2 \cdot \pi \cdot 1 \cdot 10^6 \text{ Hz})^2 \cdot 5.6 \cdot 10^{-6} \text{ H} \cdot 2.2 \cdot 10^{-9} \text{ F}}\right) = \\ &= -\arctan(0.03) \approx -0.03 \text{ rad} = -1.7^\circ. \end{aligned}$$

Check the result using simulation. For simulation, I used LTspice XVII software [17].

This simulation is clearly for case when load resistance is infinite and source resistance is zero.

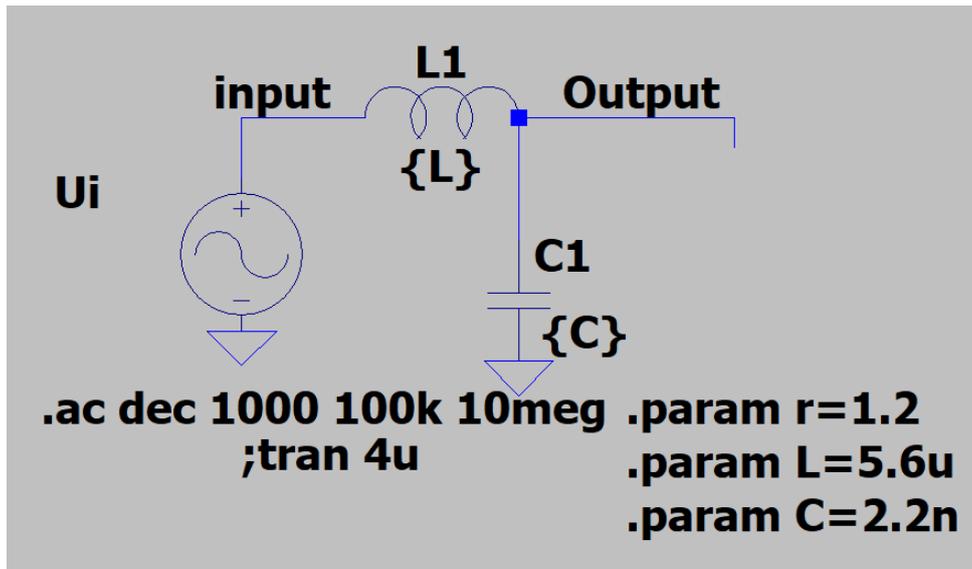


Figure 11. Simulated schematic of single segment.

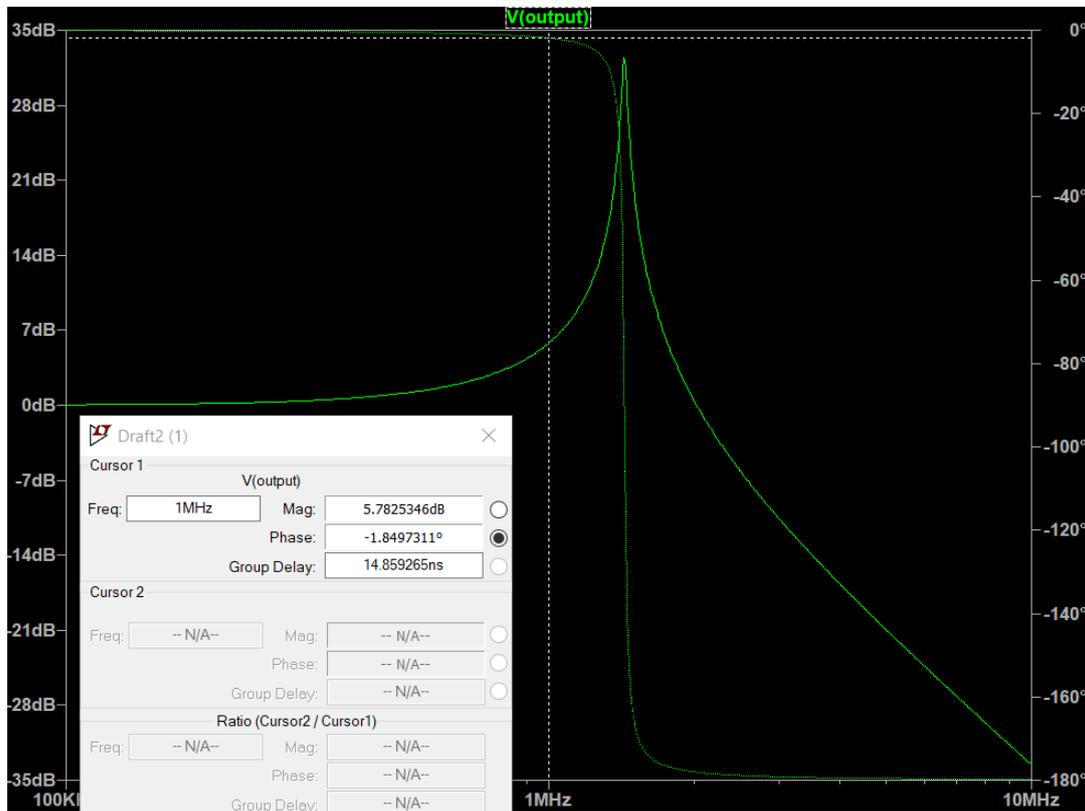


Figure 12. Frequency response of single segment simulation.

According to the results of the simulation, we see that the signal at a frequency of 1 MHz has a phase shift of -1.85° . We can conclude that all calculations performed correctly.

2.2 Single segment with source and load impedance

We are currently looking at a case where the signal source is an ideal voltage source and there is no load resistance. In a real model, the source can have a source resistance of 50Ω and the line can be loaded with the same resistance.

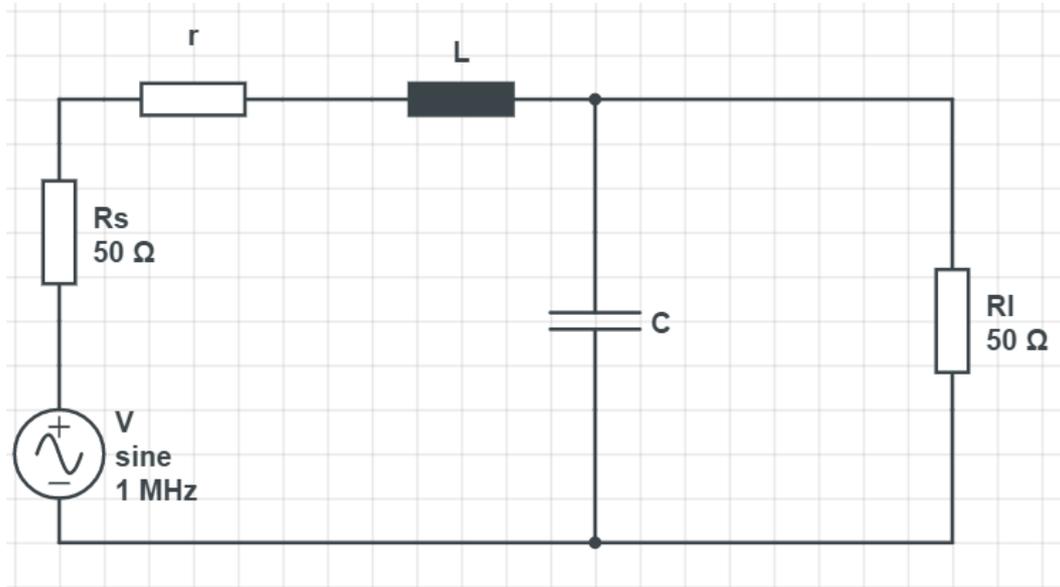


Figure 13. Equivalent RLC circuit with source and load.

Current through R_l is

$$i = \frac{E}{R_s + r + sL + \frac{R_L}{1 + sR_L C}}, \quad (2.6)$$

Output voltage is

$$U_o = i \cdot \frac{R_L}{1 + sR_L C}, \quad (2.7)$$

Input voltage at same time is

$$U_i = i \cdot \left(sL + \frac{R_L}{1 + sR_L C} \right), \quad (2.8)$$

finally the transfer function obtained is:

$$\begin{aligned}
H(s) = \frac{U_o}{U_i} &= \frac{i \cdot \frac{R_L}{1 + sR_L C}}{i \cdot \left(sL + \frac{R_L}{1 + sR_L C} \right)} = \frac{\frac{R_L}{1 + sR_L C}}{\left(sL + \frac{R_L}{1 + sR_L C} \right)} & (2.9) \\
&= \frac{R_L}{(1 + sR_L C) \left(sL + \frac{R_L}{1 + sR_L C} \right)} \\
&= \frac{R_L}{(1 + sR_L C) \left(\frac{sL + s^2 R_L C L + R_L}{(1 + sR_L C)} \right)} = \frac{R_L}{sL + s^2 R_L C L + R_L} \\
&= \frac{1}{\frac{sL}{R_L} + s^2 C L + 1}
\end{aligned}$$

and because $s^2 = (j\omega)^2 = -\omega^2$,

$H(j\omega)$ is the amplitude function:

$$H(j\omega) = \frac{1}{1 - \omega^2 LC + \frac{\omega L}{R_L}}, \quad (2.10)$$

φ is the phase response:

$$\varphi \angle H(j\omega) = -\arctan\left(\frac{\frac{\omega L}{R_L}}{1 - \omega^2 LC}\right) \quad (2.11)$$

$$\begin{aligned}
\varphi &= -\arctan\left(\frac{\frac{\omega L}{R_L}}{1 - \omega^2 LC}\right) \\
&= -\arctan\left(\frac{\frac{2 \cdot \pi \cdot f \cdot L}{50 \Omega}}{1 - (2 \cdot \pi \cdot f)^2 LC}\right) = \\
&= -\arctan\left(\frac{\frac{2 \cdot \pi \cdot 1 \cdot 10^6 \text{ Hz} \cdot 5.6 \cdot 10^{-6} \text{ H}}{50 \Omega}}{1 - (2 \cdot \pi \cdot 1 \cdot 10^6 \text{ Hz})^2 \cdot 5.6 \cdot 10^{-6} \text{ H} \cdot 2.2 \cdot 10^{-9} \text{ F}}\right) = \\
&= -\arctan(1.37) \approx -0.94 \text{ rad} = -53.9^\circ
\end{aligned}$$

Check the result using simulation.

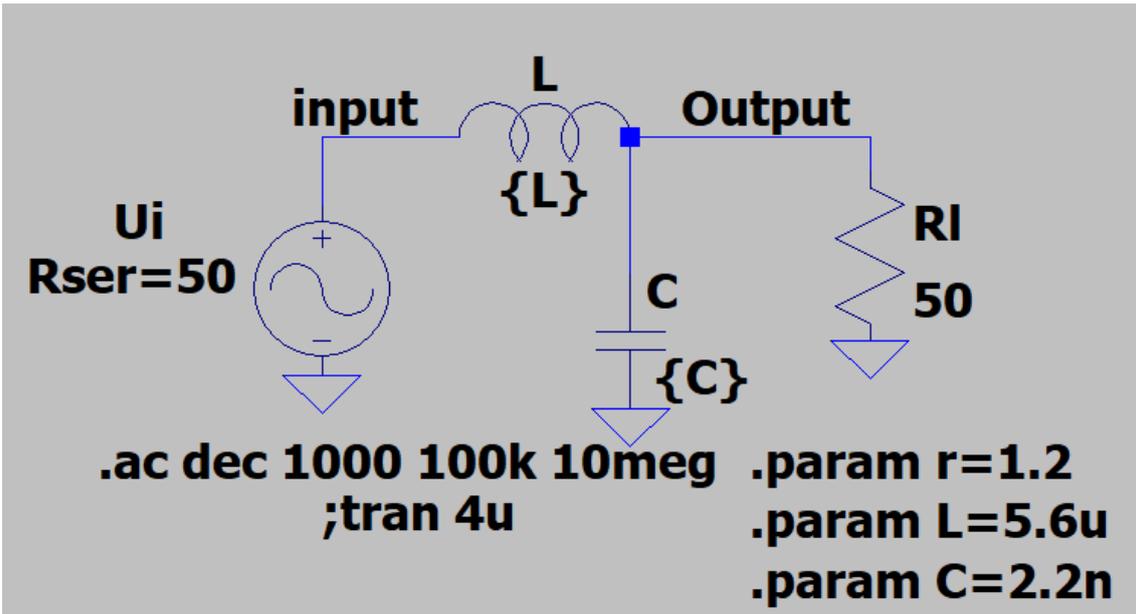


Figure 14. Single segment with source and load concept for simulation in LTspice.

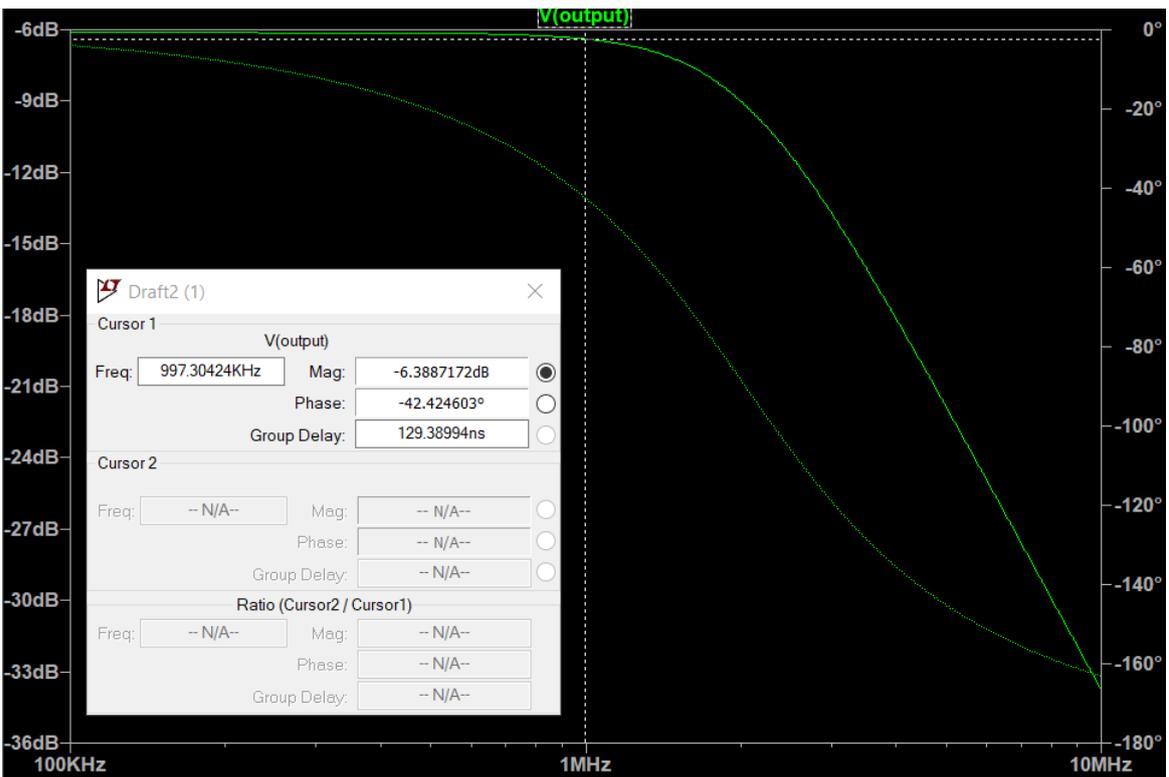


Figure 15. Frequency response at output.

We see that at the output we have a phase shift of -42° . Now we need to get a phase shift before the signal passes the first segment.

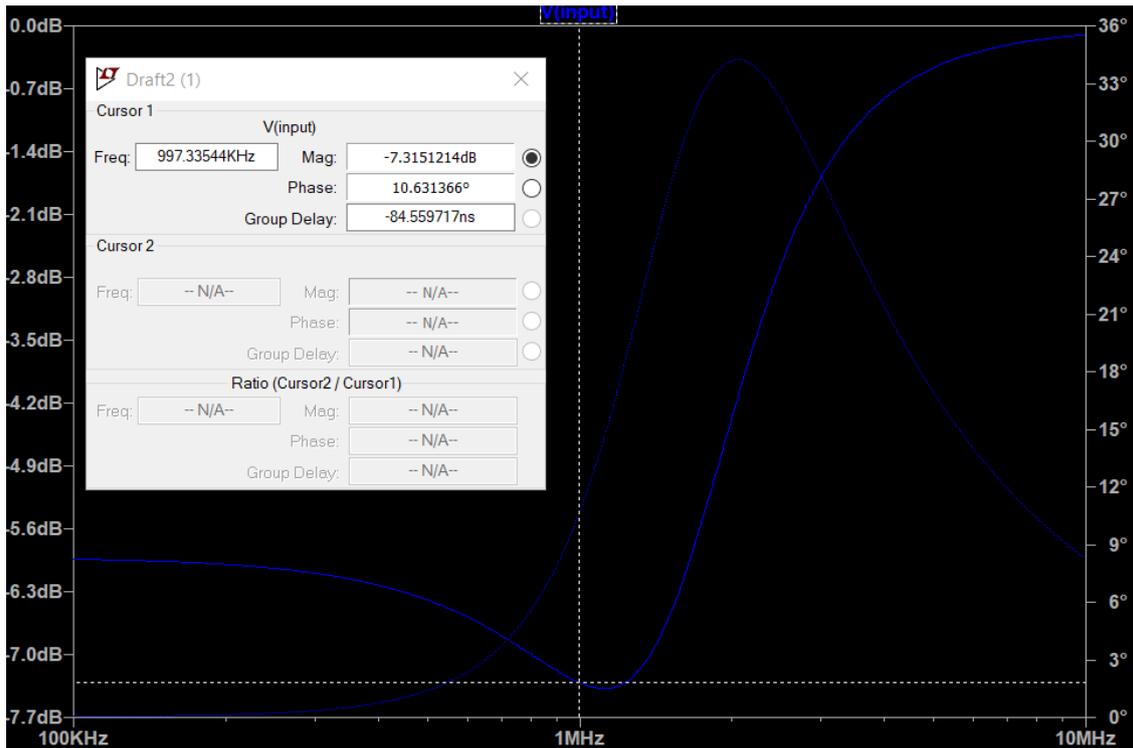


Figure 16. Frequency response of input.

We see that at the input we have a phase shift of 6.8° .

$$\varphi = \varphi_{out} - \varphi_{in} = -42^\circ - 10^\circ = -52^\circ$$

Calculations and simulations are approximately equal. We can conclude that all calculations performed correctly

2.3 Full-circuit simulation

Our transmission line model has 12 segments LC circuits. After each segment, we will measure the phase shift.

1. Simulation condition – load impedance is 50Ω . The simulation results can be found on Appendix 1. Table 2.

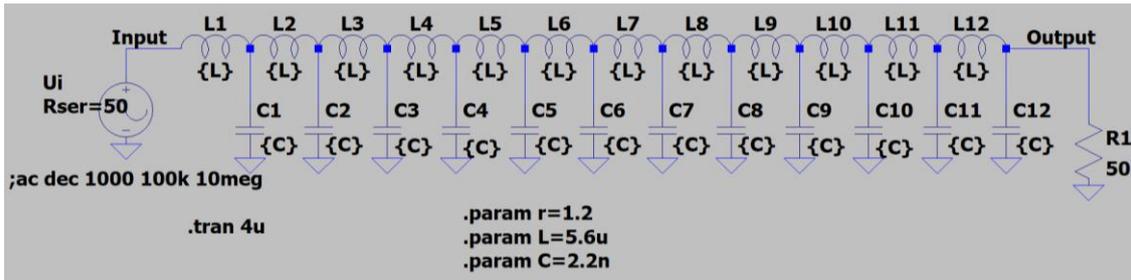


Figure 17. Full transmission line concept for simulation when $Z_L = 50\Omega$.

- Simulation condition – load impedance is 0. The simulation results can be found on Appendix 2. Table 4.

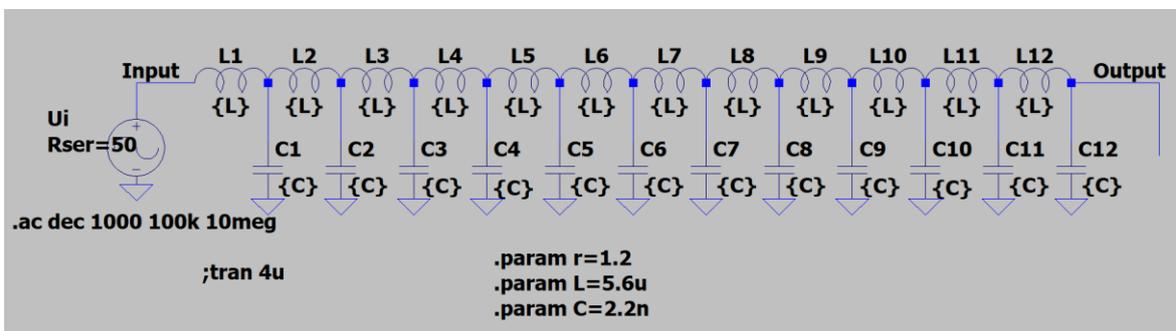


Figure 18. Full transmission line concept for simulation when $Z_L = 0\Omega$.

- Simulation condition – load impedance is infinity. The simulation results can be found on Appendix 3. Table 6.

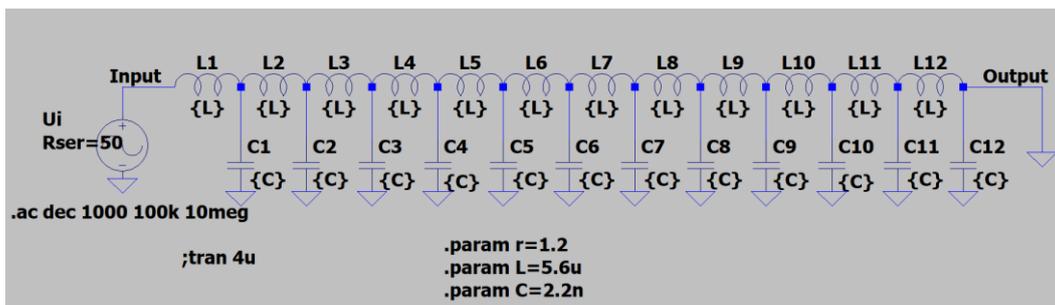


Figure 19. Full transmission line concept for simulation when $Z_L = \infty$.

Based on the data, we built a chart where can see phase shift of about 45 degrees at each test what is in line with our calculations.

According to the results, we see that there are minimal differences from the ideal line. This may be because our parameters may not be accurate and there are no ideal lines, there is always some resistance.

3 PCB design and assembly

After receiving the simulation and calculation results, I proceeded to design the PCB. For PCB modeling it was used the CircuitMaker environment ver. 1.03.0.181 [18]. The Transmission line concept, which was shown on figure 17, and used during the simulations, was recreated in the PCB modeling environment with minor changes. At the input and output of the circuit, left pads in order if it is necessary to add resistors there. For example, when in lab we have signal generator with higher output impedance than 50Ω . As for microstrip, signal traces are located on the Top Layer, the ground is located on the Top and Bottom layers and connected to each other. Using a Silk layer to label all elements on the PCB

The initial condition was that the board had the shape of a square with a side size of 10cm. As insulation material we use FR4 with a width of 1mm. For design, I used 2 BNC connectors, 14 testpoints, 12 capacitors and 12 inductors.

3D representation of this PCB with all the components is shown on a figure 22.

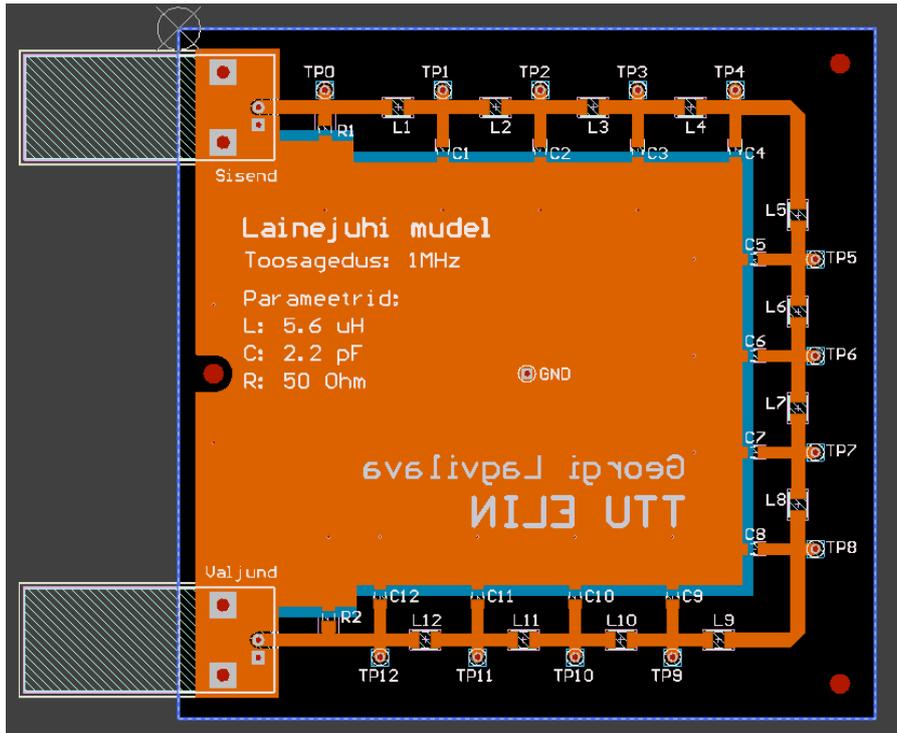


Figure 20. PCB design of the transmission line.

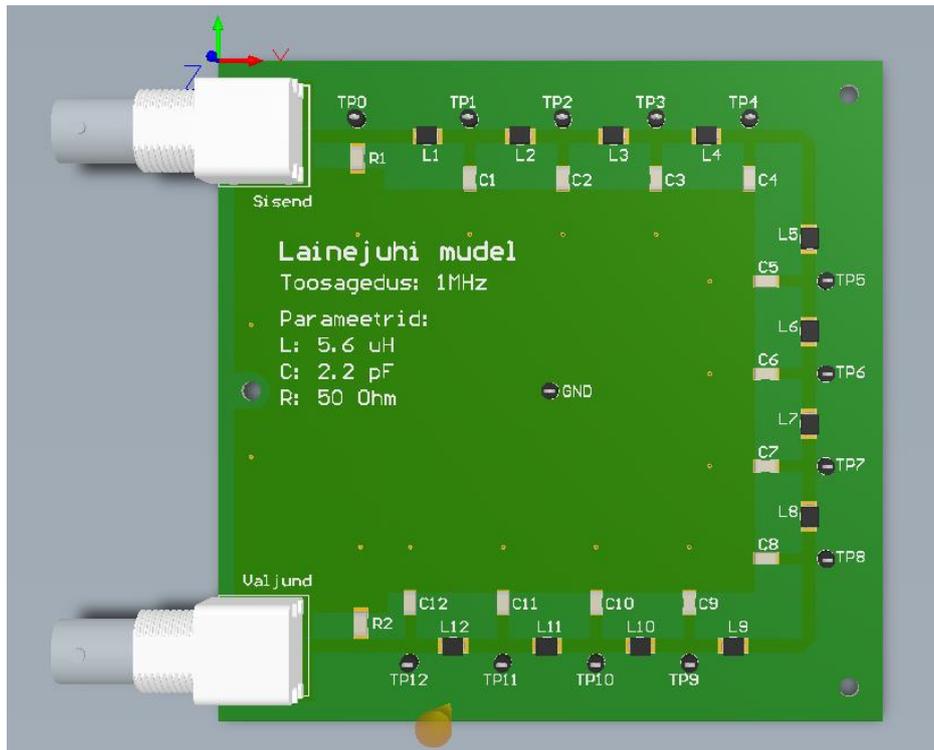


Figure 21. 3D representation of the transmission line PCB design.

After creating the PCB, Gerber files were exported and a board and components was ordered. Bill of materials can be found in Table 8.

After receiving all the components, I started assembling. During the assembly, an error was found in ordering test points. The error consisted in the fact that during the design we used the component with the number 5001, and in the ordered component with the number 5005. Fortunately, with the help of small manipulations, the components became interchangeable. The second drawback was that to fix the BSC connectors to the board, you need to use self-tapping screws, which must be ordered separately.

These problems did not prevent to collect the PCB. The assembled board can be seen on figure 24.

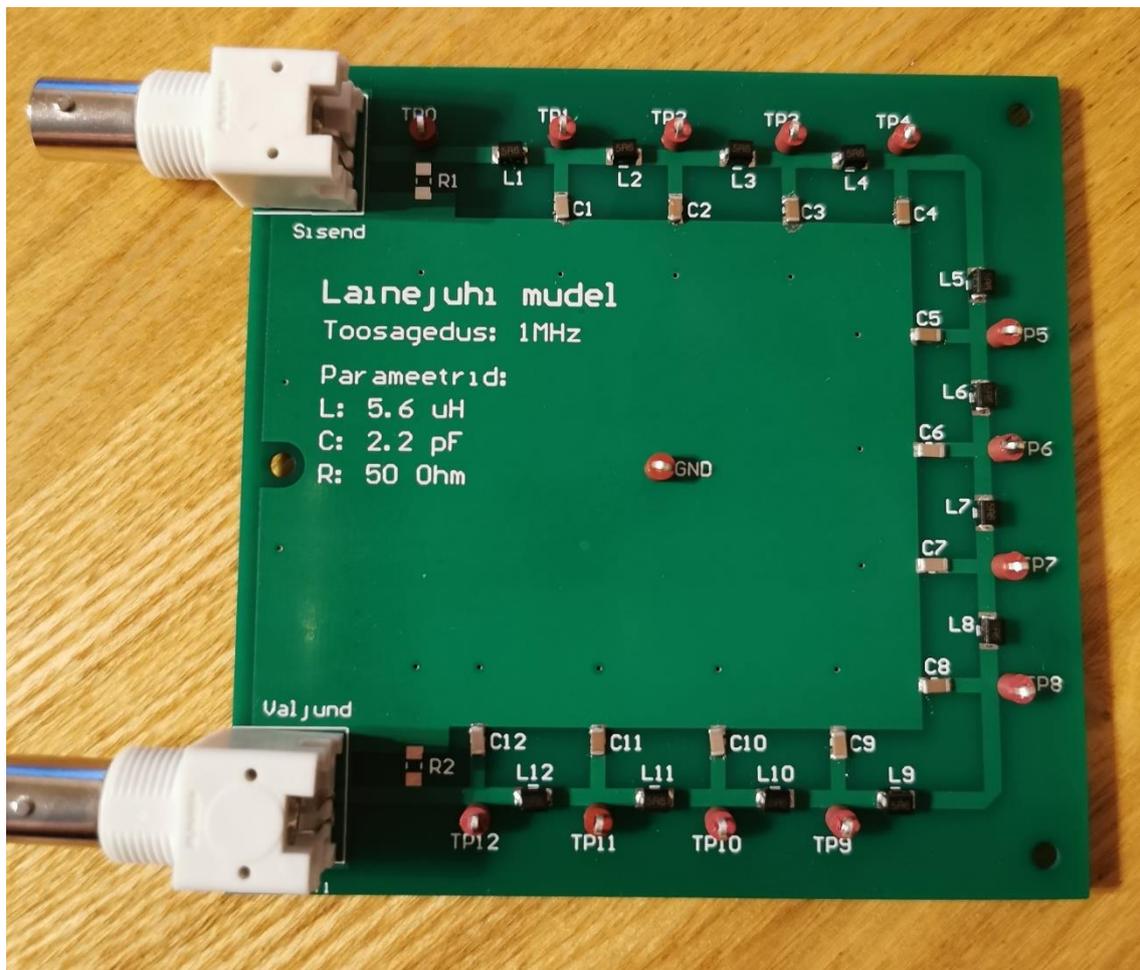


Figure 22. Designed and assembled Transmission line prototype.

4 Prototype test

The next step after assembly is testing the board. For test I used an oscilloscope Keysight DSOX1102G, also used two BNC cables, a splitter and a terminators with different ratings. Connecting everything as shown in figure 31. After that started to measure.

The goal was to measure the amplitude and phase shift at all 13 test points with different output loads. The first condition was open end line. Measured each test point and entered the results in a Table 7 that can be seen on the Appendix 3. Change the load to a short line. Measured each test point and entered the results in a Table 5 that can be seen on the Appendix 2. The last step is install a terminator with a load of 50 ohms. Measured each test point and entered the results in a Table 3 that can be seen on the Appendix 1.

Figure 23 Shows connected and properly tuned oscilloscope

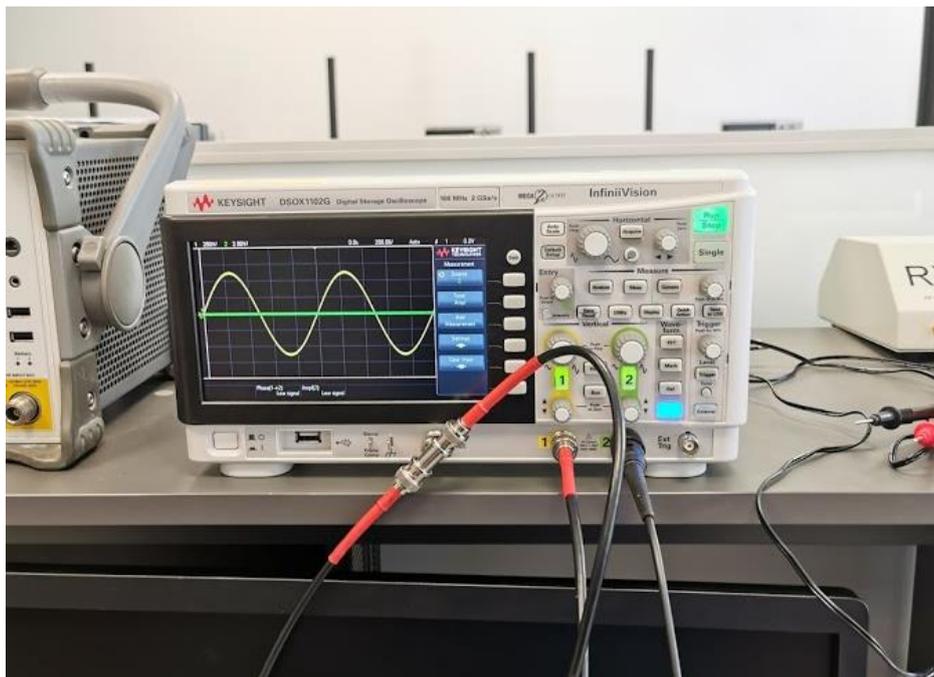


Figure 23. Connected oscilloscope during test.

In figure 24 we can see the PCB during measurements

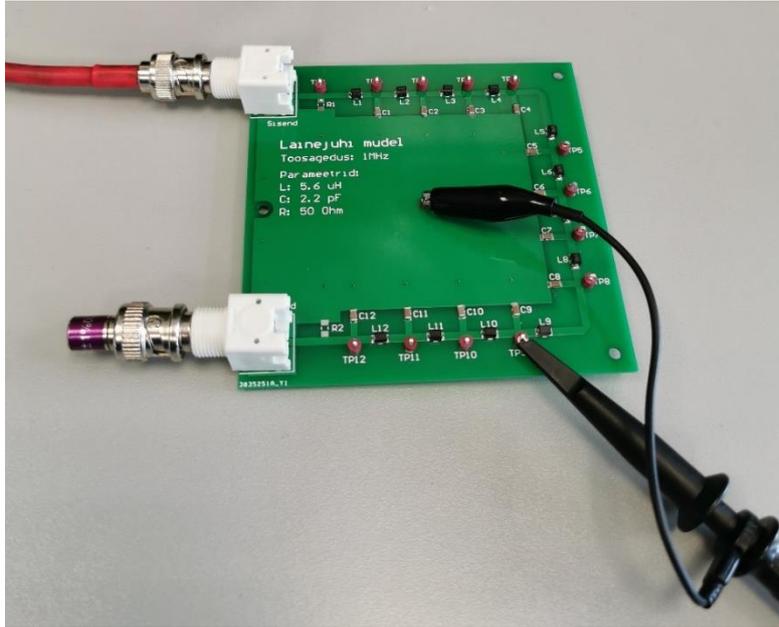


Figure 24. PCB during laboratory.

5 Results analysis

Based on the data obtained from the simulation and the PCB test, we built phase shift and amplitude graphs and analyse them.

Built graphs of the phase shift for three different load conditions: Figure 25 when $Z_l = 50 \Omega$, Figure 25 when $Z_l = 0$, Figure 25 when $Z_l = \infty$.

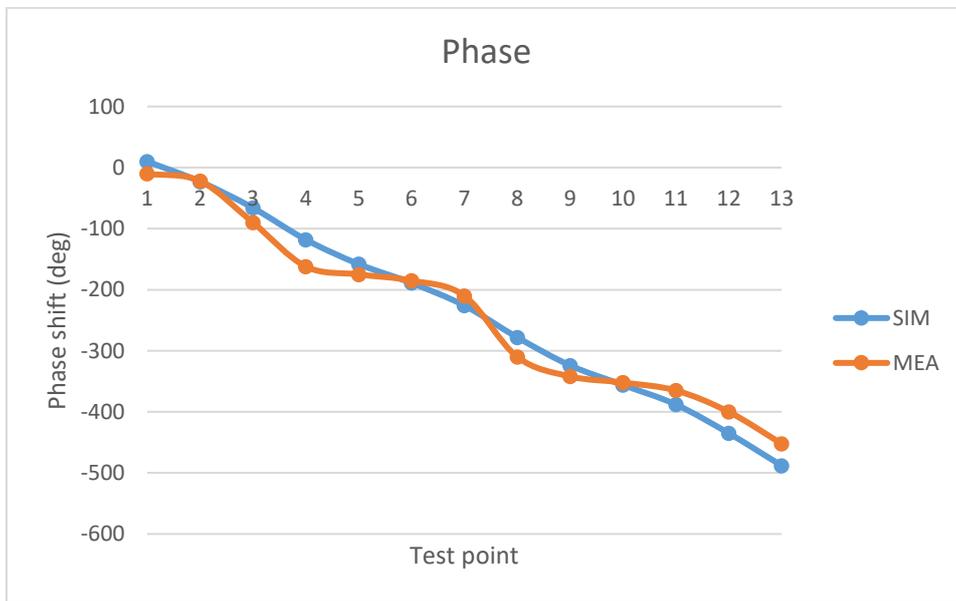


Figure 25. Phase shift chart, when load impedance is 50Ω .

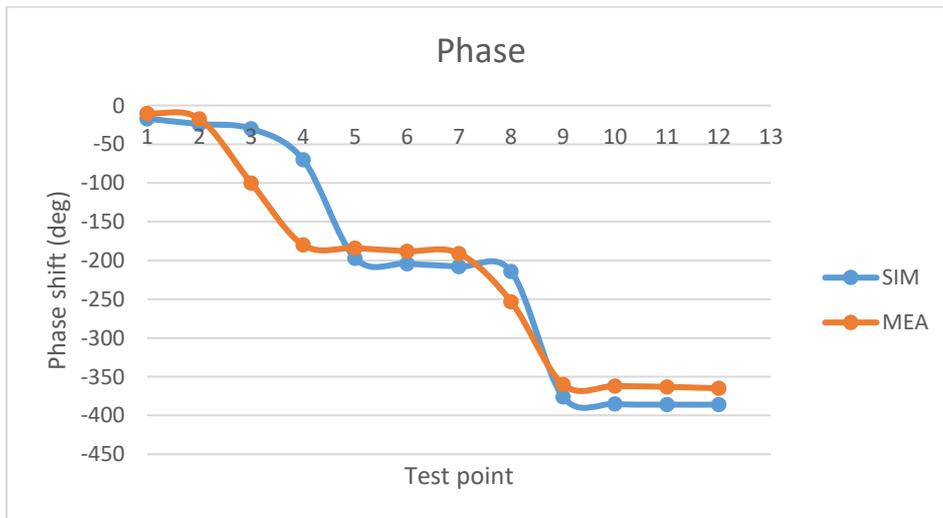


Figure 26. Phase shift chart, when load impedance is 0.

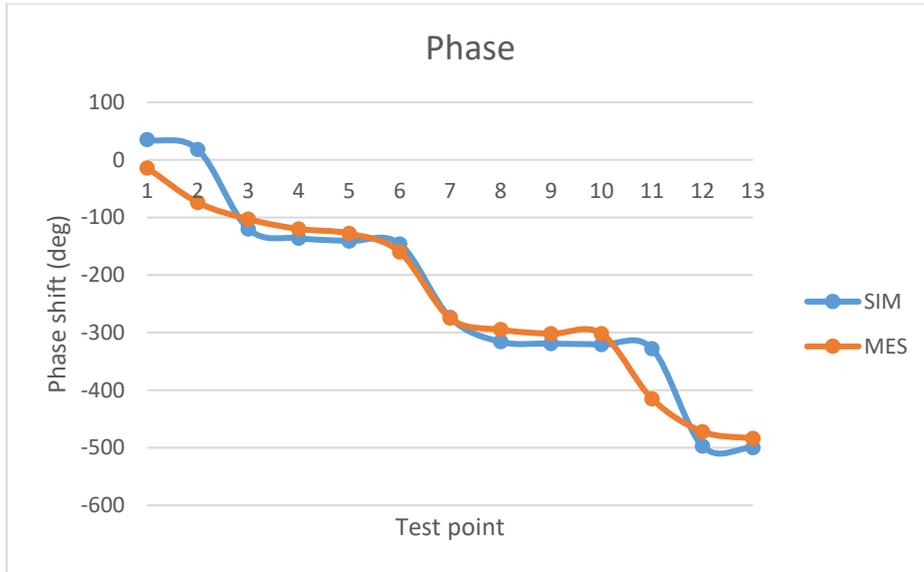


Figure 27. Phase shift chart, when load impedance is ∞ .

We see that all graphs (at different loads) practically repeat each other except for some points. The reasons for the possible differences can be as follows:

- In the model for simulation, we use the exact values of the components, in reality the elements cannot be accurate and have tolerance. Therefore, at different points, we can have different results;
- All components were selected with approximate values;
- The tracks on the board also have resistance, not significant, but we did not consider it during the simulation;
- The source of the signal and the load may differ from the 50 ohms that we used.

Based on the results, we construct the graphs of the amplitudes with different loads: Figure 28 when $Z_l = 50 \Omega$, Figure 29 when $Z_l = 0$, Figure 30 when $Z_l = \infty$.

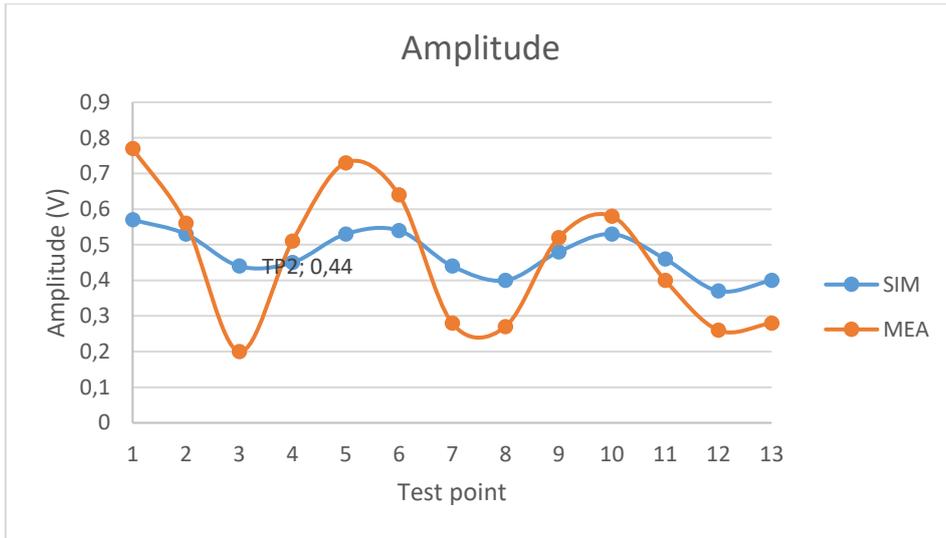


Figure 28. Amplitude chart, when load impedance is 50 Ω .

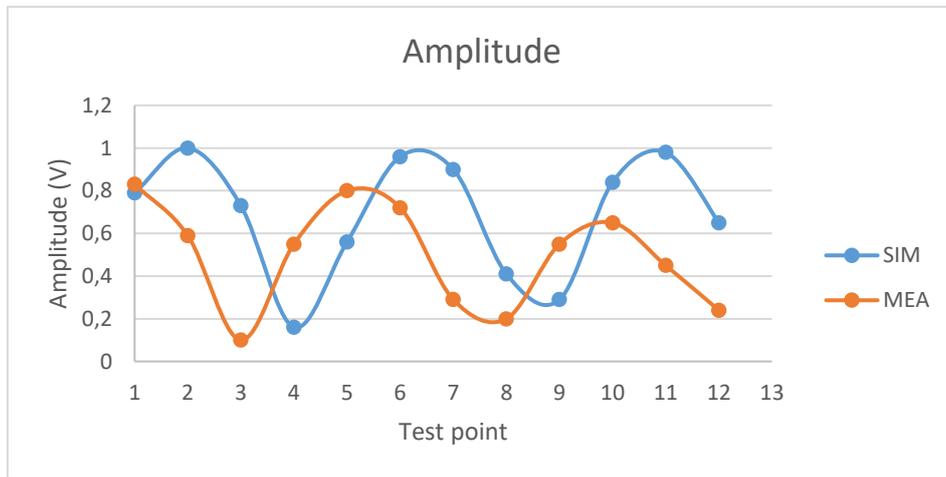


Figure 29. Amplitude chart, when load impedance is 0.

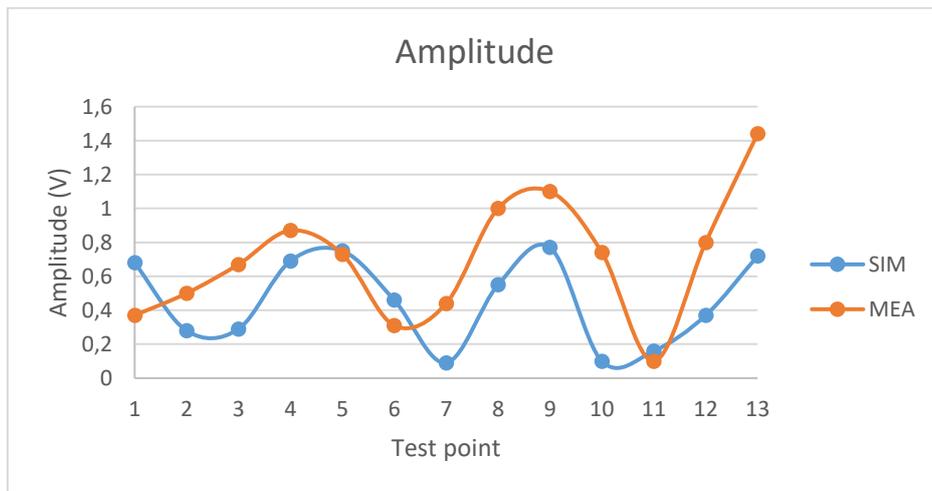


Figure 30. Amplitude chart, when load impedance is ∞ .

If you look at the graphs, you can see that all the graphs are similar but have different amplitudes at some points they are shifted from each other. The reasons for the possible differences can be as follows:

- Our transmission line is not 50Ω , because of this the reflection coefficient increases;
- Also need to take into account the tolerance of the elements;
- The tracks on the board also have resistance, not significant, but we did not consider it during the simulation;
- The source of the signal and the load may differ from the 50 ohms that we used.

6 Manual for laboratory work

The purpose of the laboratory work is to measure the phase shift and amplitude at each test point with different loads, build charts and analyse them.

For laboratory work we need:

1. Model of transmission line
2. Oscilloscope,
3. Signal generator,
4. Two BNC coaxial cables (50 Ω),
5. BNC T-splitter,
6. BNC terminators:
 - a. 50 Ω ,
 - b. 75 Ω .
7. BNC short

Connections. Schematic can be found on figure 31

1. Connect the BNC T-splitter to the output of the signal generator.
2. Take one BNC coaxial cable and connect the splitter to the oscilloscope.
3. Take the second cable and connect the second output of the splitter to the input of the PCB
4. Connect ground clip to the GND on PCB.
5. Connect the sharp tip to all test points during the laboratory work TP0-TP12.

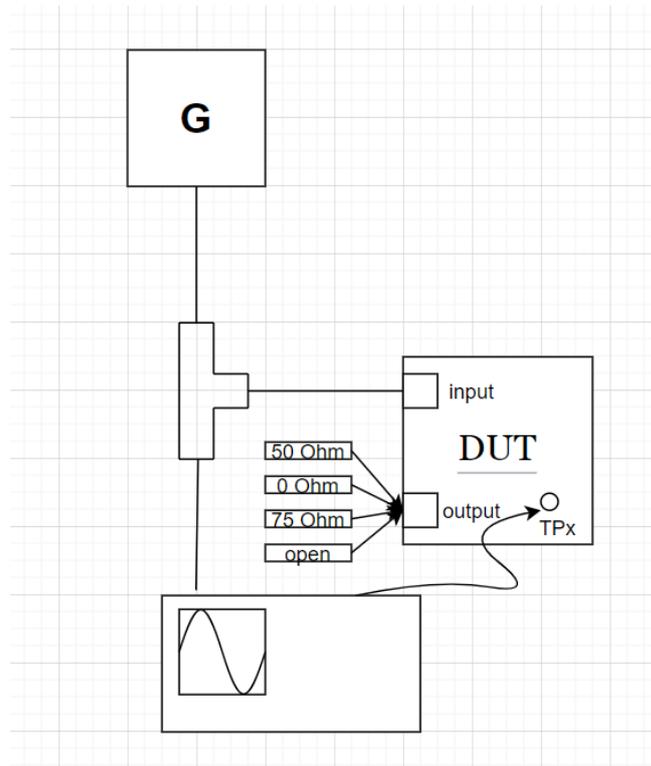


Figure 31. Connection schematic.

The next step is to set up a signal generator and oscilloscope. Turn on the oscilloscope / generator. Set the AC signal with a frequency of 1 MHz and an amplitude of 1 V and expose the values that we want to measure. In our case, this is the phase shift and the amplitude.

The purpose of the laboratory work is to measure the phase shift and amplitude at each test point with different loads:

1. $Z_L = 50 \Omega$. Install a terminator with a nominal value of 50Ω on the output connector of the board. Take measurements, enter data in a table and build a graph.
2. *Short line*. Install a BNC short line on the output connector of the board. Take measurements, enter data in a table and build a graph.
3. $Z_L = 75 \Omega$. Install a terminator with a nominal value of 75Ω on the output connector of the board. Take measurements, enter data in a table and build a graph.
4. *Open line*. Leave the output without load. Take measurements, enter data in a table and build a graph.

Compare the resulting graphs with the ideal transmission line graphs with the same loads.

Table 1. Table example for laboratory work.

	TP0	TP1	TP2	TP3	TP4	TP5	TP6	TP7	TP8	TP9	TP10	TP11	TP12
Phase													
Amplitude													

Summary

The purpose of the thesis was to create laboratory work that students can use to study the transmission line. In the course of the work, the theoretical part studied. What types of transmissions are, consider the losses in the transmission lines, as well important parameters of transmission line. Calculation's with various parameters were performed and analyzed. For example, a transmission line consisting of one RLC segment and transmission line that having matched load with source and load.

During operation, we performed various simulations with different output loads: short line, open line, 50 and 75 Ω . Got results, created graphs and analyzed them.

After that, we started designing the PCB. For this, we have chosen real components. We tried to choose cheap components, order them and PCB. When we got all the components, we started installation. For me, this was the most interesting part, since before that I had never worked with SMD components.

The next step was to conduct tests with the model. By changing the load, I measured each of 13 points and based on the received data I created graphs and compared them with the simulation results and the ideal transmission line.

And the final step, the creation of a manual for laboratory work. I expect this model and laboratory work help students study transmission lines in TalTech University.

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Appendix 1 – Phase and amplitude, $Z_L = 50 \Omega$

Table 2. Simulation results when load impedance 50 Ω .

	TP0	TP1	TP2	TP3	TP4	TP5	TP6	TP7	TP8	TP9	TP10	TP11	TP12
Phase	10°	-23°	-66°	-118°	-158°	-189°	-226°	-278°	-324°	-356°	-388°	-435°	-488°
Amplitude	0.57	0.53	0.44	0.45	0.53	0.54	0.44	0.40	0.48	0.53	0.46	0.37	0.4

Table 3. Prototype test results when load impedance is 50 Ω .

	TP0	TP1	TP2	TP3	TP4	TP5	TP6	TP7	TP8	TP9	TP10	TP11	TP12
Phase	-10°	-22°	-90°	-162°	-175°	-185°	-210°	-310°	-342°	-352°	-365°	-400°	-452°
Amplitude	0.77	0.56	0.2	0.51	0.73	0.64	0.28	0.27	0.52	0.58	0.4	0.26	0.28

Appendix 2 – Phase and amplitude, $Z_L = 0 \Omega$

Table 4. Simulation results when load impedance 0.

	TP0	TP1	TP2	TP3	TP4	TP5	TP6	TP7	TP8	TP9	TP10	TP11	TP12
Phase	-17°	-24°	-30°	-70°	-197°	-204°	-208°	-214°	-376°	-385°	-386°	-386°	
Amplitude	0.79	1	0.73	0.16	0.56	0.96	0.9	0.41	0.29	0.84	0.98	0.65	

Table 5. Prototype test results when load impedance is 0.

	TP0	TP1	TP2	TP3	TP4	TP5	TP6	TP7	TP8	TP9	TP10	TP11	TP12
Phase	-10°	-17°	-100°	-180°	-184°	-188°	-191°	-353°	-360°	-362°	-363°	-365°	
Amplitude	0.83	0.59	0.1	0.55	0.8	0.72	0.29	0.2	0.55	0.65	0.45	0.24	

Appendix 3 – Phase and amplitude, $Z_L = \infty$

Table 6. Simulation results when load impedance is infinity.

	TP0	TP1	TP2	TP3	TP4	TP5	TP6	TP7	TP8	TP9	TP10	TP11	TP12
Phase	35°	18°	-120.3°	-136°	-141°	-146°	-274°	-316°	-319°	-321°	-328°	-497°	-500
Amplitude	0.68	0.28	0.29	0.69	0.75	0.46	0.09	0.55	0.77	0.61	0.16	0.37	0.72

Table 7. Prototype test results when load impedance is infinity.

	TP0	TP1	TP2	TP3	TP4	TP5	TP6	TP7	TP8	TP9	TP10	TP11	TP12
Phase	-14°	-74°	-103°	-120°	-128°	-160°	-275°	-295°	-302°	-302°	-415°	-472°	-484°
Amplitude	0.37	0.50	0.67	0.87	0.73	0.31	0.44	0.10	0.11	0.74	0.10	0.80	1.44

Appendix 4 – Bill of materials

Table 8. Bill of materials for 1 PCB.

components	Mfr. No	quantity	price	total	link
Test Point	5005	12	0.33	3.96	https://www.mouser.ee/ProductDetail/Keystone-Electronics/5005?qs=sGAEpiMZZMsQtlBhqKq43bLWnO4HHayv
BNC connector	226990-1	2	2.24	4.48	https://www.mouser.ee/ProductDetail/TE-Connectivity-AMP/226990-1?qs=sGAEpiMZZMuLQf%252BEuFsOrvdll0QhJmy5C0Of%2FGIJtgY%3D
Resistor 50Ω SMD	TNPW120650R0BEEN07	2	0.83	1.66	https://www.mouser.ee/ProductDetail/Vishay-Dale/TNPW120650R0BEEN07?qs=sGAEpiMZZMtG0KNrPCHnjeBnwtocD3TseKyaJ%2FSuAw%3D
Capacitor 2200pF SMD	VJ1206Y222MXQCW1BC	12	0.33	3.96	https://www.mouser.ee/ProductDetail/Vishay-Vitramon/VJ1206Y222MXQCW1BC?qs=sGAEpiMZZMs0AnBnWHyRQAR7at33PD0%252Bnaf7H9i5O4Q%3D
Inductor 5.6uH SMD	CM322522-5R6KL	12	0.23	2.76	https://www.mouser.ee/ProductDetail/Bourns/CM322522-5R6KL?qs=%2Fha2pyFadujCicUdKn7vIYI6lIX1hJy5D4%252BmVs99b%2FAncxtMnACp7A%3D%3D