TALLINN UNIVERSITY OF TECHNOLOGY

Tallinn School of Economics and Business Administration

Department of Finance and Economics

Chair of Finance and Banking

Veiko Niinemäe

DYNAMIC ASSET ALLOCATION BY VOLATILITY FORECASTING

Master's Thesis

Supervisor: Professor Karsten Staehr, Ph.D.

Tallinn 2015

I declare I have written the master's thesis independently.

All works and major viewpoints of the other authors, data from other sources of literature and elsewhere used for writing this paper have been referenced.

Veiko Niinemäe ....

(Signature, date)

Student's code: 143790

Student's e-mail address: veiko.niinemae@interactiveanalysts.com

Supervisor: Professor Karsten Staehr, Ph.D.

The thesis conforms to the requirements set for the master's theses

....

(Signature, date)

Chairman of defence committee:

Permitted to defence

....

(Title, name, signature, date)

TABLE OF CONTENTS

ABSTRACT

This thesis addresses several volatility modeling and dynamic asset allocation techniques. In the first part of the paper a comprehensive overview of generalized autoregressive conditional heteroskedastic (GARCH) type volatility modeling methods and dynamic asset allocation techniques is provided. In addition, findings of previous empirical studies are discussed. The second part of this thesis, empirical analsysis, is divided into two. First, one day ahead conditional volatility is forecasted for the U.S. and European equity indexes and fixed income futures over a ten year observation period by using GARCH, EGARCH and GJR-GARCH models. Second, the portfolio simulation analysis is conducted by using the shortfall risk-based dynamic asset allocation strategy, which is compared to the portfolio based on static asset allocation.

The Akaike and Bayesan information criterions indicated that most suitable models for conditional volatility forecasting were EGARCH and GJR-GARCH models. These forecasts were used as the main inputs of the expected shortfall risk calculations, although, the further empirical analysis revealed that during the forecasting period ordinary GARCH model was superior.

It appeared from the results of portfolio simulation that, if transaction costs are below 0.38% of traded volume, then during the observation period portfolio based on dynamic strategy outperformed portfolio based on static strategy. Additionally were concluded that results are highly dependent on the dataset, observation period, transaction costs and constraints which were used.

Keywords: dynamic asset allocation, asset management, expected shortfall, CVaR, minimum variance, volatility modeling, volatility forecasting, GARCH, EGARCH, GJR-GARCH

JEL Classification: G11, G15, G17, D81

ABBREVIATIONS

ADF – Augmented Dickey-Fuller

AIC – Akaike Information Criterion

ARCH – Autoregressive Conditional Heteroskedasticity

BIC – Bayesian Information Criterion

BDS – Brock, Dechert, Scheinkma

CPPI – Constant Proportion Portfolio Insurance

CVaR – Conditional Value at Risk

DEUE – STOXX Europe 600 Index Daily Close Price Returns

DGERF – Euro-Bund Future Daily Close Price Returns

USE – S&P500 Index Daily Close Price Returns

USF – U.S. 10Y Treasury Future Daily Close Price Returns

E-GARCH – Exponential Generalized Autoregressive Conditional Heteroskedasticity

EUE – STOXX Europe 600 Index Daily Close Prices

EWMA – Exponentially Weighted Moving Average

GARCH – Generalized Autoregressive Conditional Heteroskedasticity

GERF – Euro-Bund Future Daily Close Prices

GJR – Glosten-Jagannathan-Runkle

RMSE – Root Mean Squared Error

SMA – Simple Moving Average

TGARCH – Threshold Generalized Autoregressive Conditional Heteroskedasticity

USE – S&P500 Index Daily Close Prices

USF – U.S. 10Y Treasury Future Daily Close Prices

VaR – Value at Risk

QGARCH – Quadratic Generalized Autoregressive Conditional Heteroskedasticity

INTRODUCTION

"An investment in knowledge always pays the best interest."

- Benjamin Franklin (1758)

The globalization of financial markets and the continuous development of investment products are offering investors a growing number of options to compile investment portfolios. In search of optimizing the portfolio, a lot of attention has been given to the Markowitz's (1952) mean-variance optimization framework, which is one of the cornerstones of the modern portfolio theory. Despite of the useful insight provided by mean-variance framework, the biggest financial crises during the last decades[1] have provided evidences that this framework possesses one major shortcoming: it assumes static correlations between different financial assets, which deviate from the reality. The correlation dynamics pose a major problem for the asset managers and investors who have been relying on a mean-variance framework as a tool to minimize risks of their portfolios. Numerous studies, for example Erb *et al.* (1994), Karolyi and Stulz (1996), Longin and Solnik (2001), and Ang and Beakert (2002), observed correlations between different financial assets and reached a conclusion that correlations tend to increase during more volatile periods, e.g., during financial crises. In general, correlations are considerably lower for upside movements than for downside movements. Therefore, the same investment portfolio, which is optimal during low volatility period, is not optimal during high volatility period.

The latter is also one of the reasons why Li and Sullivan (2011) claimed that portfolio management is nowadays moving toward a more flexible and dynamic approach, which is capable of capturing the dynamics in risk and return expectations among different financial assets. Under more flexible and dynamic approach Li and Sullivan (2011) considered allocation choices among different asset classes.

---

[1] For example, during the following crises: Black Monday (1987), Asian Crisis (1997-1998), Russian Crisis (1998), Brazilian Crisis (1999), Dot-com Bubble (2001), Subprime Mortgage Crisis/Global Financial Crisis (2007-2008), European Sovereign Debt Crisis (2010-2011).

In addition, the last major financial crises have shown that the traditional static asset allocation, which is widely used by most of mutual funds, may not be rational during different phases of financial market cycles. During the recessions, most of the portfolios which were based on static asset allocation lose the value of underlying assets however, it is important to emphasize that not all asset classes might be decreasing at the same time. Thus, the necessity arises for asset allocation strategies and techniques, in which the asset composition varies over time, commonly known as dynamic asset allocation strategies.

The importance of the asset allocation policy was first studied by Brinson *et al.* (1986). They found that the portfolio's asset allocation policy explains 93.6% of the monthly variance of total returns. Prompted by the intriguingly high percentage, later on, many researches started studying the importance of asset allocation policy and obtained different results. Until today, it is impossible to determine exactly how many percentages of portfolio's total returns are determined by the asset allocation, but it is assumed that it is the key success factor[2] for long-term investments.

This thesis addresses several volatility modeling and dynamic asset allocation techniques. Although correlation modeling would be important from the asset allocation perspective, there are two reasons why only volatility is modeled. First, correlation modeling is more extensive topic than volatility modeling, thus it would be too extensive research considering the scope of this thesis, and the fact that this thesis also addresses dynamic asset allocation techniques. Second, the dynamic asset allocation models, observed in this thesis, use expected volatility (or variance) as the main input. Therefore, there is no need for correlation modeling.

Despite of the fact that there are many studies on volatility modeling and several on dynamic asset allocation strategies, there is lack of studies addressing these problems together. Moreover, the majority of studies on dynamic asset allocation strategies are often not considering transaction costs. Therefore, it is difficult to compare different models and techniques, because transaction costs might influence results significantly. All in all, this topic is worth of investigation and needs further development.

The purpose of this thesis is to provide a clear overview and evaluate the effectiveness of the generalized autoregressive conditional heteroskedastic (GARCH) volatility modeling

---

[2] For short- and mid-term investments key success factors are considered to be market timing and financial instrument selection, respectively (Ibbotson 2010).

methods and dynamic asset allocation techniques. In addition to the latter, two research questions were formulated:

1. Are GARCH volatility modeling methods more precise and accurate for volatility modeling and forecasting than naïve techniques?
2. Can an investment portfolio based on dynamic asset allocation strategy generate higher absolute returns and limit more efficiently short-term losses than a portfolio based on static asset allocation?

In order to answer the research questions, different volatility forecasting techniques will be investigated and dynamic asset allocation strategy will be compared to static asset allocation in the empirical part of this thesis.

This thesis is structured as follows. In Chapter 1, the overview of literary sources in relation to dynamic asset allocation approach and strategies are presented. Chapter 2 describes different volatility modeling techniques, evaluation methods and stylized facts of financial asset returns. In Chapter 3, the results of the empirical analysis of the volatility modeling and forecasting are presented and discussed, which not only illustrate the theoretical review provided in the previous chapter, but it additionally contributes to the further investigation of this topic, by providing a comprehensive comparison of different volatility modeling methods. Chapter 4 is dedicated to the comparison of portfolios based on dynamic and static asset allocation, including implementation of one dynamic asset allocation technique based on the forecasted volatility

1. DYNAMIC ASSET ALLOCATION. A LITERATURE REVIEW

This chapter is structured as follows. In Section 1.1, some main concepts of modern portfolio theory and different asset allocation strategies are presented. In addition to latter, an overview of the importance of asset allocation is given based on empirical studies. Sections 1.2 and 1.3 introduce dynamic asset allocation approach and different strategies, respectively.

1.1. Asset Allocation

The future is uncertain, likewise returns from investments. Nobody knows with certainty what will happen in the financial markets from now on, yet investors need to invest into the future. According to Ferri (2010), successful investing requires well though-out design, implementation, and maintenance of a long-term investment strategy that is based on investor's individual and unique needs. Asset allocation is a central component of that plan. It determines most of investment portfolio risk and return in long-term horizon. The general idea of asset allocation is not to predict expected financial assets risk and return characteristics, but to reduce the need for these predictions. Sharpe (1992) defined asset allocation as the process of dividing investment portfolio between different asset classes. The aim of asset allocation is to design an investor specific asset allocation mix, which has acceptable expected risk and return ratio, so that investor's needs are satisfied. The asset allocation policy paradigm, in which a portfolio is divided up among a various asset classes and then separately managed within each asset class, is an integral part of the asset management.

In the process of making asset allocation decisions, many investors have relied on Markowitz's (1952) mean-variance optimization framework, which is without doubt the cornerstone of the modern portfolio theory. Even though this framework provides useful insight, the biggest financial crises of recent decades have cast doubt on the effectiveness of it, because it strongly depends on the correlations between different assets classes, which vary

drastically during financial crises[3]. Consequently, it might not be rational to use asset allocation based on static weights throughout different phases of financial cycles.

In order to understand the shortcomings of mean-variance optimization framework and fixed weights asset allocation, it is necessary to explain some of the main concepts behind modern portfolio theory.

1.1.1. The Modern Portfolio Theory

Modern portfolio theory was first introduced in 1952 by Harry Markowitz. According to Fabozzi *et al.* (2002), the essence of modern portfolio theory is to guide the selection and construction of investment portfolios. Markowitz (1952) assumed that investors act rationally and consequently want to maximize the discounted value of future returns. Nevertheless, those expected future returns involve an allowance of investment risk. The principle that expected return rises with an increase in risk, *ceteris paribus*, is applied. Therefore, there is always expected risk and return trade-off. (Markowitz 1952; Fabozzi *et al* 2002, 15)

Suppose there are N securities, portfolio expected return is denoted by r_p, portfolio variance is denoted by σ_p^2, portfolio weight of security i is denoted by w_i, expected return of security i is denoted by r_i and its standard deviation is denoted by σ_i, covariance between securities i and j is denoted by σ_{ij}, and correlation coefficient is denoted by ρ_{ij}. Markowitz showed that under the denotations made above, the expected return and variance of the investment portfolio can be described by the equations 1.1, 1.2 and 1.3 (Markowitz 1952, 78-80):

$$r_p = \sum_i^N w_i r_i \tag{1.1}$$

$$\sigma_p^2 = \sum_i^N \sum_j^N w_i w_j \sigma_{ij} \tag{1.2}$$

where

$$\sum_i^N w_i = 1, \qquad \sigma_{ij} = \rho_{ij} \sigma_i \sigma_j \tag{1.3}$$

---

[3] For example, see Erb *et al*. (1994), Karolyi and Stulz (1996), Longin and Solnik (2001), and Ang and Beakert (2002). They observed correlations between financial assets and concluded that correlations tend to increase during more volatile periods (e.g. crises).

Markowitz measured the investment risk by using mathematical formulations and found that risk can be reduced through the concept of diversification. According to Megginson (1996), diversification effect can be considered as the most important aspect of Markowitz's modern portfolio theory. If the investor increases the number of securities within a portfolio, their covariance relationships create a diversification effect. Diversified portfolio's total risk, measured as volatility, is due to correlations between different financial assets in some cases lower, than the sum of portfolio's individual assets.

Markowitz demonstrated a quadratic program with an objective function of maximizing an optimal portfolio through mean-variance optimization. The portfolio selection problem can be described with equation 1.4 (Markowitz 1952, 81-83):

$$Max \left(r_p - \lambda \sigma_p^2 \right) \tag{1.4}$$

where λ denotes risk aversion. While the correlation coefficient between securities pair is in range -1 to 1, then standard deviation of portfolio is always less than the simple weighted average standard deviation of these securities. (Markowitz 1952, 78 - 83)

Another Markowitz's mean-variance framework key concept was the efficient frontier. All portfolios, which are set on the efficient frontier, show higher expected return for a given level of expected risk than any other portfolio. Although, an investor can invest into any given portfolio which plots inside the circle in the mean-variance plane, then rational investor prefers portfolios which have higher expected return at the same expected risk level (Markowitz 1952, 82).

Although modern portfolio theory was further developed, and includes a few more main concepts, for instance the capital asset pricing model[4], these are not introduced, because these are irrelevant considering the context of this thesis.

To conclude, modern portfolio theory attempts to maximize portfolio expected return for a given amount of portfolio risk, or equivalently minimize risk for a given level of expected return, by carefully choosing the proportions of various assets. In other words, this framework attempts to find the best expected risk and return trade-off combination.

---

[4] James Tobin (1958) expanded the portfolio theory using Keynesian liquidity preference theory and added a risk-free asset to the analysis. Based on Markowitz (1952) and Tobin (1958) studies, William Sharpe (1964), Jack Treynor (1962), John Lintner (1965a, 1965b) and Jan Mossin (1966) independently developed the Capital Asset Pricing Model (CAPM), as it later became known. The CAPM revolutionized the modern portfolio theory and practice of investments by simplifying the asset allocation and selection processes (Sullivan 2006, 207).

1.1.2. Asset Allocation Strategies

According to Royston (2011), asset allocation strategy is an investment strategy, which aims to balance risk and reward by apportioning a portfolio's assets according to an investor's goals, risk tolerance and investment horizon. There are several different ways for classifying asset allocation strategies. One of the options is to use time-horizon based classification: long-term, medium-term and short-term asset allocation. Alternatively, strategies can be classified by different investment decision processes and rules. The latter also forms the basis for Ferri (2010) classification of asset allocation strategies, which claims that there are three different main types of asset allocation strategies:

- strategic asset allocation;
- tactical asset allocation;
- dynamic asset allocation.

Strategic asset allocation is a long-term strategy and does not require making accurate short-term predictions about the markets in order to be successful. However, tactical and dynamic asset allocations require accurate short-term market predictions in order to be successful. (Ferri 2010, 15)

Strategic asset allocation combines the investor's risk and return objectives with market expectations in order to establish the exposure to the permissible asset classes. At the center of referred strategy is selecting suitable asset classes and investments to be held for the long-term. In case of implementation of this strategy, an asset allocation will not be changed based on the alternating economic and business cycle phases. (Ferri 2010, 15) Expectation, that systematic risk is compensated in the long run, speaks in favor of strategic asset allocation. This strategy provides a framework to systematic risk exposure.

Tactical asset allocation presumes temporary divergences from strategic asset allocation weights, based on short-term market forecasts and views. These predictions are generally outputs of a function of fundamental, economic or technical variables. For instance, fundamental variables might be such as earnings or interest-rate forecasts, economic variables such as the outlook for economic growth in different countries, or technical variables such as recent price trends and charting patterns. (Ferri 2010, 15)

Dynamic asset allocation is for investors who believe they can consistently forecast major movements in the market and thus beat the market by rebalancing asset allocation weights constantly. It is tactical asset allocation in the extreme. (Ferri 2010, 15) There are no

restrictions on asset class weights, and this strategy is certainly not for spreading risk. This strategy can be very profitable when market timing is done correctly, as well as vice versa.

In general, dynamic and tactical asset allocations have a monthly or quarterly horizon, while strategic asset allocation is done for long-term horizon. The distinction is important from a governance point of view. Dynamic and tactical asset allocations can be seen as a short term corrections of strategic asset allocation, taken into account contingent market situation and involves people dealing with it on a daily basis. Strategic asset allocation involves implementing once set in place long-term goals.

1.1.3. The Importance of Asset Allocation

Asset allocation is supposedly very important from investments performance point of view. This sub-section will provide an overview of several empirical studies, which observed asset allocation policy influence to investment portfolio's return characteristics.

The literature on the importance of asset allocation is vast. Most studies in this area focus on analysis of mutual and pension funds, and explore how big percentage of the portfolio's total return is explained by deviations from an institution's policy asset class weights. One of the first attempts to determine the asset allocation importance was conducted by Brinson *et al.* (1986). They analyzed 91 U.S. pension funds' underlying assets returns from 1974 to 1983 by regressing monthly portfolio total returns against to the monthly returns to each funds' policy portfolio. As a result, they concluded that the portfolio's asset allocation policy explains 93.6% of the monthly variance in pension funds' total returns during this period. Further studies, which will be described below, highlight that the coefficient of determinations should vary probably between 33-75%. Brinson *et al.* (1986) got higher coefficient because the results depended from aggregated market movements instead of pension funds' specific asset allocation mix.

Ibbotson and Kaplan (2000) developed Brison *et al.* (1986) empirical analysis further by exploring the degree to which funds' asset allocation mix explained the cross-sectional differences in absolute returns across several funds, and whether it is an asset managers' competence that drove assets performance or asset allocation policy. Their study was based on two earlier reports by Brinson *et al.* (1986) and Brison *et al.* (1991). They carried out cross-sectional regression, using annualized cumulative returns over a 10-year observation period and found as a result that approximately 40 percent of the variation of returns was determined by asset allocation. They concluded, that the majority of pension funds'

performance can be explained by the funds' decision to choose asset classes (including holding cash) to invest. The latter creates the need for explicit rules and indicators or forecasts for choosing between asset classes.

Vardharaj and Fabozzi (2007) applied similar techniques used in Ibbotson and Kaplan (2000) report for investment funds and found out that the determination coefficients were sensitive to observation time and the asset allocation mix determined approximately 33 to 75 percent of the variance in asset returns. Also, in a recent study, Xiong *et al.* (2010) showed that the variations of returns among assets what can be determined by asset allocation policy are dependable of the sample.

All found determination coefficients in exact percentage points are results of some sort of study and therefore, consequently depending on the specific inputs and methods used. Actually for any given investment fund, the necessity and the importance of asset allocation depend on the asset owner preferences, expectations and risk tolerance.

Asset allocation provides passive return (beta return), and the remainder of the return is the active return (excess or alpha return). The alpha sums to zero, because on average asset managers do not beat the market. Thus, on average the passive asset allocation determines 100 percent of the return, only at the aggregate level. (Ibbotson 2010, 18) Active fund management reduces the importance of asset allocation, but it is difficult to say exactly how much. Depending on the asset managers' objectives, asset allocation can provide in addition to return also an opportunity to optimize mean variance and to diversify risks.

1.2. Dynamic Asset Allocation Approach

The last major financial crises have shown that the traditional static asset allocation may not be rational during different phases of financial market cycles. During the recession periods many portfolios which are based on static asset allocation lose the value of underlying assets, however, not all asset classes might be falling at the same time. Investors want to hold their assets in rising markets over the long-term, but it is also in their interest to not to fall with markets and avoid large negative returns in shorter periods. According to Herold *et al.* (2007), this has led to renewed interest in portfolio selection and asset allocation strategies, which produce absolute returns and that particularly, control downside risk, commonly known as a dynamic asset allocation.

Dynamic asset allocation determines an optimal portfolio asset allocation mix in accordance with changing market expectations and conditions (Wang *et al.* 2012, 26). This strategy involves systematic capital re-allocation among different asset classes. While strategic asset allocation uses static expectations of asset allocation policy, this framework provides flexible approach, which is capable of capturing the dynamics in risk and return expectations, across an array of asset classes (Li and Sullivan 2011, 31).

According to Herold *et al.* (2007), the main characteristic of the dynamic asset allocation approach is that the weights of different asset classes are allowed to change significantly, depending on changes in the economic climate and activity. Since, dynamic asset allocation generally does not involve market timing, and then asset classes' weights changes are driven by a set of predefined rules and indicators (Lawrence and Singh 2011, 49).

While dynamic asset allocation is implemented for individual and institutional asset management there are different dynamic asset allocation definitions. For individual asset management, the most important criteria for doing asset allocation is the time-horizon. Risk-aversion increases as the individual investor ages. At the moment, these strategies[5] are out of the scope of the thesis, herein are concerned asset allocation strategies which can be adapted by institutional investors. However, it is worth to notify, that according to Herold *et al.* (2007), most of dynamic asset allocation strategies, which are aimed for institutional investors, can be also applied a for individual asset management. According to them, in general, the aim of dynamic asset allocation is to protect the portfolio value from falling below a pre-specified floor. This is an extremely important criterion for an individual asset management, as well as for institutional asset management.

The mechanism for dynamic asset allocation is not the same as that for modern portfolio theory. For modern portfolio theory investments are diversified in order to reduce risk through a covariance term. Even though the volatility is reduced in short-term, it might not be in the long-term. In the dynamic asset allocation, an investment choice is made between a different asset classes. The essential difference between modern portfolio theory and dynamic asset allocation is that the latter is a dynamic process that presents the opportunity to increase return, while modern portfolio theory uses averaged statistics and portfolios to allocate resources across different investments at the same time. Dynamic asset

---

[5] For example, based on Xiong and Idzorek (2011) article, the most important investment decision, whether to take risk and how much, will change when the investor ages.

allocation seeks to increase risk and return trade-off by investing in a better performing asset classes. (Harloff 1998, 7)

The dynamic asset allocation differs from strategic and tactical asset allocation by the length of period when asset allocation weights are reconsidered, and by the changes which are allowed to make in asset allocation weights, respectively. The strategic asset allocation approach reviews asset allocation weights on a periodic basis, using assumption that expected asset class return, risk, and correlation can be derived from long-term historic averages (Knutzen 2011, 1). Even though tactical asset allocation allows the portfolio manager to take active positions whenever often necessary, then these are made with respect to a strategic benchmark in order to generate risk adjusted excess returns compared to the benchmark. In that case, investors usually diverge only within a narrow range from the strategic benchmark, e.g. they change weights by a couple of percentage points when they expect falling or rising prices. (Herold *et al.* 2007, 61) Dynamic asset allocation approach does not have these restrictions. Asset allocation weights can be changed whenever necessary and usually the weights changes have no limitations.

Portfolios based on dynamic asset allocation are usually aimed to produce absolute return (either total return above a pre-specified target or positive returns) rather than relative excess return over the benchmark. The latter enables the possibility to protect the portfolio value falling during market recession. Absolute return portfolios, that target a certain margin above inflation, can maintain and grow underlying assets value much more likely than relative return portfolios that aim to outperform the benchmark.

1.3. Dynamic Asset Allocation Strategies

There are many classifications of the dynamic asset allocation strategies. One criterion to distinguish the dynamic asset allocation strategies is the amount of input data needed. Some strategies, like stop loss and constant proportion portfolio insurance (CPPI), involve only observable parameters, while the shortfall risk-based (conditional value at risk) strategy makes distributional assumptions and requires estimating several parameters (Herold et al. 2007, 62). This classification is used relatively infrequently.

Another more commonly used criterion to distinguish the dynamic asset allocation strategies is the methods and rules which are used. The latter classification will be used in this

thesis. This type of classification was first used in 1988 by Perold and Sharpe. According to Perold and Sharpe (1988), there are four distinguishable dynamic strategies:

- buy-and-hold
- constant-mix
- onstant-proportion portfolio insurance (CPPI)
- option-based portfolio insurance.

Since today, there have arisen more dynamic asset allocation strategies than Perold and Sharpe (1988) considered in their article. One of the most attention drawn, and much referred to in subsequent studies, is conducted by Herold *et al.* (2007). They distinguish dynamic asset allocation strategies where forecasting is primary or which are rules-based. Strategies where forecasting is primary, depend highly on the accuracy of forecasts. These include many alternative investments, for example, this strategy is often applied by global macro hedge funds, which are based on forecasts and want to either time the market or exploit market inefficiencies through the skills of their managers (Herold *et al* 2007, 61). Since forecasting is a subjective activity and the coincidence of favorable events might often be the reason for success, these types of dynamic asset allocation strategies were not studied in detail by Herold *et al.* (2007), nor will be in this thesis. This thesis focuses on dynamic asset allocation strategies which are either risk- or rules-based, and do not rely on forecasts (or where forecasts do not play a dominant role).

According to Herold *et al.* (2007), the dynamic asset allocation strategies which are either rules- or risk-based can be divided into three groups:

- portfolio insurance,
- rainbow options,
- shortfall risk-based strategies.

All these three groups of strategies aim at dynamically managing portfolio risk through asset allocation decisions, in order to protect the portfolios' total value from falling below a pre-specified floor.

It is worth to notify that terms "total return" or "absolute return" are used, because the risk-based strategies are designed to produce either positive returns or total returns above a predetermined minimum return, not relative return compared to benchmark.

Hereinafter, this section is divided into three sub-sections, which explain the concepts of the following rules- and risk-based strategies: portfolio insurance, rainbow option and

shortfall risk-based. While previous empirical studies have proven the superiority of shortfall risk-based strategies[6], these are discussed in more detail.

1.3.1. Portfolio Insurance Strategies

Portfolio insurance techniques include three distinctive strategies: stop loss, synthetic put and constant proportion portfolio insurance (CPPI). The objective of portfolio insurance strategies is to maintain the portfolio value above a certain predetermined floor, while allowing some upside potential.

Stop loss strategy is probably the most intuitive and simplest strategy, but it is difficult to quantify in practice. In case of this strategy, the entire portfolio is initially invested into the risky asset. As soon as the risky asset drops below the predetermined floor, the entire portfolio is rebalanced totally into the risk-free asset. When the market rebounds above the floor, the entire portfolio will be rebalanced back into the risky asset. (Tankov 2009, 7-9) Stop loss strategies are not much in use in practice, because it is unrealistic to carry out transactions instantly and without costs. If portfolios' assets under management are large, then liquidation of open positions may take days, weeks or even months, depending on the assets liquidity and market depth.

The concept of option based portfolio insurance tactics, based on using either traded or synthetic options, was introduced by Leland and Rubinstein (1976). Option based portfolio insurance is based upon the work of Black and Scholes (1973), which showed that under certain assumptions the payoff of an option can be replicated through a continuously revised combination of the underlying asset and a risk-free bond. Leland and Rubinstein (1976) extended this insight by showing that a dynamic asset allocation method which increased (or decreased) stock allocation of a portfolio during rising (or falling) market period, and reinvested the remaining portion in cash, would duplicate the payoffs to a call option on an index of stocks. (Lummer and Riepe 1994, 4) The price behavior of a call option is similar to a combined position, involving the borrowing and underlying stock. If the market is normally functioning, the call option price and the stock price will change in the same direction. Moreover, Rubinstein and Leland (1981) found that the number of stocks in the replication

---

[6] For example, Herold *et al.* (2007) compared shortfall risk-based strategy with different alternative dynamic asset allocation strategies in variety of asset classes. They found that shortfall risk-based strategy protects downside risk much the same as portfolio insurance and rainbow option concepts. In addition, shortfall risk-based strategy uses the available risk budget in an effective way and enhances performance in the long-term. (Herold *et al.* 2007, 72)

portfolio must equal to the slope of the call price curve. Their concept permits to replicate, not only call options, but also other option positions. Investors and institutions can create themselves covered calls and protective puts on stocks, which do not have options available, by using replicating portfolios.

Black and Jones (1987) and Perold and Sharpe (1988) developed CPPI method, which became popular with practitioners (Karoui *et al.* 2005, 450). In case of this method, all asset allocation decisions are based on the floor value of portfolio, which the investors initially have to set. Two asset classes are used: risk-free assets and risky assets. In general, fixed income assets or money-market funds are considered as risk-free assets, and equities or mutual funds as risky assets. The asset allocation weights depends on the cushion value and multiplier coefficient, where cushion value is defined as the current portfolio value less the floor value, and a multiplier coefficient denotes the aggression of the strategy. The floor on the portfolios' value grows at the risk-free rate over time, and the exposure to the risky asset is calculated as a multiplication of the cushion value and multiplier coefficient. (Black and Jones 1987, 48)

1.3.2. Rainbow Option Strategies

A rainbow option, also known as basket option, is a derivative exposed to two or more sources of uncertainty. As opposite, regular options are exposed to one source of uncertainty, price movements in the underlying asset. In general, rainbow options are calls or puts on the best or worst of N underlying assets. Or options which pay the worst or best of N assets. (Chantnani 2010, 169) The aim of rainbow option strategy is to provide to the investor right to rebalance portfolio into better performing asset class. The difference between the performance of this strategy and the better performing asset class is called the rainbow option premium. Payoff depends on the relative price performance of chosen asset class.

Suppose an investor uses best of stocks and bonds method, and purchases a 100% bond portfolio and an exchange option at the beginning of the year. The option gives to the investor a right to exchange the performance of bonds with the performance of stocks at the end of the year. Similarly to the protective put method, this strategy is implemented by replication the exchange option. In practice, this amounts to start with portfolio allocated equally between asset classes each year, and at the end of the year, the portfolio will be invested 100% into the better performing asset class. (Herold *et al* 2007, 61-62) While best-of-two strategy cannot protect the portfolios' value from falling below a predetermined floor,

Merton *et al.* (1978), Merton *et al.* (1982), and Stulz (1982) enhanced best-of-two strategy by a floor, which protects portfolios' value from falling. Particular strategy is called best-of-two plus floor below.

Suppose, there is a portfolio, which invested 80% of its assets in money-market instruments and 20% in a diversified portfolio of stock call options, provided equity exposure on the upside with a guaranteed "floor" on the value of the portfolio. The gain from equity exposure realizes when options are in money[7] when these are exercised or sold. The protected value equals to the value of assets which are invested into risk-free asset class.

1.3.3. Shortfall Risk-Based Strategies

Shortfall risk-based strategies are also known as value at risk-based strategies. Even though Perold and Sharpe (1988) claimed that return forecasts are not a part of these strategies, as the overall target is to protect the portfolio value from falling below a pre-specified floor, then more recent studies, including Herold *et al.* (2007), classify shortfall risk-based strategies into two groups, depending on whether the method is forecast free or incorporates market views. The overall idea of shortfall risk-based strategies is to enter conditional return, volatility and correlation into the calculations of shortfall probability.

Herold *et al.* (2005) investigated a rules-based and not benchmark related shortfall risk-based approach, which can accommodate a wide variety of asset classes and at the same time, keep control for downside risk. They applied this particular approach using two asset classes: fixed-income and cash. Their empirical study indicated substantial shifts in asset classes' weights over time. They found that shortfall risk-based strategy controls portfolio's risk more efficiently than regular static asset allocation strategies. (Herold *et al.* 2005, 40)

Two years later Herold *et al.* (2007) extended the shortfall risk approach to the multi-asset case and compared results with different alternative dynamic asset allocation strategies. In addition they also provided an extensive simulation study to quantify short-run hedging effectiveness and long-run hedging costs. In conclusion they found that shortfall risk-based strategy offers downside risk protection much the same as insurance concepts, moreover, this strategy uses the available risk budget in an effective way, thus can enhance portfolios' performance in the long-term (Herold *et al.* 2007, 72).

---

[7] The strike price of a call option is lower than the market price (the strike price of a put option is higher than the market price).

In order to understand the concept behind the expected shortfall risk-based method, it is necessary to define value at risk (VaR) before. The mathematics that underlies VaR was largely developed in the context of portfolio theory by Markowitz (1952). VaR refers to the loss risk caused by uncertain changes in asset prices. (Angelovska 2013, 85)

According to Jorion (2001), VaR measures the worst expected loss over a given time horizon under normal market conditions at a given level of confidence. The fundamental variables of VaR are: confidence level, forecast horizon, and volatility. The confidence level is the probability that the expected loss is not greater than predicted. Forecast horizon is the time framework that VaR is estimated, in calculation it is generally assumed that portfolio's holdings does not change during that horizon. (Nylund 2001, 9) The mathematical definition of VaR can be described by following equation 1.5 (Angelovska 2013, 85):

$$\text{VaR} = -k(a) * P * \sigma_p \qquad (1.5)$$

where the portfolios' standard deviation is denoted by σ_p, the value of the portfolio is denoted by P, and the desirable level of confidence is denoted by $k(a)$.

Figure 1.1 illustrates the latter. On the left side there is a probability density function. In the middle of the density function is the mean return and on the left there is VaR. Investors' minimum acceptable return (MAR) is between mean return and VaR. MAR exact location between VaR and mean return, depends on the investor's risk aversion. The more risk averse investors' MAR is closer to mean return, and contrariwise, less risk averse investors' MAR is closer to VaR.

On the right side of the Figure 1.1, there is a fictional asset historical price shown from time zero to time t. After time t, further expected return is described by the probability density function.
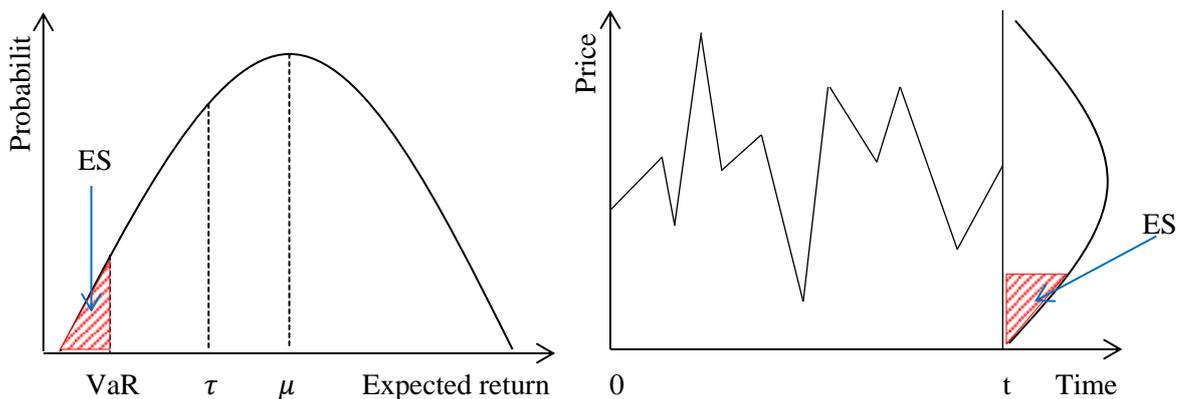
Figure 1.1 Value-at-Risk and Expected Shortfall

Source: Compiled by the author

Painted red area on the Figure 1.1, presents the expected shortfall (ES) probability, also known as conditional value at risk (CVaR). The basic idea of shortfall risk-based strategy is to control the shortfall risk probability directly. In order to do that, lower partial moment of order minimum acceptable return is calculated (Herold *et al* 2005, 34). In simpler terms, the red area is calculated by using integration.

To simplify the calculation it is assumed that returns are normally distributed, $X \sim N(\mu, \sigma^2)$. The assumption does not concern conditional mean return and conditional volatility, but particularly skewness and kurtosis (according to the assumption skewness = 0, and kurtosis = 3). In that case, expected shortfall probability process can de described by following equation 1.6[8] (Herold *et al.* 2005, 34):

$$ES(R) = \phi\left(\frac{\tau - \mu}{\sigma}\right) \tag{1.6}$$

where the portfolio expected return is denoted by R, the cumulative standard normal distribution is denoted by ϕ, the minimum acceptable return is denoted by τ, the mean return is denoted by μ, the conditional volatility of the return distribution is denoted by σ.

Based on the expected shortfall risk probability portfolio's asset allocation will be constantly revised, and if necessary, specific asset exposure will be adjusted to hold pre-

---

[8] See the VBA code in Appendix 1.

specified shortfall risk probability[9]. In case, when the expected shortfall risk probability is below pre-specified target, is possible to increase exposure over 100%, by using leverage, additional free cash, and etc. Also on the contrary, when the expected shortfall risk probability is above pre-specified target, exposure will be decreased.

---

[9] In order to illustrate the latter, an example is compiled based on fictional data, see Appendix 2.

2. VOLATILITY MODELLING AND FORECASTING. A LITERATURE REVIEW

The previous chapter covered theoretical and empirical aspects of asset allocation, its importance, and introduced different models which can be used for dynamic asset allocation. The overview of different dynamic asset allocations models revealed that the main input for most of the models is the expected volatility of different underlying assets classes. Since expected volatility is extremely important input and may change the results enormously, Chapter 2 will be dedicated on a literature review of volatility modeling and forecasting.

This chapter proceeds as follows. Firstly there is an overview of the stylized facts of financial asset returns. Section 2 is dedicated on the naïve volatility forecasting methods. Section 3 is about the development of different ARCH- and GARCH-type models. Section 4 is concentrated on the evaluation of the models forecasting performance.

2.1. Stylized Facts of Financial Asset Returns

This section is about financial asset returns distributional characteristics (heavy tails, negative skew, volatility clustering, and asymmetric dependence) which collectively are often referred to as *stylized facts* of financial asset returns[10].

The literature on financial data returns modeling methods is very rich and it dates back to 1960s. Empirical study on commodity returns volatility modeling and clustering, on the log return time series data, conducted by Mandelbrot (1963), and showed that return distributions are heavy-tailed[11]. This was an outcome that has later been found in every main asset class, including equities (Fama 1963), currencies (Westerfield 1977), fixed income (Amin and Kat 2003), and REITS (Lizieri *et al.* 2007).

---

[10] See for example Cont (2001) or McNeil *et al.* (2005)
[11] Heavy tails are also often referred as fat tails or leptokurtic distribution; it means that extreme values are more probable than under normal distribution (Cooke and Nieboer, 2011, 5).

Furthermore, Mandelbrot (1963) additionally recognized volatility clustering in commodity returns, where "large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes". As previously, this outcome find confirmation in most of major asset classes, including equities (Fama 1965), foreign exchange (Baillie and Bollerslev 1989), and fixed income (Weiss 1984).

Kraus and Litzenberger (1976) found that financial assets' returns, where large declines are more common than large inclines, are usually negatively skewed. Same results were confirmed by Beedles (1979), Alles and Kling (1994), and Harvey and Siddique (1999).

Asymmetry of the volatility of financial asset's return was observed by Black (1976), Christie (1982) and Schwert (1990), they all reached to a same outcome, that financial asset current returns are in a negative correlation with expected volatility. Chelley-Steeley and Steeley (1996), as well as many later studies[12], suggested that the asymmetric volatility phenomenon refers to a situation, where financial asset price conditional volatility caused by new positive information has smaller magnitude, than on the contrary conditional volatility which is caused by new negative information. However, it is worth noticing that the asymmetry is not present in currencies (Allen and Satchell 2014).

Wei *et al.* (2011) intuitively described the increase in expected volatility after negative news from the investor's point of view: if after negative news equity price (value) is decreasing and the proportion of debt remains same, then financial leverage of the company increases, which in turn increases the risk of holding these stocks, and thus the expected return might be more volatile. Exactly the opposite reaction is intuitively expected to occur after the release of positive news: equity price (value) increases and financial leverage decreases, which in turn, makes holding this equity less risky and expected returns less volatile[13]. Described phenomenon is called in the literature as the "leverage effect"[14]. (Wei *et al.* 2011, 83)

It is necessary to emphasize that the growth in financial leverage solely is insufficient to account for the observed increase in volatility following market recessions (Bollerslev 2010), and that behavioral factors may have influence (Allen and Satchell 2014).

---

[12] See Glosten *et al.* (1993), Hung (1997), Laopodis (1997), Yang (2000), and Hansen and Lunde (2005).

[13] Yet, not all researches agree with this assumption. For example, Lo and MacKinlay (1987) assume that the asymmetric volatility phenomenon is a result from non-synchronous trading; while Sentana and Wadhwani (1992) consider it caused due to investors' herding behavior.

[14] While Black (1976) firstly noted the volatility asymmetrical effect, the phenomenon as the leverage effect is often also referred as the Fisher-Black effect.

More recent studies which have been identifying the asymmetric tail dependence between different assets, have been reaching to results, that "when the market crashes all correlations go to one" (*Ibid.*). Many studies have confirmed that correlations tend to increase during crises and volatile periods. For instance, Erb *et al.* (1994) concluded that the correlations between the G7 country stock exchange indices are significantly higher in bull markets than in bear markets[15]. Karolyi and Stulz (1996) showed that the correlation between Japanese and U.S. equities increases during enormous market shocks. Ang and Bekaert (2002) conducted a study between major international equity indices and found that the correlations tend to increase during more volatile periods.

And these are the results not only for major indices. The same pattern is evident between individual stocks and aggregate indices. For example, Longin and Solnik (2001), as well Ang and Chen (2002), observed correlations between individual stocks and indices, particularly small capitalization and value stocks, and found that correlations are considerably lower for upside movements than for downside movements.

Considering the latter stylized fact of financial asset returns – correlations are changing substantially during market shocks – this thesis is not going to focus on correlation asymmetry, nor to the modern portfolio theory principle which involve minimizing the portfolio's risk through diversification effect[16], because it would be too extensive research and it has been by now relatively much studied. Recalling the main aim of this thesis, to provide an overview and implement different dynamic asset allocation strategies by using forecasted expected volatility as an input, the latter is also a reason, why these issues concerning correlation and portfolio's overall risk, are further no longer directly addressed in the theoretical parts of this thesis.

To conclude, generally most major financial asset classes' returns volatility is not constant, serial dependence is present in the lags of returns, distribution of the data are not Gaussian, but asymmetric and heavy-tailed. Even though some of these findings were made already in 1960s, these are confirmed also nowadays[17].

---

[15] The terms bull market and bear market describe upward and downward market trends, respectively (Preis and Stanley 2011).

[16] Due to negative correlations between different assets

[17] For instance, Duncan and Kabundi (2014) observed volatility of world equity markets during 1994-2008.

2.2. Naïve Forecasting Models

Before providing an overview of autoregressive conditional heteroskedasticity models, two naïve volatility forecasting models will be introduced. These are simple moving average (SMA) with fixed and equal weights, and exponentially weighted moving average (EWMA) volatility.

2.2.1. Simple Moving Average

The simple moving average volatility forecasting is based on a financial asset price volatility (standard deviation) formula, which can be defined with equation 2.1:

$$\sigma = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(r_t - \bar{r})^2} \tag{2.1}$$

where price volatility is denoted by σ; number of observations is denoted by T; return at a period t is denoted by r_t; and mean return is denoted by $\bar{r}$.

Further, the simple moving average forecasting method uses recent historical standard deviations to forecast next period expected volatility. This process can be described with following equation (Knight and Satchell 2007, 28):

$$\hat{\sigma}_t = \frac{(\sigma_{t-1} + \sigma_{t-1} + \ldots + \sigma_{t-\tau})}{\tau} \tag{2.2}$$

where forecasted next period price volatility is denoted by $\hat{\sigma}_t$; and the number of recent periods included in the SMA process is denoted by τ.

In (2.2), τ defines the number of rolling volatilities used in forecast. For example, if T is greater than τ, then volatility forecast considers only τ number of recent volatilities which will be equally weighted, and $T - \tau$ number volatilities will have zero weight in forecast.

2.2.2. Exponentially Weighted Moving Average

Another naïve, and improved, way for making forecast is an exponentially weighted moving average (EMWA). In this approach latest observations carry the highest weight in the volatility forecast. For particularly volatility forecasting, in financial literature RiskMetrics[TM] (1996) developed techniques are used. It is used instead of SMA, because EWMA uses a decay factor λ to assign weight to historical observations.

The EWMA financial asset price volatility can be defined by following formula (RiskMetrics[TM] 1996, 78):

$$\sigma_t = \sqrt{(1-\lambda)\sum_{t=1}^{T}\lambda^{t-1}(r_t - \bar{r})^2} \qquad (2.3)$$

where decay factor is denoted by λ. Parameter λ defines the effective amount of data used in estimating volatility and the relative weights that are applied to the observations (returns)[18].

The one period ahead EWMA volatility forecast is given by the formula (*Ibid.*, 81):

$$\hat{\sigma}_{1,t+1|t} = \sqrt{\lambda\sigma^2{}_{1,t|t-1} + (1-\lambda)r^2{}_{1,t}} \qquad (2.4)$$

In equation 2.4, the subscript "$t+1|t$" is read as "the time $t+1$ forecast given information up to and including time t." The subscript "t|$t-1$" is read in an analogous manner. This notation emphasizes the fact that the volatility is time-dependent. (*Ibid.*, 82)

Volatility forecasts based on the EWMA are more adequate than forecasts based on SMA. The EWMA volatility reacts faster to shocks in the market as recent data carries more weight, thus it incorporates external shocks better than equally weighted moving averages. In addition, following a shock (or a large return), the volatility declines also exponentially, as the weight of the shock observation falls exponentially[19].

2.3. Autoregressive Conditional Heteroskedasticity Models

Previously mentioned stylized facts of financial asset returns indicated that a random walk models with Gaussian increments are not suitable for modeling the volatility of financial data. Driven by latter, there are proposed a several different models to solve abovementioned shortcomings.

2.3.1. ARCH

Two decades after Mandelbrot (1963) and Fama (1965) published their pioneering studies, Engel (1982) introduced the ARCH (Autoregressive Conditional Heteroskedastic)

---

[18] For example, RiskMetrics[TM] has a tendency to use a lambda of 0.94, or 94%. In this case, the most recent squared periodic return is weighted by $(1-0.94)(.94)^0 = 6\%$. The next squared return is simply a lambda-multiple of the prior weight; in this case 6% multiplied by 94% = 5.64%. And the third prior day's weight equals $(1-0.94)(0.94)^2 = 5.30\%$. The process continues in the same way until the last observation.

[19] Inversely to the forecasts based on SMA, where the shocks have equal weight to the forecast, regardless how long time ago it took place, until the shock falls out of the measurement sample.

model to solve heteroskedasticity problem. Engel showed in his article the usefulness of the ARCH models for improving the performance of ordinary least squares model by supposing that the conditional variance is not constant over time and shows autoregressive structure (Engel 1982, 992-994).

The ARCH(1)[20] process can be described by equations 2.5, 2.6 and 2.7 (Ou 2014, 13):

$$r_t = \mu_t + \varepsilon_t \tag{2.5}$$

$$\varepsilon_t = \sigma_t z_t, \quad z_t \sim NID(0,1) \tag{2.6}$$

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 \tag{2.7}$$

where the continuously compounded rate of returns from time t-1 to t is denoted by r_t; the conditional mean or underlying asset volatility is denoted by μ_t; the residuals are denoted by ε_t; the probability density function with a mean of zero and unit variance is denoted by NID(0,1)[21]; ω and α are non-negative parameters.

The usefulness of the ARCH model, introduced by Engel (1982), was also confirmed by studies conducted later on by Engle and Bollerslev (1986) and (1993), as well as Ding and Granger (1996). In fact, the majority of subsequent studies on asset return's volatility have been dominated by ARCH-type models, which have been extremely successful in capturing the main characteristics of asset return's volatility.

2.3.2. GARCH

In 1986, Bollerslev introduced GARCH (Generalized Autoregressive Conditional heteroskedasticity). The latter is a natural generalization of the ARCH process, allowing for a more flexible lag structure. (Bollerslev 1986)

While in ARCH(q) model, the q is the number of lags included, and the variance is dependent on lagged squared deviations, then in GARCH(p,q) model additionally includes lagged variances, where q is the number of lags of the squared error, and p is the number of lags of the conditional variance.

---

[20] According to Curto and Pinto (2012), despite the theoretical interest of (p,q) models, the (1,1) specification is generally suitable when modeling the volatility of financial assets returns; see also Bollerslev *et al.* (1992) and more recently Hansen and Lunde (2005). Therefore, in this paper all autoregressive conditional heteroskedastic models are of the order p = 1, q = 1.

[21] Normally and independently distributed.

The conditional variance equation of the GARCH(1,1) model is given in equation 2.8 (Ou 2014, 15):

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \qquad (2.8)$$

where the conditional variance of underlying asset volatility is denoted by σ_t^2, ω, α and β are non-negative parameters, where $\alpha + \beta < 1$ to ensure that the stationary and positive conditional variance conditions are met.

While the conditional variance is identified as a weighted average of squared errors, both ARCH and GARCH models, are able to describe the phenomenon of volatility clustering of returns and they also partly describe the heavy-tails demonstrated by financial time-series.

However, the simple structure of these models causes two major drawbacks. First, GARCH model parameters have non-negativity restrictions. Another drawback is the GARCH model assumes that only the amplitude of the change determines the conditional variance, so it cannot distinguish between the sign of the volatility (the difference between negative and positive volatility). Thus, it fails to incorporate the leverage effect[22]. (Wei *et al.* 2011, 83)

Many studies, including Ou (2014), are predominately concluded that, the simplest symmetric linear GARCH models [for example GARCH(1,1)] are shown to be not accurate, because, as already mentioned herein before, the linear models requires positive volatility and in result the current volatility lag residuals are symmetric, but in reality, the negative shocks would cause larger influence on future volatility, than the same amount of positive shocks could affect (Ou 2014, 2).

2.3.3. EGARCH and QGARCH

Therefore, to overcome the limitations of symmetric linear GARCH models, many asymmetric-type non-linear GARCH models were developed. In order to distinguish the sign difference and distribute the asymmetric volatility, Nelson (1991) developed exponential GARCH model (EGARCH) and Campbell and Hentschel (1992) developed quadratic GARH model (QGARCH).

The logarithmic conditional variance equation of the EGARCH(1,1) process can be described with equation 2.09 (Ou 2014, 17):

---

[22] See Tavares *et al.* (2007).

$$ln\ \sigma_t^2 = \omega + \theta_1 \frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} + \gamma_1 \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_1 ln\ u_{t-1}^2 \tag{2.09}$$

where the asymmetric leverage coefficients[23] are denoted by θ and γ.

The conditional variance equation of the QGARCH(1,1) process is given in equation 2.10 below (Goodwin 2012, 20):

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \delta_Q \varepsilon_{t-1} \tag{2.10}$$

where the asymmetry parameter is denoted by δ_Q. The difference to the GARCH(1,1) model is the addition of $\delta_Q \varepsilon_{t-1}$, where the most recent error is multiplied by the asymmetry parameter. Additionally to the non-negativity constraints in the GARCH(1,1) model, $\sigma_t^2 < 4\alpha\bar{\omega}(1 - \alpha - \beta)$ is required to ensure positivity of σ_t^2. (*Ibid.*)

Although both of these models distinguish the sign difference and distribute the asymmetric volatility, later studies conducted by Engle and Ng (1993) and Hafner (1998) measured these two models, and found that EGARCH model is more suitable in most cases.

2.3.4. GJR-GARCH and TGARCH

A different approach to capture the leverage effect is presented by Glosten, Jagannanthan and Runkle (1993), they propose the GJR-GARCH model, which is modeling the standard deviation instead of the conditional variance. GJR-GARCH model is very similar to the threshold GARCH model (TGARCH) introduced by Zakoian (1994), which is dividing the distribution of the changes into separate intervals and then estimates a linear function for the conditional standard deviation.

The conditional variance equation of the GJR-GARCH(1,1) model is given in the equation 2.11 (Ou 2014, 19):

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \gamma_1 I_{t-1} \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{2.11}$$

where the indicator function is denoted by I ($I_{t-1} = 1$, if $\varepsilon_{t-1}^2 < 0$ and $I_{t-1} = 0$, if $\varepsilon_{t-1}^2 \geq 0$); the asymmetric leverage coefficient is denoted by γ. The indicator function equals to 1, when the asymmetric leverage coefficient is negative, and on the contrary, the indicator function equals to 0, when the asymmetric leverage coefficient is positive. In other words, there is an

---

[23] Describing the volatility leverage effect.

assumption in the GJR-GARCH model that the ε_t^2 sign (positive or negative) has a different impact on conditional variance σ_t^2.

The TGARCH(1,1) process is very similar to GJR-GARCH(1,1). The main difference lies in replacing the conditional variance with the conditional standard deviation. The conditional standard deviation equation of the TGARCH(1,1) process can be described by equation 2.12 (Zakoian 1994):

$$\sigma_t = \omega + \alpha_1^+ \varepsilon_{t-1}^+ + \alpha_1^- \varepsilon_{t-1}^- + \beta_1 \sigma_{t-1} \tag{2.12}$$

where $\varepsilon_{t-1}^+ = \varepsilon_{t-1}$ if $\varepsilon_{t-1} > 0$, and $\varepsilon_{t-1}^+ = 0$ if $\varepsilon_{t-1} \leq 0$; similarly $\varepsilon_{t-1}^- = \varepsilon_{t-1}$ if $\varepsilon_{t-1} \leq 0$, and $\varepsilon_{t-1}^- = 0$ if $\varepsilon_{t-1} > 0$.

Although TGARCH model is intuitively easier to interpret, while the sign of conditional volatility is taken into account directly, the GJR-GARCH model is based on a similar principle. The square of standard deviation eliminates the sign of return, but the leverage effect caused by the sign of ε_t^2, is taken into account through the indicator function I.

According to Ali (2013), even though, the principle ideas of GJR-GARCH and TGARCH models are very similar, there might be significant differences in the results, depending on dataset distribution, skewness, and kurtosis.

2.3.5. Conclusion

Even though, today there are numerous variations of GARCH-type models, then considering the aim of this paper, it is out of the scope and unnecessary to discuss all of these models in more depth. Many recent studies, for example Kasibhatla (2005), Liu and Hung (2010), Curto and Pinto (2012) and Ou (2014), indicate that GARCH (1,1) EGARCH(1,1) and GJR-GARCH(1,1) models are proven to be most accurate in capturing the dynamics of financial assets volatility, as well as, used in volatility forecasting. Thus, these three abovementioned models will be used in this paper to forecast expected volatility.

2.4. Evaluation of the Forecasting Performance

2.4.1. Economic Loss Functions

Considering the large number of GARCH model variations, the need arises for the evaluation criterion, upon which the decision can be based on, to choose one model among

others. The need for evaluation methods for volatility models is the reason why the literature economic loss functions has been increasing in recent decades. Even though there are several loss functions proposed, then according to Lopez (2001), the economic loss functions proposed by West *et al.* (1993) and Engle *et al.* (1993) provided the most significant forecast evaluations since these directly took account the investor's decision structure into the evaluation process (Lopez 2001, 95). West *et al.* (1993) proposed a utility-based economic loss function and Engle *et al.* (1993) proposed an economic loss function, which required forecasted volatility, based on the expected profit from an investment decisions.

In addition to Lopez (2001) findings, Bollerslev *et al.* (1994) noted that, economic loss functions which particularly involve the costs confronted by forecasted volatility users offer the most significant forecast evaluations. Later on, numerous such loss functions, which are based on explicit economic problems, have been proposed in the literature[24].

2.4.2. Statistic Loss Functions

However, since explicit economic loss functions are often unavailable, volatility forecast evaluation is usually conducted by minimizing a statistical loss function, such as root mean squared error (RMSE), Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC) (Lopez 2001, 100).

The RMSE is scale-dependent accuracy measure, which is applicable while comparing different methods on the same dataset, but should not be used, when comparing datasets with different scales. The RMSE process can be described with equation 2.13 (Hyndman 2006):

$$RMSE = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(\hat{\sigma}_t - \sigma_t)^2} \qquad (2.13)$$

where the number of observations is denoted by N, forecasted and realized volatilities are denoted by $\hat{\sigma}_t$ and σ_t, respectively.

If the comparable models have the same number of parameters, it is possible to compare the maximum value of models' likelihood functions. In case, when the models have a different number of parameters, it is necessary to use AIC, which makes adjustments to the likelihood functions to account for the number of parameters (Reider 2009, 14). AIC is described in the equation 2.14 (Akaike 1974):

---

[24] For example, see Noh *et al.* (1994) and Engle *et al.* (1996).

$$AIC(P) = 2\ln(L) - 2P \qquad (2.14)$$

where maximized value of the likelihood function for the model is denoted by L; and the number of parameters in the model is denoted by P. This means that AIC gives a penalty of 2 for an additional parameter (Reider 2009, 15). Thus, it turns out that AIC is biased for small samples and might recommend a model which has a higher number of parameters (Chatfield 2000, 77).

Therefore, an alternative, BIC is widely used together with AIC. The BIC penalizes the addition of extra parameters more severely than AIC. The BIC calculation process is described in equation 2.15 (Priestley 1981)

$$BIC(P) = n * \ln\left(\widehat{\sigma_e^2}\right) + k * ln(n) \qquad (2.15)$$

where error variance is denoted by $\widehat{\sigma_e^2}$; and number of observations is denoted by n.

Even though, statistical loss functions are widely used in evaluation of the goodness-of-fit of volatility models[25], applying these is in some cases challenging. Mainly because of two reasons: firstly, some statistical loss functions (for example RMSE) use squared returns as a proxy for the latent volatility process (*Ibid.*). Andersen and Bollerslev (1997) reached an understating that, statistical loss functions which use squared returns as a proxy limit the accessible interpretation regarding the forecast accuracy. Secondly, while statistical loss functions require often realized volatility (or returns), then these loss functions can be only used for in-sample evaluations.

2.4.3. Choosing a Proxy for Statistical Loss Function

In order to measure the performance of statistical loss function, an appropriate proxy should be selected. Patton (2008) conducted a study which used different standard volatility proxies, such as squared returns, the intra-daily range and realized volatility. Study results indicated the goodness-of-fit of realized volatility.

The latter is also a reason, why in this thesis realized volatility will be used as a proxy for statistical loss functions. Realized volatility is easily computable, provides a natural proxy for forecast evaluation, and at the same time is very intuitive and easily interpretable. The calculating process for realized volatility is in equation 2.16 (Andersen *et al.* 2003):

---

[25] See Taylor (1987), Akgiray (1989), Dimson and Marsh (1990), Pagan and Schwert (1990), Lee (1991), West and Cho (1994), Kroner *et al.* (1995), Bollerslev and Ghysels (1996), Brailsford and Faff (1996).

$$RV^{(n)} = 100 * \sqrt{\frac{252}{n} \sum_{t=1}^{n} R_t^2} \qquad (2.16)$$

where realized volatility is denoted by RV; number of trading days in the measurement period is denoted by n; counter representing a trading day is denoted by t; continuously compounded daily returns are denoted by R_t. In addition, the formula makes an assumption that, there are 252 trading days in a year.

2.4.4. Other Considerations

Financial assets' volatility forecasts are often used for investment purposes, so there are several studies which proposed new economic loss functions based on maximizing trading profits or minimizing losses[26]. While in the empirical part of this thesis dynamic asset allocation models will be implemented, it would be possible to propose an economic loss function, which is based on dynamic asset allocation model. Although it would be very intuitive and easy to interpret, economic loss function might include itself influence to the evaluation. For instance, using VaR model as an economic loss function, it includes to the forecast evaluation VaR model's structure, confidence levels, and distribution function influences. Thus, the forecasted volatility evaluations might be biased.

Nevertheless of the shortcomings of statistical loss functions, these are well suited, easy to implement and widely used for in-sample forecasts evaluations. Considering the latter and the possible weaknesses of economic loss functions, statistical loss functions are used to evaluate forecasted volatility in Chapter 3[27].

---

[26] For example, Engle *et al.* (1993) introduced an economic loss function, which was based on profits made in a options market.

[27] As stated in the introduction, only in-sample volatility forecasts will be made in the empirical part of this thesis.

3. VOLATILITY MODELLING AND FORECASTING. AN EMPIRICAL ANALYSIS

3.1. Describing Data

The empirical analysis employs daily closing prices for four financial instruments: S&P 500 Index, STOXX Europe 600 Index, U.S. Treasury Future, and Euro-Bund Future. The observation period is from 01/01/2000 to 30/04/2015. While conditional variance forecasts are made with 5 year rolling window, then forecast period is from 01/01/2005 to 30/04/2015. The information of the instruments used is shown in the table 3.1. The data are retrieved from the Bloomberg database.

Table 3.1 Financial Instruments

Symbol	Bloomberg Ticker	Instrument Name	Representative Asset Class	Region
USE	SPX INDEX	S&P 500 Index	Equity	U.S.
EUE	SXXP INDEX	STOXX Europe 600 Index	Equity	Europe
USF	US1 COMB COMDTY	U.S. 10Y Treasury Future	Fixed Income	U.S.
GERF	RX1 COMDTY	Euro-Bund Future	Fixed Income	Europe

Source: Bloomberg (2015); compiled by the author

These financial instruments were chosen because they are well known, widely traded and represent relatively well these regions asset classes. In addition, historical daily price quotes for these instruments were available for more than 15 years. The latter makes these financial instruments attractive research subjects, because two major financial crises occurred during the observation period.

Figures 3.1, 3.2, 3.3 and 3.4 illustrate daily price and return movements of different observed financial instruments from 01/01/2000 to 30/04/2015. The return for each financial instrument is calculated as the percent logarithmic difference in daily close prices, the calculation process is given in equation 3.1:

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right) * 100 \tag{3.1}$$

where R_t and P_t stand for the market return and asset price for each day, respectively. Returns for USE, EUE, USF and GERF are denoted with symbols DUSE, DEUE, DUSF and DGERF, respectively.
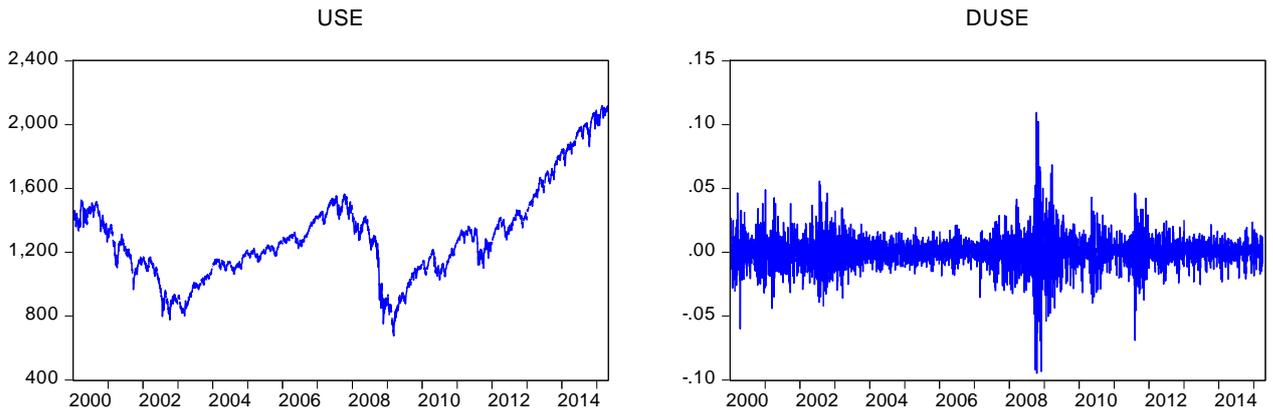
Figure 3.1 Daily prices and returns of USE from 01/01/2000 to 30/04/2015
Source: Bloomberg; compiled by the author
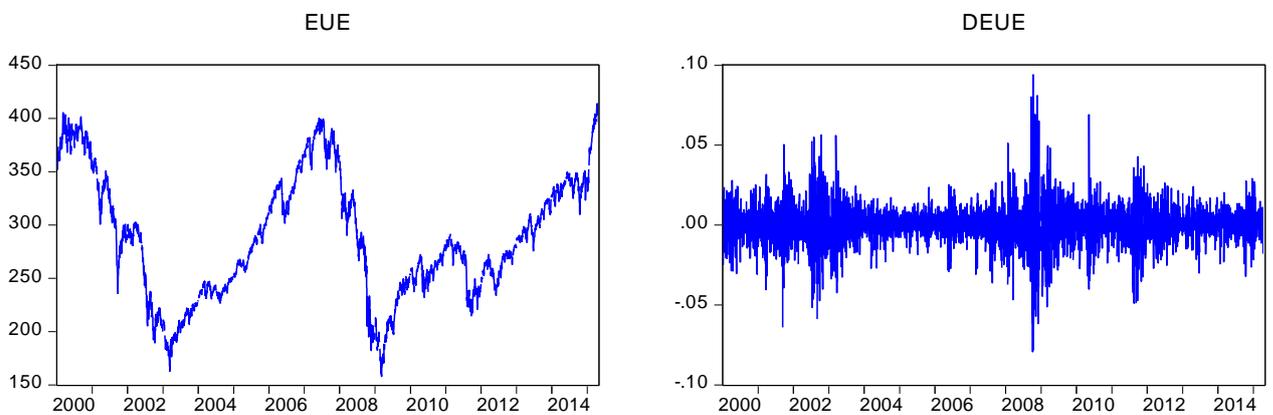
Figure 3.2 Daily prices and return of EUE from 01/01/2000 to 30/04/2015
Source: Bloomberg; compiled by the author

Evident from the figures 3.1 and 3.2, it appears that USE and EUE have behaved during observation period similarly. Both indexes experienced a fall in price levels and increase in returns volatilities during financial crises (during 2000-2002 and 2007-2009).
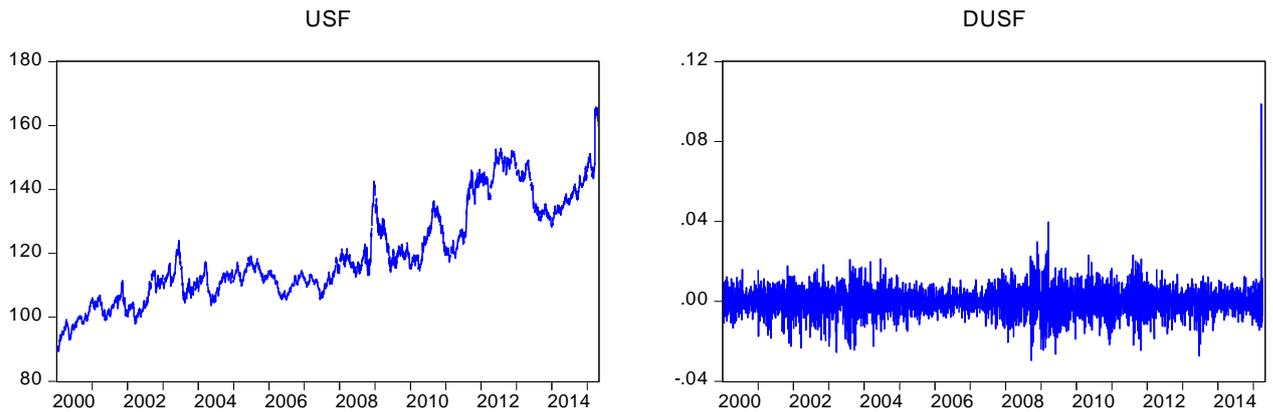
Figure 3.3 Daily prices and returns of USF from 01/01/2000 to 30/04/2015
Source: Bloomberg; compiled by the author
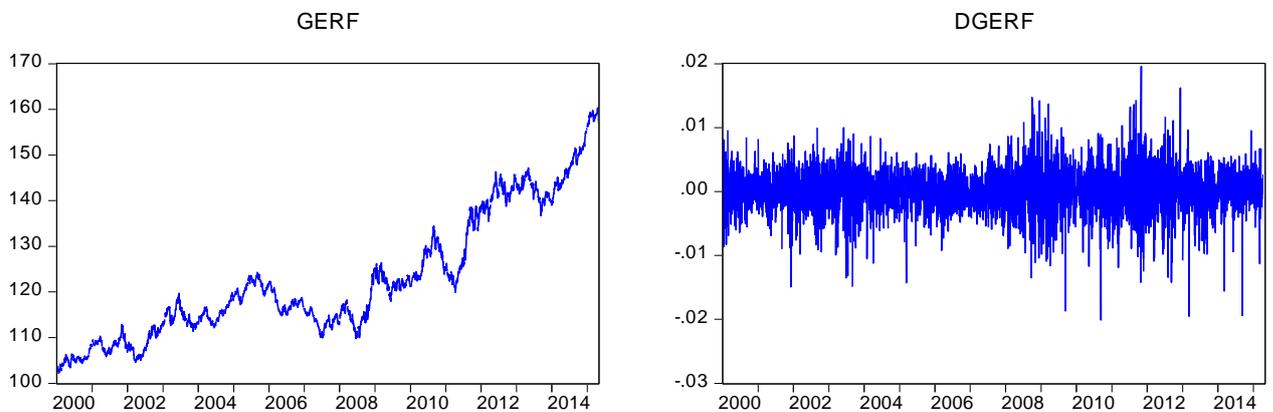
Figure 3.4 Daily prices and returns of GERF from 01/01/2000 to 30/04/2015
Source: Bloomberg; compiled by the author

From figures 3.3 and 3.4 appears that fixed income prices tend to move in same trend during the observation period, but is worth to notify that GERF returns are less volatile (see return movements amplitudes) and USF returns.

Descriptive statistics of the selected assets are presented in table 3.2. Mean and median returns for all instruments are close to zero. Standard deviations for equity returns, DUSE (0.127) and DEUE (0.127), are higher than for fixed income returns, DUSF (0.007) and DGERF (0.004). DUSE. DEUE and DGERF returns have negative skewness, which is normal for financial instrument returns[28]. The excess kurtosis statistic refers to a departure from normal distribution, that is, all series are highly leptokurtic. The latter means that the 1.96 of standard deviation of the mean is less than 95%.

---

[28] According to Ali (2013), negative skewness is a feature of many financial instrument returns.

Table 3.2 Descriptive statistics of the daily prices and returns from 01/01/2000 to 30/04/2015

Statistic \ Symbol	USE	EUE	USF	GERF	DUSE	DEUE	DUSF	DGERF
Mean	1299.521	288.409	119.750	122.811	0.00010	0.00002	0.00015	0.00011
Median	1263.510	283.000	115.688	118.590	0.00050	0.00050	0.00030	0.00020
St.deviation	290.265	57.677	15.112	13.828	0.01273	0.01263	0.00670	0.00352
Kurtosis	0.545	-0.864	-0.472	-0.366	8.111	5.289	13.328	2.527
Skewness	0.814	0.158	0.603	0.799	-0.171	-0.105	0.604	-0.306
Range	1441.160	256.090	76.594	58.300	0.204	0.173	0.128	0.040
Minimum	676.530	157.970	89.219	102.060	-0.095	-0.079	-0.030	-0.020
Maximum	2117.690	414.060	165.813	160.360	0.110	0.094	0.099	0.020
Count	3847	3847	3847	3847	3846	3846	3846	3846

Source: Compiled by the author

Although daily price level figures (see figures 3.1, 3.2, 3.3, 3.4) indicated that the financial instruments price levels are not stationary, and have a unit root, then daily return movements were varying around zero, thus it cannot be concluded with certainty whether or not returns are stationary. In order to test the stationary of return movements Augmented Dickey-Fuller (ADF) test is conducted. The latter is an extension to the stationary test proposed by Dickey and Fuller (1979). The ADF test results (see table 3.3), reveal that the t-statistics for all the series are highly negative. Thus, the t-statistic values exceed critical values and the null hypothesis, the data has a unit root, can be rejected. It can be concluded with the significance level of 1% that the data are stationary.

Table 3.3 Augmented Dickey-Fuller (ADF) results

Symbol	t-statistic	Prob.	1% level	5% level
DUSE	-46.906	0.0001	-3.432	-2.862
DEUE	-44.490	0.0000		
DUSF	-62.203	0.0001		
DGERF	-43.369	0.0000		

Source: Compiled by the author

In order to test for data nonlinearity, Brock, Dechert, Scheinkman (BDS) test is used. The BDS test was proposed by Brock, Dechert, Scheinkman (1987) to indicate noisiness of the data and the suitability for models. The BDS test results are presented in table 3.4.

Table 3.4 Brock, Dechert, Scheinkma (BDS) test results

Symbol	Dimension 2				Dimension 3			
	statistic	std. error	z-statistic	prob.	statistic	std. error	z-statistic	prob.
DUSE	0.022	0.002	14.462	0.000	0.050	0.002	20.369	0.000
DEUE	0.023	0.001	15.499	0.000	0.049	0.002	20.920	0.000
DUSF	0.011	0.001	8.874	0.000	0.020	0.002	10.088	0.000
DGERF	0.013	0.001	10.545	0.000	0.025	0.002	12.578	0.000

Source: Compiled by the author

BDS test results indicate that the test-statistics for the standardized residuals are highly significant for each time series in both dimensions. Thus the null hypothesis, the remaining residuals are identically distributed and independent, can be accepted. This means the data are nonlinear.

In order to fit the data for GARCH models, ARCH effects in the residuals should be present. Under the ARCH effect autocorrelation in the squared errors is considered. With the purpose of determine the presence of ARCH effects, a heteroskedasticity ARCH test should be conducted on the ARMA(1,1) model. The ARMA(1,1) model, first proposed by Whittle (1951), can be described by the equation 3.2:

$$\varphi(L)X_t = \mu + \theta(L)\epsilon_t \qquad (3.2)$$

where lag operator is denoted by (L), observable variable is denoted by X_t, a constant term is denoted by μ, weak white noise disturbance term is denoted by ϵ_t. If heteroskedasticity ARCH test F-statistic for ARMA(1,1) model is significant, a conclusion can be drawn that ARCH effects are present in the lags of the squared residuals. Heteroskedasticity ARCH test results are presented in table 3.5.

Table 3.5 Heteroskedasticity ARCH test results.

Symbol	F-statistic	Prob. F(2,3991)
DUSE	415.2176	0.000
DEUE	263.920	0.000
DUSF	0.959	0.383
DGERF	35.904	0.000

Source: Compiled by the author

Heteroskedasticity ARCH test results indicate that ARCH effects are present for DUSE, DEUD and DGERF time-series, and not present in DUSF. The latter indicates that ARCH-type models would be not suitable for DUSF.

3.2. Competing Volatility Modeling Methods

In section 3.3 EWMA, GARCH(1,1), EGARCH(1,1) and GJR-GARCH(1,1) models (see equations 2.4, 2.8, 2.09 and 2.11, respectively) will be used for modeling the conditional variance of DUSE, DEUE, DUSF and DGERF[29].

The choice turned out to be in favor of GARCH(1,1), EGARCH(1,1) and GJR-GARCH(1,1) models, because based on Kasibhatla (2005), Liu and Hung (2010), Curto and Pinto (2012) and Ou (2014) empirical studies, these models are most accurate in capturing the dynamics of financial instruments returns and are most precise in conditional variance (volatility) forecasting process.

Although, heteroskedasticity ARCH test results for DUSF indicated that ARCH-type models might not be appropriate, selected three GARCH models will be still used for modeling the conditional variance of DUSF. The latter will be done for further comparison purposes. While the descriptive statistics (presented in section 3.1 table 3.2), indicated that DUSE, DEUE, DUSF and DGERF distributions are leptokurtic, then in addition to normal (*Gaussian*) distribution *Student's t* distribution is used.

The naïve model, EWMA, was selected to identify if GARCH volatility modeling methods are more precise and accurate than naïve models. The EWMA was chosen instead of SMA, because EWMA uses a decay factor λ to assign adjustable weight to historical observations.

The estimated parameters of the GARCH(1,1), EGARCH(1,1), GJR-GARCH(1,1) models for DUSE, DEUE, DUSF and DGERF from 01/01/2000 to 31/12/2004 are presented in tables 3.6, 3.7, 3.8 and 3.9.

---

[29] See Matlab code in Appendix 3.

Table 3.6 Estimated parameters and evaluation of GARCH(1,1), EGARCH(1,1), GJR-GARCH(1,1) models for DUSE from 01/01/2000 to 31/12/2004

Symbol	Distribution	Model	Const.	GARCH	ARCH	Leverage	Evaluation	
			ω	α	β	γ	AIC	BIC
DUSE	Gaussian	GARCH(1,1)	1.046e-06	0.920	0.073		-7.5889	-7.5735
		St.Error	(1.080e-06)	(0.013)	(0.012)			
		EGARCH(1,1)	-0.124	0.986	0.072	-0.109	**-7.6544**	**-7.6339**
		St.Error	(0.031)	(0.003)	(0.018)	(0.013)		
		GJR(1,1)	1.179e-06	0.928		0.126	-7.6468	-7.6314
		St.Error	(9.912e-07)	(0.013)		(0.019)		
	Student t	GARCH(1,1)	1.667e-06	0.911	0.079		-7.5907	-7.5702
		St.Error	(1.337e-06)	(0.017)	(0.016)			
		EGARCH(1,1)	-0.123	0.986	0.069	-0.109	-7.6532	-7.6276
		St.Error	(0.032)	(0.004)	(0.021)	(0.014)		
		GJR(1,1)	1.177e-06	0.929		0.125	-7.6457	-7.6252
		St.Error	(1.00e-06)	(0.014)		(0.021)		

Source: Compiled by the author

Table 3.7 Estimated parameters and evaluation of GARCH(1,1), EGARCH(1,1), GJR-GARCH(1,1) models for DEUE from 01/01/2000 to 31/12/2004.

Symbol	Distribution	Model	Const.	GARCH	ARCH	Leverage	Evaluation	
			ω	α	β	γ	AIC	BIC
DEUE	Gaussian	GARCH(1,1)	1.767e-06	0.890	0.099		-7.5878	-7.5724
		St.Error	(1.098e-06)	(0.014)	(0.013)			
		EGARCH(1,1)	-0.148	0.984	0.110	-0.118	**-7.6354**	-7.6149
		St.Error	(0.029)	(0.003)	(0.020)	(0.015)		
		GJR(1,1)	1.864e-06	0.907		0.154	**-7.6354**	**-7.62**
		St.Error	(8.743e-07)	(0.013)		(0.022)		
	Student t	GARCH(1,1)	1.635e-06	0.899	0.093		-7.5889	-7.5684
		St.Error	(1.219e-06)	(0.017)	(0.016)			
		EGARCH(1,1)	-0.139	0.985	0.108	-0.119	-7.6346	-7.609
		St.Error	(0.034)	(0.003)	(0.023)	(0.016)		
		GJR(1,1)	1.781e-06	0.908		0.153	-7.634	-7.6135
		St.Error	(8.924e-07)	(0.014)		(0.024)		

Source: Compiled by the author

Table 3.8 Estimated parameters and evaluation of GARCH(1,1), EGARCH(1,1), GJR-GARCH(1,1) models for DUSF from 01/01/2000 to 31/12/2004.

Symbol	Distribution	Model	Const.	GARCH	ARCH	Leverage	Evaluation	
			Ω	α	β	γ	AIC	BIC
DUSF	Gaussian	GARCH(1,1)	4.693e-07	0.958	0.030		-8.9825	-8.9671
		St.Error	(3.824e-07)	(0.006)	(0.007)			
		EGARCH(1,1)	-0.209	0.978	0.076	-0.015	-8.9796	-8.959
		St.Error	(0.087)	(0.008)	(0.021)	(0.011)		
		GJR(1,1)	5.957e-07	0.955	0.023	0.013	-8.981	-8.9605
		St.Error	(4.050e-07)	(0.007)	(0.011	(0.012)		
	Student t	GARCH(1,1)	2.227e-06	0.9	0.05		-8.9959	**-8.9754**
		St.Error	(1.145e-06)	(0.024)	(0.015)			
		EGARCH(1,1)	-0.182	0.981	0.080	-0.010	-8.9993	-8.9736
		St.Error	(0.105)	(0.010)	(0.027)	(0.015)		
		GJR(1,1)	5.219e-07	0.955	0.030	0.005	**-9.0007**	**-8.9754**
		St.Error	(4.701e-07)	(0.010)	(0.017	(0.017)		

Source: Compiled by the author

Table 3.9 Estimated parameters and evaluation of GARCH(1,1), EGARCH(1,1), GJR-GARCH(1,1) models for DGERF from 01/01/2000 to 31/12/2004.

Symbol	Distribution	Model	Const.	GARCH	ARCH	Leverage	Evaluation	
			ω	α	β	γ	AIC	BIC
DGERF	Gaussian	GARCH(1,1)	2,00E-07	0.942	0.037		-1.084	-1.0825
		St.Error	(1.629e-07)	(0.006)	(0.007)			
		EGARCH(1,1)	-0.065	0.994	0.071	0.019	-1.0849	-1.0829
		St.Error	(0.043)	(0.003)	(0.013)	(0.010)		
		GJR(1,1)	2,00E-07	0.942	0.040	-0.005	-1.0838	-1.0818
		St.Error	(1.637e-07)	(0.007)	(0.007)	(0.015)		
	Student t	GARCH(1,1)	2,00E-07	0.942	0.037		-1.0864	-1.0843
		St.Error	(1.803e-07)	(0.010)	(0.010)			
		EGARCH(1,1)	-0.072	0.993	0.075	0.019	**-1.0871**	**-1.0845**
		St.Error	(0.062)	(0.005)	(0.020)	(0.013)		
		GJR(1,1)	2,00E-07	0.941	0.043	-0.008	-1.0862	-1.0836
		St.Error	(1.820e-07)	(0.010)	(0.018)	(0.020)		

Source: Compiled by the author

From tables 3.6 and 3.7 appears that instead of GJR-GARCH(1,1) models there are GJR-GARCH(1,0) models, because ARCH lag parameter β are not estimated. Although in

model specifications ARCH lag were set to $\beta = 1$, parameters were still excluded, because these were too close to zero[30]. It does not mean, that GJR-GARCH(1,0) models will be used for DUSE and DEUE forecasts, because during the forecasting period ARCH lag coefficient may increase and parameter might enough significant to be included into the model.

According to AIC and BIC (lowest coefficients are in bold) it appears that best model for DUSE from 01/01/2000 to 31/12/2004 is EGARCH(1,1) with normal distribution. For DEUE, DUSF and DGERF best models are GJR-GARCH(1,0) with normal distribution, GJR-GARCH(1,1) with *Student t* distribution, and EGARCH(1,1) with *Student t* distribution, respectively. It is worth to notify that these are the best models in that certain sample period. Best models for the entire observation period might be different.

3.3. Evaluation of the Forecasted Volatility

In this section forecasted conditional volatility will be evaluated according to RMSE criteria and correlation coefficient. One-day ahead conditional volatility forecasts of DUSE, DEUE, DUSF and DGERF were calculated by using GARCH(1,1), EGARCH(1,1) and GJR-GARCH(1,1) models under *Gaussian* and *Student t* distributions[31], and with EWMA naïve technique. The forecast will be dynamic with 5 year rolling window sample (1245 trading days). Each of the 24 initial GARCH type models (presented in Section 3.2) coefficients will be calibrated during forecasting process 2601 times and 2602 one-day ahead forecasts will be made. See volatility forecasts in appendixes 5-8.

The forecasts will be evaluated according to RMSE criteria and correlation coefficient. Realized volatility is used as a proxy in RMSE calculations. Evaluations are based on RMSE criteria, but if RMSE criterions of different models are equal, then correlation coefficients will be also considered.

The RMSE and correlation coefficients of volatility forecasts are presented in table 3.10. Most accurate and precise models and respective model RMSE and correlation coefficient are presented in bold text. Initially selected best models are presented in red text.

---

[30] GARCH and ARCH lags are related with an underlying lag operator polynomial, thus a near-zero tolerance exclusion test is conducted in parameters estimation process. If GARCH or ARCH lags coefficients magnitudes are equal or less than 1e-12 then these will be excluded. (The MathWorks, Inc 2015)

[31] See Matlab code in Appendix 4.

The results indicate that the most accurate and precise models for DUSE and DEUE is GARCH(1,1) under normal distribution (initially the best models were EGARCH(1,1) and GJR-GARCH(1,1), respectively, under normal distributions. Most suitable model for DUSF is GJR-GARCH(1,1) under normal distribution (initially the best model was GJR-GARCH(1,1) under *Student t* distribution), and for DGERF is EWMA technique (initially the best models was EGARCH(1,1) under *Student t* distribution). One may assume that this is caused by the short, one day ahead, forecasting period, because several studies[32], which found either EGARCH or GJR-GARCH models more precise than the ordinary GARCH, were using longer forecast periods.

Although it is worth to notify, that in many cases volatility forecasts RMSE-s differ not much or are equal, which means the accuracy and preciseness of some models is very similar during the observation period.

---

[32] For example Marcucci (2005) studied stock market volatility at horizons that range from one day to one month, and found that at forecast horizons longer than one week asymmetric GARCH models tend to be superior.

Table 3.10 RMSE and correlation of volatility forecasts

Symbol	Distribution	Model	RMSE	Correlation
DUSE	Gaussian	EWMA	0.0028	0.9498
		GARCH(1,1)	**0.0023**	**0.9602**
		EGARCH(1,1)	0.0031	0.9339
		GJR(1,1)	0.0024	0.9562
	Student t	GARCH(1,1)	**0.0023**	0.9598
		EGARCH(1,1)	0.0030	0.9297
		GJR(1,1)	0.0024	0.9546
DEUE	Gaussian	EWMA	0.0031	0.9174
		GARCH(1,1)	**0.0021**	**0.9583**
		EGARCH(1,1)	0.0032	0.8944
		GJR(1,1)	0.0025	0.9302
	Student t	GARCH(1,1)	**0.0021**	0.9581
		EGARCH(1,1)	0.0032	0.8904
		GJR(1,1)	**0.0021**	0.9580
DUSF	Gaussian	EWMA	**0.0018**	0.8441
		GARCH(1,1)	**0.0016**	0.8481
		EGARCH(1,1)	0.0020	0.7245
		GJR(1,1)	**0.0016**	**0.8482**
	Student t	GARCH(1,1)	0.0017	0.8382
		EGARCH(1,1)	0.0019	0.7701
		GJR(1,1)	**0.0016**	0.8336
DGERF	Gaussian	**EWMA**	**0.0009**	**0.8426**
		GARCH(1,1)	**0.0009**	0.8174
		EGARCH(1,1)	0.0011	0.6507
		GJR(1,1)	**0.0009**	0.7614
	Student t	GARCH(1,1)	**0.0009**	0.8176
		EGARCH(1,1)	0.0010	0.7079
		GJR(1,1)	**0.0009**	0.7840

Source: Compiled by the author

3.4. Discussion of the Results

3.4.1. Interpretation

The evaluation of the conditional volatility forecasts indicated that initially chosen models were not the most accurate and precise for volatility forecasting during the observation period. Although the RMSE-s of different GARCH type models were in some cases very similar, or even equal, RMSE-s did not indicate initially chosen models, except for DUSF.

In addition, it appeared that while considering RMSE as the only evaluation criteria then GARCH(1,1) model under normal distribution suits best for all observed assets. Even though many previous empirical studies have proved the superiority of EGARCH(1,1) and GJR-GARCH(1,1) models, GARCH(1,1) volatility forecasts seems to be more accurate. Moreover, in contrast to previous empirical studies, EGARCH(1,1) under *Gaussian* and *Student t* distributions was most inaccurate and imprecise for volatility forecasting.

Although ARCH effects were not present in DUSF, GARCH(1,1) and GJR(1,1) models under both distributions provided more precise volatility forecasts than EWMA. This indicates that even if ARCH effects are not present, ARCH/GARCH type models might provide more precise volatility forecasts than EWMA. Nevertheless, it is not correct to draw any conclusions based on the latter, because EWMA does not consider ARCH effects at all, and it can be assumed, that some other type modeling methods would give more accurate volatility forecasts than GARCH type models.

Interestingly, the result indicated that for DGERF all used forecasting methods, except EGARCH(1,1), provided same precise results. Although, the correlation coefficient between forecasted and realized volatility was highest on EWMA technique, any solid conclusions cannot be drawn based on correlation coefficients.

Considering the purpose and first research question of this thesis, it can be concluded, based on the conducted empirical analysis, that GARCH volatility modeling methods are equal or more accurate and precise than naïve volatility modeling techniques. Even if ARCH effects are not present, GARCH models tend to outperform naïve techniques.

3.4.2. Limitations and Suggestions for Further Research

Although GARCH type models possess accurate and precise forecasting qualities there are some limitations. First, long-term forecast of conditional variance will converge to

the unconditional mean variance, thus static long-term forecasts are imprecise. Secondly, GARCH type models require long time-series of data in order to be trustworthy. The latter might be a problem, if some new financial instruments are considered where long-term historical data does not exists.

Further research is required on different GARCH type models for evaluating the accuracy and preciseness of conditional volatility forecasts. Although many previous empirical studies have proven EGARCH and GJR-GARCH models to be more precise than the ordinary GARCH, this empirical analysis, in general, indicated the ordinary GARCH model as the most precise, and EGARCH model as the most imprecise. Before making any solid conclusions, further research should be made with longer time series and different financial instruments.

In addition, from volatility forecasting perspective, it might increase forecasting ability, if indicators which reflect confidence in economy are included to the model and considered as well, for example different leading indicators. Moreover, it is necessary to evaluate, if some other volatility modeling methods provide better results. For instance, models based on stochastic volatility theory, or nonparametric methods for volatility density estimation.

4. DYNAMIC ASSET ALLOCATION. AN EMPIRICAL ANALYSIS

In this chapter, an empirical analysis is conducted in order to compare investment portfolios based on dynamic and static asset allocation strategies. The data presented in Section 3.1 will be used. Although, the evaluation of forecasted volatility (see Section 3.3) indicated that, in within this specific sample and time horizon, in general, GARCH(1,1) models are most precise for volatility forecasting, the models chosen out in Section 3.2 will be used (for DUSE EGARCH(1,1) with normal distribution, for DEUE GJR-GARCH(1,1) with normal distribution, for DUSF GJR-GARCH(1,1) with *Student t* distribution, for DGERF EGARCH(1,1) with *Student t* distribution). The latter is because portfolio simulation is from 1/01/2005 to 30/04/2015 and the information about future is not available, thus the evaluation of forecasted volatility should not be considered.

While transaction costs are often dependent on negotiated terms and counterparties, these are chosen randomly, in general, for institutional investors transaction costs are below 0.5% of the traded volume, thus the chosen transaction costs are following: 0%, 0.25%, and 0.5% of trading volume.

4.1. Competing Portfolios

In order to compare portfolio based on dynamic and static strategies, suppose there are launched two dynamic and two static portfolios, for both strategies one with U.S. assets and one with European assets, with an inception value of 100. The initial asset classes' weights are 50% equity and 50% fixed income. If necessary, assets are reallocated with daily intervals, either to maintain the initial asset allocation for static portfolio, or based on the expected shortfall probability for each asset class separately. In order to minimize trading costs, asset allocation changes are applied only if they exceed at least 5% of portfolio's value.

While several empirical studies, for example Herold *et al.* (2005) and Herold *et al.* (2007), have indicated the superiority of shortfall risk based-strategies, compared to other

dynamic asset allocation strategies, the expected shortfall probability method will be implemented.

As described in the Chapter 1, the covariance between different assets is not constant and using false covariance might deflect results. Thus, in order not to deflect results and simplify the further empirical analysis, the expected shortfall risk probability is calculated for each asset class separately (by using equation 1.6). Dynamic portfolios' asset allocation weights are allowed to decrease from 0% to 100%, below 0% allocation, short selling[33], is not allowed.

In addition to equities and fixed income, holding cash is included as an option for asset allocation. Cash is considered as non-interest bearing asset without any holding risks.

Even tough, the expected shortfall probability calculations are based on assumption that the data are normally distributed with skewness 0 and kurtosis 3, the adjustment of integration equitation 1.6 for expected shortfall probability would be out of the scope of this thesis.

Expected shortfall risk probability calculations require a minimum acceptable return, and in order to take risk and invest not into a risk-free asset class, some risk has to be taken, thus the minimum acceptable return is set to -1%. This means, the return -1% or above is required each day at a 95% confidence level. The maximum allowed expected shortfall risk probability for asset allocation decisions is set to 5%, which means probability of producing a return below -1% must not exceed 5%.

4.2. Results

4.2.1. U.S. Dataset

It appears from visual assessment that dynamic asset allocation weights changed remarkably during 07/2007 to 07/2009, and 07/2011 to 01/2012. During these periods the expected shortfall risk probability of equities increased and assets were reallocated to fixed income, which appeared to be with lower expected shortfall risk probability. Although, during these periods for a short time the expected shortfall risk probability of fixed income exceeded allowable limit, and around half of the portfolio's assets were held in cash, see Figure 4.1.

---

[33] Selling assets not owned by the seller (for example borrowed), in purpose to buy assets back in future at lower price.
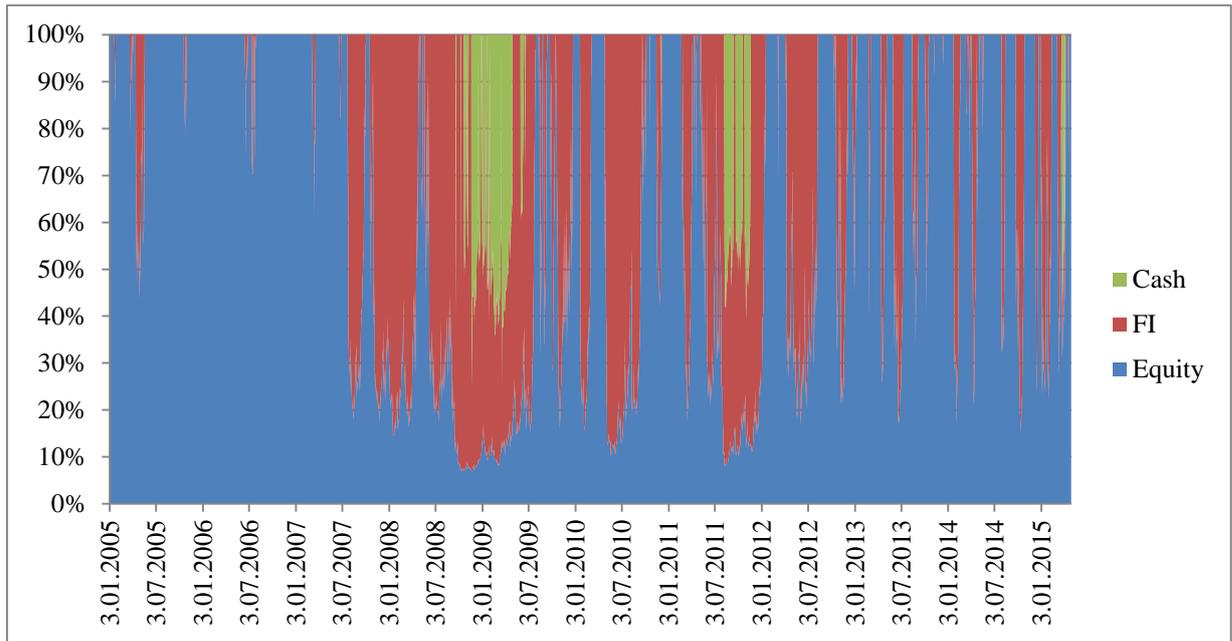
Figure 4.1 Asset allocation weights of the dynamic portfolio from 1/01/2005 to 30/04/2015
Source: Compiled by the author

The performance of dynamic and static portfolios, without trading costs, is presented in Figure 4.2. It appears from Figure 4.2 that most of the time dynamic portfolio's value was higher than static portfolio's value. In order to compare portfolios performance during different periods the relative performance[34] was calculated, presented in Figure 4.3. It appears there is an upward trend in relative performance, which indicates that dynamic portfolio is outperforming static portfolio in long-term period. Although, in some shorter periods, for instance the second half of 2009, static portfolio outperformed dynamic portfolio.

The performance and relative performance figures for portfolios with transaction costs are presented in Appendixes 9-12.

---

[34] Relative performance represents dynamic portfolio's performance over static portfolio's performance (dynamic portfolio's value is divided by static portfolio's value).
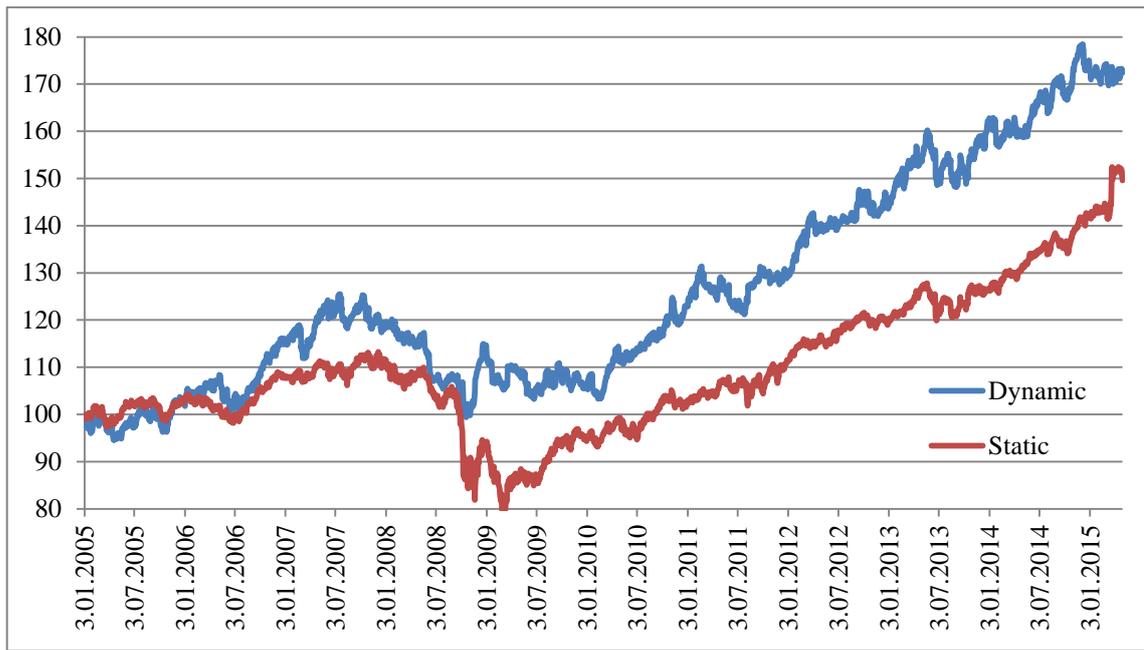
Figure 4.2 Dynamic and static portfolios performance from 1/01/2005 to 30/04/2015

Source: Compiled by the author

Figure 4.3 Dynamic and static portfolios relative performance from 1/01/2005 to 30/04/2015

Source: Compiled by the author

Table 4.1 presents portfolio values at the end, compound annual growth rates (CAGR)[35], and standard deviations of portfolios' returns. It appears that portfolios with 0% and 0.25% of transaction costs reached to higher absolute value, therefore CAGR is higher. If transaction costs are 0.5% or higher then dynamic portfolio's value is lower. Despite of the differences in portfolios' values and CAGR's, portfolios' standard deviations are all equal.

Table 4.1 Dynamic and static portfolios statistics from 1/01/2005 to 30/04/2015

Strategy	Portfolio Value	CAGR	Standard Deviation	Min. Return	Max. Return
Dynamic (0%)	170.6	5.49%	0.006	-3.59%	3.68%
Static (0%)	149.6	4.11%	0.006	-5.09%	4.74%
Dynamic (0.25%)	151.0	4.21%	0.006	-3.59%	3.66%
Static (0.25%)	144.4	3.74%	0.006	-5.11%	4.73%
Dynamic (0.5%)	133.7	2.95%	0.006	-3.59%	3.64%
Static (0.5%)	139.3	3.37%	0.006	-5.12%	4.72%

Source: Compiled by the author

4.2.1. European Dataset

The European dataset results are presented in Figures 4.4, 4.5, 4.6 and Table 4.2. Figure 4.4 indicates that assets were never held in cash, asset allocation was fluctuating between equities and fixed income. During 2008 to 2009 and 2011 to 2012 most of the assets were allocated to fixed income, rest of the observation period to equities. From Figure 4.5 appears that the dynamic portfolio's value was during the observation period constantly higher than static portfolio's value.

Relative performance reveals that dynamic strategy outperformed static asset allocation from 2005 to the end of first half of 2009. Later on, dynamic strategy has been underperforming compared to the static portfolio.

---

[35] The compound annual growth rate (CAGR) calculation process can be described with following equation:

$$CAGR = \frac{X_{tn}^{\frac{1}{tn-t0}}}{X_{t0}} - 1$$

where, last observation value is denoted by X_{tn}, first observation value is denoted by X_{to}, last year of observation is denoted by tn, first year of observation is denoted by $t0$.
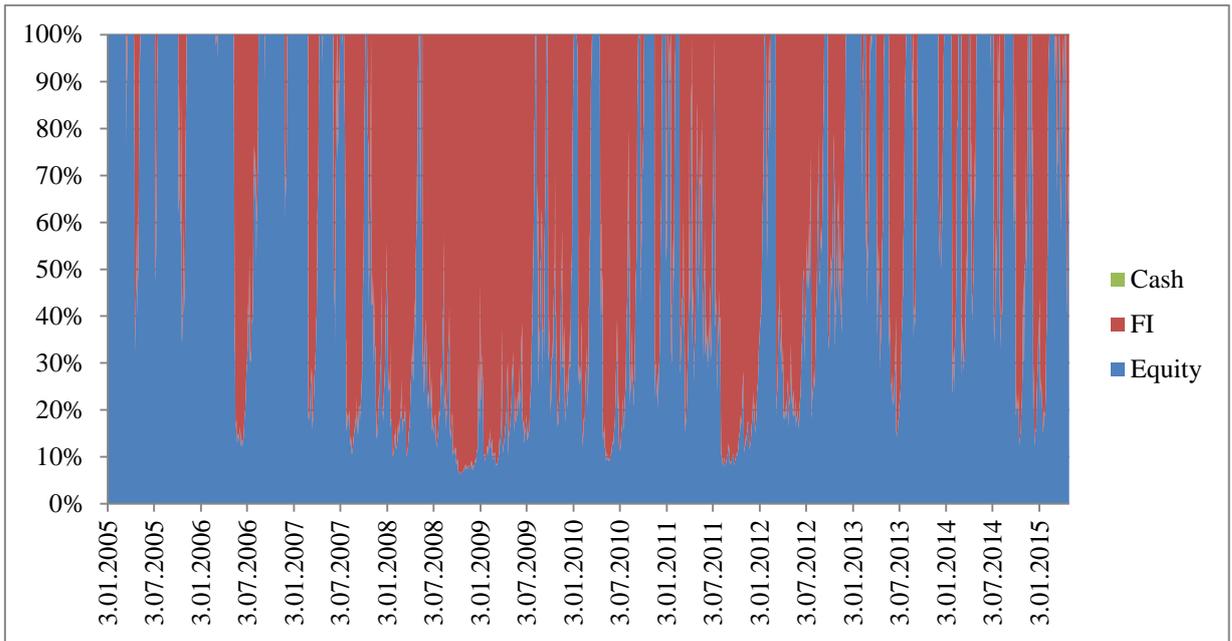
Figure 4.4 Asset allocation weights of the dynamic portfolio from 1/01/2005 to 30/04/2015

Source: Compiled by the author
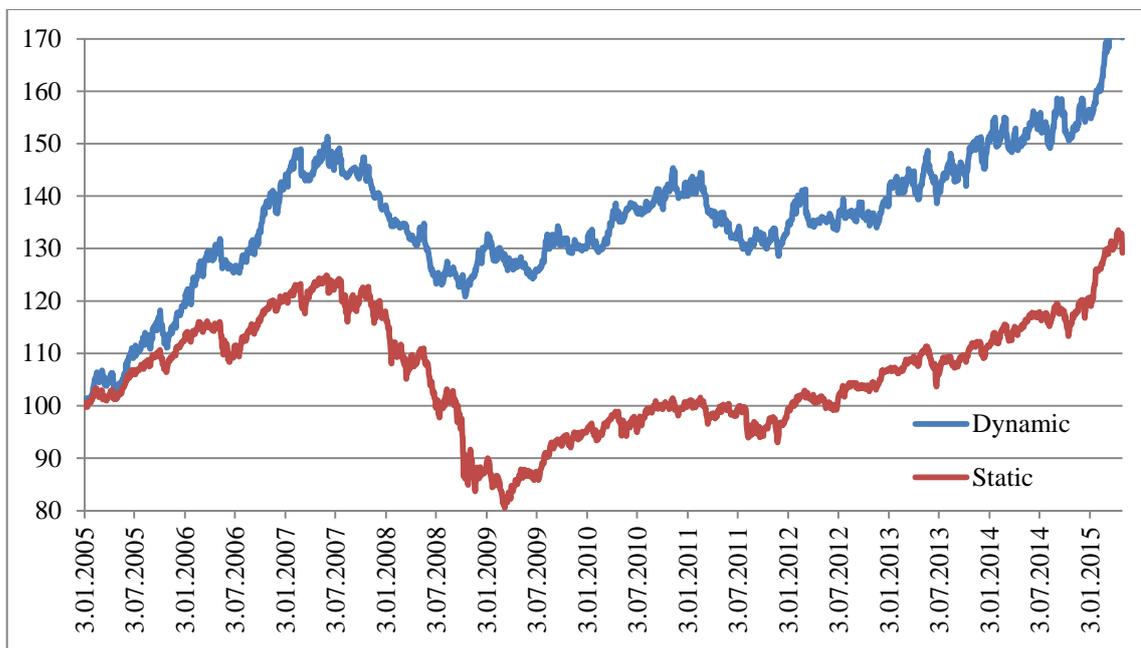
Figure 4.5 Dynamic and static portfolios performance from 1/01/2005 to 30/04/2015

Source: Compiled by the author

Figure 4.6 Dynamic and static portfolios relative performance from 1/01/2005 to 30/04/2015

Source: Compiled by the authors

Portfolio based on dynamic strategy outperformed static portfolio with all implemented transaction costs. In addition, it appears that while increase in transaction costs decreases dynamic portfolio's value, then static portfolio value is nearly not changing. Standard deviations of returns are all equal for both strategies with different transaction costs.

Table 4.2 Dynamic and static portfolios statistics from 1/01/2005 to 30/04/2015

Strategy	Portfolio Value	CAGR	Standard Deviation	Min. Return	Max. Return
Dynamic (0%)	169.2	5.40%	0.005	-3.13%	2.02%
Static (0%)	129.1	2.59%	0.005	-4.01%	3.63%
Dynamic (0.25%)	149.0	4.07%	0.005	-3.13%	2.02%
Static (0.25%)	129.1	2.59%	0.005	-4.02%	3.65%
Dynamic (0.5%)	131.2	2.75%	0.005	-3.13%	2.02%
Static (0.5%)	129.1	2.59%	0.005	-4.03%	3.66%

Source: Compiled by the author

4.3. Discussion of the Results

4.3.1. Interpretation

The purpose of the empirical analysis is to clarify whether an investment portfolio based on shortfall risk-based strategy can outperform portfolio based on static strategy during the observation period, or not. In addition to latter, to evaluate if dynamic portfolio can limit more efficiently short-term losses than static portfolio. One may find that the dynamic strategy meets partially the expectations, but is dependent on the transaction costs.

If transaction costs are set to 0% or 0.25%, then dynamic portfolio outperformed static portfolio with both datasets (the CAGR is without transaction costs with U.S. and European datasets 1.38% and 2.81% higher, respectively). While on dynamic portfolio there is more reallocation between asset classes than on static portfolio[36], dynamic portfolio is highly sensitive to transaction costs. As transaction costs increases, dynamic portfolios' absolute value and CAGR decrease significantly.

Although, if during the observation period transaction costs are 0% or 0.25%, it appears that dynamic portfolio outperforms static portfolio, it is necessary to notify that in shorter periods it is not always outperforming. With U.S. dataset, there are several short periods when dynamic portfolio underperforms static portfolio, for instance during the second half of 2009. Nevertheless, during most of the observation period it seems like dynamic portfolio is slightly outperforming static portfolio. In addition, during the financial crisis of 2008 to 2009 the value of dynamic portfolio decreased -21%, while the value of static portfolio decreased -28%. In addition, it appears that the minimum return of dynamic portfolio is higher than on static portfolio (without transaction costs -3.59% compared to -5.09%). Therefore, one may anticipate that, based on the U.S. dataset, shortfall risk-based strategy can limit short-term and extreme losses more efficiently than portfolio based on static asset allocation.

With European dataset, it appears that during the first five years, from 2005 to 2009, dynamic portfolio clearly outperformed static portfolio (in 2/03/2009 the value of dynamic portfolio is 63% higher than the value of static portfolio). Since the second half of 2009, dynamic portfolio starts to underperform static portfolio, at the end of observation period the value of dynamic portfolio is 31% higher than the value of static portfolio.

---

[36] The values of asset classes do not necessarily move in sync. Thus, in order to hold initial asset allocation weights for static portfolio, asset allocation has to be constantly reviewed during the observation period.

One may assume, that the reason why dynamic portfolio started to underperform static portfolio relies in the uncertainty of European economic climate. In addition to the sovereign debt crisis, there have been used novel methods to alleviate the crisis, e.g. quantitative easing and extremely low interest rates. These methods have eased the effects of the crisis and it seems that the markets have been mostly in rising trend after 2009, except equities in 2011. While the growth in asset prices might be leveraged by the quantitative easing and low interest rates, market participants are extremely sensitive, which makes financial asset returns very volatile. However, it might complex the forecasting of conditional volatility because, in general, volatility is low when markets are rising. Thus, the GARCH models which use 5 year historical volatility as a rolling window might not be appropriate for volatility forecasting. It is worth to emphasize, that the latter is an assumption without any evidence.

Considering the limitation of short-term losses, it appears that during the financial crisis of 2008 to 2009 the value of dynamic portfolio decreased -25%, while the value of static portfolio decreased -56%. In addition, it appears that the minimum return of dynamic portfolio is higher than on static portfolio (without transaction costs -3.13% compared to -4.01%). Thus, one may conclude that dynamic portfolio limited extreme and short-term losses more efficiently than static portfolio.

4.3.2. Limitations and Suggestions for Further Research

The implementation of one dynamic asset allocation strategy is not enough to make any general conclusions about dynamic asset allocation strategies. Although several previous empirical studies have implemented different dynamic asset allocation strategies, it is necessary to implement all different dynamic strategies with same dataset and observation period, in order to find the most suitable and profitable dynamic asset allocation strategy. In addition, it might provide additional value if dynamic models would be adjusted or some economic leading indicators would be included to the model.

It is important to emphasize that the results, are might be dependent on dataset and observation period, thus results might be different with different data and period. In order to strengthen the results, the empirical analysis should be repeatedly replicated with other assets and asset classes.

It must be taken into account that the implementation of a dynamic strategy did not take into account the liquidity problems. It was assumed, that all trades can be carried out with full amount immediately.

It is worth further research, what would be the optimal threshold level for eliminating minor changes in the asset classes' weights, in order to improve the performance of the shortfall risk-based strategy. Also in further analysis of dynamic asset allocation strategies, it would make sense to divide the observation period to bull, bear and flat markets – thus it would be possible to distinguish different dynamic strategies' behavior during different market cycles.

SUMMARY

In search of optimizing a portfolio, the vast majority of asset managers have relied on the Markowitz's (1952) portfolio selection theory. Even though Markowitz's framework is definitely the cornerstone of modern portfolio theory and provided useful insight, the financial crises of last decades have provided evidences that it is extremely difficult to implement this framework effectively in practice. Mainly because of this framework assumes correlations to be static between different financial assets, which deviate from the reality. The latter poses a major problem for the asset managers and investors, who have been relying on a mean-variance framework as to tool to optimize the risk and return trade-off of their portfolios, because during the financial crises correlations tend to increase, consequently as well as the overall risk of investment portfolio.

Moreover, the portfolio management is nowadays moving towards a more dynamic and flexible approach, because the last major financial crises have shown that the investment portfolios based on the traditional static asset allocation tend to lose a lot of the value of assets under management during the crises. Even if the relative performance of portfolio is positive, the absolute performance can be negative.

In order to solve these problems dynamic asset allocation strategies and techniques, in which the asset allocation composition varies over time, have been developed. The mechanism for these strategies is not the same as that for mean-variance optimization framework, in which portfolio's risk is reduced through a covariance term. Dynamic asset allocation seeks to increase risk and return trade-off by investing in a better performing asset class. As it appeared from the Chapter 1, most of these strategies are aimed to produce absolute return and control shortfall risk directly, by protecting the value of portfolio to fall below a pre-specified floor.

Based on the literature review and survey of previous empirical studies of different dynamic asset allocation strategies, one may find that shortfall risk-based strategy has proven to be the best performing among dynamic asset allocation technique. On this basis, the shortfall risk based strategy was implemented in the empirical part of this thesis.

While the shortfall risk-based strategy uses expected volatility as a main input, different volatility modeling methods were discussed in Chapter 2. Based on the survey of previous empirical studies, GARCH(1,1), EGARCH(1,1) and GJR-GARCH(1,1) models were selected out for forecasting the conditional volatility. In order to evaluate the effectiveness of selected volatility modeling methods, and the preciseness of volatility forecasts, these were implemented on two different datasets (U.S. and European equities and fixed income). The observation period was from 1/01/2000 to 30/04/2015. All three models were estimated with *Gaussian* and *Student t* distribution by using sample period 1/01/2000 to 31/12/2004. Based on AIC and BIC, it appeared that the best model for DUSE was EGARCH(1,1) with normal distribution, for DEUE GJR-GARCH(1,1) with normal distribution, for DUSF GJR-GARCH(1,1) with *Student t* distribution, and for DGERF EGARCH(1,1) with *Student t* distribution. In order to evaluate the preciseness of volatility forecasts, conditional volatility was forecasted were made with all three models under both distributions for all time-series from 1/01/2005 to 30/04/2015. As a base of sample period, 5 year rolling window was used. In order to compare the forecasting performance of complex GARCH type models to simple naïve techniques, the EWMA volatility forecasts were calculated as well.

Although, previously the results were in accordance with the literature, that EGARCH and GJR-GARCH models are superior in capturing the dynamics in volatilities, then the findings of the empirical analysis presented in Chapter 3 indicated the superiority of GARCH(1,1) model during the forecasting period. One may assume that this is caused by the short, 1-day ahead, forecasting period, because several studies which found either EGARCH or GJR-GARCH models more precise than the ordinary GARCH, were using longer forecast periods.

Even though it appeared that GARCH(1,1) model was the most precise for 1-day ahead conditional volatility forecasting, this information was not available in 31/12/2004. Thus, in order to carry out portfolio simulation from 1/1/2005 to 30/04/2015, previously selected EGARCH and GJR-GARCH models were used for conditional volatilities forecasting.

In Chapter 4, portfolio simulation was conducted. In order to compare shortfall risk-based asset allocation strategy was compared to static asset allocation strategy (50% equity and 50% fixed income), two dynamic and two static portfolios were launched. For both strategies one portfolio was based on U.S. and one on European dataset. Based on the

proposed research questions, dynamic portfolio's abilities to generate higher absolute returns and limit more efficiently short-term losses than static portfolio were evaluated. The results indicated that, in general, with given dataset and observation period and without considering any transaction costs, portfolio based on dynamic asset allocation strategy outperformed portfolio based on static asset allocation. Even though the returns of dynamic portfolio are same volatile, high negative returns are limited more effectively than on static portfolio. As a dynamic strategy involves continuous re-allocation between asset classes, there is a higher trading volume than on static portfolio. Higher trading volume reduces the return of portfolio if transaction costs are included. Nevertheless, in this empirical analysis, it appeared that if transaction costs are below 0.38% (the breakeven points for U.S. and European datasets were 0.38% 0.53%, respectively) then the absolute return over the observation period is for dynamic portfolio higher than for static portfolio.

Although these results are in accordance with the previous empirical studies, that dynamic portfolios tend to outperform static portfolios, especially in bear environments, one may not fully agree with that statement. The latter is because dynamic portfolio underperformed static portfolio with European dataset from 01/01/2009 to 30/04/2015, this is relatively long period, during in which markets have been rising and falling. Thus, it can be concluded, that the results are highly dependent on the observation period and portfolios based on shortfall risk-based asset allocation strategy might not be always outperforming portfolios based static asset allocation.

In addition, it is worth to notify that the shortfall risk-based strategy might provide different outcome, if the inputs of expected shortfall probability calculations, and model constraints are changed, e.g. minimum acceptable return and threshold level for eliminating minor changes in the asset classes' weights. Exactly the same way, as there is a trade-off between expected return and risk, there is a trade-off between choosing the threshold level for eliminating minor changes in the asset classes' weights. Frequent overview of asset classes' weights might help to protect portfolio value from falling and increase upside potential, but on the other side might decrease portfolio value through high transaction costs.

The purpose of this thesis was fulfilled, answers to the research questions and inferences were proposed as much as the results permitted. It is worth further investigation, whether the different volatility forecasting methods, e.g. models based on stochastic volatility theory, or nonparametric methods for volatility density estimation, could improve the

preciseness of forecasted volatility. Regarding shortfall risk-based strategy, it is necessary to research what would be the optimal threshold level for eliminating minor changes in the asset classes' weights, in order to improve the performance of this strategy. In addition, the empirical analysis on dynamic asset allocation strategies should be extended, so that all different dynamic strategies would be compared during same observation period and dataset.

REFERENCES

Ali, G. (2013). EGARCH, GJR-GARCH, TGARCH, AVGARCH, NGARCH, IGARCH and APARCH Models for Pathogens at Marine Recreational Sites. *Journal of Statistical and Econometric Methods*, Vol. 2, No.3, pp. 57-73.

Akaike, H. (1974) A New Look at the Statistical Model Identification. *IEEE Transactions on Automatic Control*, Vol. 19, No. 6, pp. 716-723.

Akgiray, V. (1989) "Conditional Heteroskedasticity in Time Series of Stock Returns: Evidence and Forecasts," *Journal of Business*, Vol. 62, pp. 55-80.

Alles, L. A., Kling, J. L. (1994) Regularities in the Variation of Skewness in Asset Returns, *Journal of Financial Research*, Vol. 17, pp. 427-438.

Allen, D., Satchell, S. (2014) "The Four Horsemen: Heavy-tails, Negative Skew, Volatility Clustering, Asymmetric Dependence." The University of Sydney, Business School Discipline of Finance. Discussion Paper, No. 4.

Amin, G. S., Kat H. M. (2003) Stocks, Bonds, and Hedge Funds - Not a Free Lunch, Journal of Portfolio Management, Vol. 29, pp. 113.

Andersen, T. G., Bollerslev, T. (1997) "Answering the Critics: Yes, ARCH Models Do Provide Good Volatility Forecasts," Department of Finance, Kellogg Graduate School of Management, Northwestern University. Working Paper. No. 227.

Andersen, T. G., Bollerslev, T., Diebold, F. X., Labys, P. (2003). Modeling and Forecasting Realized Volatility. *Econometrica*, Vol. 71, No. 2, pp. 579-625.

Angelovska, J. (2013). Managing Market Risk with VaR (Value at Risk). *Management*, Vol. 18, No. 2, pp. 81-96.

Ang, A., Bekaert, G. (2002) International Asset Allocation with Regime Shifts. *Review of Financial Studies*, Vol. 15, pp. 1137-1187.

Ang, A., Chen, J. (2002) Asymmetric Correlations of Equity Portfolios. *Journal of Financial Economics*, Vol. 63, No. 3, pp. 443-494.

Baillie, R. T., Bollerslev, T. (1989) The Message in Daily Exchange-rates – A Conditional Variance Tale. *Journal of Business & Economic Statistics*, Vol. 7, pp. 297-305.

Beedles, W. L. (1979) Asymmetry of Market Returns, Journal of Financial and Quantitative Analysis, Vol. 14, pp. 653-660.

Black, F., Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy*, Vol. 81, No. 3, pp. 637-654.

Black, F. (1976). Studies of Stock Market Price Volatility Changes. Proceedings of the 1976 Meetings of the American Statistical Association, Business and Economical Statistics Section, American Statistical Association, pp. 177-181.

Black, F., Jones, R., (1987). Simplifying portfolio insurance. *Journal of Portfolio Management*, Vol. 14, No. 1, pp. 48–51.

Bloomberg (2015). Bloomberg Terminal. Database Bloomberg L.P.

Bollerslev, T. (1986). Generalized Autoregressive Conditional Heteroskedasticity, *Journal of Econometrics*, Vol. 31, pp. 307-327.

Bollerslev, T. (1988). On the Correlation Structure for the Generalized Autoregressive Conditional Heteroskedastic Process. *Journal of Time Series Analysis,* Vol. 9, No. 2, pp. 121-131.

Bollerslev, T., Chou, R., Kroner, K. (1992). ARCH Modeling in Finance. *Journal of Econometrics*, Vol. 52, pp. 5-59.

Bollerslev, T., Engle. R. F. (1993). Common Persistence in Conditional Variances. *Econometrica,* Vol. 61, pp. 167-186.

Bollerslev, T., Engle, R. F., Nelson, D. B. (1994). "GARCH Models" Engle, R. F., McFadden, D.L. (eds.) Handbook of Econometrics, Vol. 4, pp. 2961-3038. Amsterdam: Elsevier.

Bollerslev, T., Ghysels, E. (1996) "Periodic Autoregressive Conditional Heteroskedasticity," *Journal of Business and Economic Statistics*, Vol. 14, pp. 139-157.

Bollerslev, T, (2010) Glossary to ARCH (GARCH*). *Volatility and Time Series Econometrics*, University Press, Oxford Scholarship Online. Available: http://oxfordindex.oup.com–/view/10.1093/acprof:oso/9780199549498.003.0008 (11.03.2015)

Brailsford, T. J., Faff, R. W. (1996) "An Evaluation of Volatility Forecasting Techniques," *Journal of Banking and Finance*, Vol. 20, pp. 419-438.

Brinson, G. P., Hood, L. R., Beebower, G. L. (1986). Determinants of Portfolio Performance. *Financial Analysts Journal.* Vol 42. No 4, pp. 39–44.

Brinson, G. P., Singer, B. D., Beebower, G. L. (1991). Determinants of Portfolio Performance II: An Update. *Financial Analysts Journal*, Vol. 47, No. 3, pp. 40-48.

Brock, W. A., Dechert, W. D., Scheinkman, J. A. (1987). A Test for Independence Based on the Correlation Dimension, - Department of Economic, University of Wisconsin at Madison, University of Houston, and University of Chicago.

Campbell, J. Y., Hentschel, L. (1992). No News is Good News: An Asymmetric Model of Changing Volatility in Stock Returns. *Journal of Financial Economics*, Vol. 31, No. 3, pp. 281-318.

Chantnani, N. N. (2010). Commodity Markets: Operations, Instruments, and Applications. New Delhi: Tata McGraw Hill.

Chatfield, C. (2000) Time-Series Forecasting. Florida, Boca Raton: Chapman & Hall/CRC Press LLC

Chelley-Steeley, P. L., Steeley, J. M. (1996). Volatility transmission in the UK equity market. *European Journal of Finance*, Vol. 2, No. 2, pp. 145-160.

Christie, A. A. (1982). The Stochastic Behavior of Common Stock Variances – Value, Leverage and Interest Rate Effects. *Journal of Financial Economics*, Vol. 10, pp. 407-432.

Cont, R. (2001) Empirical Properties of Asset Returns: Stylized Facts and Statistical Issues. *Quantitative Finance,* Vol. 1, pp. 223-236.

Cooke, R. M., Nieboer, D. (2011). Heavy-Tailed Distributions: Data, Diagnostics, and New Developments. *Resources for the Future Discussion Paper*, Vol. 11, No. 19.

Curto, J. D., Pinto, J. C. (2012). Predicting the Financial Crisis Volatility. *Economic Computation & Economic Cybernetics Studies & Research*, Vol. 46, No. 1, pp. 183-195.

Dickey, D. A., Fuller, W. A. (1979). Distribution of the Estimators for Autoregressive Time Series with a Unit Root, - *Journal of the American Statistical Association,* Vol. 74, No 366, pp. 427-431.

Dimson, E., Marsh, P. (1990) "Volatility Forecasting without Data-Snooping," *Journal of Banking and Finance*, Vol. 14, pp. 399-421.

Duncan, A., Kabundi, A. (2014). Global Financial Crises and Time-Varying Volatility Comovement in World Equity Markets. *South African Journal of Economics*, Vol. 82, No. 4, pp. 531-550.

Engle, R. F. (1982). Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica*, Vol. 50, No. 4, pp. 987-1007.

Engle. R. F., Bollerslev, T. (1986). Modeling the Persistence of Conditional Variances. *Econometrics Review,* Vol. 5, No. 1, pp. 1-50.

Engle, R. F., Hong, C. H., Kane, A., Noh, J. (1993). "Arbitrage Valuation of Variance Forecasts with Simulated Options," in Chance, D. M., Tripp, R. R. eds., Advances in Futures and Options Research, pp. 393-415. Greenwich: JIA Press.

Engle, R. F., Ng, V. K. (1993). Measuring and Testing the Impact of News on Volatility. *The Journal of Finance*, Vol. 48, pp. 1749-1778.

Engle, R. F., Kane, A., Noh, J. (1996) "Index-Option Pricing with Stochastic Volatility and the Value of Accurate Variance Forecasts," *Review of Derivatives Research*, Vol. 1, pp. 139-158.

Erb, C., Harvey, C., Viskanta, T. (1994) Forecasting International Equity Corrleations. *Financial Analysts Journal.* November-December, pp. 32-56.

Fabozzi, F., Gupta, F., Markowitz, H. (2002). The legacy of modern portfolio theory. *Journal of Investing*, Vol. 11, No. 3, pp 7-22.

Fama, E. F. (1965). The Behavior of Stock-Market Prices. *The Journal of Business*. Vol. 38, No. 1, pp. 34-105.

Ferri, R. A. (2010). All About Asset Allocation: The Easy Way to Get Started. New York: McGraw-Hill, Inc.

Franklin, B. (1758). Poor Richard's Almanack. Published in American colonies, No. 26.

Glosten, L., Jagannanthan, R., Runkle, R. (1993). On the Relationship between the Expected Value and the Volatility of the Nominal Excess Returns on Stocks. *Journal of Finance*, Vol. 48, pp. 1779-1801.

Goodwin, D. (2012). Modeling and Forecasting Volatility in Copper Price Returns with GARCH Models. Bachelor Thesis. School of Economics and Management, Lund University.

Hafner, C. M. (1998). Estimating High-Frequency Foreign Exchange Rate Volatility with Nonparametric ARCH Models. *Journal of Statistical Planning and Inference*, Vol. 68, pp. 247-269Harloff, G. J. (1998). Dynamic Asset Allocation: Beyond Buy and Hold. *Technical Analysis of Stocks & Commodities*, Vol. 17, No. 1, pp. 1-8.

Hansen, P., Lunde, A. (2005). A Forecast Comparison of Volatility Models: Does Anything Beat a GARCH(1,1)? *Journal of Applied Econometrics*, Vol. 20, pp. 873-889.

Harvey, C. R., Siddique, A. (1999) Autoregressive Conditional Skewness, Journal of Financial and Quantitative Analysis, Vo. 34, pp. 465-487.

Herold, U., Maurer, R., Stamos, Purschaker, N. (2005). Total Return Fixed-Income Portfolio Management. *The Journal of Portfolio Management*, Vol. 31, No. 3, pp. 32-43.

Herold, U., Maurer, R., Stamos, M., Thanh Vo, H. (2007). Total Return Strategies for Multi-Asset Portfolios. *The Journal of Portfolio Management*, Vol. 33, No. 2, pp. 60-76.

Hung, J. (1997). Intervetion Strategies and Exchange Rate Volatility. A Noise Trading Perpective. *Journal of International Money and Finance*, Vol. 16, pp. 779-793.

Hyndman, R. (2006). Another Look at Measures of Forecast Accuracy. *International Journal of Forecasting*, Vol. 22, No. 4, pp. 679-688.

Ibbotson, R. G., Kaplan, P. D. (2000). Does Asset Allocation Policy Explain 40, 90 or 100 Percent of Performance? *Financial Analysts Journal*, Vol. 56, No. 1, pp. 26–33.

Ibbotson, R. G. (2010). Perspectives: The Importance of Asset Allocation. *Financial Analysts Journal*, Vol. 66, No. 2, pp. 18-20.

Jorion, P. (2001). Value at Risk: The New Benchmark for Managing Financial Risk. 2nd. ed., New York: McGraw Hill.

Karolyi, G. A., Stulz, R. M. (1996) Why do Markets Move Together? An Investigation of US-Japan Stock Return Comovements. *Journal of Finance,* Vol. 51, No. 3, pp. 951-986.

Kroner, K. F., Kneafsey, K. P., Claessens, S. (1995) "Forecasting Volatility in Commodity Markets," *Journal of Forecasting*, Vol. 14, pp. 77-96.

Kraus, A., Litzenberger R. H. (1976) Skewness Preference and Valuation of Risk Assets, Journal of Finance, Vol. 31, pp. 1085-1100.

Karoui, N. E., Jeanblanc, M., Lacoste, V. (2005). Optimal portfolio management with American capital guarantee. *Journal of Economic Dynamics & Control*, No. 29, pp. 449-468.

Kasibhatla, K. M., Stewart, D. B., Sen, S., Malindretos, J. (2005). GARCH Modeling of Conditional Volatility and Leverage Effect in Global Equity Markets. *International Journal of Finance*, Vol. 17, No. 3, pp. 3568-3580.

Knight, J., Satchell, S. (2007). Forecasting Volatility in the Financial Markets. Third Edition. The Netherlands: Elsevier Ltd.

Knutzen, E. (2011). Investing in Volatile Times: A Dynamic Approach to Asset Allocation. *Investments and Wealth Monitor*, Vol. 11, No. 4, pp. 1-10.

Laopodis, N. T. (1997). U.S. Dollar Asymmetry and Exchange Rate Volatility. *Journal of Applied Business Research.* Vol. 13, pp. 1-8.

Lawrence, S. D., Singh, T. N. (2011). Managing DB Plan Risk: The Dynamic Asset Allocation Strategy. *The Magazine of WorldatWork*, No. 11, pp. 47-50.

Lee, K. Y. (1991) "Are the GARCH Models Best in Out-of-Sample Performance?," *Economic Letters*, Vol. 37, pp. 305-308.

Leland, H. E., Rubinstein, M. (1976). The evolution of portfolio insurance. Luskin, D. L.: Portfolio Insurance: A Guide to Dynamic Hedging, New York: Wiley, pp. 3-10.

Li, X., Sullivan, R. N. (2011). A Dynamic Future for Active Quant Investing. *Journal of Portfolio Management*, Vol. 37, No. 3, pp. 29-36.

Lintner, J. (1965a). The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. *Review of Economics and Statistics*, Vol. 47, No. 1, pp. 13–37.

Lintner, J. (1965b). Security Prices, Risk and Maximal Gains from Diversification. *Journal of Finance*, Vol. 20, No. 4, pp. 587–615.

Liu, H. C., Hung, J. C. (2010). Forecasting S&P-100 Stock Index Volatility: The Role of Volatility Asymmetry and Distributional Assumption in GARCH Models. *Expert Systems with Applications*, Vol. 37, No. 7, pp. 4928-4934.

Lizieri, C., Satchell, S., Zhang, Q. (2007) The Underlying Return-Generating Factors for REIT Returns: An Application of Independent Component Analysis, - *Real Estate Economics*, Vol. 35, pp. 569-598.

Longin, F., Solnik, B. (2001) Extreme Correlation of International Equity Markets. *Journal of Finance,* Vol. 56, pp. 649-676.

Lo, A., MacKinlay, C. (1987). An Econometric Analysis of Nonsynchronous Trading. *Journal of Econometrics,* Vol. 55, pp. 181-211.

Lopez, J. A. (2001). Evaluating the Predictive Accuracy of Volatility Models. *Journal of Forecasting,* Vol. 20, No. 2, pp. 87-109.

Lummer, S. L., Riepe, M. W. (1994). The Role of Asset Allocation in Portfolio Management. Global Asset Allocation: Techniques for Optimizing Portfolio Management, ed. Lederman, J., Klein, R. A., New York: Wiley.

Mandelbrot, B. (1963). The Variation of Certain Speculative Prices. *The Journal of Business*, Vol. 36, No. 4, pp. 394-419.

Marcucci, J. (2005). Forecasting Stock Market Volatility with Regime-Switching GARCH Models. - *Studies in Nonlinear Dynamics & Econometrics*, Vol. 9, No. 4, pp. 53-55.

Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance*, Vol. 7. No. 1, pp. 77-91.

McNeil, A. J., Frey, R. Embrechts, P. (2005) Quantitative risk management: Concepts, Techniques and Tools. Princeton University Press.

Megginson, W. (1996). A historical overview of research in finance. *Journal of Finance*, Vol. 39, No. 2, pp. 323-346.

Merton, R. C., Scholes, M. S., Gladstein, M. L. (1978). The Returns and Risk of Alternative Call Option Portfolio Investment Strategies. *Journal of Business*, Vol. 51, No. 2, pp. 183-242.

Merton, R. C., Scholes, M. S., Gladstein, M. L. (1982). The Returns and Risks of Alternative Put-Option Portfolio Investment Strategies. *Journal of Business*, Vol. 55, No. 1, pp. 1-55.

Merton, R. C. (1998). Applications of Option-Pricing Theory: Twenty-Five Years Later. *American Economic Review*, Vol. 88, No. 3, pp. 323-349.

Mossin, J. (1966). Equilibrium in a Capital Asset Market. *Econometrica*, Vol. 35, No. 4, pp. 768–83.

Nam, D. (2011). Active Portfolio Management Adapted For the Emerging Markets. Master's degree thesis. United States: Massachusetts Institute of Technology.

Nelson, D. (1991). Conditional Heteroskedasticity in Asset Returns: A New Approach. *Econometrics*, Vol. 59, pp. 347-370.

Noh, J., Engle, R. F., Kane, A. (1994). Forecasting Volatility and Option Prices of the S&P 500 Index. *Journal of Derivatives*, Vol. 2, pp. 17-30.

Nylund, S. (2001). Value-at-Risk Analysis for Heavy-Tailed Financial Returns. Master's thesis. Helsinki University of Technology, Department of Engineering Physics and Mathematics.

Ou, A. (2014). Forecasting Hong Kong Hang Seng Index (HSI) Volatility using GARCH-class Models and the Implication of Index Option Pricing. Dissertation. University of Glasgow.

Pagan, A. R., Schwert, G. W. (1990) "Alternative Models for Conditional Stock Volatility," *Journal of Econometrics*, Vol. 45, pp. 267-290.

Patton, A. J. (2011). Volatility Forecast Comparison Using Imperfect Volatility Proxies. *Journal of Econometrics*, Vol. 160, No. 1, pp. 246-256.

Perold, A. F., Sharpe, W. F. (1988). Dynamic Strategies for Asset Allocation. *Financial Analysts Journal*, Vol. 44, No. 1, pp. 16-27.

Preis, T., Stanley, H. E. (2011) "Bubble Trouble: Can a Law Describe Bubbles and Crashes in Financial Markets?" *Physics World*, Vol. 24, pp. 29-32.

Priestley, M. B. (1981) Spectral Analysis and Time Series. Vol. 1 and 2. London: *Academic Press*.

Reider, R. (2009). Volatility Forecasting I: GARCH Models. New York University Working Paper.

Riskmetrics^{TM} (1996). Riskmetrics^{TM} Technical Document. Fourth Edition. New York: J.P.Morgan and Reuters.

Royston, G. (2011). Assessing a Quantitative Approach to Tactical Asset Allocation. Master's thesis, University of Pretoria, Gordon Institute of Business Science.

Rubinstein, M., Leland, H. E. (1981). Replicating Options with Positions in Stock and Cash. *Financial Analysts Journal*, Vol. 37, No. 4, pp. 63-72.

Schwert, G. W. (1990). Stock Volatility and the Crash of 87. *Review of Financial Studies*, Vol. 3, pp. 77-102.

Sentana, E., Wadhwani, S. (1992). Feedback Traders and Stock Return Autocorrections: Evidence from a Century of Daily Data. *The Economic Journal*, Vol. 102, pp. 415-425.

Sharpe, W. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance*, Vol. 19, No. 3, pp. 425-442.

Sharpe, W. (1992). Asset Allocation: Management Style and Performance Measurement. *Journal of Portfolio Management*, Vol. 18, No. 2.

Stulz, R. M. (1982). Options on the Minimum or the Maximum of Two Risky Assets. *Journal of Financial Economics*, Vol. 10, No. 2, pp. 161-185.

Sullivan, E. J. (2006). A brief history of the Capital Asset Pricing Model. Northeastern Association of Business, Economics and Technology, Pennsylvania: Days Inn.

Tankov, P. (2009). Introduction to portfolio insurance. Centra de Mathématiques Appliquees seminar materials. France: École Polytechnique CNRS.

Taylor, S. J. (1987) "Forecasting the Volatility of Currency Exchange Rates," *International Journal of Forecasting*, Vol. 3, pp. 159-170.

The MathWorks, Inc. (2015). Matlab R2015a Documentation. Available: https://www.mathworks.com/help/pdf_doc/matlab/math.pdf (22/04/2015).

Treynor, J. L. (1962). Toward a Theory of Market Value of Risky Assets. Unpublished manuscript. Final version: Korajczyk, R. A. (1999). Asset Pricing and Portfolio Performance. London: Risk Books, pp. 15–22.

Tobin, J. (1958). Liquidity preference as behavior toward risk. *Review of Economic Studies*, Vol. 25, No. 2, pp. 65-85.

Vardharaj, R., Fabozzi, F. J. (2007). Sector, Style, Region: Explaining Stock Allocation Performance. *Financial Analysts Journal*, Vol. 63, No. 3, pp. 59–70.

Vince, R. (1990). Portfolio Management Formulas: Mathematical Trading Methods for the Futures, Options, and Stock Markets. New York: John Wiley & Sons.

Wang, P., Sullivan, R. N., Ge, Y. (2012). Risk-Based Dynamic Asset Allocation with Extreme Tails and Correlations. *Journal of Portfolio Management*, Vol. 38, No. 4, pp. 26-42.

Wei, S. Y., Chen, J. T., Yang, J. J. W., Hong, W. C. (2011). The Volatility for Pre and Post Global Financial Crisis: An Application of Computational Finance. *International Journal of Applied Evolutionary Computation*, Vol. 2, No. 2, pp. 82-95.

Weiss, A. (1984) ARMA Models with ARCH Errors, Journal of Time Series Analysis, Vol. 5, pp. 129-143

West, K. D., Cho, D. (1994) "The Predictive Accuracy of Several Models of Exchange Rate Volatility," *Journal of Econometrics*, Vol. 69, pp. 367-391.

Westerfield, J. M. (1977) Examination of Foreign-Exchange Risk under Fixed and Floating Rate Regimes, Journal of International Economics, Vol. 7, pp. 181-200.

Whittle, P. (1951). Hypothesis Testing in Time Series Analysis. Uppsala: Almquist and Wicksell.

Xiong, J., Ibbotson, R. G., Idzorek, T., Chen, P. (2010). The Equal Importance of Asset Allocation and Active Management. *Financial Analysts Journal*, Vol. 66, No. 2, pp. 22-30

Xiong, J., Idzorek, T. (2011). The Impact of Skewness and Fat Tails on the Asset Allocation Decision. *Financial Analysts Journal*, Vol. 67, No. 2, pp. 23-35.

Yang, J. J. (2000). The Leverage Effect and Herding Behavior in Taiwan's Stock Market. *Journal of Risk Management.* Vol. 2, pp. 69-86.

Zakoian, J. M. (1994). Threshold Heteroskedastic Models. *Journal of Economic Dynamics and Control*, Vol. 18, pp. 931-955.

RESÜMEE

DÜNAAMILINE VARADE JAOTUS PROGNOOSITUD VOLATIILSUSE ALUSEL

Veiko Niinemäe

Raha- ja kapitaliturgude globaliseerumine ning pidev finantstoodete areng pakuvad investoritele laialdaselt võimalusi investeerimisportfelli koostamiseks. Tänaseni on optimaalse portfelli koostamisel palju tähelepanu pööratud Markowitz'i (1952) keskmise-dispersiooni optimeerimise metodoloogiale, mis on kindlasti üks kaasaegse portfelliteooria põhialustest. Vaatamata kasulikule indikatsioonile, mida keskmise-dispersiooni optimeerimise metodoloogia pakub, on viimastel kümnenditel toimunud suuremad finantskriisid tõestanud, et see metodoloogia omab ühte suurt puudust: eeldatakse, et korrelatsioonid erinevate finantsvarade vahel on ajas konstantsed, mis erineb tegelikkusest. Korrelatsioonidünaamikad on valmistanud suurt probleemi varahalduritele ja investoritele, kes on lähtunud keskmise-dispersiooni optimeerimise metodoloogiast kui võimalusest, millega maandada portfelli riski. Arvukad varasemad uuringud, mis on uurinud korrelatsioone erinevate finantsvarade vahel, on leidnud, et finantsvarade vahelised korrelatsioonid tõusevad volatiilsematel perioodidel, näiteks finantskriiside ajal. Sellest tulenevalt ei ole lähtuvalt keskmise-dispersiooni optimeerimise metodoloogiast madala volatiilsusega perioodil optimaalseks osutund portfell ka optimaalne kõrge volatiilsusega perioodil.

Eelpool toodu on ka üheks põhjuseks, miks Li ja Sullivan (2011) väitsid, et tänapäeval on varahaldus liikumas paindlikumate ja dünaamilisemate varade jaotamise strateegiate suunas, mis oleksid võimelised võtma arvesse oodatava riski ja tootluse dünaamikaid erinevate varaklasside lõikes. Lisaks on viimased suuremad finantskriisid tõestanud, et laialt levinud fikseeritud varaklasside jaotusega portfellid ei ole ratsionaalsed erinevate majandustsüklite jooksul. Majanduslanguse ajal on enamuse fikseeritud varaklasside jaotusega portfellide tootlused negatiivsed, samas on oluline siinkohal mainida, et mitte kõik varaklassid ei pruugi majanduslanguse perioodil negatiivset tootlust pakkuda. Sellest

tulenevalt, on tekkinud vajadus dünaamiliste varade jaotamise strateegiate järele, kus varaklasside osakaalud muutuvad ajas.

Käesolevas magistritöös vaadeldi mitmeid volatiiluse modellerimise tehnikaid ning dünaamilise varade jaotamise strateegiaid. Kuigi klassikaliselt on varade jaotamise seisukohast olnud korrelatsioon üks olulisemaid tegureid, siis käesolevas töös varaklasside vahelisi korrelatsioone ei prognoosita ega kasutatata, sest enamik dünaamilisi varade jaotuse strateegiaid ei kasuta korrelatsiooni sisendina, vaid pigem tingimuslikku volatiilsust.

Kuigi on olemas mitmeid varasemaid uuringuid volatiilsuse modelleerimise kui dünaamiliste varade jaotamise strateegiate osas, on puudujääk uuringutes, mis vaatleks neid koos. Veelgi enam, paljud uuringud dünaamiliste varade jaotamise strateegiate osas ei võta arvesse tehingukulusid.

Magistritöö eesmärgiks oli anda ülevaade ja võrrelda erinevaid autoregressiivseid tingimuslikke heteroskedastiivseid volatiilsuse modelleerimise meetodeid (GARCH, *Generalized Autoregressive Conditional Heteroskedasticity*), ning dünaamilisi varade jaotamise strateegiaid. Lisaks püstitati kaks uurimisküsimust: (i) kas GARCH klassi kuuluvad volatiilsuse modelleerimise meetodid võimaldavad täpsemaid prognoose kui naiivsed meetodid; (ii) kas dünaamilisel varade jaotusel põhinev investeerimisportfell on kõrgema tootlusega ning piirab efektiivsemalt lühiajalisi negatiivseid tootluseid kui fikseeritud varaklasside jaotusega portfell. Täitmaks magistritöö eesmärki ning leidmaks vastust neile küsimustele, andis autor esimeses ja teises peatükis ülevaate dünaamilist varade jaotust ning volatiiluse modelleerimist puudutavatest teoreetilistest lähtekohtadest ning varasematest uuringutest. Varasematest uuringutest selgus, et dünaamilistest varade jaotamise strateegiatest on mitmete uuringute põhjal parimaks osutunud oodataval langusriskil põhinev strateegia (*shortfall risk-based strategy*).

Empiirilised uuringud volatiilsuse modelleerimise ning prognoosimise meetodite osas ei ole jõudnud täpselt ühesele tulemusele, kuid siiski on valdavalt osutunud täpsema prognoosimisvõimega mudeliteks GARCH(1,1), EGARCH(1,1) ja GJR-GARCJ(1,1). Järgenvalt, kolmandas peatükis, modelleeriti ning prognoositi nimetatud kolme mudeli põhjal finantsvarade volatiilsust perioodil 01.01.2000 kuni 30.04.2015. Kasutatavateks finantsvaradeks valiti nii USA kui Euroopa aktsiaindeksid ning pikaajaliste võlakirjade futuurid. Kuna finantsvarade esimest järku logaritmitud diferentsid ei allunud normaaljaotusele, kasutati modelleerimisel lisaks ka *Student t* jaotust. Prognoositi järgmise

päeva tingimuslikku volatiilsust, kasutades viie aastast libisevat baasperioodi. Peale igat päeva, kalibreeriti mudelite parameetreid uuesti ning tehti prognoos järgmiseks päevaks. Kuigi hiljem ilmnes, et terve vaatluslause perioodi põhjal sai realiseerunud volatiiluse abil leida, et üldiselt on normaaljaotusega GARCH(1,1) mudel andnud iga aegreaga täpseima prognoosi, siis esialgsete mudelite seast, 5 aastase perioodiga alates 01.01.2000 kuni 31.10.2004, osutus Akaike ja Bayesani informatsiooni kriteeriumite põhjal parimateks järgnevad: USA aktsiaindeksile normaaljaotusega EGARCH(1,1); Euroopa aktsiahindeksile normaaljaotusega GJR-GARCH(1,1); USA võlakirjafutuurile *Student t* jaotusega GJR-GARCH(1,1); ning Euroopa võlakirafutuurile *Student t* jaotusega EGARCH(1,1). Üldiselt ilmnes siiski tulemustest, et GARCH klassi mudelid võimaldavaid täpsemaid prognoose kui naiivsed meetodid.

Neljandas peatükis viidi läbi erinevate portfellide simulatsioon perioodil 01.01.2005 kuni 30.04.2015. Kuna portfelli simulatsiooni alguse hetkeks ei olnud võimalik teada, et normaaljaotusega GARCH(1,1) mudel osutub volatiiluse prognoosimisel täpsemaiks, kasutati eelnevalt Akaike ja Bayesani informatsiooni kriteeriumite põhjal parimateks osutunud EGARCH ja GJR-GARCH mudeleid. Simulatsioonis kasutati nelja portfelli, kaks põhinesid dünaamilisel ja kaks fikseeritud varaklasside jaotusel. Mõlema varaklasside jaotuse jaoks kasutati eraldi nii USA kui Euroopa andmestikku. Dünaamilisel strateegial põhinevat portfelli võrreldi fikseeritud varaklasside kaaludel (50% aktsiad ja 50% võlakirjad) põhineva portfelliga. Empiirilise analüüsi tulemustest ilmnes, et nende andmete, vaatlusaluse perioodi ja valitud sisendite põhjal (arvestamata tehingukulusid), osutus oodataval langusriskil põhinev dünaamiline portfell paremaks. Kuigi nii dünaamiliste kui fikseeritud varaklasside jaotusega portfellide tootlused on sama volatiilsed, piirab dünaamiline strateegia kõrgeid negatiivseid tootluseid efektiivsemalt, samuti on nii diskonteeritud aastane kasvumäär kui portfelli absoluutne lõppväärtus kõrgemad.

Dünaamiline strateegia hõlmab endas pidevat varade ümberjaotamist erinevate varaklasside vahel, seega mõjutavad tehingukulud dünaamilise portfelli tootlust märkimisväärselt. Ilmnes, et kui tehingukulud jäävad alla 0.38% (tasakaalupunktid USA ja Euroopa andmetega olid vastavalt 0.38% ja 0.53%) tehingute mahust, siis on dünaamilise portfelli tootlus kõrgem. Kuigi tulemused on kooskõlas varasemate empiiriliste uuringutega, et dünaamilisel varade jaotusel põhinevad portfellid pakuvad kõrgemat tootlust ja limimeerivad efektiivsemalt lühiajalisi negatiivseid tootluseid kui fikseeritud varaklasside

jaotusel põhinevad portfelli, ei ole tegemist absoluutselt tõese järeldusega igal ajahetkel. Euroopa andmestikul põhineva simulatsiooni tulemustest ilmneb, et dünaamilisel strateegial põhinev portfell on fikseeritud kaaludega portfellist alates 2009. aastast madalam olnud. Kuigi terve vaatlusaluse perioodi jooksul on dünaamilise portfelli tootlus siiski kõrgem, on selge, et tulemused on äärmiselt sõltuvad vaatlusalusest perioodist.

Magistritöö eesmärk sai täidetud, vastuseid uurimisküsimustele leiti ning järeldusi tehti nii palju, kui tulemused võimaldasid. Autor leidis, et kuigi mõlemale uurimisküsimusele andis empiiriline analüüs positiivsed vastused, on tulemused äärmiselt sõltuvad andmetest, vaatlusalusest perioodist, sisenditest ning piirangutest mida kasutatakse.

Edasisteks võimalikeks uurimisteemadeks pakkus autor välja uurida piirmäära, millest alatest on oodataval langusriskil põhineva strateegia rakendamisel otstarbekas varade ümberjaotamist teostada. Nõnda on võimalik ebaolulised väiksemahulised muutused eemaldada ning tehingutasusid alandada. Käesolevas magistritöös valiti piirmääraks 5% portfelli väärtusest. Lisaks soovitas autor uurida erinevaid volatiilsuse prognoosimise meetodeid, näiteks stohhastilise volatiilsuse mudelid ning mitteparameetrilisi meetodid volatiilsuse tiheduse hindamisel. Samuti oleks võimalik dünaamiliste varade jaotamise strateegiate osas empiirilist analüüsi laiendada, rakendades kõiki erinevaid strateegiaid samade algandmetega ning perioodile. Nõnda oleks võimalik üheselt hinnata erinevaid dünaamilisi strateegiaid, tänaseni on valdav enamus võrdlevaid empiirilisi uuringuid käistlenud vaid mõningaid strateegiaid koos.

APPENDICES

Appendix 1. VBA Code: Expected Shortfall Probability

```
Function ExpectedShortfall(MAR. Mean. Sigma)
    ER = MAR - Mean
    z = ER / Sigma
    If z > 0 Or z = 0 Then
        w = 1
    Else
        w = -1
    End If
    y = 1 / (1 + 0.2316419 * w * z)
    ES = 0.5 + w * (0.5 - (Exp(-z * z / 2) / 2.506628) * _
        (y * (0.3193815 + y * (-0.3565638 + y * _
        (1.7814779 + y * (-1.821256 + y * 1.3302744))))))
        'cumulative normal distribution (Vince 1990, 199)
    ExpectedShortfall = ES
 End Function
```

Source: Vince (1990); compiled by the author

Appendix 2. Example: Implementation of the Expected Shortfall Method

On the Figure 5.1 asset weight is fixed to 100% and expected shortfall probability is fluctuating over the period. Fluctuations are caused of the changes in inputs – conditional volatility and return. If investor is risk averse and wants to hold expected shortfall risk probability constant over time, for example 5%, then it can be done by changing asset weight (exposure) as show on the Figure 5.2.
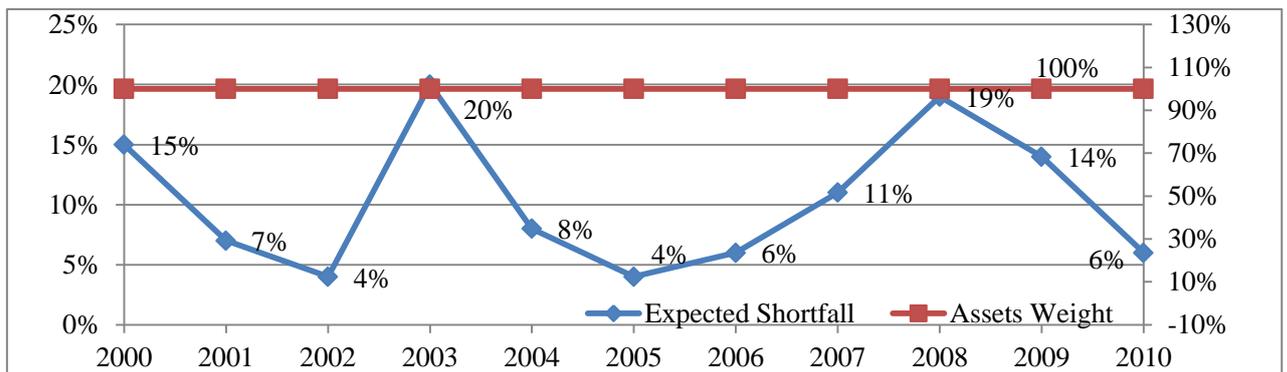

Figure 5.1 Example: Dynamic expected shortfall risk probability vs. static asset weight
Source: Compiled by the author
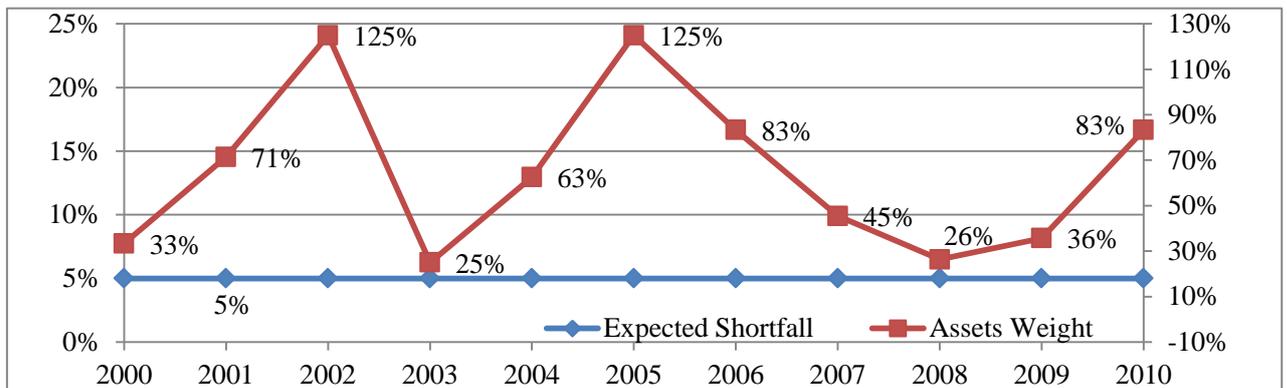
Figure 5.2. Example: Static expected shortfall risk probability vs dynamic asset weight
Source: Compiled by the author

Appendix 3. Matlab Code: Estimating Models Parameters

```
r = DEUE; % DUSE; DEUF; DGERF (datasets)
T = length(r);
logL = zeros(1,6);
numParams = logL;

Mdl1 = garch('GARCHLags',1,'ARCHLags',1);
[EstMdl1,EstParamCov1,logL(1)] = estimate(Mdl1,r);
numParams(1) = sum(any(EstParamCov1));

Mdl2 = egarch('GARCHLags',1, 'ARCHLags',1, 'LeverageLags',1);
[EstMdl2,EstParamCov2,logL(2)] = estimate(Mdl2,r);
numParams(2) = sum(any(EstParamCov2));

Mdl3 = gjr('GARCHLags',1, 'ARCHLags',1, 'LeverageLags',1);
[EstMdl3,EstParamCov3,logL(3)] = estimate(Mdl3,r);
numParams(3) = sum(any(EstParamCov3));

Mdl4 = garch('Distribution', 't','GARCHLags',1,'ARCHLags',1);
[EstMdl4,EstParamCov4,logL(4)] = estimate(Mdl4,r);
numParams(4) = sum(any(EstParamCov4));

Mdl5 = egarch('Distribution', 't', 'GARCHLags',1, 'ARCHLags',1,
'LeverageLags',1);
[EstMdl5,EstParamCov5,logL(5)] = estimate(Mdl5,r);
numParams(5) = sum(any(EstParamCov5));

Mdl6 = gjr('Distribution', 't', 'GARCHLags', 1,'ARCHLags',1, 'LeverageLags',1);
[EstMdl6,EstParamCov6,logL(6)] = estimate(Mdl6,r);
numParams(6) = sum(any(EstParamCov6));

[aic,bic] = aicbic(logL,numParams,T)
```

Source: Compiled by the author

Appendix 4. Matlab Code: 1-Day Rolling Window Conditional Variance Forecast[37]

```
r = DUSE; % DUSE; DEUF; DGERF (datasets)
T = size(r);

Mdl = egarch('Offset', NaN, 'Constant', NaN, 'GARCHLags',1,'ARCHLags',1,
'LeverageLags',1, 'Distribution', 'Gaussian');

[fit1,~,LogL1] = estimate(Mdl, y);

RW = 1245;

sample = r(1: RW ,1);
[fitS,~,~] = estimate(Mdl, sample);
RWF(1, 1) =  forecast(fitS,1, 'Y0', sample);
for t = RW:T(1,1)-1
   sample = r( t - RW +1 : t );
   [fitS,~,~] = estimate(Mdl, sample);
   RWF(t-RW+1+1, 1) = forecast(fitS,1, 'Y0', sample);
   t;
end

figure('Name', 'In-sample 1-day ahead conditional variance forecast');
hold on;
plot(RWF(2:end));
legend('Forecast','Location','SouthEast');
hold off;
```

Source: Compiled by the author

---

[37] This model (Mdl) is set to estimate EGARCH(1,1) parameters under Gaussian distribution. In order to set this model to estimate GARCH(1,1) or GJR-GARCH(1,1) parameters, "egarch" should be replaced with "garch" or "gjr", respectively. For changing normal (Gaussian) distribution with Student t distribution, 'Gaussian' needs to be replaced with 't'.

# Appendix 5. Conditional Volatility Forecasts of DUSE
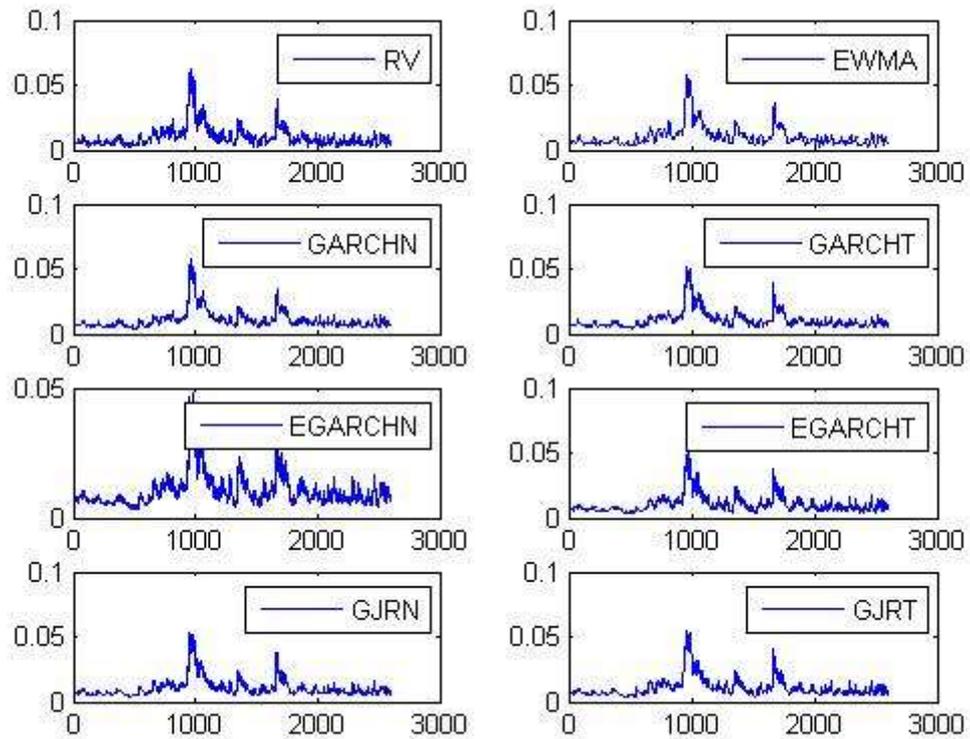
Figure 5.3 Conditional Volatility Forecasts of DUSE

Source: Compiled by the author

# Appendix 6. Conditional Volatility Forecasts of DEUE
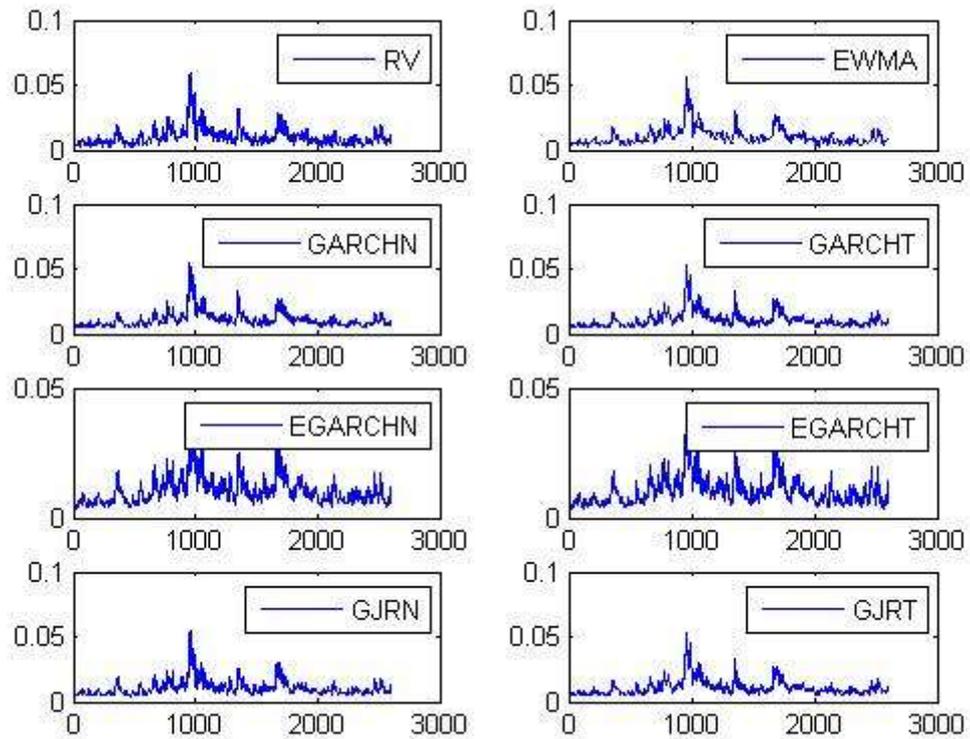
Figure 5.4 Conditional Volatility Forecasts of DEUE

Source: Compiled by the author

Appendix 7. Conditional Volatility Forecasts of DUSF
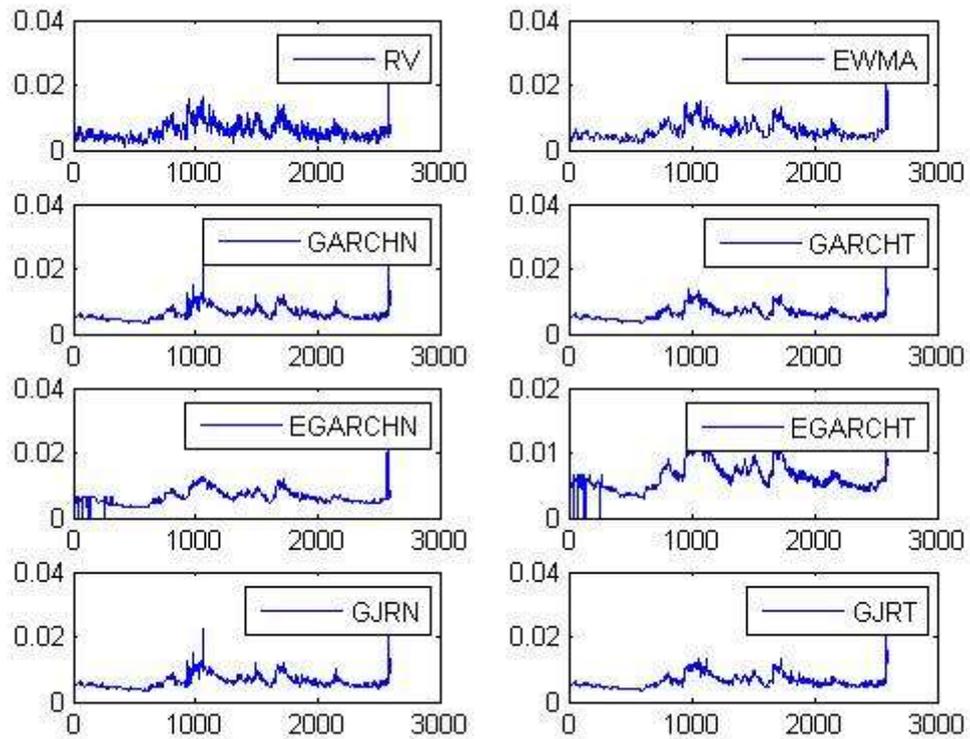
Figure 5.5 Conditional Volatility Forecasts of DUSF

Source: Compiled by the author

Appendix 8. Conditional Volatility Forecasts of DGERF
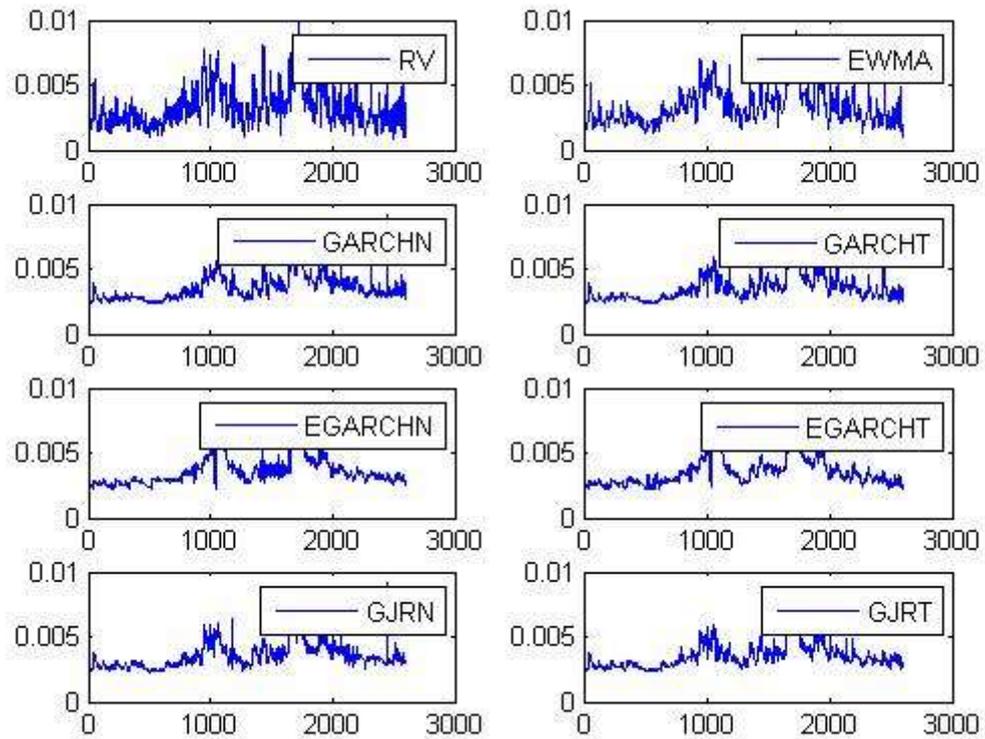
Figure 5.6 Conditional Volatility Forecasts of DGERF

Source: Compiled by the author

Appendix 9. U.S. Dataset Results (0.25% Transaction Costs)
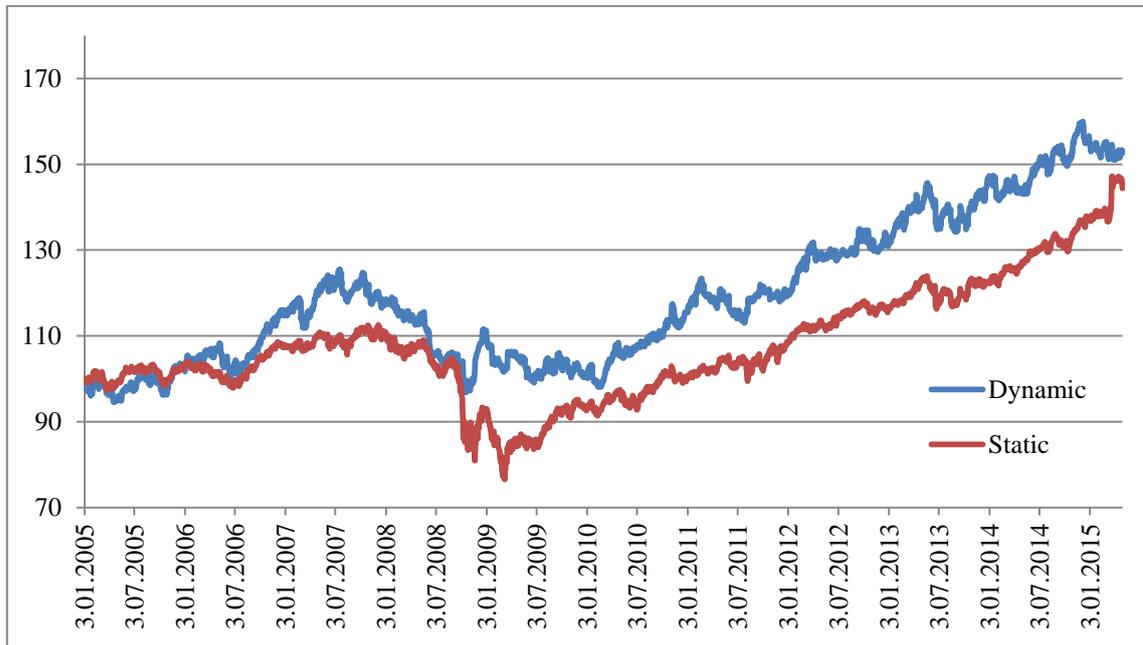
Figure 5.7 Dynamic and static portfolios performance from 1/01/2005 to 30/04/2015

Source: Compiled by the author
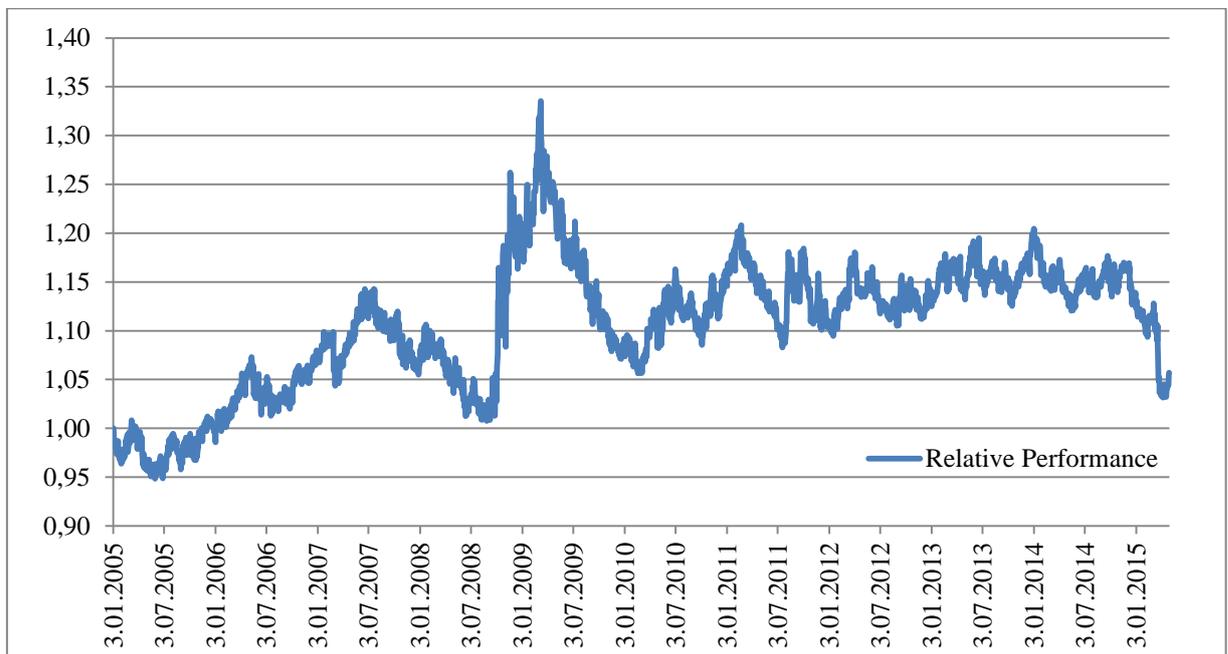
Figure 5.8 Dynamic and static portfolios relative performance from 1/01/2005 to 30/04/2015

Source: Compiled by the author

Appendix 10. U.S. Dataset Results (0.5% Transaction Costs)

Figure 5.9 Dynamic and static portfolios performance from 1/01/2005 to 30/04/2015

Source: Compiled by the author
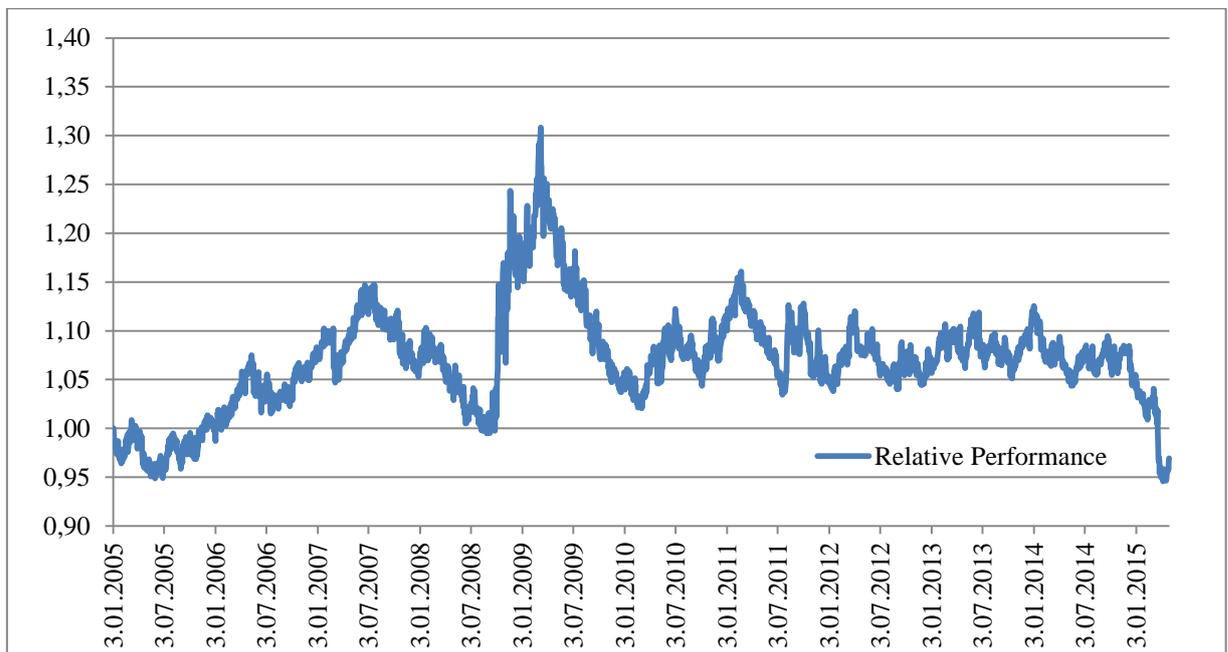
Figure 5.10 Dynamic and static portfolios relative performance from 1/01/2005 to 30/04/2015

Source: Compiled by the author

Appendix 11. European Dataset Results (0.25% Transaction Costs)
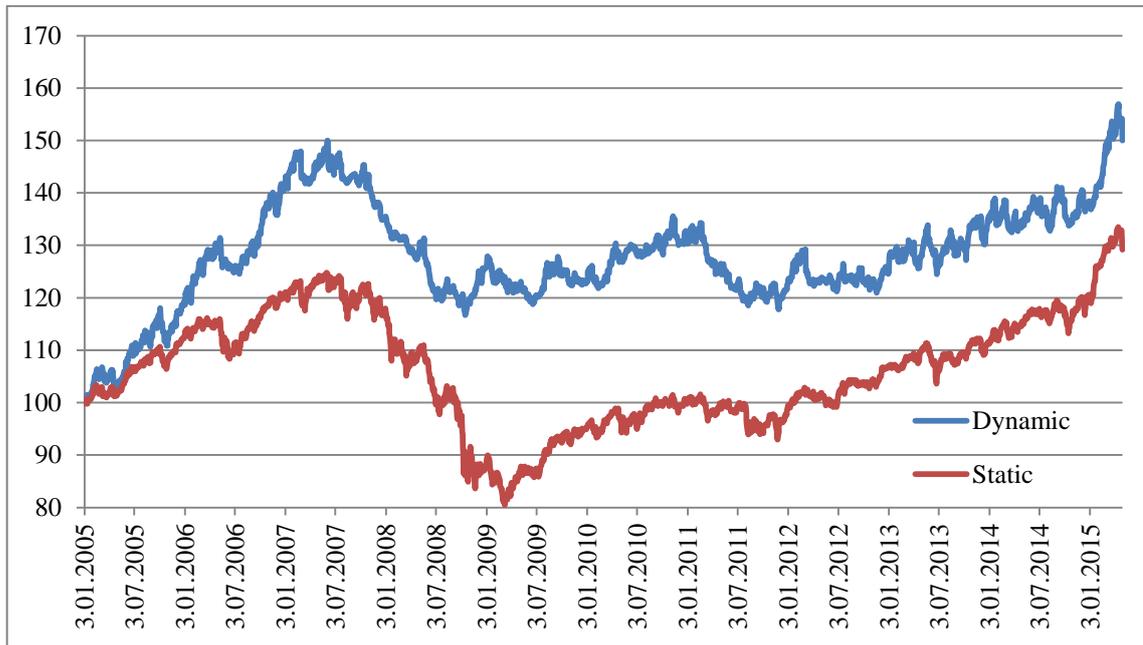
Figure 5.11 Dynamic and static portfolios performance from 1/01/2005 to 30/04/2015

Source: Compiled by the author
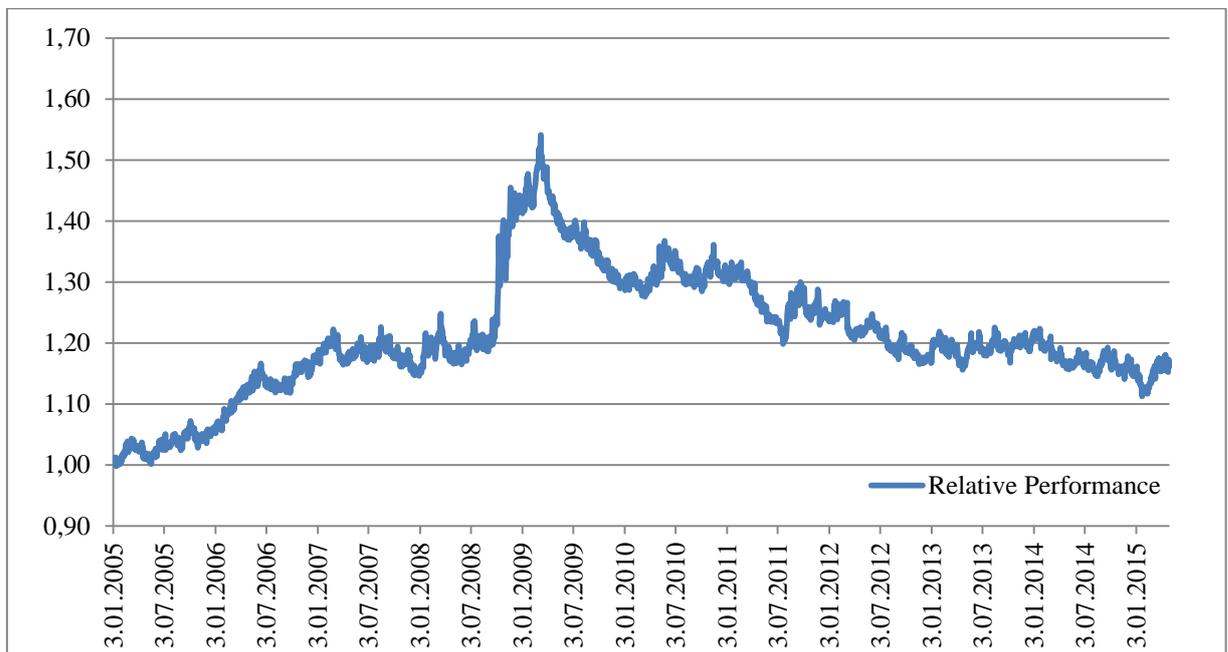
Figure 5.12 Dynamic and static portfolios relative performance from 1/01/2005 to 30/04/2015

Source: Compiled by the author

Appendix 12. European Dataset Results (0.5% Transaction Costs)
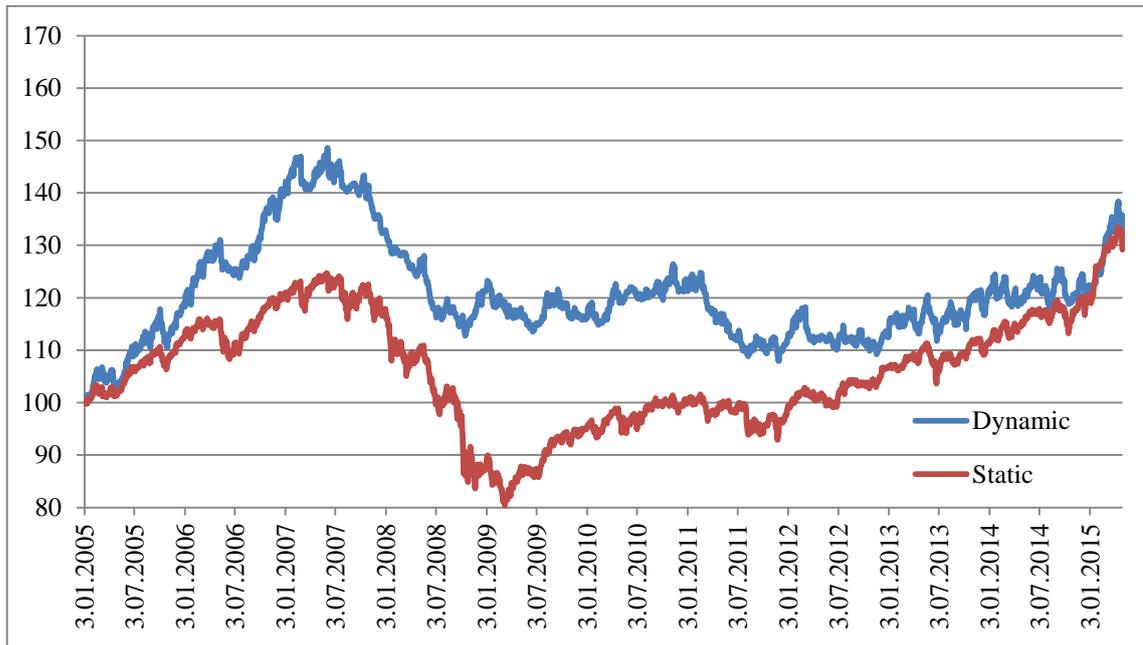
Figure 5.13 Dynamic and static portfolios performance from 1/01/2005 to 30/04/2015

Source: Compiled by the author
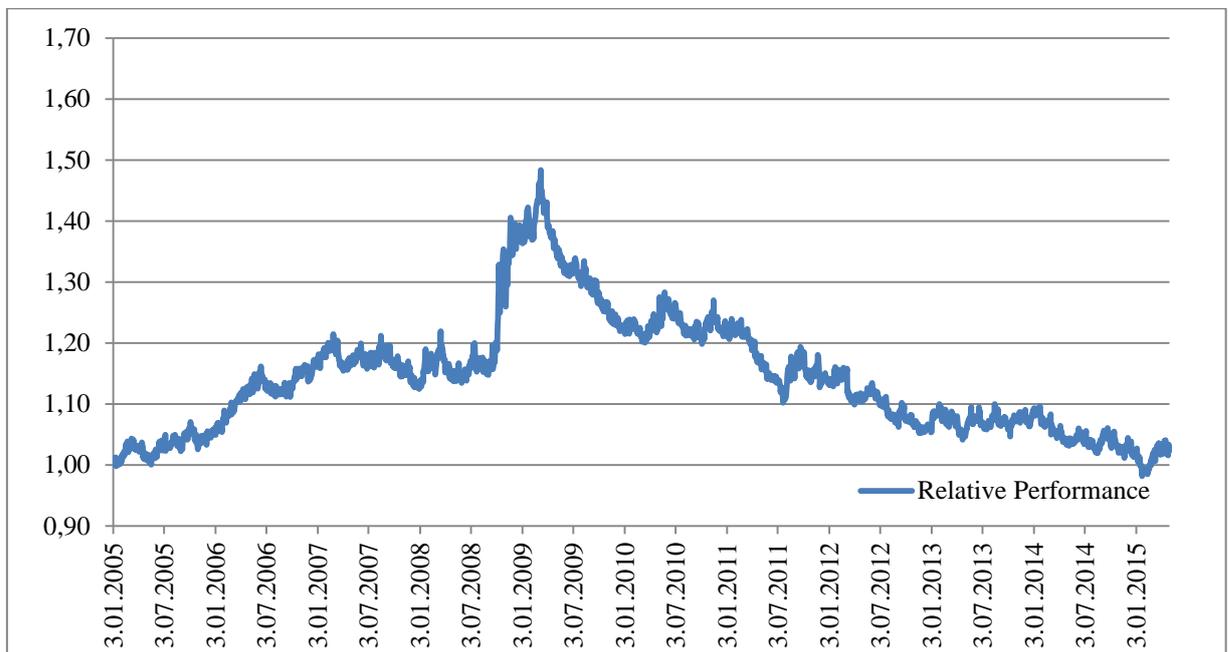
Figure 5.14 Dynamic and static portfolios relative performance from 1/01/2005 to 30/04/2015

Source: Compiled by the author