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FORECASTING OF VALUE AT RISK USING NONLINEAR AUTOREGRESSIVE VOLATILITY MODELS

Master's thesis

Supervisor: Professor Karsten Staehr, Ph.D.

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I declare I have written the master’s thesis independently.

All works and major viewpoints of the other authors, data from other sources of literature and elsewhere used for writing this paper have been referenced.

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ABSTRACT

This thesis addresses several issues in value at risk modelling and conditional variance estimation. Firstly, the aim of this thesis is to give a clear overview and evaluate the goodness-of-fit of selected nonlinear autoregressive volatility models in forecasting value at risk and to conclude whether using these models will bring significant advantages over calculating the value at risk with more naïve alternatives. Financial institutions among others could benefit from adopting a more complex method of computing value at risk, therefore this thesis is intended for further use in risk management and portfolio allocation problems. Secondly, this thesis analyses whether using Student-t distribution improves the goodness-of-fit of selected conditional variance models compared to the Gaussian distribution. Thirdly, if the sample size has a clear impact on the forecasting results. The models used in this thesis are the GARCH, EGARCH, GJR-GARCH and ARMA model.

In the first part of the paper, an overview of different value at risk and conditional variance models will be given together with their performance measurement techniques. Also, well known stylised properties of financial returns will be discussed and previous empirical studies analysed. The second part of the paper is devoted to an empirical analysis using nonlinear conditional variance models in estimating value at risk of different assets over several observation periods and distributions.

The results of the empirical analysis confirmed the benefits of using more sophisticated volatility models over naïve alternatives. Also, conditional variance models estimated with the Student-t distribution provided more accurate results compared to the Gaussian distribution. Thirdly, it was not possible to conclude that the sample size has a clear effect on the accuracy of selected models.

Keywords: volatility modelling, autoregressive models, volatility forecasting, GARCH, EGARCH, GJR, Value at Risk, VaR

JEL Classification: C13, C53, C58, G15, G17

ABBREVIATIONS

ADF – Augmented Dickey-Fuller

AGARCH – Asymmetric GARCH

AIC – Akaike Information Criterion

ARCH – Autoregressive Conditional Heteroskedasticity

ARMA – Autoregressive Moving Average

BIC – Bayesian Information Criterion

BDS – Brock, Dechert, Scheinkma

CVaR – Conditional Value at Risk

DJIA – Dow Jones Industrial Average

EGARCH – Exponential Generalised Autoregressive Conditional Heteroskedasticity

EWMA – Exponentially Weighted Moving Average

FIGARCH – Fractionally Integrated Generalised Autoregressive Conditional Heteroskedasticity

FIEGARCH – Fractionally Integrated Exponential Generalised Autoregressive Conditional Heteroskedasticity

GARCH – Generalised Autoregressive Conditional Heteroskedasticity

GJR-GARCH – Glosten-Jagannathan-Runkle GARCH

IGARCH – Integrated GARCH

ISE – Istanbul Stock Exchange

LR – likelihood-ratio test

MC Simulation– Monte Carlo Simulation

NGARCH – Nonlinear asymmetric GARCH

POF-test – Proportion of Failure test

RMSE – Root Mean Squared Error

SEC – U.S. Securities and Exchange Commission

TGARCH – Treshold Generalised Autoregressive Conditional Heteroskedasticity

TUFF – Time Until First Failure test

VaR – Value at Risk

INTRODUCTION

"*It is not the strongest or the most intelligent who will survive but those who can best manage change*."

Charles Darwin

Robert Engle stated in his Nobel Lecture that the advantage of knowing about risks is that we can change our behaviour to avoid them. Of course it is not possible to avoid all risks, since it is tied to our everyday lives, but managing risk has become an inseparable part of financial decisions. The stock market crash in the late 1980s, the dot-com bubble of 2000, the financial crisis of 2007-2008 and numerous other events proved the need for trustworthy risk management models, as volatility reached new highs. The increased focus on risk management over the past decades has led to the development of various techniques and models, some more widely used than others. Value at risk is one of the financial risk quantifying methods that has been the cornerstone of the risk management revolution in the past decade. The method measures the worst expected outcome over a given horizon under normal market conditions at a given confidence level (Jorion 2001). The model is very intuitive, which is one of the reasons why it is widely used by financial institutions. It has also been accepted by the Basel Committee on Banking Supervision as a preferred approach to measuring market risk (Basel 2006). Despite the fact that there are several ways of calculating the measure, according to a survey by Perignon and Smith, most of the commercial banks that are using such models do so by calculating the historical value at risk and putting the same weight on all observations (Perignon and Smith 2009). This can prove troublesome if volatility fluctuations increase and the estimated value at risk would most likely underestimate the computed possible loss. This provides an incentive to search for more sophisticated value at risk models that capture the changes in the variance more accurately.

Since the Autoregressive Conditional Heteroskedasticity model was developed by Engle in 1982, volatility modelling has been one of the most popular research subjects in financial time-series analysis (Bollerslev 1992). Over the years, several extensions to this popular modelling strategy have appeared. Autoregressive volatility models can be used in various areas including portfolio allocation and optimisation, risk management problems and pricing financial options to name a few. Autoregressive conditional heteroskedasticity models are more dependent on recent variance information opposed to putting the same weight on all observations as for the simple historical average volatility. Considering that conditional variance shocks are usually followed by high volatility periods, using these autoregressive models could produce more accurate results compared to the simple historical volatility. Therefore, value at risk models could also benefit from using the nonlinear autoregressive volatility methods.

This thesis addresses several issues in value at risk modelling and conditional variance estimation. The aim of this thesis is to give a clear overview and evaluate the goodness-of-fit of selected nonlinear autoregressive volatility models in forecasting value at risk and to conclude whether using these models will bring significant advantages over calculating the value at risk with more naïve alternatives. Financial institutions among others could benefit from adopting a more complex method of computing value at risk which is why this thesis is intended for further use in risk management and portfolio allocation problems. There were three research questions formulated in this thesis:

1. Do generalised ARCH volatility models show better goodness-of-fit and forecasting accuracy than naïve alternatives when estimating conditional variance and VaR?
2. Does using Student-t distribution show advantages in modelling conditional variance over Gaussian distribution?
3. Does using larger sample sizes improve the goodness-of-fit and forecasting performance of conditional variance and value at risk models?

In order to answer these research questions, different nonlinear volatility models were used in estimating value at risk, more specifically the generalised Autoregressive Conditionally Heteroscedastic (GARCH) model developed by Bollerslev (1986), the exponential GARCH (EGARCH) model introduced by Nelson (1991), the GJR-GARCH model developed by Glosten, Jagannathan and Runkle (1993) and the simple ARMA model by Whittle (1951). The reason for choosing GARCH models over several other methods used for forecasting volatility comes from previous researches including Bollerslev (1986), McMillan (2000), Goyal (2000), which show the superiority of these models over many others. The ARMA model was selected to compare the GARCH models to a simple alternative.

The empirical analysis focuses on calculating the value at risk of four different single assets over different samples and distributions. The DAX index (DAXR) represents equity, US 10Y Treasury note futures (TYR) represents fixed income, US Dollar index represents the movements in currencies and S&P GSCI Index represents the commodity asset class. The particular financial instruments were chosen since they are widely used as benchmarks in their asset class and are commonly traded. Asset classes behave differently in various economic cycles, which is why observing chosen models across several assets makes an interesting research subject. The calculations can be extended to a portfolio of assets, however in which case one should consider estimating correlations between these assets as well. Despite the fact that there are several studies on modelling conditional variance and value at risk, there are not many cross-asset analyses that combine the two.

The paper is divided into four main parts. Chapter 1 gives an overview of the theory of risk management, focusing on Value at Risk and the different methods of calculating it. Chapter 2 analyses the stylised properties of financial returns and the background literature of volatility modelling and forecasting. Chapter 3 is dedicated to discussing the previous empirical studies on estimating VaR with nonlinear volatility models to see if the results in this thesis are in line with other research done on this topic. In Chapter 4, an empirical analysis is conducted using nonlinear conditional variance models for estimating value at risk across different assets, distributions and sample sizes.

1. RISK MANAGEMENT: VALUE AT RISK – A LITERATURE REVIEW

"*We have no future because our present is too volatile. We have only risk management...*"

William Gibson (*Pattern Recognition*)[1]

The instability of financial markets provides motivation for financial institutions, corporations and individuals alike to manage risk. The increased focus on risk management over the past decades has led to the development of various techniques and models, some more widely used than others. This research focuses on a financial risk quantifying model called Value at Risk (VaR form here forward), which has been one of the cornerstones of the risk management revolution in the past decade (Jorion 2001). The choice to choose VaR over several other methods came from its wide area of use in information reporting, controlling and managing risk, and since it is being adopted *en masse* by institutions all over the world (Ibid.). Still, according to Perignon and Smith (2009), most of the commercial banks that are using VaR models, do so by using the historical simulation method (discussed in chapter 1.2). Therefore, financial institutions could benefit from adopting a more complex method of computing VaR, which is one of the possible applications of this research.

This chapter will cover the world of value at risk in four sections. The first section is devoted to the development of value at risk and the Basel framework. Section 2 discusses the different approaches to modelling value at risk, their formulas and empirical literature. Section 3 analyses the performance measurement of value at risk and conclusions will be discussed at the end of the chapter.

[1] Quote by William Gibson from his book „Pattern Recognition", a New York Times bestseller, 2005, Berkley Publishing

1.1. The story of quantifying risk

Philippe Jorion describes risk as the volatility of unexpected outcomes, generally the value of assets or liabilities of interest (Jorion 2001). The instability has been high in financial markets, which has raised the need for a measure that quantifies risk of possible loss of an asset or a portfolio. Value at Risk is one of those methods, measuring the worst expected outcome over a given horizon under normal market conditions at a given confidence level (Ibid.). The model is very intuitive, which is one of the reasons why it is widely used by financial institutions. The concept of VaR dates back to the late 1980s and early 1990s, when during the stock market crash billions of dollars were lost because of poor supervision and management of financial risk (Jorion 2001). In 1980, the SEC imposed haircuts for financial institutions due to the volatility in US interest rates, which were based on a statistical analysis of historical market data. These restrictions reflected the possible loss for a company over one month using the .95-quantile (Dale 1996). One can see that even though they were not called the value at risk at that time, it was a VaR measure. The development of RiskMetrics™ by J.P.Morgan at 1994 popularised the use of value at risk, although by then many desks were already using it (Lang 2000). The Basel Committee approved the use of VaR for large enough banks to base their required regulatory capital in 1996 (Basel 1996)[2]. Moreover, from the introduction of Basel II in 2004, VaR became the preferred approach for measuring market risk (Basel 2006).

Despite being widely used, there has been some criticism towards the model as well. Einhorm (2008) discussed the inefficiency of VaR in extreme situations, since the model focuses on given confidence levels and ignores the rare very large losses[3]. He compared the value at risk model to an airbag that works all the time, only to fail at a car accident. Although he may be somewhat right with his statement, there is still yet to be found a model that is more useful and widely adopted than VaR.

[2] It is not obligatory for US banks to use VaR as an internal regulatory capital measure (Perignon 2009).

[3] For more information on the limits of VaR, see Roundtable: The Limits of VaR. Derivatives Strategy, April 1998. It consisted of: Kolman Joe, Michael Onak, Philippe Jorion, Nassim Taleb, Emanuel Derman, Blu Putnam, Richard Sandor, Stan Jonas, Ron Dembo, George Holt, Richard Tanenbaum, William Margrabe, Dan Mudge, James Lam and Jim Rozsypal.

1.2. Value at Risk models

Value at risk measures the expected maximum (or worst) loss within a target horizon and a given confidence interval (Jorion 2001). There are three main methods to calculating value at risk, however each of them have many variations within. This section is going to give an overview of those three methods and discuss their strengths and shortcomings. Those approaches are the simple historical simulation, variance-covariance method and the Monte Carlo simulation.

To compute the VaR of an asset or a portfolio, one would need the value of the portfolio, the volatility, confidence level and a predetermined time horizon. The formula for computing VaR, assuming that asset returns are normally distributed is brought in equation 1.1 (Jorion 2001).

$$VaR_{\alpha} = \alpha * \sigma * W \tag{1.1}$$

where α is the selected confidence level, σ is the volatility or standard deviation of the portfolio and W is the initial mark-to-market value of the current portfolio. The historical method uses actual daily returns from a specified period in the past and identifies the loss that exceeds with a chosen probability.[4] The VaR estimates could then be plotted to a histogram from which the loss that is exceeded under a specified probability is easily obtainable. The model works on the assumption that the historical price changes act as a good proxy for current portfolio returns, however which may sometimes lead to distorted VaR estimates (Dowd 1998). Figure 1.1 shows the S&P500 index return histogram and value at risk with probability of 5%. The historical simulation method is often used due to its simplicity as it does not include the correlations of assets, only aggregated portfolio returns are needed. Another benefit is the ability to account for heavy tails in the data since there is no assumption of normal distribution (Jorion 2001). According to a survey conducted by Perignon and Smith on the use of VaR in commercial banks for the year 2005, 73% of banks disclose their VaR method report using the historical simulation (Perignon and Smith 2009). This can prove troublesome, as the historical simulation method estimates the future possible loss based on past returns, taking into account only the events that have occurred during the analysed period and putting the same weight on all observations. If the volatility fluctuations would increase,

[4] Usually probability of 0.1, 0.05 or 0.01 is selected, however one ought to be careful when choosing a probability too low since it can make model validation difficult (Jorion 2001).

the historical simulation model would most likely underestimate the computed possible loss. Also, the sampling variation of the historical method is usually higher than for other methods if the sample size is too short (Jorion 2001). This provides a good motivation to search for improvements to the simple historical method.

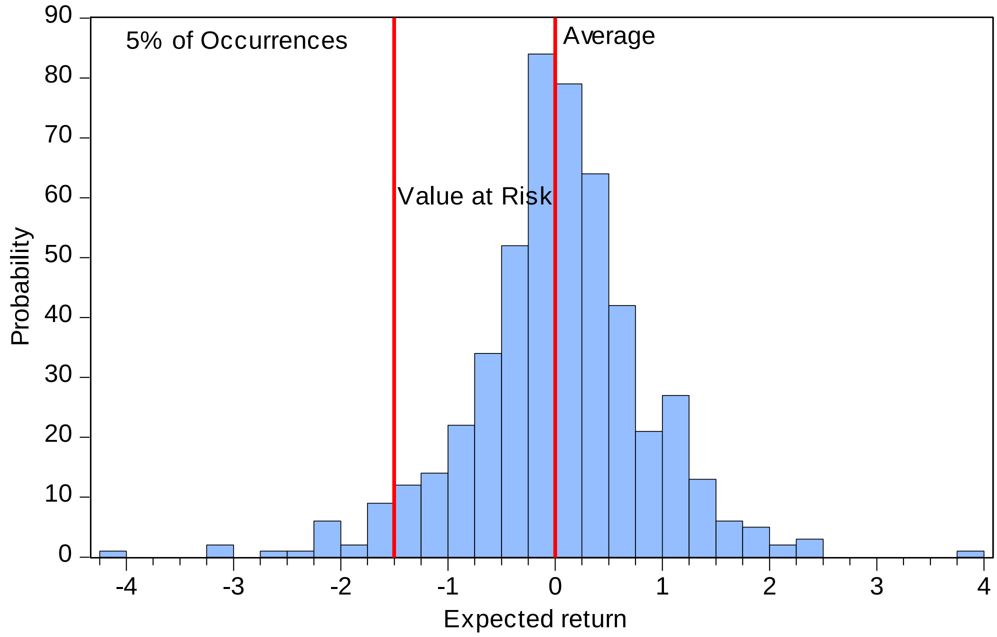

Figure 1.1 S&P500 index return histogram and Value at Risk

Source: Compiled by the author

The Monte Carlo (MC) Simulation[5] method produces random outcomes for a specified set of risks. The process starts by specifying the financial variables and parameters like volatility and correlation, which can be derived from option prices or historical data. After defining the variables, fictitious price paths are simulated which form a distribution of returns. From that generated distribution, a VaR measure can be obtained. The MC method has similar properties as the historical simulation, however the hypothetical changes in prices are created by random draws instead of sampled from past data (Jorion 2001). One of the benefits of the MC method is that the returns are not assumed to be normally distributed, which is often the case in financial data. The model is also very flexible, being capable of including numerous

[5] Statistical method dating back to the 1930s and 1940s, which entails generating random outcomes according to a probability distribution and a set of input parameters. It is used in variety of problems in many fields of sciences (Maginn 2007).

exposures, like the time variation in volatility, extreme scenarios, specified cashflows etc. (Ibid). However, the computational time can be rather large when analysing portfolios with many assets. Also, one has to be careful when specifying assumptions as the estimated results may be distorted when done wrongly (Ibid.).

The variance-covariance method (also sometimes analytical method or the delta-normal method) assumes that the financial returns are normally distributed, which makes the computation relatively easy. To calculate the VaR figures, one has to estimate the variances and covariances of the standardised instruments in the portfolio[6]. The easiest is to obtain the statistics from the historical data. Together with the portfolio weighs of the standardised positions and the covariance matrix, it is possible to calculate the VaR estimates. The problem with this approach is that most of the financial returns have heavy tailed distributions (see subsection 2.1.2.), suggesting that the extreme values are more common than normal distribution presumes, which might lead to underestimating VaR (Jorion 2001). To improve the quality of the method, it is possible to conduct the delta-gamma version of the model. This is done by taking the second order approximations of the returns and adding a convexity coefficient to the equation (Damodaran 2007). The delta-gamma approach is sometimes used when calculating the value at risk of options, however it makes the calculations a lot more complex and therefore will not be used in this research.

The three main methods were discussed referring to historical financial data, however it is possible to use forecasted volatilities and correlations in the computation of VaR for a search of improvement in the results. There have been several studies applying GARCH family models in VaR estimation, including Hung (2008), McMillan (2008), Vlaar (2000), Giot (2004) and Perignon (2008). However, it is important to keep the time horizon in mind, when using forecasted volatility in value at risk estimation. Financial institutions and investors, who have actively traded portfolios usually estimate a 1-day ahead forecast, whereas non-financial and institutional investors prefer longer horizons (Linsmeier 1996). Choosing a long-term forecast period entails some problems, which will be discussed in chapter 2, under volatility modelling.

[6] It is important for the instruments to be standardised to provide useful estimates. For example, a five-year coupon bond could be broken down into five zero-coupon bonds (Damodaran 2007).

1.3. Value at Risk performance measurement

For a model to be useful in practice, one has to know the accuracy of the outcome. The same applies for VaR models, since without proper validation we can never be sure we have reasonable risk estimates. The performance of value at risk models is measured by backtesting the results, which entails comparing the actual trading results to the risk measures generated by the model (Angelovska 2013). There are different ways to backtest a VaR model, one possibility is to use the Proportion of Failure test (POF-test) developed by Kupiec, which examines the losses that exceeded VaR (Kupiec 1995). The aim of the POF-test is to find out whether the number of exceptions is consistent with the confidence level (Ibid.). The number of parameters required to implement a POF-test is small and it is best conducted as a likelihood-ratio (LR) test[7]. The null hypothesis for this test is that the model is behaving well and the expected failure rate is not significantly different from the failure rate suggested by the confidence interval. The null hypothesis formula for the POF-test is brought in equation 1.2 (Dowd 2006):

$$H_0: p = \hat{p} = \frac{x}{T} \tag{1.2}$$

where x is the number of exceptions, T is the number of observations, $\hat{p}$ is the observed failure rate and p is the failure rate suggested by the confidence interval. The formula for computing the test statistic is brought in equation 1.3 (Kupiec 1995):

$$LR_{POF} = -2ln\left(\frac{(1-p)^{T-x}p^{x}}{\left[1-\left(\frac{x}{T}\right)\right]^{T-x}\left(\frac{x}{T}\right)^{x}}\right) \tag{1.3}$$

If the computed LR_{POF} statistic is larger than the critical value of the chi-squared distribution, the null hypothesis will be rejected and one can conclude the model to be inaccurate[8] (Jorion 2001). A shortcoming of the POF-test is that it only considers the frequency of exceptions and

[7] The Likelihood-ratio (LR) test is a statistical test to compare models by calculating the ratio between the maximum probabilities of results using two alternative hypothesis. The decision whether to accept or reject the null hypothesis is based on the value of the LR-statistic. If the value is too large compared to the critical value, the null hyphothesis is rejected. The likelihood-ratio test is widely used and very powerful according to statistical decision theory (Jorion 2001).

[8] Chi-squared distribution (χ^2 distribution) is a probability distribution that is widely used in hyphothesis testing and confirming the goodness-of-fit of statistical models (Jorion 2001).

not the time they occur. This may prove to be a problem if the model produces clustered results (Campbell 2005).

Another backtesting method, also developed by Kupiec, is the Time until first failure (TUFF) test, which measures the time it takes for the first exception to occur (Kupiec 1995). The assumptions are similar to the previously described POF-test and the likelihood-ratio test statistic is described in equation 1.3:

$$\mathrm{LR_{TUFF}} = -2\ln\left(\frac{\mathrm{p}(1-\mathrm{p})^{\mathrm{v}-1}}{\left(\frac{1}{\mathrm{v}}\right)\left(1-\frac{1}{\mathrm{v}}\right)^{\mathrm{v}-1}}\right) \tag{1.3}$$

where v denotes the time it takes for the first exception to occur. Similarly to the POF-test, if the test statistic is below the critical value, the model is accepted as accurate and if not, the null hypothesis is rejected. There has been some criticism toward the TUFF method for exhibiting too low power to identify bad VaR models (Dowd 1998), which is why it will not be used in this research.

The two backtesting models did not take into account the clustering of VaR violations[9], which is important when modelling risk with autoregressive volatility. Large losses that appear in succession tend to be more dangerous than individual exceptions (Christoffersen 2004). The Christoffersen’s Interval Forecast Test, developed by Christoffersen, takes into account the frequency of VaR violations as well as the time when they occur (Christoffersen 1998). The log-likelihood testing framework is the same as for the models proposed by Kupiec, but a separate statistic for independence of violations is added (Jorion 2001). The computational procedure[10] is slightly more difficult compared to the two previously described models as seen from equation 1.3:

$$LR_{ind} = -2ln\left(\frac{(1-\pi)^{n_{00}+n_{10}}\pi^{n_{01}+n_{11}}}{(1-\pi_0)^{n_{00}}\pi_0{}^{n_{01}}(1-\pi_1)^{n_{10}}\pi_1{}^{n_{11}}}\right) \tag{1.3}$$

where n_{ij} is the number of days when condition j occured assuming that condition i occurred on the previous period, π is the probability of registering a violation conditional on state i the day before. As one can see from the equation, the model takes into account the consistency of exceptions to the VaR model. It is also possible to combine the $\mathrm{LR_{ind}}$ statistic and the Kupiec’s POF-test, which would allow one to examine whether the violation was due to

[9] During 1998, J.P.Morgan experienced 20 exceptions to its VaR model and half of which were closely clustered. As a result, the bank revised its VaR models substantially. (Jorion 2001).

[10] Discussed in Christoffersen (1998) and Jorion (2001) in more detail

inaccurate coverage or clustered exceptions, however it is advisable to always run the tests separately as well (Campbell 2005).

2. VOLATILITY MODELLING AND FORECASTING – A LITERATURE REVIEW

Chapter 1 discussed the theory and methods of value at risk modelling and performance measurement. Three main methods for calculating value at risk were presented: the historical simulation method, the Monte Carlo simulation method and the variance-covariance method. One of the most important inputs for calculating value at risk is the standard deviation. Most commonly the historical average volatility is used, however considering that conditional variance shocks are usually followed by high volatility periods, using autoregressive models could produce more accurate results compared to the simple historical volatility. Therefore, value at risk models could also benefit from using the nonlinear autoregressive volatility methods.

This chapter will cover the vast literature of volatility modelling and forecasting from the earliest papers, dating back decades, to newer research papers with more recent data. The aim of this chapter is to discuss the stylised facts of financial returns and give an overview of the theory on volatility modelling and forecasting. This chapter will be divided into four sections. Section 2.1 discusses the stylized properties of financial returns, Section 2.2 analyses the alternative volatility modelling methods, Section 2.3 is dedicated to ARCH volatility models and Section 2.4 discusses the forecasting performance measurement.

2.1. Stylized properties and statistical issues of financial returns

Over the last decades, from the works of Mandelbrot (1963) and Fama (1965), many academics have explored the properties of financial asset dynamics, discovering regularities in the financial markets. One may think that since the information influencing the returns of different asset classes varies, the markets will exhibit different properties. Besides, how could the properties of milling wheat futures be similar to the ones of GBP/USD exchange rate or Apple Inc stock? However, numerous studies, including Cont (2001), Malmsten and

Teräsvirta (2010), show that from a statistical point of view, the asset prices do have significant similarities. Five of these stylized properties will be discussed in this section to set an important framework for this research. The five stylized facts are volatility clustering, heavy tails, return asymmetry, leverage effect and absence of autocorrelation.

2.1.1 Volatility clustering

Volatility is the measure of amplitude and frequency of price fluctuations over a time period. The research in volatility clustering dates back to Mandelbrot (1963) and Fama (1965), indicating that the volatility changes over time and showing the positive autocorrelation in the lags of daily commodity returns as well as stock index returns. This was also the finding of Engle and Bollerslev (1986) with weekly USD/CHF spot prices, Engle and Bollerslev (1993) with USD, Deutsche mark and GBP as well as Ding and Granger (1996) with S&P500 stock index returns, including numerous other researches. Therefore stating that large price changes tend to be followed by large price changes and small price changes are to be followed by small price changes. This kind of persistence has been the cornerstone of volatility modelling over the years.

2.1.2 Heavy tails

Another stylized property of financial returns is that in most cases the probability distribution is not Gaussian[11] (normal distribution) in the sense that the returns have more mass in the centre and have fatter tails. This means the unconditional probability distributions of financial returns are leptokurtic. Mandelbrot (1963) addressed the insufficiency of using a normal distribution in modelling the financial asset returns as well as many other authors, including Fama (1963) and (1965), Mandelbrot and Taylor (1967) to name a few. Analysing the heavy tails has been an important subject in risk management and is necessary to take into account when calculating Value at Risk. Mandelbrot (1963 and 1997) used a method to measure the tails by representing the sample moments as a function of sample size. Jansen and de Vries (1991), Longin (1996) and some others have used the extreme value method instead, analysing the large movements in the data. No matter the method, heavy tails and

[11] Gaussian distribution, also known as the bell curve, first published by Carl Friedrich Gauss (1809) in his monograph "*Theoria motus corporum coelestium in sectionibus conicis solem ambientium*"

leptokurtosis in financial returns is something that needs to be addressed and will be discussed later in this paper to see if our data confirms this stylised fact.

2.1.3 Return asymmetry

Return asymmetry in financial data means that large downward movements or drawdowns in asset prices do not have equally large upward movements. Lins (2009) found in his paper that for stocks and indices, it typically takes longer to gain 5% than to lose 5%. Jensen observed the returns of DJIA to see if the probability to find relatively large negative returns is higher than positive returns for short investment horizons. He concluded that the downward movements are faster than the upward ones, therefore indicating a gain-loss asymmetry in the data (Jensen 2003). There have also been articles finding that return asymmetry is less clear in emerging markets and in individual stocks compared to indices that are mostly used (Karpio 2006). Cont found in his paper that this property is not particularly strong in exchange rates as well, as there is a higher symmetry in up and down moves (Cont 2001). Despite the different findings, there are volatility forecasting models that take this stylised property into account and will later be analysed and compared to other models to see if there is significant improvement in the output.

2.1.4 Leverage effect

Another stylised property of financial returns is the leverage effect, which is related to the return asymmetry. Leverage effect means that the volatility of an asset is negatively correlated with the returns of that asset (Cont 2001). This was first analysed by Black (1973, 1976), who found that volatility tends to increase after large downward movements in stock prices. There can be many explanations for why this effect exists, one discussed by Andersen (2005) is the effect of "volatility feedback", stating that "heightened volatility requires an increase in the future expected returns to compensate for the increased risk, in turn necessitating a drop in the current price to go along with the initial increase in the volatility" (Andersen 2005). On the other hand, Bouchaud (2001) suggests that the leverage effect could in fact arise from simple panic on the market following a negative price shock. There is evidence of such an effect across assets, however Pagan (1996) found it to be less clear for interest rates. Nevertheless, this phenomenon could bring problems when modelling and forecasting volatility with simple, more naive models. However there are models developed to take the leverage effect into account, which will be discussed later in this paper.

2.1.5 Absence of autocorrelation

The last stylized fact discussed in this section is the absence of autocorrelation. That means there are no significant correlations in price increments and financial asset returns. This statistical property is often cited as support for the efficient market hypothesis (Cont 2001). The intuition for the absence of autocorrelation is discussed by Fama (1991), stating that if price changes would be correlated, then a statistical arbitrage would be available with positive expected earnings, which will therefore reduce correlations. He goes on to argue that autocorrelation may be present in very short time scales, since the market has to react to new information. Cont (1997) brings out in his research that the autocorrelation of the price changes decays to zero in a few minutes, which is why it is safe to assume it to be zero for practical purposes. However when looking at weekly and monthly returns, some autocorrelation might be present (Cont 2001).

2.2. Alternative volatility modelling methods

There are numerous methods for modelling volatility, the list of alternative volatility models described in this subsection is not conclusive. Some examples of naive models were chosen to illustrate the easiest alternatives to ARCH-family models.

The most straightforward possibility for volatility estimation is to use the simple historical volatility model, which can be measured by the variance or standard deviation of returns over a chosen period. The result can then be used as the volatility forecast for subsequent periods. This method is widely used in option pricing models for example Black and Scholes (1973). The standard deviation of returns is computed using equation 2.1

$$\sigma = \sqrt{\frac{1}{T-1} \sum_{t=1}^{T} (r_t - \mu)^2} \tag{2.1}$$

where σ is the volatility of an asset; r_t is the return of an asset over the period *t*; *T* is the number of observations and μ is the average return over period *T*.

The simple moving average process is brought in equation 2.2

$$\sigma_t = \frac{(\sigma_{t-1}+\sigma_{t-2}+\cdots+\sigma_{t-n})}{n} \tag{2.2}$$

where the forecasted volatility for the next period is σ_t; and the number of observations included in the simple moving average is n.

Several authors including Merton (1980), Potebra and Summers (1986), French, Schwert and Stambaugh (1987) and Schwert and Seguin (1990) have used this technique to compute monthly stock return variance estimates by taking the average of the squared daily returns. However, Bollerslev (1992) argues that this does not make efficient use of the data. The main shortcoming of using a simple historical model is the trade-off between examining a large sample and trying to avoid including data that is too old and is therefore obsolete (Figlewski 2004). In periods of low volatility, the historical model could produce fair results, however when shocks occur, the estimate moves further away from reality. An extension to the described simple moving average process is the autoregressive moving average (ARMA) model[12], first described by Whittle (1951), which combines the autoregressive AR(p) and moving average MA(q) parts of asset returns. The ARMA(1,1) model is brought in equation 2.3

$$\varphi(L)y_t = \mu + \theta(L)u_t \tag{2.3}$$

where L is the lag operator; y_t is the observation variable, therefore $Ly_t = y_{t-1}$; u_t is the white noise disturbance term, therefore $Lu_t = u_{t-1}$ and μ is the constant term. There are many variations to the general ARMA model[13], however which will not be discussed further in this research.

Another possibility is to use the exponentially weighted moving average of historical volatility which increases the impact of more recent observations on the forecast. The EMWA model is specified in equatiation 2.4:

$$\sigma_{t+1}^2 = \lambda\sigma_t^2 + (1-\lambda)r_t^2 \tag{2.4}$$

where lambda $0 \leq \lambda \leq 0$ is the decay factor, which typically takes a value between 0.94 and 0.97 (Ding 2010). One has to note that a single volatility forecast applies to all future time

[12] The ARMA(p,q) model was popularised by George E. P. Box and Gwilym Jenkins in 1971 (Box and Jenkins 1970, revised 1976).

[13] Mostly used variations to the original ARMA(p,q) include ARIMA, vector ARIMA (VARIMA), non-linear ARMA (NARMA), frictional ARIMA (FARIMA), seasonal ARIMA, ARIMAX and many others. For further research in this field see Hannan, Deistler and Manfred (1988), Schwert (1989) and Xiong (2002).

horizons. The EWMA volatility estimation approach is supported by RiskMetrics[TM;14,15], a set of techniques and data to measure market risks in portfolios of different assets and their derivatives (Lang 2000).

A different approach is to measure the implied volatility, which can be derived from options valuation, for example the Black and Scholes (1973) formula. Naturally, this volatility is dependent on the expectations of market participants and can therefore be called the markets forecast of volatility. This method has been considered by Hsieh Manas-Anton (1988), Jorion (1988), Lyons (1988) and Engle and Mustafa (1992) to name a few. Bollerslev (1992) states in his research, that although the volatility estimates could be rather accurate, not all assets have exchange traded options. Also, problems could arise from market inefficiencies and higher spreads.

2.3. ARCH volatility forecasting models

Autoregressive conditionally heteroscedasticity (ARCH) models, introduced by Engle (1982), are widely used non-linear models that take into account the persistence of volatility. In ARCH(q) models, the variance is dependent on lagged squared deviations where q is the number of lags included in the model. Since the introduction of the ARCH model, several hundred research papers applying this modelling strategy to financial time-series data has already appeared (Bollerslev 1992). However, empirical evidence, including Bodurtha and Mark (1991) and Attanasio (1991), show that to catch the dynamic of conditional variance, one has to select a high number of lags (high ARCH order). This can make the calculations burdensome, since a large number of parameters have to be estimated. The autoregressive conditional heteroscedasticity model is specified in equation 2.5, where the variance is forecasted as the moving average of past error terms (Engle 2012),

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 \tag{2.5}$$

$$y_t - \mu_t = \varepsilon_t = \sigma_t z_t \tag{2.6}$$

[14] RiskMetrics Technical Document description (Riskmetrics 1996).

[15] RiskMetrics[TM] was released in 1994 and was spun off from J.P.Morgan four years later in 1998. Over the years it has become an internationally-accepted standard for measuring VaR (Lang 2000).

where y_t is an asset return and z_t belongs to an i.i.d. process as an unobservable random variable, with variance equal to 1 and mean equal to $0; var(z_t) = 1\ ;\ E(z_t) = 0;$. Conditional variance is denoted by σ_t^2 in the equation and parameters $\omega > 0;\ \alpha \geq 0$. The formula for error term, denoted as ε_t, is shown in equation 2.5. The specified model is an ARCH(1) model, with one lagged residual term, however any number of lags can be used[16]. As seen from the equation, the forecasted error variance at time *t* is based on the information from the previous period *t – 1*, making the model autoregressive. Also, knowing the past errors, the model leaves no uncertainty on the estimated squared error at time *t* (Engle 2012).

2.3.1 GARCH model

Four years after the previously described ARCH model by Engle (1982), an extension was developed by Bollerslev (1986), called the generalised autoregressive conditionally heteroscedastic (GARCH) model. The proposed model was a solution to the high-order problem in ARCH, since the generalized model includes lagged squared deviations and lagged variances, reducing the number of estimated parameters. The most used form of this model is the GARCH(1,1) model, however it can be extended to a GARCH(*p,q*) formulation with q lags of the squared error and p lags of the conditional variance. But according to Bollerslev (1988) and numerous other researches, a general GARCH(1,1) model is found to suffice in most applications.

ARCH models are very widely used in time-series analysis, possibly for their simplicity and goodness-of-fit. GARCH is the generalised form of the ARCH model. In GARCH(1,1), the conditional variance is also dependent on its own previous lags, but has decreasing weights that never go completely to zero (Engle 2012). An advantage of GARCH models is that the returns are not assumed to be Gaussian, solving the heavy tails problem (see subsection 2.1.2.), also the model is able to account for volatility clustering (Bauwens 2012). The equation is brought below (equation 2.7),

$$\sigma_{t|t-1}^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1|t-2}^2 \tag{2.7}$$

[16] Empirical evidence, including Bodurtha and Mark (1991) and Attanasio (1991), show that often a large number of lags will be needed to catch the dynamic of conditional variance, however to reduce the number of parameters, an ad hoc linearly declining lag structure was often imposed in some earlier applications. For more information see Engle (1982), Engle (1983) and Bollerslev (1992).

where σ_t^2 is the conditional variance; parameters are denoted as ω, α and β, where ω is the long-term average value; $\alpha\varepsilon_{t-1}^2$ is the volatility of the previous period and $\beta\sigma_{t-1|t-2}^2$ is the fitted variance from the previous period (Andersen, 2005). The positivity of σ_t^2 is ensured by a restriction that parameters $\omega > 0$; $\alpha \geq 0$ and $\beta \geq 0$.

The estimation process of this model is fairly simple, the maximum log-likelihood will be used. Although it is an efficient model, there are some problems arising. Figlewski (2004) proposed three main shortcomings of this model: the impacts of the shocks are independent of its sign, therefore not taking into account the asymmetry in the data; the models require a large number of observations to behave well; long horizon forecasts are not as informative since the model converges to the long-run variance. These limitations of the GARCH model will also be tested with the data used in this research, to see whether these shortcomings are evident or not[17].

2.3.2 GJR-GARCH model

The GARCH model proposed by Bollerslev (1986) assumes that positive and negative shocks of the same magnitude will influence the future conditional variances identically. This comes from squaring the lagged error and therefore losing the sign of the lagged residuals. However, past researches including Nelson (1991) and Glosten (1993) show that there are asymmetric properties present in financial data, with negative shocks resulting in larger future volatility (see subsection 2.1.3. and 2.1.4.) compared to positive price shocks. There are some models that address this asymmetry and leverage effect, two of which will be used in this paper.

GJR-GARCH (GJR)[18] model developed by Glosten, Jagannathan and Runkle (1993) is an extension of GARCH where an additional term is added to account for asymmetries in the data. The additional ARCH term is with the sign of the past innovation and assigns stronger impact to the past negative volatility shocks on the future conditional variances. The formula for GJR(1,1) model is brought in equation 2.8:

[17] For more extensive research on GARCH(p,q) models, see Bollerslev et al.(1992), Bera and Higgins (1993), Bollerslev et al. (1993), Diebold and Lopez (1996), Pagan (1996), Palm (1996), Shephard (1996), Giraitis et al. (2006), Nakatani and Teräsvirta (2009).

[18] Also known as TARCH for Threshold ARCH, developed by Zakoian (1994).

$$\sigma_{t|t-1}^2 = \omega + \alpha\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 I(\varepsilon_{t-1} < 0) + \beta\sigma_{t-1|t-2}^2 \quad (2.8)$$

where I is the indicator function, a dummy variable, which takes a value of 1, when $\varepsilon_{t-1} < 0$ and value of 0, when $\varepsilon_{t-1} \geq 0$ (Andersen, 2005). The coefficient of the asymmetry term is denoted as γ, and it is seen from the equation that when $\gamma > 0$, negative shocks will have a larger impact on conditional variance than positive shocks. Also one can notice that GARCH(1,1) model is nested inside the GJR(1,1) model, therefore if there are no leverage effects (coefficients are zero), then the model reduces to the GARCH(1,1) model. That means it is possible to test these models against each other with the likelihood ratio test[19], which however is not the purpose of this research.

2.3.3 EGARCH model

Another asymmetric generalized ARCH model used in this paper is called the exponential GARCH (EGARCH) model, introduced by Nelson (1991). This model uses logarithmic form of conditional variance and accounts for both magnitude and sign of the volatility shock. The model is described as EGARCH(p,q), with q lags of the squared error and p lags of the conditional variance just like the generalized ARCH model and similarly, the EGARCH(1,1) formulation is mostly used due to less computational difficulty and goodness-of-fit based on empirical evidence (Nelson 1991). The formula for EGARCH(1,1) model is brought in equation 2.9 (Andersen and Bollerslev 2006):

$$log_e\left(\sigma_{t|t-1}^2\right) = \omega + \alpha\left(|z_{t-1}| - E(|z_{t-1}|)\right) + \gamma z_{t-1} + \beta log\,(\sigma_{t-1|t-2}^2) \quad (2.9)$$

where σ_t^2 is the conditional variance, $\beta\log\,(\sigma_{t-1|t-2}^2)$ is the fitted variance from the previous period and $z_t \equiv \sigma_{t|t-1}^{-1}\varepsilon_t$ (Andersen, 2005). Unlike the generalized ARCH model, there are no restrictions on the parameters α and β for the conditional variance to be nonnegative due to the logarithmic variance. Similarly to the previously described GJR-GARCH model, values of the leverage term $\gamma > 0$ assignes a larger impact for negative shocks on the conditional variance. According to research by Andersen (2006), EGARCH model can often be somewhat more difficult to estimate and analyze numerically compared to other models, since the interest is in point forecasts for σ_{t+h}^2, not $log_e(\sigma_{t+h}^2)$. The three previously discussed ARCH

[19] A statistical test proposed by Cai, Fan and Yao (2000) and Fan, Zhang and Zhang (2001), that is used to test the goodness-of-fit of two models through hypothesis testing.

models will be estimated on the data and results analyzed later in the paper together with the comparison of the models.

2.3.4 Other ARCH models

The number of ARCH models developed over the years is vast, some more widely used than others. The aim of this subsection is not to give an overview of all the models developed, rather than mention some more remarkable extensions to the general ARCH model.

The three ARCH models discussed in Chapters 2.3.1-2.3.3 all imply that the shocks to the conditional volatility decay at an exponential rate. They can therefore be called short memory volatility models. Since in many occasions the forecasted period sought after is longer than one period ahead, researchers have tried to model long-memory volatility models. One of those models proposed by Baillie, R. T., Bollerslev, T. and Mikkelsen, H. O. (1996) is the Fractionally Integrated GARCH (FIGARCH) model, where the volatility forecasts will decay at a slow hyperbolic rate. This makes the forecasts of longer horizons more accurate compared to regular models. Fractionally integrated EGARCH (FIEGARCH) developed by Bollerslev and Mikkelsen (1996) has similar properties, but introducing logarithmic conditional variance to the equation.

Other considerable extensions to the ARCH family models include the threshold GARCH model (TGARCH) by Zakoian (1994), the AGARCH by Engle (1990), the IGARCH by Engle and Bollerslev (1986), the NGARCH of Higgins and Bera (1992), APARCH model proposed by Ding, Granger, and Engle (1993), the H-GARCH of Hentshel (1995), NA-GARCH and the V-GARCH models suggested by Engle (1993).

2.4. Forecasting performance measurement

There are many methods to evaluate the performance of econometric models. In simple regressions, R^2, the standard forecast criteria is often used. However, this is not appropriate for volatility models according to Andersen (1998) due to the "inherent noise in the return generating process". This calls for a different approach. In order to measure the performance of selected models, an appropriate benchmark should be selected. In this thesis, realised volatility will be used for that purpose, since it provides a natural benchmark for model evaluation, it is easily computable and at the same time very intuitive. The formula for calculating realised volatility is in equation 2.10,

$$RV^{(n)} = 100 \cdot \sqrt{\frac{m}{n} \sum_{t=1}^{n} R_t^2} \qquad (2.10)$$

where *RV* is the realised volatility, *m* is the number of trading days in a year (usually 252), *t* is the counter representing a trading day, *n* is the number of trading days in the measurement period and R_t is the continuously compounded daily returns. The daily returns used in the realised volatility equation as well as estimating the models, are calculated logarithmically (equation 2.11),

$$R = ln \frac{S_T}{S_0} \qquad (2.11)$$

where *R* represents the continuous return of an asset in period *T*, S_0 is the initial price and S_T is the price at time *T*. Since realised volatility depends on past information, it is not suitable for out-of-sample forecast evaluation. However, when modelling volatility, it is possible to take that into account and decrease the period used, keeping some of the data for later evaluation or just obtain additional, more up to date information when possible.

In addition to evaluating the forecasting ability for selected models, the comparison between them is also of interest. For that reason, the root mean squared error (RMSE)[20], Akaike information criterion (AIC) and Bayesian information criterion (BIC) will be computed. The formula for RMSE is brought in equation 2.12,

$$RMSE = \sqrt{\frac{1}{N} \sum_{t=1}^{N} (\hat{\sigma}_t - \sigma_t)^2} \qquad (2.12)$$

where σ_t is the observed volatility on period *t*, $\hat{\sigma}_t$ is the volatility forecast and *N* is the number of days in the data set (Ladokhin 2009). The root mean squared error measures the error in terms of average deviations. According to Ladokhin, RMSE is a very popular error function among practitioners, but is not always the best measure when comparing models Ladokhin (2009). When doing the latter, calculating the Akaike information criterion is a widely used

[20] See Poon (2005) for more information on different error functions

method. The formulation for Akaike information criterion, introduced by Akaike (1974)[21] is in equation 2.13,

$$AIC = 2k - 2\,ln(L) \tag{2.13}$$

where k is the number of parameters in the model and L is the maximised value of the likelihood function for the model. Akaike information criterion makes adjustments to the likelihood function to take into consideration the number of parameters, which might be different across models. Another widely used method to compare models is the Bayesian information criterion, developed by Schwarz (1978). The BIC formula[22] is shown in equation 2.14,

$$BIC = -2 \cdot ln(L) + k \cdot ln(n) \tag{2.14}$$

where L is the maximised value of the likelihood function, n is the number of observations and k is the number of parameters to be estimated.

To measure the goodness-of fit of selected models as well as compare the models to each other, the correlation between the modelled volatility and realised volatility is sometimes computed. The equation for calculating the correlation for time-series data is brought in equation 2.15,

$$cor(X,Y) = \frac{E[(X-E(X)(Y-E(Y)]}{\sigma_X \sigma_Y} \tag{2.15}$$

where X and Y identifes the two time-series, σ_X is the standard deviation of values in time-series X and σ_Y is the standard deviation of values in time-series Y. $E(X)$ and $E(Y)$ represent the mean values of time-series data values from X and Y (Sayal 2004).

[21] For further reading about AIC, see Anderson (2008) and Liu, Yang (2011).
[22] For limitations of the BIC formula, see Giraud (2014).

3. RESULTS OF EMPIRICAL STUDIES

Since the early 1990s, when value at risk started to gain acceptance among practitioners, there has been a search for ways to improve the risk quantifying measure. There are numerous studies addressing this issue and backtesting different variations of VaR models across assets. Chapter 1 analysed three main methods for calculating value at risk and standard deviation of financial returns was a key variable in all of them. Instead of using simple historical average volatility, which according to Bollerslev (1992), Figlewski (2004) and numerous other studies underestimates the standard deviation in high volatility periods, one could use more sophisticated models in search of better VaR estimates. Chapter 2 discussed the developments in volatility modelling over the last three decades and confirmed the benefits of using autoregressive models over more naïve methods. This chapter is going to give an overview of the results in empirical studies estimating the value at risk with ARCH volatility models and the empirical evidence will later be compared to the results of this thesis to see whether they reach the same conclusions or not.

There have been several studies applying ARCH family models in VaR estimation, some selected research papers are brought in Table 3.1 together with the models, datasets and conclusions withdrawn from the analysis. The list is not conclusive, however these papers have been selected for their contribution on the topic, interesting results and differences in datasets. Most of the previous researches in Table 3.1 use the variance-covariance method of estimating value at risk and some do so by ignoring the covariances between the assets in the portfolio. This can prove to be a problem in some cases, however according to Lucas (2000), the more sophisticated VaR models based on the estimates of variance-covariance matrices do not perform significantly better compared to univariate value at risk models that only require conditional variance estimates.

One issue that was addressed by some of the researchers was the size of the sample. Hendricks (1996), Danielsson (2002) and Vlaar (2000) found that increased sample size generated better VaR results. On the contrary, Frey and Michaud (1997) and Hoppe (1998) argued smaller amount of data to be more accurate due to better capturing the behavioural change in the trading activity (Hoppe 1998). Angelidis, Benos and Degiannakis (2004) conducted a research that included numerous different GARCH variations in VaR estimation process on five popular equity indices, where one of the goals was to analyse the sample size problem (Angelidis 2004). A rolling sample of 500, 1000, 1500 and 2000 observations showed no significant differences in the model performance. Nevertheless this is an issue worth addressing when modelling VaR.

Table 3.1 Empirical studies on VaR estimation

Author (year)	**Model**	**Dataset**	**Conclusions**
Angelidis (2004)	GARCH variations	5 equity indices: S&P500, NIKKEI 225, DAX30, CAC40, FTSE100	EGARCH outperformed others; no significant differences in sample sizes
Aydin and Korkmaz (2002)	GARCH (1,1), EWMA	ISE-30 index	GARCH (1,1) outperformed EWMA
Bali (2007)	10 popular GARCH variations	Daily S&P 500 data from 1950-2000	EGARCH best overall performance
Berkowitz and O'Brien (2002)	ARMA (1,1), GARCH (1,1)	Historical data of six banks	GARCH outperformed ARMA
Engle (2001)	ARCH and GARCH	Portfolio of Nasdaq, DJIA, 10-year treasury bonds	Both models performed very well
Frey and Michaud (1997), Hoppe (1998)	ARCH models	Variety of datasets	Smaller sample size leads to more accurate VaR estimates
Galdi (2007)	EWMA, GARCH, Stochastic volatility	Preferred shares of Petroleo Brasileiro SA	EWMA suffered fewer violations than GARCH
Giot and Laurent (2003)	ARCH models	Six commodities	GARCH performed well, complex APARCH was superior
Hendricks (1996), Danielsson (2002), Vlaar (2000)	ARCH models	Variety of datasets	Increased sample size generated better VaR estimates
Jansky and Rippel (2011)	ARMA(q), GARCH, EGARCH	Six world stock indices from 2004–2009	GARCH and EGARCH outperformed ARMA
McMillan et al (2008)	GARCH models	Euro exchange rate intraday data	GARCH outperformed more complex models
Orhan (2011)	GARCH (1,1)	Six ISE indices before and during financial crisis	GARCH(1,1) behaved well during the crisis

Source: Compiled by the author

Majority of research papers that compared nonlinear volatility models found GARCH outperforming other alternatives. The performance measurement was often done by comparing the AIC and BIC of volatility models or POF, TUFF and Christoffersen's Interval Forecast Test of value at risk estimates (see section 1.3 and 2.4). Aydin and Korkmaz analysed the VaR estimates for Istanbul Stock exchange index (ISE-30) using GARCH(1,1) and EWMA (used in Riskmetrics^{TM}) as volatility inputs and concluded that the first outperforms the latter (Aydin and Korkmaz 2002). The same result was reached by Berkowitz and O'Brien (2002) and Jansky and Rippel (2011) when comparing the GARCH model with ARMA. These papers used the returns of several banks and indices as samples, but a different conclusion was made by Galdi (2007) when using the data of a single stock (Preferred shares of Petroleo Brasileiro SA). Galdi (2007) concluded, that although GARCH(1,1) model behaved well, the EWMA model suffered fewer violations compared to the generalised ARCH formula. There has not been many papers on estimating value at risk using the returns of single stocks, however the good performance of the GARCH volatility model seems to be evident in most cases.

Another popular extension of the ARCH family is the EGARCH model, which uses the logarithmic form of conditional variance and accounts for both magnitude and sign of the volatility shock (see subsection 2.3.3). It has been found effective in high volatility periods, where good VaR estimates are vital. Bali (2007) analysed the value at risk of S&P500 using ten most popular GARCH variations over the period of 1950-2000 (Bali 2007). The observation period is long, containing several shocks to the market and different economic cycles. The EGARCH(1,1) model outperformed other variations, capturing the conditional variance most accurately. Angelidis conducted a similar paper on five equity indices (S&P500, NIKKEI 225, DAX30, CAC40, FTSE100) and reached the same conclusion (Angelidis 2004). Despite EGARCH being often favoured during high volatility periods, Orhan found GARCH(1,1) capturing the variance effectively as well when he observed the VaR estimates on six Istanbul Stock Exchange indices before and during the financial crisis (Orhan 2011).

3.1. Conclusions

Empirical studies have not resolved the issue of appropriate sample size and do not agree on the most effective volatility models for estimating value at risk. The sampling periods and datasets are different and further research on this topic is necessary. Nevertheless, the generalised ARCH model seems to be performing well and even outperforming many alternative models according to several papers. During periods of high volatility, the exponential GARCH has proven useful, since it accounts for both magnitude and sign of the volatility shock, capturing the conditional variance more accurately than other models.

It should also be noted that there has not been many cross-asset analysis conducted on this topic. Most researchers focus on a stock index or list of indices, not comparing the results of different asset classes, which can be beneficial when computing value at risk on a well diversified portfolio.

4. VALUE AT RISK USING FORECAST VOLATILITY – AN EMPIRICAL ANALYSIS

The aim of this research was to evaluate the goodness-of-fit of nonlinear autoregressive volatility models in estimating value at risk. So far, the thesis researched and analysed the value at risk and volatility modelling techniques as follows. Chapter 1 presented the literature review and necessary tools for estimating value at risk. Chapter 2 discussed the stylized facts of financial returns and the theory of volatility modelling and forecasting. The empirical results of previous studies on modelling value at risk with forecasted nonlinear autoregressive volatility models was brought in Chapter 3, raising several interesting research problems. There are three main research questions raised in thesis. Firstly, whether using sophisticated volatility models brings significant advantages over using more naïve alternatives; secondly, if using Student-t distribution over Gaussian distribution improves the goodness-of-fit of selected models and thirdly, if the number of observations have a clear impact on the forecasting results. Numerous previous empirical analysis found GARCH(1,1) volatility model outperforming other alternatives, hence the comparison between different models in this thesis as well. Also, the sample size issue raised by some academics will be tested with 500, 1000 and 2000 observations to see whether the results differ significantly.

This chapter is divided into seven sections, from describing the data and models to evaluating the estimated conditional variance and value at risk models as well as discussing the results. In the end of this chapter, a conclusion will be made to which extent the results are in line with previous findings and suggestions for further research will be offered. To reach the desired conclusions, one day ahead out-of-sample forecasting performance of four nonlinear volatility models will be evaluated and compared. The analysis uses cross-asset datasets with different sample sizes, including one equity index, a bond index, a currency index and a commodity index, all with 500, 1000 and 2000 observations.

4.1. Describing the data

Since this research focuses on cross-asset analysis of value at risk, daily closing price data of 4 different financial instruments were chosen. These instruments are the DAX index, US 10Y Treasury note futures, US dollar index and S&P GSCI index. Table 4.1 shows the chosen instruments, their Bloomberg tickers, asset classes and regions. The sample consists of daily return data from 01/01/2006 – 01/12/2015 with sample sizes of 500, 1000 and 2000 to compare the effect of different amounts of data on estimation results. Using the selected sample, 300 step-ahead forecasts of the conditional variance will be estimated using a rolling window of 200, 700 and 1700 observations. All the data is retrieved from Bloomberg database.

Table 4.1 Financial instruments

Security name	Security code	Bloomberg ticker	Asset class	Region
DAX Stock index	DAX	DAX Index	Equity	Europe
10Y US Treasury Note Futures	TY	TY1 Comdty	Fixed Income	US
US Dollar index	DXY	DXY Curncy	Currency	World
S&P GSCI Index spot CME	SPG	SPGSCI Index	Commodity	World

Souce: Bloomberg (2015). Compiled by the author

The particular financial instruments were chosen since they are widely used as benchmarks in their asset class and are commonly traded. Asset classes behave differently in various economic cycles, which is why observing chosen models across several assets makes an interesting research subject. Figures 4.1 - 4.4 illustrate the daily price movements and returns of chosen financial instruments from 01/01/2006 – 01/12/2015. All the returns have been calculated logarithmically (see equation 2.10). It is possible to see from the figures, that high volatility periods tend to be followed by higher volatility periods and low volatility periods followed by periods of lower volatility, which was also concluded by Madelbrot (1963) and Fama (1965). This kind of persistence in the volatility of financial returns indicate that there is conditionally heteroscedastic error term present in the data which is why GARCH models would be appropriate to capture the variance (see chapter 2).

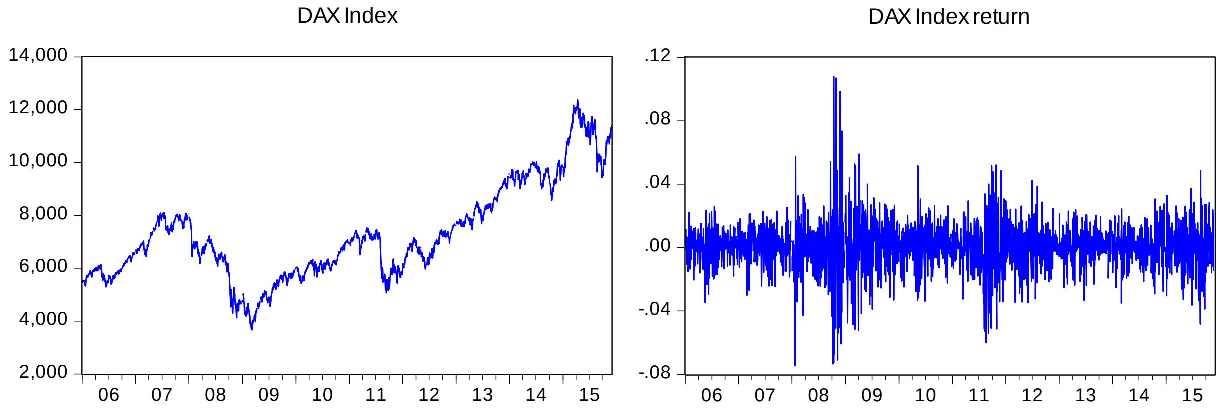

Figure 4.1 Daily prices and returns of DAX from 01/01/2006 – 01/12/2015
Source: Bloomberg (2015); Compiled by the author

The Deutsche Boerse AG German Stock index (DAX) represents the total return of 30 selected German blue chip stocks that trade on the Frankfurt Stock Exchange. The base value is 1000 as of December 31, 1987 (Bloomberg 2015). DAX index has shown significant gains over the observation period as seen from Figure 4.1 with large movements during the financial crisis of 2008 and 2009.

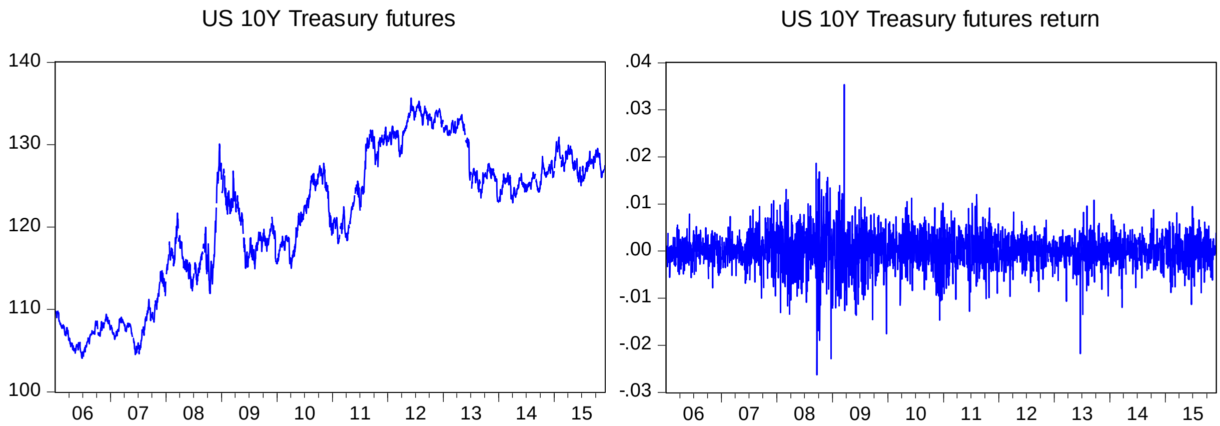

Figure 4.2 Daily prices and returns of TY from 01/01/2006 – 01/12/2015
Source: Bloomberg (2015); Compiled by the author

The fixed income asset class is represented by the 10-year US Treasury Note futures (TY), trading on the Chicago Board of Trade. The value of one contract is 100 000 USD and it consists of US Treasury Notes with a maturity of at least 6.5 years, but not more than 10 years from the first day of the delivery month (Bloomberg 2015). As one would expect, the returns in Figure 4.2 are much more stable compared to the equity index described below. Although it seems there have been negative shocks during 2008 and 2009 as well as 2013.

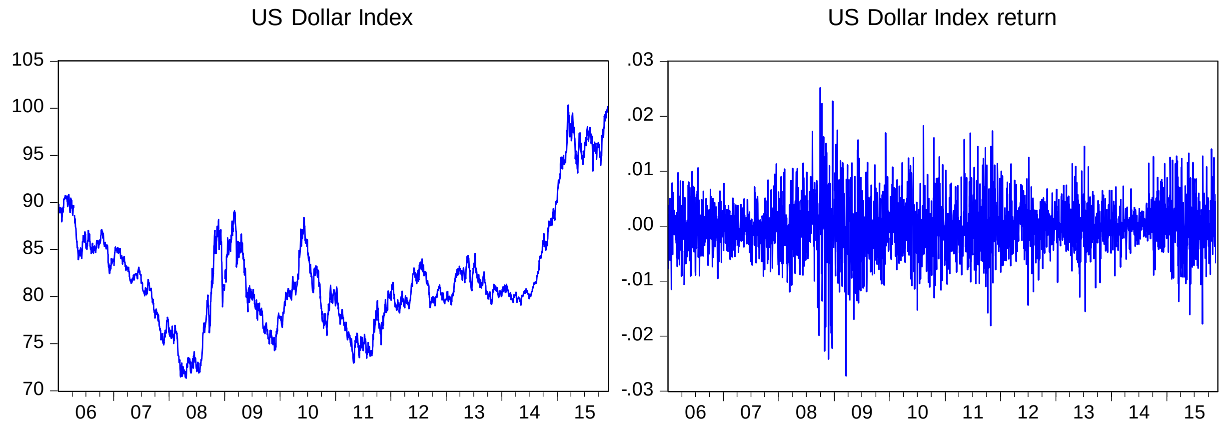

Figure 4.3 Daily prices and returns of DXY from 01/01/2006 – 01/12/2015
Source: Bloomberg (2015); Compiled by the author

The US Dollar index (DXY) indicates the general value of the USD by averaging the exchange rates between the USD and major world currencies. It is computed by the Intercontinental Exchange (ICE) and uses the rates supplied by some 500 banks (Bloomberg 2015). The return graph of DXY is rather noisy compared to the treasury futures, however there seem to be less outliers present in the data.

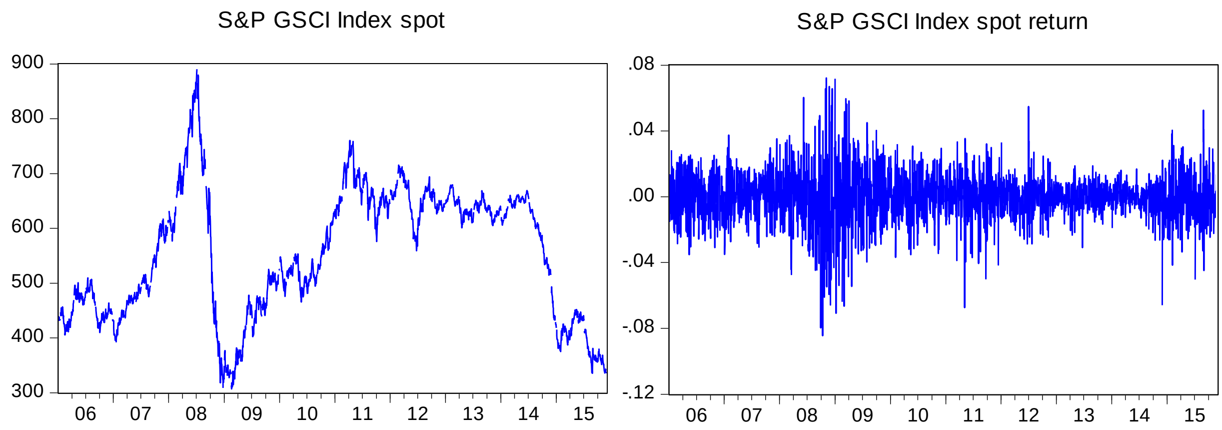

Figure 4.4 Daily prices and returns of SPG from 01/01/2006 – 01/12/2015
Source: Bloomberg (2015); Compiled by the author

S&P GSCI Index represents the general price movements in the commodity markets and is widely recognised as a leading benchmark in the asset class (Bloomberg 2015). The index, similarly to DAX, has seen large fluctuations over the observation period (Figure 4.4). During the financial crisis in 2008, the index lost nearly 60% of its value, being the single biggest movement from its creation in the beginning of 1970.

The descriptive statistics of selected financial instruments is brought in table 4.2, where DAX, TY, DXY and SPG represent the price information and DAXR, TYR, DXYR and SPGR represent the information about the respective returns. As seen from table 4.2, the mean returns of all financial assets are practically zero, even slightly negative in the case of SPGR partly due to the global financial crisis, but also general downward trend in commodity prices. The standard deviation of returns is higher for DAXR and SPGR, showing that equities and commodities have been relatively more volatile than currencies and fixed income. One can see that TYR, DXYR and SPGR have negative skewness, which is usual among financial data (see section 2.1). Also, all of the returns are leptokurtic, meaning they have more mass in the centre and have fatter tails. The return asymmetry and excess kurtosis is not problematic, however it is reasonable to take it into account when modelling volatility and estimating value at risk.

Table 4.2 Descriptive statistics of selected financial assets from 01/01/2006 – 01/12/2015

Security/ Statistic	**Mean**	**Median**	**Max**	**Min**	**Std. Dev.**	**Skewness**	**Kurtosis**	**No. Obs**
DAX	7268,75	6921,37	12374,73	3666,41	1798,17	0,7233	2,9424	2521
TY	121,35	123,67	135,66	104,08	8,53	-0,4352	2,0607	2500
DXY	82,21	80,93	100,33	71,33	5,94	0,9568	3,7946	2578
SPG	552,47	560,63	890,29	306,77	116,02	-0,0141	2,1874	2499
DAXR	0,00029	0,00102	0,1080	-0,0743	0,0144	0,0273	8,8905	2520
TYR	0,00006	0,00013	0,0354	-0,0263	0,0040	-0,1197	8,0568	2499
DXYR	0,00004	0,00000	0,0252	-0,0273	0,0051	-0,0237	5,0166	2577
SPGR	-0,00010	0,00040	0,0721	-0,0845	0,0152	-0,2711	6,2467	2498

Source: Compiled by the author

Figures 4.1 – 4.4 show that the financial asset price data are not stationary, while this might be the case for return series. However, it is possible to test this with the Augmented Dickey-Fuller test (ADF test), which is an extension to the previously developed Dickey and Fuller (1979) test. The ADF test is designed to test for the unit root in the sample and if the t-statistic is highly negative, it is possible to conclude that the data is stationary. The test results are brought in table 4.3. It can be seen from table 4.3 that the test statistic is highly negative in all cases, meaning that it is possible to reject the null hypothesis and conclude that the data is stationary with a significance level of more than 1%.

Table 4.3 The Augmented Dickey-Fuller test results

Augmented Dickey-Fuller test		
Test critical values	**1% level**	**5% level**
	-3,432792	-2,862505
Security	**t-statistic**	**Probability**
DAXR	-50,02701	0,0001
TYR	-49,67963	0,0001
DXYR	-49,99469	0,0001
SPGR	-22,76462	0,0001

Source: Compiled by the author

Since nonlinear volatility models are used in this research, it is appropriate to test for the linearity of the data to see whether nonlinear models are the best choice. One possible way of testing for non-linearity is to use the BDS test developed by Brock, Dechert and Sheinkman (1987). The BDS test detects nonlinear serial dependencies in the data and therefore can be used to ascertain whether the data are noisy as well as determining the goodness-of-fit of estimated models. However, the mentioned test will not indicate what type of non-linearity is present in the data (Brooks 2000). In this case, the test is conducted on raw financial returns which is why the goodness-of-fit is not determined, however it is possible to conclude whether the data is non-linear. The test uses first-differencing (or detrending) in order to remove the linear structure from the data.[23] The BDS test results are brought in table 4.4 and they show that the test statistics for the standardised residuals are highly significant in all cases. This indicates the non-linearity in the data and justifies the use of non-linear models.

Now that there is evidence of the non-linearity and stationarity of the data, it would be appropriate to test whether there are ARCH effects in the returns. The theory behind the autoregressive conditionally heteroscedastic effects in the data was described in chapter 2, meaning that there is autocorrelation in the squared residuals.

[23] The null hyphothesis is that the remaining residuals are i.i.d, independent and indentically distributed Brock, Dechert and Sheinkman (1987).

Table 4.4 Test results for non-linearity using Brock, Dechert, Scheinkma (BDS) test

BDS test								
	Dimension 2				**Dimension 3**			
Security	**Statistic**	**Std.Error**	**z-stat.**	**Prob**	**Statistic**	**Std.Error**	**z-stat.**	**Prob**
DAXR	0,014477	0,001834	7,894762	0,0000	0,032362	0,002907	11,13208	0,0000
TYR	0,011476	0,001708	6,718090	0,0000	0,025961	0,002708	9,586297	0,0000
DXYR	0,006601	0,001649	4,002922	0,0001	0,017140	0,002621	6,539752	0,0000
SPGR	0,016247	0,001798	9,036805	0,0000	0,034736	0,002853	12,17623	0,0000

Source: Compiled by the author

To test for the autocorrelation, a heteroscedasticity ARCH test should be performed on the ARMA(1,1) model. If the F-statistic turns out to be statistically significant, it is possible to conclude that there is ARCH effect present in the data. The ARMA(1,1) model is described in section 2.2 and the test results for the ARCH effects for selected financial assets is in table 4.5.

Table 4.5 Heteroskedasticity test results for DAXR, TYR, DXYR and SPGR

Security	**F-Statistic**	**Prob. F (2,2582)**
DAXR	110,1865	0,0000
TYR	17,37725	0,0000
DXYR	47,35708	0,0000
SPGR	82,56607	0,0000

Source: Compiled by the author

Table 4.5 shows that the F-statistic and the associated critical value allows the null hypothesis to be rejected, indicating that the lags of the squared residuals have coefficient values not significantly different from zero. This means that the ARCH effect is present in the data of all chosen financial instruments and ARCH-type models are appropriate for estimating volatility.

4.2. Describing the conditional variance models

The conditional variance models used in this analysis were selected based on previous empirical studies including Aydin and Korkmaz (2002), Berkowitz and O’Brien (2002) and Bali (2007), discussed in more length in Chapter 3. The GARCH(1,1), EGARCH(1,1) and

GJR-GARCH(1,1) outperformed other volatility models in several occasions by producing more accurate conditional variance estimates, which is why they will be compared in this analysis. Also, to conclude whether using more sophisticated volatility models outperform more naïve alternatives, simple ARMA(1,1) model will also be included. All the models will be calculated using the Student-t and Gaussian distribution with sample sizes of 500, 1000 and 2000. The estimated models for DAXR are shown in table 4.6, the models for TYR, DXYR and SPGR are shown in appendix 3. When performing another heteroscedasticity ARCH test on the estimated GARCH(1,1) models to see if there are any ARCH effects left in the residuals, it can be seen that the values of the F-statistics are very low, showing that the ARCH effects are not significant anymore. This means that the model explains all of the conditional heteroscedasticity present in the data. The corresponding values for the F-statistics are presented in Table 4.7.

One characteristic of GARCH volatility models, also observable in this case, is that the sum of the coefficients of the lagged squared error and the lagged conditional variance is close to unity. This shows the high persistency of the shocks to the conditional variance, which is useful when considering the dynamical properties of volatility. This means that high volatility in this period would be followed by high volatility in the next period, which is in line with economic literature, including Engle and Bollerslev (1986) and Ding and Granger (1996) (see subsection 2.1.1). It can also be seen that the variance intercept *C* has very low value, indicating that the long-term variance is not affecting the volatility estimates as much as recent shocks. It is not possible to interpret the parameter values of EGARCH(1,1) models as intuitively as for GARCH or GJR-GARCH models since it is in the logarithmical form. However, the signs of ARCH and GARCH coefficients have to be positive and the leverage coefficient negative (also observable in table 4.6), since large negative shocks will increase the variance in the next period. The GJR-GARCH(1,1) model, which is also an asymmetric nonlinear extension to the general ARCH model, also includes a leverage effect, however in a non-logarithmic form. Therefore, the leverage coefficient is positive ($\gamma>0$) and the lagged negative shocks will have larger impact on the conditional variances in the future. The estimated leverage coefficients for GJR-GARCH(1,1) models are rather large, which could lead to overestimating the movements in the variance and needs be tested through comparing the goodness-of-fit of selected models.

Table 4.6 Conditional variance models for DAXR from 01/01/2006 – 01/12/2015

Distribution	No. Obs	Model	Constant (ω)	GARCH (α)	ARCH (β)	Leverage (γ)
Gaussian	500	GARCH(1,1)	4,484E-06	0,8763	0,0989	
		Std error	(2,982E-06)	(0,0324)	(0,0252)	
		EGARCH(1,1)	-0,5334	0,9392	0,1465	-0,1349
		Std error	(0,1450)	(0,0169)	(0,0416)	(0,0299)
		GJR-GARCH(1,1)	6,949E-06	0,8728	0,0038	0,1584
		Std error	(1,085E-06)	(0,0282)	(0,0240)	(0,0442)
	1000	GARCH(1,1)	2,975E-06	0,9100	0,0695	
		Std error	(1,776E-06)	(0,0184)	(0,0126)	
		EGARCH(1,1)	-0,4386	0,9501	0,1450	-0,1283
		Std error	(0,0921)	(0,0103)	(0,0261)	(0,0183)
		GJR-GARCH(1,1)	4,586E-06	0,8914	0,0119	0,1356
		Std error	(1,725E-06)	(0,0193)	(0,0092)	(0,0263)
	2000	GARCH(1,1)	2,823E-06	0,9041	0,0840	
		Std error	(1,024E-06)	(0,0107)	(0,0092)	
		EGARCH(1,1)	-0,2258	0,9734	0,1269	-0,1323
		Std error	(0,0282)	(0,0033)	(0,0173)	(0,0103)
		GJR-GARCH(1,1)	3,732E-06	0,9033	0,0000	0,1590
		Std error	(8,909E-07)	(0,0117)	(0,0000)	(0,0165)
Student-t	500	GARCH(1,1)	4,075E-06	0,8590	0,1245	
		Std error	(3,224E-06)	(0,0383)	(0,0363)	
		EGARCH(1,1)	-0,5233	0,9405	0,1823	-0,1640
		Std error	(0,1917)	(0,0216)	(0,0561)	(0,0407)
		GJR-GARCH(1,1)	5,996E-06	0,8626	0,0066	0,2044
		Std error	(3,099E-06)	(0,0356)	(0,0304)	(0,0647)
	1000	GARCH(1,1)	2,643E-06	0,9004	0,0868	
		Std error	(1,940E-06)	(0,0245)	(0,0218)	
		EGARCH(1,1)	-0,4178	0,9527	0,1612	-0,1642
		Std error	(0,1139)	(0,0127)	(0,0373)	(0,0278)
		GJR-GARCH(1,1)	4,146E-06	0,8850	0,0017	0,1991
		Std error	(1,827E-06)	(0,0234)	(0,0160)	(0,0453)
	2000	GARCH(1,1)	2,508E-06	0,8941	0,1059	
		Std error	(1,299E-06)	(0,0157)	(0,0170)	
		EGARCH(1,1)	-0,2324	0,9730	0,1381	-0,1536
		Std error	(0,0394)	(0,0045)	(0,0227)	(0,0161)
		GJR-GARCH(1,1)	3,314E-06	0,9009	0,0000	0,1753
		Std error	(1,020E-06)	(0,0143)	(0,0000)	(0,0245)

Source: Compiled by the author

Table 4.7 Heteroskedasticity test results for DAXR, TYR, DXYR and SPGR

Security	F-Statistic	Prob. F (4,2578)
DAXR	1,736043	0,1392
TYR	1,442077	0,2175
DXYR	0,871353	0,4803
SPGR	1,073233	0,3681

Source: Compiled by the author

Another observation that can be seen in the variance models is the difference in estimated values of various sample sizes. Increasing the sample size from 500 to 1000 has larger effect on the estimates than increasing from 1000 to 2000 observations. However, to conclude whether larger sample sizes provide more accurate results, additional tests will be conducted in the next section.

4.3. Evaluation of the models

In order to evaluate the goodness-of-fit of estimated models, the Akaike information criterion (AIC), Bayesian information criterion (BIC) and root mean squared error (RMSE) will be computed. The latter is done by comparing the conditional variance produced by the estimated models to the realised volatility of the same period. In addition, the correlation between the modelled volatility and the realised volatility will be provided. In the end of this section, the forecasting qualities of the models will be evaluated by computing one day-ahead volatility forecasts and comparing the results to realised volatility. One goal of this thesis was to assess if using more sophisticated variance models brings significant advantage over naïve models, which is why the simple ARMA(1,1) model is also included.

Before evaluating the models, an illustrative figure of the conditional variances is shown to give an idea how the estimated variances of these models move throughout the observation period. This can be seen in figure 4.5, where the estimated conditional variances of the models for DAXR are brought, including the 200 days-ahead out-of-sample forecasts. The inferred conditional variance forecasts were made by fitting the model to the actual realised variances.

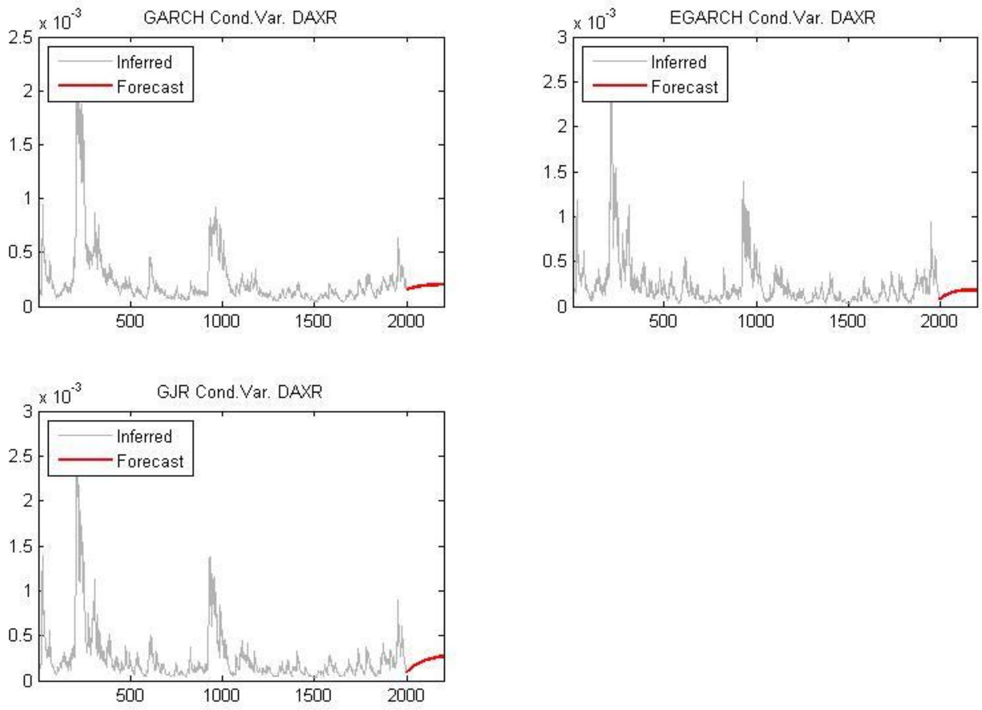

Figure 4.5 Conditional variance forecasts for DAXR
Source: Compiled by the author

Figure 4.5 illustrates the leverage effect of the asymmetric models, where the x-axis describes the number of observations and y-axis the fitted conditional variance. The 200 step-ahead forecasts might not be not very accurate, however it can already be seen that the out-of-sample results do not differ much between the models. The conditional variance seems to be lower in the case of GARCH(1,1) and higher for the GJR-GARCH(1,1) model due to the higher leverage in the latter. It can also be seen that the fitted variances of the EGARCH model seem to be sharper and die out quicker compared to the other models. The large shocks in the beginning of the inferred variance illustrate the volatility during 2007-2009, which was also seen to be higher in the return graphs.

All the computed AIC, BIC and RMSE values are brought in appendix 4 and the best models according to the criteria are shown in table 4.8. The correlation and RMSE were calculated by comparing the 300 step-ahead estimated volatility to the realised standard deviation over selected periods. The higher the correlation is, the better the goodness-of-fit of a model. The lower the values are for AIC and BIC criteria as well as RMSE, the better the model.

Table 4.8 shows that EGARCH(1,1) outperformed other models significantly, having the lowest AIC and BIC values in almost every dataset, distribution and number of observations. GARCH(1,1) was the second performing variance model, showing good results when comparing the RMSE and correlation with realised volatility, however could not compete with EGARCH(1,1) nor GJR-GARCH(1,1) in AIC and BIC tests. The simple ARMA(1,1) was not as successful in capturing the conditional variance as the other alternatives. This can be seen from significantly higher AIC and BIC values (appendix 4) and it confirms the benefit of using more sophisticated volatility models over simple naïve models. This is in line with previous research, including Bali (2007), (Angelidis 2004), Berkowitz and O'Brien (2002) and Jansky and Rippel (2011), where GARCH(1,1) and EGARCH(1,1) conditional variance models outperformed the ARMA(1,1) model.

When comparing the Gaussian and Student-t distribution, it can be seen that there was slight improvement in the performance of the models according to all criteria. The AIC and BIC values were lower in almost all cases and most of the times models estimated with Student-t distribution showed higher correlation with the realised volatility. This helps to confirm the stylized fact discussed in subsection 2.1.2, that financial returns are not normally distributed and are usually heavy tailed (leptokurtic), which is why using Student-t distribution is beneficial.

The sample size issue was analysed in this thesis as well by estimating every selected model under 500, 1000 and 2000 observations. Previous empirical studies discussed in Chapter 3 did not agree on the benefits or disadvantages of using different sample sizes. When Hendricks (1996), Danielsson (2002) and Vlaar (2000) found that increased sample size improved the performance of estimated models, Frey and Michaud (1997) and Hoppe (1998) argued that smaller amount of data could be more accurate due to better capturing the behavioural change in the trading activity (Hoppe 1998). As seen from appendix 4, there is no clear evidence that increased sample size improves or decreases the performance in estimated models. For TYR, smaller number of observations showed better correlation and RMSE values, however for DAXR, the result was opposite. DXYR and SPGR had the best results when estimating the models with 1000 observations. Therefore it is not possible to conclude that sample size has clear effect on the results of conditional variance estimates.

Table 4.8 Model evaluation and comparison, best models according to the criteria

Symbol	Distribution	No. Obs	Correlation	RMSE	AIC	BIC
DAXR	Gaussian	500	GARCH(1,1)	GARCH(1,1)	EGARCH(1,1)	EGARCH(1,1)
		1000	GARCH(1,1)	GARCH(1,1)	EGARCH(1,1)	EGARCH(1,1)
		2000	GARCH(1,1)	GARCH(1,1)	EGARCH(1,1)	EGARCH(1,1)
	Student-t	500	GARCH(1,1)	GARCH(1,1)	EGARCH(1,1)	EGARCH(1,1)
		1000	GARCH(1,1)	GARCH(1,1)	EGARCH(1,1)	EGARCH(1,1)
		2000	GARCH(1,1)	GARCH(1,1)	EGARCH(1,1)	EGARCH(1,1)
TYR	Gaussian	500	GJR(1,1)	GJR(1,1)	EGARCH(1,1)	EGARCH(1,1)
		1000	EGARCH(1,1)	GARCH(1,1)	GARCH(1,1)	GARCH(1,1)
		2000	EGARCH(1,1)	GJR(1,1)	GJR(1,1)	GJR(1,1)
	Student-t	500	GJR(1,1)	GARCH(1,1)	EGARCH(1,1)	EGARCH(1,1)
		1000	EGARCH(1,1)	GJR(1,1)	EGARCH(1,1)	GARCH(1,1)
		2000	EGARCH(1,1)	GARCH(1,1)	EGARCH(1,1)	EGARCH(1,1)
DXYR	Gaussian	500	GARCH(1,1)	GARCH(1,1)	EGARCH(1,1)	EGARCH(1,1)
		1000	EGARCH(1,1)	GJR(1,1)	EGARCH(1,1)	EGARCH(1,1)
		2000	EGARCH(1,1)	GJR(1,1)	EGARCH(1,1)	EGARCH(1,1)
	Student-t	500	GARCH(1,1)	GARCH(1,1)	EGARCH(1,1)	EGARCH(1,1)
		1000	EGARCH(1,1)	GARCH(1,1)	EGARCH(1,1)	EGARCH(1,1)
		2000	EGARCH(1,1)	GJR(1,1)	EGARCH(1,1)	EGARCH(1,1)
SPGR	Gaussian	500	GARCH(1,1)	GARCH(1,1)	EGARCH(1,1)	EGARCH(1,1)
		1000	GARCH(1,1)	GARCH(1,1)	EGARCH(1,1)	GARCH(1,1)
		2000	GARCH(1,1)	GARCH(1,1)	EGARCH(1,1)	GARCH(1,1)
	Student-t	500	GARCH(1,1)	GJR(1,1)	EGARCH(1,1)	EGARCH(1,1)
		1000	EGARCH(1,1)	GARCH(1,1)	EGARCH(1,1)	EGARCH(1,1)
		2000	GARCH(1,1)	GARCH(1,1)	GJR(1,1)	GARCH(1,1)
Count			**ARMA(1,1)**	**GARCH(1,1)**	**EGARCH(1,1)**	**GJR(1,1)**
			0	36	48	12

Source: Compiled by the author

4.4. VaR forecast evaluation

The value at risk of chosen assets was calculated using the conditional variance models, mean returns and the z-values of selected confidence intervals. The observation period is the same as for conditional variance models, from 01/01/2006 – 01/12/2015 using the daily asset returns. The value at risk violations were computed from the previously estimated 300 step-ahead conditional variance forecasts. Table 4.9 illustrates the best volatility models for calculating value at risk according to each asset, distribution and number of observations. The number of violations to the VaR estimates for each asset is shown in appendix 5. The calculations were done using the variance-covariance method of estimating value at risk and since the assets were analysed separately instead of in a portfolio, no covariances between the assets were needed. The estimated VaR values can also be used in a portfolio, since according to Lucas (2000), the more sophisticated value at risk models based on the estimates of variance-covariance matrices do not perform significantly better compared to univariate value at risk models that only require conditional variance estimates.

The value at risk evaluation results are in line with conditional variance results shown in section 4.3. Comparing all selected models in different distributions, samples and asset returns, GARCH(1,1) and EGARCH(1,1) volatility models outperform the GJR(1,1) model, however the differences were often rather small. In section 4.3, the correlation and RMSE values of DAXR conditional variance forecasts favoured the GARCH(1,1) model strongly similarly as in Table 4.9. The results for the other assets were slightly more mixed. When comparing the estimated value at risk violations across different distributions, the Student-t distribution appears to outperform the Gaussian distribution, having slightly less violations across the samples. This is in line with the stylised fact discussed in subsection 2.1.2, that financial returns are not normally distributed and it proved beneficial taking this into account when modelling VaR. When comparing the differences in value at risk violations across various sample sizes, no significant pattern emerges. Therefore it cannot be concluded that the number of observations have clear impact on the goodness-of-fit of value at risk models and on the accuracy of its forecasts.

Table 4.9 Evaluation of VaR estimates using conditional variance forecasts

Symbol	Distribution	No. Obs	VaR (90%)	VaR (95%)	VaR (99%)
DAXR	Gaussian	500	GARCH(1,1)	GARCH(1,1)	GARCH(1,1)
		1000	GARCH(1,1)	GARCH(1,1)	GARCH(1,1)
		2000	GARCH(1,1)	GARCH(1,1)	GARCH(1,1)
	Student-t	500	GARCH(1,1)	GARCH(1,1)	GARCH(1,1)
		1000	GJR(1,1)	GARCH(1,1)	GARCH(1,1)
		2000	GARCH(1,1)	GARCH(1,1)	GARCH(1,1)
TYR	Gaussian	500	EGARCH(1,1)	EGARCH(1,1)	EGARCH(1,1)
		1000	GJR(1,1)	GJR(1,1)	GJR(1,1)
		2000	EGARCH(1,1)	EGARCH(1,1)	EGARCH(1,1)
	Student-t	500	GARCH(1,1)	GARCH(1,1)	GARCH(1,1)
		1000	GARCH(1,1)	GARCH(1,1)	GARCH(1,1)
		2000	EGARCH(1,1)	EGARCH(1,1)	EGARCH(1,1)
DXYR	Gaussian	500	EGARCH(1,1)	EGARCH(1,1)	GJR(1,1)
		1000	GJR(1,1)	GARCH(1,1)	GJR(1,1)
		2000	GARCH(1,1)	EGARCH(1,1)	EGARCH(1,1)
	Student-t	500	EGARCH(1,1)	EGARCH(1,1)	GARCH(1,1)
		1000	GARCH(1,1)	GARCH(1,1)	GARCH(1,1)
		2000	GARCH(1,1)	EGARCH(1,1)	EGARCH(1,1)
SPGR	Gaussian	500	GJR(1,1)	GJR(1,1)	EGARCH(1,1)
		1000	EGARCH(1,1)	EGARCH(1,1)	EGARCH(1,1)
		2000	EGARCH(1,1)	EGARCH(1,1)	GARCH(1,1)
	Student-t	500	GJR(1,1)	EGARCH(1,1)	EGARCH(1,1)
		1000	EGARCH(1,1)	EGARCH(1,1)	EGARCH(1,1)
		2000	EGARCH(1,1)	EGARCH(1,1)	EGARCH(1,1)
Count			**GARCH(1,1)**	**EGARCH(1,1)**	**GJR(1,1)**
			31	31	10

Source: Compiled by the author

4.5. Conclusions

This empirical analysis compared the volatility models in estimating Value at Risk of four different assets. DAX index (DAXR) represented equity, US 10Y Treasury note futures (TYR) represented fixed income, US Dollar index represented the movements in currencies and S&P GSCI Index represented the commodity asset class. The dataset was selected from 01/01/2006 – 01/12/2015 daily asset returns with samples of 500, 1000 and 2000 observations. Four conditional variance models were used in this analysis: GARCH(1,1), EGARCH(1,1), GJR-GARCH(1,1) and ARMA(1,1) model, all estimated with Gaussian as well as Student-t distribution. There were three main research questions raised in thesis. Firstly, whether using sophisticated volatility models brings significant advantages over using more naïve alternatives, secondly if using Student-t distribution improves the goodness-of-fit of selected models and thirdly if the number of observations have a clear impact on the forecasting results.

The evaluation of estimated conditional variances of selected models favoured sophisticated volatility models over the more naïve alternative. The AIC and BIC values were significantly better for GARCH(1,1) and EGARCH(1,1) models compared to the simple ARMA(1,1) model. Overall, the EGARCH(1,1) and GARCH(1,1) models outperformed the GJR(1,1) model in all criteria, however the differences were less significant when comparing value at risk estimates. When comparing different distributions, the Student-t distribution managed to capture the changes in the variance more accurately compared to the Gaussian distribution, which is in line with previous empirical analysis. The value at risk violations and conditional variance performance results varied across different sample sizes which is why it is not possible to conclude that the sample size has a clear effect on the accuracy of selected models.

4.6. Suggestions for further research

Although using nonlinear autoregressive volatility models to estimate Value at Risk showed advantages over naïve alternatives, there are some certain limitations. As seen from figure 4.5, when the forecasting horizon increases, the benefits of selected models decrease. However, financial institutions are often interested in estimating longer period forecasts. The decrease in benefits of selected models was also expected since the GARCH models converge to the long-run variance when the forecasting period is increased enough (see subsection 2.3.4). This is called the dying-out effect and it puts significant restrictions on the models when there is a desire to produce longer forecasts. To illustrate this shortcoming, the long-run (400 days step-ahead) conditional variance forecasts for DAXR using the selected models are brought in figure 4.6, where the x-axis describes the number of observations and y-axis the conditional variance forecast. It can be seen that the dying-out effect happens in a rather short period of time. The value is very close to the average already in about 200 step-ahead forecasts for the GARCH model and in about 100 step-ahead forecasts for asymmetric models. Therefore, concluding that the leverage effect of EGARCH and GJR model increase the dying-out effect further. This indicates that in order to get better results for long-run volatility forecasting, it could be appropriate to consider different models where this effect is dampened, like FIGARCH and FIEGARCH models (see subsection 2.3.4).

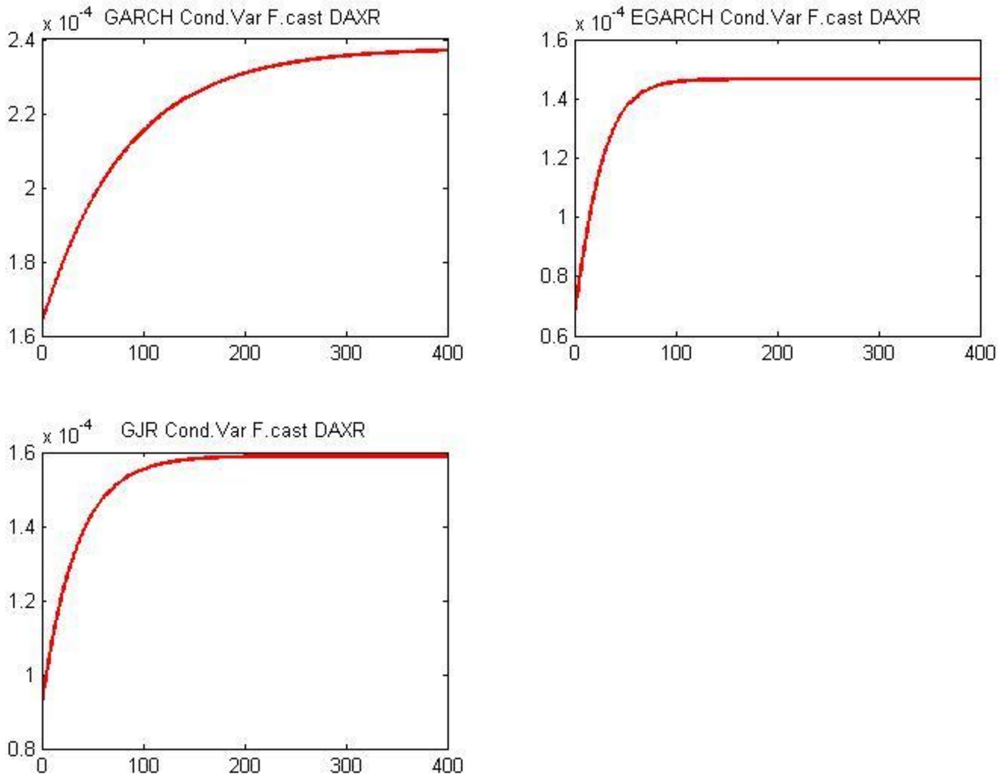

Figure 4.6 Long-run conditional variance forecasts for DAXR
Source: Compiled by the author

Another suggestion for further research would be to use impulse-response analysis on the VaR estimates that use forecasted volatility. This would show how the selected models behave under large volatility shocks, which is an important feature considering risk management problems. Section 4.3 illustrated the fitted variances of the volatility models during calm periods as well as financial recession and the variances differed across models noticeably. During large downward movements the conditional variances of EGARCH models were higher due to the leverage effect, however possibly overestimating the variance in some occasions. The GARCH model on the other hand seemed to react more slowly to volatility shocks, which could lead to not capturing the whole variance in the returns. All in all, more research on this issue would be appropriate.

This thesis focused on estimating value at risk of single assets, however in many occasions, risk measures of a portfolio is required. In that case, forecasted conditional variances could be beneficial as well, but one should also consider forecasting asset correlations. There are several models for this purpose, like VEC-GARCH model developed by Bollerslev, Engle, and Wooldridge (1988); BEKK model by Engle and Kroner (1995); Orthogonal GARCH by Alexander and Chibumba (1997) and MGARCH models proposed by Bollerslev (1990) to name a few. The conditional correlation models differ in complexity, however advantages over using only conditional variance forecasts could be achieved when estimating value at risk for several asset portfolios.

SUMMARY

In light of several stock market crashes and financial crises over the past decades, financial practitioners and academics alike have been searching for ways to improve risk measures and value at risk models have been the cornerstone of this risk management revolution. The increased focus on risk management over the past decades has led to the development of various techniques and models, some more widely used than others. Since the development of Autoregressive Conditional Heteroscedasticity models by Engle in 1982, volatility modelling has become one the most popular research subjects in financial time-series analysis with several extensions to the original model appeared. This thesis analysed the use of nonlinear autoregressive volatility models on estimating value at risk across different asset classes.

The thesis presented several conditional variance and value at risk modelling concepts as follows. Chapter 1 gave an overview of the theory of risk management, focusing on Value at Risk and the different methods of calculating it. Chapter 2 analysed the stylised properties of financial returns and the background literature of volatility modelling and forecasting. The stylised facts were the volatility clustering in financial data, the heavy tails and return asymmetry, the leverage effect and the absence of autocorrelation. Chapter 3 was dedicated to discussing the previous empirical studies on estimating VaR with nonlinear volatility models to see if the results in this thesis are in line with other research done on this topic. In Chapter 4, an empirical analysis was conducted using nonlinear conditional variance models for estimating value at risk across different assets, distributions and sample sizes.

Three research questions were raised in the beginning of this thesis. Firstly, whether using sophisticated conditional variance models brings significant advantages over using more naïve alternatives; secondly, if using Student-t distribution over Gaussian distribution improves the goodness-of-fit of selected models and thirdly, if the number of observations have a clear impact on the forecasting results. Previous empirical evidence discussed in Chapter 3 showed the superiority of EGARCH and GARCH models over more simple alternatives. When comparing the different distributions on conditional variance models then

according to previous analysis, the Student-t distribution outperforms Gaussian distribution by providing more accurate results. The evidence on the sample size is twofold, while some papers found that increased sample size improved the performance of estimated models, others argued that smaller amount of data could be more accurate due to better capturing the changes in the financial returns.

To answer these research questions, four autoregressive volatility models were selected to estimate value at risk. The models are the GARCH(1,1), EGARCH(1,1), GJR-GARCH(1,1) and ARMA(1,1) model, which were chosen based on previous empirical evidence. The ARMA(1,1) model was included to compare the GARCH variations to a more naïve alternative. All the selected models were estimated using the Student-t as well as Gaussian distribution. The empirical analysis was conducted on four different financial assets, the DAX index (DAXR) which represents the equity returns, US 10Y Treasury note futures (TYR) which represents the returns of fixed income assets, US Dollar index which represents the movements in currencies and the S&P GSCI Index which represents the commodity asset class. The sample used in the empirical analysis consisted of daily returns from 01/01/2006 – 01/12/2015 with sample sizes of 500, 1000 and 2000 to compare the effect of different amounts of data on estimation results. Using the selected sample, 300 step-ahead forecasts of the conditional variance were estimated using a rolling window of 200, 700 and 1700 observations.

In order to evaluate the goodness-of-fit of estimated conditional variance models, the Akaike information criterion (AIC), Bayesian information criterion (BIC) and root mean squared error (RMSE) were computed. The latter was done by comparing the conditional variance produced by the estimated models to the realised volatility of the same period. In addition, the correlation between the modelled volatility and the realised volatility was provided. To evaluate the value at risk estimates, daily violations across samples were computed.

The evaluation of estimated conditional variances of selected models favoured sophisticated volatility models over the more naïve alternative. The results showed that GARCH(1,1) and EGARCH(1,1) models outperformed the ARMA(1,1) and GJR(1,1) models in nearly every criteria. When comparing different distributions across models, the Student-t distribution managed to capture the changes in the variance more accurately compared to the Gaussian distribution. Both of these results are in line with previous studies discussed in

Chapter 3. The conditional variance performance results and value at risk violations varied across different sample sizes, which is why it was not possible to conclude that the sample size has a clear effect on the accuracy of selected models. The previous empirical evidence showed different results as well, which is why more research on this topic is needed.

The purpose of this research was fulfilled and two of the three questions raised in the beginning of the thesis were answered. However, this thesis used only one step-ahead conditional variance forecasts, which is not always the desired estimation period. Financial institutions are often interested in longer-term value at risk forecasts. Section 4.3 and 4.6 illustrated the problems in long-run volatility forecasting and concluded that GARCH volatility models might not be appropriate for this task. There are long-memory volatility models like FIGARCH and FIEGARCH, which could be beneficial, however further research on this topic is required.

Another suggestion to improve the value at risk models was to use the impulse-response analysis on VaR estimates. This would show how the selected models behave under large volatility shocks, which is an important feature considering risk management problems. Estimated conditional variances showed higher values for asymmetric models during large negative volatility shocks, however to better quantify the benefits in these periods, more research would be appropriate.

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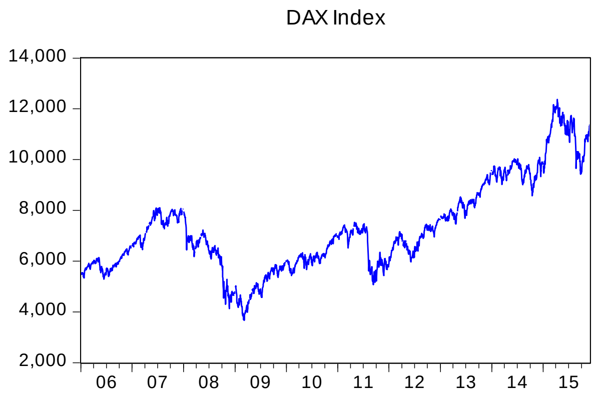
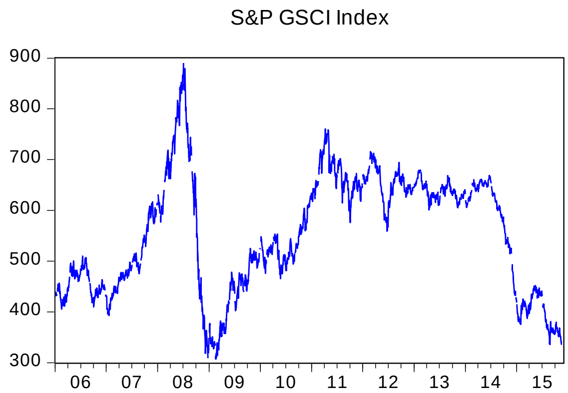
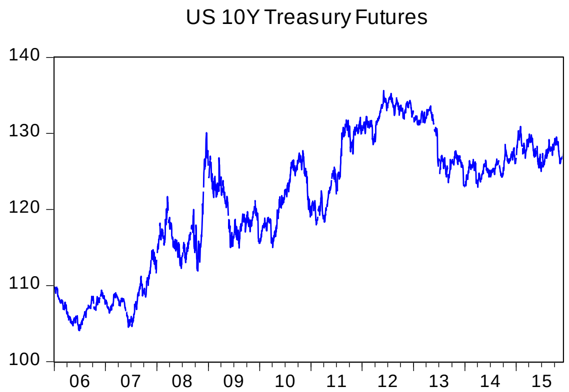
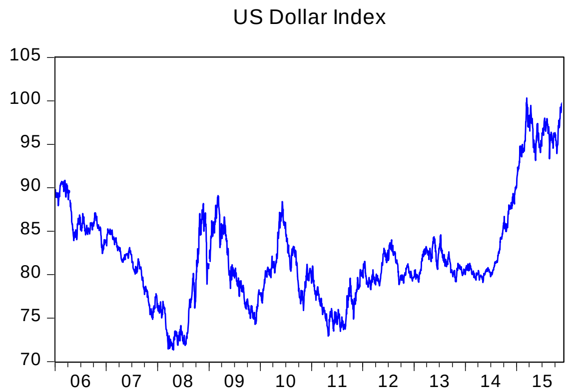
APPENDICES

Appendix 1. Index price graphs









Source: Compiled by the author

Appendix 2. Matlab code example for estimating and evaluating conditional variance models for DAXR

Import the financial data under the name DAXR

```
r=DAXR(2001:2500);
r2=DAXR(1501:2500);
r3=DAXR(501:2500);
N=length(r);
N2=length(r2);
N3=length(r3);
```

For estimating with Gaussian distribution:

```
model111 = garch(1,1);
[fit111,~,LogL111] = estimate(model111,r);
model112 = egarch(1,1);
[fit112,~,LogL112] = estimate(model112,r);
model113 = gjr(1,1);
[fit113,~,LogL113] = estimate(model113,r);
[aic,bic] = aicbic([LogL111,LogL112,LogL113],[3,4,4],N);
model121 = garch(1,1);
[fit121,~,LogL121] = estimate(model121,r2);
model122 = egarch(1,1);
[fit122,~,LogL122] = estimate(model122,r2);
model123 = gjr(1,1);
[fit123,~,LogL123] = estimate(model123,r2);
[aic2,bic2] = aicbic([LogL121,LogL122,LogL123],[3,4,4],N2);
model131 = garch(1,1);
[fit131,~,LogL131] = estimate(model131,r3);
model132 = egarch(1,1);
[fit132,~,LogL132] = estimate(model132,r3);
model133 = gjr(1,1);
[fit133,~,LogL133] = estimate(model133,r3);
[aic3,bic3] = aicbic([LogL131,LogL132,LogL133],[3,4,4],N3);
```

Source: Compiled by the author

Appendix 2 continues

For estimating with Student-t distribution:

```
model211 = garch('Distribution', 't','GARCHLags',1,'ARCHLags',1);
[fit211,~,LogL211] = estimate(model211,r);
model212 = egarch('Distribution', 't','GARCHLags',1,'ARCHLags',1,'LeverageLags',1);
[fit212,~,LogL212] = estimate(model212,r);
model213 = gjr('Distribution', 't','GARCHLags',1,'ARCHLags',1,'LeverageLags',1);
[fit213,~,LogL213] = estimate(model213,r);
[aic4,bic4] = aicbic([LogL211,LogL212,LogL213],[3,4,4],N);
model221 = garch('Distribution', 't','GARCHLags',1,'ARCHLags',1);
[fit221,~,LogL221] = estimate(model221,r2);
model222 = egarch('Distribution', 't','GARCHLags',1,'ARCHLags',1,'LeverageLags',1);
[fit222,~,LogL222] = estimate(model222,r2);
model223 = gjr('Distribution', 't','GARCHLags',1,'ARCHLags',1,'LeverageLags',1);
[fit223,~,LogL223] = estimate(model223,r2);
[aic5,bic5] = aicbic([LogL221,LogL222,LogL223],[3,4,4],N2);
model231 = garch('Distribution', 't','GARCHLags',1,'ARCHLags',1);
[fit231,~,LogL231] = estimate(model231,r3);
model232 = egarch('Distribution', 't','GARCHLags',1,'ARCHLags',1,'LeverageLags',1);
[fit232,~,LogL232] = estimate(model232,r3);
model233 = gjr('Distribution', 't','GARCHLags',1,'ARCHLags',1,'LeverageLags',1);
[fit233,~,LogL233] = estimate(model233,r3);
[aic6,bic6] = aicbic([LogL231,LogL232,LogL233],[3,4,4],N3);
```

Estimating the ARMA(1,1) model:

```
model001 = arima(1,0,1);
[fit001,~,LogL001] = estimate(model001,r);
[aic01,bic01] = aicbic([LogL001,LogL112,LogL113],[3,4,4],N);
model002 = arima(1,0,1);
[fit002,~,LogL002] = estimate(model002,r2);
[aic02,bic02] = aicbic([LogL002,LogL122,LogL123],[3,4,4],N2);
model003 = arima(1,0,1);
[fit003,~,LogL003] = estimate(model003,r3);
[aic03,bic03] = aicbic([LogL003,LogL132,LogL133],[3,4,4],N3);
```

Source: Compiled by the author

Appendix 2 continues

<u>The rolling-window process:</u>

```
RW1 = 200;
sample = r(1: RW1,1);
[fitS111,~,~] = estimate(model111, sample);
RWF(1, 1) = forecast(fitS111,1, 'Y0', sample);
for t = RW1:N(1,1)-1;
sample = r( t - RW1 +1 : t );
[fitS111,~,~] = estimate(model111, sample);
RWF(t-RW1+1+1, 1) = forecast(fitS111,1, 'Y0', sample);
t;
end;

RW1 = 700;
sample = r(1: RW1,1);
[fitS111,~,~] = estimate(model111, sample);
RWF(1, 1) = forecast(fitS111,1, 'Y0', sample);
for t = RW1:N(1,1)-1;
sample = r( t - RW1 +1 : t );
[fitS111,~,~] = estimate(model111, sample);
RWF(t-RW1+1+1, 1) = forecast(fitS111,1, 'Y0', sample);
t;
end;

RW1 = 1700;
sample = r(1: RW1,1);
[fitS111,~,~] = estimate(model111, sample);
RWF(1, 1) = forecast(fitS111,1, 'Y0', sample);
for t = RW1:N(1,1)-1;
sample = r( t - RW1 +1 : t );
[fitS111,~,~] = estimate(model111, sample);
RWF(t-RW1+1+1, 1) = forecast(fitS111,1, 'Y0', sample);
t;
end;
```

<u>Forecasting:</u>

```
Vf1 = forecast(fit231,200,'Y0',r4);
V1 = infer(fit231,r4);
Vf400g = forecast(fit131,400,'Y0',r4);
Vf2 = forecast(fit232,200,'Y0',r4);
V2 = infer(fit232,r4);
Vf400e = forecast(fit222,400,'Y0',r4);
Vf3 = forecast(fit233,200,'Y0',r4);
V3 = infer(fit233,r4);
Vf400j = forecast(fit123,400,'Y0',r4);
```

Source: Compiled by the author

Appendix 2 continues

<u>Plotting the figures:</u>

```
figure(1)
subplot(2,2,1)
plot(V1,'Color',[.7,.7,.7])
hold on
plot(N4+1:N4+200,Vf1,'r','LineWidth',2)
xlim([1,N4+200])
legend('Inferred','Forecast','Location','Northwest')
title('GARCH Cond.Var. DAXR')
hold off
subplot(2,2,2)
plot(V2,'Color',[.7,.7,.7])
hold on
plot(N4+1:N4+200,Vf2,'r','LineWidth',2)
xlim([1,N4+200])
legend('Inferred','Forecast','Location','Northwest')
title('EGARCH Cond.Var. DAXR')
hold off
subplot(2,2,3)
plot(V3,'Color',[.7,.7,.7])
hold on
plot(N4+1:N4+200,Vf3,'r','LineWidth',2)
xlim([1,N4+200])
legend('Inferred','Forecast','Location','Northwest')
title('GJR Cond.Var. DAXR')
hold off

figure(2)
subplot(2,2,1)
plot(Vf400g,'r','LineWidth',2)
title('GARCH Cond.Var F.cast DAXR')
subplot(2,2,2)
plot(Vf400e,'r','LineWidth',2)
title('EGARCH Cond.Var F.cast DAXR')
subplot(2,2,3)
plot(Vf400j,'r','LineWidth',2)
title('GJR Cond.Var F.cast DAXR')
```

Source: Compiled by the author

Appendix 3. Conditional variance models for TYR, DXYR and SPGR

TYR						
Distribution	No. Obs	Model	Constant (ω)	GARCH (α)	ARCH (β)	Leverage (γ)
Gaussian	500	GARCH(1,1)	4,770E-07	0,900000	0,050000	
		Std error	3,698E-07	0,023489	0,022914	
		EGARCH(1,1)	-0,175630	0,984782	0,073325	0,003671
		Std error	0,174883	0,015053	0,041273	0,019961
		GJR-GARCH(1,1)	4,770E-07	0,900000	0,050000	0,000000
		Std error	3,701E-07	0,024020	0,031783	0,000000
	1000	GARCH(1,1)	4,804E-07	0,900000	0,050000	
		Std error	2,568E-07	0,013748	0,014649	
		EGARCH(1,1)	-10,000000	0,134608	0,162462	-0,079335
		Std error	2,496200	0,215570	0,070179	0,033505
		GJR-GARCH(1,1)	4,804E-07	0,900000	0,050000	0,000000
		Std error	2,569E-07	0,014221	0,022672	0,000000
	2000	GARCH(1,1)	2,831E-07	0,906662	0,081200	
		Std error	2,064E-07	0,008043	0,009788	
		EGARCH(1,1)	-0,094146	0,991199	0,106415	-0,007429
		Std error	0,018598	0,001671	0,012375	0,006090
		GJR-GARCH(1,1)	2,000E-07	0,944304	0,053340	-0,017646
		Std error	1,570E-07	0,005395	0,007050	0,007749
Student t	500	GARCH(1,1)	4,770E-07	0,900000	0,050000	
		Std error	4,123E-07	0,029057	0,028164	
		EGARCH(1,1)	-0,135445	0,988354	0,065797	0,011940
		Std error	0,184165	0,015830	0,042638	0,024453
		GJR-GARCH(1,1)	4,770E-07	0,900000	0,050000	0,000000
		Std error	4,126E-07	0,029493	0,039718	0,000000
	1000	GARCH(1,1)	2,000E+00	0,958165	0,020130	
		Std error	1,703E-07	0,008691	0,008615	
		EGARCH(1,1)	-0,164946	0,985847	0,057605	-0,017707
		Std error	0,134959	0,011592	0,027444	0,016614
		GJR-GARCH(1,1)	2,000E-07	0,957525	0,023466	-0,004756
		Std error	1,719E-07	0,009809	0,018115	0,019286
	2000	GARCH(1,1)	2,000E-07	0,943827	0,044073	
		Std error	1,788E-07	0,008126	0,008637	
		EGARCH(1,1)	-0,051157	0,995479	0,085511	-0,002028
		Std error	0,029488	0,002643	0,016427	0,010931
		GJR-GARCH(1,1)	2,000E-07	0,943990	0,053567	-0,018613
		Std error	1,785E-07	0,008183	0,013015	0,015097

Source: Compiled by the author

Appendix 3 continues

DXYR						
Distribution	No. Obs	Model	Constant (ω)	GARCH (α)	ARCH (β)	Leverage (γ)
Gaussian	500	GARCH(1,1)	2,000E-07	0,935389	0,053158	
		Std error	3,246E-07	0,013015	0,014587	
		EGARCH(1,1)	-0,011597	1,000000	-0,056832	0,084257
		Std error	0,000027	0,000001	0,000765	0,003492
		GJR-GARCH(1,1)	9,914E-07	0,824160	0,130054	-0,056020
		Std error	6,396E-07	0,027737	0,032438	0,036670
	1000	GARCH(1,1)	2,000E-07	0,942186	0,046702	
		Std error	2,357E-07	0,008061	0,008857	
		EGARCH(1,1)	-0,037458	0,996733	0,047724	0,047174
		Std error	0,024978	0,002264	0,012297	0,007335
		GJR-GARCH(1,1)	2,000E-07	0,947775	0,055908	-0,034363
		Std error	2,204E-07	0,007362	0,011224	0,013071
	2000	GARCH(1,1)	2,287E-07	0,941473	0,058527	
		Std error	2,315E-07	0,007661	0,009583	
		EGARCH(1,1)	-0,036599	0,996398	0,072099	0,024132
		Std error	0,016193	0,001534	0,011989	0,005194
		GJR-GARCH(1,1)	2,000E-07	0,950732	0,047695	-0,011542
		Std error	1,946E-07	0,005761	0,007917	0,008827
Student t	500	GARCH(1,1)	2,000E-07	0,930208	0,064958	
		Std error	4,165E-07	0,021262	0,025894	
		EGARCH(1,1)	-0,018114	0,999132	-0,044717	0,087543
		Std error	0,001328	0,000037	0,032716	0,005366
		GJR-GARCH(1,1)	2,000E-07	0,941312	0,089288	-0,083866
		Std error	3,821E-07	0,020316	0,033209	0,035508
	1000	GARCH(1,1)	2,000E-07	0,939579	0,051775	
		Std error	2,904E-07	0,012524	0,014211	
		EGARCH(1,1)	-0,010712	0,999348	0,041045	0,056229
		Std error	0,024851	0,002232	0,015831	0,010722
		GJR-GARCH(1,1)	2,000E-07	0,946929	0,064537	-0,046091
		Std error	2,680E-07	0,011675	0,017856	0,020677
	2000	GARCH(1,1)	2,000E-07	0,947209	0,046624	
		Std error	2,274E-07	0,008346	0,009072	
		EGARCH(1,1)	-0,020303	0,998127	0,072477	0,026315
		Std error	0,020593	0,001938	0,015135	0,007693
		GJR-GARCH(1,1)	2,000E-07	0,949314	0,052489	-0,016971
		Std error	2,236E-07	0,008287	0,011148	0,012993

Source: Compiled by the author

Appendix 3 continues

SPGR						
Distribution	No. Obs	Model	Constant (ω)	GARCH (α)	ARCH (β)	Leverage (γ)
Gaussian	500	GARCH(1,1)	4,867E-06	0,824328	0,143715	
		Std error	2,402E-06	0,036043	0,032704	
		EGARCH(1,1)	-0,017037	0,998818	0,026108	-0,082621
		Std error	0,013379	0,001572	0,017701	0,007821
		GJR-GARCH(1,1)	4,867E-06	0,824328	0,143715	0,000000
		Std error	2,416E-06	0,036456	0,041856	0,000000
	1000	GARCH(1,1)	6,592E-07	0,944481	0,052497	
		Std error	7,606E-07	0,008989	0,008212	
		EGARCH(1,1)	-0,011635	0,999077	0,011146	-0,062393
		Std error	0,007383	0,000782	0,007166	0,004408
		GJR-GARCH(1,1)	2,000E-07	0,968281	0,000000	0,056312
		Std error	4,968E-07	0,006087	0,000000	0,008563
	2000	GARCH(1,1)	5,325E-07	0,952113	0,046976	
		Std error	4,954E-07	0,005651	0,005812	
		EGARCH(1,1)	-0,030430	0,996243	0,090931	-0,042526
		Std error	0,012931	0,001464	0,011090	0,005581
		GJR-GARCH(1,1)	3,217E-07	0,960125	0,015352	0,046225
		Std error	4,752E-07	0,004952	0,005179	0,006883
Student t	500	GARCH(1,1)	4,867E-06	0,824328	0,143715	
		Std error	2,801E-06	0,051507	0,047599	
		EGARCH(1,1)	-0,013701	0,999175	0,034883	-0,073597
		Std error	0,019976	0,002228	0,029838	0,016783
		GJR-GARCH(1,1)	4,867E-06	0,824328	0,143715	0,000000
		Std error	2,802E-06	0,051509	0,060173	0,000000
	1000	GARCH(1,1)	6,423E-07	0,928649	0,071351	
		Std error	9,611E-07	0,016977	0,018109	
		EGARCH(1,1)	-0,009378	0,999331	0,011950	-0,057371
		Std error	0,011795	0,001227	0,010778	0,007671
		GJR-GARCH(1,1)	2,000E-07	0,970986	0,000000	0,050252
		Std error	5,405E-07	0,008066	0,000000	0,015012
	2000	GARCH(1,1)	3,580E-07	0,954333	0,045636	
		Std error	5,660E-07	0,007796	0,008143	
		EGARCH(1,1)	-0,016755	0,998188	0,081649	-0,039992
		Std error	0,014153	0,001620	0,014643	0,008290
		GJR-GARCH(1,1)	2,000E-07	0,963654	0,014476	0,041763
		Std error	4,995E-07	0,006363	0,008050	0,010926

Source: Compiled by the author

Appendix 4. Conditional variance model evaluation by asset class

DAXR						
Distribution	No. Obs	Model	Correlation	RMSE	AIC	BIC
Gaussian	500	ARMA (1,1)			-2963,30	-2950,6597
		GARCH(1,1)	**0,78580**	**0,00238**	-3015,28	-3002,64
		EGARCH(1,1)	0,59494	0,00403	**-3030,77**	**-3013,91**
		GJR-GARCH(1,1)	0,69682	0,00454	-3028,41	-3011,55
	1000	ARMA (1,1)			-6019,075	-6004,35
		GARCH(1,1)	**0,60441**	**0,00297**	-6110,92	-6096,20
		EGARCH(1,1)	0,46800	0,00374	**-6152,09**	**-6132,46**
		GJR-GARCH(1,1)	0,48568	0,00362	-6137,67	-6118,04
	2000	ARMA (1,1)			-11008,67	-10991,87
		GARCH(1,1)	**0,76434**	**0,00249**	-11616,65	-11599,85
		EGARCH(1,1)	0,58641	0,00484	**-11729,36**	**-11706,95**
		GJR-GARCH(1,1)	0,64895	0,00492	-11716,07	-11693,67
Student t	500	GARCH(1,1)	**0,73605**	**0,00456**	-3023,46	-3010,81
		EGARCH(1,1)	0,56083	0,00499	**-3040,11**	**-3023,25**
		GJR-GARCH(1,1)	0,61642	0,00542	-3037,61	-3020,76
	1000	GARCH(1,1)	**0,58826**	**0,00310**	-6143,32	-6128,60
		EGARCH(1,1)	0,42253	0,00412	**-6185,15**	**-6165,52**
		GJR-GARCH(1,1)	0,44816	0,00396	-6173,99	-6154,36
	2000	GARCH(1,1)	**0,75791**	**0,00459**	-11672,34	-11655,53
		EGARCH(1,1)	0,55953	0,00532	**-11769,26**	**-11746,86**
		GJR-GARCH(1,1)	0,64179	0,00535	-11756,79	-11734,39

Source: Compiled by the author

Appendix 4 continues

TYR						
Distribution	No. Obs	Model	Correlation	RMSE	AIC	BIC
Gaussian	500	ARMA (1,1)			-4355,3418	-4342,70
		GARCH(1,1)	0,378499	0,000403	-4362,50	-4349,86
		EGARCH(1,1)	0,533917	0,000473	**-4368,02**	**-4351,17**
		GJR-GARCH(1,1)	**0,543618**	**0,000325**	-4360,50	-4343,64
	1000	ARMA (1,1)			-8709,7658	-8695,04
		GARCH(1,1)	0,536083	**0,000561**	**-8720,55**	**-8705,83**
		EGARCH(1,1)	**0,588369**	0,000652	-8715,93	-8696,30
		GJR-GARCH(1,1)	0,482271	0,000573	-8718,55	-8698,92
	2000	ARMA (1,1)			-16104,35	-16087,55
		GARCH(1,1)	0,276146	0,000663	-16437,70	-16420,90
		EGARCH(1,1)	**0,337499**	0,000715	-16445,93	-16423,52
		GJR-GARCH(1,1)	0,272029	**0,000620**	**-16448,56**	**-16426,16**
Student t	500	GARCH(1,1)	0,544667	**0,000246**	-4367,39	-4354,75
		EGARCH(1,1)	0,739928	0,000260	**-4373,31**	**-4356,46**
		GJR-GARCH(1,1)	**0,782087**	0,000252	-4365,39	-4348,54
	1000	**GARCH(1,1)**	0,503038	0,000715	-8778,91	**-8764,18**
		EGARCH(1,1)	**0,679442**	0,000570	**-8783,57**	-8763,94
		GJR-GARCH(1,1)	0,514831	**0,000555**	-8776,98	-8757,35
	2000	GARCH(1,1)	0,305634	**0,000612**	-16541,53	-16524,73
		EGARCH(1,1)	**0,392830**	0,000668	**-16548,76**	**-16526,35**
		GJR-GARCH(1,1)	0,293816	0,000701	-16540,99	-16518,59

Source: Compiled by the author

Appendix 4 continues

DXYR						
Distribution	No. Obs	Model	Correlation	RMSE	AIC	BIC
Gaussian	500	ARMA (1,1)			-4043,36	-4030,72
		GARCH(1,1)	**0,8776**	**0,0007**	-4145,23	-4132,59
		EGARCH(1,1)	0,8761	0,0010	**-4185,91**	**-4169,05**
		GJR-GARCH(1,1)	0,8300	0,0007	-4116,29	-4099,43
	1000	ARMA (1,1)			-8048,0846	-4030,72
		GARCH(1,1)	0,8836	0,0008	-8171,67	-8156,95
		EGARCH(1,1)	**0,9181**	0,0008	**-8181,84**	**-8162,21**
		GJR-GARCH(1,1)	0,8879	**0,0007**	-8174,20	-8154,57
	2000	ARMA (1,1)			-15211,99	-15195,19
		GARCH(1,1)	0,4158	0,0016	-15538,65	-15521,84
		EGARCH(1,1)	**0,5129**	0,0014	**-15562,96**	**-15540,55**
		GJR-GARCH(1,1)	0,4790	**0,0014**	-15553,72	-15531,32
Student t	500	GARCH(1,1)	**0,8939**	**0,0007**	-4169,90	-4173,16
		EGARCH(1,1)	0,8900	0,0011	**-4198,28**	**-4181,42**
		GJR-GARCH(1,1)	0,8353	0,0008	-4173,16	-4156,30
	1000	GARCH(1,1)	0,8837	**0,0008**	-8210,42	-8195,69
		EGARCH(1,1)	**0,9263**	0,0009	**-8223,12**	**-8203,49**
		GJR-GARCH(1,1)	0,8785	0,0008	-8213,22	-8193,59
	2000	GARCH(1,1)	0,3557	0,0015	-15592,57	-15575,77
		EGARCH(1,1)	**0,5359**	0,0015	**-15604,26**	**-15581,85**
		GJR-GARCH(1,1)	0,4636	**0,0014**	-15592,30	-15569,89

Source: Compiled by the author

Appendix 4 continues

SPGR						
Distribution	No. Obs	Model	Correlation	RMSE	AIC	BIC
Gaussian	500	ARMA (1,1)			-2997,26	-2984,61
		GARCH(1,1)	**0,6073**	**0,0037**	-3103,23	-3101,23
		EGARCH(1,1)	0,4460	0,0052	**-3143,24**	**-3126,38**
		GJR-GARCH(1,1)	0,5511	0,0040	-3101,23	-3084,37
	1000	ARMA (1,1)			-6180,3324	-6165,61
		GARCH(1,1)	**0,6841**	**0,0037**	-6368,70	**-6394,78**
		EGARCH(1,1)	0,6329	0,0043	**-6413,30**	-6393,67
		GJR-GARCH(1,1)	0,6238	0,0040	-6394,78	-6375,15
	2000	ARMA (1,1)			-10929,33	-10912,53
		GARCH(1,1)	**0,4722**	**0,0044**	-11613,21	**-11634,36**
		EGARCH(1,1)	0,3942	0,0050	**-11635,65**	-11613,25
		GJR-GARCH(1,1)	0,3877	0,0048	-11634,36	-11611,96
Student t	500	GARCH(1,1)	**0,6644**	0,0035	-3145,68	-3133,04
		EGARCH(1,1)	0,5439	0,0045	**-3170,50**	**-3153,65**
		GJR-GARCH(1,1)	0,6335	**0,0035**	-3143,68	-3126,82
	1000	GARCH(1,1)	0,6705	**0,0037**	-6430,09	-6415,36
		EGARCH(1,1)	**0,6713**	0,0042	**-6449,32**	**-6429,69**
		GJR-GARCH(1,1)	0,6385	0,0039	-6442,27	-6422,64
	2000	**GARCH(1,1)**	**0,4920**	**0,0043**	-11697,32	**-11709,32**
		EGARCH(1,1)	0,4377	0,0048	-11708,52	-11686,11
		GJR-GARCH(1,1)	0,4143	0,0047	**-11709,32**	-11686,91

Source: Compiled by the author

Appendix 5. Value at risk evaluation by asset class

DAXR Violations					
Distribution	No. Obs	Model	VaR (90%)	VaR (95%)	VaR (99%)
Gaussian	500	**GARCH(1,1)**	**65**	**41**	**13**
		EGARCH(1,1)	69	46	19
		GJR-GARCH(1,1)	69	44	17
	1000	**GARCH(1,1)**	**34**	**21**	**9**
		EGARCH(1,1)	36	26	13
		GJR-GARCH(1,1)	35	24	11
	2000	**GARCH(1,1)**	**57**	**35**	**10**
		EGARCH(1,1)	63	36	15
		GJR-GARCH(1,1)	62	40	15
Student t	500	**GARCH(1,1)**	**59**	**39**	**11**
		EGARCH(1,1)	64	43	20
		GJR-GARCH(1,1)	63	41	17
	1000	**GARCH(1,1)**	**35**	**22**	**9**
		EGARCH(1,1)	36	26	13
		GJR-GARCH(1,1)	35	24	11
	2000	**GARCH(1,1)**	**57**	**35**	**11**
		EGARCH(1,1)	62	40	16
		GJR-GARCH(1,1)	61	37	15

TYR Violations					
Distribution	No. Obs	Model	VaR (90%)	VaR (95%)	VaR (99%)
Gaussian	500	GARCH(1,1)	36	23	6
		EGARCH(1,1)	**33**	**22**	**4**
		GJR-GARCH(1,1)	38	25	7
	1000	GARCH(1,1)	34	20	4
		EGARCH(1,1)	34	21	4
		GJR-GARCH(1,1)	33	20	4
	2000	GARCH(1,1)	32	17	3
		EGARCH(1,1)	30	17	3
		GJR-GARCH(1,1)	30	17	3
Student t	500	**GARCH(1,1)**	39	23	6
		EGARCH(1,1)	46	24	8
		GJR-GARCH(1,1)	44	25	7
	1000	**GARCH(1,1)**	31	18	3
		EGARCH(1,1)	34	20	5
		GJR-GARCH(1,1)	32	20	5
	2000	GARCH(1,1)	31	17	3
		EGARCH(1,1)	29	17	3
		GJR-GARCH(1,1)	30	16	3

Source: Compiled by the author

Appendix 5 continues

DXYR Violations					
Distribution	No. Obs	Model	VaR (90%)	VaR (95%)	VaR (99%)
Gaussian	500	GARCH(1,1)	51	32	14
		EGARCH(1,1)	45	31	14
		GJR-GARCH(1,1)	49	33	12
	1000	**GARCH(1,1)**	47	31	10
		EGARCH(1,1)	53	33	19
		GJR-GARCH(1,1)	47	32	10
	2000	**GARCH(1,1)**	27	15	6
		EGARCH(1,1)	29	14	3
		GJR-GARCH(1,1)	31	14	6
Student t	500	GARCH(1,1)	49	32	13
		EGARCH(1,1)	43	29	15
		GJR-GARCH(1,1)	48	31	10
	1000	GARCH(1,1)	47	32	10
		EGARCH(1,1)	51	33	19
		GJR-GARCH(1,1	47	32	10
	2000	**GARCH(1,1)**	28	13	4
		EGARCH(1,1)	29	13	3
		GJR-GARCH(1,1)	30	14	5

SPGR Violations					
Distribution	No. Obs	Model	VaR (90%)	VaR (95%)	VaR (99%)
Gaussian	500	GARCH(1,1)	31	18	7
		EGARCH(1,1)	31	17	6
		GJR-GARCH(1,1	29	15	7
	1000	GARCH(1,1)	32	16	7
		EGARCH(1,1)	26	14	7
		GJR-GARCH(1,1)	29	15	7
	2000	GARCH(1,1)	27	14	6
		EGARCH(1,1)	25	13	7
		GJR-GARCH(1,1)	25	14	8
Student t	500	GARCH(1,1)	34	19	7
		EGARCH(1,1)	32	16	7
		GJR-GARCH(1,1	30	16	8
	1000	GARCH(1,1)	32	16	7
		EGARCH(1,1)	26	14	7
		GJR-GARCH(1,1)	28	16	7
	2000	GARCH(1,1)	27	14	7
		EGARCH(1,1)	25	13	7
		GJR-GARCH(1,1)	25	14	7

Source: Compiled by the author