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Distributed Signal Processing in Cognitive Radio Networks

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Declaration:
Hereby I declare that this doctoral thesis, my original investigation and achievement, submitted for the doctoral degree at Tallinn University of Technology has not been submitted for doctoral or equivalent academic degree elsewhere.

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signature

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Hajutatud signaalitöötlus kognitiivse raadio võrgus

AHTI AINOMÄE

To my parents.

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LIST OF PUBLICATIONS

The work of this thesis is based on the following publications:

P1. A. Ainomäe, T. Trump, and M. Bengtsson, "Distributed recursive energy detection," in *IEEE WCNC 2014 Conference Proceedings*, Istanbul, Turkey, 2014. ETIS 3.1.

P2. A. Ainomäe, T. Trump, and M. Bengtsson, "CTA diffusion based recursive energy detection," in *Latest Trends in Circuits, System Signal Processing and Automatic Control*, Salerno, Italy, 2014, pp. 38 – 47. ETIS 3.2.

P3. A. Ainomäe, T. Trump, and M. Bengtsson, "Distributed diffusion LMS based energy detection," in *IEEE 6th International Congress on Ultra Modern Telecommunications and Control Systems and Workshops (ICUMT)*, St. Petersburg, Russia, 2014, pp. 176 – 183. ETIS 3.1.

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P5. A. Ainomäe, M. Bengtsson, and T. Trump, "Distributed largest eigenvalue based spectrum sensing using diffusion adaptation," *IEEE Transactions on Signal and Information Processing over Networks*, Sep. 2017. ETIS 1.1.

P6. A. Ainomäe, T. Trump, and Y. L. Moullec, "SNR weighted distributed largest eigenvalue based spectrum sensing," in *Submitted to 16th Biennial Baltic Electronics Conference (BEC 2018)*, Tallinn, Estonia, 2018. ETIS 3.1.

OTHER RELATED PUBLICATIONS

The following papers were written by the author of the thesis during his PhD studies at TUT, but which are not included in this thesis:

P7. A. Ainomäe, "LMS based distributed variance estimation for signal energy detection," in *Info- ja kommunikatsioonitehnoloogia doktorikooli IKTDK viienda aastakonverentsi artiklite kogumik: 25.-26. novembril 2011, Nelijärve*, Nelijärve, Estonia, 2011, pp. 1 – 4. ETIS 3.5.

P8. A. Ainomäe, I. Rokk, and E. Lossmann, "Teaching of telecommunication-specific university course in cooperation with partners from industry," *Elektronika ir Elektrotechnika*, vol. 110, no. 4, 2011. ETIS 1.1.

AUTHOR'S CONTRIBUTIONS TO THE PUBLICATIONS

The contributions of the author of this thesis to the included papers are the outcome of the author′s own work, in collaboration with the co-authoring academical advisors Dr. Tõnu Trump, Prof. Mats Bengtsson and Prof. and Yannick Le Moullec. The author of this thesis is the first and only non-supervising author of the papers P1. to P6.

The contribution of the author to the papers in this thesis are:

P1. The author proposed the use of distributed, ring-around topology based recursive power estimation scheme for implementing the distributed energy detection algorithm in CR networks. The author performed the literature research, developed and implemented the proposed algorithm, proposed the formulas of the theoretical moments of the proposed adaptive estimates and performed the theoretical analysis. The author created the MATLAB simulation scripts, performed the computer experiments and processed the

results. The author wrote the paper, supervisors Dr. Tõnu Trump and Prof. Mats Bengtsson helped with paper revision.

P2. The author proposed the use of distributed, Combine And Adapt (CTA) type of diffusion LMS based adaptive power estimation scheme for implementing the distributed energy detection algorithm in CR networks. The author performed the literature research, developed and implemented the proposed algorithm, proposed the general framework for the performance analysis of the algorithm and performed the theoretical analysis. The author created the MATLAB simulation scripts, performed the computer experiments and processed the results. The author wrote the paper, supervisors Dr. Tõnu Trump and Prof. Mats Bengtsson helped with paper revision.

P3. The author generalized the approach, presented in paper P2., by proposing the usage of generic Adapt and Combine (ATC) type of diffusion LMS based power estimation scheme for implementing the distributed energy detection algorithm in CR networks. The author performed the literature research, developed and implemented the proposed algorithm, extended the framework for the performance analysis of the algorithm, performed the theoretical analysis and verified the theoretical results. The author created the MATLAB simulation scripts, performed the computer experiments and processed the results. Additional simulations were run by the author to study the observation exchange effect of these algorithms. The author wrote the paper, supervisors Dr. Tõnu Trump and Prof. Mats Bengtsson helped with paper revision.

P4. The author proposed the use of distributed, Combine And Adapt (CTA) type of diffusion LMS based adaptive correlation matrix estimation scheme for implementing the distributed largest eigenvalue detection algorithm in CR networks. The author performed the literature research, developed and implemented the proposed algorithm, proposed the framework for the performance analysis of the algorithm including the needed distribution approximations for the theoretical analysis and performed the theoretical analysis. The author created the MATLAB simulation scripts, performed the computer experiments and processed the results. The author wrote the paper, supervisors Dr. Tõnu Trump and Prof. Mats Bengtsson helped with paper revision.

P5. The author extended the topic, introduced in paper P4., by focusing on the ATC type of diffusion LMS based CM estimation algorithm, by extending the theoretical analysis, by proposing an additional CM distribution approximation method and by adding an

additional signal model for a comparison. The author performed an additional literature research, developed and implemented the proposed algorithm, extended the framework for the performance analysis and performed the theoretical analysis. The author created the MATLAB simulation scripts, performed the computer experiments and processed the results. The author wrote the paper, supervisors Dr. Tõnu Trump and Prof. Mats Bengtsson helped with paper revision.

P6. The author extended the topic, introduced in papers P4. and P5., by proposing the use of local SNR based weighted input observations in the previously proposed CTA type of adaptive CM estimation algorithm. The author proposed an adaptive local SNR estimation and local observation exchange and weighting strategy for the usage in CR networks. The author created the MATLAB simulation scripts, performed the computer experiments and processed the results of probability of detection largest eigenvalue method together with local SNR based observation exchange. The author wrote the paper, supervisors Dr. Tõnu Trump and Yannick Le Moullec helped with paper revision.

Thus, as specified in Section 1.4, material presented in this thesis is based on the author's previous work which is published or submitted to conferences and journals held by or sponsored by IEEE and WSEAS publishers. They hold the copyright of the published papers and will hold the copyright of the recently accepted papers. Confirmation of the usage of these papers in current PhD thesis is to be asked.

ABBREVIATIONS

3G	Third Generation
3GPP	3rd Generation Partnership Project
4G	Fourth Generation
5G	Fifth Generation
ATC	Adapt and Then Combine
AWGN	Additive White Gaussian Noise
CDF	Cumulative Distribution Function
CM	Correlation Matrix
CR	Cognitive Radio
CSCG	Circular Symmetric Complex Gaussian Distribution
CTA	Combine and Then Adapt
dB	Decibel
ED	Energy Detection
FC	Fusion Center
GSM	Global System for Mobile Communications
IEEE	Institute of Electrical and Electronics Engineers
IID	Independent and Identically Distributed
IoT	Internet of Things
LE	Largest Eigenvalue Detection
LMS	Least Mean Square
LTE	Long-Term Evolution
MIMO	Multiple-Input Multiple-Output
MMSE	Minimum Mean Squared Error
MSE	Mean of the Squared Error
OFDM	Orthogonal Frequency Division Multiplexing
PD	Probability of Detection
PDF	Probability Density Function
PFA	Probability of False Alarm
PSD	Power Spectral Density
PU	Primary User in a Cognitive Radio Network
RF	Radio Frequency
RLS	Recursive Least Squares
SNR	Signal to Noise Ratio
WSN	Wireless Sensor Networks

SYMBOLS

a	Scalars are written in normal font
$\|a\|$	The absolute value of the scalar a
$\mathbf{a}$	Vectors are written in lower-case bold font
$\|\mathbf{x}\|_p$	p-norm of $\mathbf{x}$
$\mathbb{R}^N$	N-dimensional real field
$\mathbb{C}^N$	N-dimensional complex field
$\|\mathbf{x}\|$	2-norm of $\mathbf{x}$
$\mathbf{A}$	Matrices are written in upper-case bold font
$\mathbf{a}_{i_k,n}$	The nth column of the matrix $\mathbf{A}_{i_k}$
$\mathbf{A}^{-1}$	Matrix Inverse of $\mathbf{A}$
$[\mathbf{A}]_{i,j}$	Element corresponding to i row and j column
$\mathbf{A}^{\mathrm{T}}$	Matrix transpose
$\mathbf{A}^*$	Conjugate transpose
$\mathbf{A}^{\mathbf{H}}$	Hermitian transpose
$\otimes$	Kronecker product
$\mathbf{I}_n$	The identity matrix of size $n \times n$
$\lambda_{\max}(\mathbf{A})$	The eigenvalue of the matrix $\mathbf{A}$ with the largest magnitude
$\lambda_{\min}(\mathbf{A})$	The eigenvalue of the matrix $\mathbf{A}$ with the smallest magnitude
$\Pr(\mathcal{E})$	Probability of the event $\mathcal{E}$
$p(x)$	PDF of x
$\mathcal{CN}(\mathbf{m}, \mathbf{C})$	The circularly-symmetric complex Gaussian probability distribution with mean $\mathbf{m}$ and covariance $\mathbf{C}$
$\mathcal{CW}_M(N, \mathbf{\Sigma})$	Complex Wishart distribution of dimension M with degree of freedom N and population covariance matrix $\mathbf{\Sigma}$
$a \sim b$	The random variable a is distributed according to the probability distribution b
$\mathrm{E}[\cdot]$	Expectation of a random variable
$\mathrm{Var}[\cdot]$	Variance of a random variable
$\mathrm{Cov}[\cdot]$	Covariance of a random vector
$\mathrm{vec}[\cdot]$	Vectorization of a matrix
$\det[\cdot]$	Determinant of a matrix
$\mathrm{diag}[\cdot]$	Vector of diagonal elements of a matrix
$\mathrm{Tr}[\cdot]$	Trace of a matrix
$\{a, b, c\}$	Set with elements a, b and c

$[a, b]$	Closed interval between a and b
$l \in \mathcal{N}_k / \{k\}$	Variable l belongs to the set of $\mathcal{N}_k$ by excluding the element of k from the set
$\mathbf{Q}(\cdot)$	Tail Probability of standard normal function
argmin	The argument that achieves minimum
$\binom{n}{k}$	The (n, k)th binomial coefficient
$\mathcal{C}$	The set of complex numbers
$\mathrm{Re}(c)$	Real part of the complex number $c \in \mathcal{C}$
$\mathrm{Im}(c)$	Imaginary part of the complex number $c \in \mathcal{C}$

LIST OF FIGURES

1. INTRODUCTION

Future communication networks will have seamless and ubiquitous connectivity among several communicating devices using different radio technologies. In the year 2021, it is predicted that there will be 16 billion devices that will be connected [9]. These devices could include cell phones, TVs, computers, tablets, etc. Wireless sensor networks (WSN) play an important role in the future of Internet of Things (IoT) systems. Several applications, such as Smart Grids, Smart Homes, Intelligent control systems, are built upon wireless sensor networks. As a result, sensing and information processing in the sensor networks becomes more and more important. The increasing trend of more connected devices via wireless channels leads to the potential problem of lack of free and usable radio frequencies (as a national resource) and brings up the dilemma for allowing an opportunistic spectrum usage. Special solutions are needed to handle that problem.

1.1. Cognitive Radio in Wireless Communications

Cognitive telecommunication systems are a relatively new direction in telecommunication research. Traditionally, the radio frequencies have been divided between the interested parties by licensing. The party who has a license to use a given frequency band has exclusive rights to the band and no one else can use this band. Nowadays we are reaching the situation where the attractive frequency bands are full and there are no more frequencies available to license out new and innovative applications. This situation makes the development and implementation of new radio-based services more challenging all over the world. Recent studies have shown that the available licensed radio spectrum is becoming more occupied, while the assigned spectrum is significantly underutilized [10]. The licensed users do not use their spectrum in all locations and at all times and it is possible to utilize the available spectrum more fully and effectively. Cognitive radio [11–13] is a technology that was proposed in 1998 by Joseph Mitola to solve this problem [14]. Within this paradigm the radio equipment will search unused frequencies by itself and sense the spectrum area of interest to detect the presence of a licensed user. The proposed solution poses both technical and legal problems, which are currently dealt with. Cognitive radio is seen as a new promising technology and the research topic is providing interests to great

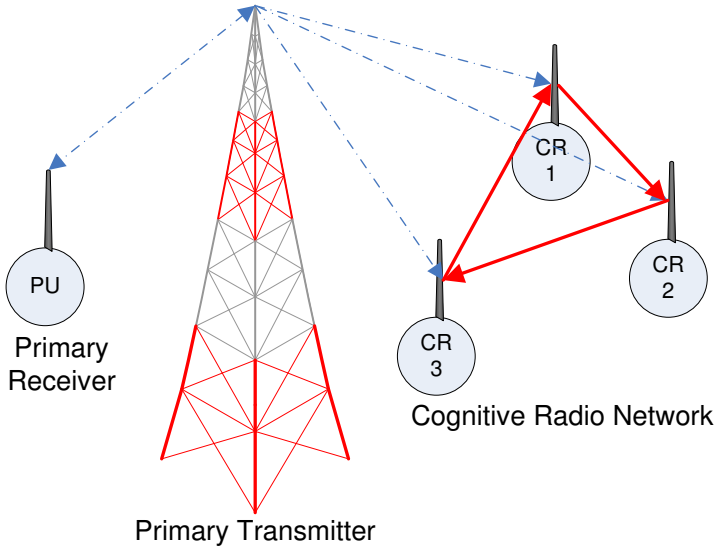

Figure 1.1 Basic layout of CR network

amount of universities in spectrum sensing and signal detection, estimation, and communication areas [15, 16].

More specifically, spectrum utilization can be improved by allowing secondary (unlicensed) users to opportunistically access the licensed spectrum area when the primary user (PU) is not present. A cognitive radio (CR) technology is able to serve the secondary users (SUs) for detecting and utilizing so called spectrum holes by sensing and adapting to the environment without causing harmful effects or interference to the licensed PUs. It is expected that CR systems are able to systematically detect the presence of a primary user (while the CR system usually does not have the *a priori* knowledge that the channel is free) by continuously sensing the spectrum area. If a PU signal is detected, the CR system has to immediately stop operating in this specific frequency area and has to adapt and find new free spectrum area or channel for continuing its operation. PUs may use different kind of modulations, transmission rates and powers, which makes the spectrum sensing more complicated. A CR network is illustrated in Fig. 1.1.

Since the active work-pattern of a PU is usually not known for the CR system, then adaptive signal processing methods could be used for spectrum sensing, which are able to learn and track the changes in the statistical properties of the underlying process.

One of the examples with Cognitive Radio technology is the usage of TV White Space. The unoccupied TV UHF band may be used for secondary services

during time periods when the primary TV stations are switched off [12, 17]. Support for opportunistic spectrum access has for example been proposed initially for the LTE (4G) standard [18] and also for the 5G [19, 20]. The topics related to cognitive radio technology are providing interests to world leading mobile access technology providers, including Ericsson [21, 22], etc.

The research in this thesis focuses on the methods of statistical signal processing. As widely known, statistical signal processing is a research area of applied mathematics and (digital) signal processing, where signals are treated as (discrete time) stochastic processes and the processing methods utilize the statistical properties of the signals to perform signal processing tasks. Statistical techniques are widely used in wireless signal processing applications.

In this thesis we investigate distributed cooperative detection algorithms that the radio equipment can use to determine whether a frequency is usable or not, i.e. whether the primary user is using the frequency for its own purposes or not. A single CR node may not be in a good location to detect the presence of a PU with high probability because of the effects of radio propagation like fading and shadowing of radio waves. A more reliable decision can be obtained if several cognitive users work together sharing information. In the thesis we investigate two cooperative detection techniques, that do not need any fusion center (which would be a single point of failure), but are rather similar to those used in adaptive filtering to share the information. The individual nodes share the information directly with each other.

The aim of the work is to develop algorithms usable for both individual and cooperative detection that can be used in cognitive radio networks to detect the presence of PU users. In this thesis we assume that there is only one PU signal present; however, the current work can be logically extended also to the cases were more PU signals are present, by updating the measurement signal model and by choosing or designing most optimal detector (module) for these specific cases.

1.2. Adaptive Distributed Signal Processing and Optimization

Several classical distributed detection methods have been proposed and studied in the literature [23]. Most of the classical solutions are, however, based on the ”close to or ideal” *a priori* knowledge about the statistical properties of the observations and the detection hypotheses. In the CR application area, we have usually limited information about the PU signal and about the prior probabilities of the detection hypotheses. The CR system usually has limited information about transmission parameters, modes and functions of the PU system. Thus, in the CR context we usually can not design an optimal detector in the sense of classical detection theory since the parameters of the conditional distributions of the measurements are not ideally known (but have to be estimated, where an estimation error is always present). Also in the CR context, it is not that practical to limit the detection solutions with the assumption that the prior probabilities of

the detection hypotheses are known and fixed over a period of time. Thus, the classical detection methods based on the Bayesian approach are not that practical in the CR context and we instead use the Neyman-Pearson type of detectors.

Adaptive filters [24, 25] have been used extensively in the systems where the parameter to be estimated has a dynamic nature. Several applications in the literature use non-adaptive estimation methods (based on collected amount of samples) to estimate a parameter of interest. Adaptive (recursive) algorithms are, however, able to react to the changes in the statistical properties of the measurements ”on line” and during the time when the recursive algorithm is kept running. In comparison, classical non-adaptive estimation methods usually have to be restarted when the maximum amount of samples have been collected and when the value of the estimate has been calculated. This leads to design issues related to the size of measurement data windows for a specific application and there is a higher probability to miss the start moments of the transitions in the statistical processes of the measurements. Moreover, adaptive algorithms usually do not require large amounts of system memory since only the data from the previous time instant should be stored into the memory. These mentioned aspects make the usage of adaptive estimation algorithms in the CR application context more practical.

Distributed adaptive estimation and detection schemes have been studied before in several papers [26, 27]. An optimal, matched filter based distributed detection scheme has been studied in [28]. However, in most cases we do not have any information about the waveform of the PU signals and hence we cannot design a matched filter based solution [28]. LMS (Least Mean Square) based distributed estimation schemes have been investigated for example in [26, 27]. In this thesis, LMS (Least Mean Square) based adaptive estimation algorithms (which is a stohastic gradient based algorithm) are chosen due to the simplicity, robustness and good tracking abilities, compared to e.g. RLS (Recursive Least Squares) [29].

Some recent developments in adaptation, learning, and optimization over networks have been published for example in [30, 31]. Diffusion Optimization strategies [32, 33] can be seen as a generalizations of diffusion LMS estimation algorithm [26, 27].

CONTRIBUTIONS OF THE THESIS

The current thesis is composed of the papers, listed in the publication section. The corresponding research work was performed in accordance to the cooperation memorandum, agreed and signed between Tallinn Institute of Technology, Estonia and KTH Royal Institute of Technology, Stockholm, Sweden in September 2012. This cooperation memorandum, stated the plan of additional studied at KTH, joint co-supervision of my research work and the defence strategy of the jointly published material. More specifically, the jointly prepared papers P1-P5 i.e. the Chapters 3 to 4 are also included in my Licenciate thesis (Lic. Tech.) [34], which was defended successfully on September 28, 2017. The Chapters 1, 2 and 6 of this thesis are based on the structure which is similar to the corresponding chapters of my Lic. Tech. thesis, but which is updated with relevant information. This thesis includes the additional Chapter 5, which is based on the corresponding latest publication, prepared after September 28, 2017.

1.3. Motivation and Research Statements

Based on the discussion in previous sections, in this thesis we consider a scenario with a number of CR nodes in the network, which sense a spectrum area of interest. We additionally assume that the Gaussian noise floor is constant over the nodes. Several solutions have been proposed that make use of a central processing unit to collect all the measurements over a sensing period from all the nodes and make decisions about presence or absence of PU, for example [17, 35, 36]. Instead of this, we expect that the measurements or estimates are exchanged between the CR nodes directly, without involvement of any central processing unit (fusion center). At every time instant new measurements or estimates from the neighbouring nodes become available. Thus CR nodes estimate the elements of the test statistics in their own location and make individual decisions about the detection hypotheses. Depending on the exact topology of the network, with such a solution communication in the network can be reduced as compared to solutions where nodes send their measurements to a fusion center, which sends the collected estimates back to the nodes after an iteration of the estimation process. This method saves energy, required for the data transmission of the single nodes (transmitting function usually consumes most of the power of a node [37]). In addition, this method enhances network failure resistance (in case of FC stops operating).

The above discussion naturally leads to the following main research topics which are addressed in this thesis:

1. Cooperative signal processing in CR Networks.
2. Distributed estimation and detection in Cognitive Radio, without using a FC.
3. Distributed Energy and Largest Eigenvalue detection in Cognitive Radio. Resulting detection performance analysis.

The main research objectives in the thesis are the following:

1. Removal of the central processing unit – FC – from the domain of estimation and detection in the CR network. It is expected that the CR network is able to estimate the test statistic of a detector and to detect the presence of the PU signal without the usage of any FC.

   The solutions in this thesis are based on the idea that distributed estimation schemes are used for designing distributed detection schemes, with no use of a FC. Thus, the distributed detection schemes are based on the underlying distributed estimation strategies and topology in the CR Network.

2. We assume to have limited information about the type and properties of the PU signal and therefore an energy detection method becomes a usable solution. The energy detection method is implemented in a distributed way in the CR network. Secondly, several types of correlation matrix based detection methods exist in the literature. We have chosen to study the Largest Eigenvalue detection method, which is similarly implemented in a distributed way in the CR network.

3. Least Mean Square (LMS) type of adaptive estimation algorithms are based on the stochastic gradient descent and the LMS estimates are modelled as random variables. Thus, LMS type of algoritms are suitable for the estimation of a statistical moment based detection test statistics. Distributed Diffusion LMS algorithms have been already proposed and studied in the literature. We adapt the Diffusion LMS algorithms to estimate the statistical moment based detection test statistics directly.

4. Since the PU signal is assumed to be slowly fading, we design the usage of distributed adaptive estimation schemes so that approximately equal statistical properties of the estimates are achieved in every CR node in the network. In such a way, an averaged detection performance in every CR node is achieved regardless of the actual channel conditions of each single node.

5. Usage of adaptive and recursive estimation schemes. We are interested in the online tracking ability of the statistical properties of the estimates

to react to the changes in the presence of PU signal – i.e to the changes of the underlying detection hypothesis, over the iteration period of the distributed estimation algorithm.

6. As common in the area of statistical signal processing, a (statistical) performance analysis for the proposed algorithms is performed. Since the detection performance of the proposed distributed detection schemes depends on the statistical properties of the underlying estimates, we propose to use a generic framework for studying the performance of the proposed estimation schemes in the CR network level. We focus on the analysis of the theoretical statistical moments of the estimates to study the resulting detection performance.

7. In the simulation sections we compare the theoretical findings with the results obtained via Monte-Carlo based computer simulations. A good match between theory and practice allows us to use computationally much faster theoretical calculations to evaluate the performance of the proposed algorithms in different use cases. We mainly use the probability of detection versus averaged SNR type of computer simulations to study the detection performance of the proposed algorithms and to evaluate the ranges when the detection methods fail to provide perfect detection results.

1.4. Thesis Outline

This section provides an outline of the thesis with a brief summary of the material presented in each chapter. This thesis consists of six chapters: Introduction, Preliminaries, three Contribution Chapters and the Summary Chapter, of which are summarized as follows.

Chapter 2

Chapter 2 provides background information, a brief discussion on the concepts and tools is given, which is essential to follow the rest of the thesis. We give a short summary of the theory of statistical signal processing in connection to the material in this thesis, where we discuss the basics of detection and estimation theory. We provide a generic introduction for the derivation of diffusion LMS types of algorithms. Also, we provide a short summary about the literature on CR.

Chapters 3, 4 and 5 discuss the main contributions of this thesis. Each chapter follows the structure of the corresponding published papers and thus is complete by itself – the reader does not need the content of the previous or subsequent chapters to follow the material. However, the chapters themselves address problems and solutions which are partly related. Each chapter begins with a background section, which gives the overall context to the discussion that

follows and ends with a conclusion section which summarizes the chapter along with the main concepts from that chapter.

Chapter 3

More specifically, Chapter 3 addresses the distributed energy detection problem in CR networks. Often we have limited information about the signal received by the cognitive radio nodes and such signal flow can not be modelled as a deterministic process. Since the radio signals contain information when the PU signal is present, it is often more suitable to model the PU signal also by a random process, in addition to the radio channel noise process. In such cases, energy detection becomes a usable solution. We are interested to remove a potential single point of error - a central processing unit from the cognitive radio network. Each CR node should be able to rely only on the communication between the neighbour CR nodes. We use distributed recursive estimation schemes to estimate the power of the received signal in a distributed way.

We propose the usage of distributed, Diffusion LMS type of power estimation algorithms and three different static network topologies: Ring-Around, Combine And Adapt and Adapt and Combine are studied. We provide a generic framework for studying the detection performance of the proposed schemes by using the statistical properties of these distributed estimates. In case of the Ring-Around topology, a generic recursive signal power (statistical variance) estimation algorithm is proposed and more specific results about the moment estimation of the distributed estimates can be given. These results have been integrated into the same chapter. The theoretical findings are verified by MATLAB based simulations.

This chapter is based on the following 3 papers:

P1. A. Ainomäe, T. Trump, and M. Bengtsson, “Distributed recursive energy detection,” in *IEEE WCNC 2014 Conference Proceedings*, Istanbul, Turkey, 2014. ETIS 3.1.

P2. A. Ainomäe, T. Trump, and M. Bengtsson, “CTA diffusion based recursive energy detection,” in *Latest Trends in Circuits, System Signal Processing and Automatic Control*, Salerno, Italy, 2014, pp. 38 – 47. ETIS 3.2.

P3. A. Ainomäe, T. Trump, and M. Bengtsson, “Distributed diffusion LMS based energy detection,” in *IEEE 6th International Congress on Ultra Modern Telecommunications and Control Systems and Workshops (ICUMT)*, St. Petersburg, Russia, 2014, pp. 176 – 183. ETIS 3.1.

Chapter 4

Chapter 4 deals with distributed correlation matrix (CM) based signal detection in Cognitive Radio network. The PU signal is assumed to be temporally correlated. Similarly as in the previous chapter, we study the usage of diffusion LMS based estimation strategies for estimating the elements of the correlation matrices, used for PU signal detection. Two static network topologies Combine and Adapt (CTA) and Adapt and Combine (ATC), are used in this chapter and we run some simulations with Consensus and FC based network topologies for comparison. The estimation and detection solution does not rely on any central processing unit in the network. The estimation strategies and the section of performance analyses have been adapted and extended to deal with vector estimates and block-covariance matrices. Several correlation matrix based detection solutions have been proposed in the literature and in this research work we have chosen the Largest Eigenvalue based detection solution, where in case of Primary user signal exists in the network we assume that the PU signal has a rank one correlation matrix. In order to obtain analytic results on the detection performance, the exact distribution of the CM estimates are approximated by a Wishart distribution, by matching the moments. The theoretical findings are similarly verified by MATLAB based simulations.

This chapter is based on the following 2 papers:

P4. A. Ainomäe, T. Trump, and M. Bengtsson, "Distributed largest eigenvalue detection," in *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP) 2017*, New Orleans, United States, 2017, pp. 176 – 183. ETIS 3.1.

P5. A. Ainomäe, M. Bengtsson, and T. Trump, "Distributed largest eigenvalue based spectrum sensing using diffusion adaptation," *IEEE Transactions on Signal and Information Processing over Networks*, Sep. 2017. ETIS 1.1.

Chapter 5

Chapter 5 deals with distributed correlation matrix (CM) based signal detection in Cognitive Radio network, where diffusion LMS based estimation strategies for estimating the elements of the correlation matrices are applied for PU signal detection and no FC unit is used. Compared to the solutions in our papers [4] and [38], in this chapter, an additional local observation exchange and combination strategy is introduced and studied, which is based on the local SNR estimates and is adapted to the context of binary hypothesis testing. It is shown that when the PU signal is present and when the local SNR estimates are available, then the network-wise PU signal detection performance can be slightly improved, compared to the standard case with no observation exchange studied in our paper [4]. The theoretical analysis is performed and the theoretical findings are similarly verified by MATLAB-based simulations.

This chapter is based on the following paper:

P6. A. Ainomäe, T. Trump, and Y. L. Moullec, “SNR weighted distributed largest eigenvalue based spectrum sensing,” in *Submitted to 16th Biennial Baltic Electronics Conference (BEC 2018)*, Tallinn, Estonia, 2018. ETIS 3.1.

Chapter 6

Finally, Chapter 6 summarizes the author’s contribution results in this thesis and lists possible directions for future research.

2. PRELIMINARIES

In this chapter, some basic concepts, that are essential to follow the rest of thesis, are introduced.

2.1. Summary on Cognitive Radio

In this section a brief summary about the aspects of Cognitive Radio Networks is provided, which are essential in the context of the thesis. The section is based mainly on the material from [12], [39].

It was already briefly mentioned in Chapter

The concepts of software−defined radio and cognitive radio have been introduced to enhance the efficiency of frequency spectrum usage in next generation wireless and mobile computing systems. Cognitive ratio, which can be implemented through software−defined radio, is able to observe, learn, optimize, and intelligently adapt to achieve optimal frequency band usage.

Dynamic spectrum access (DSA) or opportunistic spectrum access (OSA) is the key approach in a cognitive radio network and has emerged as a new design paradigm for next generation wireless networks. Therefore, a new spectrum licensing paradigm also needs to be initiated by the national frequency regulation institutions, for being more flexible in allowing unlicensed (or secondary) users to access the spectrum as long as the licensed (or primary) users are not interfered with. In such a way, the utilization of the frequency spectrum could be improved. In general the development of dynamic spectrum access-based cognitive radio technology has to deal with technical and practical considerations as well as regulatory requirements.

The main frequency bands for CR are considered as follows

1. UHF band, typically 470-790 MHz;
2. Cellular bands, typically 800-900 MHz, 1.8-1-9 GHz, 2.1 GHz, 2.3 GHz, and 2.5 GHz;
3. Wireless access bands, typically 2.5-3.5 GHz.

The main functions of CR to support DSA can be listed as follows [12]:

1. Periodical spectrum sensing, which can be centralized (FC based) or distributed, to determine if the frequency area of interest is free;

2. Spectrum analysis, to process the information obtained from previously listed step, plan the spectrum access and optimize the transmission parameters;

3. Spectrum access, with the help of a cognitive medium access control (MAC) protocol;

4. Spectrum mobility, to change the operating frequency band of CR users.

Three major models of dynamic spectrum access are considered: common-use, shared-use, and exclusive-use models. In the first case, the spectrum is open for access to all users. In the second case, licensed users (i.e. PUs) are assigned to the frequency bands which are opportunistically accessed by the unlicensed users. In the latter case, a PU can grant access of a particular frequency band to an unlicensed user for a spectrum leasing (for a certain period of time).

CR has to use a frequency area without causing interference to the PUs. There are three main approaches for opportunistic spectrum access [39]:

1. Spectrum Interweave;

2. Spectrum Overlay;

3. Spectrum Underlay.

The spectrum interweave paradigm was the original motivation for the idea of CR. The requirement is that the CRs should not interfere with the communication between the already active PUs. Thus, the CRs should be able to detect (sense), with very high probability, the primary user transmissions in the network. Once the CR successfully detects the PU transmissions, it can opportunistically communicate only if it is able to do so without causing any harmful effects to the PU transmissions. This requires spectrum agility or the ability to transmit at different frequencies. The temporary space−time−frequency gap in the transmission of PUs is referred to as a spectrum hole or a white space. The spectrum overlay paradigm is more advanced. The CR needs to know the channel properties between the primary transmitter and the primary and secondary receivers, as well as the channel between the secondary transmitter and the primary receiver. With the channel knowledge of both the primary and CRs, the CR can then choose appropriate transmission strategies so that the communication in the secondary network causes least interference to the primary network. In the spectrum underlay paradigm, the secondary transmitter keeps the interference levels below a certain threshold. The primary receiver sees a higher noise level if the primary and secondary transmission overlap in the same band. Possible methods include transmission power control, beam-forming and spread spectrum techniques.

A combination of these methods may be also considered. In practice estimation of channel gains is often a complicated task and the main CR detection

algorithm could be constructed so that knowledge about the channel gains is not required. Thus, although the overlay and the interweave approaches are similar, in this thesis I focus on the detection methods which follow the interweave approach, where it is assumed the detectors are not aware about the channel gains of PU signal.

2.1.1. Spectrum Sensing in Cognitive Radio

This section focuses on the spectrum sensing task of the CR. The objective is to detect the presence of transmissions from licensed users. Three major types of spectrum sensing types are: non−cooperative, cooperative and interference−based sensing [12].

As already discussed in Chapter 1, a CR can accomodate an independent detector function. Thus a CR nodes monitors a spectrum area of interest (of bandwidth B) by processing the received PU signal in an additive noise (including filtering, discretizing, calculation of the function of test statistics etc.).

Usually the model for signal detection is given based on the following idea

$$\begin{aligned} H_0 &: x(n) = v(n) \\ H_1 &: x(n) = \alpha s(n) + v(n), \end{aligned} \tag{2.1}$$

where $x(n)$ is the received signal (and is modelled by a stochastic process) of a CR user at a discrete time instant n, $s(n)$ is the transmitted signal of the PU, $v(n)$ is the additive white Gaussian noise (AWGN), and α is the channel constant (gain). The test statistic of a detector is a function of the received signal samples $x(n)$ and the test statistics is compared with a predefined detection threshold (see Chapter 2.2).

Three classical and one class of additional detection methods in *non-cooperative sensing* are for example:

1. Matched filter detection or coherent detection;
2. PU transmitter energy detection;
3. Cyclostationary feature detection;
4. Correlation based detection.

The matched filter is designed to detect a signal by correlating a known (transmitted) signal with the received signal. A matched filter will maximize the received SNR for the measured signal [40]. If the information of the signal from a licensed user is known, then a matched filter is an optimal detector in stationary Gaussian noise environment [41]. Thus, when a signal template is perfectly known and in case of the correlation is achieved between the transmitted and received signal, then a matched filter requires only a small amount of time to operate. On the other hand, when this template is not available or is incorrect, the

performance of spectrum sensing degrades significantly. Matched filter detection is suitable when the PU signal has a pilot, preambles, synchronization word or spreading codes which can be used to construct the template for spectrum sensing [12].

Energy detection is the optimal method for spectrum sensing when the information from a PU (i.e. signal type, pattern, etc.) is unavailable [41]. The output signal from a bandpass filter is squared and integrated over the observation interval. A decision algorithm compares the integrator output with a threshold to decide whether a licensed user exists or not [12]. It is widely known, that the energy detection performance deteriorates when the SNR decreases. The Energy detection method is studied further in Chapter 3 of this thesis.

The PU signal has often a cyclostationarity (periodic) pattern, and this property could be used to detect the presence of a PU user. A signal is cyclostationary (in the wide sense) if its autocorrelation is a periodic function. With such a periodic pattern, the transmitted PU signal can be distinguished from noise, which is a wide-sense stationary signal without correlation. In general, cyclostationary detection can provide a more accurate sensing result and it is robust to variations in noise power. However, the detection is complex and requires long observation periods to obtain the sensing result [12].

Let us note that in [42, Chapter 2] a brief overview of these three classical detectors with some detailed signal models and processing block schemes is given and thus we are omitting such summary in this thesis.

A second large group of detectors for spectrum sensing are based on eigenvalue properties of an estimated correlation matrix (CM) [43–45]. When the PU signal exploits certain type of low rank correlation, then this feature can be used to detect the presence of a PU signal. Several CM based detectors have been proposed in the literature: the largest eigenvalue (LE) method, the volume based detector (VD), the covariance based detector (CAV), which have been studied for example in [46, 47] and [48]. So called robust detectors do not require noise power value in the threshold calculation. Eigenvalue Arithmetic to Geometric Mean (AGM) [49], the Maximum to Minimum eigenvalue ratio (MME), the Energy to Minimum Eigenvalue ratio (EME) [45], the Eigenvalue Moment ratio (EMR) [49], and the Hadamard [50] robust detectors have been proposed in the literature. The LE method is studied further in Chapter 4 of this thesis.

A CR node may not always be able to detect the signal from a licensed transmitter due to its geographic separation (a shadowing problem) and channel fading (a multipath fading problem). In *cooperative sensing*, spectrum sensing information from multiple CRs are exchanged among each other to detect the presence of a PU. The cooperative spectrum sensing is usually performed in a centralized or distributed manner. Obviously cooperative sensing will increase the communication and computation overhead compared with non-cooperative sensing. However in case of cooperative sensing, the detection probability can usually be significantly improved [10]. In this thesis we assume that fully

distributed CR nodes perform spectrum sensing and no central processing unit (a fusion center) is used in estimation and detection domain.

We also mention, that in case of *Interference based sensing*, the noise/interference level (from all sources of signals) at the receiver of the primary user is measured. This information is used by a CR to control the spectrum access (e.g. by computing expected interference level) without violating the interference temperature limit. Alternatively, an unlicensed transmitter may observe the feedback signal from a licensed receiver to gain knowledge on the interference level.

Finally, we briefly list the potential application areas of CR [12, 39, 51] etc.:

1. Next Generation Wireless Networks, IoT, Machine-to-machine communications, Dynamic spectrum access in cellular systems;
2. Wireless broadband for distribution and backhaul, Data boost for mobile networks;
3. Coexistence of different wireless technologies, Cognitive digital home;
4. Intelligent transportation system, Long range vehicle-to-vehicle networks;
5. eHealth services;
6. Emergency networks;
7. Military networks.

Regarding the two last application areas, we could add that CR technology is proposed for military radars and maritime monitoring in wide costal areas.

2.1.2. Common Research areas in cognitive radio

For an overview, we list some main CR research areas and aspects, which follow the function areas of CR:

1. Spectrum sensing;
2. Spectrum management;
3. Spectrum mobility;
4. Network layer and transport layer;
5. Cross-layer design for cognitive radio networks;
6. Artificial intelligence approach in cognitive radio.

Since this thesis focuses on the area of spectrum sensing, it could be specified, that generic research issues can be categorized for example as follows:

1. Sensing interference limit,
2. Spectrum sensing in multiuser and multichannel networks,
3. Optimizing the period of spectrum sensing,
4. Spectrum management issues,

where obviously the research in this thesis is related to the second topic (and with the focus on the physical layer).

From another perspective, by following the recently emphasised interests in the world-level scientific conferences of communication systems, such as IEEE GLOBECOM 2017 but also IEEE ICCASP 2017, IEEE WCNC 2017, the following can be added. In the research area of embedded (electronic) systems a continuing interest is on the design of energy efficient and failure resistant hardware platform and architectures for testing and implementing the CR technology. On the other hand, in the research area of applications and services of CR, the continuing interest are in the areas of cognitive networking in TV whitespaces, adaptation and integration with newest access technologies (incl. massive MIMO and full-duplex). Also aspects related to the (cyber-) security and privacy in CR radio networks are gaining an interest.

Since the development of new generation 5G access technology is closely related to the IoT (Internet of Things) concept, then recently the research area of CR in the 5G/IoT technologies has gained increasing interest. It is expected that 5G will become the backbone for IoT devices by forming an ecosystem of so called smart devices. For example [52, Chapters 4 and 2] give an overview about the challenges related to the implementation of IoT using CR capabilities in the future 5G Mobile Networks. As also initially planned for 4G, in 5G technology, the CR technology is expected to improve the handling of resources of the future smart environments - such as improving the utilization of available radio spectrum.

2.1.3. Standardization in cognitive radio

In this section some comments about the standardization in CR area are given, based on [39].

In May 2004 US Federal Communications Commission (FCC) initiated the proposal to provide more efficient and effective use of the TV spectrum (i.e in the VHF and UHF band). As a result, IEEE 802.22 Working Group (WG) was formed to define a standardized air interface based on CRs. The IEEE 802.22 standard for Wireless Regional area Networks requires that CR nodes sense the spectrum to detect the presence or absence of active primary transmitters. In November 2008, the FCC issued a second report to adopt rules to allow unlicensed radio transmitters to operate in TV white spaces in order to make a significant amount of spectrum available for new and innovative products

and services, including broadband data and other services for businesses and consumers. FCC expects that a database and active spectrum sensing is used by the solution. In September 2010, the FCC released a third report that finalized the rules for using unused TV bands for unlicensed wireless devices, where mandatory sensing requirements were removed.

Some of the other IEEE standards related to white space networks are as follows

1. The IEEE 802.11af WG (first approved standard in 2014) for channel access and coexistence in TV White Spaces (TVWS);

2. The IEEE P1900 WG (since 2005, first approved standard in 2008) for developing supporting standards dealing with new technologies and techniques being developed for cognitive radio and advanced spectrum management;

3. The IEEE SCC41 (since 2005) for of checking, whether reusing the IEEE 802 PHY/MAC is optimal for white space operation and to estimate how far the performance of the system could benefit from a tailored PHY/MAC system;

4. The IEEE 802.19 WG (since 2005) focuses on developing standards for coexistence between wireless standards of CR devices. The first standard was formed to minimize the interference between different networks belonging to various wireless standards in the unlicensed band. The TVWS project 802.19.1 (since 2009) focuses on the coexistence of unlicensed devices in the TV White Space.

The International Telecommunication Union (ITU) has formed the following study groups that discuss cognitive radio networks.

1. ITU-R Study Group 1 on Spectrum Management, dynamic spectrum issues was covered by working part 1B;

2. ITU-R Study Group 5 on Terrestrial Services, working part 5A, which described the potential application of cognitive radio systems in the land mobile service;

3. ITU-R Study Group 5, working party 5D, where the scope of this work is to consider the inclusion of CRS into the IMT family of technologies.

In Europe:

1. The European Communications Committee (ECC) has a special Task Group working on operation of cognitive radio systems in the white spaces of the UHF frequency band. The initial focus is on opportunistic use of radio spectrum in TV White Spaces.

2. The End−to−End Efficiency is a German Large Scale Integrating Project for integrating cognitive wireless systems. The key objective of the E3 project is to design, develop, prototype, and showcase solutions to guarantee interoperability, flexibility, and scalability between existing legacy and future wireless systems.

2.2. Detection and Estimation Theory

This section summarizes some elements of detection and estimation theory and is written based mainly on materials from [53], [40] and [54, Ch.2]. Detection theory deals with the problem of determining a particular hypothesis from the observation, $\mathbf{x}$. Typically a hypothesis maps to a particular phenomenon that is being detected. For example, in the context of a CR, we can formulate a hypothesis for whether a PU signal is present or not. If there are only two hypotheses, H_0 and H_1 for a phenomenon, then the detection problem reduces to a binary hypothesis test. For a binary hypothesis, the following types of errors can occur when deciding based on the observation:

- A type-1 error or false alarm, which occurs when the observation is decoded as H_1, for an H_0 event. Probability of false alarm, $P_{FA} = \Pr(H_1; H_0)$[1].
- A type-2 error or miss, which occurs when the observation is decoded as H_0, for an H_1 event. Probability of miss, $P_M = \Pr(H_0; H_1)$.

For an optimal design, both type-1 and type-2 errors cannot be reduced simultaneously. A typical, Neyman-Pearson (NP) approach to hypothesis testing is to fix the false alarm (type-1 error) and seek an optimal detector to minimize the type-2 error. Note that minimizing the type-2 error is the same as maximizing the detection probability, $P_D = (1 - \Pr(H_0; H_1)) = \Pr(H_1; H_1)$. We can formalize this into a equation as follows:

Theorem 2.2.1. *For a given false alarm, $P_{FA} = \alpha$, to maximize P_D, decide toward H_1 if,*

$$L(\mathbf{x}) = \frac{p(\mathbf{x}; H_1)}{p(\mathbf{x}; H_0)} > \gamma, \tag{2.2}$$

where the threshold, γ, is obtained from

$$P_{FA} = \int_{\mathbf{x}:L(\mathbf{x})>\gamma} p(\mathbf{x}; H_0) d\mathbf{x} = \alpha. \tag{2.3}$$

Equation 2.2 is called the likelihood ratio test [40]. Let us note that the formula for P_D is obviously given as

$$P_D = \int_{\mathbf{x}:L(\mathbf{x})<\gamma} p(\mathbf{x}; H_1) d\mathbf{x}. \tag{2.4}$$

[1]We define $\Pr(H_i; H_j)$ as the probability of choosing hypothesis H_i when H_j has occurred.

In practice and given the specific signal model, the conditional probability density functions $p(\mathbf{x}; H_1)$ and $p(\mathbf{x}; H_1)$ of the observation variable x are specified. By following the standard derivation procedure, then usually all the constant variables in 2.2 are moved on the right hand side of the inequality and the observation data dependant variables on the left hand side. In general the detection formula can be given as follows

$$\begin{aligned} H_0 &: \mathbf{T}_x < \gamma, \\ H_1 &: \mathbf{T}_x \geq \gamma, \end{aligned} \tag{2.5}$$

where after the mentioned steps the left hand side of the likelihood ratio is made equal to the variable $\mathbf{T}_x$, which is called a test statistics of the detector. The exact or approximate conditional probability density functions are assigned for the variable $\mathbf{T}_x$ as mentioned above. Throughout this thesis, the threshold γ is determined based on the desired P_{FA} value. Often, the detection performance of a NP detector is studied with the help of P_D versus P_{FA} graphs, called Receiver Operation Charateristics (ROC) [40, Chapter 3.4].

The details for the Energy and Largest Eigenvalue Detectors are given in the corresponding sections of Chapters 3 and 4, 5, respectively. In this thesis we use the P_D versus the network average SNR graphs to study the areas where the detection method fails to provide perfect detection results.

The Estimation theory deals with arriving at a quantitative conclusion about a parameter, $\boldsymbol{\theta}$, from the observation, $\mathbf{x}$. An example of this is estimating the power of the PU signal (which is modelled as a Circularly Symmetric Complex Gaussian (CSCG) process) in CR network from a function of received PU signal samples. The joint probability distribution function, $p(\boldsymbol{\theta}, \mathbf{x})$, denotes the complete statistical description of the parameters and observations. The parameter $\boldsymbol{\theta}$ can be random and unknown. However, in certain estimation problems $\boldsymbol{\theta}$, can be deterministic. Under these conditions, good estimators can be designed by mathematically modelling the observation $\mathbf{x}$ through the parametrized PDF, $p(\mathbf{x}; \boldsymbol{\theta})$.

Typical estimation methods depend on the model assumptions. In this thesis mainly the mean and variance estimation tasks are considered. The details are described in Section 3, 4 and 5, respectively.

Let us note that in case of a PU signal detection problem, the usage of Bayes approach, both in detection [40] and in estimation [53] domains, is rather impractical, since usually the CR system does not obtain sufficiently accurate and *a priori* data about the (longer time) statistical behaviour of the PU signal(s) and thus about the parameters of the distributions of the corresponding random processes. It is more practical to view the PU behaviour as a dynamic process, where the statistical parameters of interest may change inexplicably during the observation time. Thus, we rather need to look for the adaptive estimation solutions to implement the detectors of interest, as discussed in what follows.

2.3. Adaptive Distributed Signal Processing and Optimization

”An adaptive filter is a system with a linear filter that has a transfer function controlled by variable parameters and a means to adjust those parameters according to an optimization algorithm” [24, 25]. Usually the adaptive filters are digital filters and are suitable for the applications where ”some parameters of the desired processing operation” are not known in advance or are changing over the time instant.

Stochastic optimization methods generate and use random variables. For stochastic problems, the random variables appear in the formulation of the optimization problem, which involve ”for example” random objective functions. Stochastic gradient descent is a stochastic approximation of the gradient descent optimization method for minimizing an objective function [24] − i.e by finding a minima or maxima by iteration. A popular stochastic gradient descent algorithm is the least mean squares (LMS) adaptive filter.

Thus the concepts of adaptive filtering and stochastic optimization are connected. Usually in both cases a parameter of interest is found from the realizations of random inputs variables iteratively by solving an optimization problem with the minima search.

In recent years, the research area of distributed optimization has gained increasing interest [30, 55]. Distributed estimation algorithms are useful in several contexts, including wireless and wired sensor networks, where scalability, robustness, and low power consumption are desirable. Since diffusion cooperation schemes (such as diffusion LMS) have been shown to provide good performance, robustness to node, and link failure and are amenable to distributed implementations [27], then in this thesis we have used diffusion LMS type of algorithms for designing and implementing the distributed Energy and Largest Eigenvalue detection solutions.

2.3.1. Diffusion LMS Algorithm

In this overview section the idea and the derivation steps of the distributed Diffusion Least Mean Square type of algorithm are briefly described in general form, by summarising the material from [27]. This section provides some brief background information for the reader to follow the re−derivation and implementation steps of the diffusion LMS type of algorithms in Chapters 3 and 4, 5.

Distributed Estimation Problem Formulation

Let us assume we have K nodes in a CR network. Let $\mathcal{N}_k$ denote the neighborhood group of node $k \in K$, i.e $\mathcal{N}_k$ defines the set of nodes l which can send data unidirectionally to the node k. In general, at time instant n, every node k receives:

1. a scalar measurement $d_k(n)$ and a $1 \times M$ row regression vector $\mathbf{u}_{k,n}$ or

2. a $M \times 1$ vector measurement $\mathbf{d}_k(n)$ while the row regression vector $\mathbf{u}_{k,n}$ is neglected from the derivations.

$d_k(n), \mathbf{d}_k(n), \mathbf{u}_k(n)$ are realizations of corresponding complex random processes. On page 24 we explain that in this thesis we adapt and apply the theory of diffusion LMS for two different measurement and estimation dimension sets. In the first case, every node k, using data set $\{d_k(n), \mathbf{u}_k(n)\}$ estimates an optimal parameter p^o. In the second case, an optimal $M \times 1$ vector $\mathbf{p}^o$ is estimated based on the set $\{\mathbf{d}_k(n)\}$. Thus, for the generic notation in this overview section, we use boldface notation $\mathbf{d}_k(n)$ for the measurement parameter and $\mathbf{p}^o$ for the optimal vector, respectively, and show the row regression parameter $\mathbf{u}_{k,n}$ in the derivations.

Global Optimization

We seek the $M \times 1$ optimal linear estimator $\mathbf{p}^o$ that minimizes the following **global cost** function

$$J^{glob}(\mathbf{p}) \triangleq \sum_{k=1}^{K} \mathrm{E}\,|\mathbf{d}_k(n) - \mathbf{u}_{k,n}\mathbf{p}|^2. \tag{2.6}$$

In case of the so called desired process $\mathbf{d}_k(n)$ and the so called regressor process $\mathbf{u}_{k,n}$ are wide sense stationary (WSS), then the optimal solution is given as

$$\mathbf{p}^o = \left(\sum_{k=1}^{K} \mathbf{R}_{u,k}\right)^{-1} \left(\sum_{k=1}^{K} \mathbf{R}_{du,k}\right), \tag{2.7}$$

where $\mathbf{R}_{u,k} = \mathrm{E}\left[\mathbf{u}_{k,n}^* \mathbf{u}_{k,n}\right]$ and $\mathbf{R}_{du,k} = \mathrm{E}\left[\mathbf{d}_k(n)\mathbf{u}_{k,n}^*\right]$ are the corresponding covariance matrices.

Steepest Descent Solution

For the minimization of the global cost function, the standard iterative Steepest-Descent algorithm can be used and we have

$$\mathbf{p}_n = \mathbf{p}_{n-1} - \mu \left[\bigtriangledown_w J^{glob}(\mathbf{p}_{n-1})\right]^*, \tag{2.8}$$

whwith scalar step size parameter $\mu > 0$ and where $\mathbf{p}$ is the estimate of $\mathbf{p}^o$ at time iteration i. The complex gradient is given as follows

$$\left[\bigtriangledown_{\mathbf{p}} J^{glob}(\mathbf{p}_{n-1})\right]^* = \sum_{k=1}^{K} \left(\mathbf{R}_{u,k}\mathbf{p} - \mathbf{R}_{du,k}\right), \tag{2.9}$$

and we get the steepest descent recursion as

$$\mathbf{p}_n = \mathbf{p}_{n-1} - \mu \sum_{k=1}^{K} \left(\mathbf{R}_{du,k} - \mathbf{R}_{u,k}\mathbf{p}_{n-1} \right). \tag{2.10}$$

Since usually the second order moments in (2.10) are not known *a-priori*, then the following approximations can be used instead: $\mathbf{R}_{u,k} \approx \mathbf{u}_{k,n}^{*}\mathbf{u}_{k,n}$ and $\mathbf{R}_{du,k} \approx \mathbf{d}_k(n)\mathbf{u}_{k,n}^{*}$. As a result, we get a non−distributed Global [2] LMS type of algorithm

$$\mathbf{p}_n = \mathbf{p}_{n-1} - \mu \sum_{k=1}^{K} \mathbf{u}_{k,n}^{*} \left(\mathbf{d}_k(n) - \mathbf{u}_{k,n}\mathbf{p}_{n-1} \right). \tag{2.11}$$

Local Optimization

Let us introduce a matrix $\mathbf{C}$ with elements $\{c_{l,k}\}$, where the element $c_{l,k}$ defines whether observation from node l is available for the node k. $\mathbf{C}$ is usually considered to be a doubly-stochastic $K \times K$ non-negative real matrix with entries $c_{l,k}$ and $c_{l,k} = 0$ if $l \notin \mathcal{N}_k$ and thus $\mathbf{C1} = \mathbf{1}$, $\mathbf{1}^T\mathbf{C} = \mathbf{1}^C$. The **local cost** at node k is given as

$$J_k^{loc}(\mathbf{p}) = c_{l,k}\, \mathrm{E}\, |\mathbf{d}_l(n) - \mathbf{u}_{l,n}\mathbf{p}|^2. \tag{2.12}$$

The optimal solution can therefore be updated as

$$\mathbf{p}_k^{loc} = \left(\sum_{l \in \mathcal{N}_k} c_{l,k}\mathbf{R}_{u,l} \right)^{-1} \left(\sum_{l \in \mathcal{N}_k} c_{l,k}\mathbf{R}_{du,l} \right). \tag{2.13}$$

Let us define additionally the matrix $\Gamma_k \triangleq \sum_{l \in \mathcal{N}_k} c_{l,k}\mathbf{R}_{u,l}$. By completing the squares, we get that J_k^{loc} can be alternatively rewritten in terms of $\mathbf{p}_k^{loc}$ as

$$J_k^{loc}(\mathbf{p}) = \|\mathbf{p} - \mathbf{p}_k^{loc}\|_{\Gamma_k}^2 + \mathbf{MMSE}, \tag{2.14}$$

where the Minimum Mean Square Error, denoted as **MMSE**, is a constant part. With the usage of matrix $\mathbf{C}$, minimizing the global cost $J^{glob}(\mathbf{p})$ is equivalent to minimizing the following cost function for any $k \in K$

$$J^{glob}(\mathbf{p}) = \sum_{l=1}^{K} J_l^{loc}(\mathbf{p}) = J_k^{loc}(\mathbf{p}) + \sum_{l \neq k}^{K} J_l^{loc}(\mathbf{p}) \tag{2.15}$$

$$J^{glob}(\mathbf{p}) = \sum_{l \in \mathcal{N}_k} c_{l,k}\, \mathrm{E}\, |\mathbf{d}_l(n) - \mathbf{u}_{l,n}\mathbf{p}|^2 + \sum_{l \neq k}^{K} \|\mathbf{p} - \mathbf{p}_k^{loc}\|_{\Gamma_l}^2 \tag{2.16}$$

We have now an alternative global cost representation in terms of local estimates $\{\mathbf{p}_k^{loc}\}$.

[2] The term Global means that the algorithm requires data from all the nodes in the network.

MSE Minimization

Minimization of $J^{glob}(\mathbf{p})$ on every node k still requires access to the global information $\{\mathbf{p}_l^{loc}\}$ and matrices Γ_l in the other nodes in the network. A fully distributed solution is derived in what follows and this is based on the diffusion LMS strategy.

Let us replace $\mathbf{\Gamma}_l$ with $\mathbf{\Gamma}_l = b_{l,k}\mathbf{I}_M$, where $\mathbf{I}_M$ is $M \times M$, $b_{l,k} = 0$ if $l \notin \mathcal{N}_k$, $\mathbf{1}^T\mathbf{B} = \mathbf{1}^T$. Let us introduce a new $K \times K$ matrix $\mathbf{B}$. Also we replace $\mathbf{p}_k^{loc}$ with the intermediate estimate $\boldsymbol{\psi}_l$ at node l. Then the following approximation of J^{glob} is proposed so that each node k can minimize modified cost as

$$J_k^{dist}(\mathbf{p}) = \sum_{l\in\mathcal{N}_k} c_{l,k}\,\mathrm{E}\,\|\mathbf{d}_l(n) - \mathbf{u}_{l,n}\mathbf{p}\|^2 + \sum_{l\in\mathcal{N}_k/\{k\}} b_{l,k}\|\mathbf{p} - \boldsymbol{\psi}_l\|^2 \tag{2.17}$$

The complex gradient is given as:

$$\left[\bigtriangledown_{\mathbf{p}} J_k^{dist}(\mathbf{p}_{n-1})\right]^* = \sum_{l\in\mathcal{N}_k} c_{l,k}\left(\mathbf{R}_{u,l}\mathbf{p} - \mathbf{R}_{du,l}\right) + \sum_{l\in\mathcal{N}_k/\{k\}} b_{l,k}\left(\mathbf{p} - \boldsymbol{\psi}_l\right). \tag{2.18}$$

We can use $J_k^{dist}(\mathbf{p})$ to obtain the recursion for the estimate of $\mathbf{p}$ at node k in two steps:

$$\begin{aligned}\boldsymbol{\psi}_{k,n} &= \mathbf{p}_{k,n-1} + \mu_k \sum_{l\in\mathcal{N}_k} c_{l,k}\left(\mathbf{R}_{du,l} - \mathbf{R}_{u,l}\mathbf{p}\mathbf{p}_{k,n-1}\right)\\ \mathbf{p}_{k,n} &= \boldsymbol{\psi}_{k,n} + \nu_k \sum_{l\in\mathcal{N}_k/\{k\}} b_{l,k}\left(\boldsymbol{\psi}_l - \mathbf{p}_{k,n-1}\right).\end{aligned} \tag{2.19}$$

In the second equation of (2.19) two replacements are performed: $\boldsymbol{\psi}_l$ is replaced by the intermediate estimate $\boldsymbol{\psi}_{l,n}$, available at node l, at time n, and secondly $\mathbf{p}_{k,n-1}$ is replaced by $\boldsymbol{\psi}_{k,n}$. As a result we get

$$\begin{aligned}\boldsymbol{\psi}_{k,n} &= \mathbf{p}_{k,n-1} + \mu_k \sum_{l\in\mathcal{N}_k} c_{l,k}\left(\mathbf{R}_{du,l} - \mathbf{R}_{u,l}\mathbf{p}_{k,n-1}\right)\\ \mathbf{p}_{k,n} &= \boldsymbol{\psi}_{k,n} + \nu_k \sum_{l\in\mathcal{N}_k/\{k\}} b_{l,k}\left(\boldsymbol{\psi}_{l,n} - \boldsymbol{\psi}_{k,n}\right).\end{aligned} \tag{2.20}$$

The second recursion of (2.19) can be rearranged again. First recall that

$$\mathbf{p}_{k,n} = (1 - \nu_k + \nu_k b_{k,k})\boldsymbol{\psi}_{k,n} + \nu_k \sum_{l\in\mathcal{N}_k/\{k\}} b_{l,k}. \tag{2.21}$$

Let us define $K \times K$ matrix left stochastic $\mathbf{A}$ containing the elements as coefficients $a_{k,k} = (1 - \nu_k + \nu_k b_{k,k})$ and $a_{l,k} = (\nu_k b_{l,k})$ for $l \neq k$. We get the following recursion

$$\boldsymbol{\psi}_{k,n} = \mathbf{p}_{k,n-1} + \mu_k \sum_{l \in \mathcal{N}_k} c_{l,k} \left(\mathbf{R}_{du,l} - \mathbf{R}_{u,l} \mathbf{p}_{k,n-1} \right)$$

$$\mathbf{p}_{k,n} = \sum_{l \in \mathcal{N}_k} a_{l,k} \boldsymbol{\psi}_{l,n}. \tag{2.22}$$

Let us note that $c_{l,k} = a_{l,k} = 0$ if $l \notin K$, $\mathbf{1}^T C = \mathbf{1}^T$, $C\mathbf{1} = \mathbf{1}$, and (obviously) $\mathbf{1}^T A = \mathbf{1}^T$.

ATC and CTA Diffusion LMS algorithms

Two common versions of the Diffusion LMS algorithm are considered in the literature: **Adapt and Combine** (ATC) and **Combine and Adapt** (CTA) types of Diffusion LMS algorithms. By inserting the approximations of the covariance matrices we have that

ATC Diffusion LMS

Init: $\mathbf{p}_{k,0} = 0$ for all $k \in K$. Given the non-negative real coefficients $\{c_{l,k}, a_{l,k}\}$ for each time $n \geq 0$ and for all nodes k:

$$\begin{cases} \boldsymbol{\psi}_{k,n} = \mathbf{p}_{k,n-1} + \mu_k \sum_{l \in \mathcal{N}_k} c_{l,k} \mathbf{u}_{l,n}^* \left(\mathbf{d}_l(n) - \mathbf{u}_{l,n} \mathbf{p}_{k,n-1} \right), & \text{(incremental step)}, \\ \mathbf{p}_{k,n} = \sum_{l \in \mathcal{N}_k} a_{l,n} \boldsymbol{\psi}_{l,n} & \text{(diffusion step)}. \end{cases} \tag{2.23}$$

CTA Diffusion LMS

Init: $\mathbf{p}_{k,0} = 0$ for all l. Given the non-negative real coefficients $\{c_{l,k}, a_{l,k}\}$ for each time $n \geq 0$ and for all nodes $k \in K$:

$$\begin{cases} \boldsymbol{\psi}_{k,n-1} = \sum_{l \in \mathcal{N}_k} a_{l,k} \mathbf{p}_{l,n-1} & \text{(diffusion step)}, \\ \mathbf{p}_{k,n} = \boldsymbol{\psi}_{k,n-1} + \mu_k \sum_{l \in N_k} c_{l,k} \mathbf{u}_{l,n}^* \left(\mathbf{d}_l(n) - \mathbf{u}_{l,n} \boldsymbol{\psi}_{k,n-1} \right), & \text{(incremental step)}. \end{cases} \tag{2.24}$$

We note that detailed performance analysis of the Diffusion LMS algorithms is performed in [27] but in the estimation domain only and based on the estimation error recursions.

Comments on the implementation and usage in the CR context

In Chapters 3 and 4 we use the Diffusion LMS algorithm derivation framework for deriving a diffusion LMS based scalar (power) estimation solution for the distributed Energy detection solution and a diffusion LMS based vector (vectorized correlation matrix) estimation solution for the distributed Largest Eigenvalue detection solution. For deriving these estimation algorithms, we need to introduce modifications in the standard derivation flow of the Diffusion LMS algorithms.

The considerations are the following.

1. Depending on the application of an adaptive filter [24, Chapter 1.7], the regressor variable $\mathbf{u}_{k,n}$ can be seen as a variable which can contain some *a priori* information for the estimation process. In a practical PU signal detection task, a CR system usually can not use *a priori* data, which can be incorporated in the estimation process of the elements of test statistics $-$ i.e the signal sequence of the PU user for implementing a matched filter detection solution. For the Energy and Largest Eigenvalue detection solutions, proposed in this thesis, the regressor variable is expendable (i.e $\mathbf{u}_k(n) = 1$ constantly) and thus can be excluded from the derivations. The secondary statistics becomes then $\mathbf{R}_{u,k} = 1$ and $\mathbf{R}_{du,k} = \mathrm{E}\left[\mathbf{d}_k(n)\right]$. Thus, in our solutions the "desired" variable $\mathbf{d}_k(n)$ is connected with the observations for the estimation process.

2. Due to the previous point and for the power estimation algorithm in Chapter 3, $\mathbf{p}^o$ and $\mathbf{d}_k(n)$ are both selected as scalars and the derivation of diffusion LMS type of algorithm can be slightly simplified. These details are shown in Chapter 3.

3. For the vector estimation algorithm in Chapter 4, the variables $\mathbf{d}_k(n)$ and $\mathbf{p}^o$ are taken as a $M \times 1$ vectors. The derivation of diffusion LMS type algorithm is slightly modified and these details are shown in Chapter 4.

4. In this thesis we do not proceed with the performance analysis of the Diffusion LMS type algoritm, based on the estimation error measures. Instead we are interested in the analysis of the statistical moments of the estimates directly, to be able to proceed with the analysis of the detection performance of the proposed distributed detection solutions.

Thus, in Chapter 3 and 4 we skip some of the standard derivation steps and focus on the differences from the standard derivation flow of diffusion LMS type of algorithms.

3. DISTRIBUTED DIFFUSION LMS BASED ENERGY DETECTION

CR systems need to detect the presence of a primary user by continuously sensing the spectrum area of interest. Radiowave propagation effects like fading and shadowing often complicate sensing of spectrum holes because the PU signal can be weak in a particular area. Cooperative spectrum sensing is seen as a prospective solution to enhance the detection of PU signals. This chapter studies distributed spectrum sensing in a cognitive radio context based on the results in [3] and [2]. We investigate distributed energy detection schemes without using any fusion center. Due to reduced communication, such a topology is more energy efficient. We propose the usage of distributed, diffusion least mean square (LMS) type of power estimation algorithms. In this chapter, an Adapt and Combine (ATC) diffusion based power estimation scheme is proposed and the performance is compared with the Combine and Adapt (CTA) and ring-around schemes in a common framework. Additionally we show in this chapter also the results from the first paper [1] for a recursive and distributed power estimation scheme with a ring around topology, which does not necessarily have to be related with the Diffusion LMS context. In this case specific theoretical results for the performance analysis of that algorithm can be given. The power to be estimated for the energy detection is a scalar quantity. The PU signal is assumed to be slowly fading. We analyse the resulting energy detection performance and verify the theoretical findings through simulations.

3.1. Background

The cognitive radio (CR) system is dynamic. Often in practice the statistical information (for example conditional probability density of observations, prior probabilities of detection hypotheses, longer time statistical behaviour of primary user (PU)) is not available *a priori* for constructing a PU signal detection solution. The properties of the test statistics (for making a detection decision) may change in time.

In cognitive radio context we would like to avoid interference to the PU user and find free spectrum opportunities as fast as possible. On-line distributed network learning methods are able to learn the statistical information based on

observations received by the nodes in the network. These methods can react to possible changes in the properties of estimated statistics in real time.

Several proposed distributed spectrum sensing solutions make use of a central fusion center [12], [17], [35], [36]. A fusion center is, however, seen as a single point of failure in the network since a malfunction in this unit affects the performance of the whole distributed solution. We propose a power estimation solution where the available power estimates (and measurements) are fused in cognitive radio network nodes, to allow all nodes to make detection decisions based on data from the neighbour nodes and without involvement of any central processing unit. Such a solution enhances network failure resistance (at the cost of slightly increased information overhead in the network).

Several distributed adaptive estimation and detection schemes have been studied in the past. Least mean square (LMS) and recursive least squares (RLS) based estimation schemes are analysed for example in [26], [27], [56], [29] and consensus based schemes in [57], [58], [59], [60]. Optimal, matched filter distributed detection, based on diffusion type LMS and RLS estimation schemes, was studied in [28]. Here, we make the assumption that the CR network does not have any prior information about the waveform of the PU signal in the secondary nodes and hence we cannot design a matched filter. Therefore energy detection becomes a practical solution.

A ring network topology for distributed energy detection without a fusion centre has been suggested in [61]. In [1] we proposed and analysed an estimation based recursive calculation of the test statistics for the energy detectors in cognitive radio network with ring topology. The test statistic in form of a converged power estimate is the soft information used for making the detection decision at every node. Ring networks are, however, sensitive to link failures. Combine and Adapt (CTA) diffusion based recursive calculation of the test statistics for the energy detectors was proposed and studied in [2]. In this chapter we focus mainly on the analysis of the Adapt and Combine (ATC) version of diffusion LMS type of received power estimation algorithm. The performance of the ATC diffusion based distributed power estimator is compared with the previously proposed CTA [2] and ring [1] schemes to complete the analysis. The resulting energy detection performance is studied and is dependent on the performance of the used distributed recursive power estimation algorithm.

We organize the remainder of the chapter as follows. In Section 3.2 we review the system model and the basics of energy detection. We derive an ATC type received signal power estimation algorithm based on diffusion LMS strategy and summarize the CTA based version. In Section 3.3 we analyse the performance of the proposed distributed power estimation algorithm (using a common model) and the resulting energy detection. In Section 3.4 we present our simulations results.

3.2. Distributed power estimation and detection

According to classical detection theory, an energy detector can be used for detecting random signals in additive noise. For energy detection in a cognitive radio context, the type of PU signal can be completely unknown. During a sensing time t, an energy detector (ED) receives N samples of a signal $x(n)$ from a specific frequency band [17]. The average energy of the received data samples is the test statistic $T(x)$ of the ED, which compares $T(x)$ to a predefined threshold γ and decides which of the hypotheses H_0 or H_1 is more likely.

We assume the following signal model at node k:

$$\begin{aligned} H_0 &: E[|x_k(n)|^2] = E[|v_k(n)|^2] \\ H_1 &: E[|x_k(n)|^2] = E[|\alpha_k|^2|s(n)|^2] + E[|v_k(n)|^2], \end{aligned} \tag{3.1}$$

where $k = 1, 2, ..., K$ is the node number and $n = 1, 2, ...N$ is the sample index. $v_k(n)$ is independent and identically distributed (*i.i.d*) circularly symmetric complex Gaussian (CSCG) noise with zero mean and variance $E[|v_k(n)|^2] = \sigma_{v,k}^2$, i.e. $v(n) \sim \mathcal{C}N(0, \sigma_{v,k}^2)$. The power of the emitted PU signal $s(n)$ is denoted as $E[|s(n)|^2] = S$, under H_1. The primary signal $s(n)$ and the noise $v_k(n)$ are assumed to be statistically independent. The PU signal passes through a slowly fading channel with gain $\alpha_k(n)$. The gain α_k is considered to be constant. Note, that for implementing the energy detector, only the noise variance is needed to determine the detection threshold γ, therefore estimates of the channel gains are not required in practical implementations. Noise power estimation is not considered in this research work. In this chapter we make the following assumptions:

- (AS 1) The $x(n)$ is sensed by *K* nodes in the CR network.
- (AS 2) The additive noise $v_k(n)$ is uncorrelated in time and space and has the same power level over all the nodes in the CR network.
- (AS 3) The number of performed iterations N is large enough.
- (AS 4) The links between the CR nodes are ideal and not capacity restricted (no need to quantize the soft information).

In the literature on distributed detection, for example in [23], a fusion center, which collects all the local soft information, hard or soft binary decisions from the sensors, is often used in distributed detection networks. Similarly a central processing unit has been used in distributed estimation schemes, see e.g. [26]. However, such a central processing unit can potentially be a single point of failure in the detection system. Secondly it may require frequent data exchange between the nodes and the centre and thus drain system energy resources, since usually most of the energy is spent for powering up the transmitter to exchange the data with neighbour nodes.

A distributed and recursive estimation scheme is one of the possible solutions for removing the central processing unit from the system and thus the network is able to calculate the global estimates based on the local observations collected by the CR nodes. Then based on the estimated test statistic, the detector at each CR node can locally make its own decision if the PU signal is present or not. We denote the power estimate at node k and at iteration n as $\hat{p}_k(n)$. The network topology is assumed to be fixed over the sensing time. We consider a linear, fixed combination of neighbour estimates and measurements at every node k.

Next we shortly review the global model for estimating the received signal power in cooperative manner (as proposed in [2]). Then we derive an ATC type power estimation algorithm, where the nodes can observe the measurements and share the estimates (and measurements) only with their neighbour nodes, according a to predefined network topology. Finally we propose a data exchange and combination strategy for ATC diffusion algorithm.

3.2.1. Global estimation

According to model (3.1), the power of the PU signal is attenuated at every node k. The locally estimated power varies between nodes k. Therefore if the channel gain at node k is low, the resulting energy detection performance is low. The result is opposite, when the node has a good channel gain. When nodes cooperate to estimate a common parameter P^o, the resulting detection performance will improve. As in [2] we recommend the following form of p^o

$$p^o = \frac{1}{K}\sum_{k=1}^{K} \mathrm{E}\left[|x_k(n)|^2\right] = S\frac{1}{K}\sum_{k=1}|\alpha_k|^2 + \sigma_v^2. \tag{3.2}$$

The p^o is the average of the received power across the nodes $k \in K$ in the network. The second equation in (3.2) follows from the signal model (3.1) if the PU signal is present and from the assumption AS 2. When we have sufficient number of nodes in the CR network, the effect of varying channel gains is averaged over nodes $k \in K$.

The corresponding global cost function is given as:

$$J^{glob}(p) = \sum_{k=1}^{K} \mathrm{E}\left[|x_k(n)|^2 - p\right]^2, \tag{3.3}$$

where we have used the form of global cost as proposed in [31], [28], [26]. Minimization of the mean square error across the network (3.3) with respect to P results in the optimal solution, which is given by (4.10).

3.2.2. Distributed ATC Diffusion LMS estimation

Suppose that K nodes in the CR network are interested in estimating the scalar parameter p^0 in a distributed manner, where the nodes rely only on the

information, that is available to them. Depending on network topology, nodes are connected only to selected neighbour nodes and do not have access to any global data. The global cost (3.3) needs to be approximated in a distributed manner. The derivation of the ATC diffusion power estimation algorithm follows the ideas in [55], [27].

Let $\mathcal{N}_k$ denote the neighbourhood group of node $k \in K$, i.e $\mathcal{N}_k$ consists of nodes l which can communicate with node k. We assume that the network is connected and the connection between nodes l and k is unidirectional.

Let us define $K \times K$ doubly stochastic matrix $\mathbf{C}$ containing non-negative elements $c_{l,k}$ and $c_{l,k} = 0$ if $l \neq \mathcal{N}_k$ (i.e when data from node l is not available for node k). Let us note that for a doubly stochastic matric $\mathbf{C}$ it holds that $\mathbf{C1} = \mathbf{1}$ and $\mathbf{1}^T\mathbf{C} = \mathbf{1}^T$. The local cost and the corresponding local optimal solution in the neighbourhood of node k can be expressed with the help of coefficients $c_{l,k}$ as follows

$$J_k^{loc}(p) = \sum_{l \in \mathcal{N}_k} c_{l,k}\, \mathrm{E}\left[|x_l(n)|^2 - p\right]^2, \tag{3.4}$$

$$p_k^{loc} = \sum_{l \in \mathcal{N}_k} c_{l,k}\, \mathrm{E}\left[|x_l(n)|^2\right]. \tag{3.5}$$

The global cost can be fractioned into the local cost of over the neighbourhood of node k and local costs over the neighbourhood of other nodes. Using the completion of squares argument [31] to relate variable P and local optimal solution P_l^{loc}, secondly ignoring the mmse part which is not dependant on p, the global cost function can be expressed as follows

$$J^{glob'}(p) = \sum_{l \in N_k} c_{l,k}\, \mathrm{E}\left[|x_l(n)|^2 - p\right]^2 + \sum_{l \neq k}^{K} \|p - p_l^{loc}\|^2. \tag{3.6}$$

Node k may not have access to all the data p_l^{loc} in the network. We modify the second member of right hand side (RHS) of (3.6) by replacing the summation $\sum_{l \neq k}^{K}$ with $\sum_{l \in \mathcal{N}_k / \{k\}}$. Next we replace $\|p - p_l^{loc}\|^2 \approx b_{l,k}\|p - p_l^{loc}\|^2$ ([55, Eq. 117]). We collect the non-negative coefficients $b_{l,k}$ in a $K \times K$ matrix $\mathbf{B}$ and assume $b_{l,k} = 0$ if $l \neq \mathcal{N}_k$. Also we replace the unknown p_l^{loc} with an intermediate estimate $\hat{\psi}_l$ available at node l. Then the approximation of (3.6) at node k is given as

$$\begin{aligned} J_k^{dist}(p) = & \sum_{l \in N_k} c_{l,k}\, \mathrm{E}\left[|x_l(n)|^2 - p\right]^2 \\ & + \sum_{l \in \mathcal{N}_k / \{k\}} b_{l,k}\|p - \hat{\psi}_l\|^2 \end{aligned} \tag{3.7}$$

and the derivative of the cost function is (3.7) is

$$\nabla_p J_k^{dist}(p) = 2 \sum_{l \in N_k} c_{l,k} \left[p - \mathrm{E}\left[|x_l(n)|^2 \right] \right] + 2 \sum_{l \in \mathcal{N}_k / \{k\}} b_{l,k} \left[p - \hat{\psi}_l \right]. \tag{3.8}$$

The cost (4.23) can be used to obtain a recursion for the estimate of p at node k, denoted as $\hat{p}_k(n)$. Using the steepest descent method, which is divided into two parts, we get an iterative solution for (3.7) as follows:

$$\begin{aligned} \hat{\psi}_k(n+1) &= \hat{p}_k(n) + \mu_k \sum_{l \in N_k} c_{l,k} \left[\mathrm{E}\left[|x_l(n)|^2 \right] - \hat{p}_k(n) \right] \\ \hat{p}_k(n+1) &= \hat{\psi}_k(n+1) + \nu_k \sum_{l \in \mathcal{N}_k / \{k\}} b_{l,k} \left[\psi_l - \hat{p}_k(n) \right]. \end{aligned} \tag{3.9}$$

Different step sizes μ_k and ν_k at the nodes k have been assigned and the constants 2 has been incorporated into μ_k and ν_k. In the second equation of (3.9) we replace $\hat{\psi}_l$ with time dependant $\hat{\psi}_l(n+1)$, $\hat{p}_k(n)$ with $\hat{\psi}_k(n+1)$ and we get

$$\hat{p}_k(n+1) = \left[1 - \nu_k \sum_{l \in \mathcal{N}_k / \{k\}} b_{l,k} \right] \hat{\psi}_k(n+1) + \nu_k \sum_{l \in \mathcal{N}_k / \{k\}} b_{l,k} \hat{\psi}_l(n+1). \tag{3.10}$$

Next we introduce the coefficients $a_{l,k} = 0$ if $l \neq \mathcal{N}_k$, $a_{l,k} = \nu_k b_{l,k}$ if $l \neq k$ and $a_{k,k} = 1 - \nu_k \sum_{l \in \mathcal{N}_k / \{k\}} b_{l,k}$ if $l = k$. If we collect the coefficients $a_{l,k}$ into a $K \times K$ matrix $\mathbf{A}$, it is straightforward to see that $\sum_{l \in \mathcal{N}_k} a_{l,k} = 1$ for every $k \in K$ and thus $\mathbf{A}$ is a left stochastic matrix [1] (but $\mathbf{A}$ can be also doubly stochastic). We replace $\mathrm{E}\,|x_l(n)|^2$ with $|x_l(n)|^2$ and finally arrive to the Adapt and Combine (ATC) recursions that we summarise with energy detection as Algorithm 1.

In the ATC diffusion algorithm, during the incremental step, at time instant n, the estimate $\hat{\psi}_k(n+1)$ at node k is calculated using the estimate $\hat{p}_k(n)$ at node k and the new observation available for node k. The coefficients $c_{l,k}$ define how the measurements are exchanged between the nodes. During the diffusion step the estimate $\hat{p}_k(n+1)$ at every node k is calculated using a linear combination of the estimates $\hat{\psi}_l(n+1)$ available for node k. The elements $a_{l,k}$ specify the combination strategy of estimates.

Note that in practice the non-negative coefficients $a_{l,k}$ and $c_{l,k}$ can be chosen freely under the conditions, that $\mathbf{C1} = \mathbf{1}$, $\mathbf{1}^T\mathbf{C} = \mathbf{1}^T$, $\mathbf{1}^T\mathbf{A} = \mathbf{1}^T$, $a_{l,k} = 0$, if $l \neq \mathcal{N}_k$ and $c_{l,k} = 0$ if $l \neq \mathcal{N}_k$. The coefficients $b_{l,k}$ are absorbed into coefficients $a_{l,k}$ and do not have to be considered in practice.

[1] For a left stochastic matric $\mathbf{A}$ it holds $\mathbf{1}^T\mathbf{A} = \mathbf{1}^T$.

Algorithm 1 Distributed ATC Diffusion Power Estimation

Start with $\hat{p}_k(0) = p(0)$.
Given non-negative real coefficients $a_{l,k}, c_{l,k}$
for every time instant $n \geq 1$ **do**
  for every node $k = 1, ..., K$ **do**
    1. Power estimation:
    $\hat{\psi}_k(n+1) = \hat{p}_k(n)$
      $+\mu_k \sum_{l \in N_k} c_{l,k} \left(|x_l(n)|^2 - \hat{p}_k(n) \right)$
    $\hat{p}_k(n+1) = \sum_{l \in N_k} a_{l,k} \hat{\psi}_l(n+1)$.
    2. Detection decision:
    $H_0 : \hat{p}_k(n+1) < \gamma$ or $H_1 : \hat{p}_k(n+1) > \gamma$.
    (Refer to (3.55) for selecting the threshold).
  end for
end for

We also add that if we replace the order of adaptation and fusion equations in (3.9) as follows

$$\begin{aligned} \hat{\psi}_k(n) &= \hat{p}_k(n) + \nu_k \sum_{l \in \mathcal{N}_k / \{k\}} b_{k,l} \left[\psi_l - \hat{p}_k(n) \right] \\ \hat{p}_k(n+1) &= \hat{\psi}_k(n) + \mu_k \sum_{l \in N_k} c_{k,l} \left[\mathrm{E} \left| x_l(n) \right|^2 - \hat{p}_k(n) \right]. \end{aligned} \tag{3.11}$$

By skipping the standard steps, we arrive to the CTA (Combine and Adapt) version of the diffusion LMS algorithm, which is summarised at next In the

Algorithm 2 Distributed CTA Diffusion Power Estimation

Start with $\hat{p}_k(0) = p(0)$.
Given non-negative real coefficients $a_{k,l}, c_{k,l}$
for every time instant $n \geq 1$ **do**
  for every node $k = 1, ..., K$ **do**
    1. Power estimation:
    $\hat{\psi}_k(n) = \sum_{l \in N_k} a_{k,l} \hat{p}_l(n)$
    $\hat{p}_k(n+1) = \hat{\psi}_k(n)$
      $+\mu_k \sum_{l \in N_k} c_{k,l} \left(|x_l(n)|^2 - \psi_k(n) \right)$.
    2. Detection decision:
    $H_0 : \hat{p}_k(n+1) < \gamma$ or $H_1 : \hat{p}_k(n+1) > \gamma$.
    (Refer to (3.55) for selecting the threshold).
  end for
end for

CTA diffusion algoritm, the estimates $\{\hat{p}_k(n)\}_{k \in \mathcal{N}_k}$ including the $\hat{p}_k(n)$ from

node k are combined together at every node k. This is the diffusion step. Then the combined estimate $\hat{\psi}_k(n)$ at node k is used to calculate the new estimate $\hat{p}_k(n+1)$ at node k, using the new observation available for node k, at time instant n. This is the incremental step.

3.2.3. Recursive ring-around topology

As shown in paper [1], a recusive estimator can be interpreted also as a counterpart of a non-recursive sample variance estimator. By taking into account the suggestions in [53] for a local, non-cooperating estimator for sample variance, the distributed estimator using a circular estimation topology can be constructed as follows

$$\hat{p}_k(n) = \frac{1}{n}\sum_{i=1}^{n} |x_{(k-i+1) \bmod K}(n-i+1)|^2. \tag{3.12}$$

A recursive equivalent to (3.12) is given by

$$\begin{aligned} \hat{p}_k(n) &= \hat{p}_{(k-1) \bmod K}(n-1) + \mu(n)(|x_k(n)|^2 \\ &\quad -\hat{p}_{(k-1) \bmod K}(n-1)), \end{aligned} \tag{3.13}$$

where $n \geq 1$ and with step size: $\mu(n) = \frac{1}{n}$

The usage of step size $\mu(n) = \frac{1}{n}$, however, expects that the received signal $x_k(n)$ over $n \in N$ stays under a fixed hypotheses: H_0 or H_1. This fact makes its direct use in real-time spectrum sensing problematic. As a solution, a positive constant step size $\mu(n) = \mu$ can be used in recursive power estimation algorithm and then (3.13) is able to track the possible changes in power of the received signal $x_k(n)$. As common in the literature of adaptive filtering, the step size of the algorithm is user defined.

The estimated power level $\hat{p}_k(n)$ is used as the test statistic of the recursive ED. i.e. $T(x) = \hat{p}_k(n)$. Since there is no fusion centre and for system redundancy purposes, information overhead is allowed in the network. Thus there are K circular estimation processes running in parallel to provide a global estimate for every node $k \in K$. Every node can then perform the energy detection at any time instant. The algorithm can in principle run infinitely (no window for sample processing is required). The proposed algorithm is summarized in Algorithm 3. Let us note, that with the suggested algorithm, only one-directional communication with the adjacent node is required for exchanging the soft information, compared to the schemes, where a central processing unit is used and thus two way communication direction is needed to also send the global soft information back to the nodes at every iteration n. An example with $K = 2$ nodes and thereby 2 estimation processes (red and blue) is illustrated in Fig. 3.1 with nodes $k = 1, 2$ receiving samples $n = 1, ..., 3$.

According to AS3 it is assumed that the number of iterations performed with the recursive algorithm is larger than the number of nodes in the network. The estimator needs to converge to steady state before the detection decision

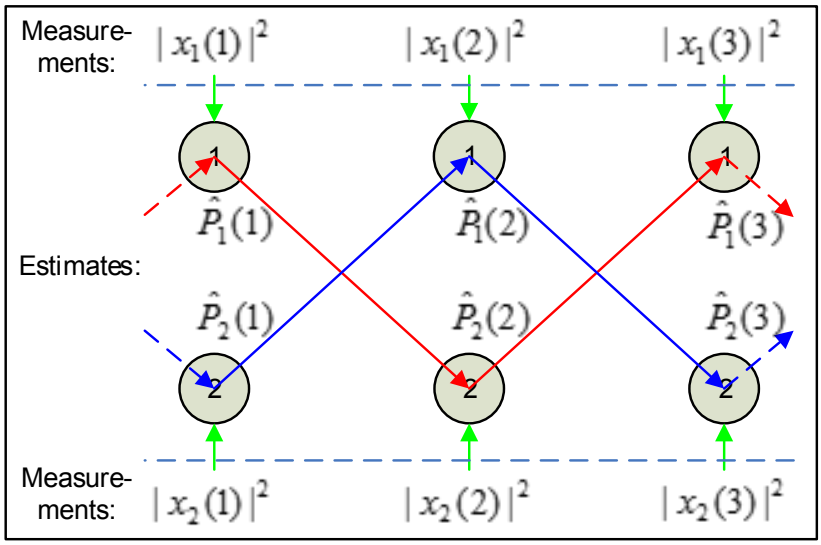

Figure 3.1 Distributed Power Estimation with 2 nodes.

is made and for the convergence a sufficient number of samples are required. In slow fading the channel coherence time is large and the convergence is achievable. Secondly, in the performance section of the proposed algorithm the Central Limit Theorem (CLT) is applied so enough samples are required also for this approximation to hold. The minimum number of samples for the CLT approximation has been evaluated in the literature, e.g. in [62].

3.2.4. Network topologies

In the ring-around topology [1], the power estimates are exchanged circularly between the nodes. At time instant n, node k has access only to one estimate $\hat{p}_{(k-1)\mathrm{mod}K}(n)$ from the node $(k-1)\mathrm{mod}K$ for calculating $\hat{p}_k(n+1)$. The local estimate $\hat{p}_k(n)$ is ignored. The algorithm uses only locally observed measurements (i.e $\mathbf{C} = \mathbf{I}$). Thus, K estimates have to be sent over the wireless links at time instant n.

For improving the link failure resistance but keep the need for exchanging the data over wireless links in the network minimal, we compose the diffusion topology from the local ($\mathbf{A}, \mathbf{C} = \mathbf{I}$) and ring-around topologies. At time instant n, at node k the local estimate $\hat{p}_k(n)$ and the estimate $\hat{p}_{(k-1)\mathrm{mod}K}(n)$ from node $(k-1)\mathrm{mod}K$ are fused together using equal, constant weight 0.5 for calculating $\hat{p}_k(n+1)$. For example when $K = 3$ and keeping the same notation and

Algorithm 3 Distributed Ring−Around Power Estimation

Start with $\hat{P}_k(0) = P_0$.
for every time instant $n \geq 1$ **do**
  for every node $k = 1, ..., K$ **do**
    1. Power estimation:
    $\hat{P}_k(n) = \hat{P}_{(k-1) \bmod K}(n-1) +$
    $\mu(|x_k(n)|^2 - \hat{P}_{(k-1) \bmod K}(n-1))$.
    2. Detection decision:
    $H_0 : \hat{P}_k(n) < \gamma$ or $H_1 : \hat{P}_k(n) > \gamma$.
    (Refer to (3.55) for selecting the threshold).
  end for
end for

conditions for the elements of matrix $\mathbf{A}$, the ring around and diffusion topologies are given as follows

$$\mathbf{A}_{\text{ring}}^T = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad \mathbf{A}_{\text{diff}}^T = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix} \tag{3.14}$$

and is illustrated in Fig. 3.2 If measurements are exchanged between the nodes, then we set $\mathbf{C} = \mathbf{A}_{\text{diff}}^T$. Hence at time instant n, additionally K measurements have to be exchanged in the network. Otherwise $\mathbf{C} = \mathbf{I}$. Therefore, in the subsequent sections we assume, that both matrices $\mathbf{C}$ and $\mathbf{A}$ are doubly stochastic (i.e we have additionally $\mathbf{A1} = \mathbf{1}$) and all the conditions for selecting elements $a_{l,k}$ and $c_{l,k}$, listed in last subsection, are satisfied.

3.3. Performance analysis

The performance analysis of the proposed algorithms is divided into two parts. First we derive a general model for analysing the mean and variance of the estimates of the ATC, CTA [2] and ring-around [1] algorithms in one framework. Next we analyse the resulting energy detection performance. Let us note that for the theoretical performance analysis we need to know the values of the channel gains.

For more convenient notation we stack the estimates and observations from all the nodes $k \in K$ into $K \times 1$ time dependent vectors $\hat{\mathbf{p}}(n) = [\mathbf{p}_1(n) \dots \mathbf{p}_K(n)]^T$ and $\mathbf{x}(n) = \left[|x_1(n)|^2 \dots |x_K(n)|^2\right]^T$ respectively.

Let us define additional matrix $\boldsymbol{\mathcal{M}} = \operatorname{diag}\{\mu_1, \dots, \mu_K\}$, which contain the algorithm step size parameters. We introduce also two additional $K \times K$ matrices $\mathbf{L}_1$ and $\mathbf{L}_2$ for being able to represent all the 3 algorithms using one framework. Then we can write the recursion in the following general form

$$\hat{\mathbf{p}}(n+1) = \mathbf{L}_2 \left(\mathbf{I} - \boldsymbol{\mathcal{M}}\right) \mathbf{L}_1 \hat{\mathbf{p}}(n) + \mathbf{L}_2 \boldsymbol{\mathcal{M}} \mathbf{C} \mathbf{x}(n). \tag{3.15}$$

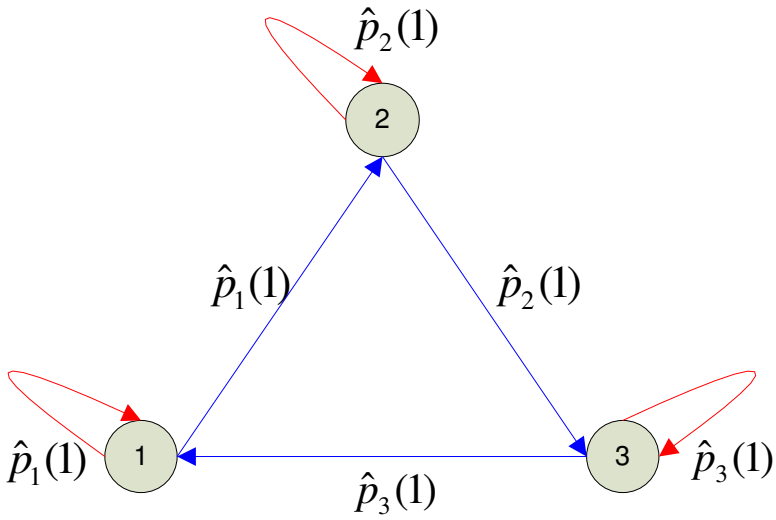

Figure 3.2 Distributed Power Estimation with 3 nodes.

The initial estimate is $\hat{\mathbf{p}}(0)$. It follows, that we get the ATC algorithm, when we take $\mathbf{L}_2 = \mathbf{A}_{\text{diff}}^T$, $\mathbf{L}_1 = \mathbf{I}$, $\mathbf{C} = \mathbf{I}$ or $\mathbf{C} = \mathbf{A}_{\text{diff}}^T$ in case of the measurements are exchanged between the nodes. For the CTA algorithm we take $\mathbf{L}_1 = \mathbf{A}_{\text{diff}}^T$, $\mathbf{L}_2 = \mathbf{I}$, $\mathbf{C} = \mathbf{I}$ or $\mathbf{C} = \mathbf{A}_{\text{diff}}^T$. The ring around topology is selected when $\mathbf{L}_2 = \mathbf{I}$, $\mathbf{L}_1 = \mathbf{A}_{\text{ring}}^T$ and $\mathbf{C} = \mathbf{I}$. Note that to keep the matching notation with Algorithm 1, we use transposed matrices in the general recursion. The local, non-cooperative received power estimation is represented by $\mathbf{L}_1 = \mathbf{L}_2 = \mathbf{C} = \mathbf{I}$.

For evaluating the performance of the estimation algorithms and the resulting energy detection, we first evaluate the mean and variance of estimates $\mathbf{p}_k(n)$.

3.3.1. Mean of estimates

Following the signal model (3.1), let us denote the conditional expectation of the observation vector as $\mathrm{E}\left[\mathbf{x}(n)|H_i\right]$, where $i = 1$ denotes the case when PU signal is present and $i = 0$ the case when PU signal is absent. In this section we assume that the environment is stationary. The conditional means are thus constant over time.

Considering the general recursion (5.11), we have

$$\begin{aligned}\mathrm{E}\left[\hat{\mathbf{p}}(n+1)|H_i\right] = \mathbf{L}_2\left(\mathbf{I} - \boldsymbol{\mathcal{M}}\right)\mathbf{L}_1\,\mathrm{E}\left[\hat{\mathbf{p}}(n)|H_i\right] \\ + \mathbf{L}_2\boldsymbol{\mathcal{M}}\mathbf{C}E\left[\mathbf{x}(n)|H_i\right],\end{aligned} \tag{3.16}$$

for $i = 0, 1$, where the initial value is given as $\mathrm{E}\left[\hat{\mathbf{p}}(0)|H_i\right]$. Let us note that the conditional mean of $\hat{p}_k(n)$ under hypothesis H_i, $i = 0, 1$ at node k is

$$\mathrm{E}\left[\hat{p}_k(n)|H_i\right] = w_k^T \mathrm{E}\left[\mathbf{x}(n)|H_i\right] \quad \textit{for i=0,1}, \tag{3.17}$$

where $w_k = \mathrm{col}(0 \ldots, [w_k(k) = 1], \ldots 0)$ at node k.

After iterating we see, that the mean recursion can be given in the following equivalent form

$$\begin{aligned}\mathrm{E}\left[\mathbf{x}(n)|H_i\right] = &\left[\mathbf{L}_2\left(\mathbf{I} - \boldsymbol{\mathcal{M}}\right)\mathbf{L}_1\right]^n \hat{\mathbf{p}}(0) \\ &+ \left[\sum_{i=0}^{n-1}\left[\mathbf{L}_2\left(\mathbf{I} - \boldsymbol{\mathcal{M}}\right)\mathbf{L}_1\right]^i\right]\mathbf{L}_2\boldsymbol{\mathcal{M}}\mathbf{C}E\left[\mathbf{x}(n)|H_i\right].\end{aligned} \tag{3.18}$$

We are interested in finding the mean of the estimates when the filter has converged to a steady state, i.e. when $n \to \infty$. Thus according to (3.18) we need to analyse the asymptotic behaviour of $\left[\mathbf{L}_2\left(\mathbf{I} - \boldsymbol{\mathcal{M}}\right)\mathbf{L}_1\right]^n$ and the limit of the geometric series $\sum_{i=0}^{n-1}\left[\mathbf{L}_2\left(\mathbf{I} - \boldsymbol{\mathcal{M}}\right)\mathbf{L}_1\right]^i$.

According to [63, Lemma 5.6.11], if for a matrix norm it holds that

$$\left\|\mathbf{L}_2\left(\mathbf{I} - \boldsymbol{\mathcal{M}}\right)\mathbf{L}_1\right\| < 1 \tag{3.19}$$

then $\lim_{n\to\infty}[\mathbf{L}_2(\mathbf{I} - \boldsymbol{\mathcal{M}})\mathbf{L}_1]^n \to 0$. Thus given the doubly stochastic matrices $\mathbf{L}_1$, $\mathbf{L}_2$ and $\mathbf{C}$, the choice of step sizes in $\boldsymbol{\mathcal{M}}$ should guarantee that the stability condition (3.19) holds. Using the matrix 2-norm and the submultiplicativity property of a matrix norm, we have that

$$\left\|\mathbf{L}_2\left(I - \boldsymbol{\mathcal{M}}\right)\mathbf{L}_1\right\|_2 \leq \left\|\mathbf{L}_2\right\|_2\left\|\left(I - \boldsymbol{\mathcal{M}}\right)\right\|_2\left\|\mathbf{L}_1\right\|_2 < 1. \tag{3.20}$$

The spectral norm of a doubly stochastic matrix is 1 [2]. Since the matrix $(\mathbf{I} - \boldsymbol{\mathcal{M}})$ is diagonal, we have that

$$\left\|\mathbf{L}_2\left(\mathbf{I} - \boldsymbol{\mathcal{M}}\right)\mathbf{L}_1\right\|_2 \leq \left|\left|\left(\mathbf{I} - \boldsymbol{\mathcal{M}}\right)\right|\right|_2 = \max_k |1 - \mu_k| < 1. \tag{3.21}$$

We conclude that for the (3.19) to hold, we must select the μ_k, $k = 1 \ldots K$ in $\boldsymbol{\mathcal{M}}$ so that the diagonal matrix $(\mathbf{I} - \boldsymbol{\mathcal{M}})$ is stable. Thus we have the following condition

$$\left|\lambda_k\left[\left(\mathbf{I} - \boldsymbol{\mathcal{M}}\right)\right]\right| = |1 - \mu_k| < 1 \quad \textit{for all k=1...K}. \tag{3.22}$$

Since in our model we have only one mode of convergence of the filter [25], μ_k should be selected in the range:

$$0 < \mu_k < 2. \tag{3.23}$$

The geometric series $S_n = \sum_{i=0}^{n-1}\left[\mathbf{L}_2\left(\mathbf{I} - \boldsymbol{\mathcal{M}}\right)\mathbf{L}_1\right]^i$, which is generated by matrix $\left[\mathbf{L}_2\left(\mathbf{I} - \boldsymbol{\mathcal{M}}\right)\mathbf{L}_1\right]$, converges if and only if the condition (3.19) holds

[2] See [63, Problem 8.7.P5]

for all λ_i. The condition (3.19) guarantees that the $[\mathbf{I} - [\mathbf{L}_2 (\mathbf{I} - \boldsymbol{\mathcal{M}}) \mathbf{L}_1]]$ is invertible. Thus we can write the geometric series as follows

$$S_n = [\mathbf{I} - [\mathbf{L}_2 (\mathbf{I} - \boldsymbol{\mathcal{M}}) \mathbf{L}_1]]^{-1} [\mathbf{I} - [\mathbf{L}_2 (\mathbf{I} - \boldsymbol{\mathcal{M}}) \mathbf{L}_1]^n]. \tag{3.24}$$

Hence according to (3.19) as $n \to \infty$ the geometric series converges to

$$S_n = [\mathbf{I} - [\mathbf{L}_2 (\mathbf{I} - \boldsymbol{\mathcal{M}}) \mathbf{L}_1]]^{-1}. \tag{3.25}$$

Thus by noting the mean of $\hat{\mathbf{p}}(n)$ in steady state and under both hypotheses H_i, $i = 0, 1$ as $\mathrm{E}[\hat{\mathbf{p}}(\infty)|H_i]$, we can write

$$\begin{aligned} \mathrm{E}[\hat{\mathbf{p}}(\infty)|H_i] = &[\mathbf{I} - [\mathbf{L}_2 (\mathbf{I} - \boldsymbol{\mathcal{M}}) \mathbf{L}_1]]^{-1} \\ &\times \mathbf{L}_2 \boldsymbol{\mathcal{M}} \mathbf{C} \, \mathrm{E}[\mathbf{x}(n)|H_i], \end{aligned} \tag{3.26}$$

where the conditional expectations of observations $\mathrm{E}[\mathbf{x}(n)|H_i]$ follow (3.1). Similarly to (3.17) we have that the mean of $\hat{p}_k(n)$ in steady state is

$$\mathrm{E}[\hat{p}_k(\infty)|H_i] = w_k^T \, \mathrm{E}[\hat{\mathbf{p}}(\infty)|H_i] \quad \textit{for i=0,1.} \tag{3.27}$$

Mean of Ring-Around estimates

Since the iteration cycle of the ring-around estimation structure can be easily tracked, specific results for the ring-round estimates can be given.

The mean of the global estimation recursion (3.13) can be found directly. Dropping the mod$K$ notation, we have

$$\mathrm{E}[\hat{p}_k(n)] \quad = \quad (1-\mu)\mathrm{E}[\hat{p}_{k-1}(n-1)] + \mu \mathrm{E}[|x_k(n)|^2]. \tag{3.28}$$

The initial condition is $p_0 = \hat{p}_k(0)$. Due to the circular estimation topology we have that $N = KM + m$, where $M = \lfloor N/K \rfloor$ and where m denotes additional iterations after full cycles. Let $\mathrm{E}[\hat{p}_k(N)|H_1]$ denote the mean when PU signal present and $\mathrm{E}[\hat{p}_k(N)|H_0]$ the mean when only noise is present. By iterating recursion (3.28), using the proposed notation and replacing the expectations using model (3.1), we can write

$$\begin{aligned} \mathrm{E}[\hat{p}_k(N)|H_1] = & \\ & \mu S[\frac{1-(1-\mu)^{KM}}{1-(1-\mu)^K} \left[\sum_{l=0}^{K-1} (1-\mu)^l |\alpha_{k-l}|^2 \right] \\ & + \sigma_v^2 \left[1 - (1-\mu)^{KM+m} \right] \\ & + p_0 (1-\mu)^{KM+m} \\ & + \mu S \left[(1-\mu)^{KM} \left[\sum_{i=0}^{m-1} (1-\mu)^i |\alpha_{k-i}|^2 \right] \right]. \end{aligned} \tag{3.29}$$

In line 2 of (3.29), the geometric series $\sum_{i=0}^{M-1}(1-\mu)^{Ki}$ has been replaced with its sum. Let us note, that according to lines 2 and 5 of (3.29), the mean differs from node to node due to the values and processing order of $|\alpha_k|^2$. When only noise is present then $S = 0$ and

$$\begin{aligned}&\mathrm{E}[\hat{p}_k(N)|H_0] = \\ &\quad p_0(1-\mu)^{KM+m} + \sigma_v^2\left[1-(1-\mu)^{KM+m}\right].\end{aligned} \tag{3.30}$$

According to AS3, $M >> K$ and in steady state of the estimator, when $M \to \infty$, the exponential factors $(1-\mu)^{KM+m}$ and $(1-\mu)^{KM}$ in (3.29) converge to 0 if $0 < \mu < 1$. In steady-state, formula (3.29) goes to

$$\begin{aligned}&\mathrm{E}[\hat{p}_k(\infty)|H_1] = \\ &\quad \sigma_v^2 + \frac{\mu S}{1-(1-\mu)^K}\left[\sum_{l=0}^{K-1}(1-\mu)^l|\alpha_{k-l}|^2\right]\end{aligned} \tag{3.31}$$

and in the noise only case correspondingly to

$$\mathrm{E}[\hat{p}_k(\infty)|H_0] = \sigma_v^2 \quad . \tag{3.32}$$

3.3.2. Variance of estimates

Let us denote the conditional covariance of the estimates under the hypothesis H_i, $i = 0, 1$ as $\mathrm{Cov}\,[\hat{\mathbf{p}}(n+1)|H_i]$. Similarly, let $\mathrm{Cov}\,[\mathbf{x}(n)|H_i]$ denote the conditional covariance of the observations. By using recursions (5.11), (4.28) and standard definition of covariance, taking mathematical expectation and considering the fact that $\hat{\mathbf{p}}(n)$ is independent of the observation vector $\mathbf{x}(n)$, it can be shown that the covariance recursion is

$$\begin{aligned}\mathrm{Cov}\,[\hat{\mathbf{p}}(n+1)|H_i] = {}& \mathbf{L}_2\left(\mathbf{I}-\boldsymbol{\mathcal{M}}\right)\mathbf{L}_1\,\mathrm{Cov}\,[\hat{\mathbf{p}}(n)|H_i] \\ &\times \mathbf{L}_1^T\left(\mathbf{I}-\boldsymbol{\mathcal{M}}\right)\mathbf{L}_2^T \\ &+ \mathbf{L}_2\boldsymbol{\mathcal{M}}\mathbf{C}\,\mathrm{Cov}\,[\mathbf{x}(n)|H_i]\,\mathbf{C}^T\boldsymbol{\mathcal{M}}\mathbf{L}_2^T.\end{aligned} \tag{3.33}$$

where initial estimate of covariance matrix is noted by $Cov\,[\hat{\mathbf{p}}(0)|H_i]$, $i = 0, 1$. The covariance matrix of observations $Cov\,[\mathbf{x}(n)|H_i]$ is constant over time n.

Next we derive the structure of $K \times K$ covariance matrix $\mathrm{Cov}\,[\mathbf{x}(n)|H_i]$. By considering the model (3.1), when PU signal is present the main diagonal elements of matrix $\mathrm{Cov}\,[\mathbf{x}(n)|H_1]$ – the variances of observations at node $k \in K$ can be shown to be:

$$\mathrm{Var}\left[|x_k(n)|^2|H_1\right] = \left(|\alpha_k|^2\sigma_s^2 + \sigma_{v,k}^2\right)^2. \tag{3.34}$$

Similarly when the PU signal is not present and according to AS 2 the variances of observations at node $k \in K$ are given as

$$\mathrm{Var}\left[|x_k(n)|^2|H_0\right] = \sigma_{v,k}^4. \tag{3.35}$$

When the PU signal is present, the off diagonal elements of matrix $\mathrm{Cov}\left[\mathbf{x}(n)|H_1\right]$ - the covariance of observations at nodes k and j if $k, j \in K$ and $i \neq j$ can be shown to be:

$$\mathrm{Cov}\left[|x_k(n)|^2, |x_j(n)|^2 | H_1\right] = |\alpha_k|^2 |\alpha_j|^2 \sigma_s^4. \tag{3.36}$$

According to AS 2 the noise realizations $v_k(n)$ and $v_j(n)$ are uncorrelated in time and space for $k, j \in K$ and $i \neq j$. Thus when the PU signal is absent the covariance of observations is

$$\mathrm{Cov}\left[|x_k(n)|^2, |x_j(n)|^2 | H_0\right] = 0, \tag{3.37}$$

for $k, j \in K$ and $i \neq j$.

The variance of $\hat{p}_k(n)$ at node k, given the hypothesis H_i, $i = 0, 1$, can be found by multiplying the recursion (4.37) with vector w_k^T from the left and with vector w_k from the right

$$\begin{aligned}\mathrm{Var}\left[\hat{p}_k(n+1)|H_i\right] = w_k^T \mathbf{L}_2 \left(\mathbf{I} - \boldsymbol{\mathcal{M}}\right) \mathbf{L}_1 \mathrm{Cov}\left[\hat{\mathbf{p}}(n)|H_i\right] \\ \times \mathbf{L}_1^T \left(\mathbf{I} - \boldsymbol{\mathcal{M}}\right) \mathbf{L}_2^T w_k \\ + w_k^T \mathbf{L}_2 \boldsymbol{\mathcal{M}} \mathbf{C} \, \mathrm{Cov}\left[\mathbf{x}(n)|H_i\right] \\ \times \mathbf{C}^T \boldsymbol{\mathcal{M}} \mathbf{L}_2^T w_k.\end{aligned} \tag{3.38}$$

Note that (4.37) is in the form of a discrete time Lyapunov's equation , [64, App. E]. The steady state variance $\mathrm{Var}\left[\hat{p}_k(\infty)H_i\right]$, $i = 0, 1$, at node $k \in K$ can be recovered by selecting the $\{k, k\}$ element of the steady state covariance matrix $\mathrm{Cov}\left[\hat{\mathbf{p}}(\infty)|H_i\right]$, which has been found as a solution to the Lyapunov's equation. Since the Lyapunov's equation can be solved using standard methods, we skip the details here. We have finally

$$\mathrm{Var}\left[\hat{p}_k(\infty)|H_i\right] = w_k^T \left[\mathrm{Cov}(\hat{\mathbf{p}}(\infty)|H_i)\right] w_k. \tag{3.39}$$

To find the solution, we use the Kronecker product property

$$\mathrm{vec}(U\Sigma V) = \left(V^T \otimes U\right) \mathrm{vec}\left(\Sigma\right) \tag{3.40}$$

to vectorize the covariance recursion (4.37). The notation $\mathrm{vec}(\mathbf{A})$ stacks the columns of its matrix argument $\mathbf{A}$ on top of each other, while $\mathrm{vec}^{-1}(\mathrm{vec}(\mathbf{A}))$ denotes the inverse operation to recover the matrix argument from the vector input. Thus we can write:

$$\begin{aligned}\mathrm{vec}(\mathrm{Cov}(\hat{\mathbf{p}}(n+1)|H_i)) \\ = \left[\mathbf{L}_2 \left(\mathbf{I} - \boldsymbol{\mathcal{M}}\right) \mathbf{L}_1 \otimes \mathbf{L}_2 \left(\mathbf{I} - \boldsymbol{\mathcal{M}}\right) \mathbf{L}_1\right] \\ \times \mathrm{vec}(Cov(\hat{\mathbf{p}}(n)|H_i)) \\ + \left[\mathbf{L}_2 \boldsymbol{\mathcal{M}} \mathbf{C} \otimes \ \mathbf{L}_2 \boldsymbol{\mathcal{M}} \mathbf{C}\right] \mathrm{vec}(\mathrm{Cov}(\mathbf{x}(n)|H_i).\end{aligned} \tag{3.41}$$

In steady state, when $n \to \infty$, the $\mathrm{Cov}(\hat{\mathbf{p}}(n+1)|H_i)$ and $\mathrm{Cov}(\hat{\mathbf{p}}(n)|H_i)$ have converged to the same value.

The solution for $\mathrm{vec}(\mathrm{Cov}(\hat{\mathbf{p}}(\infty))|H_i)$, $i = 0, 1$, leads to the following result

$$\begin{aligned}
&\mathrm{vec}(\mathrm{Cov}(\hat{\mathbf{p}}(\infty)|H_i)) \\
&\quad = [\mathbf{I} - [\mathbf{L}_2 (\mathbf{I} - \boldsymbol{\mathcal{M}}) \mathbf{L}_1 \otimes \mathbf{L}_2 (\mathbf{I} - \boldsymbol{\mathcal{M}}) \mathbf{L}_1]]^{-1} \\
&\qquad \times [\mathbf{L}_2 \boldsymbol{\mathcal{M}} \mathbf{C} \otimes \ \mathbf{L}_2 \boldsymbol{\mathcal{M}} \mathbf{C}] \, \mathrm{vec}(\mathrm{Cov}(x(\infty)|H_i)).
\end{aligned} \tag{3.42}$$

Thus, the steady state variance $\mathrm{Var}(\hat{p}_k(\infty)H_i)$, $i = 0, 1$, at node $k \in K$ can be recovered by selecting the $\{k, k\}$ element of covariance matrix $\mathrm{Cov}(\hat{\mathbf{p}}(\infty)|H_i)$, which has been found as a solution to Lyapunov's equation and is given by (3.42). We have finally

$$\mathrm{Var}(\hat{p}_k(\infty)|H_i) = w_k^T \left[\mathrm{vec}^{-1} \left(\mathrm{vec}(\mathrm{Cov}(\hat{\mathbf{p}}(\infty y)|H_i))\right)\right] w_k. \tag{3.43}$$

Variance of Ring-Around estimates

Similarly, as for the mean of ring-around estimates, specific results for the variance of ring-round estimates can be given. Since $\hat{p}_{k-1}(n)$ and $|x_k(n)|^2$ are uncorrelated and by dropping the mod$K$ notation, we have

$$\begin{aligned}
\mathrm{Var}[\hat{p}_k(n)] \quad &= \quad (1-\mu)^2 \mathrm{Var}[\hat{p}_{k-1}(n-1)] \\
&+ \quad \mu^2 \mathrm{Var}[|x_k(n)|^2].
\end{aligned} \tag{3.44}$$

Replacing the expectations using model (3.1)

$$\begin{aligned}
&\mathrm{Var}[\hat{p}_k(KM+m+1)] = \\
&\mu^2 \sum_{i=0}^{M-1} (1-\mu)^{2Ki} \sum_{l=0}^{K-1} (1-\mu)^{2l} \mathrm{Var}(|x_{k-l}(n)|^2) \\
&+ \mu^2 (1-\mu)^{2KM} \sum_{i=0}^{m-1} (1-\mu)^{2i} \mathrm{Var}(|x_{k-i}(n)|^2)
\end{aligned} \tag{3.45}$$

Since $x_k(n)$ is CSCG, then according to model (3.1) the PU signal is present, $\mathrm{Var}(|x_k(n)|^2) = (S|\alpha_k|^2 + \sigma_v^2)^2$. Let $\mathrm{Var}[\hat{p}_k(N)|H_1]$ denote the variance when the PU signal present and $\mathrm{Var}[\hat{p}_k(N)|H_0]$ the variance when received signal contains only noise. By iterating (3.44), replacing the variances and using the proposed notation, we have that

$$
\begin{aligned}
&\mathrm{Var}[\hat{p}_k(N)|H_1] = \\
&\quad \mu\sigma_v^4 \frac{1-(1-\mu)^{2(KM+m)}}{2-\mu} \\
&\quad + \mu^2 \frac{1-(1-\mu)^{2KM}}{1-(1-\mu)^{2K}} \\
&\qquad \cdot \left[\sum_{l=0}^{K-1} (1-\mu)^{2l} \left[S^2 |\alpha_{k-l}|^4 + 2S|\alpha_{k-l}|^2 \sigma_v^2 \right] \right] \\
&\quad + \mu^2 (1-\mu)^{2KM} \\
&\qquad \cdot \left[\sum_{i=0}^{m-1} (1-\mu)^{2i} \left[S^2 |\alpha_{k-i}|^4 + 2S|\alpha_{k-i}|^2 \sigma_v^2 \right] \right].
\end{aligned}
\tag{3.46}
$$

In line 3 of (3.46), the geometric series $\sum_{i=0}^{M-1}(1-\mu)^{2Ki}$ has been replaced with its sum. Similarly to the mean, the variance differs over the nodes. When only noise is present, the resulting variance is given as

$$
\mathrm{Var}[\hat{p}_k(N)|H_0] = \frac{\mu\sigma_v^4}{2-\mu} \left[1-(1-\mu)^{2(KM+m)} \right]. \tag{3.47}
$$

In steady state of the estimator, when $M \to \infty$, the exponential factors $(1-\mu)^{2(KM+m)}$ and $(1-\mu)^{2KM}$ in (3.46) converge to 0 if the constant step size μ is taken sufficient. Thus, the variance tends to

$$
\begin{aligned}
&\mathrm{Var}[\hat{p}_k(\infty)|H_1] = \frac{\mu\sigma_v^4}{2-\mu} \\
&+ \frac{\mu^2}{1-(1-\mu)^{2K}} \left[\sum_{l=0}^{K-1} (1-\mu)^{2l} \left[S^2 |\alpha_{k-l}|^4 + 2S|\alpha_{k-l}|^2 \sigma_v^2 \right] \right]
\end{aligned}
\tag{3.48}
$$

under H_1 and in the noise only case to

$$
\mathrm{Var}[\hat{p}_k(\infty)|H_0] = \frac{\mu\sigma_v^4}{2-\mu}. \tag{3.49}
$$

The residual variance of the fixed step size power estimation algorithm depends on the value of μ. We observe that smaller μ causes smaller residual variance and thus more precise estimation results. On the other hand, it is known from the literature of adaptive filtering that smaller μ causes slower convergence in the mean.

3.3.3. Detection Performance Analysis

The test statistic of the energy detector at node k at time instant n is estimated using distributed received signal power estimation algorithms. Thus, the resulting detection performance is dependent on the performance of the underlying estimation process. For deriving the formulas of probability of detection (P_D) and probability of false alarm (P_{FA}) we need to evaluate the probability density function (PDF) of the test statistic $\hat{p}_k(n+1)$ under both hypotheses H_0 and H_1.

The input signal is CSCG and in case $K = 1$, the test statistic of ED $\hat{p}_k(n+1)$ is local and under both hypothesis a Chi-Square distributed random variable with 2N degrees of freedom. The test statistic $\hat{p}_k(n+1)$ is obtained as a sum of a number of identically distributed variables and hence the CLT can be applied to approximate the Chi square distribution by a Gaussian distribution [62]. According to AS 3 the number of samples is large enough, and the CLT is expected to apply.

The global test statistic $\hat{p}_k(n+1)$ in case of hypothesis H_1, is, however, estimated over independent, but not identically distributed variables. In such a case the Lyapunov CLT [65] can still be applied over a large number of samples to result in a Gaussian approximation.

Let Q be the complementary distribution function of the standard Gaussian

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{t^2}{2}\right) dt. \tag{3.50}$$

The conditional mean $\mathrm{E}(\hat{p}_k(n+1)|H_i)$ and the conditional variance $\mathrm{Var}(\hat{p}_k(n+1)|H_i)$ at node k (for $i = 0, 1$) can be easily obtained from previously derived (4.28) and (4.37) respectively. The conditional moments in steady state can be obtained similarly from the corresponding steady state results. At next we provide approximate formulas for the resulting energy detection performance. The probability of false alarm P_{FA} of the energy detector under hypothesis H_0 is found by

$$P_{FA}(\gamma, t) = Pr(T(x) > \gamma | H_0) = \int_\gamma^\infty p_x(x|H_0) dx \tag{3.51}$$

Substituting the estimation mean and variance under H_0, we get

$$P_{FA} = Q\left(\frac{\gamma - \mathrm{E}(\hat{p}_k(n+1)|H_0)}{\sqrt{\mathrm{Var}(\hat{p}_k(n+1)|H_0)}}\right), \tag{3.52}$$

which according to AS 2, holds for every node $k \in K$.

The probability of detection of an energy detector under hypothesis H_1 is correspondingly

$$P_D(\gamma, t) = Pr(T(x) > \gamma | H_1) = \int_\gamma^\infty p_x(x|H_1) dx. \tag{3.53}$$

Let the probability of detection at node k be: $P_{D,k}$. Similarly substituting the mean and variance under H_1, we get

$$P_{D,k} = Q\left(\frac{\gamma - \mathrm{E}(\hat{p}_k(n+1)|H_1)}{\sqrt{\mathrm{Var}(\hat{p}_k(n+1)|H_1)}}\right). \tag{3.54}$$

The sensing threshold is found from (3.52) by fixing the desired value of P_{FA}. Thus

$$\begin{aligned}\gamma &= \mathrm{E}[\hat{p}_k(n+1)|H_0] \\ &+ Q^{-1}(P_{FA})\sqrt{\mathrm{Var}[\hat{p}_k(n+1)|H_0]}.\end{aligned} \tag{3.55}$$

Due to AS 2 [2] the thresholds for every CR node k are equal.

Calculation of the threshold requires, however, knowledge of the moments of the estimation algorithm in case of hypothesis H_0 and these moments are dependent on the algorithm parameters (especially the step size). In practice the required moments can be calculated in advance using (4.28) and (4.37), known values of the step size and the noise power and then substituting these results into (3.55).

3.4. Simulation results

In the numerical simulation section we firstly investigate the ATC power estimation algorithm and compare the results with the CTA [2] and ring-around [1] versions. Secondly, we view the resulting energy detection performance. In all these simulations the PU signal $s(n)$ is taken as QPSK with unit power S, under the active hypothesis H_1, the step size is: $\mu = 0.01$.

3.4.1. Local and distributed power estimation

We start with the investigation of the estimation algorithms. The channel gains are assumed to be constant, fixed during the simulations and obtained by: $\alpha_k \sim \mathcal{C}N(0,1)$.

Ring-Around

We first investigate the estimates of (3.13) under two modes - *local*: if the nodes are not cooperating with each other (i.e. every node acts as a stand alone energy estimator/detector) and -*global*: if the nodes cooperate. In the next two examples all nodes receive $N = 1200$ samples. To illustrate the tracking feature, we examine how the algorithm reacts if the power level changes at sample 601. Thus, during samples $n = 1, ..., 600$ hypothesis H_1 is present (the source signal power S is attenuated by channel gain $|\alpha_k|^2$). Due to slow fading, α_k is

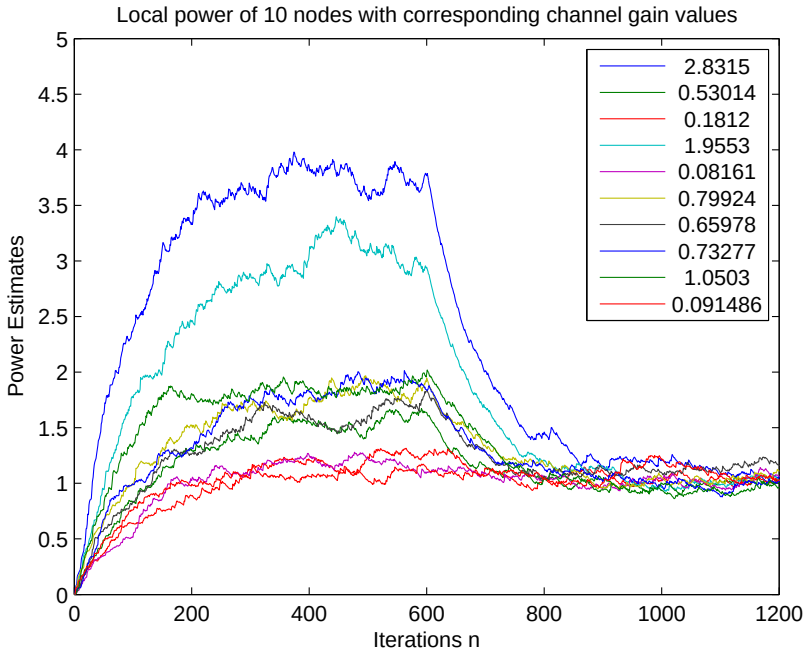

Figure 3.3 Local power estimation, fixed step , for the recursive ring-round topology

assumed to be constant and is obtained by: $\alpha_k \sim \mathcal{C}N(0,1)$. In sample range $n = 601, ..., 1200$, the PU signal is absent and only background noise power $\sigma_v^2 = 1$ is present at every node k.

Using recursion (3.13) the local, non-cooperative power estimate is plotted in Fig. 3.3, with 10 nodes in the CR network. The channel gain values $|\alpha_k|^2$ are given in the figure. Obviously, the estimation result using local information is dependant on the channel coefficient of the specific node. From $n = 601$ the algorithm is starting to converge to the noise only power level $\sigma_v^2 = 1$. If we for instance chose n dependant step size $\mu = \frac{1}{n}$, then from $n = 601$ the algorithm would obviously not reach to noise level during 600 samples.

In Fig. 3.4 we investigate the cooperative scheme. Exactly the same channel gains are used as in the local simulation. Since the mean and variance differ at nodes k, then for illustration we plot only the global power estimation result of node $k = 10$ in the network with $K = 10$. The corresponding mean and ± 3 times standard deviation are given in Fig. 3.4.

In Fig. 3.4 the global estimate is converging around the mean. Due to the proposed circular estimation topology, the recursion (3.13) can reduce the effect of random gain caused by channel coefficients. We see that the global estimate stays within the ± 3 times standard deviation limits from the mean, which is expected in case of a Gaussian distribution.

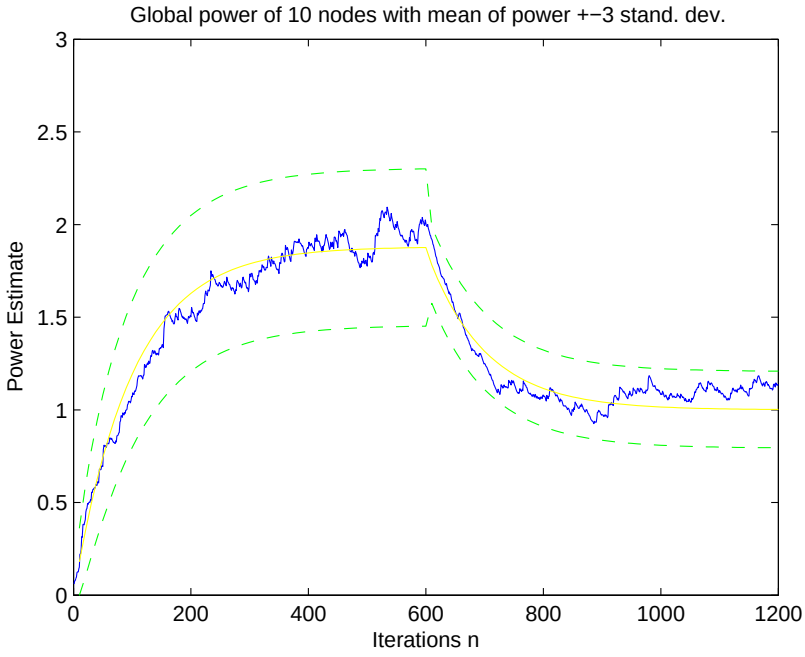

Figure 3.4 Global power estimation, fixed step, for the recursive ring-round topology

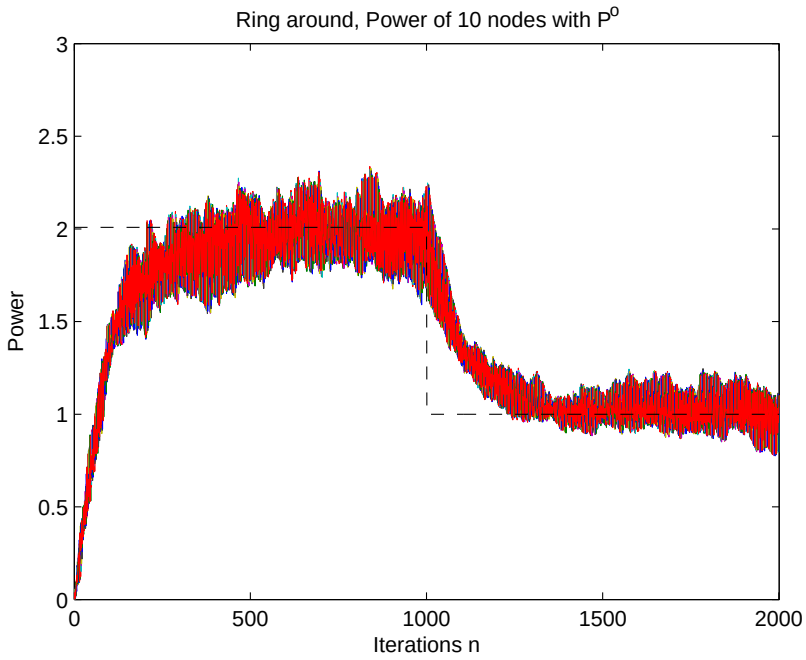

Figure 3.5 Local power estimation.

ATC and CTA

In the comparison of algorithms we use the same channel gains for all the algorithms. In this subsection, all the nodes in the network receive $N = 2000$ samples. To illustrate how the proposed adaptive algorithms react to changes in the underlying stochastic process, we have changed the active detection hypothesis at sample $n = 1001$. During samples $n = 1 \ldots 1000$ the PU signal with constant unit power S is present. In sample range $n = 1001 \ldots 2000$ the PU signal is absent and only noise is present. Under both detection hypotheses the noise power is $\sigma_v^2 = 1$ and assumed to be the same in all the nodes. In this subsection, it is assumed, that no measurements are exchanged between the nodes, $\mathbf{C} = \mathbf{I}$.

For illustration purpose, all the estimated power values in the CR network of 10 nodes are first plotted in Fig. 3.6. Using the ATC algorithm the estimates of the received power together with the optimal solution P^o have been plotted in Fig. 3.6. All the estimated power values in the CR network of the 10 nodes are plotted in one figure. When we use the CTA algorithm we obtain the results, which are given in Fig. 3.7. The value of the optimal solution P^o in figure Fig. 3.6 and in Fig. 3.7 is shown as the black dashed line and is calculated according to (4.10) using the present channel gains values.

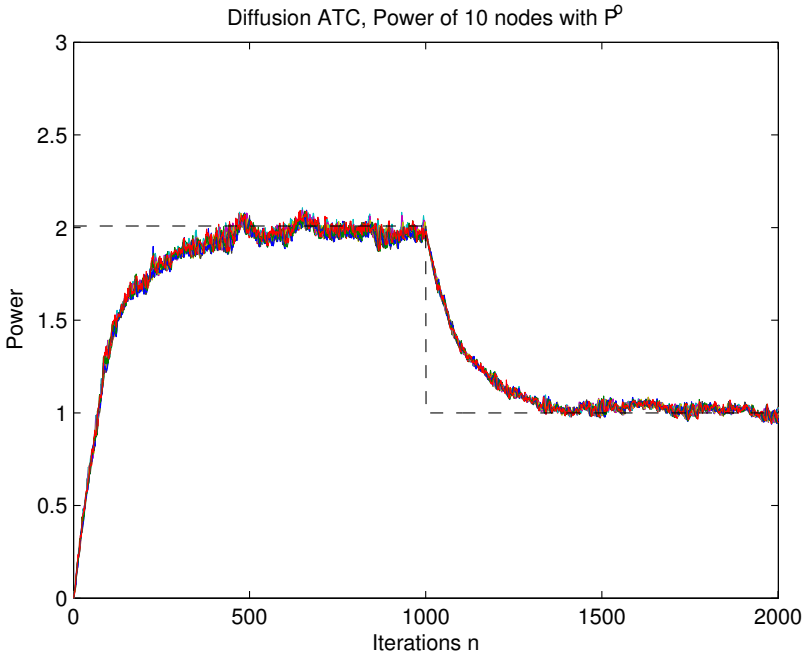

Figure 3.6 Local power estimation using ATC

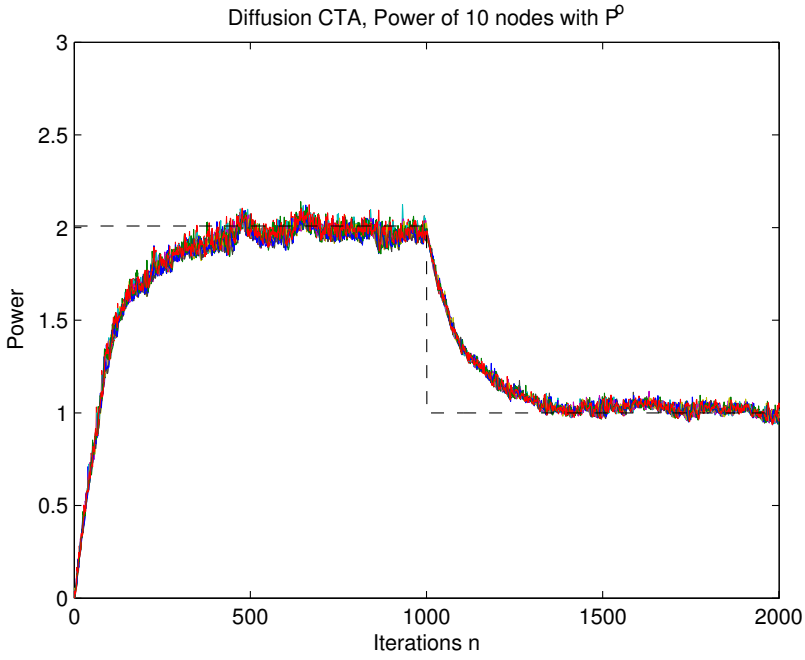

Figure 3.7 Local power estimation using CTA

Compared to the ring round topology in diffusion strategies, more information is processed at every node k, since neighbour estimate $(k-1)\text{mod}K$ is fused with the local estimate of node k. It was shown in [2] that the variance of the estimates of the CTA algorithm is lower than the variance of estimates of the ring around algorithm. Based on Fig. 3.6 and in Fig. 3.7 we observe that the variance of the estimates of the ATC algorithm is even slightly lower than the variance of estimates of the CTA algorithm.

The smallest value of steady state variance is achieved using the ATC algorithm. Compared to the ring around algorithm, since the precision of power estimates increases when the diffusion estimation strategies are used, the resulting detection performance increases as well.

3.4.2. Probability of detection

Next we investigate the probability of detection using the proposed distributed power estimation algorithms. In the following simulations we compare the performance of 5 different network sizes: $K = 1, 3, 10, 30, 50$ nodes. More specifically, the estimated and theoretical results of P_D of the last nodes in the set are compared, i.e $k = K$. In the simulations the converged power estimate is used for detection i.e. $\hat{p}_k(\infty)$. The theoretical mean and variance of the power estimates are calculated using directly the steady state formulas.

We set the desired $P_{FA} = 10^{-4}$. The thresholds of the energy detectors at nodes $k \in K$ are calculated using (3.55) and the corresponding steady state theoretical mean and variance of the power estimates (of algorithms CTA, ATC and ring around, respectively) under detection hypothesis H_0.

For estimating the P_D we use the Monte Carlo method [40] and run 1000 experiments with the same fixed set of channel constants and noise power for all the algorithms. The estimated P_D is compared with the theoretical P_D. The theoretical P_D is calculated using (3.54) and the corresponding steady state mean and variance of the power estimates of the three algorithms under detection hypothesis H_1. In the following figures the continuous lines represent the theoretical P_D and the corresponding signs the estimated P_D. First we set $\mathbf{C} = \mathbf{I}$. The detection performance of ATC, CTA and the ring around algorithms are shown in Fig. 3.8, in Fig. 3.10 and in Fig. 3.9 respectively. We see that there is a good match between the estimated and theoretical P_D. The PDF of the test statistic is approximated by a Gaussian distribution and the CLT approximation applies even with small K and when the underlying stochastic process is cyclostationary (since the variance of the sample of received signal is changing periodically over n). As we noticed in [2], the CTA algorithm outperforms the ring around algorithm. The P_D of the set with few nodes is more influenced by the given values of channel constants. According to simulation data when $K = 1$ the PU signal is in deep fading and this explains the worse P_D result. In case of non-distributed estimation and detection, not much can be done to improve the P_D. As the number of nodes in the network increases, about 4 dB is gained

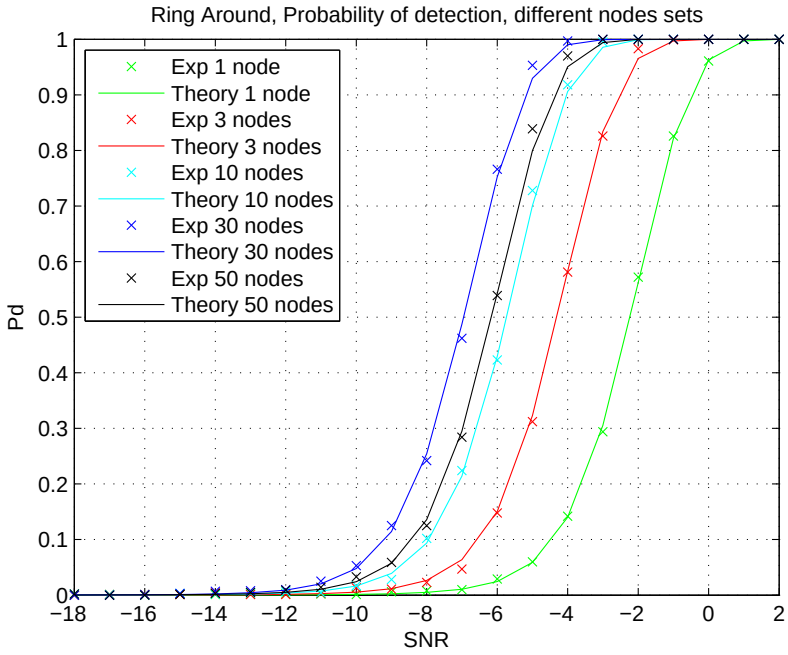

Figure 3.8 Probability of detection, ring around, $\boldsymbol{C} = \boldsymbol{I}$

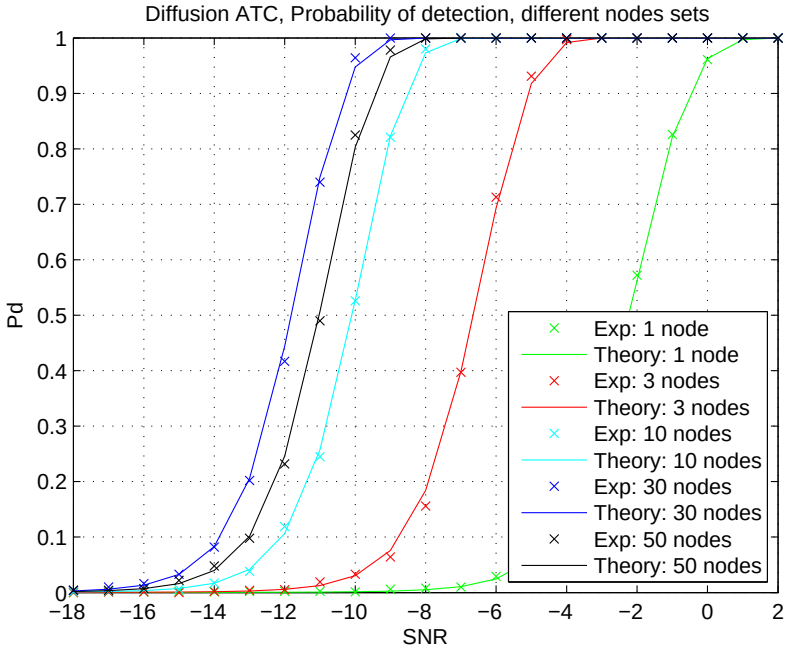

Figure 3.9 Probability of detection, ATC, $\boldsymbol{C} = \boldsymbol{I}$

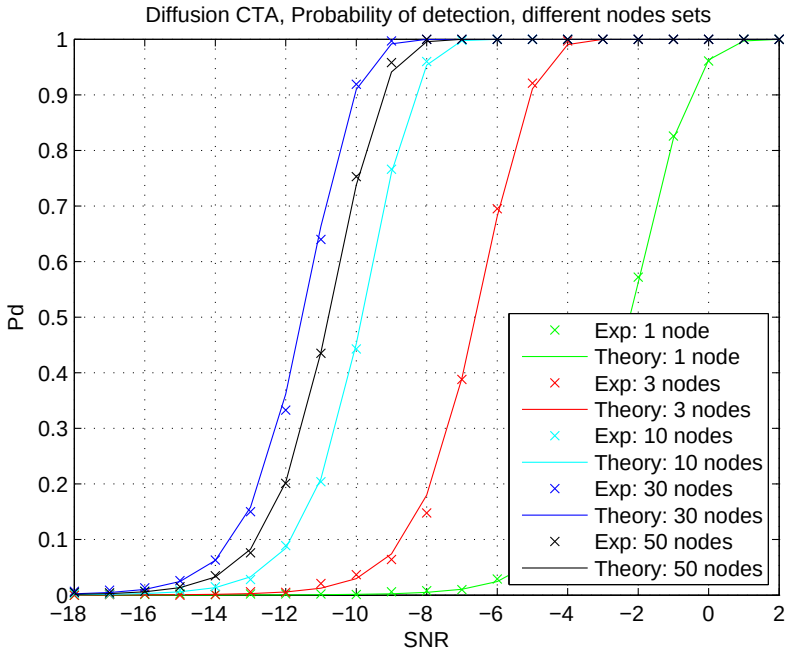

Figure 3.10 Probability of detection, CTA topology, $\boldsymbol{C} = \boldsymbol{I}$

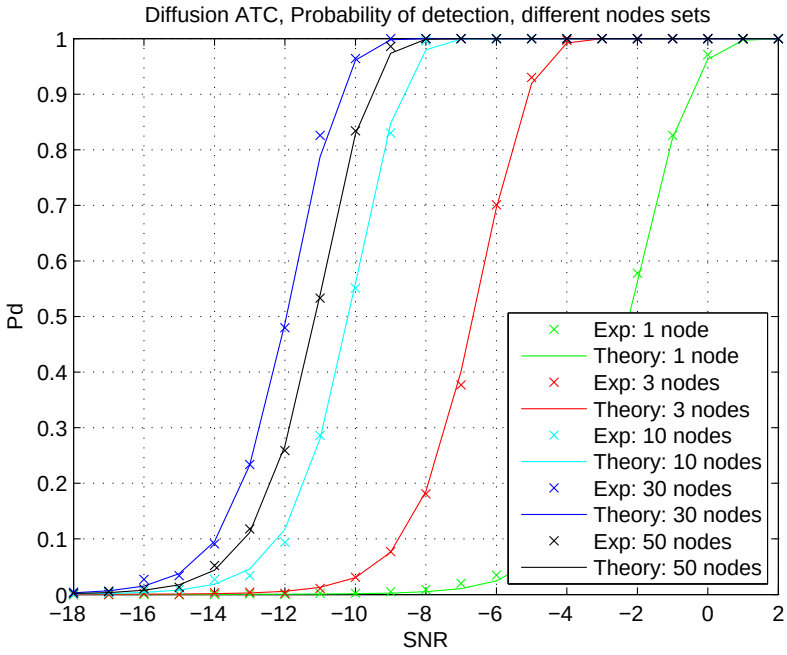

Figure 3.11 Probability of detection, ATC topology, $\boldsymbol{C} = \boldsymbol{A}_{diff}^{T}$

with respect to the noise power. Based on Fig. 3.9 we see that the ATC slightly outperforms the CTA. As the number of nodes K increases, from about $K = 30$, the P_D result stabilizes close to the theoretical P_D plot of the no fading case.

We then consider that also measurements from a neighbour node are available and we set $\mathbf{C} = \mathbf{A}_{\text{diff}}^{T}$ for the CTA and ATC algorithms; the results are shown in Fig. 3.12 and in Fig. 3.11, respectively. We note that ATC performs slightly better when more nodes in the network. While ATC fuses more data than CTA, the difference of detection performance with CTA is rather small. We see minor increase in the detection performance when additionally measurements are exchanged between the nodes. Thus, we conclude that the best detection results are obtained using ATC algorithm; however, the difference between ATC and CTA is quite small. On the other hand, for exchanging measurements between the nodes in a neighbourhood of a node in the CR network, additional data has to be broadcast and processed; this requires additional energy. Thus, the usage of measurement exchange may not be justified in a practical implementation.

3.5. Conclusion

In this chapter we studied a diffusion based distributed power estimation approach, that is applicable for CR networks for detecting the presence of

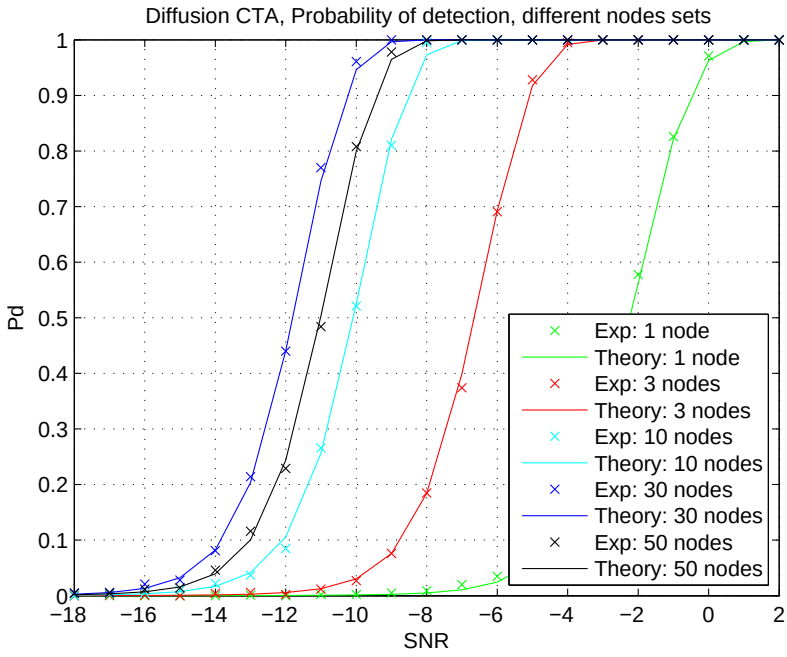

Figure 3.12 Probability of detection, CTA topology, $\boldsymbol{C} = \boldsymbol{A}_{diff}^{T}$

PU signal. We derived Ring-Around, CTA and ATC diffusion based energy detection algorithms for energy detection. We proposed a general framework for analysing the performance of the ATC diffusion, previously studied CTA and ring-around power estimation algorithms and compared the resulting energy detection performances. Our simulation study demonstrated that both diffusion LMS based energy detection algorithms outperform the previously proposed ring around algorithm and that the ATC diffusion algorithm slightly outperforms the CTA diffusion algorithm, and that the CTA diffusion algorithm outperforms the ring-around algorithm. It was also observed that the effect of exchanging measurements in addition to the estimates in CTA and ATC type of algorithms is rather small. All the three proposed algorithms with fixed step size are able to track changes in the received signal power and are usable in cognitive radio systems.

4. DISTRIBUTED LARGEST EIGENVALUE BASED SPECTRUM SENSING USING DIFFUSION LMS

In this chapter we propose a distributed detection scheme for cognitive radio (CR) networks, based on the largest eigenvalues (LEs) of adaptively estimated correlation matrices (CMs), assuming that the primary user signal is temporally correlated. The proposed algorithm is fully distributed, thereby avoiding the potential single point of failure that a fusion centre (FC) would imply. Different forms of diffusion least mean square (LMS) algorithms are used for estimating and averaging the CMs over the CR network for the LE detection and the resulting estimation performance is analyzed using a common framework. In order to obtain analytic results on the detection performance, the exact distribution of the CM estimates are approximated by a Wishart distribution, by matching the moments. The theoretical findings are verified through simulations.

4.1. Background

We consider the interweave CR paradigm [39], where CR systems detect the presence of a primary user (PU) signal by sensing the spectrum area of interest. The binary detection problem is studied: PU signal is present or absent [12, 13, 66]. In the interweave paradigm it is expected that the CR system should accurately detect the transmission of a PU system, when the latter is operating.

As already described in Chapter 3, in the literature several type of detectors for spectrum sensing have been proposed. When the PU signal waveform, channel and additive noise properties are known *a priori*, then the matched filter detector (MFD) is optimal [40]. The MFD requires perfect synchronization between the PU signal waveform and the received signal. However in practice such required knowledge is often not available, which makes the usage of the MFD detector impractical. The cyclostationary feature detection method [67] requires a priori knowledge about the cyclic frequencies of the PU signals, which often is a too strong assumption for practical implementation - in general it is complicated to implement and it requires that knowledge about the type, modulation and configuration properties of the PU signal is available. The Energy Detection (ED) method [40] models the PU signal as a random process and does not require knowledge about the PU signal, modulation type and channel properties. In such

a case, when the received PU signal is white, the ED is optimal. However, setting the detection threshold requires knowledge of the noise power value. It has been shown, that if there is uncertainty in the noise power or if the received PU signal is correlated, the ED performance decreases and it is no more optimal [17].

A second large group of detectors for spectrum sensing are based on eigenvalue properties of an estimated correlation matrix [43–45]. Detection based on the largest eigenvalue (LE) of estimated CMs [43] is optimal when the observations are zero mean Gaussian distributed, we do not have specific information about the PU signal and the channel gains, and when the PU signal is rank one correlated [68]. The LE method uses knowledge about the additive noise power to determine the detection threshold. Random Matrix Theory has been used to study the performance of the CM eigenvalue based detectors [69]. We note, that when linear estimation of CM is used, more sophisticated detectors: the volume based detector (VD) and the covariance based detector (CAV), which avoid eigenvalue or singular value decomposition, have been studied in [46, 47] and [48] respectively. Similarly, when linear estimation of a CM is used, several eigenvalue based detectors are robust in the sense, that the noise power value does not influence the test statistics or threshold of the detectors. For example the Eigenvalue Arithmetic to Geometric Mean (AGM) [49], the Maximum to Minimum eigenvalue ratio (MME), the Energy to Minimum Eigenvalue ratio (EME) [45], the Eigenvalue Moment ratio (EMR) [49], and the Hadamard [50] detectors have been proposed in the literature. A method for blind and optimal combination of observations for the ED has been proposed in [70]. For these detectors, the performance analysis is based on the assumption that the sample CM is Wishart distributed with known degrees of freedom (DoF), an assumption that does not hold when exponentially weighted (adaptive) CM estimation is used. Also, the proposed approximate or asymptotic analysis of the theoretical detection performance for EME, MME, CAV detectors tend to be inaccurate in the low SNR regime, as seen in [45, 48]. Such potential inaccuracy is not well usable for studying the accuracy of distribution parameter approximations of adaptive CM estimates in a low SNR region.

From the previous chapters we know, that in cognitive radio (CR) contexts we would like to avoid creating interference to the PU user and find free spectrum opportunities as fast as possible. On the other hand the active detection hypothesis may change during the processing time. Distributed, adaptive network learning methods, based on exponential averaging estimation, are able to learn the statistical information based on observations received by the nodes in the network. These methods can react to possible changes in the properties of estimated statistics in real time. Several proposed distributed spectrum sensing solutions make use of a central FC. A FC will however form a single point of failure in the network since a malfunction in this unit affects the performance of the whole distributed solution. We therefore propose a CM estimation solution, where the available CM estimates (and corresponding measurements) are fused in cognitive radio network nodes, to allow all nodes to make detection decisions

based on data from the neighboring nodes and without involvement of any central processing unit. Such a solution enhances the network failure resistance.

Also in the Chapter 3 we mentioned that several distributed adaptive estimation schemes have been studied in the past. Consensus based schemes are analyzed for example in [57–60]. Diffusion estimation schemes are studied for instance in [71, 72], while Least mean square (LMS) and recursive least squares (RLS) schemes in [26, 27, 29, 56]. It has been shown, that distributed diffusion strategies can often perform better (in terms of faster convergence and lower Mean Square Deviation) and be more stable compared to consensus algorithms [30, 73]. Several detection solutions, based on distributed estimation, have been studied for example in [28, 74–76]. A ring network topology for distributed energy detection without a FC has been suggested in [61]. In [1] we proposed and analyzed a diffusion LMS based recursive calculation of the test statistics with ring topology for the energy detectors in cognitive radio network. Ring networks are however sensitive to communication link failures. Combine and Adapt (CTA) LMS diffusion based calculation of the test statistics for the energy detectors was studied in [2] and an Adapt and Combine (ATC) based version was investigated further in [3].

In this chapter we study the performance of LE detection in a distributed CR network, based on adaptively, distributively estimated CMs, using the completely distributed diffusion LMS strategy. We focus on the distributed detection problem and the analysis of dynamics of the diffusion estimation process is beyond the scope of the chapter and this thesis. We make the assumption that the CR network does not have prior information about the waveform of the PU signal and about the channel gains in the secondary nodes. We assume that the received PU signals samples are temporally correlated. Secondly in general we assume the noise power level is known. Noise power estimation procedures and analysis of the sensitivity to estimation errors falls outside the scope of this chapter. To analyze the detection performance and determine the threshold value, we follow the ideas of [77–79] and approximate the distribution of the exponentially averaged CM estimate by a Wishart distribution by moment matching. The resulting DoF for the approximate Wishart distribution will depend both on the step size, the network topology, and under H_1 detection hypothesis will depend also on the value of the noise variance parameter. We have therefore focused on the LE based detection, since under H_1 the robustness of alternative detectors like EME, MME, CAV in case of adaptively estimated CMs, is lost anyway. We however provide a simulation with the MME detector, which is a robust detector. In the distributed CR network, every node acts as an independent detector in terms of detection decision making based on the available CM estimates. Due to limited information about the PU signal and the communication channel, the theoretical global estimation model is proposed as a network-average CM (while in practice the CR nodes have only access to the subset of data from the neighbor nodes). We consider the control-level analysis of the proposed distributed CM estimation and LE detection algorithm to be out of scope of the chapter.

We organize the remainder of the chapter as follows. In Section 4.2 we describe the motivation, specify the system models which are analysed further in this chapter and we motivate the usage of the LE detector. In Section 4.3 we derive an adaptive, distributed CM estimation algorithm based on diffusion LMS strategy and summarize the versions of it. In Section 4.4 we analyse the performance of the proposed distributed CM estimation algorithm using a common framework for moment based analysis for all the versions of the Diffusion LMS algorithm. We propose the usage of Total and General Variance based approximations for being able to model the distributions of adaptive CM estimates under both detection hypotheses. Using these results the theoretical false alarm and the detection performance of the LE detector are studied. In Section 4.5 we present our simulations results and verify the theoretical findings.

Notation. In this chapter we use the following notations. Boldface uppercase and lowercase letters denote matrices and vectors, respectively. $\mathrm{E}[\cdot]$, $\mathrm{Var}[\cdot]$, $\mathrm{Cov}[\cdot]$ denote expectation, variance (of a scalar) and covariance operators, respectively. $\mathrm{vec}[\cdot]$ and $\mathrm{vec}^{-1}[\cdot]$ denote conversion from matrix to vector and from vector to matrix. $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^c$ denote the vector or matrix transpose, the Hermitian transpose and the complex conjugate, respectively. $\otimes$ denotes the Kronecker product.

4.2. Problem formulation and background

4.2.1. Signal model and assumptions

Assume that K single-antenna CR nodes are independently sensing the communication band of a PU. Let the observation bandwidth of the communication band be denoted as B. A collection of samples of the down converted continuous time signal $z_s(t)$ are collected every T_s seconds, with sampling period $\delta_s < T_s$. As a result every node individually obtains a vector

$$\mathbf{y}_k(n) = [z_s(nT_s), z_s(nT_s - \delta_s), \ldots, z_s(nT_s - (M-1)\delta_s)]\,, \tag{4.1}$$

which gives the following observation model for both detection hypotheses

$$\begin{aligned} H_0 &: \mathbf{y}_k(n) = \mathbf{v}_k(n), \\ H_1 &: \mathbf{y}_k(n) = \alpha_k \mathbf{s}(n) + \mathbf{v}_k(n), \end{aligned} \tag{4.2}$$

where $k = 1, 2, ..., K$ is the node number, M is the length of the observation vector, and $n = 1, 2, ...N$ is the sample discrete time index. The primary signal $\mathbf{s}(n)$, the noise $\mathbf{v}_k(n)$ and channel gains α_k at node k are assumed to be statistically independent. We additionally assume that the PU signal follows

$$\mathbf{s}(n) \sim CN_M\left(\mathbf{m}_s, \mathbf{\Sigma}_s\right). \tag{4.3}$$

Due to the one communication channel assumption between a CR and PU, temporally correlation models of CMs are justified by the signal model 5.1. In

the performance analysis of the LE detection scheme, the following assumption will be used.

AS 1. The additive noise $\mathbf{v}_k(n)$ is independently and identically distributed (*i.i.d*) circularly symmetric complex Gaussian (CSCG) noise with zero mean and covariance $\mathbf{\Sigma}_{v,k} = \sigma_{v,k}^2 \mathbf{I}_M$. In the CR network $\mathbf{v}_k(n)$ is uncorrelated in time and space. We assume the noise power is known a *priori* and has the same power level for all nodes in the CR network.

Under H_1 we have the following $M \times M$ CM model

$$\mathbf{R}_k = \mathbf{R}_{s,k} + \mathbf{\Sigma}_{v,k}. \tag{4.4}$$

Let us denote the actually occupied bandwidth (within the observation bandwidth B) as b. Thus the ratio between occupation and observation bandwidths is denoted as $\beta = b/B$ [80] and the rank of the PU signal matrix can be then approximated as $\text{rank}(\mathbf{R}_{s,k}) \approx \lceil \beta M \rceil$. We assume $M > 1$, $\beta < 1$ and then $\mathbf{R}_{s,k}$ has in general a low rank (see also [81]), while $\mathbf{\Sigma}_{v,k}$ is a scaled identity matrix. This property can be used for detecting the PU signal.

4.2.2. Largest Eigenvalue detection

In this chapter, we focus on the LE detector, which is known to follow from the General Likelihood Ratio approach, when AS 1 holds, the received observation vectors obey a Multivariate Complex Gaussian distribution with zero mean, and when the PU signal population covariance matrix $\mathbf{R}_{s,k}$ is rank one [68]. The LE detector requires low computational complexity and the detection performance analysis is easy to conduct. As seen in [43] and in Section 4.4, there exist usable theoretical results for the conditional distributions without asymptotic approximations, which predict the true performance well both in low and high SNR. The LE method is optimal for one PU signal. In the case of higher rank PU signals (i.e more than one PU signal in the network), then the LE detector is no longer optimal, but still usable. We note that all these existing results from the literature for the LE detector hold when estimating the CM using a standard non-weighted sample covariance matrix, resulting in a complex Wishart distribution.

For the distributed adaptive estimation scheme considered here, this latter assumption is no longer true, but as will be shown in Sections 4.4 and 4.5, the distribution can still be well approximated by a complex Wishart distribution. The DoF approximations depend on the parameters of the distributed and adaptive CM estimation algorithm step-size and under H_1 also on the preciseness of the noise power value (AS 1). Extending the analysis to other type of detectors can therefore be done using the existing results in the literature, for example from [46, 47, 49, 50]. As seen in Section 4.5, a noise power uncertainty under the detection hypothesis H_1 causes an inaccuracy to the approximated $\text{DoF}|H_1$ value. This effect causes a potential inaccuracy in the theoretical detection performance formula of a detector, which requires the $\text{DoF}|H_1$ value. However since the

threshold of a robust detector is not affected by the noise power perturbations, then such a detector can still be used in the framework of this chapter. Thus to keep the focus of the chapter, we have limited our study to the LE detector, where AS 1 is necessary for the threshold calculation and to illustrate the effect of accuracy of the DoF approximations under both detection hypotheses. Since the LE detector is vulnerable to the noise power value uncertainty, then in Section 4.5 we also provide a simulation with the robust MME detector in the proposed distributed and adaptive CM estimation framework.

Thus an estimate $\hat{\mathbf{R}}_k(n)$ of the CM $\mathbf{R}_k$ is assumed to be available for every node $k \in K$ at time index n. Let us define the eigenvalues of $\hat{\mathbf{R}}_k(n)$ in non-increasing order as $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M$. Every node k detects the presence of a PU signal by independently determining the LE of the locally available estimate $\hat{\mathbf{R}}_k(n)$ and performing the following detection test

$$\lambda_1 \left[\hat{\mathbf{R}}_k(n)\right] \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_{LE,k}, \tag{4.5}$$

using a threshold $\gamma_{LE,k}$, which is given in Section 4.4.3 by (5.14) or (4.57).

Next we implement the diffusion LMS based method to derive a distributed adaptive CM based LE detector in the CR network, so that the algorithm: A) is able to react to a possible change in the statistics of observations on line (i.e when the detection hypothesis changes during the observation time) and B) estimates the CMs in a cooperative manner with an averaging effect over the CR network. CR nodes can have access only to a subset of neighbor nodes and no FC unit is used in the CR network.

4.3. Adaptive, Distributed CM estimation and LE detection

Obviously one of the most simple cooperation strategies is where all the CR nodes are able to exchange their local data (estimates or observations) with all the other nodes in the CR network, i.e the network global data is available at every node. However in practice it means that all nodes have to be within hearing distance of all the other nodes and significant amount of data needs to be exchanged and processed over the CR network. Secondly transmitting and processing of (global) data consumes energy, which may drain the batteries of the CR nodes. In this chapter we assume to have a more general network topology model, where nodes only share data with a subset of neighbor nodes and thus no global data is available. Thus we assume that the CR nodes use low power transmitters (i.e a low energy communication, to save the batteries) we also would like to save some energy required for local data processing. This means that while every CR node k still needs to transmit its estimate or observation at a time instant n, other nodes use data of pre-selected neighbor nodes and in such a way some energy can be saved by processing (in an adaptive manner) less data at every CR node.

We first describe local CM estimation, when the CR nodes in the network do not cooperate. Then we propose a global (theoretical) cost function for estimating the CM in a cooperative manner. We assume, that the K nodes in the CR network estimate a vector parameter $\mathbf{p}^o$ in a distributed manner, where nodes rely only on the information, that is available to them. The network topology is assumed to be fixed over the sensing time. We consider a linear, fixed combination of neighbor estimates and measurements at every node k and time instant n. The proposed global cost needs to be approximated in a distributed manner, where no FC, as a potential single point of failure in the system, is used. The derivation of the ATC and CTA type CM estimation algorithm diffusion power estimation algorithm follows the ideas in [3, 27, 55].

4.3.1. Local estimation

When CR nodes do not cooperate, then according to (5.2) $\mathbf{\Sigma}_{v,k} = \mathrm{E}\left[\mathbf{v}_k(n)\mathbf{v}_k(n)^H\right]$ and $\mathbf{R}_{s,k} = \mathrm{E}\left[|\alpha_k|^2\mathbf{s}(n)\mathbf{s}(n)^H\right]$. The estimate $\hat{\mathbf{R}}_k(N)$ of CM $\mathbf{R}_k$ based on the observations $n = 1, \ldots, N$ can be obtained (independently, non-adaptively) at every node k for example as

$$\hat{\mathbf{R}}_k(N) = \frac{1}{N}\sum_{n=1}^{N}\mathbf{y}_k(n)\mathbf{y}_k(n)^H, \tag{4.6}$$

We continue with the notation, suitable for the adaptive processing, i.e the estimate $\hat{\mathbf{R}}_k(n)$, available at node k at time instant n. In the light of the signal model cases in [82], we consider two specific PU signal models under the detection hypothesis H_1, where $\mathbf{s}(n)$ is a constant or a random variable. Under the different detection hypotheses, the $\hat{\mathbf{R}}_k(n)$ therefore follows the following Wishart distributions [43, 44, 83]

$$\begin{aligned} &H_0 : \hat{\mathbf{R}}_k(n) \sim CW_M(N, \tfrac{1}{N}\mathbf{\Sigma}_{v,k}),\\ &H_1 : \hat{\mathbf{R}}_k(n) \sim CW_M(N, \tfrac{1}{N}\mathbf{\Sigma}_{v,k}, \tfrac{1}{N}\mathbf{\Omega}_k) \quad \text{if} \quad \mathbf{m}_s \neq 0,\\ &H_1 : \hat{\mathbf{R}}_k(n) \sim CW_M(N, \tfrac{1}{N}\mathbf{\Sigma}'_k) \quad \text{if} \quad \mathbf{m}_s = 0, \end{aligned} \tag{4.7}$$

where N is the degree of freedom (DoF) parameter, $\mathbf{\Sigma}'_k = \mathbf{R}_{s,k} + \mathbf{\Sigma}_{v,k}$, by following the notation in [83, Th. 3.5.2] $\frac{1}{N}\mathbf{\Omega}_k = \left[\frac{1}{N}\mathbf{\Sigma}_{v,k}\right]^{-1}\left[\frac{1}{N}\mathbf{E}_k\mathbf{E}_k^H\right]$, and where the non-zero column n of $M \times N$ mean matrix $\mathbf{E}_k$ equals $\mathrm{E}\left[\alpha_k\right]\mathbf{m}_s$. The first case corresponds to the Complex Central Wishart (CCW) under detection hypothesis H_0, with population covariance matrix $\frac{1}{N}\mathbf{\Sigma}_{v,k}$. The second case with the non-centrality matrix $\frac{1}{N}\mathbf{\Omega}$ corresponds to the Complex Non-central Wishart distribution (NCW) under H_1. We denote it as Case 1. The third case corresponds to the Complex Central Correlated Wishart (CCCW) under H_1 with population covariance matrix $\frac{1}{N}\mathbf{\Sigma}'_k$. We denote it as Case 2.

According to (5.2), every node k has a unique channel gain α_k from the PU source, which is not known *a priori* for the nodes. When the nodes in the CR

network estimate $\mathbf{R}_k$ without cooperating with other nodes, then the estimates of $\mathbf{R}_k$ are (locally) influenced by the individual channel gains of the corresponding nodes. The local SNR at node k is given by

$$\mathbf{SNR}_k = \frac{\mathrm{Tr}\left[|\alpha_k|^2 \left(\mathbf{R}_{s,k} + \mathbf{m}_s \mathbf{m}_s^H\right)\right]}{\mathrm{Tr}\left[\mathbf{\Sigma}_{v,k}\right]}. \tag{4.8}$$

As seen, some CR nodes achieve better detection performance due to higher channel gains (i.e due to better position in the space) than the other. We are interested in a scheme, where all nodes can achieve similar detection performance, despite of their individual channel gains. The method (4.6) expects that N samples are available for calculation of the estimate and is not adaptive in its nature, i.e the CR system is unable to react quickly to a possible change of a detection hypothesis during the observation time N. This may increase the possibility of false alarm or a miss-detection of the PU user and thus also an interference to the PU user. As seen in next chapters, we find an adaptive, exponential (non-equal weighed) averaging based method for estimated the CMs, which is able to learn and react to the changes in the statistics of the CM in real time and needs to store only data from previous iteration.

4.3.2. Global estimation

The CR nodes could cooperate via internal communication links to enhance the detection performance (of the PU signal(s)) at every node k. In the distributed CR network we assume:

- **AS 2**. There is a common control channel available for the CR system for transferring the network level control messages. The communication links between the CR nodes are ideal and not capacity restricted.
- **AS 3**. The CR network is strongly connected (however nodes can directly communicate only with a subset of neighbor nodes).

We propose a model where nodes jointly (and in case of either detection hypothesis) estimate the network average CM, which is denoted as $\mathbf{R}^o$ and defined as follows

$$\mathbf{R}^o = \frac{1}{K} \sum_{k=1}^{K} \mathbf{R}_k^o. \tag{4.9}$$

For notational convenience, introduce $M^2 \times 1$ $\mathbf{r}^o = \mathrm{vec}(\mathbf{R}^o)$. Thus we can write

$$\mathbf{r}^o = \frac{1}{K} \sum_{k=1}^{K} \mathrm{vec}(\mathbf{R}_k^o) = \frac{1}{K} \sum_{k=1}^{K} \mathrm{E}\left[\mathrm{vec}\left[\mathbf{y}_k(n)\mathbf{y}_k(n)^H\right]\right]. \tag{4.10}$$

Let us define the Hermitian rank one observation matrix $\mathbf{D}_{R,k}(n) = \mathbf{y}_k(n)\mathbf{y}_k(n)^H$ (under both hypothesis) at node k at time instant n. Its $M^2 \times 1$ vectorized form

is $\mathbf{d}_{R,k}(n) = \mathrm{vec}\,[\mathbf{D}_{R,k}(n)]$. We can decompose the $\mathbf{d}_{R,k}(n)$ into the product of a $M^2 \times M^2$ constant (invertible) complex matrix $\mathbf{T}$ and a $M^2 \times 1$ real vector $\mathbf{d}_k(n)$ as $\mathbf{d}_{R,k}(n) = \mathbf{T}\mathbf{d}_k(n)$, to keep the dimension of the estimated vector minimal in the adaptive recursions. For example, when $M = 2$, then

$$\mathbf{T}\mathbf{d}_k(n) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -i & 0 \\ 0 & 1 & i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{D}_{R,k}(n)(1,1) \\ \Re[\mathbf{D}_{R,k}(n)(1,2)] \\ \Im[\mathbf{D}_{R,k}(n)(1,2)] \\ \mathbf{D}_{R,k}(n)(2,2) \end{bmatrix}. \tag{4.11}$$

We denote the estimate of the real valued $\mathrm{E}\,[\mathbf{d}_k(n)]$ as $\hat{\mathbf{p}}_k(n)$. To construct an adaptive distributed estimation algorithm, we first relate the estimates of $\mathbf{R}_k^o$ and $\mathbf{R}^o$ in (4.9) with the minimization of the following global (network-wise) cost function

$$\mathbf{p}^o = \underset{\mathbf{p}}{\mathrm{argmin}} \sum_{k=1}^{K} J_k(\mathbf{p}) = \underset{\mathbf{p}}{\mathrm{argmin}} \sum_{k=1}^{K} \mathrm{E}\,\|\mathbf{d}_k(n) - \mathbf{p}\|^2, \tag{4.12}$$

where the vector $\mathbf{p} \in R^{M^2}$ represents the real valued parameters of the CM, to be estimated. Thus $\mathbf{p}^o$ represents the optimal (real valued) CM estimate or is the optimal solution for the minimization of the Mean Square Error (MSE) type of global aggregate cost function $J^{glob}(\mathbf{p})$, which is given as

$$\begin{aligned} J^{glob}(\mathbf{p}) &= \sum_{k=1}^{K} J_k(\mathbf{p}) \\ &= \sum_{k=1}^{K} \mathrm{E}\left[\|\mathbf{d}_k(n)\|^2 - \mathbf{d}_k^T(n)\mathbf{p} - \mathbf{p}^T\mathbf{d}_k(n) + \mathbf{p}^T\mathbf{p}\right]. \end{aligned} \tag{4.13}$$

Let us note that compared to the models in [27, 28, 31], in (4.13) both the observation and estimation variables are vectors. By differentiating $J^{glob}(\mathbf{p})$ in (4.13) with respect to $\mathbf{p}$ and setting the result to zero, we get

$$\nabla_{\mathbf{p}} J^{glob}(\mathbf{p}) = -\sum_{k=1}^{K} \mathrm{E}\left[\mathbf{d}_k^T(n)\right] + K\mathbf{p}^T = 0. \tag{4.14}$$

It follows that

$$\mathbf{p}^o = \frac{1}{K} \sum_{k=1}^{K} \mathrm{E}\,[\mathbf{d}_k(n)]. \tag{4.15}$$

The Hessian of the aggregate cost function is

$$\nabla_{\mathbf{p}}^2 J^{glob}(\mathbf{p}) = 2\mathbf{I}_M. \tag{4.16}$$

Obviously $J^{glob}(\mathbf{p})$ in (4.13) is strongly convex [30, C.18] with the unique solution $\mathbf{p}^o$. Also, in case of one node in the CR system ($K = 1$) or when

the nodes do not cooperate, then the individual cost $J_k(\mathbf{p})$ is minimized at the point $\mathbf{p}_k^o = \mathrm{E}\left[\mathbf{d}_k(n)\right]$. Since $\nabla^2_{\mathbf{p}} J_k^{loc}(\mathbf{p}) = 2\mathbf{I}_M$ and the individual cost $J_k^{loc}(\mathbf{p})$ is strongly convex, thus $\mathbf{p}_k^o$ is unique as well.

Compared to [27,28,31], in this chapter the local costs $J_k(\mathbf{p})$ are individually not minimized at the same global point $\mathbf{p}^o$ due to different channel conditions. However the derivation of the diffusion LMS algorithm still follows the procedure as proposed in these papers. The proposed optimal solution (4.12) is similar to the Pareto model, which is analysed in [33].

Note that

$$\begin{aligned} \mathbf{R}_k^o &= \mathrm{vec}^{-1}\left[\mathbf{T}\mathbf{p}_k^o\right] \\ \mathbf{R}^o &= \mathrm{vec}^{-1}\left[\mathbf{T}\mathbf{p}^o\right]. \end{aligned} \tag{4.17}$$

We seek an iterative solution to estimate the $\mathbf{p}_k^o$ and $\mathbf{p}^o$ in a manner, which is adaptive in time, and is fully distributed (cooperative). We propose to use diffusion LMS based distributed solution.

4.3.3. Iterative Diffusion solutions

Let $\mathcal{N}_k$ denote the neighborhood group of node $k \in K$, i.e $\mathcal{N}_k$ defines the set of nodes l which can send data unidirectionally the node k. The node k is assumed to be always connected to itself. For deriving the diffusion LMS algorithm, we define and use the standard matrices $\mathbf{A}$, $\mathbf{C}$ and $\mathbf{C}$ similarly to [27], with non-negative elements $a_{l,k}$, $b_{l,k}$ and $c_{l,k}$, that describe how data is exchanged and combined in the network.

Let us start by defining the $K \times K$ right stochastic matrix $\mathbf{C}$ with non-negative elements so that

$$c_{l,k} = 0 \quad \text{if} \quad l \notin \mathcal{N}_k, \quad \mathbf{C}\mathbf{1} = \mathbf{1}, \tag{4.18}$$

where $c_{l,k} = 1$ if node l is connected to the node k. The global cost (4.13) can be divided into the local cost of over the neighborhood of node k and the sum of local costs of other nodes over their corresponding neighborhoods, and can be given in the following form

$$J^{glob}(\mathbf{p}) = J_k^{loc}(\mathbf{p}) + \sum_{l \neq k}^{K} J_l^{loc}(\mathbf{p}). \tag{4.19}$$

The local cost at every node k can be expressed as a weighted combination of the costs of the neighbors of every node k. Thus with the help of non-negative coefficients $c_{l,k}$ the local cost can be given as follows

$$J_k^{loc}(\mathbf{p}) = \sum_{l \in \mathcal{N}_k} c_{l,k} J_l(\mathbf{p}) \tag{4.20}$$

and is minimized at the location $\mathbf{p}_k^{loc}$. The following relation $J_l^{loc}(\mathbf{p}) \approx J_l^{loc}(\mathbf{p}^{loc}) + \|\mathbf{p} - \mathbf{p}_l^{loc}\|^2$ [32] can be used for the second part of right hand side

(RHS) of (4.19) to relate the variable $\mathbf{p}$ and the $\mathbf{p}_l^{loc}$. Here the $J_k^{loc}(\mathbf{p}_l^{loc})$, can be ignored, since it is independent on the variable $\mathbf{p}$. Thus we have the modified global cost function $J^{glob'}$ as follows

$$J^{glob'}(\mathbf{p}) = J_k^{loc}(\mathbf{p}) + \sum_{l \neq k}^{K} \|\mathbf{p} - \mathbf{p}_l^{loc}\|^2. \tag{4.21}$$

Note that it is not assumed, that node k has access to all the $\mathbf{p}_l^{loc}$ in the network. Thus we need to approximate the $J^{glob'}(\mathbf{p})$ locally at every node k and the standard steps follow. We use the non-negative coefficients $b_{l,k}$ to define if $\mathbf{p}_l^{loc}$ is available for the node k. Thus the elements $b_{l,k}$ take the following values

$$\text{if} \quad l \notin \mathcal{N}_k \quad \text{then} \quad b_{l,k} = 0 \quad \text{else} \quad b_{l,k} = 1. \tag{4.22}$$

Then, we limit the summation $\sum_{l \neq k}^{K} \|\mathbf{p} - \mathbf{p}_l^{loc}\|^2$ on the RHS of (4.21) to the neighbors of node k i.e $\sum_{l \in \mathcal{N}_k / \{k\}} b_{l,k} \|\mathbf{p} - \mathbf{p}_l^{loc}\|^2$. Secondly, we replace the (only theoretically available) $\mathbf{p}_l^{loc}$ with an intermediate estimate $\hat{\boldsymbol{\psi}}_l$, which is available at node l.

After these steps the approximation of (4.21) at node k is given as

$$\begin{aligned} J_k^{dist}(\mathbf{p}) = & \sum_{l \in N_k} c_{l,k} \, \mathrm{E} \, \|\mathbf{d}_l(n) - \mathbf{p}\|^2 \\ & + \sum_{l \in \mathcal{N}_k / \{k\}} b_{l,k} \|\mathbf{p} - \hat{\boldsymbol{\psi}}_l\|^2. \end{aligned} \tag{4.23}$$

The steepest descent algorithm [25] can be used to obtain a recursion for the estimate of $\mathbf{p}^o$ at time instant n, at node k, denoted as $\hat{\mathbf{p}}_k(n)$. By skipping the derivation steps, as in [27], the two-step steepest descent recursions are then given as

$$\begin{aligned} \hat{\boldsymbol{\psi}}_k(n+1) &= \hat{\mathbf{p}}_k(n) + \mu_k \sum_{l \in N_k} c_{l,k} \left[\mathbf{d}_l(n) - \hat{\mathbf{p}}_k(n)\right] \\ \hat{\mathbf{p}}_k(n+1) &= \left[1 - \nu_k \sum_{l \in \mathcal{N}_k / \{k\}} b_{l,k}\right] \hat{\boldsymbol{\psi}}_k(n+1) \\ &\quad + \nu_k \sum_{l \in \mathcal{N}_k / \{k\}} b_{l,k} \hat{\boldsymbol{\psi}}_l(n+1), \end{aligned} \tag{4.24}$$

where μ_k and ν_k are a positive step sizes, $\hat{\boldsymbol{\psi}}_k(n+1)$ is an intermediate estimate at node k at time n.

The coefficients in front of $\hat{\boldsymbol{\psi}}_l(n+1)$, $l = 1, \ldots, K$ in the second equation of (4.24) can be incorporated into the non-negative coefficients $a_{l,k}$. Let us introduce the $K \times K$ matrix $\mathbf{A}$, whose elements satisfy

$$a_{l,k} = 0 \quad \text{if} \quad l \notin \mathcal{N}_k, \quad \mathbf{1}^T \mathbf{A} = \mathbf{1}^T. \tag{4.25}$$

Thus we take $a_{k,k} = 1 - \nu_k \sum_{l \in \mathcal{N}_k / \{k\}} b_{l,k}$ and $a_{l,k} = \nu_k b_{l,k}$ for $l \neq k$. It is straightforward to see that $\sum_{l \in \mathcal{N}_k} a_{l,k} = 1$ for every $k \in K$ and thus $\mathbf{A}$ is a left stochastic matrix. Finally we obtain the Adapt and Combine (ATC) recursions as

$$\begin{aligned} \hat{\boldsymbol{\psi}}_k(n+1) &= \hat{\mathbf{p}}_k(n) + \mu_k \sum_{l \in N_k} c_{l,k} \left(\mathbf{d}_l(n) - \hat{\mathbf{p}}_k(n) \right) \\ \hat{\mathbf{p}}_k(n+1) &= \sum_{l \in N_k} a_{l,k} \hat{\boldsymbol{\psi}}_l(n+1). \end{aligned} \tag{4.26}$$

In similar manner the Combine and Adapt (CTA) version can be derived, following the ideas from [27]. In the ATC and CTA algorithms the coefficients $c_{l,k}$ and $a_{l,k}$ define respectively how the measurements $\mathbf{d}_l(n)$ and $\hat{\mathbf{p}}_l(n)$ are (unidirectionally) available for the node k. Thus the matrices $\mathbf{A}$ and $\mathbf{C}$ specify the combination strategy of the measurements and the estimates respectively in the CR network.

In Algorithm 4 we present the ATC and CTA based CM estimation recursions and the detection step in a common form. For this we define an additional intermediate estimate $\hat{\boldsymbol{\phi}}_k(n)$ and denote the $K \times K$ matrix $\mathbf{A}$ as $\mathbf{A}_1$ or $\mathbf{A}_2$, with the elements $a_{1,l,k}$ and $a_{2,l,k}$ correspondingly. The selection options of the matrices $\mathbf{A}_1$ and $\mathbf{A}_2$ and $\mathbf{C}$ based on [27] are given in Table 1. In practice the non-negative coefficients $a_{1,l,k}$, $a_{2,l,k}$, $c_{l,k}$ can be chosen freely under the conditions (5.7) and (5.8) respectively. The coefficients $b_{l,k}$ are absorbed into coefficients $a_{l,k}$ and do not have to be considered in practice. For comparison in Section 4.5, we list also a topology, where every node acts as a FC, denoted as Global FC LMS in Table 1. In such case CR nodes estimate the CM adaptively and independently (without sharing estimates), all the measurements from all the CR nodes are available and equally weighted for every node in the network.

Table 4.1 Choices of Matrices $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ and $\boldsymbol{C}$ for different LMS algorithms

Algorithm	$\mathbf{A}_1$	$\mathbf{A}_2$	$\mathbf{C}$
No Cooperation LMS	**I**	**I**	**I**
Global FC LMS [27]	**I**	**I**	$(1/K)\mathbf{1}\mathbf{1}^T$
CTA diffusion LMS [27]	**A**	**I**	**C**
ATC diffusion LMS (4.26)	**I**	**A**	**C**

Thus we observe that according to (4.17), Table 1 and the CM estimation recursions in Algorithm 4, when the nodes in the CR network do not cooperate, then the adaptive estimate $\hat{\mathbf{p}}_k(n)$ at time instant n at node k defines the individual (local) adaptive estimate of $\mathbf{R}_k^o$. When nodes cooperate by following the proposed cost (4.12), Table 1 and the CM estimation recursions in Algorithm 4, then the adaptive estimate $\hat{\mathbf{p}}_k(n)$ at time instant n at node k defines the adaptive estimate of $\mathbf{R}^o$ in (4.9), within acceptable mean square error bounds [27, 28]. Thus

Algorithm 4 Distributed LMS based CM Estimation and Detection

Start with $\hat{\mathbf{p}}_k(0) = \mathbf{p}(0)$ for every k .
Given non-negative real coefficients $a_{1,l,k}, a_{2,l,k}, c_{l,k}$
for every time instant $n \geq 1$ **do**
  for every node $k = 1, ..., K$ **do**
    1. CM estimation recursions:
    $\hat{\boldsymbol{\phi}}_k(n) = \sum_{l=1}^{K} a_{1,l,k}\hat{\mathbf{p}}_l(n))$.
    $\hat{\boldsymbol{\psi}}_k(n+1) = \hat{\boldsymbol{\phi}}_k(n)$
      $+\mu_k \sum_{l=1}^{K} c_{l,k}\left[\mathbf{T}^{-1}\mathbf{d}_{R,l}(n) - \hat{\boldsymbol{\phi}}_k(n)\right]$
    $\hat{\mathbf{p}}_k(n+1) = \sum_{l=1}^{K} a_{2,l,k}\hat{\boldsymbol{\psi}}_l(n)$
    2. LE detection decision:
    $H_0 : \lambda_1\left[\mathrm{vec}^{-1}\left[\mathbf{T}\hat{\mathbf{p}}_k(n+1)\right]\right] < \gamma_{LE,k}$ or
    $H_1 : \lambda_1\left[\mathrm{vec}^{-1}\left[\mathbf{T}\hat{\mathbf{p}}_k(n+1)\right]\right] > \gamma_{LE,k}$.
    (Refer to (5.14) or (4.57) for selecting the $\gamma_{LE,k}$).
  end for
end for

after several iterations, the adaptive estimate $\hat{\mathbf{R}}_k(n)$ of $\mathbf{R}^o$ is available (via the transformation (4.11) and de-vectorization) for every node in the CR network. Therefore depending on the cooperation model of the nodes, the node k at time instant n can perform independently the LE detection based on the available matrix estimate $\hat{\mathbf{R}}_k(n) = \mathrm{vec}^{-1}\left[\mathbf{T}\hat{\mathbf{p}}_k(n)\right]$.

Regarding the communication cost of Algorithm 4, then based on Table 1 it is obvious, that when $\mathbf{A} \neq \mathbf{I}$, then from the transmission point of view still every node $k \in K$ needs to broadcast its $M^2 \times 1$ estimation vector $\hat{\mathbf{p}}_k(n)$ at time instant n to the neighbours of hearing distance of the node k. However from the receiving point of view the number of estimates $\hat{\mathbf{p}}_k(n)$ required for the fusion by every node k is determined by the selection of matrix $\mathbf{A}$. Similarly, every node k obtains at time instant n a $M^2 \times 1$ observation vector $\hat{\mathbf{d}}_k(n)$ and when $\mathbf{C} \neq \mathbf{I}$ broadcasts it at time instant n to the neighbours of hearing distance of the node k. Thus on the receiving side, the exact selection of $\mathbf{C}$ determines the number $\hat{\mathbf{d}}_k(n)$ required by every node k at time instant n for observation fusion. In Section 4.5.1 we comment our selection of $\mathbf{A}$ and $\mathbf{C}$ for the simulations.

Finally we note that in addition to AS 2, obviously the CR system needs some control layer protocol to establish a connection between the nodes. The details of this operation is outside the scope of this chapter. We note, the exact control layer model and implementation of the CR system is out of scope or the paper. In general a protocol needs to be implemented to control the (iteration) time of reliable spectrum sensing and the time of transmission of secondary (CR) system. Thus based on assumption AS 4, the Algorithm 4 is started and running on-line, until stopped or , re-initiated by the system.

4.4. Performance analysis

The performance analysis of the proposed algorithm is divided into three parts. First we derive a general model for analyzing the mean and (co-)variance of the adaptive CM estimates of recursions in Algorithm 4 in one framework. Secondly we study the statistical properties of the adaptive CM estimates. For studying the LE detection performance of the adaptive CM estimate, the distribution of the adaptive CM estimate is approximated by a CCCW distribution. We propose the usage of the Total and General Variance methods for approximation the DoF and mean matrix parameters for the corresponding CCCW distributions, based on the moments of adaptive CM estimates. Thirdly we provide theoretical results for the LE detector. Let us note that for the theoretical performance analysis of the LE detector, we need to know the values of the channel gains and the noise power.

4.4.1. Moment analysis of adaptive CM estimates

For the analysis of the moments of the spatio-temporal adaptive CM estimates, we propose to use a more general vector/matrix recursion model.

We stack first the $M^2 \times 1$ estimates and observations from all the nodes $k \in K$ into a $KM^2 \times 1$ column vector $\hat{\mathbf{p}}(n)|H_i = [\hat{\mathbf{p}}_1(n)|H_i \dots \hat{\mathbf{p}}_K(n)|H_i]^T$ and $\mathbf{d}(n)|H_i = [\mathbf{d}_1(n)|H_i \dots \mathbf{d}_K(n)|H_i]^T$ respectively, where $i = 1$ denotes the case when the PU signal is present and $i = 0$ the case when the PU signal is absent. The initial estimate is noted as $\hat{\mathbf{p}}(0)|H_i$.

Secondly we define an additional $K \times K$ matrix $\boldsymbol{\mathcal{M}} = \operatorname{diag}\{\mu_1, \dots, \mu_K\}$, which contains the positive step size parameters of the algorithms for every node $k \in K$. The matrix $\boldsymbol{\mathcal{M}}$ is then be extended to another $KM^2 \times KM^2$ matrix as $\overline{\boldsymbol{\mathcal{M}}} = \boldsymbol{\mathcal{M}} \otimes \mathbf{I}_{M^2}$. For the purpose of comparison with the Consensus algorithm [73], let the $K \times K$ matrix $\mathbf{A}_0$ specify the fusion strategy of estimates of the consensus algorithm.

The $K \times K$ network topology matrices $\mathbf{A}_0$, $\mathbf{A}_1$, $\mathbf{A}_2$ and $\mathbf{C}$ are extended to $KM^2 \times KM^2$ matrices as follows, $\overline{\mathbf{A}}_0 = \mathbf{A}_0^T \otimes \mathbf{I}_{M^2}$, $\overline{\mathbf{A}}_1 = \mathbf{A}_1^T \otimes \mathbf{I}_{M^2}$, $\overline{\mathbf{A}}_2 = \mathbf{A}_2^T \otimes \mathbf{I}_{M^2}$ and $\overline{\mathbf{C}} = \mathbf{C}^T \otimes \mathbf{I}_{M^2}$.

Proposition 1. The distributed LMS algorithms in Table 1 and the consensus algorithm [73] can be described by the following spatio-temporal recursion

$$\hat{\mathbf{p}}(n+1)|H_i = \overline{\mathbf{A}}_2 \left(\overline{\mathbf{A}}_0 - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1 \hat{\mathbf{p}}(n)|H_i + \overline{\mathbf{A}}_2 \overline{\boldsymbol{\mathcal{M}}\mathbf{C}} \mathbf{d}(n)|H_i. \tag{4.27}$$

In case of LMS algorithms $\mathbf{A}_0 = \mathbf{I}_K$ and for example we get the ATC algorithm with no measurement exchange, when we take additionally $\mathbf{A}_1 = \mathbf{C} = \mathbf{I}_K$ and $\mathbf{A}_2 \neq \mathbf{I}_K$, according to the selected network topology. Thus $\overline{\mathbf{A}}_0 = \overline{\mathbf{A}}_1 = \mathbf{I}_K \otimes \mathbf{I}_{M^2}$, $\overline{\mathbf{A}}_2 = \mathbf{A}_2^T \otimes \mathbf{I}_{M^2}$ and $\overline{\mathbf{C}} = \mathbf{I}_K \otimes \mathbf{I}_{M^2}$. For CTA algorithm we take $\overline{\mathbf{A}}_1 = \mathbf{A}_{\text{diff}}^T \otimes \mathbf{I}_{M^2}$, $\overline{\mathbf{A}}_2 = \mathbf{I}_K \otimes \mathbf{I}_{M^2}$, $\overline{\mathbf{C}} = \mathbf{I}_K \otimes \mathbf{I}_{M^2}$ or $\overline{\mathbf{C}} = \mathbf{A}_{\text{diff}}^T \otimes \mathbf{I}_{M^2}$. Note that to keep the matching notation with Algorithm 1, we use transposed matrices in the general

spatio-temporal vector recursion. For the Consensus algorithm [73], we take $\mathbf{A}_1 = \mathbf{A}_2 = \mathbf{C} = \mathbf{I}_K$, $\mathbf{A}_0 \neq \mathbf{I}_K$ according to the network topology and thus we have $\overline{\mathbf{A}}_0 = \mathbf{A}_0^T \otimes \mathbf{I}_{M^2}$ and $\overline{\mathbf{A}}_1 = \overline{\mathbf{A}}_2 = \mathbf{I}_K \otimes \mathbf{I}_{M^2}$. Note, that the proposed Kronecker extension retains the stochastic property of the extended matrix and due to the transpose, the matrices $\overline{\mathbf{A}}_1$ and $\overline{\mathbf{A}}_2$ are now right stochastic and $\overline{\mathbf{C}}$ is left stochastic.

For studying the performance of the LMS algorithms, we first need to evaluate the moments - mean and covariance of the stacked estimates $\hat{\mathbf{p}}(n)$ and we provide the corresponding recursions for evaluating these moments.

Mean of estimates

Let us denote the conditional expectation of the observation vector as $\mathrm{E}\left[\mathbf{d}(n)|H_i\right]$, where $i = 0, 1$. We specify these values in the Section 4.4.2.

Proposition 2. The general recursion (5.11), can be expressed as

$$\begin{aligned}\mathrm{E}\left[\hat{\mathbf{p}}(n+1)|H_i\right] = \overline{\mathbf{A}}_2 \left(\overline{\mathbf{A}}_0 - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1 \mathrm{E}\left[\hat{\mathbf{p}}(n)|H_i\right] \\ + \overline{\mathbf{A}}_2 \overline{\boldsymbol{\mathcal{M}}\mathbf{C}} E\left[\mathbf{d}(n)|H_i\right], \end{aligned} \tag{4.28}$$

for $i = 0, 1$, where the initial value for the mean vector is given as $\mathrm{E}\left[\hat{\mathbf{p}}(0)|H_i\right]$, $i = 0, 1$.

After iterating we see, that the mean recursion can be given in the following equivalent form

$$\begin{aligned}\mathrm{E}\left[\hat{\mathbf{p}}(n)|H_i\right] = \left[\overline{\mathbf{A}}_2 \left(\overline{\mathbf{A}}_0 - \overline{\mathcal{M}}\right) \overline{A\mathbf{1}}\right]^n \hat{\mathbf{p}}(0) \\ + \left[\sum_{i=0}^{n-1} \left[\overline{\mathbf{A}}_2 \left(\overline{\mathbf{A}}_0 - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1\right]^i\right] \\ \times \overline{\mathbf{A}}_2 \overline{\boldsymbol{\mathcal{M}}\mathbf{C}} E\left[\mathbf{d}(n)|H_i\right]. \end{aligned} \tag{4.29}$$

For the asymptotic analysis of the mean recursion (4.29), we need to analyse the asymptotic behavior of $\left[\overline{\mathbf{A}}_2 \left(\overline{\mathbf{A}}_0 - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1\right]^n$ and the limit of the geometric series $\sum_{i=0}^{n-1} \left[\overline{\mathbf{A}}_2 \left(\overline{\mathbf{A}}_0 - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1\right]^i$, when $n \to \infty$.

According to [63, Theorem 5.6.12], the convergence $\lim_{n\to\infty}[\overline{\mathbf{A}}_2 \left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1]^n \to 0$ happens if and only if the spectral radius of the matrix $\overline{\mathbf{A}}_2 \left(\overline{\mathbf{A}}_0 - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1$ satisfies

$$\rho\left(\overline{\mathbf{A}}_2 \left(\overline{\mathbf{A}}_0 - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1\right) < 1. \tag{4.30}$$

As also noted in [73], the stability of the consensus algorithm is dependent not only on the selection of step sizes but also on the estimation exchange topology $\mathbf{A}_0$. This fact limits the usage of consensus algorithm in practice.

For the diffusion LMS based algorithms, the choice of step sizes in the $\overline{\boldsymbol{\mathcal{M}}}$ of the block diagonal matrix $\left(\mathbf{I}-\overline{\boldsymbol{\mathcal{M}}}\right)$ should guarantee that the stability condition (4.30) holds, given the left stochastic matrices $\mathbf{A}_1$ and $\mathbf{A}_2$ and by considering the proposed Kronecker extensions. It was shown in [55, Lemma D.6], that by using the block maximum norm, denoted as $\|.\|_{b,\infty}$, then for the matrix of type $\overline{\mathbf{A}}_2\left(\mathbf{I}-\overline{\boldsymbol{\mathcal{M}}}\right)\overline{\mathbf{A}}_1$, it holds that

$$\begin{aligned}\rho\left(\overline{\mathbf{A}}_2\left(\mathbf{I}-\overline{\boldsymbol{\mathcal{M}}}\right)\overline{\mathbf{A}}_1\right) &\leq \|\overline{\mathbf{A}}_2\left(\mathbf{I}-\overline{\boldsymbol{\mathcal{M}}}\right)\overline{\mathbf{A}}_1\|_{b,\infty}\\ &\leq \|\overline{\mathbf{A}}_2\|_{b,\infty}\|\left(\mathbf{I}-\overline{\boldsymbol{\mathcal{M}}}\right)\|_{b,\infty}\|\overline{\mathbf{A}}_1\|_{b,\infty}\\ &= \|\left(\mathbf{I}-\overline{\boldsymbol{\mathcal{M}}}\right)\|_{b,\infty}\\ &= \rho\left(\mathbf{I}-\overline{\boldsymbol{\mathcal{M}}}\right). \end{aligned} \tag{4.31}$$

Since the matrix $\left(\mathbf{I}-\overline{\boldsymbol{\mathcal{M}}}\right)$ is diagonal we impose to have that

$$\rho\left(\mathbf{I}-\overline{\boldsymbol{\mathcal{M}}}\right)=\max_k|1-\bar{\mu}_k|<1, \tag{4.32}$$

where the $\bar{\mu}_k$, $k=1,\ldots,KM^2$ are the diagonal elements of $\overline{\boldsymbol{\mathcal{M}}}$. Thus based on (4.32), the sufficient condition for the (4.30) to hold (i.e to make the power component in the (4.29) to zero) is to select every $\bar{\mu}_k$ in $\overline{\boldsymbol{\mathcal{M}}}$ so that the diagonal matrix $\left(\mathbf{I}-\overline{\boldsymbol{\mathcal{M}}}\right)$ is stable - i.e all the eigenvalues of $\left(\mathbf{I}-\overline{\boldsymbol{\mathcal{M}}}\right)$ are inside the unit circle. Since $\overline{\boldsymbol{\mathcal{M}}}=\boldsymbol{\mathcal{M}}\otimes\mathbf{I}_{M^2}$, the step size condition (4.32) applies for the diagonal elements μ_k of the $K\times K$ diagonal matrix $\boldsymbol{\mathcal{M}}$ directly. Thus for every $k=1\ldots K$ we should have

$$0<\mu_k<2. \tag{4.33}$$

The CR system designer can choose the step size(s) of the nodes (freely) in the range (4.33), by taking into account the CR system design considerations (which are however out of the scope of this work). Usually the step sizes are taken quite small to get more precise estimates (and thus better detection performance) i.e $\mu_k\ll 2$, but with the cost of longer convergence time of the adaptive estimations. We illustrate the effect of convergence in Section 4.5.

Next we analyse the convergence condition of the second component on the RHS of (4.29). Based on the result of [63, Corollary 5.6.16] the geometric series $S_n=\sum_{i=0}^{n-1}\left[\overline{\mathbf{A}}_2\left(\mathbf{I}-\overline{\boldsymbol{\mathcal{M}}}\right)\overline{\mathbf{A}}_1\right]^i$ is generated by the matrix $\left[\overline{\mathbf{A}}_2\left(\mathbf{I}-\overline{\boldsymbol{\mathcal{M}}}\right)\overline{\mathbf{A}}_1\right]$ and converges if for a matrix norm it holds that $\|\overline{\mathbf{A}}_2\left(\mathbf{I}-\overline{\boldsymbol{\mathcal{M}}}\right)\overline{\mathbf{A}}_1\|<1$. This condition guarantees that $\left[\mathbf{I}-\left[\overline{\mathbf{A}}_2\left(\mathbf{I}-\overline{\boldsymbol{\mathcal{M}}}\right)\overline{\mathbf{A}}_1\right]\right]$ is invertible. Since from (4.31) we have $\rho\left(\overline{\mathbf{A}}_2\left(\mathbf{I}-\overline{\boldsymbol{\mathcal{M}}}\right)\overline{\mathbf{A}}_1\right)\leq\|\left(\mathbf{I}-\overline{\boldsymbol{\mathcal{M}}}\right)\|_{b,\infty}=\rho\left(\mathbf{I}-\overline{\boldsymbol{\mathcal{M}}}\right)$, then the sufficient condition for the convergence of the series is given by (4.32).

Hence when the condition (4.32) is satisfied, then as $n \to \infty$ the geometric series converges to

$$S_n = \left[\mathbf{I} - \left[\overline{\mathbf{A}}_2 \left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1\right]\right]^{-1}. \tag{4.34}$$

Thus by noting the mean of $\hat{\mathbf{p}}(n)$ in steady state and under both hypothesis H_i, $i = 0, 1$ as $\mathrm{E}\left[\hat{\mathbf{p}}(\infty)|H_i\right]$, we have that

$$\begin{aligned}\mathrm{E}\left[\hat{\mathbf{p}}(\infty)|H_i\right] = &\left[\mathbf{I} - \left[\overline{\mathbf{A}}_2 \left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1\right]\right]^{-1} \\ &\times \overline{\mathbf{A}}_2 \overline{\boldsymbol{\mathcal{M}}\mathbf{C}}\, \mathrm{E}\left[\mathbf{d}(n)|H_i\right],\end{aligned} \tag{4.35}$$

where the conditional expectations of observations $\mathrm{E}\left[\mathbf{d}(n)|H_i\right]$ are given in the Section 4.4.2.

The steady state result (4.35) is asymptotically biased. Let us note, that the mean error (or bias) in steady state is given as

$$\overline{\mathrm{E}\left[\hat{\mathbf{p}}(\infty)|H_i\right]} = \|(\mathbf{1}_K \otimes \mathbf{p}^o|H_i) - \mathrm{E}\left[\hat{\mathbf{p}}(\infty)|H_i\right]\|^2, \tag{4.36}$$

for, $i = 0, 1$, where $\mathbf{p}^o|H_i$ denotes the optimal solution (4.15) and $\mathrm{E}\left[\hat{\mathbf{p}}(\infty)|H_1\right]$ follows from (4.35). Since the global solution (4.15) follows the Pareto model, we refer in this chapter to the generic result [33, Th. 3] for characterizing the bias term, such as (4.36). The referred theorem determines that under certain conditions (for example when we have the same step-sizes and a doubly-stochastic matrix $\mathbf{A}$), a lower step-size makes the bias term also lower - i.e the estimates are closer to the optimal solution. Thus in practice, when very low step-size values are used, the bias term can be ignored.

Covariance of estimates

Let us denote the conditional covariance of the estimates under the hypothesis H_i, $i = 0, 1$ as $\mathrm{Cov}\left[\hat{\mathbf{p}}(n+1)|H_i\right]$. Similarly let $\mathrm{Cov}\left[\mathbf{d}(n)|H_i\right]$ denote the conditional covariance of the observations.

Proposition 3. By using recursions (5.11), (4.28), the definition of covariance and by considering the fact that $\hat{\mathbf{p}}(n)|H_i$ is independent of the stacked observation vector $\mathbf{d}(n)|H_i$, it can be shown that the covariance recursion is

$$\begin{aligned}\mathrm{Cov}\left[\hat{\mathbf{p}}(n+1)|H_i\right] = &\overline{\mathbf{A}}_2 \left(\overline{\mathbf{A}}_0 - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1\, \mathrm{Cov}\left[\hat{\mathbf{p}}(n)|H_i\right] \\ &\times \overline{\mathbf{A}}_1^T \left(\overline{\mathbf{A}}_0^T - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_2^T \\ &+ \overline{\mathbf{A}}_2 \overline{\boldsymbol{\mathcal{M}}\mathbf{C}}\, \mathrm{Cov}\left[\mathbf{d}(n)|H_i\right] \overline{\mathbf{C}}^T \overline{\boldsymbol{\mathcal{M}}\mathbf{A}}_2^T.\end{aligned} \tag{4.37}$$

where initial estimate of covariance matrix is noted by $Cov\left[\hat{\mathbf{p}}(0)|H_i\right]$, $i = 0, 1$.

The covariance matrix of the observations, $Cov\left[\mathbf{d}(n)|H_i\right]$, is constant over time n and we provide the values in the Section 4.4.2. Note that (4.37) is in

the form of a discrete time algebraic Lyapunov's equation. Thus the covariance results in steady state (i.e the solution to Lyapunov's equation), can be found by using standard procedures, such as [64, App. E].

Finally we note, that according to the theory of adaptive filtering it is generically known that a smaller step size causes lower co-variance of an adaptive estimate in steady state [25] and this leads to better detection result.

4.4.2. Statistical modeling of adaptive CM estimates

In this section we first find the theoretical moments for the rank one (Hermitian) observations $\mathbf{d}_{R,k}(n)$, which are then transformed to real domain for the spatio-temporal moment recursions of CM estimate $\hat{\mathbf{p}}_k(n)$, described in the previous subsection. Then we describe the statistical modelling of adaptive CM estimates. Thirdly we propose two methods for approximating the adaptive CM estimates by a Wishart distribution.

Moments of rank one observations

First we summarize the generic and known results about the moments of $M \times M$ NCW and CCCW matrices $\hat{\mathbf{R}}_k$, based on [84].

When a $M \times M$ matrix $\hat{\mathbf{R}}_k$ follows a NCW distribution with a DoF parameter $\bar{N}$, a noise population covariance matrix $\bar{\mathbf{\Sigma}}_{v,k}$ and a non-centrality matrix $\bar{\mathbf{\Omega}}_k = \left[\bar{\mathbf{\Sigma}}_{v,k}\right]^{-1} \bar{\boldsymbol{T}}_k$, where $\bar{\boldsymbol{T}}_k = \bar{\mathbf{E}}_k \bar{\mathbf{E}}_k^H$ and where the non-zero column k of $M \times N$ mean matrix $\bar{\mathbf{E}}_k$ is $\mathrm{E}\left[\mathbf{y}_k(n)\right]$, i.e $\hat{\mathbf{R}}_k \sim CW_M(\bar{N}, \bar{\mathbf{\Sigma}}_{v,k}, \bar{\mathbf{\Omega}})$, then the first and vectorized second moments are given as

$$\begin{aligned} \mathrm{E}\left[\hat{\mathbf{R}}_k\right] &= \bar{N}\bar{\mathbf{\Sigma}}_{v,k} + \bar{\boldsymbol{T}}_k, \\ \mathrm{Cov}\left[\mathrm{vec}(\hat{\mathbf{R}}_k)\right] &= (\bar{\mathbf{\Sigma}}_{v,k}^T \otimes \bar{\boldsymbol{T}}_k) + (\bar{\boldsymbol{T}}_k^T \otimes \bar{\mathbf{\Sigma}}_{v,k}) \\ &\quad + \bar{N}(\bar{\mathbf{\Sigma}}_{v,k}^T \otimes \bar{\mathbf{\Sigma}}_{v,k}). \end{aligned} \tag{4.38}$$

As a special case, when the matrix $\hat{\mathbf{R}}_k$ follows a CCCW distribution with a population covariance matrix $\bar{\mathbf{\Sigma}}_k$, i.e $\hat{\mathbf{R}}_k \sim CW_M\left(\bar{N}, \bar{\mathbf{\Sigma}}_k\right)$, then the matrix $\bar{\boldsymbol{T}}_k$ equals zero and we get

$$\begin{aligned} \mathrm{E}\left[\hat{\mathbf{R}}_k\right] &= \bar{N}\bar{\mathbf{\Sigma}}_k, \\ \mathrm{Cov}\left[\mathrm{vec}(\hat{\mathbf{R}}_k)\right] &= \bar{N}(\bar{\mathbf{\Sigma}}_k^T \otimes \bar{\mathbf{\Sigma}}_k). \end{aligned} \tag{4.39}$$

These results in [84] are based on the characteristic functions of the corresponding Wishart distributions and apply for $\bar{N} \geq 1$. We note that $\bar{\mathbf{\Sigma}}_k^T = \bar{\mathbf{\Sigma}}_k^c$ for a Hermitian matrix and then (4.39) also follows from [85] and [86]. Thus the moments of $\mathbf{d}_{R,k}(n)$ can be found by using the results (4.38) and (4.39) with

$\bar{N} = 1$, $\hat{\mathbf{R}}_k = \mathbf{y}_k(n)\mathbf{y}_k(n)^H = \mathbf{D}_{R,k}(n)$, $\bar{\mathbf{\Sigma}}_{v,k} = \sigma_v^2\mathbf{I}_M$, $\bar{\mathbf{\Sigma}}_k = \mathbf{R}_{s,k} + \sigma_v^2\mathbf{I}_{M^2}$, $\mathbf{R}_{s,k} = \mathrm{E}\left[|\alpha_k|^2\right]\mathbf{\Sigma}_s$ and where in NCW case $\bar{\boldsymbol{T}}_k = \mathrm{E}\left[|\alpha_k|^2\right]\mathbf{m}_s\mathbf{m}_s^H$.

Based on the signal model (5.1) and on the AS 1, obviously under H_0 we have that

$$E\left[\mathbf{d}_{R,k}(n)|H_0\right] = \mathrm{vec}\left[\sigma_v^2\mathbf{I}_M\right]. \tag{4.40}$$

Under H_1 the mean at node k is given as

$$E\left[\mathbf{d}_{R,k}(n)|H_1\right] = \mathrm{vec}\left[\mathbf{R}_{s,k} + \sigma_v^2\mathbf{I}_M\right]. \tag{4.41}$$

Given the network size K, the stacked $KM^2 \times 1$ vector $E\left[\mathbf{d}_R(n)|H_i\right]$ over $k = 1 \ldots K$ and for $i = 0, 1$ can be formed based on the results (4.40) and (4.41) respectively.

Due to the AS 1, the k,k ($k \in K$) diagonal block of the $KM^2 \times KM^2$ network-wise covariance matrix $\mathrm{Cov}\left[\mathbf{d}_R(n)|H_0\right]$ is given as

$$\mathrm{Cov}\left[\mathbf{d}_{R,k}(n)|H_0\right] = \sigma_v^4\mathbf{I}_{M^2}, \tag{4.42}$$

while the off-diagonal blocks are zeros, since the observation noise is not correlated over the CR nodes.

The $KM^2 \times KM^2$ network-wise $\mathrm{Cov}\left[\mathbf{d}_R(n)|H_1\right]$ is constructed as follows. Firstly, when $\mathbf{m}_s = 0$ and $\mathbf{\Sigma}_s \neq 0$ (i.e Case 2 type) it can be verified, that the $k,j \in K$ blocks of the $\mathrm{Cov}\left[\mathbf{d}_R(n)|H_1\right]$ are given as

$$\mathrm{Cov}\left[\mathbf{d}_{R(k,j)}(n)|H_1\right] = \begin{cases} \left[\left(\bar{\mathbf{\Sigma}}_k\right)^c \otimes \bar{\mathbf{\Sigma}}_k\right], & k = j \\ \left[(\mathbf{R}_{s,k,j})^c \otimes \mathbf{R}_{s,k,j}\right], & k \neq j \end{cases} \tag{4.43}$$

where $\bar{\mathbf{\Sigma}}_k = \mathrm{E}\left[|\alpha_k|^2\right]\mathbf{\Sigma}_s + \sigma_v^2\mathbf{I}_{M^2}$ and where for $k \neq j$ $\mathbf{R}_{s,k,j} = \mathrm{E}\left[\mathbf{y}_k(n)\mathbf{y}_j(n)^H\right] = \mathrm{E}\left[\alpha_k\alpha_j^c\right]\mathbf{\Sigma}_s$, since due to (AS 1) in this case the observations $\mathbf{y}_k(n)$, $\mathbf{y}_j(n)$ are zero mean Gaussian vectors with independent noise processes. Secondly, when $\mathbf{m}_s \neq 0$ and $\mathbf{\Sigma}_s = 0$ (i.e Case 1 type) and $k = j$, then the k,k on-diagonal block of $\mathrm{Cov}\left[\mathbf{d}_R(n)|H_1\right]$ is given as

$$\begin{aligned} \mathrm{Cov}\left[\mathbf{d}_{R(k,k)}(n)|H_1\right] = &\left[\left(\sigma_v^2\mathbf{I}_{M^2}\right)^T \otimes \sigma_v^2\mathbf{I}_{M^2}\right] \\ &+ \left[\left(\mathrm{E}\left[|\alpha_k|^2\right]\mathbf{m}_s\mathbf{m}_s^H\right)^T \otimes \sigma_v^2\mathbf{I}_{M^2}\right] \\ &+ \left[\left(\sigma_v^2\mathbf{I}_M^2\right)^T \otimes \left(\mathrm{E}\left[|\alpha_k|^2\right]\mathbf{m}_s\mathbf{m}_s^H\right)\right]. \end{aligned} \tag{4.44}$$

When $k \neq j$, then due to (AS 1) the observation noise is not correlated over the CR nodes and it can be verified, that for the k,j off-diagonal blocks, $\mathrm{Cov}\left[\mathbf{d}_{R(k,j)}(n)|H_1\right] = 0$. Given the network size K, the network-wise covariance matrix $\mathrm{Cov}\left[\mathbf{d}_R(n)|H_1\right]$ can be composed by using (4.43) and (4.44) respectively.

Finally the moments of the real observations (as the inputs for the moment recursions of the estimates $\hat{\mathbf{p}}_k(n)$, provided in the previous subsection) can be given for $i = 0, 1$ as

$$E\left[\mathbf{d}(n)|H_i\right] = \left[\mathbf{T}^{-1} \otimes \mathbf{I}_{M^2}\right] E\left[\mathbf{d}_R(n)|H_i\right], \tag{4.45}$$

and

$$\begin{aligned}\mathrm{Cov}\left[\mathbf{d}(n)|H_i\right] = &\left[\mathbf{T}^{-1} \otimes \mathbf{I}_{M^2}\right] \\ &\times \mathrm{Cov}\left[\mathbf{d}_R(n)|H_i\right] \left[(\mathbf{T}^H)^{-1} \otimes \mathbf{I}_{M^2}\right].\end{aligned} \tag{4.46}$$

Distributions of the adaptive estimates

To study the detection performance of the proposed distributed, adaptive LE detector, we need to specify the conditional distributions for the detection test statistics - the LE of

$$\hat{\mathbf{R}}_k(n) = \mathrm{vec}^{-1}\left[\mathbf{T}\hat{\mathbf{p}}_k(n)\right] \tag{4.47}$$

under both detection hypothesis. As summarized in (4.7), when the estimate $\hat{\mathbf{R}}_k(n)$ is obtained by using the linear, equal weighting based method (4.6) in a non-distributed and non-cooperative manner, then according to the definition of Wishart matrices [69, Chapter 2], $\mathbf{R}_k(n)$ follows a Wishart distribution. Based on the literature, several results exist for the distributions of the LE of Wishart distributed matrices under both detection hypotheses.

The non-asymptotic cumulative distribution function (CDF) model of the LE of a NCW distributed CM matrix is more complicated for practical and numerical evaluation, compared to the corresponding model of a CCCW distribution. Thus often a NCW distribution is approximated by a CCCW distribution, where the non-centrality part of the NCW distribution is incorporated into the population covariance matrix parameter of the CCCW distribution [43, 79, 87].

When the estimate $\hat{\mathbf{R}}_k(n)$ is obtained by using the exponential type of averaging (as used in LMS type of algorithms), then due to different weights at every $n \in N$, it can be seen, that a sum of non-equally weighted Wishart matrices over N is not Wishart distributed [79, Theorem 3.3.1, 3.5.2]. Based on (5.11) it is easy to verify, that the adaptive CM estimate $\hat{\mathbf{R}}_k(n)$ is an average over non-equally weighted vectorized observation matrices. At iteration step n, at node k the elements of the vectors $\hat{\mathbf{p}}_k(n)$ are weighted equally and fused without changing or mixing the order of the elements of $\hat{\mathbf{p}}_k(n)$. The Hermitian property of the estimated CMs is not affected. Thus we need to seek generic CC(C)W approximations for studying the conditional CDFs of LE of adaptively estimated CMs.

Total and General variance approximations

We propose the usage of two methods for approximating the adaptive CM estimates $\hat{\mathbf{R}}_k(n)$ (4.47) by conditional approximate CC(C)W distributions. Thus

based on (4.39) and we assume that

$$\hat{\mathbf{R}}_k(n)|H_i \sim CW_M\left(\bar{N}_i, \bar{\mathbf{\Sigma}}_{k,i}\right), \tag{4.48}$$

for $i = 0, 1$, and where $\sim$ denotes an approximate distribution, $\bar{N}_i$ is the approximating DoF and $\bar{\mathbf{\Sigma}}_{k,i}$ is the approximating population covariance matrix parameter of the corresponding CC(C)W distribution. As shown at next, the values for $\bar{N}_i$ and $\bar{\mathbf{\Sigma}}_{k,i}$ are found by matching the mean and trace or determinant of moments of $\hat{\mathbf{R}}_k(n)|H_i$ with the corresponding moments of the devectorized adaptive estimate $\text{vec}^{-1}\left[\mathbf{T}\hat{\mathbf{p}}_k(n)\right]$ under both detection hypothesis.

Proposition 4. For the approximation (4.48), $\bar{\mathbf{\Sigma}}_{k,i}$ is found as

$$\bar{\mathbf{\Sigma}}_{k,i} = \frac{1}{\bar{N}_i}\,\text{E}\left[\hat{\mathbf{R}}_k(n)|H_i\right] \tag{4.49}$$

and $\bar{N}_i$ can be found using the Total Variance (TV) or General Variance (GV) method, respectively, as

$$\bar{N}_{TV,i} = \left\lceil \frac{\text{Tr}\left[\text{E}\left[\hat{\mathbf{R}}_k(n)|H_i\right]^c \otimes \text{E}\left[\hat{\mathbf{R}}_k(n)|H_i\right]\right]}{\text{Tr}\left[\mathbf{T}\,\text{Cov}\left[\mathbf{p}_k(n)|H_i\right]\mathbf{T}^H\right]} \right\rceil \tag{4.50}$$

or

$$\bar{N}_{GV,i} = \left\lceil \sqrt[M^2]{\frac{\det\left[\text{E}\left[\hat{\mathbf{R}}_k(n)|H_i\right]^c \otimes \text{E}\left[\hat{\mathbf{R}}_k(n)|H_i\right]\right]}{\det\left[\mathbf{T}\,\text{Cov}\left[\mathbf{p}_k(n)|H_i\right]\mathbf{T}^H\right]}} \right\rceil, \tag{4.51}$$

where $\text{E}\left[\hat{\mathbf{R}}_k(n)|H_i\right] = \text{vec}^{-1}\left[\mathbf{T}\,\text{E}\left[\mathbf{p}_k(n)|H_i\right]\right]$ for $i = 0, 1$.

These results are found as follows. Firstly we insert the $\bar{\mathbf{\Sigma}}_{k,i} = \text{E}\left[\hat{\mathbf{R}}_k(n)|H_i\right]/\bar{N}_i$ from the first equation of (4.39) into the RHS of the second equation of (4.39) and we have that

$$\text{Cov}\left[\text{vec}(\hat{\mathbf{R}}_k(n)|H_i)\right] = \frac{1}{\bar{N}_i}\Big[\,\text{E}\left[\hat{\mathbf{R}}_k(n)|H_i\right]^c \otimes \text{E}\left[\hat{\mathbf{R}}_k(n)|H_i\right]\Big]. \tag{4.52}$$

Based on (4.28) or (4.35) and the first equation of (4.39), we equalize the means of matrices $\hat{\mathbf{R}}_k(n)|H_i$ and $\text{vec}^{-1}[\mathbf{T}\mathbf{p}_k(n)|H_i]$ and get (4.49). For the DoF, $\bar{N}_i$, to use in the approximation, we adapt the idea proposed in [77, 78] and equalize the total variances (i.e the traces of corresponding covariance matrices) of the matrices $\hat{\mathbf{R}}_k(n)|H_i$ and $\text{vec}^{-1}[\mathbf{T}\mathbf{p}_k(n)|H_i]$. Thus based on (4.52) we require that $\text{Tr}\left[\text{Cov}\left[\text{vec}(\hat{\mathbf{R}}_k(n)|H_i)\right]\right] = \text{Tr}\left[\mathbf{T}\,\text{Cov}\left[\mathbf{p}_k(n)|H_i\right]\mathbf{T}^H\right]$ for $i = 0, 1$. By solving

for $\bar{N}_i$ we have the total variance (TV) type of DoF approximation as given by (4.50). An alternative for finding the approximation for $\bar{N}_i$ is to equalize the determinants of both matrices [79]. Thus based on (4.52), we require that $\det\left[\mathrm{Cov}\left[\mathrm{vec}(\hat{\mathbf{R}}_k(n)|H_i)\right]\right] = \det\left[\mathbf{T}\,\mathrm{Cov}\left[\mathbf{p}_k(n)|H_i\right]\mathbf{T}^H\right]$. Similarly, by solving for $\bar{N}_i$ the general variance (GV) type of DoF approximation is given by (4.51).

Obviously the total variance method takes into account only the variances of the elements of the corresponding matrices, while the general variance method includes also the covariances of the elements of the corresponding matrices into the approximation of parameter $\bar{N}_i$. AS observed, by using the proposed TV or GV procedures under hypothesis H_1, a NCW matrix is approximated by the CCCW distribution, by matching the moments of NCW matrix into the CCCW model. This is a desired effect, as we explain in the next section. Based on these results we can proceed with the detection performance analysis.

It can be verified, that under H_0 the DoF value approximations (4.50) and (4.51) are, via the moment analysis of the adaptive estimate $\mathbf{p}_k(n)$, dependant on the step size parameter μ_k and on the full network topology. Since the same noise power value σ_v^2 is present both in the mean and covariance formulas of the adaptive estimate $\mathbf{p}_k(n)$, then a change in the $\sigma_{v,k}^2$ value does not affect the DoF value under H_0. However under H_1 both the DoF approximations are additionally dependant on the noise power value $\sigma_{v,k}^2$. This effect is illustrated in Section 4.5.

Since under H_0, the DoF parameter does not affect the threshold calculation, then a robust detector can also be applied in Algorithm 4, by changing the detection module accordingly. We give an example with the MME detector in Section 4.5. On the other hand, since under H_1 the DoF parameter is affected by the uncertainty in the noise power value, then this effect possibly makes the formula of the theoretical detection performance of a robust detector inaccurate as well, but that robust detector can still be used.

4.4.3. Detection Performance Analysis

In this section we provide formulas for studying the probability of false alarm (P_{FA}) and probability of detection (P_D) of the proposed, adaptive LE detector. For this, we need to evaluate the conditional CDFs of the LE of adaptive CM estimate $\hat{\mathbf{R}}_k(n)$ (4.48) under both detection hypotheses and under the assumption that $\hat{\mathbf{R}}_k(n)$ is approximated by a CC(C)W distribution as proposed in Section 4.4.2. The resulting detection performance of LE detector is dependent on the performance of the underlying adaptive, distributed CM estimation. Let the eigenvalues of $\bar{\mathbf{\Sigma}}_{k,i}$ in (4.48) be denoted in non-increasing order as $\nu_{1,i} \geq \nu_{2,i} \geq \cdots \geq \nu_{M,i}$.

LE under H_0 Hypothesis

Based on [43,88], the $\hat{\mathbf{R}}_k(n)|H_0$ (4.48) is assumed to follow the CCW distribution and the eigenvalues of $\bar{\mathbf{\Sigma}}_{k,0}$ are $\nu_{1,0} = \cdots = \nu_{M,0} = \sigma_v^2/\bar{N}_0$. The $P_{FA,e}$, based on the non-asymptotic CDF model of the $\hat{\mathbf{R}}_k(n)|H_0$, is given by

$$\begin{aligned} F_{H_0,e}(x) &= |\det(\hat{\mathbf{A}})| \\ P_{FA,e}(\gamma_{LE,k,e}) &= 1 - F_{H_0,e}(\gamma_{LE,k,e}) \end{aligned} \tag{4.53}$$

where the $M \times M$ matrix $\hat{\mathbf{A}}_{i,j} = \binom{\bar{N}_0 - j - i - 1}{i-1} \gamma_R(\bar{N}_0 + i - j, \frac{x}{\nu_{1,0}})$, for $i, j = 1, \ldots, M$ and where $\gamma_R(k,u) = \frac{1}{\Gamma(k)} \int_0^u x^{k-1} e^{-x} dx$ is the regularized incomplete Gamma function. The (ideal) detection threshold $\gamma_{LE,k,e}$, based on the non-asymptotic model is expressed as

$$\gamma_{LE,k,e} = F_{H_0,e}^{-1}(1 - P_{FA,e}) \tag{4.54}$$

and can be evaluated in terms of a numerical inversion of the exact CDF formula at a desired $P_{FA,e}$ value. An asymptotic CDF based on the Gaussian approximation of Tracy-Widom distribution is proposed in [43]. When $\bar{N}_0 \to \infty$, $M \to \infty$ and $M/\bar{N}_0 \in (0,1)$, the approximate CDF under H_0 can be given as

$$\begin{aligned} F_{H_0,g}(x) &= \Phi\left(\frac{x - \mathrm{E}[\lambda_1]|H_0}{\sqrt{\mathrm{Var}[\lambda_1]|H_0}}\right), \\ \mathrm{E}[\lambda_1]|H_0 &= \nu_1 \left(a_{LE} + (b_{LE}(-1.7711))\right), \\ \mathrm{Var}[\lambda_1]|H_0 &= (\nu_1 b_{LE})^2 (0.8132), \\ a_{LE} &= (\sqrt{M} + \sqrt{\bar{N}_0})^2, \\ b_{LE} &= (\sqrt{M} + \sqrt{\bar{N}_0})(\frac{1}{M} + \frac{1}{\bar{N}_0})^{1/3}. \end{aligned} \tag{4.55}$$

This leads to the $P_{FA,g}$ formula

$$P_{FA,g}(\gamma_{LE,k,e}) = Q\left(\frac{\gamma_{LE,k,g} - \mathrm{E}[\lambda_1]|H_0}{\sqrt{Var[\lambda_1]|H_0}}\right), \tag{4.56}$$

where Q is the complementary distribution function of the standard Gaussian and to the threshold formula is

$$\gamma_{LE,k,g} = \mathrm{E}[\lambda_1]|H_0 + \sqrt{\mathrm{Var}[\lambda_1]|H_0} Q^{-1}(P_{FA,g}). \tag{4.57}$$

As seen in Section 4.4, the calculation of the threshold of the LE detector at node k and time index n requires knowledge of the moments of adaptive CM estimates (present at the reference node k) under hypothesis H_0 i.e $\hat{\mathbf{R}}_k(n)|H_0$. Thus based on the values of step sizes, the noise power, the desired P_{FA}, the provided moment recursions and the distribution parameter approximations

models for the $\hat{\mathbf{R}}_k(n)|H_0$ in Section 4.4 can be applied, to evaluate the detection threshold at node k and at time instant n. As seen, CR nodes need to know the noise power value(s) to evaluate the moments of $\hat{\mathbf{R}}_k(n)|H_0$. In practice every node k needs to calculate its own threshold by using the provided procedure. While the threshold at node k can be updated iteratively based on the exact moments of $\hat{\mathbf{R}}_k(n)|H_0$, the steady state moments are preferred in practice.

LE under H_1 Hypothesis

Next we obtain a common model for the non-asymptotic CDF$|H_1$ of the LE of adaptively estimated CM matrix. As explained in Section 4.4.2, we approximate the NCW matrix by a CCCW matrix by matching the moments of the matrices. In Section 4.5 we show this approximation works quite well.

Thus we assume the $\hat{\mathbf{R}}_k(n)|H_1$ is distributed by a CCCW distribution. The CDF of the LE of a CCCW matrix $\hat{\mathbf{R}}_k(n)|H_1$ is given by [89] as follows

$$F_{H_1,e}(x) = K_{CC}\left|\left\{\nu_i^{\bar{N}_1-M+j}\bar{\Gamma}\left(\bar{N}_1-M+j,\frac{x}{\nu_{i,1}}\right)\right\}_{i,j}\right|,$$
$$K_{CC} = \left[\prod_{i=1}^{M}(\bar{N}_1-i)!\prod_{j=1}^{M}(M-i)!\right]^{-1}\prod_{k=1}^{M}(k-1)! \tag{4.58}$$

for $i,j=1,\ldots,M$ and where $\bar{\Gamma}(k,u)=\int_0^u x^{k-1}e^{-x}dx$ is the lower incomplete gamma function [90, 8.350].

This result follows from [91, Eq. 1] by integrating the joint PDF of ordered eigenvalues of a CCCW matrix, by using [91, Corollary 2]. It should be emphasized, that as explained in [91, Chapter II. B], when some of the eigenvalues of $\bar{\mathbf{\Sigma}}_{k,1}$ are coincident, then [92, Lemma. 2] needs to be used to study the limit [91, Eq. 3].

However we note, that the direct numerical evaluation of (4.58) is complicated and (4.58) needs to be simplified due to the possibly large $\bar{N}$ values and large arguments of $\bar{\Gamma}(k,u)$. In case of the matrix dimension is $M=2$, the eigenvalues of the population covariance matrix are naturally not coincident under H_1 (i.e $\nu_{1,1}>\nu_{2,1}$). It can be shown, that when $M=2$, the following simplified version of (4.58) can be used to evaluate the CDF numerically

$$
\begin{aligned}
F_{H_1,e}(x) &= \frac{\bar{D}}{\left(\frac{1}{\bar{a}} - \frac{1}{\bar{b}}\right)\bar{a}\bar{b}}, \\
\bar{a} &= \nu_{1,1}\bar{N}_1, \\
\bar{b} &= \nu_{2,1}\bar{N}_1, \\
\bar{D} &= \bar{b}\gamma_R(\bar{N}_1 - 1, \frac{x}{\nu_{1,1}})\gamma_R(\bar{N}_1, \frac{x}{\nu_{2,1}}) \\
&\quad - \bar{a}\gamma_R(\bar{N}_1 - 1, \frac{x}{\nu_{2,1}})\gamma_R(\bar{N}_1, \frac{x}{\nu_{1,1}}),
\end{aligned} \tag{4.59}
$$

where $\gamma_R(k, u)$ is the regularized incomplete gamma function.

Finally the probability of detection of the LE of a CCW matrix under H_1 using the exact CDF model is

$$
P_{D,e}(\gamma_{LE,k,e}) = 1 - F_{H_1,e}(\gamma_{LE,k,e}). \tag{4.60}
$$

As earlier, we observe that the channel gain values and the noise power value are required to complete the chain of approximations for the theoretical detection performance analysis.

4.5. Simulation results

In this numerical simulation Section we investigate the detection performance of the ATC type of distributed, adaptive LE detection algorithm. We describe the exact signal model, used in the simulations and then investigate the probability of false alarm (P_{FA}) and the probability of detection P_D of the proposed algorithms.

4.5.1. Simulation model

The channel gains in the following simulations are assumed to be constant over N and M dimension and are sampled for the CR node $k \in K$ as $\alpha_k \sim CN(0, 1)$. We assume there is only one PU signal present in the CR network i.e $\mathbf{s}(n) = s(n)\mathbf{1}$, where $s(n) \sim CN(0, P_s)$ and $P_s = 1$. Using the same examples as in [82], we use for Case 1: $\mathbf{m}_s = s\mathbf{1}$, $\mathbf{\Sigma}_s = 0$, where s is a complex signal realization, and for Case 2: $\mathbf{m}_s = 0$ and $\mathbf{\Sigma}_s = P_s\mathbf{1}\mathbf{1}^H$. Obviously rank($\mathbf{1}\mathbf{1}^H$)=1. Also in (4.43) and (4.44) we have $\mathbf{R}_{s,k} = |\alpha_k|^2 P_s \mathbf{1}\mathbf{1}^H$, $\mathbf{R}_{s,k,j} = \alpha_k \alpha_j^c P_s \mathbf{1}\mathbf{1}^H$ and $\bar{\boldsymbol{T}}_k = |\alpha_k|^2 P_s \mathbf{1}\mathbf{1}^H$.

When the CR nodes do not cooperate, the local correlation matrix $\mathbf{R}_k$ (5.2) is given as follows

$$
\mathbf{R}_k = \left[|\alpha_k|^2 \frac{\mathrm{E}\left[\|\mathbf{s}\|^2\right]}{N}\right] \mathbf{1}\mathbf{1}^H + \sigma_{v,k}^2 \mathbf{I}_M. \tag{4.61}
$$

For Case 1 we assume $|s|^2 = P_s$, where s is a complex signal realization. Then we get $\mathrm{E}\left[\|\mathbf{s}\|^2\right] = NP_s$ for both Case 1 and Case 2. The first moment of the rank one input for these two cases is given as

$$E\left[\mathbf{d}_{R,k}(n)|H_1\right] = \mathrm{vec}\left[|\alpha_k|^2 P_s \mathbf{1}\mathbf{1}^H + \sigma_v^2 \mathbf{I}_M\right]. \tag{4.62}$$

Network topology selection

To improve the communication link failure resistance in the CR network, but to keep the need for processing the data from neighbor nodes minimal, we propose to select the diffusion topology of the estimates in the CR network, i.e the $\mathbf{A}$ matrix, as a combination of the local ($\mathbf{A}, \mathbf{C} = \mathbf{I}$) and ring-around ($\mathbf{A} = \mathbf{A}_{\text{ring}}^T, \mathbf{C} = \mathbf{I}$) topologies [3, Eq. 11]. Thus at time instant n, at every node k two $M^2 \times 1$ estimates: the local estimate $\hat{\mathbf{p}}_k(n)$ and the estimate $\hat{\mathbf{p}}_{(k-1)\mathrm{mod}K}(n)$ from node $(k-1)\mathrm{mod}K$ are fused together using equal, constant weight 0.5. Therefore, in the subsequent sections we assume, that $\mathbf{C} = \mathbf{I}$, the matrix $\mathbf{A}$ is in such case doubly stochastic (i.e we have additionally $\mathbf{A}\mathbf{1} = \mathbf{1}$) and all the conditions for selecting elements $a_{l,k}$ and $c_{l,k}$, as listed in the Section 5.2.2, are satisfied.

For example when $K = 3$ and by keeping the same notation and conditions for the elements of matrix $\mathbf{A}$, the ring around and diffusion topologies are given as follows

$$\mathbf{A}_{\text{ring}}^T = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad \mathbf{A}_{\text{diff}}^T = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix}. \tag{4.63}$$

A schematic view of the proposed diffusion and incremental steps for the ATC type of algorithm with $K = 2$ is illustrated in Fig. 4.1.

In the next sections we select the dimension of the estimated matrix as $M = 2$ and use (4.11) and (4.59). The step size of the algorithms in all the simulations is selected to be $\mu = 0.001$ for all the nodes, unless stated otherwise. Given the step-size value, all the nodes in the network receive $N = 7000$ $[2 \times 1]$ vector-samples to get converged adaptive CM estimates at the last iteration/sample. These CM estimates are used in the simulations to obtain the LE observations. A system designer can choose other values for μ and N (depending on the system requirements).

In Fig. 4.2 we illustrate the change of the LE of adaptively estimated CM with respect to the threshold (5.14). We set the noise power to one. After the initialization, the algorithm first tracks and then converges to the steady state level of LE under the H_1 hypothesis. At time instant 7001 the PU signal switches off, the algorithm adapts and convergences to the H_0 level of the LE value.

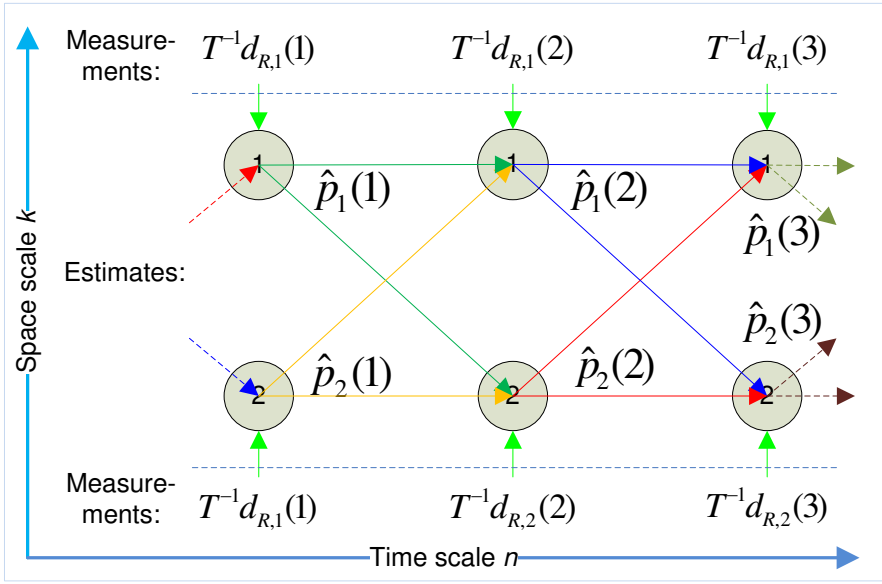

Figure 4.1 Proposed diffusion method

DoF values under noise uncertainty

In Fig. 4.3 we illustrate of the effect of the noise power uncertainty to the TV based DoF approximation under Case 2 and H_1. The network sizes are $K = 1, 3, 10, 30$ and the results are taken from the last node in the network. The horizontal axis represents the (network averaged) SNR, which is changed by scaling the noise power value σ_v^2. We use the noise perturbation model [45, Eq. 8] and denote the $\bar{\alpha}$ as the noise uncertainty factor. Two noise value perturbations are added to the non-perturbed case 0 dB ($\bar{\alpha} = 1$): -1 dB ($\bar{\alpha} = 0.796$) and 2 dB ($\bar{\alpha} = 1.585$). As we see, in case of σ_v^2 is inaccurate, then the TV approximated DoF$|H_1$ values are shifted in accordance to the value of $\bar{\alpha}$. For GV based DoF$|H_1$ values, the results are very similar. Also as we already mentioned in Section 4.4.2 that changes, and thus also the pertubations, in $\sigma_{v,k}^2$ do not affect the TV and GV based DoF$|H_0$ approximations. Thus we skip these to latter simulations here.

Next we investigate the performance of the proposed LE algorithms by studying the P_{FA} in case of PU signal is missing and the P_D, when the PU signal is present. Both the P_{FA} and P_D based on adaptive CM estimates are estimated using the Monte Carlo (MC) method [40]. To have an equal comparison between the node sets in one plot, we take all the reference results from the last node in the network. Obviously, based on the global estimation model (4.9), when we

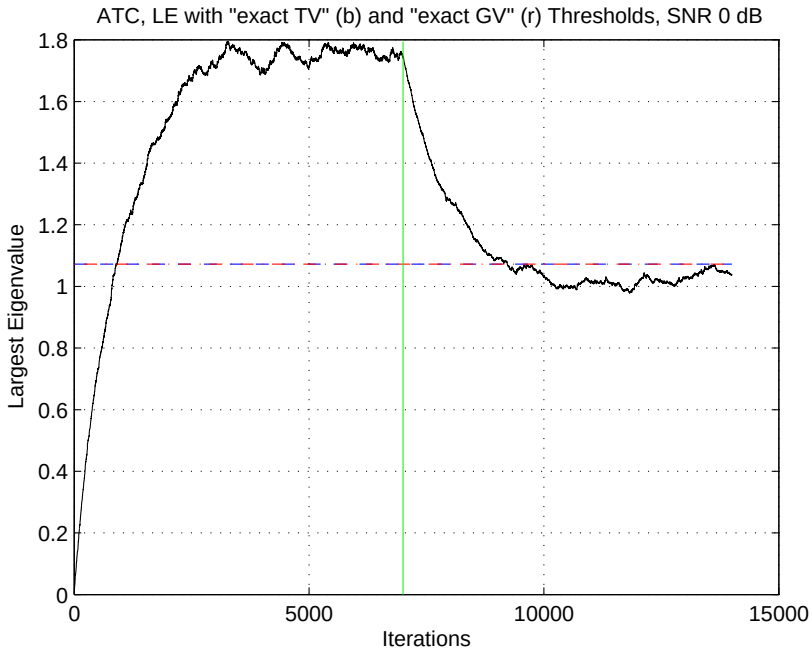

Figure 4.2 LE Adaptive Principle

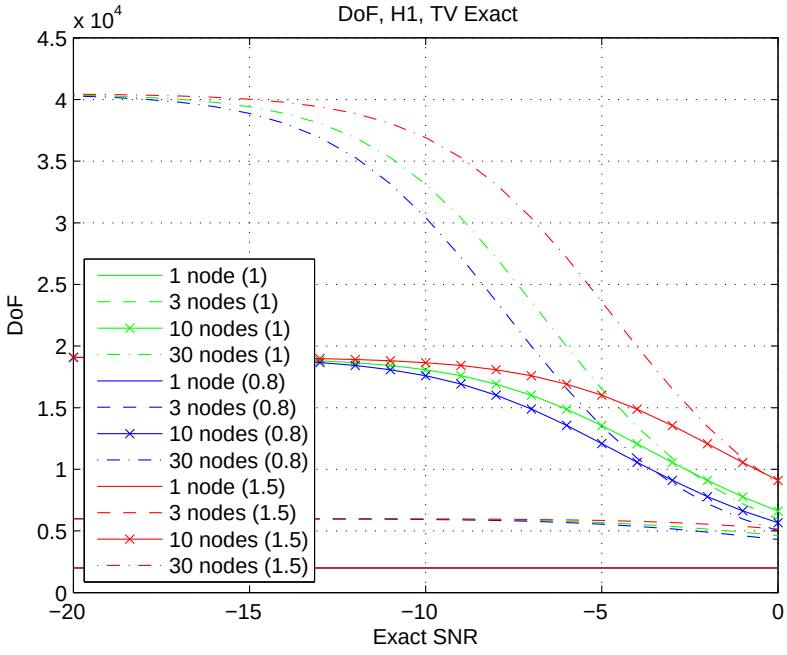

Figure 4.3 ATC, DoF $|H_1$ values with perturbations 0 dB, -1 dB and 2 dB

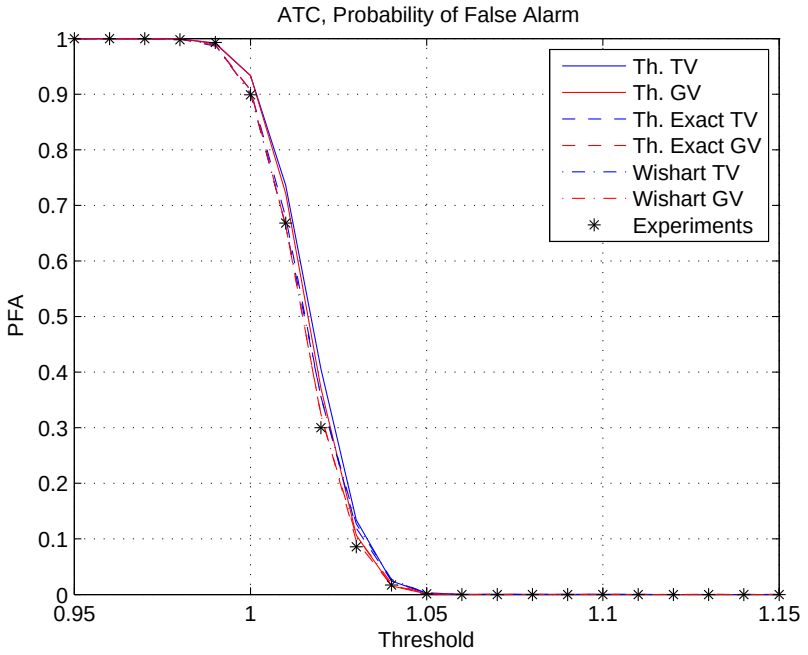

Figure 4.4 P_{FA} versus threshold using ATC

have more nodes in the network, then the CM estimates at every node have been better averaged over the channel gain values of the nodes in the CR network.

4.5.2. Probability of false alarm

We start the investigation of the proposed algorithms by studying the P_{FA}. Under the detection hypothesis H_0 we assume $\sigma_v^2 = 1$. We select 21 threshold points in the range of σ_v^2 and determine the LE realizations of adaptive CMs estimates. Then we estimate the P_{FA} over 1000 experiments at every threshold point. The estimated P_{FA} is denoted as **Experiments** in the Fig. 4.4. We compare the estimated P_{FA} with the theoretical P_{FA} models when the Total variance (TV) or the General variance (GV) method are used for determining an approximately equivalent CCW matrix. The results using (4.55) are denoted as **Th. TV** and **Th. GV** respectively. Similarly the results using (5.13) are denoted as **Th. Exact TV** and **Th. Exact GV** respectively. Finally, based on the moments of the adaptive CM estimates, we generate the approximate CCW matrices (by using Cholesky decomposition method), and study the P_{FA} performance based on those matrices in addition (denoted as **Wishart TV** and **Wishart GV** respectively). The P_{FA} versus threshold results are given in Fig. 4.4 for the ATC algorithm. We note that the performance of the TV and GV methods are almost equal and the TV/GV

approximations are sufficient for studying the P_{FA} of the adaptive CM estimates. We see a good match between the estimated P_{FA} and the theoretical P_{FA} models are achieved. The Gaussian approximate P_{FA} model (which is easier to use in numerical analyses compared to non-asymptotic P_{FA} model), follows the estimated P_{FA} results quite well and can therefore be used to characterize the P_{FA} of the adaptive estimates. Therefore by knowing the noise power value, the theoretical Gaussian approximate P_{FA} model can be also used for deriving the detection threshold, when we fix a desired P_{FA} value.

4.5.3. Probability of detection

Next we investigate the probability of detection under different noise power conditions using the proposed distributed and adaptive LE detection algorithms with signal models Case 1 or 2. In Case 1 we select one complex PU signal realization, while in Case 2 we set $P_s = 1$ for all the simulations. We note, that the performance of the moment estimation framework of adaptively estimated CMs is well illustrated by the P_D versus SNR analysis. In the P_D/SNR simulations, the change in the (network averaged) SNR is achieved by changing the noise power value σ_v^2. In the comparison of algorithms we use the same individual channel gains of the nodes in all the simulations performed under hypothesis H_1. We set the desired $P_{FA} = 10^{-2}$ for all the nodes. The thresholds of the LE detectors at nodes $k \in K$ are calculated using (5.14) with both the TV and GV approximation. Simulations studies showed, that the performance of the Gaussian CDF$|H_1$ based threshold (4.57) is almost equal to the performance of the non-asymptotic threshold (5.14) and thus not shown in this work.

In the following simulations we compare the performance of 4 different network sizes: $K = 1, 3, 10, 30$ nodes, while the comparable results are taken from the last node in the set. The P_D is estimated over 1000 experiments on a given noise power value. We compare the MC estimated P_D results (based on the adaptively estimated CMs and denoted as **Ad. Exp.** in the figures) with the non-asymptotic theoretical model (4.60) (denoted as **Theory**) and with the P_D results based on approximately equivalent CCW matrices (denoted as **W. Exp.**). These latter matrices are generated based on the respective moments under H_1. For the signal model Case 1, the P_D/SNR results are given in Fig. 4.5 and Fig. 4.6 when the TV approximation is used and in Fig. 4.7 and Fig. 4.8 when the GV approximation is used, respectively for the CTA and ATC algorithm. Similarly for the signal model Case 2, the P_D versus SNR results are given in Fig. 4.9 and Fig. 4.10 when TV approximation is used and in Fig. 4.11 and Fig. 4.12 when GV approximation is used, respectively for the CTA and ATC algorithm.

For comparison, the MC estimated P_D/SNR performance of the MME detector [45] under Case 2 is shown additionally in Fig. 4.10 and Fig. 4.12 (where denoted as **MME. Exp.**). The threshold of the MME detector is calculated by using [45, Eq. 29], where in our case $L = 1$ and $N_s = \bar{N}_{TV,0}$ or $N_s = \bar{N}_{GV,0}$. Based on the discussion in Section 4.4.2, it is obvious, that since the noise

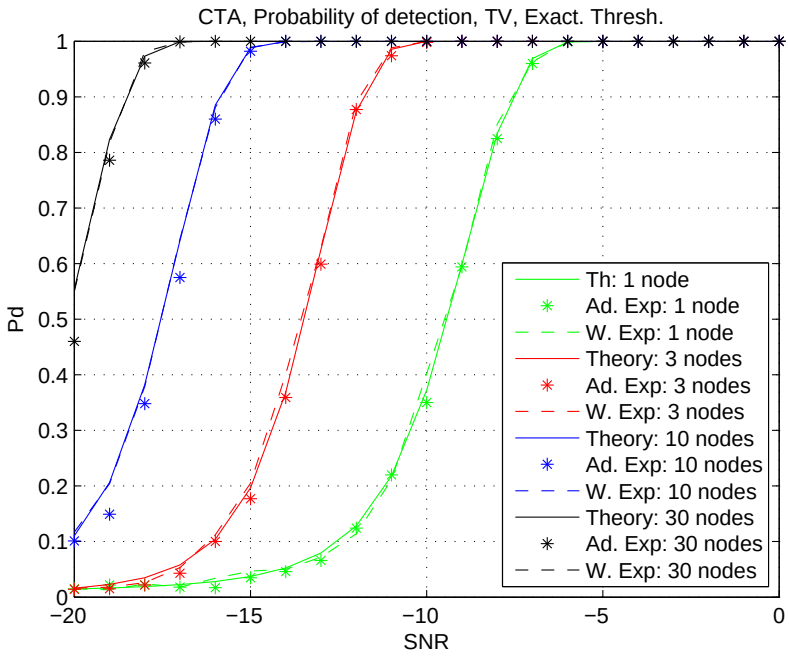

Figure 4.5 Probability of detection, CTA, TV, Case 1

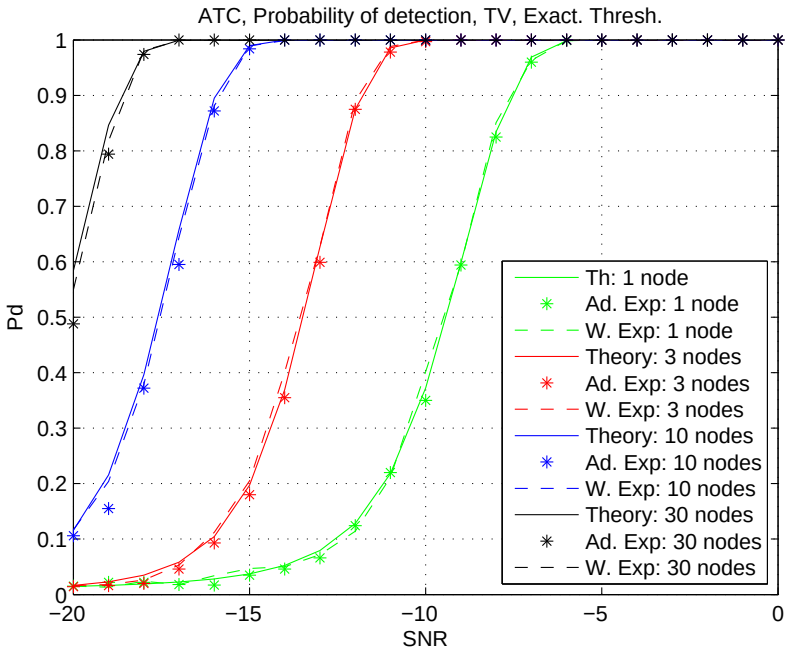

Figure 4.6 Probability of detection, ATC, TV, Case 1

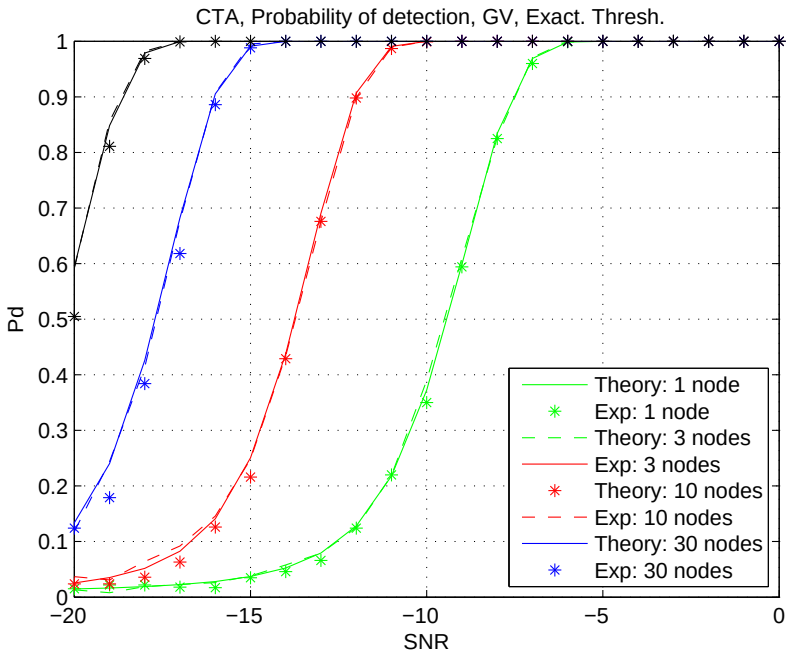

Figure 4.7 Probability of detection, CTA, GV, Case 1

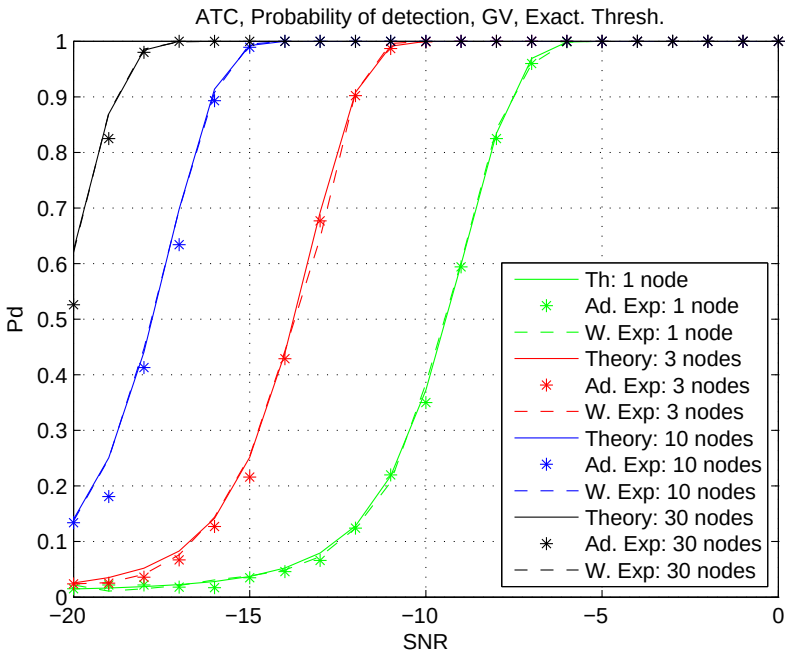

Figure 4.8 Probability of detection, ATC, GV, Case 1

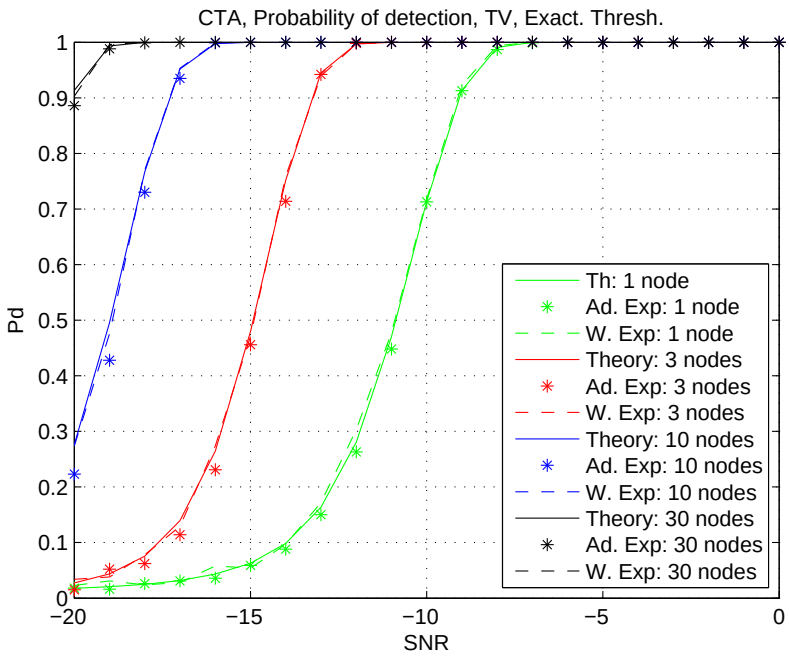

Figure 4.9 Probability of detection, CTA, TV, Case 2

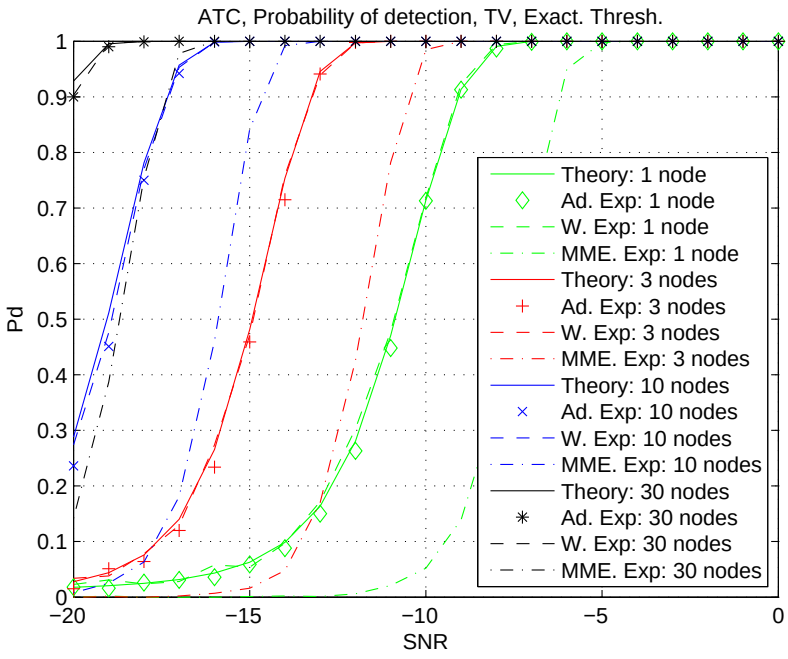

Figure 4.10 Probability of detection, ATC, TV, Case 2

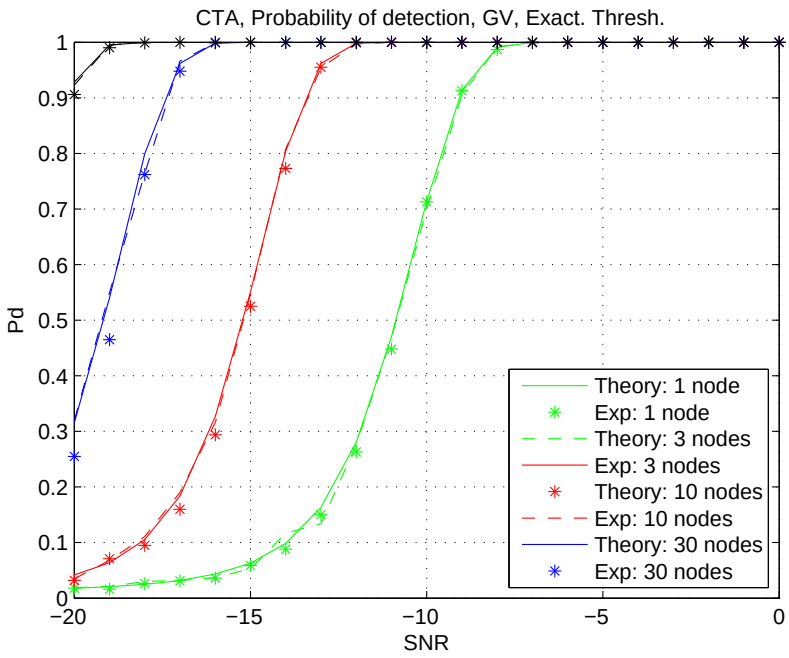

Figure 4.11 Probability of detection, CTA, GV, Case 2

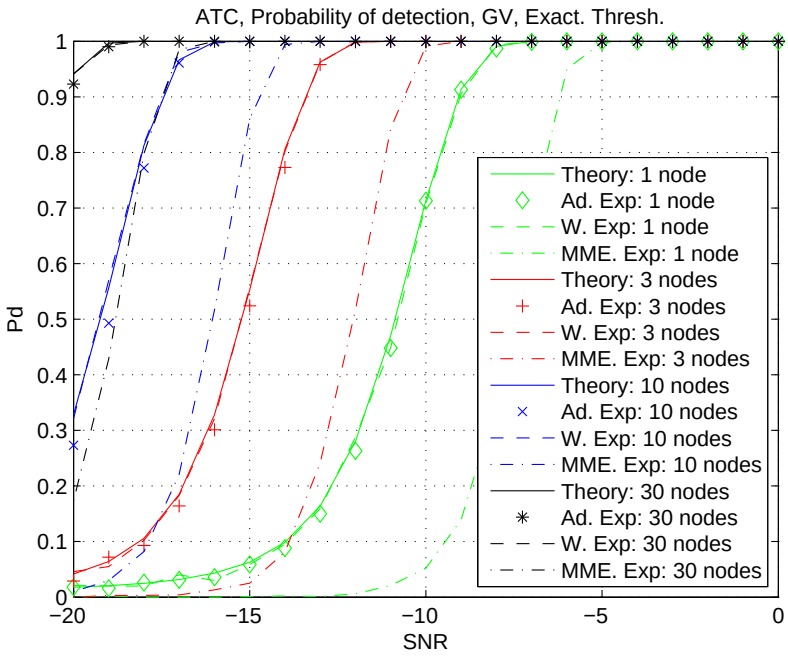

Figure 4.12 Probability of detection, ATC, GV, Case 2

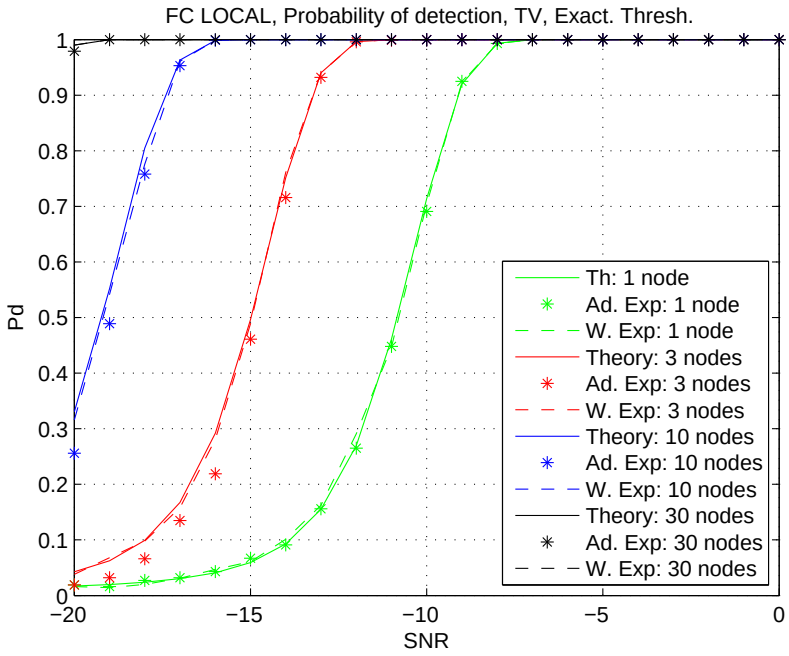

Figure 4.13 Probability of detection, FC, TV, Case 2

value perturbations are not affecting the threshold of the MME detector, then the corresponding MC based P_D/SNR performance is not affected as well. In Fig. 4.13 we show a comparison of P_D/SNR performance of the LE detector by using the FC based algorithm in Table 1, TV approximation based exact threshold, and Case 2 model only. In such case the observations of every CR nodes are available for all the CR nodes in the CR network and the CR networks can (independently and adaptively) estimate the CM. In Fig. 4.14 we provide similar comparison of the P_D/SNR performance of the LE detection scheme in Fig. 4.14, by using the consensus algorithm ([73]), TV approximation based exact threshold and Case 2 model only and we select $\mathbf{A}_0 = \mathbf{A}_{\text{diff}}^T$.

We note that the non-asymptotic theoretical P_D model describes the detection performance of adaptively estimated CMs well, also in the low SNR regime. The performance of TV and GV methods is almost equal and thus the TV approximation is computationally less demanding method for the numerical performance analysis of the LE detector. In terms of the P_D versus SNR values, the Case 1 signal model is well approximated by the signal model of Case 2 (CCCW), via the TV and GV based mean and DoF parameter matching.

We observe that as the number of nodes in the network increases, the point where the P_D starts to decrease from one and moves to the left. In case of one node in the CR network (or in case of the non-cooperating nodes) the P_D is

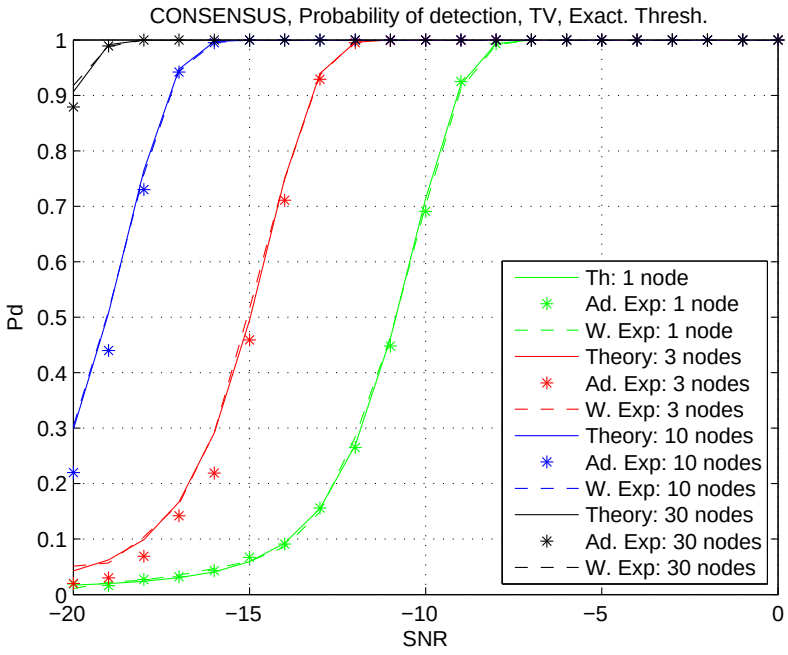

Figure 4.14 Probability of detection, Consensus, TV, Case 2

highly dependent on the channel constant of that node. As the number of nodes increases, more channel gain realizations are involved in the network-averaged CM estimation process and thus the P_D results are more equalized over the nodes.

It can be seen, that the LE detector performs better than the MME in terms of perfect detection ($P_D = 1$) in the low SNR region and in case of non-perturbed noise power values.

The detection performance of LE detector, when the FC based diffusion LMS algorithm is used, is slightly better, compared with the case of ATC type of LMS. The difference is however not significant. So in the ATC case, where only two exchanges of estimates are allowed for a CR node at time instant n, we can save energy in terms of processing less data at a node k. Also in case of ATC we are not limited to the specific network topology. The detection performance of LE detector, when the consensus algorithm is used, is very similar to the case of the ATC algorithm. As argued in Section 4.4, the usage of ATC type algorithm is less limited by the estimate exchange topology, while this is not the case with the consensus algorithm.

It is clear that the detection performance of the MME detector is not affected by noise power uncertainty also when we use the Diffusion LMS based CM estimation scheme.

Additionally we note that in [3] we showed with scalar estimates ($M = 1$) in Case 2, that when there are more nodes in the network, then the ATC performs better, compared to the CTA type of algorithm. While ATC fuses more data than CTA [27], the difference of detection performance with CTA is rather small and thus we also skip these comparisons in this work. We also observed in [1–3] that for $K > 30$, P_D does not improve significantly any more.

For illustrating the closeness of the detection results of different CR nodes, we use the theoretical results and plot the P_D/SNR performances of all the CR nodes in the network of size K, in Fig. 4.15, by using the ATC algorithm, the TV based exact threshold and the Case 2 model. The four groups of P_D/SNR results from right to the left in Fig. 4.15 correspond to the network of sizes $K = 1, 3, 10, 30$ accordingly, i.e. the leftmost group shows the P_D/SNR results of all the 30 nodes in the CR network. It can be seen that the detection performances of the CR nodes in the CR network are quite close to each other. In practice we are more concerned about the point where the P_D starts to decrease from 1. In case of 30 nodes in the network, the deviation slightly increases, but is still sufficiently close.

We observe that the non-asymptotic CDF models, the TV/GV approximations, and the CCCW based approximation of NCW type of CMs are usable for studying the performance of the LE detection of adaptively estimated CMs - for determining the threshold and for evaluating the theoretical P_D of the LE detector. When the nodes cooperate in estimating the network-wise CM (while nodes are able to communicate directly only with limited subset of neighbor nodes) then the resulting LE detection performance is equalized and stabilized

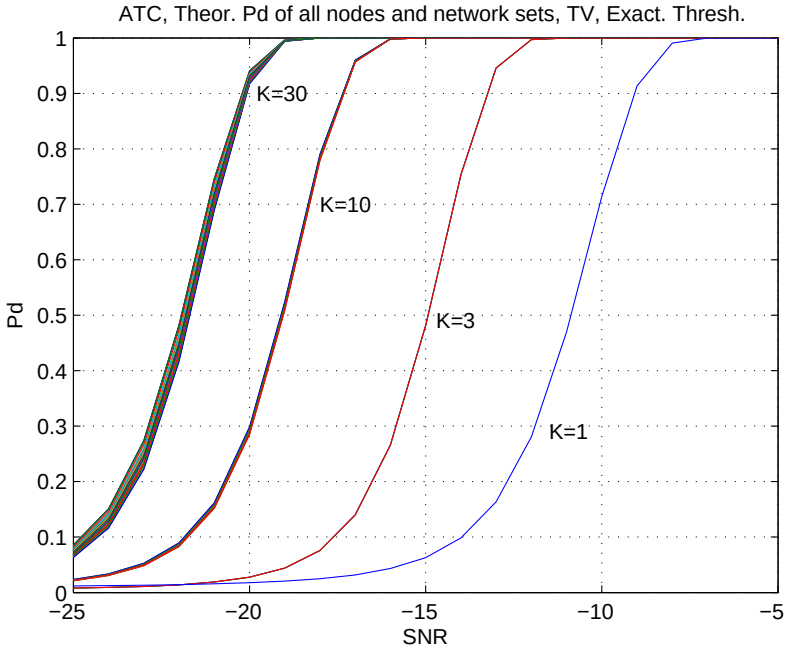

Figure 4.15 Probability of detection, ATC, TV, Case 2; All

over the individual CR nodes. We note that other distributed eigenvalue based detection schemes can be studied in similar manner by using the proposed framework in this chapter.

4.6. Conclusion

In this chapter we studied distributed and adaptive diffusion LMS based LE detection algorithms, which are applicable in CR networks for detecting the presence of a PU signal. We proposed a network-wise CM estimation model and derived ATC and CTA type of diffusion based LE detection algorithms. We proposed a general framework for analyzing the performance of the diffusion LMS based LE detection schemes. In our simulation study we demonstrated that the proposed framework and the approximations used for studying the detection performance of the proposed distributed and adaptive LE detection schemes provided matching results between the theory and simulations. The proposed algorithms are able to learn the statistical changes in the LE in real-time.

5. DISTRIBUTED ADAPTIVE LARGEST EIGENVALUE DETECTION WITH SNR WEIGHTED OBSERVATIONS

In this chapter we explore a distributed spectrum sensing approach that exploits the largest eigenvalue of CMs that are adaptively estimated; local SNR values are used to assign weights to the input observations. More specifically, CR nodes exchange also observations with a subset of neighbouring nodes and combine the neighbouring observations based on the locally estimated SNR values. We propose a mean vector estimation mechanism that is based on combine and adapt least mean square diffusion and that does not require a FC. We analyse the resulting detection performance and verify the theoretical findings through simulations.

5.1. Background

Radio spectrum is a scare resource. It has been found that even if the licensed radio spectrum becomes nominally more crowded, there is significant underutilization of the resource [10]. Cognitive radio (CR) technology has been proposed to provide an opportunistic access for cognitive radio systems to the licensed spectrum areas [13, 14]. In CR context, it is highly desirable to detect the PU and identify free spectrum opportunities as rapidly as possible and create no disturbances for the (licenced) PU communication. Distributed, adaptive network learning methods can be used to track the changes in the statistical information of the observations received by the CR nodes in real time to enhance the detection of PU signals.

Three main types of classic detection schemes for spectrum sensing in CR networks have been considered in the literature: the matched filter detector (MFD) [40], the energy detector (ED) [40], [93], and the cyclostationary detector [66]. A second large group of detectors for CR networks are based on the properties of an estimated signal correlation matrix [43], [44], [45]. The largest eigenvalue (LE) method [43] uses *a priori* knowledge about the additive noise power to determine the detection threshold.

Distributed, adaptive estimation and detection research area has gained an increasing interest over the last decade and many algorithms have been proposed in the literature. For example, least mean square (LMS) based estimation schemes

were studied in [26], [27], [56], where good properties of these algorithms were shown. On the other hand optimal, distributed MFD, based on diffusion type LMS estimation schemes were studied in [28]. In [1], [2] and [3] we proposed and analysed diffusion LMS based energy detectors in a CR network and in [4] and [38] we studied diffusion LMS based Largest Eigenvalue (LE) detectors.

Various SNR estimation methods have been proposed and studied in the literature, for example [94–96]. In the PU signal detection context, SNR estimation methods can provide additional information about the quality of the input observations and that knowledge can be used for enhancing the distributed estimation process and thus the overall detection results of the main spectrum sensing method.

In the above-mentioned papers, a constant and equal weighting method for the data fusion between the nodes has been used. Specifically, the matrix $\mathbf{A}$ and $\mathbf{C}$ are taken to be constant and with equal weights over the time instance. In the literature several methods have been proposed for weighting the communication between the nodes in an adaptive manner or for optimizing the combination weights in accordance to a selected optimization criteria. A list of such methods has been given for example in [30, Chapter 14]. It has been shown in the literature that properly optimized weights can improve the properties of the underlying estimation process – for instance providing better error measures of the estimates. However, for approaches such as for instance non−adaptive relative variance rule as seen in [73], would (in detection context) require knowledge of the channel gains under detection hypothesis H_1 and thus cannot be directly adopted to the detection context. Estimation of these gains would at least require that we know H_1 to be true for a given period of time. The possible implementation of the adaptive combination rule [73] in detection context requires an analysis of several additional aspects, which can potentially affect the total detection performance of the main estimation algorithm.

In this chapter we explore a distributed spectrum sensing approach that exploits the largest eigenvalue of CMs that are adaptively estimated. No FC unit (as a potential single point of failure) is used. Compared to the solutions in our papers [4] and [38], in this chapter, we study additionally the local observation exchange and combination strategy, which is based on the local SNR estimates and is adapted to the context of binary hypothesis testing. We show, that when the PU signal is present and when the local SNR estimates are available, then the network-wise PU signal detection performance can be slightly improved as compared to the standard case with no observation exchange, studied in our paper [4].

We assume that the CM of the PU signal is of low rank. On the other hand, the CR network operates without prior information about the PU signal's waveform and the secondary nodes' channel gains. We assume that while the PU signal may be absent for a time period, the radio channel properties under the detection hypothesis H_1 do not change over the time of interest and that long time statistics are usable in enhancing the overall PU signal detection performance.

For example, the classical TV White space model [13, Chapt. 1.2.4] could be considered, where the on/off working patterns of the PUs (i.e. TV transmitters) are quite static, the power of the PUs is constant, and where the CR nodes have fixed positions in the nearby space. In the distributed CR network, we assume that every node acts as an independent detector in terms of detection decision making based on the available CM estimates.

5.2. Distributed Adaptive Largest Eigenvalue Detection

5.2.1. Signal model and assumptions

Let us follow the same signal model as in [4], where

$$\begin{aligned} H_0 &: \mathbf{y}_k(n) = \mathbf{v}_k(n), \\ H_1 &: \mathbf{y}_k(n) = \alpha_k \mathbf{s}(n) + \mathbf{v}_k(n) \end{aligned} \tag{5.1}$$

and the detection hypothesis is denoted by H_i, $i = 0, 1$, the CR node index by $k = 1, 2, ..., K$, and the sample discrete time index by $n = 1, 2, ...N$. The noise $\mathbf{v}_k(n)$ and channel gains α_k at node k are assumed to be statistically independent. The PU signal follows $\mathbf{s}(n) \sim CN_M(0, \mathbf{\Sigma}_s)$. The noise follows $\mathbf{v}_k(n) \sim CN_M\left(0, \sigma_v^2 \mathbf{I}_M\right)$ and is assumed to be independently and identically distributed, uncorrelated in time and space. The theoretical $M \times M$ dimensional CM $\mathbf{R}_k$ at every node is given as

$$\mathbf{R}_k = \mathrm{E}\left[\mathbf{y}_k(n)\mathbf{y}_k(n)^H\right] = |\alpha_k|^2 \mathbf{R}_{s,k} + \sigma_{v,k}^2 \mathbf{I}_M. \tag{5.2}$$

Firstly, we assume, that the noise power σ_v^2 is assumed to be known a priori and to be identical at every node. Secondly, that $\mathbf{R}_{s,k}$ has a low rank. Thirdly, that when H_1 is present, the PU signal power and the channel constants do not change over the time of interest (of slow fading channel). Fourthly, internal communication channels between the CR nodes are assumed to be error free and the communication capacity is not limited.

For summarizing the LE detection method, let the eigenvalues of the estimate $\hat{\mathbf{R}}_k(n)$ of CM $\mathbf{R}_k$ be denoted in non-increasing order as $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M$. Every node k detects the presence of a PU signal by determining the largest eigenvalue λ_1 of $\hat{\mathbf{R}}_k(n)$ as follows

$$\lambda_1\left[\hat{\mathbf{R}}_k(n)\right] \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_{LE}. \tag{5.3}$$

Here the threshold γ_{LE} is given by (5.14).

For improving the detection performance of the LE detector, we introduce a second parameter - local SNR of the received observations at node k. The theoretical form is given as

$$\mathrm{SNR}_k = \frac{\mathrm{Tr}\left[|\alpha_k|^2 \mathbf{R}_{s,k}\right]}{\mathrm{Tr}\left[\sigma_v^2 \mathbf{I}_M\right]}. \tag{5.4}$$

In the SNR estimation phase, each node k estimates locally the SNR of the received signal, denoted as $\widehat{\text{SNR}}_k$, based on the locally estimated sample covariance matrix (SCM). The SCM $\hat{\mathbf{R}}_k$ at node $k \in K$ and over the N_{SNR} samples can be estimated linearly as

$$\hat{\mathbf{R}}_k(n) = \frac{1}{N_{SNR}} \sum_{k=n-N_{SNR}}^{n} \left[\mathbf{y}_k(n) \mathbf{y}_k(n)^H \right]. \tag{5.5}$$

Local SNR can be estimated separately from the (cooperative) CM estimation phase for the LE detection (5.3) and it can be considered as a "goodness measure" of the received observations in CR network. Similarly to the standard Maximum ratio combining (MRC) method, i.e. [97], local SNR estimates can be used for weighting up the observations with strong PU signal in the neighbourhood of CR nodes (and for weighting down the available neighbouring observations flows, where PU signal is more strongly attenuated). Since the LE detector requires knowledge of the σ_v^2, then according to (5.1) and the assumptions, the estimation of the local SNR reduces to the estimation of the (attenuated) PU signal power at node k. Thus in this chapter we propose the usage of the following simple SNR estimation method

$$\widehat{\text{SNR}}_k(n) = \frac{\text{Tr}\left[\hat{\mathbf{R}}_k(n)\right]}{M\sigma_{v,k}^2} - 1. \tag{5.6}$$

For avoiding negative and zero SNR values in the upcoming calculations, when the PU signal is very weak or no signal subspace is present, we assign that if $\widehat{\text{SNR}}_k(n) < 0.0001 \Rightarrow \widehat{\text{SNR}}_k(n) = 0.0001$.

Compared, for example, to the more sophisticated minimum description length (MDL) and akaike information criterion (AIC) criterion based SNR estimation methods [94], [95], [96], (5.6) is computationally simpler, and based on experiments, requires significantly less samples N_{SNR} in SCM to detect the PU signal. Once the local SNR estimates are obtained, these corresponding results could be deterministically used in the distributed adaptive CM estimation phase for the LE detection.

5.2.2. Adaptive, Distributed LE detection with SNR weighted observations

The first part of this section summarizes the adaptive, distributed CTA type of Diffusion LMS based CM estimation algorithm, which was derived in [4, Chapter II]. The second part focuses on the usage of local SNR estimates for observation weighting and exchange in the mentioned Diffusion estimation strategy.

It was shown in [4] that when CR nodes cooperate in the estimation of $\hat{\mathbf{R}}_k(n)$ in (5.3) by means of the system internal communication links, then the detection performance (of the PU signal(s)) at every node k can be enhanced. In this

chapter we continuously assume that 1) K nodes in the CR network can rely only on the subset of global information that is available to them and 2) that the CR network topology is assumed to be fixed over the sensing time and strongly connected.

Let us denote $\mathcal{N}_k$ as the neighbourhood group of node $k \in K$, i.e. $\mathcal{N}_k$ and μ_k be a positive step size of node k. We introduce the $K \times K$ matrix $\mathbf{C}_{\text{SNR}}$ with non-negative elements as follows

$$c_{l,k} = 0 \quad \text{if} \quad l \notin \mathcal{N}_k, \quad \mathbf{C}_{\text{SNR}}\mathbf{1} = \mathbf{1}. \tag{5.7}$$

For simplicity we have dropped the time index n in $\mathbf{C}_{\text{SNR}}$. Similarly, let us have constant $K \times K$ matrix $\mathbf{A}_{\text{diff}}^T$ as

$$a_{l,k} = 0 \quad \text{if} \quad l \notin \mathcal{N}_k, \quad \mathbf{1}^T\mathbf{A}_{\text{diff}}^T = \mathbf{1}^T. \tag{5.8}$$

The coefficients $c_{l,k}$ and $a_{l,k}$ define respectively how the measurements $\mathbf{d}_l(n)$ and estimates $\hat{\mathbf{p}}_l(n)$ are available for the node k in the CR network (unidirectionally). Let μ_k be positive step size of node k.

Similarly, as in [4], for keeping the dimension of the estimated vector minimal in the adaptive recursions, we decompose the observation at node k at time instant n as $\mathbf{d}_{R,k}(n) = \text{vec}\left[\mathbf{y}_k(n)\mathbf{y}_k(n)^H\right] = \mathbf{T}\mathbf{d}_k(n)$, where $M^2 \times M^2$ dimensional constant, the complex invertible matrix $\mathbf{T}$ is given in [5, Eq. 11]) and $M^2 \times 1$ vector $\mathbf{d}_k(n)$ is real valued. Thus, $\mathbf{d}_k(n) = \mathbf{T}^{-1}\,\text{vec}\left[\mathbf{y}_k(n)\mathbf{y}_k(n)^H\right]$. By denoting the $M^2 \times 1$ dimensional estimate of the real valued $\text{E}\left[\mathbf{d}_k(n)\right]$ as $\hat{\mathbf{p}}_k(n)$, then with the help of $\mathbf{T}$, we can re-define $\hat{\mathbf{R}}_k(n) = \text{vec}^{-1}\left[\mathbf{T}\mathbf{p}_k(n)\right]$.

We skip the derivation details of the adaptive, fully distributed CTA diffusion type of LMS based LE detection algorithm and show the result in Algorithm 6.

One of the disadvantages of (5.5) is that it is not adaptive and requires significant amout of memory since all the N_{SNR} samples have to be present for estimating $\hat{\mathbf{R}}_k(n)$. Since (5.6) does not use N_{SNR} directly for SNR estimation, we propose to also use local exponential averaging based adaptive estimation method for calculating the CM for local SNR estimation. In light of the previously showed decompositions, let us denote the real time adaptive local CM estimate (for local SNR estimaton) at node k at time instant n as $\hat{\mathbf{w}}_k(n)$, while $\hat{\mathbf{R}}_k(n) = \text{vec}^{-1}\left[\mathbf{T}\mathbf{w}_k(n)\right]$. Let the step sizes for all the nodes k be equal and denoted as μ_{SNR}. The step-size μ_{SNR} needs to be selected so that converged estimates are achieved after the expected number of samples N_{SNR} (which determines also the accuracy of the estimates), while the algorithm can continue running after N_{SNR} have been processed.

Each node k communicates the signal observations and $\widehat{\text{SNR}}_k$ value in real time to the neighbouring nodes, which are connected to the node k. For defining the network connections in $\mathbf{C}_{\text{SNR}}$ (5.7) we introduce a non-negative matrix $\mathbf{C}_{\text{O}}^T$ with the element of $c_{O,l,k}$, which are formed as follows

$$c_{O,l,k} = \begin{cases} 1, & l \in \mathcal{N}_k \\ 0, & \text{otherwise,} \end{cases} \tag{5.9}$$

to define if node l is connected to node k. The combination weights $\mathbf{C}_{\text{SNR}}$ at each node $k \in K$ are formed as follows

$$c_{l,k} = \frac{c_{O,l,k}\widehat{\text{SNR}}_k}{\sum_{k=1}^{K}\left[c_{O,l,k}\widehat{\text{SNR}}_k\right]}, \tag{5.10}$$

We note that the rows of $\mathbf{C}_{\text{SNR}}$ are normalized to 1, which is useful in the PU signal detection context. In case of detection hypothesis H_1 is present and sufficiently accurate SNR estimates are available, then the observations with higher SNRs are slightly weighted up in the observation exchange in the neighbourhood of the nodes and observations. In such a way, the observations with higher channel gains are more dominating in the adaptive estimation algorithm and this property can enhance the overall detection performance of the LE detector in Algorithm 6. On the other hand, when H_0 or in case a weak PU signal is present, the SNR estimates are set equal to 0.0001. If the local SNR estimates are not available at every node, then it is easy to verify that equal weights $c_{l,k}$ (5.10) are obtained in the neighbourhood of the CR nodes. In such a way, the existing threshold determination solution (for the LE detector) under H_0, as presented in [4], can still be used. After $\mathbf{C}_{\text{SNR}}$ is formed, then this matrix can be used in the adaptive CTA type of LMS based LE detection method, proposed in [4], summarized in Algorithm 6. The local SNR estimation steps are given in Algorithm 5.

Algorithm 5 Local SNR Estimation

1. Local CM estimation:
Start with $\hat{\mathbf{w}}_k(0) = \mathbf{w}(0)$ for every k .
for every time instant $n \geq 1$ **do**
  for every node $k = 1, ..., K$ **do**
    $\hat{\mathbf{w}}_k(n+1) = \hat{\mathbf{w}}_k(n) + \mu_{SNR}\left[\mathbf{d}_k(n) - \hat{\mathbf{w}}_k(n)\right]$
  end for
end for
2. SNR estimation:
for every node $k = 1, ..., K$ **do**
  $\hat{\mathbf{R}}_k(n+1) = \text{vec}^{-1}\left[\mathbf{T}\mathbf{w}_k(n+1)\right] \Rightarrow \widehat{\text{SNR}}_k$ (5.6)
  if $\widehat{\text{SNR}}_k \leq 0.0001$ **set** $\widehat{\text{SNR}}_k = 0.0001$
  $c_{l,k} = \frac{c_{O,l,k}\widehat{\text{SNR}}_k}{\sum_{k=1}^{K}\left[c_{O,l,k}\widehat{\text{SNR}}_k\right]}$ (5.10)
end for

The algorithms have been presented separately and in principle the system designer can study these two algorithms in different combinations (also in separate time scales), depending on the system requirements and noise properties.

Algorithm 6 CTA type of LE Detection with SNR weighted observations [4]

Start with $\hat{\mathbf{p}}_k(0) = \mathbf{p}(0)$ for every k .
Given non-negative real coefficients $a_{l,k}, c_{l,k}$
for every time instant $n \geq 1$ **do**
  for every node $k = 1, ..., K$ **do**
    1. CTA type of CM estimation recursions:
    $\hat{\boldsymbol{\psi}}_k(n) = \sum_{l \in N_k} a_{l,k} \hat{\mathbf{p}}_l(n)$.
    $\hat{\mathbf{p}}_k(n+1) = \hat{\boldsymbol{\psi}}_k(n)$
      $+\mu_k \sum_{l \in N_k} c_{l,k} \left[\mathbf{d}_l(n) - \hat{\boldsymbol{\psi}}_k(n) \right]$
    2. LE detection decision:
    $H_0 : \lambda_1 \left[\text{vec}^{-1} \left[\mathbf{T} \hat{\mathbf{p}}_k(n+1) \right] \right] < \gamma_k$ or
    $H_1 : \lambda_1 \left[\text{vec}^{-1} \left[\mathbf{T} \hat{\mathbf{p}}_k(n+1) \right] \right] > \gamma_k$.
    (Refer to (5.14) for selecting the γ_k).
  end for
end for

5.3. Theoretical Detection Performance

In this section we summarize the steps needed to set the detection threshold of the LE detector with SNR weighted observation exchange and to evaluate the theoretical detection performance (for verifying the Monte-Carlo based simulation results). The performance analysis of the proposed algorithm can in general be performed based on the same framework, that was developed in [4] and [5]. Thus, we skip the details and shortly summarize the main steps. For the theoretical performance analysis of the LE detector, we assume that the channel gains are known. The analysis is divided into three parts.

Firstly, the moments of the adaptive CM estimates of Algorithm 6 are studied. As shown in [4], $K \times K$ matrices $\mathbf{A}_1 = \mathbf{A}_{\text{diff}}^T$, $\mathbf{A}_2$, $\mathbf{C}_{\text{SNR}}$ and $\boldsymbol{\mathcal{M}} = \text{diag}\{\mu_1, \ldots, \mu_K\}$ are in CR network extended to $KM^2 \times KM^2$ matrices for the CTA type of algorithm as follows: $\overline{\mathbf{A}}_1 = \mathbf{A}_{\text{diff}}^T \otimes \mathbf{I}_{M^2}$, $\overline{\mathbf{A}}_2 = \mathbf{I}_K \otimes \mathbf{I}_{M^2}$, $\overline{\mathbf{C}}(n) = \mathbf{C}_{\text{SNR}} \otimes \mathbf{I}_{M^2}$ and $\overline{\boldsymbol{\mathcal{M}}} = \boldsymbol{\mathcal{M}} \otimes \mathbf{I}_{M^2}$. Let us note that with respect to Chapter 5.4 and for simplifying the analysis, matrix $\mathbf{C}_{\text{SNR}}$ (5.7) is considered as constant and deterministic. Thus, the CTA based estimation recursion can be given as

$$\begin{aligned} \hat{\mathbf{p}}(n+1)|H_i = \overline{\mathbf{A}}_2 \left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}} \right) \overline{\mathbf{A}}_1 \hat{\mathbf{p}}(n)|H_i \\ + \overline{\mathbf{A}}_2 \overline{\boldsymbol{\mathcal{M}}} \overline{\mathbf{C}} \mathbf{d}(n)|H_i. \end{aligned} \tag{5.11}$$

For determining the threshold of the LE detector (under H_0) and for studying the theoretical detection performance (under H_1), the moments $\text{E}\left[\hat{\mathbf{p}}(n+1)|H_i\right]$ and $\text{Cov}\left[\hat{\mathbf{p}}(n+1)|H_i\right]$ for $i = 0, 1$ need to be determined [4].

Secondly, the adaptively estimated $\mathbf{T}\hat{\mathbf{p}}(n)|H_i$ is approximated by Complex Central (Correlated) Wishart distributions (CC(C)W) as

$$\mathbf{T}\hat{\mathbf{p}}(n)|H_i = \hat{\mathbf{R}}_k(n)|H_i \sim CW_M\left(\bar{N}_{TV,i}, \bar{\mathbf{\Sigma}}_{k,i}\right), \tag{5.12}$$

for being able to find the conditional CDFs of LE of adaptively estimated CMs. The values for $\bar{N}_{TV,i}$ (for approximating the DoF parameter) and $\bar{\mathbf{\Sigma}}_{k,i}$ (for approximating the population covariance matrix parameter) can be found based on the Total Variance (TV) approximation method, shown in [4, Chapt. 3].

Thirdly, since under H_0, $\hat{\mathbf{R}}_k(n)|H_0$ is assumed to follow the CCW distribution, then $P_{FA,e}$ of the largest eigenvalue of $\hat{\mathbf{R}}_k(n)|H_0$ is given as

$$P_{FA,e}(\gamma_{LE,e}) = 1 - F_{H_0,e}(\gamma_{LE,e}) \tag{5.13}$$

where the CDF$|H_0$, denoted as $F_{H_0,e}(x)$, is given in [4, Chapt. 3].

The detection threshold $\gamma_{LE,e}$, based on the non-asymptotic model is given as

$$\gamma_{LE,e} = F^{-1}_{H_0,e}(1 - P_{FA,e}) \tag{5.14}$$

and numerical inversion method can be used to determine the exact CDF formula at a desired $P_{FA,e}$ value .

Under H_1, the $\hat{\mathbf{R}}_k(n)|H_1$ is assumed to be distributed by a CCCW distribution. The P_D formula, based on the non-asymptotic CDF$|H_1$ of the LE of a CCCW matrix $\hat{\mathbf{R}}_k(n)|H_1$ is given as

$$P_{D,e}(\gamma_{LE,e}) = 1 - F_{H_1,e}(\gamma_{LE,e}), \tag{5.15}$$

where the CDF$|H_1$, denoted as $F_{H_1,e}(x)$, is also given in [4, Chapt. 3].

5.4. Simulation results

In the numerical simulation section we investigate the probability of detection P_D of the CTA type of distributed, adaptive LE detection algorithm together with the SNR weighted observations. The algorithm performance is presented in terms of the P_D versus (network averaged) SNR analysis; in the SNR, the noise power value changes. In this example we consider a rather ideal use-case, which on the other hand illustrates well the achieved LE detection performance gain $-$ we assume H_1 is known to be present during the SNR estimation and thus good local SNR estimates have been obtained over longer time. The channel gains are assumed to be constant over the simulation time and are sampled as $\alpha_k \sim CN(0,1)$. We assume to have one PU signal $\mathbf{s}(n) = s(n)\mathbf{1}$, $s(n) \sim CN(0,1)$ and $\mathbf{\Sigma}_s = \mathbf{1}\mathbf{1}^H$.

For the local SNR estimation step, we select $N_{SNR} = 50000$ and $\mu_{SNR} = 0.00015$ for getting an estimate also in the highest noise power region of interest, as seen in Fig. 5.1. The matrix $\mathbf{C}_{\text{SNR}}$ is constructed based on the

local SNR estimate realizations at every SNR point and used in the LE detection algorithm for exchanging and weighting the measurements in the CR network.

For the adaptive LE detection, we select the following simulation parameters: $M = 2$, $N = 7000$, $\boldsymbol{\mathcal{M}} = \mu \mathbf{1}_K$, $\mu = 0.001$ and $P_{FA} = 10^{-2}$ for all the nodes. The thresholds of the LE detectors at nodes $k \in K$ are found by using (5.14) with the TV approximation. We select the diffusion topology of the estimates in the CR network similarly as in [3, Eq. 11]. For example, when $K = 3$ and by keeping the same notation and conditions for the elements of matrix $\mathbf{A}$, then the diffusion topology is given as follows

$$\mathbf{A}_{\text{diff}}^T = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix}, \tag{5.16}$$

while $\overline{\mathbf{A}}_1 = \mathbf{A}_{\text{diff}}^T \otimes \mathbf{I}_{M^2}$.

In the following simulations we compare the performance of three different network sizes: $K = 1, 3$ and 10 nodes. All the results are taken from the last node in the set. The P_D versus SNR results are given in Fig. 5.1 when TV approximation is used for the CTA algorithm. The M-C estimated P_D results, based on the adaptively estimated CMs with local SNR based weighting of observations, are denoted as **Ad. Obs. Ex.** in the figure. These M-C based results are compared with the non-asymptotic theoretical model (5.15) (denoted as **Th. Obs. Ex.**). In addition, for reference we plot the The Monte Carlo estimated P_D results, based on the observation weighting with the theoretical SNR values (5.4), denoted as **Th. SNR Ex.**. Finally we add the M-C based P_D results based on adaptively estimated CMs with equal observations exchange $\mathbf{C}_{\text{SNR}} = \mathbf{A}_{\text{diff}}^T$ for the reference (denoted as **Ad. Eq. Ex**).

We note that in [3] we found that for the case of $M = 1$, an equal weighing of the observations does not improve significantly the resulting detection performance. We see that when CR nodes in addition to sharing the estimates share also their observations, while these observations in the neighbourhood are weighted not equally, but based on the locally estimated SNR values, then an observable increase in the P_D is seen. As the number of nodes in the network increases, the point where the P_D starts to decrease from 1 is moved to the left about 1.5-2dB. In general, the values of N_{SNR} and μ_{SNR} need to be determined experimentally, based on the PU activity patterns, CR system requirements and the noise power conditions. I.e. in a low SNR region, $N_S NR \gg 1$ and usually more samples have to be collected to get a reasonably accurate local SNR estimates, while less samples need to be processed in a high SNR region.

We see that when the SNR estimates with sufficient accuracy are available, then the SNR based observation weighting solution can be used for improving the performance of the adaptive, distributed LE detection algorithm. When the nodes cooperate in estimating the network-wise CM (while nodes are able to communicate directly only with limited subset of neighbour nodes) then the resulting LE detection performance is improved in the CR network.

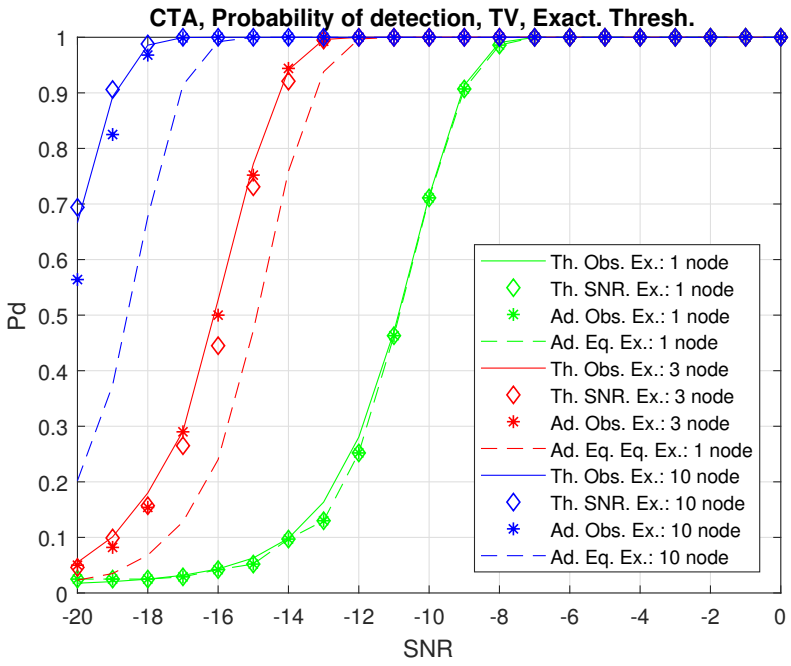

Figure 5.1 Probability of detection, CTA, TV, SNR Weighted Observations

5.5. Conclusions

In this chapter we studied a distributed and adaptive, CTA diffusion LMS based LE detection algorithm, which uses local SNR estimates for additional observation exchange between the CR nodes for PU signal detection. We analysed the performance of the proposed diffusion LMS based LE detection scheme and verified the theoretical results with the simulations. With the proposed algorithm, and in terms of the P_D versus SNR values, the PU signal detection performance is enhanced in CR network in about 1.5-2dB.

6. SUMMARY, CLAIMS AND FUTURE RESEARCH

6.1. Summary of thesis

This thesis studied two distributed and adaptive detection methods in wireless sensor networks, which are based on a distributed estimation process. The design, implementation and performance study of the proposed algorithms has been done by taking the Cognitive Radio application area into account. In this thesis we studied the algorithms from the estimation and detection domain point of view.

The objective of the current thesis was to design and implement three fully distributed and adaptive detection solutions for Cognitive Radio Networks, namely distributed Energy and distributed Largest Eigenvalue based detection solutions. This objective has been achieved successfully. We have proposed algorithms for each detector that can be practically implemented. Let us note, that hardware implementation aspects are not studied in this thesis. As common in the research area of estimation and detection, theoretical analytical performance of the proposed solutions was evaluated. We assigned statistical models for the corresponding adaptive estimates and for the detector test statistics to proceed with the moment and detection performance analysis of the proposed algorithms. The theoretical results were verified by computer simulation experiments and good matches between the theoretical performance measures and corresponding experiments were obtained. Thus we proposed and studied three main cooperative, fully distributed and adaptive spectrum sensing methods for a Cognitive Radio Network.

6.2. The Claims

Below is a summary of the theses/claims of novelty that this PhD thesis made. The claims correspond the papers P1. - P6. as detailed. Thus, thematically, the main contributions of the thesis are:

1. **An adaptive and fully distributed Energy Detection solution proposed and evaluated**. We derived and proposed the usage of distributed, diffusion least mean square (LMS) type of power estimation algorithms and three different static network topologies: Ring-Around, Combine and Adapt, Adapt and Combine are studied. The signal power estimation and

energy detection solution is not dependant on any Fusion center and the detection decisions can be made independently in every CR nodes or in a selecting a CR node for a network wise decision making. The signal power estimation solution is able to track the changes between the underlying detection hypotheses so that the usage of such algorithms is more practical in CR network. The detection performance of the proposed schemes was performed by using the statistical properties of these distributed, adaptive estimates. In case of the Ring-Round topology, due to the mathematical tractability of the estimate functions in time, more specific results about the moment estimation of the distributed estimates were given. With the help of the Central Limit Theorem the distribution of the test statistics of the energy detector was approximated by a CSCG process, by using the moments of the adaptive power estimates. The theoretical findings were verified by MATLAB based simulations. The PU signal, received by a individual CR node may be in deep fading and thus the detection results are dependant on the signal gain value (which is usually unknown for the receiver). We showed that when nodes cooperate in the estimation process of the test statistics, then the resulting detection performance can be significantly improved and stabilized. We also observed that the best detection results (also in terms of lowest variance of the power estimates) are obtained with the ATC type of estimate fusion method and especially when we have about 30 nodes in the network. It was observed that measurement fusion in the diffusion LMS estimation process did not notably improve the resulting detection results.

To the knowledge of the authors, such distributed energy detection method has not been proposed and studied before. This corresponds to the Chapter 3 and papers P1. - P3.

2. **An adaptive and fully distributed Largest Eigenvalue Detection solution proposed and evaluated**. We selected the Largest Eigenvalue detection method from the domain of correlation matrix based detection methods and designed, implemented an adaptive, fully distributed LE detection solution. Diffusion LMS type of algorithm was implemented with ATC, CTA topologies and with no Fusion Center. In order to study the resulting detection performance we extended the framework of the theoretical performance analysis, from the energy detection solution for the vector estimates. The correlation matrix estimates were vectorized for the distributed adaptive estimation process and after the estimation process re-matrizised. The distribution of the resulting CM matrix estimates was approximated by a Complex Wishard distribution and we implemented the Total and General Variance methods for approximating the Complex Wishart distribution parameters for the mentioned CM distribution approximation. These results were used to proceed with the study of the distribution of the test statistics - the largest eigenvalue of

the adaptively estimated CM. The theoretical results were verified by MATLAB based simulations. Similarly we observed that the resulting LE detection performance is more stabilized and equalized over the CR network, when nodes cooperate in the estimation process. Best results were observed with the ATC type of estimate fusion method and there was no notable difference in the performance of the Total and General Variance approximation methods. We justified that the proposed distributed estimation algorithm could be used also with some blind type of detectors, where the noise variance is not needed for the threshold calculation.

To the knowledge of the authors, such distributed largest eigenvalue detection method has not been proposed and studied before. This corresponds to the Chapter 4 and papers P4. - P5.

3. **An adaptive and fully distributed Largest Eigenvalue Detection solution with local SNR based observation exchange and weighting proposed and evaluated**.

   In addition to the solution, described in previous claim, we proposed and studied additionally the local observation exchange and combination strategy, which is based on the local SNR estimates and is adapted to the context of binary hypothesis testing. Local SNR can be estimated adaptively and independently from the (cooperative) CM estimation phase for the LE detection (5.3) and it can be considered as a ”goodness measure” of the received observations in CR network. We proposed the weighting method of received observations, were non-equal weights (based on local SNR estimates) are assigned only under the detection hypothesis H_1, so that the threshold determination solution of LE detection (under H_0), proposed in previous section, is still usable. We showed, that when the PU signal is present and when the local SNR estimates are available, then the network-wise PU signal detection performance can be slightly improved, compared to the standard case with no observation exchange, studied previously by us.

   To the knowledge of the authors, such distributed, adaptive, largest eigenvalue based detection method with such local SNR based observation exchange and weighting method has not been proposed and studied previously. This corresponds to the Chapter 5 and papers P6.

Throughout the thesis and in the distributed estimation domain we proposed and studied:

4. **Distributed diffusion LMS based scalar and vector estimation algorithms** for estimating the elements of the test statistics of Energy and Largest eigenvalue detector respectively. The algorithms were implemented so that CR nodes jointly participate in the estimation of scalar or vector quantities, where these latter quantities follow the model

of network average (to reduce the effect of channel gains), while the CR nodes individually are able to communicate only with a subset of neighbour nodes. Initially the distributed optimization concept for scalar measurements and estimates were derived for the energy detection method. Then a vectorized estimation model of the elements of correlation matrix was proposed. A network topology with minimum number of data fusions in CR network was proposed.

To the knowledge of the authors, such exact types of distributed diffusion LMS based scalar and vector estimation algorithms have not been proposed and studied before. This corresponds to the Chapters 3, 4, 5 and papers P1. - P6.

5. **A common framework for the performance analysis of the estimation algorithms and resulting detection performance in CR network**. We derived and proposed the usage of a framework for the performance analysis of the statistical moments of the distributed, adaptive estimates so that several common network topologies and data fusion types are supported. Mean stability analysis for the algorithms was performed. The statistical moments of the distributed estimates were used further in the statistical modelling the test statistics of the selected detectors and then for studying the resulting detection performance.

   To the knowledge of the authors, such type of framework for the performance analysis of distributed estimation algorithms have not been proposed before. This corresponds to the Chapters 3, 4, 5 and papers P1. - P6.

To conclude, this thesis has shown the benefits of adaptive and fully distributed energy and largest eigenvalue detection solutions for cognitive radio networks. The task of the current thesis, to derive fully distributed versions of two most widely used detectors for cognitive radio, was completed successfully. The obtained results are of practical interest, as the need for opportunistic spectrum sharing in urban areas is increasingly fast.

6.3. Future Research

We now highlight the aspects that might be worthy of future study.

1. Other signal models with various correlation structures and detection methods could be studied together with adaptive and fully distributed estimations methods, other hand diffusion LMS algorithm. The work presented in this thesis did not include comparisons for example with distributed RLS (Recursive Least Squares) method.

2. Usage of Change Detection methods could be studied further together with distributed diffusion estimation algorithms. It may be interesting to study

if a change in the statistical properties of the estimates of these algorithms could be detected based on the transitions of estimation algorithm to or from a steady-state and if this effect could be used in Cognitive Radio networks. Such a way an additional detection method could be implemented in parallel or instead of the detection algorithms, studied so far in current thesis.

3. It could be interesting to bring in more hardware aspects and constraints to the current research. The current work in this thesis is based on the MATLAB simulations and no major hardware platform specific limitations or aspects have been included so far in our research. It could be studied if algorithm properties could be tuned for reducing processing load in a hardware model, but without losing much in the PU detection performance at same time.

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ABSTRACT
Distributed Signal Processing in Cognitive Radio Networks

The lack of available radio frequencies is seen to be an increasing problem for implementing new modern radio communication solutions. Recent studies have shown that, while the available licensed radio spectrum becomes more occupied, the assigned spectrum is significantly underutilized. To alleviate the situation, cognitive radio (CR) technology has been proposed to provide an opportunistic access to the licensed spectrum areas. Secondary CR systems need to cyclically detect the presence of a primary user (PU) by continuously sensing the spectrum area of interest. Radiowave propagation effects like fading and shadowing often complicate sensing of spectrum holes. When spectrum sensing is performed in a cooperative manner, then the resulting sensing performance can be improved and stabilized.

In this thesis, three fully distributed and adaptive cooperative PU detection solutions for CR networks are studied.

In Chapter 3 of this thesis we study a distributed energy detection scheme without using any fusion center. Due to reduced communication such a topology is more energy efficient. We propose the usage of distributed, diffusion least mean square (LMS) type of power estimation algorithms with different network topologies. We analyze the resulting energy detection performance by using a common framework and verify the theoretical findings through simulations.

In Chapter 4 of this thesis we propose a fully distributed detection scheme, based on the largest eigenvalue of adaptively estimated correlation matrices, assuming that the primary user signal is temporally correlated. Different forms of diffusion LMS algorithms are used for estimating and averaging the correlation matrices over the CR network. The resulting detection performance is analyzed using a common framework. In order to obtain analytic results on the detection performance, the adaptive correlation matrix estimates are approximated by a Wishart distribution. The theoretical findings are verified through simulations.

In Chapter 5 of this thesis we propose a fully distributed largest eigenvalue detection scheme, where the observations of the elements of correlation matrices are weighted by independently estimated local SNR values. The resulting detection performance is analysed by using a common framework. The theoretical findings are verified through MATLAB simulations.

Keywords: Cognitive Radio, Distributed Estimation, Distributed Detection, Adaptive Networks, Diffusion Networks, Diffusion LMS, Spectrum Sensing, Energy Detection, Random Matrix, Largest Eigenvalue Detection, SNR Estimation.

KOKKUVÕTE
Hajutatud signaalitöötlus kognitiivse raadio võrgus

Kaasaegsete raadiosidesüsteemide arendamisel on tekkimas tendents, et tulevikus võib väheneda vabade raadiokanalite hulk. Mitmed raadiosageduskanalid on regulatiivselt antud litsentseeritud raadiosüsteemide kasutusse. Mitmed uuringud on näidanud, et kui vajadus teatud litsentseeritud raadiospektri kasutamiseks kasvab, siis võib litsentseeritud raadiospekter samas olla olulisel määral alakasutatud. Olukorra leevendamiseks on välja pakutud kognitiivse raadio (CR) tehnoloogia, mis võimaldab kognitiivse raadio (CR) süsteemidele oportunistlikku juurdepääsu litsentsitud spektrivaldkondadele. Sekundaarsed CR süsteemid peavad tsükliliselt tuvastama litsentseeritud kasutaja olemasolu, jälgides pidevalt huvipakkuvat spektriala. Raadiolainete leviefektid, nagu peegeldused, raskendavad sageli nn. spektriaukude tuvastamist. Kui spektri tuvastamine viiakse CR sõlmede poolt läbi ühiselt, siis saab saadud primaarallika tuvastustõenäosust CR võrgus parandada ja stabiliseerida.

Käesolevas väitekirjas uuritakse kolme täielikult hajutatud ja adaptiivset primaarse kasutaja (PU) tuvastamise lahendust CR võrkude jaoks.

Antud väitekirja kolmandas peatükis uuritakse hajutatud energiatuvastamise skeemi ilma keskset võrgusõlme kasutamata. Vähendatud võrgukommunikatsiooni tõttu on selline topoloogia energiasäästlikum. Vajaliku detektori teststatistika elementide hindamiseks pakutakse välja hajutatud difuusiooni LMS vähimruutude meetodil põhinev hindamise algoritm, mida vaadeldakse erinevate võrgupopulatsioonidega. Analüüsitakse väljapakutud energiatuvastamise meetodiga saavutatavat detektsioonitõenäosust, kasutades ühist analüüsiraamistikku ja kontrollides arvutisimulatsioonide abil teoreetiliste tulemuste paikapidavust.

Antud väitekirja neljandas peatükis pakutakse välja täielikult hajutatud primaarsignaali tuvastusskeem, mis põhineb adaptiivselt hinnatud korrelatsioonimaatriksite suurimal omaväärtusel, eeldades, et primaarkasutaja signaal on ajaliselt korreleeritud. CR-võrgu korrelatsioonimaatriksite hindamiseks ja keskmistamiseks kasutatakse erinevat tüüpi LMS-i algoritme. Signaalituvastustõenäosust analüüsitakse täiendatud, ühise raamistiku abil. Teoreetilise detektsioonitõenäosuse mudeli jaoks pakutaks välja kaks meetodit, millega saab adaptiivselt hinnatud korrelatsioonimaatriksit aproksimeerida Wishart'i jaotusega baasil. Teoreetilisi tulemusi kontrollitakse arvutisimulatsioonide abil.

Antud väitekirja viiendas peatükis pakutakse välja täielikult hajutatud, adaptiivne suurimal omaväärtusel põhinev primaarsignaali tuvastusskeem, kus sisendmõõteandmeid kaalutakse naabersõlmede abil, baseerudes iseseisvalt hinnantud lokaalsete SNR-i väärtuseid. CR-võrgu korrelatsioonimaatriksite hindamiseks ja keskmistamiseks kasutatakse CTA tüüpi diffusion LMS-i algoritmi. Signaalituvastustõenäosust analüüsitakse täiendatud, ühise raamistiku abil. Teoreetilisi tulemusi kontrollitakse arvutisimulatsioonide abil.

Märksõnad: Kognitiivne raadio, hajutatud hindamine, hajutatud raadiosignaali tuvastamine, Diffusioon LMS, difusioonvõrgud, adaptiivsed võrgud, spektrituvastus, energia detektsiooni meetod, juhuslikud maatriksid, suurima omaväärtuse detektsiooni meetod.

APPENDIX A

Publication P1.

A. Ainomäe, T. Trump, and M. Bengtsson, "Distributed recursive energy detection," in IEEE WCNC 2014 Conference Proceedings*, Istanbul, Turkey, 2014, ETIS 3.1.*

Distributed Recursive Energy Detection

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***Abstract*—Recent studies have shown that, while the available licensed radio spectrum becomes more occupied, the assigned spectrum is significantly underutilized. To alleviate the situation, cognitive radio (CR) technology has been proposed to provide an opportunistic access to the licensed spectrum areas. CR systems are able to serve the secondary users for detecting and utilizing so called spectrum holes by sensing and adapting to the environment without causing harmful effects or interference to the licensed primary users (PU). CR systems need to detect the presence of a primary user by continuously sensing the spectrum area of interest. Radiowave propagation effects like fading and shadowing often complicate sensing of spectrum holes because the PU signal can be weak in a particular area. Cooperative spectrum sensing is seen as a prospective solution to enhance the detection of PU signals. This paper studies distributed spectrum sensing in a cognitive radio context. We investigate a distributed energy detection scheme without using any fusion center. Due to reduced communication such a topology is more energy efficient. The PU signal is assumed to be in slow fading. A recursive distributed power estimation and detection scheme is proposed. The theoretical findings are verified through simulations.**

***Index Terms*—Cognitive radio, distributed estimation, distributed detection, adaptive networks, energy detection.**

I. Introduction

In this paper, we study a distributed detection problem, where we have a number of nodes in the network sensing the spectrum area of interest. Nodes estimate the received power in their own location.

Several solutions have been proposed that make use of a central processing unit to collect all the measurements over the sensing period from all the nodes and make decisions about presence or absence of PU [1],[2],[3],[4]. Instead we propose to let the power measurements circulate around the cognitive radio network nodes, to allow all these nodes to make decisions based on data from all the nodes without involvement of any central processing unit. At every time instant measurements with different spatial profiles become available and nodes in the network make individual decisions about the present signal detection hypotheses. Such a solution needs less communication in the CR network (therefore saves energy resources) and enhances network failure resistance since there is no need to exchange estimates with the central processing unit at every iteration.

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Distributed adaptive estimation and detection schemes have been studied before in several papers, for example the least mean square (LMS) and recursive least squares (RLS) based estimation schemes in [5],[6] and the consensus based schemes in [7],[8],[9],[10]. Optimal distributed detection, based on diffusion type LMS and RLS estimation schemes, has been studied in [11]. These two schemes rely on matched filter detection. However, here we make an assumption, that we do not have any prior information about the waveform of the PU signal in the secondary nodes and hence we cannot design a matched filter. Therefore the energy detection becomes a practical solution.

A ring network topology for distributed energy detection without a fusion centre has been suggested in [12]. Compared to [12], in the current paper we propose an estimation based recursive calculation of the test statistics for the energy detectors (in cognitive radio network with ring topology). The test statistic in form of a converged power estimate is the soft information used for making the detection decision at every node. Usage of a recursive power estimation scheme with fixed step size provides the ability to track changes in the power of the received signal in time, as new samples are received. We also show that in Rayleigh fading channel, processing of the samples of the received signal over sufficient number of nodes in the ring topology averages and diminishes the effect of the different channel gains at the CR nodes. Thus the detection scheme becomes more robust to fading, compared to the case, where the test statistic is calculated over locally received signal samples only or over few nodes. The resulting energy detection performance is dependant on the performance of the recursive power estimation algorithm.

We organize the remainder of the paper as follows. In section II we review the bases of energy detection and then derive a recursive and distributed received signal power estimation algorithm. In section III we analyse the performance of the proposed distributed power estimation algorithm and the resulting energy detection. In section IV we present our simulations results.

II. Distributed power estimation and detection

According to classical detection theory, an energy detector can be used for detecting random signals in additive noise. For

energy detection in a cognitive radio context, the type of PU signal can be completely unknown. During a sensing time t, an energy detector (ED) receives N samples of a signal $x(n)$ from a specific frequency band [2]. The average energy of the received data samples is the test statistic $T(x)$ of the ED, which compares $T(x)$ to a predefined threshold γ and decides which of the hypotheses H_0 or H_1 is more likely.

We assume the following signal model at node k:

$$\begin{aligned} H_0 &: E[|x_k(n)|^2] = E[|v(n)|^2] \\ H_1 &: E[|x_k(n)|^2] = E[|\alpha_k|^2|s(n)|^2] + E[|v(n)|^2], \end{aligned} \tag{1}$$

where $k = 1, 2, ..., K$ is the node number and $n = 1, 2, ...N$ is the sample index. $v(n)$ is independent and identically distributed (*i.i.d*) circularly symmetric complex Gaussian (CSCG) noise with zero mean and variance $E[|v(n)|^2] = \sigma_v^2$, i.e. $v(n) \sim \mathcal{C}N(0, \sigma_v^2)$. The power of the emitted PU signal $s(n)$ is denoted as $E[|s(n)|^2] = S$, under H_1. The primary signal $s(n)$ and the noise $v(n)$ are assumed to be statistically independent. The PU signal passes through a slowly fading channel with gain $\alpha_k(n)$. The gain α_k is considered to be constant. Note, that for implementing the energy detector, knowledge about the values of channel constants is not required. The constant noise variance is assumed to be known *a priori* for being able to set the threshold of the energy detector. Noise power estimation is not considered in this paper.

In the literature on distributed detection, for example in [13], a fusion center, which collects all the local soft information, hard or soft binary decisions from the sensors, is often used in distributed detection networks. Similarly a central processing unit has been used in distributed estimation schemes, see e.g. [5]. However, such a central processing unit can potentially be a single point of failure in the detection system. Secondly it may require frequent data exchange between the nodes and the centre and thus drain system energy resources, since usually most of the energy is spent for powering up the transmitter to exchange the data with neighbour nodes.

A distributed and recursive estimation scheme is one of the possible solutions for removing the central processing unit from the system and thus the network is able to calculate the global estimates based on the local observations collected by the CR nodes. Then based on the estimated test statistic, the detector at each CR node can locally make its own decision if the PU signal is present or not.

We denote the estimate at node k and at iteration n as $\hat{P}_k(n+1)$. We employ a circular topology for the distributed power estimation, as also suggested in [12]. At every time instant n, every CR node k is communicating with the $(k-1)\text{mod}K$ immediate neighbour node only. We use the modulo notation with the convention $1 \leq k \bmod K \leq K$. Such an estimation method incorporates the incoming measurements periodically over all the nodes and can potentially diminish the effect of the varying channel gains on these nodes, thus it is more robust to fading and enhances the resulting detection performance. In this paper we make the following assumptions:

- (AS1) The $x(n)$ is sensed by *K* nodes in the CR network.
- (AS2) The additive noise $v_k(n)$ is uncorrelated in time and space and has the same power level over all the nodes in the CR network.
- (AS3) For the number of performed iterations N we have that $N \gg K$.
- (AS4) The links between the CR nodes are ideal and not capacity restricted (no need to quantize the soft information).

By taking into account the suggestions in [14] for a local, non-cooperating estimator for sample variance, the distributed estimator using a circular estimation topology can be constructed as follows

$$\hat{P}_k(n) = \frac{1}{n} \sum_{i=1}^{n} |x_{(k-i+1)\text{mod}K}(n-i+1)|^2. \tag{2}$$

A recursive equivalent to (2) is given by

$$\begin{aligned} \hat{P}_k(n) \quad = \quad & \hat{P}_{(k-1)\text{mod}K}(n-1) + \mu(n)(|x_k(n)|^2 \\ & -\hat{P}_{(k-1)\text{mod}K}(n-1)), \end{aligned} \tag{3}$$

where $n \geq 1$ and with step size: $\mu(n) = \frac{1}{n}$

The usage of step size $\mu(n) = \frac{1}{n}$ however, expects that the received signal $x_k(n)$ over $n \in N$ stays under a fixed hypotheses: H_0 or H_1. This fact makes its direct use in real-time spectrum sensing problematic. As a solution, a positive constant step size $\mu(n) = \mu$ can be used in recursive power estimation algorithm and then (3) is able to track the possible changes in power of the received signal $x_k(n)$. As common in the literature of adaptive filtering, the step size of the algorithm is user defined.

The estimated power level $\hat{P}_k(n)$ is used as the test statistic of the recursive ED. i.e. $T(x) = \hat{P}_k(n)$. Since there is no fusion centre and for system redundancy purposes, information overhead is allowed in the network. Thus there are K circular estimation processes running in parallel to provide a global estimate for every node $k \in K$. Every node can then perform the energy detection at any time instant. The algorithm can in principle run infinitely (no window for sample processing is required). The proposed algorithm is summarized in Algorithm 1. Let us note, that with the suggested algorithm, only one-directional communication with the adjacent node is required for exchanging the soft information, compared to the schemes, where a central processing unit is used and thus two way communication direction is needed to also send the global soft information back to the nodes at every iteration n. An example with $K = 2$ nodes and thereby 2 estimation processes (red and blue) is illustrated in Fig. 1 with nodes $k = 1, 2$ receiving samples $n = 1, ..., 3$.

According to AS3 it is assumed, that the number of iterations performed with the recursive algorithm is larger than the number of nodes in the network. The estimator needs to converge to steady state before the detection decision is made and for the convergence a sufficient number of samples are required. In slow fading the channel coherence time is large and the convergence is achievable. Secondly, in the performance section of the proposed algorithm the Central Limit Theorem

(CLT) is applied so enough samples are required also for this approximation to hold. The minimum number of samples for the CLT approximation has been evaluated in the literature, e.g. in [15].

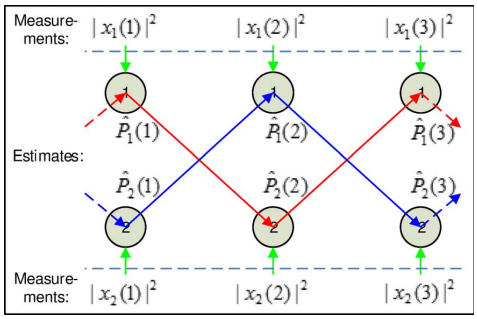

Fig. 1. Distributed Power Estimation with 2 nodes.

Algorithm 1 Distributed Circular Power Estimation

Start with $\hat{P}_k(0) = P_0$.
for every time instant $n \geq 1$ **do**
  for every node $k = 1, ..., K$ **do**
    1. Power estimation:
    $\hat{P}_k(n) = \hat{P}_{(k-1) \bmod K}(n-1) +$
    $\mu(|x_k(n)|^2 - \hat{P}_{(k-1) \bmod K}(n-1))$.
    2. Detection decision:
    $H_0 : \hat{P}_k(n) < \gamma$ or $H_1 : \hat{P}_k(n) > \gamma$.
    (Refer to (19) for selecting the threshold).
  end for
end for

III. Performance Analysis

To design the Neyman-Pearson type of detector of the proposed algorithm, we need to characterize the probability density function (PDF) of the test statistic of the recursive ED. Due to the usage of a circular estimation topology and due to the varying channel gains observed in different nodes, the distribution of the received and processed samples $|x_k(n)|^2$ differ from node to node. At first we find the mean and variance of the power estimation algorithm with fixed step size. From these the performance of the recursive energy detection is then derived, using a Gaussian approximation. The values of α_k are assumed to be known for the performance analysis.

A. Mean of Estimation

The mean of the global estimation recursion (3) can be found directly. Dropping the mod$K$ notation, we have

$$\mathrm{E}[\hat{P}_k(n)] = (1-\mu)\mathrm{E}[\hat{P}_{k-1}(n-1)] + \mu\mathrm{E}[|x_k(n)|^2]. \quad (4)$$

The initial condition is $P_0 = \hat{P}_k(0)$. Due to the circular estimation topology we have that $N = KM+m$, where $M = \lfloor N/K \rfloor$ and where m denotes additional iterations after full cycles. Let $\mathrm{E}[\hat{P}_k(N)|H_1]$ denote the mean when PU signal present and $\mathrm{E}[\hat{P}_k(N)|H_0]$ the mean when only noise is present. By iterating recursion (4), using the proposed notation and replacing the expectations using model (1), we can write

$$\begin{aligned}\mathrm{E}[\hat{P}_k(N)|H_1] = \\ &\mu S[\frac{1-(1-\mu)^{KM}}{1-(1-\mu)^K}\left[\sum_{l=0}^{K-1}(1-\mu)^l|\alpha_{k-l}|^2\right] \\ &+ \sigma_v^2\left[1-(1-\mu)^{KM+m}\right] \\ &+ P_0(1-\mu)^{KM+m} \\ &+ \mu S\left[(1-\mu)^{KM}\left[\sum_{i=0}^{m-1}(1-\mu)^i|\alpha_{k-i}|^2\right]\right]. \quad (5)\end{aligned}$$

In line 2 of (5), the geometric series $\sum_{i=0}^{M-1}(1-\mu)^{Ki}$ has been replaced with its sum. Let us note, that according to lines 2 and 5 of (5), the mean differs from node to node due to the values and processing order of $|\alpha_k|^2$. When only noise is present then $S = 0$ and

$$\begin{aligned}\mathrm{E}[\hat{P}_k(N)|H_0] = \\ &P_0(1-\mu)^{KM+m} + \sigma_v^2\left[1-(1-\mu)^{KM+m}\right]. \quad (6)\end{aligned}$$

According to AS3, $M >> K$ and in steady state of the estimator, when $M \to \infty$, the exponential factors $(1-\mu)^{KM+m}$ and $(1-\mu)^{KM}$ in (5) converge to 0 if $0 < \mu < 1$. In steady-state, formula (5) goes to

$$\begin{aligned}\mathrm{E}[\hat{P}_k(\infty)|H_1] = \\ &\sigma_v^2 + \frac{\mu S}{1-(1-\mu)^K}\left[\sum_{l=0}^{K-1}(1-\mu)^l|\alpha_{k-l}|^2\right] \quad (7)\end{aligned}$$

and in the noise only case correspondingly to

$$\mathrm{E}[\hat{P}_k(\infty)|H_0] = \sigma_v^2 \quad . \quad (8)$$

B. Variance of Estimation

Since $\hat{P}_{k-1}(n)$ and $|x_k(n)|^2$ are uncorrelated and by dropping the mod$K$ notation, we have

$$\begin{aligned}\mathrm{Var}[\hat{P}_k(n)] &= (1-\mu)^2\mathrm{Var}[\hat{P}_{k-1}(n-1)] \\ &+ \mu^2\mathrm{Var}[|x_k(n)|^2]. \quad (9)\end{aligned}$$

Since $x_k(n)$ is CSCG, then according to model (1) the PU signal is present, $\mathrm{Var}(|x_k(n)|^2) = (S|\alpha_k|^2 + \sigma_v^2)^2$. Let $\mathrm{Var}[\hat{P}_k(N)|H_1]$ denote the variance when the PU signal present and $\mathrm{Var}[\hat{P}_k(N)|H_0]$ the variance when received signal contains only noise. By iterating (9), replacing the variances and using the proposed notation, we have that

$$\begin{aligned}\mathrm{Var}[\hat{P}_k(N)|H_1] = & \\ & \mu\sigma_v^4 \frac{1-(1-\mu)^{2(KM+m)}}{2-\mu} \\ & + \mu^2 \frac{1-(1-\mu)^{2KM}}{1-(1-\mu)^{2K}} \\ & \cdot \left[\sum_{l=0}^{K-1}(1-\mu)^{2l}\left[S^2|\alpha_{k-l}|^4 + 2S|\alpha_{k-l}|^2\sigma_v^2\right]\right] \\ & + \mu^2(1-\mu)^{2KM} \\ & \cdot \left[\sum_{i=0}^{m-1}(1-\mu)^{2i}\left[S^2|\alpha_{k-i}|^4 + 2S|\alpha_{k-i}|^2\sigma_v^2\right]\right].\end{aligned} \tag{10}$$

In line 3 of (10), the geometric series $\sum_{i=0}^{M-1}(1-\mu)^{2Ki}$ has been replaced with its sum. Similarly to the mean, the variance differs over the nodes. When only noise is present, the resulting variance is given as

$$\mathrm{Var}[\hat{P}_k(N)|H_0] = \frac{\mu\sigma_v^4}{2-\mu}\left[1-(1-\mu)^{2(KM+m)}\right]. \tag{11}$$

In steady state of the estimator, when $M \to \infty$, the exponential factors $(1-\mu)^{2(KM+m)}$ and $(1-\mu)^{2KM}$ in (10) converge to 0 if the constant step size μ is taken sufficient. Thus the variance tends to

$$\begin{aligned}\mathrm{Var}[\hat{P}_k(\infty)|H_1] = & \frac{\mu\sigma_v^4}{2-\mu} \\ & + \frac{\mu^2}{1-(1-\mu)^{2K}}\left[\sum_{l=0}^{K-1}(1-\mu)^{2l}\left[S^2|\alpha_{k-l}|^4 + 2S|\alpha_{k-l}|^2\sigma_v^2\right]\right]\end{aligned} \tag{12}$$

under $H1$ and in the noise only case to

$$\mathrm{Var}[\hat{P}_k(\infty)|H_0] = \frac{\mu\sigma_v^4}{2-\mu}. \tag{13}$$

The residual variance of the fixed step size power estimation algorithm depends on the value of μ. We observe, that smaller μ causes smaller residual variance and thus more precise estimation results. On the other hand it is known from the literature of adaptive filtering, that smaller μ causes slower convergence in the mean.

C. Detection Performance Analysis

In order to derive the probability of detection (P_D) and probability of false alarm (P_{FA}), using the Algorithm 1, we need to evaluate the PDF of the test statistic $\hat{P}_k(n)$ under both hypotheses H_0 and H_1. As mentioned earlier the test statistic (for making the detection decision using threshold test) at node k at time instant n is found with the proposed distributed signal power estimation scheme. Thus the detection performance is dependant on the performance of the underlying estimation process.

As the input $x_k(n)$ is CSCG and when we have $K = 1$ only, then the test statistic of ED $\hat{P}_k(n)$ is local and under both hypothesis a Chi-Square distributed random variable with 2N degrees of freedom. The test statistic $\hat{P}_k(n)$ is obtained as a sum of a number of identically distributed variables and hence the CLT can be applied to approximate the Chi square distribution by a Gaussian distribution [15]. According to AS3, the number of samples and nodes in the network are large enough, thus the CLT is expected to apply.

The cooperative, global estimation statistic $\hat{P}_k(n)$ in (3) in case of hypothesis H_1, is however found over independent, but not identically distributed variables (due to the node specific channels). In such a case the Lyapunov CLT can still be applied over a large number of samples to result in a Gaussian approximation.

By taking into account the previously derived (5), (6), (10) and (11), we can provide approximate formulas for the recursive energy detection performance.

Let Q be the complementary distribution function of the standard Gaussian

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp\left(-\frac{t^2}{2}\right)dt. \tag{14}$$

The probability of false alarm P_{FA} of the energy detector when signal $x(n)$ contains only noise i.e. under hypothesis H_0 is found by

$$P_{FA}(\gamma, t) = Pr(T(x) > \gamma|H_0) = \int_\gamma^\infty p_x(x|H_0)dx \tag{15}$$

Substituting the estimation mean and variance under H_0, we get

$$P_{FA} = Q\left(\frac{\gamma - \mathrm{E}[\hat{P}_k(N)|H_0]}{\sqrt{\mathrm{Var}[\hat{P}_k(N)|H_0]}}\right). \tag{16}$$

Based on the assumption AS3, we observe that the formula holds for every node $k \in K$.

The probability of detection of an energy detector under hypothesis H_1 is correspondingly

$$P_D(\gamma, t) = Pr(T(x) > \gamma|H_1) = \int_\gamma^\infty p_x(x|H_1)dx. \tag{17}$$

Let the probability of detection at node k be: $P_{D,k}$. Similarly substituting the mean and variance under H_1, we get

$$P_{D,k} = Q\left(\frac{\gamma - \mathrm{E}[\hat{P}_k(N)|H_1]}{\sqrt{\mathrm{Var}[\hat{P}_k(N)|H_1]}}\right). \tag{18}$$

The sensing threshold can be found from (16) by fixing the desired value of P_{FA}. Since under hypothesis H_0 we have only noise power present,

$$\begin{aligned}\gamma &= \mathrm{E}[\hat{P}_k(N)|H_0] \\ &+ Q^{-1}(P_{FA})\sqrt{\mathrm{Var}[\hat{P}_k(N)|H_0]}.\end{aligned} \tag{19}$$

Due to the assumption AS2 the thresholds for every CR node k are equal.

However calculation of the threshold requires knowledge about the moments of the estimation algorithm in case of hypothesis H_0 and these moments are dependant on the algorithm parameters (especially the step size). In practice for the threshold calculation, the required moments can be calculated in advance using (6),(11), known values of step size, noise power and then inserting these results into (19).

IV. Simulation results

First we investigate the power estimation algorithm (with constant step size). Secondly we view the energy detection performance of proposed algorithm. In all these simulations the PU signal $s(n)$ is taken as QPSK with unit power S, under the active hypothesis H_1, the step size is: $\mu = 0.01$ and $m = 0$.

A. Local and global power estimation

We first investigate the estimates of (3) under two modes - *local*: if the nodes are not cooperating to each-other (i.e. every node acts as a stand alone energy estimator/detector) and *global*: if nodes are in cooperation. In the next two examples all nodes receive $N = 1200$ samples. To illustrate the tracking feature, we examine how the algorithm reacts if the power level changes at sample 601. Thus during samples $n = 1, ..., 600$ hypothesis H_1 is present (the source signal power S is attenuated by channel gain $|\alpha_k|^2$). Due to slow fading the α_k is assumed to be constant and is obtained by: $\alpha_k \sim \mathcal{C}N(0, 1)$. In sample range $n = 601, ..., 1200$, the PU signal is absent and only background noise power $\sigma_v^2 = 1$ is present at every node k.

Using recursion (3) the local, non-cooperative power estimate is plotted in Fig. 2, with 10 nodes in the CR network. The channel gain values $|\alpha_k|^2$ are given on the figure. Obviously,

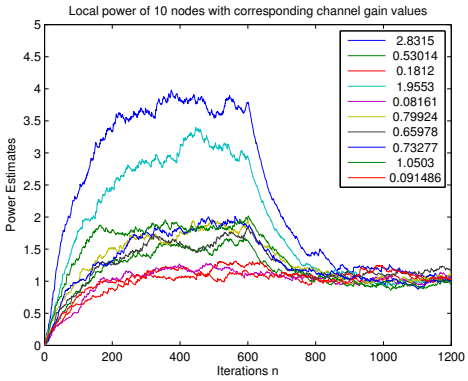

Fig. 2. Local power estimation, fixed step

the estimation result using local information is depending on the channel coefficient of the specific node. From $n = 601$ the algorithm is starting to converge to the noise only power level $\sigma_v^2 = 1$.

TABLE I
Mean and Var. ,last node in the set, $\sigma_v^2 = 1$

K	M	Mean	Variance
1	6000	1.39	0.0097
3	2000	1.63	0.0098
10	600	1.87	0.02
30	200	1.93	0.0189
50	120	2.01	0.0203

In Fig. 3 we investigate the cooperative scheme. Exactly the same channel gains are used as in the local simulation. Since the mean and variance differ at nodes k, then for illustration we plot only the global power estimation result of node $k = 10$, in the network with $K = 10$. The corresponding mean and ± 3 times standard deviation are given in Fig. 3.

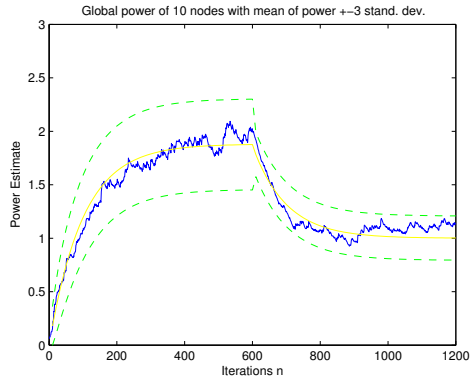

Fig. 3. Global power estimation, fixed step

In Fig. 3 the global estimate is converging around the mean. Due to the proposed circular estimation topology, the recursion (3) can reduce the effect of random gain caused by channel coefficients. We see that the global estimate stays within the ± 3 times standard deviation limits from the mean, which is expected in case of a Gaussian distribution.

B. Probability of detection

Next we investigate the probability of detection using the proposed global power estimation algorithm. We compare the performance of 5 different network sizes: $K = 1, 3, 10, 30, 50$ nodes. More specifically, the simulated and theoretical results of P_D of the last nodes in the set are compared. Table I describes the data sets with chosen fixed channel constants for this particular simulation: the mean and variance at the last node in the set, when $\sigma_v^2 = 1$. $N = 600$. The threshold is calculated using (19). We set $P_{FA} = 10^{-4}$. 1000 experiments are made with given fixed set of channel constants and noise power. The power estimate of the received signal at iteration N is compared with the threshold. Theoretical P_D is calculated using (18). In addition, the theoretical P_D with $|\alpha_k|^2 = 1$ has

been plotted. This particular result corresponds to the case with no-fading. Fig. 4 shows the results with different noise power levels.

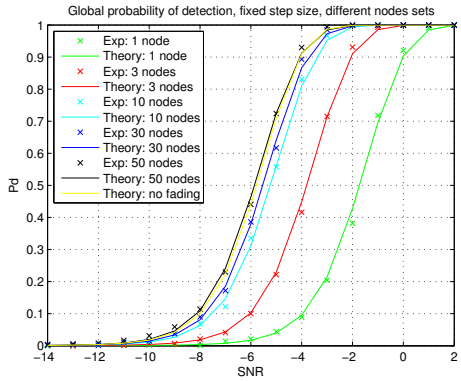

Fig. 4. Probability of detection, known constants, fixed step

As can be seen, there is a good match between simulated and theoretical P_D in every node set. The PDF of the test statistic is approximated by a Gaussian distribution and the CLT approximation applies even with small K and when the underlying stochastic process is cyclostationary (since the variance of the sample of received signal is changing periodically over n). The P_D of the set with few nodes is more influenced by the given values of channel constants. According to simulation data when $K = 1$ the PU signal is in deep fading and this explains the worse P_D result. In case of non-distributed estimation and detection, not much can be done to improve the P_D. For the $K = 3$ and $K = 10$ the attenuation of PU signal appears to be smaller and thus detection probability increases on higher noise power values. As the number of nodes K increases, from about $K = 30$, the P_D result stabilizes close to the theoretical P_D plot of the no fading case.

V. Conclusions

In this paper we proposed a recursive distributed power estimation approach, that is applicable for CR networks for detecting the presence of PU signal. The performance analysis of the derived algorithm was carried out and simulations were run. It was shown, that the proposed signal power estimation algorithm with a circular topology estimates the power of received signal $x(n)$ samples by diminishing the effect of varying channel gains at specific nodes. The proposed algorithm with fixed step size is able to track changes in received signal power and usable in cognitive radio systems.

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APPENDIX B

Publication P2.

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CTA diffusion based recursive energy detection

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Abstract: Cognitive radio (CR) has been seen as a promising technology to make radio spectrum usage more effective by providing an opportunistic access for secondary users to the licensed spectrum areas. CR systems need to detect the presence of a primary user (PU) signal by continuously sensing the spectrum area of interest. Radiowave propagation effects like fading and shadowing often complicate the detection of PU because the PU signal can be weak in a particular area. By sensing the radio spectrum area of interest in a cooperative manner, the detection performance of PU signals can be increased and made more robust to channel fading. This paper studies distributed spectrum sensing in a cognitive radio context. We propose and analyse a distributed, combine and adapt type (CTA) of diffusion energy detection scheme where a central data processing unit is not needed and the required test statistics is estimated across the network. CR nodes fuse the power estimates of several neighbour nodes in the network. The PU signal is assumed to be in slow fading. The theoretical findings are verified through simulations. The proposed CTA algorithm is compared with the ring around distributed power estimation algorithm.

Key–Words: Cognitive radio, distributed estimation, distributed detection, adaptive networks, energy detection.

1 Introduction

In this paper, we study a distributed detection problem, where we have a number of nodes in the network sensing the spectrum area of interest.

The cognitive radio system is dynamic. The PU signal can be absent or present at any time. Often in practice the statistical information (for example conditional probability density of observations, prior probabilities of detection hypotheses, statistical behaviour of PU) may not available *a priori* for constructing a PU signal detection solution. The properties of the detection statistics may change in time.

We need to look for estimation and detection strategies which are able to react to possible changes in the properties of detection statistics and to learn the statistical information based on observations received by the nodes in the network. To reduce the computational complexity (memory requirements) and increase the learning speed, we look for methods, which support real time processing. One of the possible direction is to consider adaptive, on-line network learning methods. As new observations arrive, the estimated parameter is updated directly, without a need to re-run the network averaging process using all the observations from the past. This is a reasonable approach in cognitive radio, since we would like to avoid interference to the PU user and react to changes in a channel usage as soon as possible. On the other hand we would like to find the free spectrum opportunities as fast as possible.

Several proposed distributed spectrum sensing solutions make use of a central processing unit to collect together all the observations from all the nodes and make decisions about presence or absence of PU [1],[2],[3],[4]. We would like to remove such a node from the network. Instead we propose a power estimation solution, where the power estimates and measurements are fused in every cognitive radio network node, to allow the node to make decisions based on the data, which is available for the node. At every time instant new measurements and estimates become available, nodes in the network fuse the information and then make individual decisions about the present signal detection hypotheses.

Distributed adaptive estimation and detection schemes have been studied in several papers. The least mean square (LMS) and recursive least squares (RLS) based estimation schemes were analysed in [5],[6] and the consensus based schemes were handled in [7],[8],[9],[10]. Optimal distributed detection, based on diffusion type LMS and RLS estimation schemes, was studied in [11]. These two schemes rely on matched filter detection. However, in CR network

here we make an assumption, that we do not have any prior information about the waveform of the PU signal in the secondary nodes and hence we cannot design a matched filter. Therefore energy detection becomes a practical solution.

A ring network topology for recursive distributed energy detection without a fusion centre has been analysed in [12]. Compared to [12], in the current paper we propose a CTA diffusion based recursive calculation of the test statistics for the energy detectors. The test statistic in form of a converged power estimate is the soft information used for making the detection decision independently at every node. Such estimation strategy is able to track changes in the power of the received signal in time, as new samples are received. We propose a distributed parameter estimation model, which in Rayleigh fading channel becomes more robust to fading, compared to the case, where the test statistic is calculated over locally received signal samples only or over few nodes. The resulting energy detection performance is dependant on the performance of the recursive power estimation algorithm.

We organize the remainder of the paper as follows. In section II we review the system model and the bases of energy detection. In section III we derive an adaptive and distributed signal power estimation algorithm based on diffusion LMS strategy. In section IV we analyse the performance of the proposed distributed power estimation algorithm and the resulting energy detection. In section V we present our simulations results.

2 Distributed power estimation and detection

In classical detection theory, an energy detector can be used for detecting random signals in additive noise. For energy detection in a cognitive radio context, the type of PU signal can be completely unknown. The common assumption is that the noise power is known for being able to set the detection threshold. During a sensing time t, an energy detector (ED) receives N samples of a signal $x(n)$ from a specific frequency band [2]. The average energy of the received data samples is the test statistic $T(x)$ of the ED, which compares $T(x)$ to a predefined threshold γ and decides which of the hypotheses H_0 or H_1 is more likely.

We assume the following signal model at node k:

$$\begin{aligned} H_0 &: \mathrm{E}[|x_k(n)|^2] = \mathrm{E}[|v_k(n)|^2] \\ H_1 &: \mathrm{E}[|x_k(n)|^2] = \mathrm{E}[|\alpha_k|^2|s(n)|^2] + \mathrm{E}[|v_k(n)|^2], \end{aligned} \tag{1}$$

where $k = 1, 2, ..., K$ is the node number and $n = 1, 2, ...N$ is the sample index. $v_k(n)$ is independent and identically distributed (*i.i.d*) circularly symmetric complex Gaussian (CSCG) noise with zero mean and variance $\mathrm{E}[|v_k(n)|^2] = \sigma^2_{v,k}$, i.e. $v(n) \sim \mathcal{C}N(0, \sigma^2_{v,k})$. The power of the emitted PU signal $s(n)$ is denoted as $\mathrm{E}[|s(n)|^2] = S$, under H_1. The primary signal $s(n)$ and the noise $v_k(n)$ are assumed to be statistically independent. The PU signal passes through a slowly fading channel with gain $\alpha_k(n)$. Note, that for implementing the energy detector, knowledge about the values of channel constants is not required. Noise power estimation is not considered in this paper. In this paper we make the following assumptions:

- (AS1) The $x(n)$ is sensed by *K* nodes in the CR network.
- (AS2) The additive noise $v_k(n)$ is uncorrelated in time and space and has the same power level over all the nodes in the CR network.
- (AS3) The number of performed iterations N is large enough.
- (AS4) The links between the CR nodes are ideal and not capacity restricted (no need to quantize the soft information).

We denote the received power estimate at node k and at iteration n as $\hat{P}_k(n)$. For estimating the received signal power, we consider the CTA diffusion strategy [13]. In this strategy every CR node k shares the estimates (and also measurement is set so) with the neighbour nodes which are the connected to node k. Every node k fuses the estimates (and measurements) from the neighbour nodes with the estimates (and measurements) from itself and updates its own estimate. In this work we assume the network topology to be fixed over the sensing time. Also we consider linear, fixed combinations of neighbour estimates and measurements at every node k. We derive the CTA diffusion algorithm in three phases. First we consider local processing, when the nodes do not cooperate to estimate the received signal power jointly. Secondly we propose a global model for estimating the received signal power in cooperative manner, where all the observations are collected together to a FC for central processing. Finally we propose a fully distributed power estimation algorithm, where the nodes can observe the measurements and share the data only with their neighbour nodes.

2.1 Local and Global estimation

We start with the estimation of local received power when the nodes do not cooperate between each other.

We are interested in estimating the parameter, denoted as P_k^o, in the form

$$P_k^o = \mathrm{E}[|x_k(n)|^2]. \quad (2)$$

By considering the standard cost function of Least Mean Square (LMS) type of filter [14], the mean square error of estimating the local received power adaptively, given the signal model (1), is

$$J_k(P) = \mathrm{E}[||x_k(n)|^2 - P|^2]. \quad (3)$$

Minimization of (3) with respect to parameter P independently at every node k results in the local solution, which is given in (2) and noted as P_k^o. The latter has the desired form of a test statistic of an energy detector. The local solution P_k^o is optimal in the sense of minimum mean square error. P_k^o will vary at every node k, since the expectation of $\mathrm{E}\,||x_k(n)|^2$ varies due to channel gains. Using the standard steepest descent procedure we can find an iterative solution to the (3). The derivative of cost function (3) is

$$\nabla_P J_k(P) = 2\left(P - \mathrm{E}\left[|x_k(n)|^2\right]\right). \quad (4)$$

We include the constant 2 into the step size μ. Since usually the expected value in (4) is not known in practice, we replace the $E|x_k(n)|^2$ with its approximations $|x_k(n)|^2$ and we get the local LMS recursion

$$\hat{P}_k(n+1) = \hat{P}_k(n) + \mu(|x_k(n)|^2 - \hat{P}_k(n)), \quad (5)$$

which is in the form of an exponential smoother.

According to model (1), the power of the PU signal is attenuated at every node k. The locally estimated power varies between nodes k. Therefore if the channel gain at node k is low, the resulting energy detection performance is low. The result is opposite, when the node has a good channel gain. When nodes cooperate to estimate a common parameter P^o, the resulting detection performance can become more stable and robust to channel fading. To accomplish this purpose, we propose the following global parameter P^o in the form

$$P^o = \frac{1}{K}\sum_{k=1}^{K} \mathrm{E}\left[|x_k(n)|^2\right], \quad (6)$$

which is the average of the received power across the nodes $k \in K$ in the network. According to (1) and the assumption about the distribution of channel constants we observe that if the PU signal is present and when we have sufficient number of nodes in the CR network, the effect of varying channel gains is averaged out. The sum over channel gains converges close to its variance 1.

Similarly to the local cost (3), the corresponding global cost function can be given as:

$$J^{global}(P) = \sum_{k=1}^{K} \mathrm{E}\left[|x_k(n)|^2 - P\right|^2]. \quad (7)$$

where we have used the form of global cost as proposed in [13], [11], [5]. Minimization of the mean square error across the network (7) with respect to P results in the optimal solution, which is denoted as P^o and is given by (6). The observation process $|x_k(n)|^2$ is assumed to be stationary at node k but the distributions of the observations vary across the nodes in the network.

An iterative solution for minimizing (7) can similarly be found using steepest descent method. The derivative of cost function (7) is

$$\nabla_P J^{global}(P) = 2\sum_{k=1}^{K}\left(P - \mathrm{E}\left[|x_k(n)|^2\right]\right) \quad (8)$$

Similarly the constant 2 can be included in step size μ. By replacing the moment $\mathrm{E}\,|x_k(n)|^2$ with its approximation $|x_k(n)|^2$ we get the global LMS type recursion:

$$\hat{P}(n+1) = \hat{P}(n) + \mu\left[\sum_{k=1}^{K}\left[|x_k(n)|^2 - \hat{P}(n)\right]\right], \quad (9)$$

The algorithm (9) requires that all the observations are collected together to a fusion center for updating the recursion to compute a new estimate $\hat{P}(n+1)$. Thus global information - data collected from all the nodes in the network is needed to be present for the algorithm to operate. Since the algorithm (9) is not distributed, we propose next the distributed strategy for the nodes to estimate P^o based on the information what is available to the nodes.

2.2 Distributed Diffusion LMS estimation

We assume that K nodes in the CR network are interested to estimate the scalar parameter P^0 in a distributed manner, where nodes can rely only on the information, what is available to them. Nodes do not have access to a global data. We need to find a way to approximate the global cost (7) in a distributed manner. The estimate $\hat{P}_k(n)$ of optimal (6) should be present at every node in the network for nodes being able independently to perform an energy detection.

The derivation of the CTA diffusion LMS type of algorithm follows the idea proposed in [6]. Let $\mathcal{N}_k$ denote the neighbourhood group of node $k \in K$. We assume the connections between the nodes in the

neighbourhood are directed. Let us define a $K \times K$ matrix C, which is doubly stochastic (i.e its rows and columns should sum up to 1). The non-negative element $c_{k,l}$ of matrix C defines if a measurement from node l (including node k) is available for node k. It holds that the element $c_{k,l} = 0$ if $l \notin \mathcal{N}_k$.

By using the elements $c_{k,l}$ we can express the local cost and the corresponding local optimal solution in the neighbourhood of node k as follows

$$J_k^{loc}(P) = \sum_{l \in N_k} c_{k,l}\, \mathrm{E}[||x_k(n)|^2 - P|^2], \qquad (10)$$

$$P_k^{loc} = \sum_{l \in N_k} c_{k,l}\, \mathrm{E}[|x_k(n)|^2. \qquad (11)$$

Similarly as in [6] the global cost can be fractioned into the local cost of over the neighbourhood of node k and local costs over the neighbourhood of other nodes. Using the completion of squares method to relate parameter P and local optimal solution P_k^{loc} the global cost function can be expressed as:

$$J^{glob}(P) = \sum_{l \in N_k} c_{k,l}\, \mathrm{E}\left[|x_k(n)|^2 - P\right|^2] + \sum_{l \neq k}^{K} \|P - P_l^{loc}\|^2, \qquad (12)$$

where we have used the fact that rows of C sum up to 1. Let us define a $K \times K$ doubly stochastic matrix B. The non-negative element $b_{k,l}$ of matrix B defines if data from from node l (including node k) is available for node k. It holds that $b_{k,l} = 0$ if $l \notin \mathcal{N}_k$. With the help of elements $b_{k,l}$ the corresponding approximation of (12) in case of distributed estimation is given as

$$J_k^{dist}(P) = \sum_{l \in N_k} c_{k,l}\, \mathrm{E}\left[|x_k(n)|^2 - P\right|^2] + \sum_{l \in \mathcal{N}_k/\{k\}} b_{k,l} \|P - \psi_l\|^2. \qquad (13)$$

In (13) the P_l^{loc} has been replaced with the intermediate estimate ψ_l available at node l. The derivate of the cost function is (13) is

$$\nabla_P J_k^{dist}(P) = 2 \sum_{l \in N_k} c_{k,l} \left[P - \mathrm{E}\left[|x_l(n)|^2\right]\right] + 2 \sum_{l \in \mathcal{N}_k/\{k\}} b_{k,l} \left[P - \psi_l\right]. \qquad (14)$$

The cost (13) can be used to obtain a recursion for the estimate of P at node k, denoted as $\hat{P}_k(n)$. Using the steepest descent method, which is divided into two parts we get an iterative solution for (13) as follows:

$$\hat{\psi}_k(n) = \hat{P}_k(n) + \nu_k \sum_{l \in \mathcal{N}_k/\{k\}} b_{k,l} \left[\psi_l - \hat{P}_k(n)\right]$$
$$\hat{P}_k(n+1) = \hat{\psi}_k(n) + \mu_k \sum_{l \in N_k} c_{k,l} \left[\mathrm{E}\,|x_l(n)|^2 - \hat{P}_k(n)\right]. \qquad (15)$$

Here different step size at the nodes k have been assigned and the constant 2 has been taken inside step sizes μ_k and ν_k. In the first row of (15) we replace $\hat{\psi}_l$ with $\hat{P}_l(n)$, which is available at time n. In the second row of (15) we replace $\hat{P}_k(n)$ by $\hat{\psi}_k(n)$. Thus the second row leads to

$$\hat{\psi}_k(n) = \left[1 - \nu_k \sum_{l \in \mathcal{N}_k/\{k\}} b_{k,l}\right] \hat{\psi}_k(n) + \nu_k \sum_{l \in \mathcal{N}_k/\{k\}} b_{k,l} \hat{\psi}_l(n). \qquad (16)$$

Let us finally define a $K \times K$ doubly stochastic matrix A. The non-negative element $a_{k,l}$ defines if estimate from node l (including node k) is available for node k. Thus for the elements of A it holds that $a_{k,l} = 0$ if $l \notin \mathcal{N}_k$. By taking $a_{k,k} = \left[1 - \nu_k \sum_{l \in \mathcal{N}_k/\{k\}} b_{k,l}\right]$ and $a_{k,l} = \nu_k b_{k,l}$ for $l \neq k$ we arrive to LMS type recursion what is called combine and adapt (CTA). We summarise it together with energy detection as Algorithm 1. In the CTA diffusion algoritm, the estimates

Algorithm 1 Distributed CTA Diffusion Power Estimation

Start with $\hat{P}_k(0) = P(0)$.
Given non-negative real coefficients $a_{k,l}, c_{k,l}$
for every time instant $n \geq 1$ **do**
  for every node $k = 1, ..., K$ **do**
    1. Power estimation:
    $\hat{\psi}_k(n) = \sum_{l \in N_k} a_{k,l} \hat{P}_l(n)$
    $\hat{P}_k(n+1) = \hat{\psi}_k(n)$
      $+\mu_k \sum_{l \in N_k} c_{k,l} \left(|x_l(n)|^2 - \psi_k(n)\right).$
    2. Detection decision:
    $H_0 : \hat{P}_k(n+1) < \gamma$ or $H_1 : \hat{P}_k(n+1) > \gamma$.
    (Refer to (43) for selecting the threshold).
  end for
end for

$\left\{\hat{P}_k(n)\right\}_{k \in \mathcal{N}_k}$ including the $\hat{P}_k(n)$ from node k are combined together at every node k. This is the diffusion step. Then the combined estimate $\hat{\psi}_k(n)$ at node

k is used to calculate the new estimate $\hat{P}_k(n+1)$ at node k, using the new observation available for node k, at time instant n. This is the incremental step.

3 Performance analysis

The performance analysis of the proposed algorithms is divided into three parts. First we analyse the mean and variance of CTA power estimates. Next we analyse the resulting energy detection performance. Let us note that for the theoretical analysis we need to know the values of the channel gains. For implementing the Algorithm 1 in practice, this knowledge is not required.

For more convenient notation we stack the estimates and observations from all the nodes into $K \times 1$ vectors as follows:

$$\hat{P}(n) = \begin{bmatrix} \hat{P}_1(n) \\ \vdots \\ \hat{P}_K(n) \end{bmatrix}, \quad X(n) = \begin{bmatrix} |x_1(n)|^2 \\ \vdots \\ |x_K(n)|^2 \end{bmatrix}. \quad (17)$$

Let us define additional matrix $\mathcal{M}$ which holds the LMS algorithm step size parameters as follows

$$\mathcal{M} = \mathrm{diag}\left(\mu_1, \ldots, \mu_K\right). \quad (18)$$

Then we can write the recursion in the following form

$$\hat{P}(n+1) = (I - \mathcal{M})\, A\hat{P}(n) + \mathcal{M}CX(n). \quad (19)$$

The initial estimate is $\hat{P}(n) = \left[\hat{P}_k(0) \ldots \hat{P}_k(0)\right]^T$. For the CTA algorithm $A = A_{\text{diff}}$, $C = I$ or $C = A_{\text{diff}}$ in case the measurements are exchanged between the nodes. Observe, that we can use the same recursion also for analysing the results in ring around topology [12], where $A = A_{\text{ring}}$ and $C = I$. For example when we have 3 nodes in the network, the corresponding ring around and diffusion topologies are given as follows

$$A_{\text{ring}} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad A_{\text{diff}} = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix}. \quad (20)$$

Both the A_{ring} and A_{diff} are doubly stochastic. In this paper the diffusion topology is composed of local ($A = I$) and ring-around (A_{diff}) topologies, where we assign constant equal weights 0.5 for every allowed transition.

For evaluating the performance of the estimation algorithms and the resulting energy detection, we need to evaluate first the theoretical mean and variance of estimates $\hat{P}(n)$.

3.1 Mean of estimation

According to signal model (1) let $\mathrm{E}(X(n)|H_i)$, $i = 0, 1$ denote the conditional mean under hypotheses H_0 (the mean when PU signal is absent) or under H_1 (PU signal is present) respectively. Considering the recursion (19), we have

$$\mathrm{E}(\hat{P}(n+1)) = (I - \mathcal{M})\, A\, \mathrm{E}(\hat{P}(n)) + \mathcal{M}C\, \mathrm{E}\left(X(n)\right), \quad (21)$$

where the initial value is given as $\mathrm{E}(\hat{P}(0)) = \left[\hat{P}_k(0) \ldots \hat{P}_k(0)\right]^T$.

We define a column vector w_k, with dimension $K \times 1$ and which elements are zero, except the element k of vector $w_k(k)$ is one, i.e

$$w_k = \mathrm{col}(0 \ldots, (w_k(k) = 1), \ldots 0). \quad (22)$$

The conditional mean of $\hat{P}_k(n)$ under hypothesis H_i, for $i = 0, 1$ at node k can be found with the help of vector w_k as follows

$$\mathrm{E}(\hat{P}_k(n)|H_i) = w_k^T\, \mathrm{E}(\hat{P}(n)|H_i) \quad \textit{for i=0,1.} \quad (23)$$

After iterating (21), the mean recursion can be given in the following equivalent form

$$\mathrm{E}(\hat{P}(n)) = \left[(I - \mathcal{M})\, A\right]^n \hat{P}(0) + \left[\sum_{i=0}^{n-1} \left[(I - \mathcal{M})\, A\right]^i\right] \mathcal{M}C\, \mathrm{E}[X(n)]. \quad (24)$$

We are interested in the mean of the estimates in steady state, when the filter has converged, i.e when $n \to \infty$. Thus according to (24) we need to analyse the asymptotic behaviour of the power of matrix $\left[(I - \mathcal{M})\, A\right]^n$ and the limit of geometric series $\sum_{i=0}^{n-1} \left[(I - \mathcal{M})\, A\right]^i$. Considering [15, Theorem 5.6.12], the power of matrix $\left[(I - \mathcal{M})\, A\right]$ converges asymptotically to zero if the matrix is stable. The matrix is stable if and only if the eigenvalues λ_i of matrix $\left[(I - \mathcal{M})\, A\right]$ are strictly inside the unit circle, i.e

$$|\lambda_k \left[(I - \mathcal{M})\, A\right]| < 1 \quad \textit{for all k=1...K.} \quad (25)$$

Thus given the diffusion strategies with a doubly stochastic matrices A and C the convergence of power of the matrix $\left[(I - \mathcal{M})\, A\right]$ to zero is dependant on the selected step sizes in matrix $\mathcal{M}$. The choice of step sizes should guarantee that the condition (25) holds.

In the recursion the matrix A is equal to doubly stochastic matrix A_{ring}, A_{diff} or identity matrix I. The matrix C is doubly stochastic or equals to I if no

measurements are fused. According to matrix spectral norm we can write for CTA algorithm that

$$\| (I - \mathcal{M}) A\|_2 \leq \| (I - \mathcal{M}) \|_2 \|A\|_2. \quad (26)$$

The spectral norm of doubly stochastic matrices A, C and identity matrix I is 1. Since the matrix $(I - \mathcal{M})$ is symmetric, we have that $|| (I - \mathcal{M}) ||_2 = |\lambda_{max} ((I - \mathcal{M}))|$. If the matrix on the LHS of (26) is also symmetric, then we can also replace the LHS with $|| (I - \mathcal{M}) A||_2 = |\lambda_{max} ((I - \mathcal{M}) A)|$. We have

$$|\lambda_{max} ((I - \mathcal{M}) A)| \leq |\lambda_{max} (I - \mathcal{M})| . \quad (27)$$

We conclude that in general if the step sizes μ_k in matrix M are selected so that the spectrum of matrix $(I - \mathcal{M})$ is inside the unit circle, it holds also that the matrix $(I - \mathcal{M}) A$ is stable.

For the selection of μ_k, $k = 1 \dots K$ so that the diagonal matrix $(I - \mathcal{M})$ is stable, we have the following condition

$$|\lambda_k [(I - \mathcal{M}) A]| = |1 - \mu_k| < 1 \quad \textit{for all k=1...K.} \quad (28)$$

Since in our model we have only one mode of convergence of the filter [14], μ_k should be selected in the range:

$$0 < \mu_k < 2. \quad (29)$$

The geometric series $S_n = \sum_{i=0}^{n-1} [(I - \mathcal{M}) A]^i$ is generated by matrix $[(I - \mathcal{M}) A]$ and converges if and only if the condition (25) holds for all λ_i. When it holds we can write the geometric series as follows

$$S_n = [I - [(I - \mathcal{M}) A]]^{-1} [I - [(I - \mathcal{M}) A]^n] . \quad (30)$$

Hence according to (25) as $n \to \infty$ the power of matrix $[(I - \mathcal{M}) A]^n$ converges to zero. Thus by using the coverged result of the geometric series and by noting the mean of $\hat{P}(n)$ in steady state and under both hypotheses H_i, $i = 0, 1$ as $E(\hat{P}(\infty)|H_i)$, we can write

$$\mathrm{E}(\hat{P}(\infty)|H_i) = [I - [(I - \mathcal{M}) A]]^{-1} \times \mathcal{M}C \, \mathrm{E}[X(n)|H_i], \quad (31)$$

where the conditional expectations of observations are given by (1). Similarly to (23) we have that the mean of $\hat{P}_k(n)$ in steady state is

$$\mathrm{E}(\hat{P}_k(\infty)|H_i) = w_k^T \, \mathrm{E}(\hat{P}(\infty)|H_i) \quad \textit{for i=0,1.} \quad (32)$$

3.2 Variance of estimation

To find the recursion for the variance of $\hat{P}_k(n)$, at node $k \in K$, we start from the recursions (19), (21) and derive first the recursion for covariance matrix $Cov(\hat{P}(n))$. The covariance of $\hat{P}(n)$ is defined as

$$\mathrm{Cov}(\hat{P}(n)) = E \left(\hat{P}(n) - \mathrm{E}[\hat{P}(n)] \right) \times \left(\hat{P}(n) - \mathrm{E}[\hat{P}(n)] \right)^T . \quad (33)$$

Let us note the conditional covariance of estimates under the hypothesis H_i, $i = 0, 1$ as $\mathrm{Cov}(\hat{P}(n+1)|H_i)$. Similarly let $\mathrm{Cov}(X(n)|H_i)$ denote the conventional covariance of observations. After substituting (19) and (21) into (33), taking expectation and considering the fact that $\hat{P}(n)$ is independent of observation vector $X(n)$, it can be shown that the covariance recursion is

$$\mathrm{Cov}(\hat{P}(n+1)|H_i) = (I - \mathcal{M}) A \, \mathrm{Cov}(\hat{P}(n|H_i)) \times A^T (I - \mathcal{M}) + \mathcal{M}C \, \mathrm{Cov}(X(n)|H_i) C^T \mathcal{M}. \quad (34)$$

where initial estimate of covariance matrix is noted by $\mathrm{Cov}(\hat{P}(0)|H_i)$, $i = 0, 1$. The covariance matrix of observations $\mathrm{Cov}(X(n)|H_i)$ is constant over time n.

The covariance matrix $\mathrm{Cov}(X(n)|H_i)$ of $K \times K$ has the following structure. When PU signal is present the main diagonal elements of matrix $\mathrm{Cov}(X(n)|H_1)$ - the variances of observations at node $k \in K$ can be shown to be:

$$\mathrm{Var}(|x_k(n)|^2|H_1) = \left(|\alpha_k|^2 \sigma_s^2 + \sigma_{v,k}^2 \right)^2 . \quad (35)$$

When PU signal is not present and according to Assumption 2 the variances of observations at node $k \in K$ are given as

$$\mathrm{Var}(|x_k(n)|^2|H_0) = \sigma_{v,k}^4. \quad (36)$$

When PU signal is present, the off diagonal elements of matrix $\mathrm{Cov}(X(n)|H_1)$ - the covariance of observations at nodes k and j if $k, j \in K$ and $i \neq j$ can be shown to be:

$$\mathrm{Cov}(|x_k(n)|^2, |x_j(n)|^2|H_1) = |\alpha_k|^2 |\alpha_j|^2 \sigma_s^4. \quad (37)$$

According to Assumption 2 the noise realizations $v_k(n)$ and $v_j(n)$ are uncorrelated in time and space for $k, j \in K$ and $i \neq j$. Thus when PU signal is absent the covariance of observations is

$$\mathrm{Cov}(|x_k(n)|^2, |x_j(n)|^2|H_0) = 0, \quad (38)$$

for $k, j \in K$ and $i \neq j$.

The variance of $\hat{P}_k(n)$ at node k under the hypothesis H_i, $i = 0, 1$, can be found by multiplying the recursion (34) with vector w_k^T from the left and with vector w_k from the right

$$\begin{aligned}\mathrm{Var}(\hat{P}_k(n+1)|H_i) = w_k^T\,(I-\mathcal{M})\,A\,\mathrm{Cov}(\hat{P}(n)|H_i)\\ \times A^T\,(I-\mathcal{M})\,w_k\\ + w_k^T\mathcal{M}C\,\mathrm{Cov}(X(n)|H_i)\\ \times C^T\mathcal{M}w_k. \qquad (39)\end{aligned}$$

To derive the steady state value for $\mathrm{Var}(\hat{P}_k(n)|H_i)$ we note that (34) is in a form of discrete time algebraic Riccati equation (DARE), [16, App. E]. Due to space constraints we skip the derivation details and note that the steady state variance $\mathrm{Var}(\hat{P}_k(\infty)H_i)$, $i = 0, 1$, at node $k \in K$ can be recovered by selecting the $\{k, k\}$ element of the steady state covariance matrix $\mathrm{Cov}(\hat{P}(\infty)|H_i)$, which has been found as a solution to DARE. We have finally

$$\mathrm{Var}(\hat{P}_k(\infty)|H_i) = w_k^T\left[\mathrm{Cov}(\hat{P}(\infty)|H_i)\right]w_k. \quad (40)$$

3.3 Detection Performance Analysis

As mentioned earlier the test statistic of the energy detector at node k at time instant n is estimated using CTA signal power estimation algorithms. Thus the resulting detection performance is dependant on the performance of the underlying estimation process. For deriving the formulas of probability of detection (P_{D}) and probability of false alarm (P_{FA}) we need to evaluate the probability density function (PDF) of the test statistic $\hat{P}_k(n)$ under both hypotheses H_0 and H_1.

Since the input signal is CSCG and in case $K = 1$, the test statistic of ED $\hat{P}_k(n)$ is local and under both hypothesis a Chi-Square distributed random variable with 2N degrees of freedom. The test statistic $\hat{P}_k(n)$ is obtained as a sum of a number of identically distributed variables and hence the CLT can be applied to approximate the Chi square distribution by a Gaussian distribution [17]. According to AS3 the number of samples is large enough, and the CLT is expected to apply.

The global test statistic $\hat{P}_k(n)$ in case of hypothesis H_1, is however estimated over independent, but not identically distributed variables. In such a case the Lyapunov CLT [18] still be applied over a large number of samples to result in a Gaussian approximation. We found in previous section that the variance $\hat{P}_k(n)$ in steady state is bounded. In [12] the formulas for the P_{FA} and $P_{\mathrm{D,k}}$ of the energy detector have been derived.

Using these results and by taking into account the (23) and (39), we provide approximate formulas for the resulting energy detection performance.

The probability of false alarm P_{FA} of the energy detector under hypothesis H_0 (using the theoretical mean and variance of estimates under H_0) is given as follows

$$P_{\mathrm{FA}} = Q\left(\frac{\gamma - \mathrm{E}(\hat{P}_k(n)|H_0)}{\sqrt{\mathrm{Var}(\hat{P}_k(n)|H_0)}}\right). \qquad (41)$$

Based on the assumption AS2, we observe that the formula holds for every node $k \in K$.

The probability of detection of an energy detector under hypothesis H_1 (using the mean and variance of estimates under H_1) is correspondingly given as follows

$$P_{\mathrm{D,k}} = Q\left(\frac{\gamma - \mathrm{E}(\hat{P}_k(n)|H_1)}{\sqrt{\mathrm{Var}(\hat{P}_k(n)|H_1)}}\right). \qquad (42)$$

The sensing threshold is found from (41) by fixing the desired value of P_{FA}. Thus

$$\begin{aligned}\gamma \;&=\; \mathrm{E}[\hat{P}_k(n)|H_0]\\ &+\; Q^{-1}\left(P_{\mathrm{FA}}\right)\sqrt{\mathrm{Var}[\hat{P}_k(n)|H_0]}.\end{aligned} \qquad (43)$$

Due to the assumption AS2 the thresholds for every CR node k are equal.

However calculation of the threshold requires knowledge about the moments of the estimation algorithm under hypothesis H_0 and these moments are dependant on the algorithm parameters (especially on the step size). In practice for the threshold calculation, the required moments can be calculated in advance using (23) and (39), known values of step size, noise power and then inserting these results into (43).

4 Simulation results

In the numerical simulation section we investigate the CTA type of power estimation algorithm (with constant step size). We compare the results with the previously proposed ring around [12] power estimation algorithm. Secondly we view the resulting energy detection performance of the CTA diffusion algorithm and compare to the ring around algorithm. In all these simulations the PU signal $s(n)$ is taken as QPSK with unit power S, under the active hypothesis H_1, the step size is: $\mu = 0.01$.

4.1 Local and distributed power estimation

We start with the investigation of the estimation algorithms. The channel gain is assumed to be constant and obtained by: $\alpha_k \sim \mathcal{C}N(0,1)$. We use the same channel gain values for all the algorithms. All the nodes in the network receive $N = 2000$ samples. During samples $n = 1 \dots 1000$ the PU signal with constant unit power S is present. The power S is attenuated by the channel gain $|\alpha_k|^2$. In sample range $n = 1001 \dots 2000$ the PU signal is absent and only noise is present. Under both detection hypothesis the noise power is $\sigma_v^2 = 1$ and assumed to be the same in all the nodes. In the following simulations no measurements are exchanged thus, $C = I$. When nodes do not cooperate, then the estimation results are highly dependant on the given channel gains and therefore vary across the nodes in the network. When nodes cooperate using to ring around topology, then the corresponding power estimates are given in the Fig. 1. All the estimated power values in the CR network of 10 nodes are plotted in one figure. In addition the optimal solution P^o has been calculated according to (6) using the given channel gains values and is added in the figure as a back dashed line. We see that the estimates of P^o fluctuate around the optimal solution, under both active detection hypotheses. Let us note, that it can be numerically verified, that the theoretical mean and variance of the ring around estimates, which are calculated using the formulas (23) and (39), match with the results, which are found using the formulas [12, Eq. 5] and [12, Eq. 10].

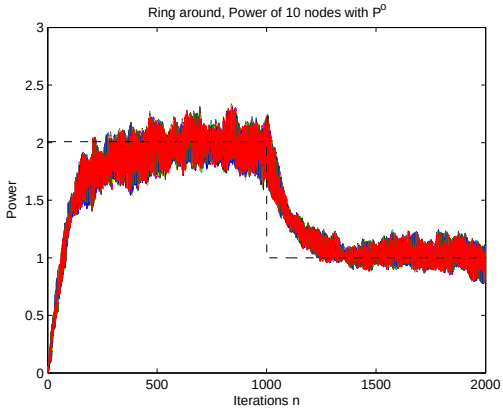

Figure 1: Local power estimation

The CTA diffusion algorithm the estimates of the received power together with optimal solution P^o have been plotted in Fig. 2. Compared to the ring round, the variance of estimates of CTA algorithm is lower than the variance of estimates of ring around algorithm. We can conclude that the preciseness of power estimates increases when CTA algorithm is used. Thus also the resulting detection performance increases, what we show in next subsection.

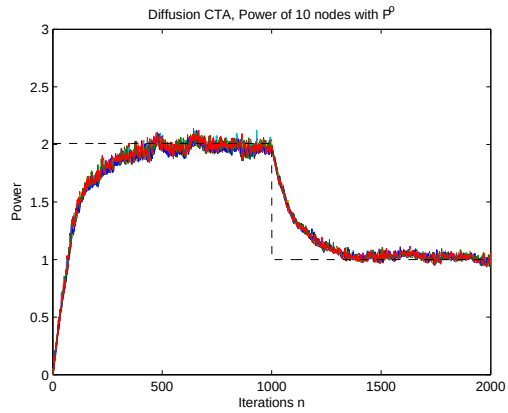

Figure 2: Local power estimation

4.2 Probability of detection

Next we investigate the probability of detection using the proposed distributed CTA power estimation algorithm. We compare the performance of 5 different network sizes: $K = 1, 3, 10, 30, 50$ nodes. The estimated and theoretical results of P_{D} of the last nodes in the set are compared, i.e at nodes $k = K$. In the simulations the converged power estimate is used for detection i.e $\hat{P}_k(\infty)$. The theoretical mean and variance of power estimates can hence be calculated directly using the steady state formulas (32), (40). The mean and covariance of the observation vector is taken under the detection hypothesis H_1 and the choices of values of matrices A and C to define CTA or ring around algorithms.

We set the desired $P_{\mathrm{FA}} = 10^{-4}$. The thresholds of the energy detectors at nodes $k \in K$ are calculated using (43) and by using the corresponding theoretical steady state mean and variance of the power estimates under detection hypothesis H_0.

For estimating the P_{D} we use the Monte Carlo method [19]. The estimated P_{D} is compared with the theoretical P_D. The latter is calculated using (42) and using the corresponding steady state mean and variance of the power estimates of the two algorithms under detection hypothesis H_1.

First we set $C = I$. The detection performances of the ring around, and CTA algorithms are shown in Fig. 3, and in Fig. 4 respectively. We see that there is

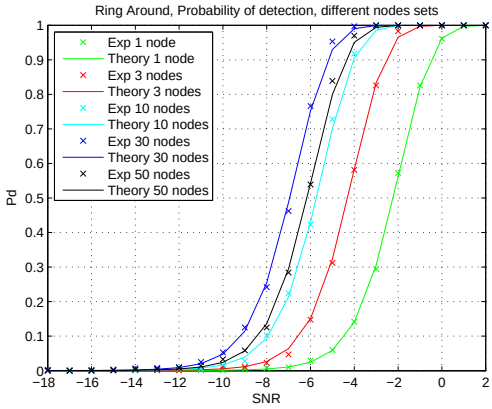

Figure 3: Probability of detection, ring around, $C = I$

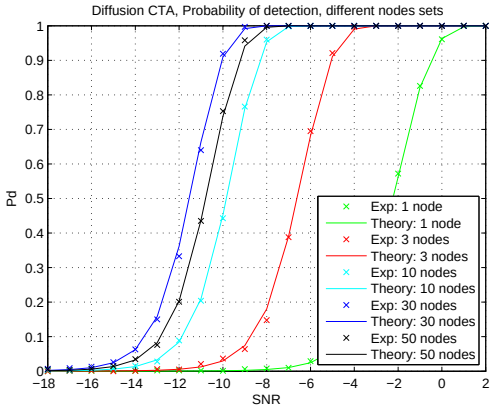

Figure 4: Probability of detection, CTA, $C = I$

a good match between estimated and theoretical P_{D}. Due to the smaller variance, the CTA algorithm outperforms the ring around algorithm. As the number of nodes in the network increases, about 4 dB is gained with respect to the noise power.

When also measurements from a neighbour nodes are available and we set $C = A_{\mathrm{diff}}$ for CTA algorithm. The result is shown in Fig. 5. We see minor increase in the detection performance when additionally measurements are exchanged between the nodes. When the data transfer and processing capacity at the nodes is limited (energy constants etc), then the measurement exchange does not give significant improvement in resulting detection performance. However by fusing more estimates compared to the simplest ring around algorithm, we see notable improvement in resulting detection performance.

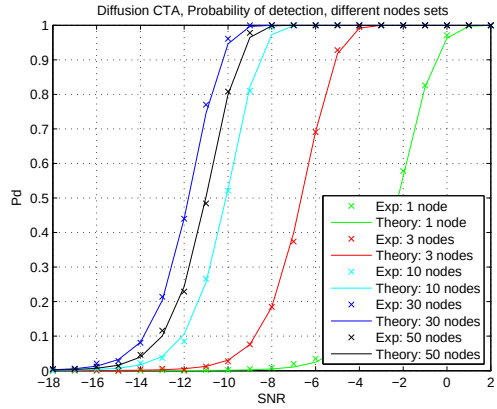

Figure 5: Probability of detection, CTA topology, $C = A_{diff}$

5 Conclusion

In this paper we proposed a diffusion based distributed power estimation approach, that is applicable for CR networks for detecting the presence of PU signal. We derived CTA diffusion based power estimation algorithm for energy detection. The performance analysis of the derived algorithm was carried out and simulations were run. It was shown that the CTA diffusion power estimation algorithm outperforms the previously proposed ring around algorithm, while the effect of exchanging also measurements is rather small. The proposed algorithm is able to track changes in received signal power and is usable in cognitive radio systems.

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APPENDIX C

Publication P3.

A. Ainomäe, T. Trump, and M. Bengtsson, "Distributed diffusion LMS based energy detection," in IEEE 6th International Congress on Ultra Modern Telecommunications and Control Systems and Workshops (ICUMT)*, St. Petersburg, Russia, 2014, pp. 176 – 183, ETIS 1.1.*

This paper is the generalized ATC version of the previous conference paper, including derivation and simulations examples, not included in the previous paper.

Distributed Diffusion LMS based Energy Detection

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***Abstract*—Cognitive radio (CR) is seen as a promising technology to make radio spectrum usage more effective by providing an opportunistic access for secondary users to the licensed spectrum areas. CR systems need to detect the presence of a primary user (PU) signal by continuously sensing the spectrum area of interest. Radiowave propagation effects like fading and shadowing often complicate sensing of spectrum holes because the PU signal can be weak in a particular area. Cooperative spectrum sensing is seen as a prospective solution to enhance the detection of PU signals. This paper studies distributed spectrum sensing in a cognitive radio context. We investigate distributed energy detection schemes without using any fusion center. We propose the usage of distributed, diffusion least mean square (LMS) type of power estimation algorithms. In this paper an Adapt and Combine (ATC) diffusion based power estimation scheme is proposed and the performance is compared with the Combine and Adapt (CTA) and ring-around schemes in a common framework. The PU signal is assumed to be slowly fading. We analyse the resulting energy detection performance and verify the theoretical findings through simulations.**

***Index Terms*—Cognitive radio, distributed estimation, diffusion LMS, diffusion networks, distributed detection, energy detection.**

I. Introduction

The cognitive radio (CR) system is dynamic. Often in practice the statistical information (for example conditional probability density of observations, prior probabilities of detection hypotheses, longer time statistical behaviour of primary user (PU)) is not available *a priori* for constructing a PU signal detection solution. The properties of the test statistics (for making a detection decision) may change in time.

In cognitive radio context we would like to avoid interference to the PU user and find free spectrum opportunities as fast as possible. On-line distributed network learning methods are able to learn the statistical information based on observations received by the nodes in the network. These methods can react to possible changes in the properties of estimated statistics in real time.

Several proposed distributed spectrum sensing solutions make use of a central fusion center [1], [2], [3], [4]. A fusion center is however seen as a single point of failure in the network since a malfunction in this unit affects the performance of the whole distributed solution. We propose a power estimation solution, where the available power estimates (and measurements) are fused in cognitive radio network nodes, to allow all nodes to make detection decisions based on data from the neighbour nodes and without involvement of any central processing unit. Such a solution enhances network failure resistance (at the cost of slightly increased information overhead in the network).

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Several distributed adaptive estimation and detection schemes have been studied in the past. Least mean square (LMS) and recursive least squares (RLS) based estimation schemes are analysed for example in [5], [6], [7], [8] and consensus based schemes in [9], [10], [11], [12]. Optimal, matched filter distributed detection, based on diffusion type LMS and RLS estimation schemes, was studied in [13]. Here, we make the assumption that the CR network does not have any prior information about the waveform of the PU signal in the secondary nodes and hence we cannot design a matched filter. Therefore energy detection becomes a practical solution.

A ring network topology for distributed energy detection without a fusion centre has been suggested in [14]. In [15] we proposed and analysed an estimation based recursive calculation of the test statistics for the energy detectors in cognitive radio network with ring topology. The test statistic in form of a converged power estimate is the soft information used for making the detection decision at every node. Ring networks are however sensitive to link failures. Combine and Adapt (CTA) diffusion based recursive calculation of the test statistics for the energy detectors was proposed and studied in [16]. In this paper we analyse the Adapt and Combine (ATC) version of diffusion LMS type of received power estimation algorithm. The performance of the ATC diffusion based distributed power estimator is compared with the previously proposed CTA [16] and ring [15] schemes to complete the analysis. The resulting energy detection performance is studied and is dependent on the performance of the used distributed recursive power estimation algorithm.

We organize the remainder of the paper as follows. In section II we review the system model and the basics of energy detection. We derive an ATC type received signal power estimation algorithm based on diffusion LMS strategy and summarize the CTA based version. In section III we analyse the performance of the proposed distributed power estimation

algorithm (using a common model) and the resulting energy detection. In section IV we present our simulations results.

II. Distributed power estimation and detection

We assume the following signal model at node k:

$$\begin{aligned} H_0 &: E[|x_k(n)|^2] = E[|v_k(n)|^2] \\ H_1 &: E[|x_k(n)|^2] = E[|\alpha_k|^2|s(n)|^2] + E[|v_k(n)|^2], \end{aligned} \quad (1)$$

where $k = 1, 2, ..., K$ is the node number and $n = 1, 2, ...N$ is the sample index. $v_k(n)$ is independent and identically distributed (*i.i.d*) circularly symmetric complex Gaussian (CSCG) noise with zero mean and variance $E[|v_k(n)|^2] = \sigma_{v,k}^2$, i.e. $v(n) \sim \mathcal{CN}(0, \sigma_{v,k}^2)$. The power of the emitted PU signal $s(n)$ is denoted as $E[|s(n)|^2] = S$, under H_1. The primary signal $s(n)$ and the noise $v_k(n)$ are assumed to be statistically independent. The PU signal passes through a slowly fading channel with gain $\alpha_k(n)$. The gain α_k is considered to be constant. Note, that for implementing the energy detector, only the noise variance is needed to determine the detection threshold γ, therefore estimates of the channel gains are not required in practical implementations. Noise power estimation is not considered in this paper. In this paper we make the following assumptions:

- (AS 1) The $x(n)$ is sensed by *K* nodes in the CR network.
- (AS 2) The additive noise $v_k(n)$ is uncorrelated in time and space and has the same power level over all the nodes in the CR network.
- (AS 3) The number of performed iterations N is large enough.
- (AS 4) The links between the CR nodes are ideal and not capacity restricted (no need to quantize the soft information).

We denote the power estimate at node k and at iteration n as $\hat{P}_k(n)$. The network topology is assumed to be fixed over the sensing time. We consider a linear, fixed combination of neighbour estimates and measurements at every node k.

Next we shortly review the global model for estimating the received signal power in cooperative manner (as proposed in [16]). Then we derive an ATC type power estimation algorithm, where the nodes can observe the measurements and share the estimates (and measurements) only with their neighbour nodes, according a to predefined network topology. Finally we propose a data exchange and combination strategy for ATC diffusion algorithm.

A. Global estimation

According to model (1), the power of the PU signal is attenuated at every node k. The locally estimated power varies between nodes k. Therefore if the channel gain at node k is low, the resulting energy detection performance is low. The result is opposite, when the node has a good channel gain. When nodes cooperate to estimate a common parameter P^o, the resulting detection performance will improve. As in [16] we recommend the following form of P^o

$$P^o = \frac{1}{K}\sum_{k=1}^{K} \mathrm{E}\left[|x_k(n)|^2\right] = S\frac{1}{K}\sum_{k=1}^{K}|\alpha_k|^2 + \sigma_v^2. \quad (2)$$

The P^o is the average of the received power across the nodes $k \in K$ in the network. The second equation in (2) follows from the signal model (1) if the PU signal is present and from the assumption AS 2. When we have sufficient number of nodes in the CR network, the effect of varying channel gains is averaged over nodes $k \in K$.

The corresponding global cost function is given as:

$$J^{glob}(P) = \sum_{k=1}^{K} \mathrm{E}\left[|x_k(n)|^2 - P\right]^2, \quad (3)$$

where we have used the form of global cost as proposed in [17], [13], [5]. Minimization of the mean square error across the network (3) with respect to P results in the optimal solution, which is given by (2).

B. Distributed ATC Diffusion LMS estimation

Suppose that K nodes in the CR network are interested in estimating the scalar parameter P^0 in a distributed manner, where nodes rely only on the information, that is available to them. Depending on network topology, nodes are connected only to selected neighbour nodes and do not have access to any global data. The global cost (3) needs to be approximated in a distributed manner. The derivation of the ATC diffusion power estimation algorithm follows the ideas in [18], [6].

Let $\mathcal{N}_k$ denote the neighbourhood group of node $k \in K$, i.e $\mathcal{N}_k$ consists of nodes l which can communicate with node k. We assume that the network is connected and the connection between nodes l and k is unidirectional.

Let us define $K \times K$ doubly stochastic matrix C [1] containing non-negative elements $c_{l,k}$ and $c_{l,k} = 0$ if $l \neq \mathcal{N}_k$ (i.e when data from node l is not available for node k). The local cost and the corresponding local optimal solution in the neighbourhood of node k can be expressed with the help of coefficients $c_{l,k}$ as follows

$$J_k^{loc}(P) = \sum_{l\in\mathcal{N}_k} c_{l,k}\,\mathrm{E}\left[|x_l(n)|^2 - P\right]^2, \quad (4)$$

$$P_k^{loc} = \sum_{l\in\mathcal{N}_k} c_{l,k}\,\mathrm{E}\left[|x_l(n)|^2\right]. \quad (5)$$

The global cost can be fractioned into the local cost of over the neighbourhood of node k and local costs over the neighbourhood of other nodes. Using the completion of squares argument [17] to relate variable P and local optimal solution P_l^{loc}, secondly ignoring the mmse part which is not dependant on P, the global cost function can be expressed as follows

$$J^{glob'}(P) = \sum_{l\in N_k} c_{l,k}\,\mathrm{E}\left[|x_l(n)|^2 - P\right]^2 + \sum_{l\neq k}^{K} \|P - P_l^{loc}\|^2. \quad (6)$$

Node k may not have access to all the data P_l^{loc} in the network. We modify the second member of right hand side (RHS) of (6) by replacing the summation $\sum_{l\neq k}^{K}$ with $\sum_{l\in\mathcal{N}_k/\{k\}}$. Next we replace $\|P - P_l^{loc}\|^2 \approx b_{l,k}\|P - P_l^{loc}\|^2$ ([18, Eq. 117]). We

[1] For a doubly stochastic matric C it holds that $C\mathbb{1} = \mathbb{1}$ and $\mathbb{1}^T C = \mathbb{1}^T$.

collect the non-negative coefficients $b_{l,k}$ in a $K \times K$ matrix B and assume $b_{l,k} = 0$ if $l \neq \mathcal{N}_k$. Also we replace the unknown P_l^{loc} with an intermediate estimate $\hat{\psi}_l$ available at node l. Then the approximation of (6) at node k is given as

$$J_k^{dist}(P) = \sum_{l \in N_k} c_{l,k} \, \mathrm{E}\left[|x_l(n)|^2 - P\right]^2 + \sum_{l \in \mathcal{N}_k / \{k\}} b_{l,k} \|P - \hat{\psi}_l\|^2 \quad (7)$$

and derivative of the cost function is (7) is

$$\nabla_P J_k^{dist}(P) = 2 \sum_{l \in N_k} c_{l,k} \left[P - \mathrm{E}\left[|x_l(n)|^2\right]\right] + 2 \sum_{l \in \mathcal{N}_k / \{k\}} b_{l,k} \left[P - \hat{\psi}_l\right]. \quad (8)$$

The cost (7) can be used to obtain a recursion for the estimate of P at node k, denoted as $\hat{P}_k(n)$. Using the steepest descent method, which is divided into two parts, we get an iterative solution for (7) as follows:

$$\begin{aligned} \hat{\psi}_k(n+1) &= \hat{P}_k(n) + \mu_k \sum_{l \in N_k} c_{l,k} \left[\mathrm{E}\left[|x_l(n)|^2\right] - \hat{P}_k(n)\right] \\ \hat{P}_k(n+1) &= \hat{\psi}_k(n+1) + \nu_k \sum_{l \in \mathcal{N}_k / \{k\}} b_{l,k} \left[\psi_l - \hat{P}_k(n)\right]. \end{aligned} \quad (9)$$

Different step sizes μ_k and ν_k at the nodes k have been assigned and the constants 2 has been incorporated into μ_k and ν_k. In the second equation of (9) we replace $\hat{\psi}_l$ with time dependant $\hat{\psi}_l(n+1)$, $\hat{P}_k(n)$ with $\hat{\psi}_k(n+1)$ and we get

$$\hat{P}_k(n+1) = \left[1 - \nu_k \sum_{l \in \mathcal{N}_k / \{k\}} b_{l,k}\right] \hat{\psi}_k(n+1) + \nu_k \sum_{l \in \mathcal{N}_k / \{k\}} b_{l,k} \hat{\psi}_l(n+1). \quad (10)$$

Next we introduce the coefficients $a_{l,k} = 0$ if $l \neq \mathcal{N}_k$, $a_{l,k} = \nu_k b_{l,k}$ if $l \neq k$ and $a_{k,k} = 1 - \nu_k \sum_{l \in \mathcal{N}_k / \{k\}} b_{l,k}$ if $l = k$. If we collect the coefficients $a_{l,k}$ into a $K \times K$ matrix A, it is straightforward to see that $\sum_{l \in \mathcal{N}_k} a_{l,k} = 1$ for every $k \in K$ and thus A is a left stochastic matrix [2] (but A can be also doubly stochastic). We replace $\mathrm{E}\,|x_l(n)|^2$ with $|x_l(n)|^2$ and finally arrive to the Adapt and Combine (ATC) recursions that we summarise with energy detection as Algorithm 1.

In the ATC diffusion algorithm, during the incremental step, at time instant n, the estimate $\hat{\psi}_k(n+1)$ at node k is calculated using the estimate $\hat{P}_k(n)$ at node k and the new observation available for node k. The coefficients $c_{l,k}$ define how the measurements are exchanged between the nodes. During the diffusion step the estimate $\hat{P}_k(n+1)$ at every node k is calculated using a linear combination of the estimates $\hat{\psi}_l(n+1)$ available for node k. The elements $a_{l,k}$ specify the combination strategy of estimates.

[2]For a left stochastic matric A it holds $\mathbb{1}^T A = \mathbb{1}^T$.

Algorithm 1 Distributed ATC Diffusion Power Estimation

Start with $\hat{P}_k(0) = P(0)$.
Given non-negative real coefficients $a_{l,k}, c_{l,k}$
for every time instant $n \geq 1$ **do**
  for every node $k = 1, ..., K$ **do**
    1. Power estimation:
    $\hat{\psi}_k(n+1) = \hat{P}_k(n) + \mu_k \sum_{l \in N_k} c_{l,k} \left(|x_l(n)|^2 - \hat{P}_k(n)\right)$
    $\hat{P}_k(n+1) = \sum_{l \in N_k} a_{l,k} \hat{\psi}_l(n+1)$.
    2. Detection decision:
    $H_0 : \hat{P}_k(n+1) < \gamma$ or $H_1 : \hat{P}_k(n+1) > \gamma$.
    (Refer to (32) for selecting the threshold).
  end for
end for

Note that in practice the non-negative coefficients $a_{l,k}$ and $c_{l,k}$ can be chosen freely under the conditions, that $C\mathbb{1} = \mathbb{1}$, $\mathbb{1}^T C = \mathbb{1}^T$, $\mathbb{1}^T A = \mathbb{1}^T$, $a_{l,k} = 0$, if $l \neq \mathcal{N}_k$ and $c_{l,k} = 0$ if $l \neq \mathcal{N}_k$. The coefficients $b_{l,k}$ are absorbed into coefficients $a_{l,k}$ and do not have to be considered in practice.

C. Network topologies

In the ring-around topology [15], the power estimates are exchanged circularly between the nodes. At time instant n, node k has access only to one estimate $\hat{P}_{(k-1) \bmod K}(n)$ from the node $(k-1) \bmod K$ for calculating $\hat{P}_k(n+1)$. The local estimate $\hat{P}_k(n)$ is ignored. The algorithm uses only locally observed measurements (i.e $C = I$). Thus K estimates have to be sent over the wireless links at time instant n.

To improve the link failure resistance but keep the need for exchanging the data over wireless links in the network minimal, we compose the diffusion topology from the local $(A, C = I)$ and ring-around topologies. At time instant n, at node k the local estimate $\hat{P}_k(n)$ and the estimate $\hat{P}_{(k-1) \bmod K}(n)$ from node $(k-1) \bmod K$ are fused together using equal, constant weight 0.5 for calculating $\hat{P}_k(n+1)$. For example when $K = 3$ and keeping the same notation and conditions for the elements of matrix A, the ring around and diffusion topologies are given as follows

$$A_{\text{ring}}^T = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad A_{\text{diff}}^T = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix}. \quad (11)$$

If measurements are exchanged between the nodes, then we set $C = A_{\text{diff}}^T$. Hence at time instant n additionally K measurements have to be exchanged in the network. Otherwise $C = I$. Therefore in the subsequent sections we assume, that both matrices C and A are doubly stochastic (i.e we have additionally $A\mathbb{1} = \mathbb{1}$) and all the conditions for selecting elements $a_{l,k}$ and $c_{l,k}$, listed in last subsection, are satisfied.

III. Performance analysis

The performance analysis of the proposed algorithms is divided into two parts. First we derive a general model for

analysing the mean and variance of the estimates of the ATC, CTA [16] and ring-around [15] algorithms in one framework. Next we analyse the resulting energy detection performance. Let us note that for the theoretical performance analysis we need to know the values of the channel gains.

For more convenient notation we stack the estimates and observations from all the nodes $k \in K$ into $K \times 1$ time dependent vectors $\hat{P}(n) = \left[\hat{P}_1(n) \dots \hat{P}_K(n)\right]^T$ and $X(n) = \left[|x_1(n)|^2 \dots |x_K(n)|^2\right]^T$ respectively.

Let us define additional matrix $\mathcal{M} = \mathrm{diag}\{\mu_1, \dots, \mu_K\}$, which contain the algorithm step size parameters. We introduce also two additional $K \times K$ matrices L_1 and L_2 for being able to represent all the 3 algorithms using one framework. Then we can write the recursion in the following general form

$$\hat{P}(n+1) = L_2 (I - \mathcal{M}) L_1 \hat{P}(n) + L_2 \mathcal{M} C X(n). \quad (12)$$

The initial estimate is $\hat{P}(0)$. It follows, that we get the ATC algorithm, when we take $L_2 = A_{\text{diff}}^T$, $L_1 = I$, $C = I$ or $C = A_{\text{diff}}^T$ in case of the measurements are exchanged between the nodes. For CTA algorithm we take $L_1 = A_{\text{diff}}^T$, $L_2 = I$, $C = I$ or $C = A_{\text{diff}}^T$. The ring around topology is selected when $L_2 = I$, $L_1 = A_{\text{ring}}^T$ and $C = I$. Note that to keep the matching notation with Algorithm 1, we use transposed matrices in the general recursion. The local, non-cooperative received power estimation is represented by $L_1 = L_2 = C = I$.

For evaluating the performance of the estimation algorithms and the resulting energy detection, we first evaluate the mean and variance of estimates $\hat{P}(n)$.

A. Mean of estimates

Following the signal model (1), let us denote the conditional expectation of the observation vector as $\mathrm{E}[X(n)|H_i]$, where $i = 1$ denotes the case when PU signal is present and $i = 0$ the case when PU signal is absent. In this section we assume that the environment is stationary. The conditional means are thus constant over time.

Considering the general recursion (12), we have

$$\mathrm{E}\left[\hat{P}(n+1)|H_i\right] = L_2 (I - \mathcal{M}) L_1 \mathrm{E}\left[\hat{P}(n)|H_i\right] + L_2 \mathcal{M} C E[X(n)|H_i], \quad (13)$$

for $i = 0, 1$, where the initial value is given as $\mathrm{E}\left[\hat{P}(0)|H_i\right]$.

After iterating we see, that the mean recursion can be given in the following equivalent form

$$\mathrm{E}\left[\hat{P}(n)|H_i\right] = \left[L_2 (I - \mathcal{M}) L_1\right]^n \hat{P}(0) + \left[\sum_{i=0}^{n-1} \left[L_2 (I - \mathcal{M}) L_1\right]^i\right] L_2 \mathcal{M} C E[X(n)|H_i]. \quad (14)$$

We are interested in finding the mean of the estimates, when the filter has converged to a steady state, i.e when $n \to \infty$. Thus according to (14) we need to analyse the asymptotic behaviour of $\left[L_2 (I - \mathcal{M}) L_1\right]^n$ and the limit of the geometric series $\sum_{i=0}^{n-1} \left[L_2 (I - \mathcal{M}) L_1\right]^i$.

According to [19, Lemma 5.6.11], if for a matrix norm it holds that

$$\|L_2 (I - \mathcal{M}) L_1\| < 1 \quad (15)$$

then $\lim_{n\to\infty}[L_2(I - M)L_1]^n \to 0$. Thus given the doubly stochastic matrices L_1, L_2 and C, the choice of step sizes in $\mathcal{M}$ should guarantee that the stability condition (15) holds. Using the matrix 2-norm and the submultiplicativity property of a matrix norm, we have that

$$\|L_2 (I - \mathcal{M}) L_1\|_2 \leq \|L_2\|_2 \| (I - \mathcal{M}) \|_2 \|L_1\|_2 < 1. \quad (16)$$

The spectral norm of a doubly stochastic matrix is 1 [3]. Since the matrix $(I - \mathcal{M})$ is diagonal, we have that

$$\|L_2 (I - \mathcal{M}) L_1\|_2 \leq || (I - \mathcal{M}) ||_2 = \max_k |1 - \mu_k| < 1. \quad (17)$$

We conclude that for the (15) to hold, we must select the μ_k, $k = 1 \dots K$ in $\mathcal{M}$ so that the diagonal matrix $(I - \mathcal{M})$ is stable. Since in our model we have only one mode of convergence of the filter [20], μ_k should be selected in the range:

$$0 < \mu_k < 2. \quad (18)$$

The geometric series $S_n = \sum_{i=0}^{n-1} \left[L_2 (I - \mathcal{M}) L_1\right]^i$ generated by matrix $\left[L_2 (I - \mathcal{M}) L_1\right]$ converges if and only if the condition (15) holds for all λ_i. The condition (15) guarantees that the $[I - \left[L_2 (I - \mathcal{M}) L_1\right]]$ is invertible. Thus we can write the geometric series as follows

$$S_n = \left[I - \left[L_2 (I - \mathcal{M}) L_1\right]\right]^{-1} \left[I - \left[L_2 (I - \mathcal{M}) L_1\right]^n\right]. \quad (19)$$

Hence according to (15) as $n \to \infty$ the geometric series converges to

$$S_n = \left[I - \left[L_2 (I - \mathcal{M}) L_1\right]\right]^{-1}. \quad (20)$$

Thus by noting the mean of $\hat{P}(n)$ in steady state and under both hypotheses H_i, $i = 0, 1$ as $\mathrm{E}\left[\hat{P}(\infty)|H_i\right]$, we can write

$$\mathrm{E}\left[\hat{P}(\infty)|H_i\right] = \left[I - \left[L_2 (I - \mathcal{M}) L_1\right]\right]^{-1} \times L_2 \mathcal{M} C \, \mathrm{E}[X(n)|H_i], \quad (21)$$

where the conditional expectations of observations $\mathrm{E}[X(n)|H_i]$ follow (1).

B. Variance of estimates

Let us denote the conditional covariance of the estimates under the hypothesis H_i, $i = 0, 1$ as $\mathrm{Cov}\left[\hat{P}(n+1)|H_i\right]$. Similarly let $\mathrm{Cov}[X(n)|H_i]$ denote the conditional covariance of the observations. By using recursions (12), (13) and standard definition of covariance, taking expectation and considering

[3] See [19, Problem 8.7.P5]

the fact that $\hat{P}(n)$ is independent of the observation vector $X(n)$, it can be shown that the covariance recursion is

$$\begin{aligned}\mathrm{Cov}\left[\hat{P}(n+1)|H_i\right] = L_2\left(I-\mathcal{M}\right)L_1\,\mathrm{Cov}\left[\hat{P}(n)|H_i\right] \\ \times L_1^T\left(I-\mathcal{M}\right)L_2^T \\ + L_2\mathcal{M}C\,\mathrm{Cov}\left[X(n)|H_i\right]C^T\mathcal{M}L_2^T.\end{aligned} \tag{22}$$

where initial estimate of covariance matrix is noted by $Cov\left[\hat{P}(0)|H_i\right]$, $i=0,1$. The covariance matrix of observations $Cov\left[X(n)|H_i\right]$ is constant over time n.

Next we derive the structure of $K\times K$ covariance matrix $\mathrm{Cov}\left[X(n)|H_i\right]$. By considering the model (1), when PU signal is present the main diagonal elements of matrix $\mathrm{Cov}\left[X(n)|H_1\right]$ – the variances of observations at node $k\in K$ can be shown to be:

$$\mathrm{Var}\left[|x_k(n)|^2|H_1\right] = \left(|\alpha_k|^2\sigma_s^2+\sigma_{v,k}^2\right)^2. \tag{23}$$

Similarly when the PU signal is not present and according to AS 2 the variances of observations at node $k\in K$ are given as

$$\mathrm{Var}\left[|x_k(n)|^2|H_0\right] = \sigma_{v,k}^4. \tag{24}$$

When the PU signal is present, the off diagonal elements of matrix $\mathrm{Cov}\left[X(n)|H_1\right]$ - the covariance of observations at nodes k and j if $k,j\in K$ and $i\neq j$ can be shown to be:

$$\mathrm{Cov}\left[|x_k(n)|^2,|x_j(n)|^2|H_1\right] = |\alpha_k|^2|\alpha_j|^2\sigma_s^4. \tag{25}$$

According to AS 2 the noise realizations $v_k(n)$ and $v_j(n)$ are uncorrelated in time and space for $k,j\in K$ and $i\neq j$. Thus when the PU signal is absent the covariance of observations is

$$\mathrm{Cov}\left[|x_k(n)|^2,|x_j(n)|^2|H_0\right] = 0, \tag{26}$$

for $k,j\in K$ and $i\neq j$.

Note that (22) is in the form of a discrete time algebraic Riccati equation (DARE), [21, App. E]. The steady state variance $\mathrm{Var}\left[\hat{P}_k(\infty)H_i\right]$, $i=0,1$, at node $k\in K$ can be recovered by selecting the $\{k,k\}$ element of the steady state covariance matrix $\mathrm{Cov}\left[\hat{P}(\infty)|H_i\right]$, which has been found as a solution to the DARE. Since the DARE can be solved using standard methods, we skip the details here.

C. Detection Performance Analysis

The test statistic of the energy detector at node k at time instant n is estimated using distributed received signal power estimation algorithms. Thus the resulting detection performance is dependent on the performance of the underlying estimation process. For deriving the formulas of probability of detection (P_D) and probability of false alarm (P_{FA}) we need to evaluate the probability density function (PDF) of the test statistic $\hat{P}_k(n+1)$ under both hypotheses H_0 and H_1.

The input signal is CSCG and in case $K=1$, the test statistic of ED $\hat{P}_k(n+1)$ is local and under both hypothesis a Chi-Square distributed random variable with 2N degrees of freedom. The test statistic $\hat{P}_k(n+1)$ is obtained as a sum of a number of identically distributed variables and hence the CLT can be applied to approximate the Chi square distribution by a Gaussian distribution [22]. According to AS 3 the number of samples is large enough, and the CLT is expected to apply.

The global test statistic $\hat{P}_k(n+1)$ in case of hypothesis H_1, is however estimated over independent, but not identically distributed variables. In such a case the Lyapunov CLT [23] can still be applied over a large number of samples to result in a Gaussian approximation.

Let Q be the complementary distribution function of the standard Gaussian

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp\left(-\frac{t^2}{2}\right)dt. \tag{27}$$

The conditional mean $\mathrm{E}(\hat{P}_k(n+1)|H_i)$ and the conditional variance $\mathrm{Var}(\hat{P}_k(n+1)|H_i)$ at node k (for $i=0,1$), can be easily obtained from previously derived (13) and (22) respectively. The conditional moments in steady state can be obtained similarly from the corresponding steady state results. We provide at next approximate formulas for the resulting energy detection performance. The probability of false alarm P_{FA} of the energy detector under hypothesis H_0 is found by

$$P_{FA}(\gamma,t) = Pr(T(x)>\gamma|H_0) = \int_\gamma^\infty p_x(x|H_0)dx \tag{28}$$

Substituting the estimation mean and variance under H_0, we get

$$P_{FA} = Q\left(\frac{\gamma-\mathrm{E}(\hat{P}_k(n+1)|H_0)}{\sqrt{\mathrm{Var}(\hat{P}_k(n+1)|H_0)}}\right), \tag{29}$$

which according to AS 2, holds for every node $k\in K$.

The probability of detection of an energy detector under hypothesis H_1 is correspondingly

$$P_D(\gamma,t) = Pr(T(x)>\gamma|H_1) = \int_\gamma^\infty p_x(x|H_1)dx. \tag{30}$$

Let the probability of detection at node k be: $P_{D,k}$. Similarly substituting the mean and variance under H_1, we get

$$P_{D,k} = Q\left(\frac{\gamma-\mathrm{E}(\hat{P}_k(n+1)|H_1)}{\sqrt{\mathrm{Var}(\hat{P}_k(n+1)|H_1)}}\right). \tag{31}$$

The sensing threshold is found from (29) by fixing the desired value of P_{FA}. Thus

$$\begin{aligned}\gamma &= \mathrm{E}[\hat{P}_k(n+1)|H_0] \\ &+ Q^{-1}\left(P_{FA}\right)\sqrt{\mathrm{Var}[\hat{P}_k(n+1)|H_0]}.\end{aligned} \tag{32}$$

Due to the AS 2 [16] the thresholds for every CR node k are equal.

Calculation of the threshold requires, however, knowledge of the moments of the estimation algorithm in case of hypothesis H_0 and these moments are dependent on the algorithm parameters (especially the step size). In practice the required moments can be calculated in advance using (13) and (22),

known values of the step size and the noise power and then substituting these results into (32).

IV. Simulation results

In the numerical simulation section we investigate the ATC power estimation algorithm and compare the results with the CTA [16] and ring-around [15] versions. Secondly we view the resulting energy detection performance. In all these simulations the PU signal $s(n)$ is taken as QPSK with unit power S, under the active hypothesis H_1, the step size is: $\mu = 0.01$.

A. Local and distributed power estimation

We start with investigation of the estimation algorithms. The channel gains are assumed to be constant, fixed during the simulations and obtained by: $\alpha_k \sim \mathcal{C}N(0,1)$. In the comparison of algorithms we use the same channel gains for all the algorithms. All the nodes in the network receive $N = 2000$ samples. To illustrate how the proposed adaptive algorithms react to changes in the underlying stochastic process, we have changed the active detection hypothesis at sample $n = 1001$. During samples $n = 1 \dots 1000$ the PU signal with constant unit power S is present. The power S is attenuated by the channel gain $|\alpha_k|^2$. In sample range $n = 1001 \dots 2000$ the PU signal is absent and only noise is present. Under both detection hypothesis the noise power is $\sigma_v^2 = 1$ and assumed to be the same in all the nodes. In this subsection, it is assumed, that no measurements are exchanged between the nodes, $C = I$.

Using the ATC algorithm the estimates of the received power together with the optimal solution P^o have been plotted in Fig. 1. All the estimated power values in the CR network of the 10 nodes are plotted in one figure. When we use the CTA

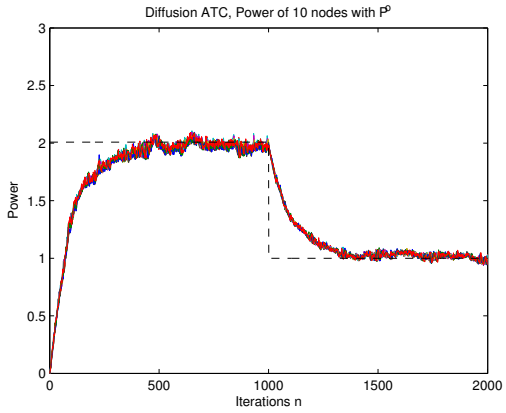

Fig. 1. Local power estimation using ATC

algorithm we obtain the results, which are given in Fig. 2. The value of optimal solution P^o in figure Fig. 1 and in Fig. 2 is shown as the black dashed line and is calculated according to (2) using the present channel gains values.

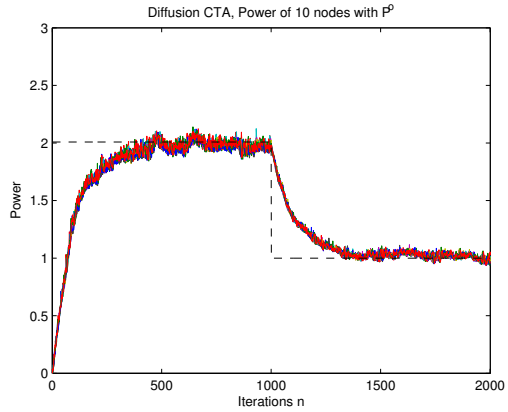

Fig. 2. Local power estimation using CTA

Compared to the ring round topology in diffusion strategies more information is processed at every node k, since neighbour estimate $(k-1)\text{mod}K$ is fused with the local estimate of node k. It was shown in [16] that the variance of the estimates of the CTA algorithm is lower than the variance of estimates of the ring around algorithm. Based on Fig. 1 and in Fig. 2 we observe that the variance of the estimates of the ATC algorithm is even slightly lower than the variance of estimates of the CTA algorithm.

The smallest value of steady state variance is achieved using the ATC algorithm. Compared to the ring around algorithm, since the preciseness of power estimates increases when the diffusion estimation strategies are used, the resulting detection performance will increase as well.

B. Probability of detection

Next we investigate the probability of detection using the proposed distributed power estimation algorithms. In the following simulations we compare the performance of 5 different network sizes: $K = 1, 3, 10, 30, 50$ nodes. More specifically, the estimated and theoretical results of P_D of the last nodes in the set are compared, i.e $k = K$. In the simulations the converged power estimate is used for detection i.e $\hat{P}_k(\infty)$. The theoretical mean and variance of the power estimates are calculated using directly the steady state formulas.

We set the desired $P_{FA} = 10^{-4}$. The thresholds of the energy detectors at nodes $k \in K$ are calculated using (32) and the corresponding steady state theoretical mean and variance of the power estimates (of algorithms CTA, ATC and ring around respectively) under detection hypothesis H_0.

For estimating the P_D we use the Monte Carlo method [24] and run 1000 experiments with the same fixed set of channel constants and noise power for all the algorithms. The estimated P_D is compared with the theoretical P_D. The theoretical P_D is calculated using (31) and the corresponding steady state mean and variance of the power estimates of the three algorithms under detection hypothesis H_1. In the

following figures the continuous lines represent the theoretical P_D and the corresponding signs the estimated P_D. First we set $C = I$. The detection performance of ATC, CTA and the ring around algorithms are shown in Fig. 3, in Fig. 4 and in Fig. 5 respectively.

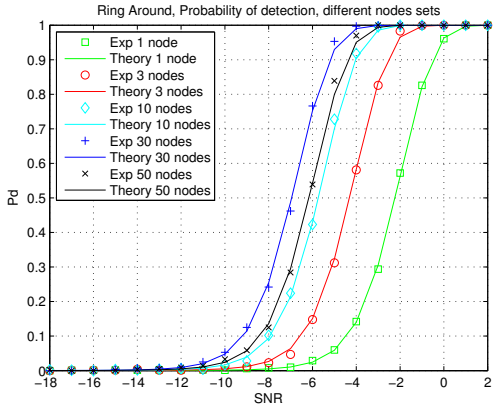

Fig. 3. Probability of detection, ring around, $C = I$

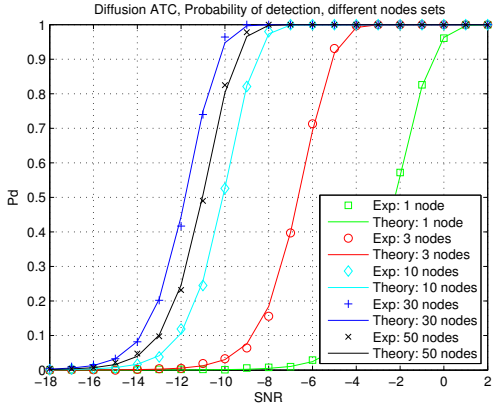

Fig. 4. Probability of detection, ATC, $C = I$

We see that there is a good match between estimated and theoretical P_D. As we noticed in [16] the CTA algorithm outperforms the ring around algorithm. As the number of nodes in the network increases, about 4 dB is gained with respect to the noise power. Based on Fig. 5 we see that the ATC slightly outperforms the CTA.

When also measurements from a neighbour node are available and we set $C = A^T_{\text{diff}}$ for the CTA and ATC algorithms, then the results are shown in Fig. 6 and in Fig. 7 respectively.

We note that ATC performs slightly better, when more nodes in the network. While ATC fuses more data than CTA, the

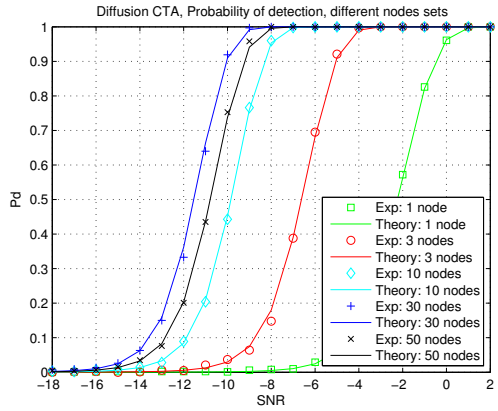

Fig. 5. Probability of detection, CTA topology, $C = I$

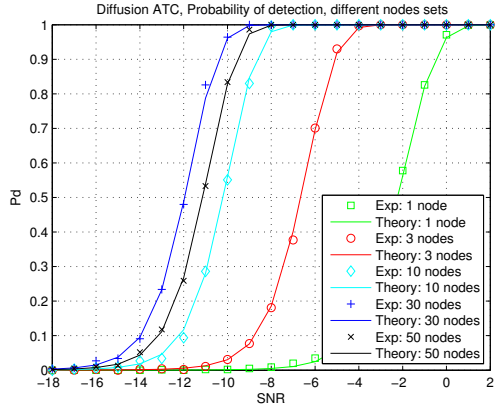

Fig. 6. Probability of detection, ATC topology, $C = A^T_{\text{diff}}$

difference of detection performance with CTA is rather small. We see minor increase in the detection performance when additionally measurements are exchanged between the nodes. Thus we conclude that the best detection results are obtained using ATC algorithm, however the difference between ATC and CTA is quite small. On the other hand since exchanging measurements between the nodes in a neighbourhood of a node in the CR network, additional data has to be broadcast, processed and this requires additional energy. Thus the usage of measurement exchange may not be justified in practical implementation.

V. Conclusions

In this paper we studied a diffusion based distributed power estimation approach, what is applicable for CR networks for detecting the presence of PU signal. We derived an ATC diffusion based energy detection algorithm for energy detection. We proposed a general framework for analysing the

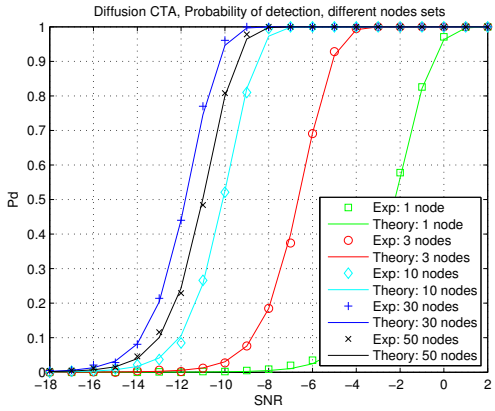

Fig. 7. Probability of detection, CTA topology, $C = A_{\text{diff}}^T$

performance of the ATC diffusion, previously studied CTA and ring-around power estimation algorithms and compared the resulting energy detection performances. Our simulation study demonstrated that both diffusion LMS based energy detection algorithms outperform the previously proposed ring around algorithm and that the ATC diffusion algorithm slightly outperforms the CTA diffusion algorithm. In addition it was observed that the effect of exchanging measurements in addition to the estimates is rather small. The proposed algorithms are able to track changes in received signal power and are usable in cognitive radio systems.

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APPENDIX D

Publication P4.

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DISTRIBUTED LARGEST EIGENVALUE DETECTION

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ABSTRACT

Cognitive radio (CR) systems need to detect the presence of a primary user (PU) signal by continuously sensing the spectrum area of interest. Radiowave propagation effects like fading and shadowing often complicate sensing of spectrum holes because the PU signal can be weak in a particular area. Cooperative spectrum sensing is seen as a prospective solution to enhance the detection of PU signals. In this paper we study distributed spectrum sensing, based on the largest eigenvalue of adaptively estimated correlation matrices (CMs) of received signals. The PU signal is assumed to be temporally correlated. In this paper an Combine and Adapt (CTA) least mean square (LMS) diffusion based mean vector estimation scheme is proposed. No fusion center (FC) for estimation or detection is used. We analyse the resulting detection performance and verify the theoretical findings through simulations.

Index Terms— Cognitive radio, distributed estimation, diffusion LMS, distributed detection, Spectrum Sensing.

1. INTRODUCTION

In cognitive radio (CR) contexts we would like to avoid creating interference to the PU user and find free spectrum opportunities as fast as possible. On the other hand the active detection hypothesis may change during the processing time. Distributed, adaptive network learning methods are able to learn the statistical information based on observations received by the nodes in the network. These methods can react to possible changes in the properties of estimated statistics in real time. Cooperative spectrum sensing is seen as a prospective solution to address these problems and to enhance the detection of PU signals [1].

Depending on the signal model assumptions, several type of detectors for spectrum sensing have been proposed in the literature such as the Matched filter detector [2], the Energy Detector [2], [3], and the Cyclostationary detector [4]. A second large group of detectors are based on the properties of an estimated signal correlation matrix eigenvalues [5], [6], [7]. The Largest Eigenvalue (LE) method [5] uses *a priori* knowledge about the additive noise power to determine the detection threshold.

Several distributed adaptive estimation and detection schemes have been studied in the past. Consensus based schemes are analysed for example in [8], [9], [10], [11]. Least mean square (LMS) and recursive least squares (RLS) based estimation schemes in [12], [13], [14], [15]. Optimal, distributed MFD, based on diffusion type LMS and RLS estimation schemes, were studied in [16], where good properties of diffusion LMS algorithms where shown. In [17], [18] and [19] we proposed and analysed diffusion LMS based energy detectors in a CR network.

In this paper we propose and study the performance of LE detection in a distributed CR network, based on adaptively, distributively estimated CMs, using the completely distributed CTA type of diffusion LMS strategy (with no central processing unit as a potential single point of failure). We make the assumption that the CR network does not have prior information about the waveform of the PU signal and about the channel gains in the secondary nodes except that the CM of the PU signal is low rank (due to temporal correlation). In the distributed CR network, every node acts as an independent detector in terms of detection decision making based on the available CM estimates.

We organize the remainder of the paper as follows. In section II we specify the system models for the LE detection method and derive an adaptive, distributed CM estimation algorithm based on the CTA diffusion LMS strategy. In section IV we analyse the performance of the proposed distributed CM estimation algorithm (using a common framework) and the detection performance of the distributed LE detection method. In section V we present our simulations results.

Notation. In the paper we use the following notations. Boldface uppercase and lowercase letters denote matrices and vectors, respectively. $\mathrm{E}[\cdot]$, $\mathrm{Cov}[\cdot]$ denote expectation and covariance operators, respectively. $\mathrm{vec}[\cdot]$ and $\mathrm{vec}^{-1}[\cdot]$ denote conversion from matrix to vector and from vector to matrix. $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^c$ denote the vector or matrix transpose, the Hermitian transpose and the complex conjugate, respectively. $\otimes$ denotes the Kronecker product.

2. DISTRIBUTED ADAPTIVE LARGEST EIGENVALUE DETECTION

2.1. Signal model and assumptions

Let the K CR nodes independently sense a communication band of a PU. Every CR node obtains individually a $M \times 1$ observation vector

$$\mathbf{y}_k(n) = [z_s(nT_s), z_s(nT_s - \delta_s), \ldots, z_s(nT_s - (M-1)\delta_s)]\,, \tag{1}$$

which contain a bunch of samples of the down converted continuous time signal $z_s(t)$, which are collected every T_s seconds with the sampling period $\delta_s < T_s$. Thus in general we have the following signal model under both detection hypotheses

$$\begin{aligned} H_0 &: \mathbf{y}_k(n) = \mathbf{v}_k(n), \\ H_1 &: \mathbf{y}_k(n) = \alpha_k \mathbf{s}(n) + \mathbf{v}_k(n), \end{aligned} \tag{2}$$

where $k = 1, 2, ..., K$ is the node number, M is the length of observation vector, and $n = 1, 2, ...N$ is the sample discrete time index. The primary signal $\mathbf{s}(n) \sim CN_M\,(0, \mathbf{\Sigma}_s)$, the noise $\mathbf{v}_k(n)$ and channel gains α_k at node k are assumed to be statistically independent. The additive noise $\mathbf{v}_k(n)$ is assumed to be independently and

identically distributed (*i.i.d*) Circularly Symmetric Complex Gaussian noise with zero mean and covariance $\mathbf{\Sigma}_{v,k} = \sigma^2_{v,k}\mathbf{I}_M$ and uncorrelated in time and space. We assume the noise power is known a *priori* and has the same power level over all the nodes in the CR network.

Each node in the CR network estimates the $M \times M$ CM $\mathbf{R}_k$ as

$$\mathbf{R}_k = \mathrm{E}\left[\mathbf{y}_k(n)\mathbf{y}_k(n)^H\right] = \mathbf{R}_{s,k} + \mathbf{\Sigma}_{v,k}. \quad (3)$$

We additionally assume that $\mathbf{R}_{s,k}$ has a low rank (see also [20], [21]), while $\mathbf{\Sigma}_{v,k} = \sigma^2_{v,k}\mathbf{I}_M$. This property can be used for detecting a PU signal.

Let us define the eigenvalues of the estimate $\hat{\mathbf{R}}_k(n)$ of CM $\mathbf{R}_k$ in non-increasing order as $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M$. Every node k detects the presence of a PU signal by independently determining the LE of the locally available estimate $\hat{\mathbf{R}}_k(n)$ and by performing the following detection test

$$\lambda_1\left[\hat{\mathbf{R}}_k(n)\right] \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_{LE}, \quad (4)$$

using a threshold γ_{LE}, which is given by (24).

2.2. Adaptive, Distributed CM estimation and LE detection

CR nodes could cooperate via internal communication links to enhance the detection performance (of the PU signal(s)) at every node k. We assume, that the K nodes in the CR network can rely only on the subset of global information, that is available to them. The CR network topology is assumed to be fixed over the sensing time and strongly connected. We consider a linear, fixed combination of neighbour estimates and measurements at every node k.

We propose a global (theoretical) model for estimating the CM in a cooperative manner, where the CR nodes jointly estimate the network average CM, which is denoted as $\mathbf{R}^o$ and in vectorized form defined as follows

$$\mathbf{r}^o = \frac{1}{K}\sum_{k=1}^{K}\mathrm{vec}(\mathbf{R}_k^o) = \frac{1}{K}\sum_{k=1}^{K}\mathrm{E}\left[\mathrm{vec}\left[\mathbf{y}_k(n)\mathbf{y}_k(n)^H\right]\right], \quad (5)$$

where the $M^2 \times 1$ $\mathbf{r}^o$ is the vectorized form of $\mathbf{R}^o$.

We can vectorize the observation $\mathbf{d}_{R,k}(n) = \mathrm{vec}\left[\mathbf{y}_k(n)\mathbf{y}_k(n)^H\right]$ at node k at time instant n and decompose it into the product of a $M^2 \times M^2$ constant (invertible) complex matrix $\mathbf{T}$ (whose elements take the values 0, 1 and $\pm i$, where i denotes the imaginary unit) and a $M^2 \times 1$ real vector $\mathbf{d}_k(n)$ as $\mathbf{d}_{R,k}(n) = \mathbf{T}\mathbf{d}_k(n)$, to keep the dimension of the estimated vector minimal in the adaptive recursions. We denote the estimate of the real valued $\mathrm{E}[\mathbf{d}_k(n)]$ as $\hat{\mathbf{p}}_k(n)$ and propose to relate the estimation of the $\mathbf{R}_k^o$ and $\mathbf{R}^o$ in (5) with the minimization of the following Mean Square Error (MSE) type of global cost function

$$\mathbf{p}^o = \underset{\mathbf{p}}{\mathrm{argmin}}\sum_{k=1}^{K}J_k(\mathbf{p}) = \underset{\mathbf{p}}{\mathrm{argmin}}\sum_{k=1}^{K}\mathrm{E}\left\|\mathbf{d}_k(n) - \mathbf{p}\right\|^2, \quad (6)$$

where $M^2 \times 1$ dimensional $\mathbf{p} \in R^M$. By using standard derivation steps on (6) we get the optimal solution

$$\mathbf{p}^o = \frac{1}{K}\sum_{k=1}^{K}\mathrm{E}\left[\mathbf{d}_k(n)\right]. \quad (7)$$

Thus with help of the transformation matrix $\mathbf{T}$, the previously introduced minimization framework can be used to re-define the $\mathbf{R}_k^o$ and $\mathbf{R}^o$ as follows

$$\mathbf{R}^o = \mathrm{vec}^{-1}\left[\mathbf{T}\mathbf{p}^o\right] \quad \text{and} \quad \mathbf{R}_k^o = \mathrm{vec}^{-1}\left[\mathbf{T}\mathbf{p}_k^o\right]. \quad (8)$$

We need to seek an iterative solution to estimate the $\mathbf{p}_k^o$ and $\mathbf{p}^o$ in a manner, which is adaptive in time and is fully distributed (cooperative).

2.3. Iterative Diffusion solutions

In this paper we skip the derivation details of the CTA type of diffusion LMS mean vector estimation algorithm (provided in [22], following the ideas of [13]). Let $\mathcal{N}_k$ denote the neighbourhood group of node $k \in K$, i.e $\mathcal{N}_k$. Let μ_k be a positive step size of node k. We introduce the $K \times K$ matrix $\mathbf{C}$ with non-negative elements satisfying

$$c_{l,k} = 0 \quad \text{if} \quad l \notin \mathcal{N}_k, \quad \mathbf{C}\mathbf{1} = \mathbf{1}. \quad (9)$$

Similarly let the $K \times K$ matrix $\mathbf{A}$ satisfy

$$a_{l,k} = 0 \quad \text{if} \quad l \notin \mathcal{N}_k, \quad \mathbf{1}^T\mathbf{A} = \mathbf{1}^T. \quad (10)$$

We summarize the CTA based CM estimation recursions and the detection step in a common form in Algorithm 1. The coefficients $c_{l,k}$

Algorithm 1 Distributed LMS based CM Estimation and Detection

Start with $\hat{\mathbf{p}}_k(0) = \mathbf{p}(0)$ for every k .
Given non-negative real coefficients $a_{l,k}, c_{l,k}$
for every time instant $n \geq 1$ **do**
  for every node $k = 1, ..., K$ **do**
    1. CTA type of CM estimation recursions:
    $\hat{\boldsymbol{\psi}}_k(n) = \sum_{l\in N_k} a_{l,k}\hat{\mathbf{p}}_l(n)$.
    $\hat{\mathbf{p}}_k(n+1) = \hat{\boldsymbol{\psi}}_k(n)$
      $+\mu_k \sum_{l\in N_k} c_{l,k}\left[\mathbf{d}_l(n) - \hat{\boldsymbol{\psi}}_k(n)\right]$
    2. LE detection decision:
    $H_0 : \lambda_1\left[\mathrm{vec}^{-1}\left[\mathbf{T}\hat{\mathbf{p}}_k(n+1)\right]\right] < \gamma_k$ or
    $H_1 : \lambda_1\left[\mathrm{vec}^{-1}\left[\mathbf{T}\hat{\mathbf{p}}_k(n+1)\right]\right] > \gamma_k$.
    (Refer to (24) for selecting the γ_k).
  end for
end for

and $a_{l,k}$ define respectively how the neighbouring measurements $\mathbf{d}_l(n)$ and estimates $\hat{\mathbf{p}}_l(n)$ are (unidirectionally) available for the node k in the CR network. Thus after several iterations the adaptive estimate $\hat{\mathbf{R}}_k(n)$ of $\mathbf{R}^o$ is available for every node in the CR network, while the FC is not used. The node k at time instant n can independently perform the LE detection based on the available matrix estimate $\hat{\mathbf{R}}_k(n) = \mathrm{vec}^{-1}\left[\mathbf{T}\hat{\mathbf{p}}_k(n)\right]$.

As a result the proposed LE detection scheme is able to react to a possible change in the statistics of observations on line (i.e when the detection hypothesis changes during the observation time) and estimates the CMs in a cooperative manner with an averaging effect over the CR network.

3. PERFORMANCE ANALYSIS

The performance analysis of the proposed algorithm is divided into three parts: analysis of the moments of the adaptive CM estimates of recursions in Algorithm 1 in one framework, analysis of the statistical properties of the adaptive CM estimates and analysis of the

detection performance of the LE of the adaptive CM estimates. Let us note that for the theoretical performance analysis of the LE detector, we need to know the values of the channel gains.

3.1. Moment analysis of adaptive CM estimates

Let us stack the $M^2 \times 1$ estimates and observations from all the nodes $k \in K$ into a $KM^2 \times 1$ column vector $\hat{\mathbf{p}}(n)|H_i = [\hat{\mathbf{p}}_1(n)|H_i \dots \hat{\mathbf{p}}_K(n)|H_i]^T$ and $\mathbf{d}(n)|H_i = [\mathbf{d}_1(n)|H_i \dots \mathbf{d}_K(n)|H_i]^T$ respectively, where $i = 1$ denotes the case when PU signal is present and $i = 0$ the case when PU signal is absent. For the positive step sizes we define additional $K \times K$ matrix $\boldsymbol{\mathcal{M}} = \text{diag}\{\mu_1, \dots, \mu_K\}$. Let $\otimes$ denote the Kronecker product. The $K \times K$ matrices $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{C}$ and $\boldsymbol{\mathcal{M}}$ are in CR network extended to $KM^2 \times KM^2$ matrices $\overline{\mathbf{A}}_1 = \mathbf{A}_1^T \otimes \mathbf{I}_{M^2}$, $\overline{\mathbf{A}}_2 = \mathbf{A}_2^T \otimes \mathbf{I}_{M^2}$, $\overline{\mathbf{C}} = \mathbf{C}^T \otimes \mathbf{I}_{M^2}$ and $\overline{\boldsymbol{\mathcal{M}}} = \boldsymbol{\mathcal{M}} \otimes \mathbf{I}_{M^2}$. Then we can write the CM estimation recursion in the following general form

$$\hat{\mathbf{p}}(n+1)|H_i = \overline{\mathbf{A}}_2 \left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1 \hat{\mathbf{p}}(n)|H_i + \overline{\mathbf{A}}_2 \overline{\boldsymbol{\mathcal{M}}} \overline{\mathbf{C}} \mathbf{d}(n)|H_i. \quad (11)$$

For example for CTA algorithm we take $\overline{\mathbf{A}}_1 = \mathbf{A}_{\text{diff}}^T \otimes \mathbf{I}_{M^2}$, $\overline{\mathbf{A}}_2 = \mathbf{I}_K \otimes \mathbf{I}_{M^2}$, $\overline{\mathbf{C}} = \mathbf{I}_K \otimes \mathbf{I}_{M^2}$ or $\overline{\mathbf{C}} = \mathbf{A}_{\text{diff}}^T \otimes \mathbf{I}_{M^2}$.

By denoting the conditional expectation of the observation vector as $\text{E}[\mathbf{d}(n)|H_i]$ for $i = 0, 1$, then based on (11), we have that

$$\begin{aligned} \text{E}[\hat{\mathbf{p}}(n+1)|H_i] &= \overline{\mathbf{A}}_2 \left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1 \text{E}[\hat{\mathbf{p}}(n)|H_i] \\ &\quad + \overline{\mathbf{A}}_2 \overline{\boldsymbol{\mathcal{M}}} \overline{\mathbf{C}} E[\mathbf{d}(n)|H_i], \end{aligned} \quad (12)$$

for $i = 0, 1$, where the initial value is given as $\text{E}[\hat{\mathbf{p}}(0)|H_i]$. It can be shown that a sufficient condition for the algorithm to be stable is to select the step size for every $k = 1 \dots K$ as

$$0 < \mu_k < 2. \quad (13)$$

Similarly by denoting the conditional covariance of the observations and estimates under the hypothesis H_i, $i = 0, 1$ as $\text{Cov}[\mathbf{d}(n)|H_i]$ and $\text{Cov}[\hat{\mathbf{p}}(n+1)|H_i]$ we have

$$\begin{aligned} \text{Cov}[\hat{\mathbf{p}}(n+1)|H_i] &= \overline{\mathbf{A}}_2 \left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1 \text{Cov}[\hat{\mathbf{p}}(n)|H_i] \\ &\quad \times \overline{\mathbf{A}}_1^T \left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_2^T \\ &\quad + \overline{\mathbf{A}}_2 \overline{\boldsymbol{\mathcal{M}}} \overline{\mathbf{C}} \, \text{Cov}[\mathbf{d}(n)|H_i] \, \overline{\mathbf{C}}^T \overline{\boldsymbol{\mathcal{M}}} \overline{\mathbf{A}}_2^T. \end{aligned} \quad (14)$$

where initial value is noted by $Cov[\hat{\mathbf{p}}(0)|H_i]$, $i = 0, 1$.

The moments $\text{E}[\mathbf{d}(n)|H_i]$ and $Cov[\mathbf{d}(n)|H_i]$ of the measurements are provided in 3.2.

3.2. Statistical modelling of adaptive CM estimates

Based on 2.1, for the rank one observations $\mathbf{d}_{R,k}(n)$ under H_1 we have that

$$E[\mathbf{d}_{R,k}(n)|H_1] = \text{vec}\left[\mathbf{R}_{s,k} + \sigma_v^2 \mathbf{I}_M\right]. \quad (15)$$

and the stacked $KM^2 \times 1$ vector $E[\mathbf{d}_R(n)|H_i]$ over $k = 1 \dots K$ and for $i = 0, 1$ can be formed based on (15) respectively.

It can be shown, that the $k, j \in K$ blocks of the $KM^2 \times KM^2$ network-wise covariance matrix $\text{Cov}[\mathbf{d}_R(n)|H_1]$ are given as

$$\text{Cov}\left[\mathbf{d}_{R(k,j)}(n)|H_1\right] = \begin{cases} \left[\left(\overline{\boldsymbol{\Sigma}}_k\right)^c \otimes \overline{\boldsymbol{\Sigma}}_k\right], & k = j \\ \left[\left(\mathbf{R}_{s,k,j}\right)^c \otimes \mathbf{R}_{s,k,j}\right], & k \neq j \end{cases} \quad (16)$$

where $\overline{\boldsymbol{\Sigma}}_k = \text{E}\left[|\alpha_k|^2\right] \boldsymbol{\Sigma}_s + \sigma_v^2 \mathbf{I}_M$ and where for $k \neq j$ $\mathbf{R}_{s,k,j} = \text{E}\left[\mathbf{y}_k(n)\mathbf{y}_j(n)^H\right] = \text{E}\left[\alpha_k \alpha_j^c\right] \boldsymbol{\Sigma}_s$ and $(.)^c$ denotes a complex conjugate. Obviously the $\text{Cov}[\mathbf{d}_R(n)|H_0]$ is given as

$$\text{Cov}[\mathbf{d}_{R,k}(n)|H_0] = \sigma_v^4 \mathbf{I}_{M^2}. \quad (17)$$

Thus the $E[\mathbf{d}(n)|H_i]$ for (12) and $\text{Cov}[\mathbf{d}(n)|H_i]$ for (14) can be given for $i = 0, 1$ as

$$E[\mathbf{d}(n)|H_i] = \left[\mathbf{T}^{-1} \otimes \mathbf{I}_{M^2}\right] E[\mathbf{d}_R(n)|H_i], \quad (18)$$

and

$$\begin{aligned} \text{Cov}[\mathbf{d}(n)|H_i] &= \left[\mathbf{T}^{-1} \otimes \mathbf{I}_{M^2}\right] \\ &\quad \times \text{Cov}[\mathbf{d}_R(n)|H_i] \left[\left(\mathbf{T}^H\right)^{-1} \otimes \mathbf{I}_{M^2}\right]. \end{aligned} \quad (19)$$

When the $\hat{\mathbf{R}}_k(n) = \text{vec}^{-1}[\mathbf{T}\hat{\mathbf{p}}_k(n)]$ is obtained by using the exponential type of averaging (as used in LMS type of algorithms), then it is not Wishart distributed [23, Theorem 3.3.1., 3.5.2.]. We propose the usage of Total Variance method [24] for approximating the $\hat{\mathbf{R}}_k(n)$ by conditional approximative Complex Central (Correlated) Wishart distributions (CC(C)W), for studying the conditional CDFs of LE of adaptively estimated CMs. Thus we use the approximation

$$\hat{\mathbf{R}}_k(n)|H_i \sim CW_M\left(\bar{N}_i, \bar{\boldsymbol{\Sigma}}_{k,i}\right), \quad (20)$$

for $i = 0, 1$ and where $\sim$ denotes an approximative distribution, $\bar{N}_i$ is the approximating DoF and $\bar{\boldsymbol{\Sigma}}_{k,i}$ is the approximating population covariance matrix parameter of the corresponding CC(C)W distribution. The values for $\bar{N}_i$ and $\bar{\boldsymbol{\Sigma}}_{k,i}$ can be found by matching the mean and trace of the moments of $\hat{\mathbf{R}}_k(n)|H_i$ with the corresponding moments of the devectorized adaptive estimate $\text{vec}^{-1}[\mathbf{T}\hat{\mathbf{p}}_k(n)]$. This gives (see [22] for details), by using the TV method,

$$\bar{\boldsymbol{\Sigma}}_{k,i} = \frac{1}{\bar{N}_i} \text{E}\left[\hat{\mathbf{R}}_k(n)|H_i\right] = \frac{1}{\bar{N}_i} \left(\text{vec}^{-1}\left[\mathbf{T}\,\text{E}\left[\mathbf{p}_k(n)|H_i\right]\right]\right). \quad (21)$$

and

$$\bar{N}_{TV,i} = \left\lceil \frac{\text{Tr}\left[\text{E}\left[\hat{\mathbf{R}}_k(n)|H_i\right]^c \otimes \text{E}\left[\hat{\mathbf{R}}_k(n)|H_i\right]\right]}{\text{Tr}\left[\mathbf{T}\,\text{Cov}\left[\mathbf{p}_k(n)|H_i\right]\mathbf{T}^H\right]} \right\rfloor, \quad (22)$$

where $\text{E}\left[\hat{\mathbf{R}}_k(n)|H_i\right] = \text{vec}^{-1}\left[\mathbf{T}\,\text{E}\left[\mathbf{p}_k(n)|H_i\right]\right]$.

3.3. Detection Performance Analysis

Let the eigenvalues of $\bar{\boldsymbol{\Sigma}}_{k,i}$ in (20) be denoted in non-increasing order as $\nu_{1,i} \geq \nu_{2,i} \geq \dots \geq \nu_{M,i}$.

Based on the [5], [25], the $\hat{\mathbf{R}}_k(n)|H_0$ (20) is assumed to follow the CCW distribution and the eigenvalues of $\bar{\boldsymbol{\Sigma}}_{k,0}$ are $\nu_{1,0} = \dots = \nu_{M,0} = \sigma_v^2/\bar{N}_0$. The $P_{FA,e}$, based on the non-asymptotic CDF model of the $\hat{\mathbf{R}}_k(n)|H_0$, is given as

$$\begin{aligned} F_{H_0,e}(x) &= |\det(\hat{\mathbf{A}})| \\ P_{FA,e}(\gamma_{LE,e}) &= 1 - F_{H_0,e}(\gamma_{LE,e}) \end{aligned} \quad (23)$$

where the $M \times M$ matrix $\hat{\mathbf{A}}_{i,j} = \binom{\bar{N}_0 - j - i - 1}{i-1} \gamma_R(\bar{N}_0 + i - j, \frac{x}{\nu_{1,0}})$, for $i, j = 1, \dots, M$ and where $\gamma_R(k, u) = \frac{1}{\Gamma(k)} \int_0^u x^{k-1} e^{-x} dx$ is the regularized incomplete Gamma function. The detection threshold $\gamma_{LE,e}$, based on the non-asymptotic model is given as

$$\gamma_{LE,e} = F_{H_0,e}^{-1}(1 - P_{FA,e}) \quad (24)$$

and can be evaluated in terms of a numerical inversion of the exact CDF formula at a desired $P_{FA,e}$ value.

Since the $\hat{\mathbf{R}}_k(n)|H_1$ is assumed to be distributed by a CCCW distribution, the P_D based on the non-asymptotic CDF$|H_1$ of the LE of a CCCW matrix $\hat{\mathbf{R}}_k(n)|H_1$ is given by [26] as follows

$$F_{H_1,e}(x) = K_{CC}\left|\left\{\nu_i^{\bar{N}_1-M+j}\bar{\Gamma}\left(\bar{N}_1-M+j,\frac{x}{\nu_{i,1}}\right)\right\}_{i,j}\right|,$$

$$K_{CC} = \left[\prod_{i=1}^{M}(\bar{N}_1-i)!\prod_{j=1}^{M}(M-i)!\right]^{-1}\prod_{k=1}^{M}(k-1)!,$$

$$P_{D,e}(\gamma_{LE,e}) = 1 - F_{H_1,e}(\gamma_{LE,e}). \qquad (25)$$

for $i,j = 1,\ldots,M$ and where $\bar{\Gamma}(k,u) = \int_0^u x^{k-1}e^{-x}dx$ is the lower incomplete gamma function [27, 8.350].

4. SIMULATION RESULTS

In this section we investigate the probability of detection P_D of the the CTA type of distributed, adaptive LE detection algorithm. The performance of the algorithm is well illustrated by the P_D versus SNR analysis, where the change in the (network averaged) SNR is achieved by changing the noise power value σ_v^2. The channel gains are assumed to be constant and are sampled for the CR node $k \in K$ as $\alpha_k \sim CN(0,1)$. We assume to have one PU signal $\mathbf{s}(n) = s(n)\mathbf{1}$, $s(n) \sim CN(0,1)$ and $\mathbf{\Sigma}_s = \mathbf{1}\mathbf{1}^H$. Obviously rank($\mathbf{1}\mathbf{1}^H$)=1. We select the $M = 2$, $N = 7000$, $\mu = 0.001$ and $P_{FA} = 10^{-2}$ for all the nodes. The thresholds of the LE detectors at nodes $k \in K$ are found by using (24) with the TV approximation. Also we select the diffusion topology of the estimates in the CR network, i.e the $\mathbf{A}$ matrix, as a combination of the local ($\mathbf{A}, \mathbf{C} = \mathbf{I}$) and ring-around ($\mathbf{A} = \mathbf{A}_{\text{ring}}^T, \mathbf{C} = \mathbf{I}$) topologies, similarly as in [19, Eq. 11].

In the following simulations the performance of 4 different network sizes: $K = 1, 3, 10, 30$ nodes are compared, while the comparable results are taken from the last node in the set. The Monte Carlo estimated P_D results (based on the adaptively estimated CMs and denoted as **Ad. Exp.** in the figures) are compared with the non-asymptotic theoretical model (25) (denoted as **Theory**) and with the P_D results based on approximately equivalent CCW matrices (denoted as **W. Exp.**). These latter matrices are generated based on the respective moments under H_1. The P_D versus SNR results are given in Fig. 1 when TV approximation is used for the CTA algorithm.

It is seen that the non-asymptotic theoretical P_D model describes the detection performance of adaptively estimated CMs well, also when the noise power is high relative to the PU signal power (SNR). As the number of nodes in the network increases, the point where the P_D starts to decrease from one, converges to the left by equalizing and averaging the P_D on every CR node.

It can be concluded that the TV approximation for the non-asymptotic CDF$|H_1$ is usable for studying the performance of the LE detection of adaptively estimated CMs. When the nodes cooperate in estimating the network-wise CM (while nodes are able to communicate directly only with limited subset of neighbour nodes) then the resulting LE detection performance is equalized and stabilized over the individual CR nodes.

5. CONCLUSIONS

In this paper a distributed and adaptive, CTA diffusion LMS based LE detection algorithm was studied, which is applicable in CR networks for detecting the presence of a PU signal. We proposed a general framework for analysing the performance of the diffusion LMS based LE detection scheme and we demonstrated that the theoretical results are matching with the simulations. It was shown that the cooperative estimation and detection scheme enhances the detection performance.

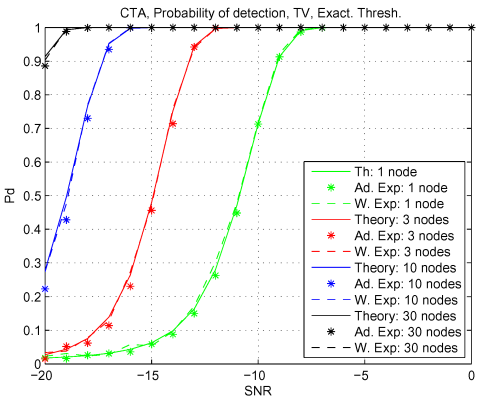

Fig. 1. Probability of detection, CTA, TV, Case 2

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APPENDIX E

Publication P5.

A. Ainomäe, M. Bengtsson, and T. Trump, "Distributed largest eigenvalue based spectrum sensing using diffusion adaptation," IEEE Transactions on Signal and Information Processing over Networks*, Sep. 2017, ETIS 1.1.*

Distributed Largest Eigenvalue Based Spectrum Sensing using Diffusion LMS

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***Abstract*—In this paper we propose a distributed detection scheme for cognitive radio (CR) networks, based on the largest eigenvalues (LEs) of adaptively estimated correlation matrices (CMs), assuming that the primary user signal is temporally correlated. The proposed algorithm is fully distributed, thereby avoiding the potential single point of failure that a fusion centre (FC) would imply. Different forms of diffusion least mean square (LMS) algorithms are used for estimating and averaging the CMs over the CR network for the LE detection and the resulting estimation performance is analyzed using a common framework. In order to obtain analytic results on the detection performance, the exact distribution of the CM estimates are approximated by a Wishart distribution, by matching the moments. The theoretical findings are verified through simulations.**

***Index Terms*—Cognitive radio, distributed estimation, diffusion LMS, diffusion networks, distributed detection, Spectrum Sensing, Random Matrix.**

I. Introduction

Cognitive radio (CR) is seen as a promising technology to make radio spectrum usage more effective by providing an opportunistic access for secondary users to the licensed spectrum areas. We consider the interweave CR paradigm [4], where CR systems detect the presence of a primary user (PU) signal by sensing the spectrum area of interest. The binary detection problem is studied: PU signal is present or absent [5], [6], [7]. In the interweave paradigm it is expected that the CR system should accurately detect the transmission of a PU system, when the latter is operating. On the other hand the radiowave propagation effects like fading and shadowing often complicate sensing of spectrum holes because the PU signal can be weak in a particular area. Cooperative spectrum sensing is seen as a prospective solution to address these problems and to enhance the detection of PU signals [8].

In the literature several type of detectors for spectrum sensing have been proposed. When the PU signal waveform, channel and additive noise properties are known *a priori*, then the matched filter detector (MFD) is optimal [9]. The MFD requires perfect synchronization between the PU signal waveform and the received signal. However in practice such required knowledge is often not available, which makes the usage of the MFD detector impractical. The cyclostationary feature detection method [10] requires a priori knowledge about the cyclic frequencies of the PU signals, which often is a too strong assumption for practical implementation. The Energy Detection (ED) method [9] models the PU signal as a random process and does not require knowledge about the PU signal, modulation type and channel properties. In such a case, when the received PU signal is white, the ED is optimal. However, setting the detection threshold requires knowledge of the noise power value. It has been shown, that if there is uncertainty in the noise power or if the received PU signal is correlated, the ED performance decreases and it is no more optimal [11].

A second large group of detectors for spectrum sensing are based on eigenvalue properties of an estimated correlation matrix [12], [13], [14]. Detection based on the largest eigenvalue (LE) of estimated CMs [12] is optimal when the observations are zero mean Gaussian distributed, we do not have specific information about the PU signal and the channel gains, and when the PU signal is rank one correlated [15]. The LE method uses knowledge about the additive noise power to determine the detection threshold. Random Matrix Theory has been used to study the performance of the CM eigenvalue based detectors [16]. We note, that when linear estimation of CM is used, more sophisticated detectors: the volume based detector (VD) and the covariance based detector (CAV), which avoid eigenvalue or singular value decomposition, have been studied in [17], [18] and [19] respectively. Similarly, when linear estimation of a CM is used, several eigenvalue based detectors are robust in the sense, that the noise power value does not influence the test statistics or threshold of the detectors. For example the Eigenvalue Arithmetic to Geometric Mean (AGM) [20], the Maximum to Minimum eigenvalue ratio (MME), the Energy to Minimum Eigenvalue ratio (EME) [14], the Eigenvalue Moment ratio (EMR) [20], and the Hadamard [21] detectors have been proposed in the literature. A method for blind and optimal combination of observations for the ED has been proposed in [22]. For these detectors, the performance analysis is based on the assumption that the sample CM is Wishart distributed with known degrees of freedom (DoF), an assumption that does not hold when exponentially weighted (adaptive) CM estimation is used. Also, the proposed approximate or asymptotic analysis of the theoretical detection performance for EME, MME, CAV detectors tend to be inaccurate in the low SNR regime, as seen in [14], [19].

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In cognitive radio (CR) contexts we would like to avoid creating interference to the PU user and find free spectrum opportunities as fast as possible. On the other hand the active detection hypothesis may change during the processing time. Distributed, adaptive network learning methods, based on exponential averaging estimation, are able to learn the statistical information based on observations received by the nodes in the network. These methods can react to possible changes in the properties of estimated statistics in real time.

Several proposed distributed spectrum sensing solutions make use of a central FC. A FC will however form a single point of failure in the network since a malfunction in this unit affects the performance of the whole distributed solution. We therefore propose a CM estimation solution, where the available CM estimates (and corresponding measurements) are fused in cognitive radio network nodes, to allow all nodes to make detection decisions based on data from the neighboring nodes and without involvement of any central processing unit. Such a solution enhances the network failure resistance.

Several distributed adaptive estimation schemes have been studied in the past. Consensus based schemes are analyzed for example in [23], [24], [25], [26]. Diffusion estimation schemes are studied for instance in [27], [28], while Least mean square (LMS) and recursive least squares (RLS) schemes in [29], [30], [31], [32]. It has been shown, that distributed diffusion strategies can often perform better (in terms of faster convergence and lower Mean Square Deviation) and be more stable compared to consensus algorithms [33], [34]. Several detection solutions, based on distributed estimation, have been studied for example in [35], [36], [37], [38]. A ring network topology for distributed energy detection without a FC has been suggested in [39]. In [3] we proposed and analyzed a diffusion LMS based recursive calculation of the test statistics with ring topology for the energy detectors in cognitive radio network. Ring networks are however sensitive to communication link failures. Combine and Adapt (CTA) LMS diffusion based calculation of the test statistics for the energy detectors was studied in [2] and an Adapt and Combine (ATC) based version was investigated further in [1].

In this paper we study the performance of LE detection in a distributed CR network, based on adaptively, distributively estimated CMs, using the completely distributed diffusion LMS strategy. We make the assumption that the CR network does not have prior information about the waveform of the PU signal and about the channel gains in the secondary nodes. We assume that the received PU signals samples are temporally correlated. Secondly in general we assume the noise power level is known. Noise power estimation procedures and analysis of the sensitivity to estimation errors falls outside the scope of this paper. To analyze the detection performance and determine the threshold value, we follow the ideas of [40], [41], [42] and approximate the distribution of the exponentially averaged CM estimate by a Wishart distribution by moment matching. The resulting DoF for the approximate Wishart distribution will depend both on the step size, the network topology, and under H_1 detection hypothesis will depend also on the value of the noise variance parameter. We have therefore focused on the LE based detection, since under H_1 the robustness of alternative detectors like EME, MME, CAV in case of adaptively estimated CMs, is lost anyway. We however provide a simulation with the MME detector, which is a robust detector. In the distributed CR network, every node acts as an independent detector in terms of detection decision making based on the available CM estimates. Due to limited information about the PU signal and the communication channel, the theoretical global estimation model is proposed as a network-average CM (while in practice the CR nodes have only access to the subset of data from the neighbor nodes). We consider the control-level analysis of the proposed distributed CM estimation and LE detection algorithm to be out of scope of the paper.

We organize the remainder of the paper as follows. In Section II we describe the motivation, specify the system models which are analysed further in this paper and we motivate the usage of the LE detector. In Section III we derive an adaptive, distributed CM estimation algorithm based on diffusion LMS strategy and summarize the versions of it. In Section IV we analyse the performance of the proposed distributed CM estimation algorithm using a common framework for moment based analysis for all the versions of the Diffusion LMS algorithm. We propose the usage of Total and General Variance based approximations for being able to model the distributions of adaptive CM estimates under both detection hypotheses. Using these results the theoretical false alarm and the detection performance of the LE detector are studied. In Section V we present our simulations results and verify the theoretical findings.

Notation. In the paper we use the following notations. Boldface uppercase and lowercase letters denote matrices and vectors, respectively. $\mathrm{E}[\cdot]$, $\mathrm{Var}[\cdot]$, $\mathrm{Cov}[\cdot]$ denote expectation, variance (of a scalar) and covariance operators, respectively. $\mathrm{vec}[\cdot]$ and $\mathrm{vec}^{-1}[\cdot]$ denote conversion from matrix to vector and from vector to matrix. $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^c$ denote the vector or matrix transpose, the Hermitian transpose and the complex conjugate, respectively. $\otimes$ denotes the Kronecker product.

II. Problem formulation and background

A. Signal model and assumptions

Assume that K single-antenna CR nodes are independently sensing the communication band of a PU. Let the observation bandwidth of the communication band be denoted as B. A collection of samples of the down converted continuous time signal $z_s(t)$ are collected every T_s seconds, with sampling period $\delta_s < T_s$. As a result every node individually obtains a vector

$$\mathbf{y}_k(n) = [z_s(nT_s), z_s(nT_s - \delta_s), \ldots, z_s(nT_s - (M-1)\delta_s)]\,, \tag{1}$$

which gives the following observation model for both detection hypotheses

$$\begin{aligned} H_0 &: \mathbf{y}_k(n) = \mathbf{v}_k(n), \\ H_1 &: \mathbf{y}_k(n) = \alpha_k \mathbf{s}(n) + \mathbf{v}_k(n), \end{aligned} \tag{2}$$

where $k = 1, 2, ..., K$ is the node number, M is the length of the observation vector, and $n = 1, 2, ...N$ is the sample discrete time index. The primary signal $\mathbf{s}(n)$, the noise $\mathbf{v}_k(n)$

and channel gains α_k at node k are assumed to be statistically independent. We additionally assume that the PU signal follows

$$\mathbf{s}(n) \sim CN_M\left(\mathbf{m}_s, \mathbf{\Sigma}_s\right). \tag{3}$$

In the performance analysis of the LE detection scheme, the following assumption will be used.

AS 1. The additive noise $\mathbf{v}_k(n)$ is independently and identically distributed (*i.i.d*) circularly symmetric complex Gaussian (CSCG) noise with zero mean and covariance $\mathbf{\Sigma}_{v,k} = \sigma^2_{v,k}\mathbf{I}_M$. In the CR network $\mathbf{v}_k(n)$ is uncorrelated in time and space. We assume the noise power is known a *priori* and has the same power level for all nodes in the CR network.

Under H_1 we have the following $M \times M$ CM model

$$\mathbf{R}_k = \mathbf{R}_{s,k} + \mathbf{\Sigma}_{v,k}. \tag{4}$$

Let us denote the actually occupied bandwidth (within the observation bandwidth B) as b. Thus the ratio between occupation and observation bandwidths is denoted as $\beta = b/B$ [43] and the rank of the PU signal matrix can be then approximated as $\text{rank}(\mathbf{R}_{s,k}) \approx \lceil \beta M \rceil$. We assume $M > 1$, $\beta < 1$ and then $\mathbf{R}_{s,k}$ has in general a low rank (see also [44]), while $\mathbf{\Sigma}_{v,k}$ is a scaled identity matrix. This property can be used for detecting the PU signal.

B. Largest Eigenvalue detection

In this paper, we focus on the LE detector, which is known to follow from the General Likelihood Ratio approach, when AS 1 holds, the received observation vectors obey a Multivariate Complex Gaussian distribution with zero mean, and when the PU signal population covariance matrix $\mathbf{R}_{s,k}$ is rank one [15]. The LE detector requires low computational complexity and the detection performance analysis is easy to conduct. As seen in [12] and in Section IV, there exist usable theoretical results for the conditional distributions without asymptotic approximations, which predict the true performance well both in low and high SNR. The LE method is optimal for one PU signal. In the case of higher rank PU signals (i.e more than one PU signal in the network), then the LE detector is no longer optimal, but still usable. We note that all these existing results from the literature for the LE detector hold when estimating the CM using a standard non-weighted sample covariance matrix, resulting in a complex Wishart distribution.

For the distributed adaptive estimation scheme considered here, this latter assumption is no longer true, but as will be shown in Sections IV and V, the distribution can still be well approximated by a complex Wishart distribution. The DoF approximations depend on the parameters of the distributed and adaptive CM estimation algorithm step-size and under H_1 also on the preciseness of the noise power value (AS 1). Extending the analysis to other type of detectors can therefore be done using the existing results in the literature, for example from [17], [18], [20], [21]. As seen in Section V, a noise power uncertainty under the detection hypothesis H_1 causes an inaccuracy to the approximated DoF$|H_1$ value. This effect causes a potential inaccuracy in the theoretical detection performance formula of a detector, which requires the DoF$|H_1$ value. However since the threshold of a robust detector is not affected by the noise power perturbations, then such a detector can still be used in the framework of this paper. Thus to keep the focus of the paper, we have limited our study to the LE detector, where AS 1 is necessary for the threshold calculation and to illustrate the effect of accuracy of the DoF approximations under both detection hypotheses. Since the LE detector is vulnerable to the noise power value uncertainty, then in Section V we also provide a simulation with the robust MME detector in the proposed distributed and adaptive CM estimation framework.

Thus an estimate $\hat{\mathbf{R}}_k(n)$ of the CM $\mathbf{R}_k$ is assumed to be available for every node $k \in K$ at time index n. Let us define the eigenvalues of $\hat{\mathbf{R}}_k(n)$ in non-increasing order as $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M$. Every node k detects the presence of a PU signal by independently determining the LE of the locally available estimate $\hat{\mathbf{R}}_k(n)$ and performing the following detection test

$$\lambda_1\left[\hat{\mathbf{R}}_k(n)\right] \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_{LE,k}, \tag{5}$$

using a threshold $\gamma_{LE,k}$, which is given in the Section IV-C1 by (54) or (57).

Next we implement the diffusion LMS based method to derive a distributed adaptive CM based LE detector in the CR network, so that the algorithm: A) is able to react to a possible change in the statistics of observations on line (i.e when the detection hypothesis changes during the observation time) and B) estimates the CMs in a cooperative manner with an averaging effect over the CR network. CR nodes can have access only to a subset of neighbor nodes and no FC unit is used in the CR network.

III. Adaptive, Distributed CM estimation and LE detection

Obviously one of the most simple cooperation strategies is where all the CR nodes are able to exchange their local data (estimates or observations) with all the other nodes in the CR network, i.e the network global data is available at every node. However in practice it means that all nodes have to be within hearing distance of all the other nodes and significant amount of data needs to be exchanged and processed over the CR network. Secondly transmitting and processing of (global) data consumes energy, which may drain the batteries of the CR nodes. In this paper we assume to have a more general network topology model, where nodes only share data with a subset of neighbor nodes and thus no global data is available. Thus we assume that the CR nodes use low power transmitters (i.e a low energy communication, to save the batteries) we also would like to save some energy required for local data processing. This means that while every CR node k still needs to transmit its estimate or observation at a time instant n, other nodes use data of pre-selected neighbor nodes and in such a way some energy can be saved by processing (in an adaptive manner) less data at every CR node.

We first describe local CM estimation, when the CR nodes in the network do not cooperate. Then we propose a global (theoretical) cost function for estimating the CM in a cooperative manner. We assume, that the K nodes in the CR network

estimate a vector parameter $\mathbf{p}^o$ in a distributed manner, where nodes rely only on the information, that is available to them. The network topology is assumed to be fixed over the sensing time. We consider a linear, fixed combination of neighbor estimates and measurements at every node k and time instant n. The proposed global cost needs to be approximated in a distributed manner, where no FC, as a potential single point of failure in the system, is used. The derivation of the ATC and CTA type CM estimation algorithm diffusion power estimation algorithm follows the ideas in [45], [30], [1].

A. Local estimation

When CR nodes do not cooperate, then according to (4) $\mathbf{R}_{s,k} = \mathrm{E}\left[|\alpha_k|^2 \mathbf{s}(n)\mathbf{s}(n)^H\right]$ and $\mathbf{\Sigma}_{v,k} = \mathrm{E}\left[\mathbf{v}_k(n)\mathbf{v}_k(n)^H\right]$. The estimate $\hat{\mathbf{R}}_k(N)$ of CM $\mathbf{R}_k$ based on the observations $n = 1, \ldots, N$ can be obtained (independently, non-adaptively) at every node k for example as

$$\hat{\mathbf{R}}_k(N) = \frac{1}{N}\sum_{n=1}^{N} \mathbf{y}_k(n)\mathbf{y}_k(n)^H, \tag{6}$$

We continue with the notation, suitable for the adaptive processing, i.e the estimate $\hat{\mathbf{R}}_k(n)$, available at node k at time instant n. In the light of the signal model cases in [46], we consider two specific PU signal models under the detection hypothesis H_1, where $\mathbf{s}(n)$ is a constant or a random variable. Under the different detection hypotheses, the $\hat{\mathbf{R}}_k(n)$ therefore follows the following Wishart distributions [12], [13], [47]

$$\begin{aligned} &H_0 : \hat{\mathbf{R}}_k(n) \sim CW_M(N, \tfrac{1}{N}\mathbf{\Sigma}_{v,k}),\\ &H_1 : \hat{\mathbf{R}}_k(n) \sim CW_M(N, \tfrac{1}{N}\mathbf{\Sigma}_{v,k}, \tfrac{1}{N}\mathbf{\Omega}_k) \quad \text{if} \quad \mathbf{m}_s \neq 0,\\ &H_1 : \hat{\mathbf{R}}_k(n) \sim CW_M(N, \tfrac{1}{N}\mathbf{\Sigma}'_k) \quad \text{if} \quad \mathbf{m}_s = 0, \end{aligned} \tag{7}$$

where N is the degree of freedom (DoF) parameter, $\mathbf{\Sigma}'_k = \mathbf{R}_{s,k} + \mathbf{\Sigma}_{v,k}$, by following the notation in [47, Th. 3.5.2] $\frac{1}{N}\mathbf{\Omega}_k = \left[\frac{1}{N}\mathbf{\Sigma}_{v,k}\right]^{-1}\left[\frac{1}{N}\mathbf{E}_k\mathbf{E}_k^H\right]$, and where the non-zero column n of $M \times N$ mean matrix $\mathbf{E}_k$ equals $\mathrm{E}[\alpha_k]\,\mathbf{m}_s$. The first case corresponds to the Complex Central Wishart (CCW) under detection hypothesis H_0, with population covariance matrix $\frac{1}{N}\mathbf{\Sigma}_{v,k}$. The second case with the non-centrality matrix $\frac{1}{N}\mathbf{\Omega}$ corresponds to the Complex Non-central Wishart distribution (NCW) under H_1. We denote it as Case 1. The third case corresponds to the Complex Central Correlated Wishart (CCCW) under H_1 with population covariance matrix $\frac{1}{N}\mathbf{\Sigma}'_k$. We denote it as Case 2.

According to (4), every node k has a unique channel gain α_k from the PU source, which is not known *a priori* for the nodes. When the nodes in the CR network estimate $\mathbf{R}_k$ without cooperating with other nodes, then the estimates of $\mathbf{R}_k$ are (locally) influenced by the individual channel gains of the corresponding nodes. The local SNR at node k is given by

$$\mathbf{SNR}_k = \frac{\mathrm{Tr}\left[|\alpha_k|^2\left(\mathbf{R}_{s,k} + \mathbf{m}_s\mathbf{m}_s^H\right)\right]}{\mathrm{Tr}\left[\mathbf{\Sigma}_{v,k}\right]}. \tag{8}$$

As seen, some CR nodes achieve better detection performance due to higher channel gains (i.e due to better position in the space) than the other. We are interested in a scheme, where all nodes can achieve similar detection performance, despite of their individual channel gains. The method (6) expects that N samples are available for calculation of the estimate and is not adaptive in its nature, i.e the CR system is unable to react quickly to a possible change of a detection hypothesis during the observation time N. This may increase the possibility of false alarm or a miss-detection of the PU user and thus also an interference to the PU user. As seen in next chapters, we find an adaptive, exponential (non-equal weighed) averaging based method for estimated the CMs, which is able to learn and react to the changes in the statistics of the CM in real time and needs to store only data from previous iteration.

B. Global estimation

The CR nodes could cooperate via internal communication links to enhance the detection performance (of the PU signal(s)) at every node k. In the distributed CR network we assume:

- **AS 2**. There is a common control channel available for the CR system for transferring the network level control messages. The communication links between the CR nodes are ideal and not capacity restricted.
- **AS 3**. The CR network is strongly connected (however nodes can directly communicate only with a subset of neighbor nodes).

We propose a model where nodes jointly (and in case of either detection hypothesis) estimate the network average CM, which is denoted as $\mathbf{R}^o$ and defined as follows

$$\mathbf{R}^o = \frac{1}{K}\sum_{k=1}^{K}\mathbf{R}_k^o. \tag{9}$$

For notational convenience, introduce $M^2 \times 1$ $\mathbf{r}^o = \mathrm{vec}(\mathbf{R}^o)$. Thus we can write

$$\mathbf{r}^o = \frac{1}{K}\sum_{k=1}^{K}\mathrm{vec}(\mathbf{R}_k^o) = \frac{1}{K}\sum_{k=1}^{K}\mathrm{E}\left[\mathrm{vec}\left[\mathbf{y}_k(n)\mathbf{y}_k(n)^H\right]\right]. \tag{10}$$

Let us define the Hermitian rank one observation matrix $\mathbf{D}_{R,k}(n) = \mathbf{y}_k(n)\mathbf{y}_k(n)^H$ (under both hypothesis) at node k at time instant n. Its $M^2 \times 1$ vectorized form is $\mathbf{d}_{R,k}(n) = \mathrm{vec}\left[\mathbf{D}_{R,k}(n)\right]$. We can decompose the $\mathbf{d}_{R,k}(n)$ into the product of a $M^2 \times M^2$ constant (invertible) complex matrix $\mathbf{T}$ and a $M^2 \times 1$ real vector $\mathbf{d}_k(n)$ as $\mathbf{d}_{R,k}(n) = \mathbf{T}\mathbf{d}_k(n)$, to keep the dimension of the estimated vector minimal in the adaptive recursions. For example, when $M = 2$, then

$$\mathbf{T}\mathbf{d}_k(n) = \begin{bmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & -i & 0\\ 0 & 1 & i & 0\\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \mathbf{D}_{R,k}(n)(1,1)\\ \Re[\mathbf{D}_{R,k}(n)(1,2)]\\ \Im[\mathbf{D}_{R,k}(n)(1,2)]\\ \mathbf{D}_{R,k}(n)(2,2) \end{bmatrix}. \tag{11}$$

We denote the estimate of the real valued $\mathrm{E}\left[\mathbf{d}_k(n)\right]$ as $\hat{\mathbf{p}}_k(n)$. To construct an adaptive distributed estimation algorithm, we first relate the estimates of $\mathbf{R}_k^o$ and $\mathbf{R}^o$ in (9) with the minimization of the following global (network-wise) cost function

$$\mathbf{p}^o = \underset{\mathbf{p}}{\mathrm{argmin}}\sum_{k=1}^{K} J_k(\mathbf{p}) = \underset{\mathbf{p}}{\mathrm{argmin}}\sum_{k=1}^{K}\mathrm{E}\left\|\mathbf{d}_k(n) - \mathbf{p}\right\|^2, \tag{12}$$

where the vector $\mathbf{p} \in R^{M^2}$ represents the real valued parameters of the CM, to be estimated. Thus $\mathbf{p}^o$ represents the optimal (real valued) CM estimate or is the optimal solution for the minimization of the Mean Square Error (MSE) type of global aggregate cost function $J^{glob}(\mathbf{p})$, which is given as

$$\begin{aligned} J^{glob}(\mathbf{p}) &= \sum_{k=1}^{K} J_k(\mathbf{p}) \\ &= \sum_{k=1}^{K} \mathrm{E}\left[\|\mathbf{d}_k(n)\|^2 - \mathbf{d}_k^T(n)\mathbf{p} - \mathbf{p}^T\mathbf{d}_k(n) + \mathbf{p}^T\mathbf{p}\right]. \end{aligned} \tag{13}$$

Let us note that compared to the models in [48], [35], [30], in (13) both the observation and estimation variables are vectors. By differentiating $J^{glob}(\mathbf{p})$ in (13) with respect to $\mathbf{p}$ and setting the result to zero, we get

$$\nabla_{\mathbf{p}} J^{glob}(\mathbf{p}) = -\sum_{k=1}^{K} \mathrm{E}\left[\mathbf{d}_k^T(n)\right] + K\mathbf{p}^T = 0. \tag{14}$$

It follows that

$$\mathbf{p}^o = \frac{1}{K}\sum_{k=1}^{K} \mathrm{E}\left[\mathbf{d}_k(n)\right]. \tag{15}$$

The Hessian of the aggregate cost function is

$$\nabla^2_{\mathbf{p}} J^{glob}(\mathbf{p}) = 2\mathbf{I}_M. \tag{16}$$

Obviously $J^{glob}(\mathbf{p})$ in (13) is strongly convex [34, C.18] with the unique solution $\mathbf{p}^o$. Also, in case of one node in the CR system ($K = 1$) or when the nodes do not cooperate, then the individual cost $J_k(\mathbf{p})$ is minimized at the point $\mathbf{p}_k^o = \mathrm{E}[\mathbf{d}_k(n)]$. Since $\nabla^2_{\mathbf{p}} J_k^{loc}(\mathbf{p}) = 2\mathbf{I}_M$ and the individual cost $J_k^{loc}(\mathbf{p})$ is strongly convex, thus $\mathbf{p}_k^o$ is unique as well.

Compared to [48], [35], [30], in our paper the local costs $J_k(\mathbf{p})$ are individually not minimized at the same global point $\mathbf{p}^o$ due to different channel conditions. However the derivation of the diffusion LMS algorithm still follows the procedure as proposed in these papers. The proposed optimal solution (12) is similar to the Pareto model, which is analysed in [49].

Note that

$$\begin{aligned} \mathbf{R}_k^o &= \mathrm{vec}^{-1}\left[\mathbf{T}\mathbf{p}_k^o\right] \\ \mathbf{R}^o &= \mathrm{vec}^{-1}\left[\mathbf{T}\mathbf{p}^o\right]. \end{aligned} \tag{17}$$

We seek an iterative solution to estimate the $\mathbf{p}_k^o$ and $\mathbf{p}^o$ in a manner, which is adaptive in time, and is fully distributed (cooperative). We propose to use diffusion LMS based distributed solution.

C. Iterative Diffusion solutions

Let $\mathcal{N}_k$ denote the neighborhood group of node $k \in K$, i.e $\mathcal{N}_k$ defines the set of nodes l which can send data unidirectionally the node k. The node k is assumed to be always connected to itself. For deriving the diffusion LMS algorithm, we define and use the standard matrices $\mathbf{A}$, $\mathbf{C}$ and $\mathbf{C}$ similarly to [30], with non-negative elements $a_{l,k}$, $b_{l,k}$ and $c_{l,k}$, that describe how data is exchanged and combined in the network.

Let us start by defining the $K \times K$ right stochastic matrix $\mathbf{C}$ with non-negative elements so that

$$c_{l,k} = 0 \quad \text{if} \quad l \notin \mathcal{N}_k, \quad \mathbf{C}\mathbf{1} = \mathbf{1}, \tag{18}$$

where $c_{l,k} = 1$ if node l is connected to the node k. The global cost (13) can be divided into the local cost of over the neighborhood of node k and the sum of local costs of other nodes over their corresponding neighborhoods, and can be given in the following form

$$J^{glob}(\mathbf{p}) = J_k^{loc}(\mathbf{p}) + \sum_{l\neq k}^{K} J_l^{loc}(\mathbf{p}). \tag{19}$$

The local cost at every node k can be expressed as a weighted combination of the costs of the neighbors of every node k. Thus with the help of non-negative coefficients $c_{l,k}$ the local cost can be given as follows

$$J_k^{loc}(\mathbf{p}) = \sum_{l\in\mathcal{N}_k} c_{l,k} J_l(\mathbf{p}) \tag{20}$$

and is minimized at the location $\mathbf{p}_k^{loc}$. The following relation $J_l^{loc}(\mathbf{p}) \approx J_l^{loc}(\mathbf{p}^{loc}) + \|\mathbf{p} - \mathbf{p}_l^{loc}\|^2$ [50] can be used for the second part of right hand side (RHS) of (19) to relate the variable $\mathbf{p}$ and the $\mathbf{p}_l^{loc}$. Here the $J_k^{loc}(\mathbf{p}_l^{loc})$, can be ignored, since it is independent on the variable $\mathbf{p}$. Thus we have the modified global cost function $J^{glob'}$ as follows

$$J^{glob'}(\mathbf{p}) = J_k^{loc}(\mathbf{p}) + \sum_{l\neq k}^{K} \|\mathbf{p} - \mathbf{p}_l^{loc}\|^2. \tag{21}$$

Note that it is not assumed, that node k has access to all the $\mathbf{p}_l^{loc}$ in the network. Thus we need to approximate the $J^{glob'}(\mathbf{p})$ locally at every node k and the standard steps follow. We use the non-negative coefficients $b_{l,k}$ to define if $\mathbf{p}_l^{loc}$ is available for the node k. Thus the elements $b_{l,k}$ take the following values

$$\text{if} \quad l \notin \mathcal{N}_k \quad \text{then} \quad b_{l,k} = 0 \quad \text{else} \quad b_{l,k} = 1. \tag{22}$$

Then, we limit the summation $\sum_{l\neq k}^{K} \|\mathbf{p} - \mathbf{p}_l^{loc}\|^2$ on the RHS of (21) to the neighbors of node k i.e $\sum_{l\in\mathcal{N}_k/\{k\}} b_{l,k}\|\mathbf{p} - \mathbf{p}_l^{loc}\|^2$. Secondly, we replace the (only theoretically available) $\mathbf{p}_l^{loc}$ with an intermediate estimate $\hat{\boldsymbol{\psi}}_l$, which is available at node l.

After these steps the approximation of (21) at node k is given as

$$\begin{aligned} J_k^{dist}(\mathbf{p}) = &\sum_{l\in N_k} c_{l,k}\,\mathrm{E}\,\|\mathbf{d}_l(n) - \mathbf{p}\|^2 \\ &+ \sum_{l\in\mathcal{N}_k/\{k\}} b_{l,k}\|\mathbf{p} - \hat{\boldsymbol{\psi}}_l\|^2. \end{aligned} \tag{23}$$

The steepest descent algorithm [51] can be used to obtain a recursion for the estimate of $\mathbf{p}^o$ at time instant n, at node k,

denoted as $\hat{\mathbf{p}}_k(n)$. By skipping the derivation steps, as in [30], the two-step steepest descent recursions are then given as

$$\hat{\boldsymbol{\psi}}_k(n+1) = \hat{\mathbf{p}}_k(n) + \mu_k \sum_{l\in N_k} c_{l,k}\left[\mathbf{d}_l(n) - \hat{\mathbf{p}}_k(n)\right]$$

$$\hat{\mathbf{p}}_k(n+1) = \left[1 - \nu_k \sum_{l\in\mathcal{N}_k/\{k\}} b_{l,k}\right]\hat{\boldsymbol{\psi}}_k(n+1) + \nu_k \sum_{l\in\mathcal{N}_k/\{k\}} b_{l,k}\hat{\boldsymbol{\psi}}_l(n+1), \quad (24)$$

where μ_k and ν_k are a positive step sizes, $\hat{\boldsymbol{\psi}}_k(n+1)$ is an intermediate estimate at node k at time n.

The coefficients in front of $\hat{\boldsymbol{\psi}}_l(n+1)$, $l = 1,\ldots,K$ in the second equation of (24) can be incorporated into the non-negative coefficients $a_{l,k}$. Let us introduce the $K \times K$ matrix $\mathbf{A}$, whose elements satisfy

$$a_{l,k} = 0 \quad \text{if} \quad l \notin \mathcal{N}_k, \quad \mathbf{1}^T\mathbf{A} = \mathbf{1}^T. \quad (25)$$

Thus we take $a_{k,k} = 1 - \nu_k \sum_{l\in\mathcal{N}_k/\{k\}} b_{l,k}$ and $a_{l,k} = \nu_k b_{l,k}$ for $l \neq k$. It is straightforward to see that $\sum_{l\in\mathcal{N}_k} a_{l,k} = 1$ for every $k \in K$ and thus $\mathbf{A}$ is a left stochastic matrix. Finally we obtain the Adapt and Combine (ATC) recursions as

$$\hat{\boldsymbol{\psi}}_k(n+1) = \hat{\mathbf{p}}_k(n) + \mu_k \sum_{l\in N_k} c_{l,k}\left(\mathbf{d}_l(n) - \hat{\mathbf{p}}_k(n)\right)$$

$$\hat{\mathbf{p}}_k(n+1) = \sum_{l\in N_k} a_{l,k}\hat{\boldsymbol{\psi}}_l(n+1). \quad (26)$$

In similar manner the Combine and Adapt (CTA) version can be derived, following the ideas from [30]. In the ATC and CTA algorithms the coefficients $c_{l,k}$ and $a_{l,k}$ define respectively how the measurements $\mathbf{d}_l(n)$ and $\hat{\mathbf{p}}_l(n)$ are (unidirectionally) available for the node k. Thus the matrices $\mathbf{A}$ and $\mathbf{C}$ specify the combination strategy of the measurements and the estimates respectively in the CR network.

In Algorithm 1 we present the ATC and CTA based CM estimation recursions and the detection step in a common form. For this we define an additional intermediate estimate $\hat{\boldsymbol{\phi}}_k(n)$ and denote the $K \times K$ matrix $\mathbf{A}$ as $\mathbf{A}_1$ or $\mathbf{A}_2$, with the elements $a_{1,l,k}$ and $a_{2,l,k}$ correspondingly. The selection options of the matrices $\mathbf{A}_1$ and $\mathbf{A}_2$ and $\mathbf{C}$ based on [30] are given in Table 1. In practice the non-negative coefficients $a_{1,l,k}$, $a_{2,l,k}$, $c_{l,k}$ can be chosen freely under the conditions (18) and (25) respectively. The coefficients $b_{l,k}$ are absorbed into coefficients $a_{l,k}$ and do not have to be considered in practice. For comparison in Section V, we list also a topology, where every node acts as a FC, denoted as Global FC LMS in Table 1. In such case CR nodes estimate the CM adaptively and independently (without sharing estimates), all the measurements from all the CR nodes are available and equally weighted for every node in the network.

Thus we observe that according to (17), Table 1 and the CM estimation recursions in Algorithm 1, when the nodes in the CR network do not cooperate, then the adaptive estimate $\hat{\mathbf{p}}_k(n)$ at time instant n at node k defines the individual (local) adaptive estimate of $\mathbf{R}_k^o$. When nodes cooperate by following the proposed cost (12), Table 1 and the CM estimation recursions in Algorithm 1, then the adaptive estimate $\hat{\mathbf{p}}_k(n)$ at time instant n at node k defines the adaptive estimate of $\mathbf{R}^o$ in (9), within acceptable mean square error bounds [35], [30]. Thus after several iterations, the adaptive estimate $\hat{\mathbf{R}}_k(n)$ of $\mathbf{R}^o$ is available (via the transformation (11) and de-vectorization) for every node in the CR network. Therefore depending on the cooperation model of the nodes, the node k at time instant n can perform independently the LE detection based on the available matrix estimate $\hat{\mathbf{R}}_k(n) = \text{vec}^{-1}\left[\mathbf{T}\hat{\mathbf{p}}_k(n)\right]$.

Algorithm 1 Distributed LMS based CM Estimation and Detection

Start with $\hat{\mathbf{p}}_k(0) = \mathbf{p}(0)$ for every k .
Given non-negative real coefficients $a_{1,l,k}, a_{2,l,k}, c_{l,k}$
for every time instant $n \geq 1$ **do**
  for every node $k = 1, ..., K$ **do**
    1. CM estimation recursions:
    $\hat{\boldsymbol{\phi}}_k(n) = \sum_{l=1}^K a_{1,l,k}\hat{\mathbf{p}}_l(n))$.
    $\hat{\boldsymbol{\psi}}_k(n+1) = \hat{\boldsymbol{\phi}}_k(n)$
      $+\mu_k \sum_{l=1}^K c_{l,k}\left[\mathbf{T}^{-1}\mathbf{d}_{R,l}(n) - \hat{\boldsymbol{\phi}}_k(n)\right]$
    $\hat{\mathbf{p}}_k(n+1) = \sum_{l=1}^K a_{2,l,k}\hat{\boldsymbol{\psi}}_l(n)$
    2. LE detection decision:
    $H_0 : \lambda_1\left[\text{vec}^{-1}\left[\mathbf{T}\hat{\mathbf{p}}_k(n+1)\right]\right] < \gamma_{LE,k}$ or
    $H_1 : \lambda_1\left[\text{vec}^{-1}\left[\mathbf{T}\hat{\mathbf{p}}_k(n+1)\right]\right] > \gamma_{LE,k}$.
    (Refer to (54) or (57) for selecting the $\gamma_{LE,k}$).
  end for
end for

TABLE I
CHOICES OF MATRICES $\mathbf{A}_1$ AND $\mathbf{A}_2$ AND $\mathbf{C}$ FOR DIFFERENT LMS ALGORITHMS

Algorithm	$\mathbf{A}_1$	$\mathbf{A}_2$	$\mathbf{C}$
No Cooperation LMS	**I**	**I**	**I**
Global FC LMS [30]	**I**	**I**	$(1/K)\mathbf{1}\mathbf{1}^T$
CTA diffusion LMS [30]	**A**	**I**	**C**
ATC diffusion LMS (26)	**I**	**A**	**C**

Regarding the communication cost of Algorithm 1, then based on Table 1 it is obvious, that when $\mathbf{A} \neq \mathbf{I}$, then from the transmission point of view still every node $k \in K$ needs to broadcast its $M^2 \times 1$ estimation vector $\hat{\mathbf{p}}_k(n)$ at time instant n to the neighbours of hearing distance of the node k. However from the receiving point of view the number of estimates $\hat{\mathbf{p}}_k(n)$ required for the fusion by every node k is determined by the selection of matrix $\mathbf{A}$. Similarly, every node k obtains at time instant n a $M^2 \times 1$ observation vector $\hat{\mathbf{d}}_k(n)$ and when $\mathbf{C} \neq \mathbf{I}$ broadcasts it at time instant n to the neighbours of hearing distance of the node k. Thus on the receiving side, the exact selection of $\mathbf{C}$ determines the number $\hat{\mathbf{d}}_k(n)$ required by every node k at time instant n for observation fusion. In Section V-A we comment our selection of $\mathbf{A}$ and $\mathbf{C}$ for the simulations.

Finally we note that in addition to AS 2, obviously the CR system needs some control layer protocol to establish a connection between the nodes. The details of this operation is outside the scope of this paper.

IV. Performance analysis

The performance analysis of the proposed algorithm is divided into three parts. First we derive a general model for analyzing the mean and (co-)variance of the adaptive CM estimates of recursions in Algorithm 1 in one framework. Secondly we study the statistical properties of the adaptive CM estimates. For studying the LE detection performance of the adaptive CM estimate, the distribution of the adaptive CM estimate is approximated by a CCCW distribution. We propose the usage of the Total and General Variance methods for approximation the DoF and mean matrix parameters for the corresponding CCCW distributions, based on the moments of adaptive CM estimates. Thirdly we provide theoretical results for the LE detector. Let us note that for the theoretical performance analysis of the LE detector, we need to know the values of the channel gains and the noise power.

A. Moment analysis of adaptive CM estimates

For the analysis of the moments of the spatio-temporal adaptive CM estimates, we propose to use a more general vector/matrix recursion model.

We stack first the $M^2 \times 1$ estimates and observations from all the nodes $k \in K$ into a $KM^2 \times 1$ column vector $\hat{\mathbf{p}}(n)|H_i = [\hat{\mathbf{p}}_1(n)|H_i \dots \hat{\mathbf{p}}_K(n)|H_i]^T$ and $\mathbf{d}(n)|H_i = [\mathbf{d}_1(n)|H_i \dots \mathbf{d}_K(n)|H_i]^T$ respectively, where $i = 1$ denotes the case when the PU signal is present and $i = 0$ the case when the PU signal is absent. The initial estimate is noted as $\hat{\mathbf{p}}(0)|H_i$.

Secondly we define an additional $K \times K$ matrix $\boldsymbol{\mathcal{M}} = \operatorname{diag}\{\mu_1, \dots, \mu_K\}$, which contains the positive step size parameters of the algorithms for every node $k \in K$. The matrix $\boldsymbol{\mathcal{M}}$ is then be extended to another $KM^2 \times KM^2$ matrix as $\overline{\boldsymbol{\mathcal{M}}} = \boldsymbol{\mathcal{M}} \otimes \mathbf{I}_{M^2}$. For the purpose of comparison with the Consensus algorithm [33], let the $K \times K$ matrix $\mathbf{A}_0$ specify the fusion strategy of estimates of the consensus algorithm.

The $K \times K$ network topology matrices $\mathbf{A}_0$, $\mathbf{A}_1$, $\mathbf{A}_2$ and $\mathbf{C}$ are extended to $KM^2 \times KM^2$ matrices as follows, $\overline{\mathbf{A}}_0 = \mathbf{A}_0^T \otimes \mathbf{I}_{M^2}$, $\overline{\mathbf{A}}_1 = \mathbf{A}_1^T \otimes \mathbf{I}_{M^2}$, $\overline{\mathbf{A}}_2 = \mathbf{A}_2^T \otimes \mathbf{I}_{M^2}$ and $\overline{\mathbf{C}} = \mathbf{C}^T \otimes \mathbf{I}_{M^2}$.

Proposition 1. *The distributed LMS algorithms in Table 1 and the consensus algorithm [33] can be described by the following spatio-temporal recursion*

$$\hat{\boldsymbol{p}}(n+1)|H_i = \overline{\boldsymbol{A}}_2 \left(\overline{\boldsymbol{A}}_0 - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\boldsymbol{A}}_1 \hat{\boldsymbol{p}}(n)|H_i + \overline{\boldsymbol{A}}_2 \overline{\boldsymbol{\mathcal{M}}} \overline{\boldsymbol{C}} \boldsymbol{d}(n)|H_i. \tag{27}$$

In case of LMS algorithms $\mathbf{A}_0 = \mathbf{I}_K$ and for example we get the ATC algorithm with no measurement exchange, when we take additionally $\mathbf{A}_1 = \mathbf{C} = \mathbf{I}_K$ and $\mathbf{A}_2 \neq \mathbf{I}_K$, according to the selected network topology. Thus $\overline{\mathbf{A}}_0 = \overline{\mathbf{A}}_1 = \mathbf{I}_K \otimes \mathbf{I}_{M^2}$, $\overline{\mathbf{A}}_2 = \mathbf{A}_2^T \otimes \mathbf{I}_{M^2}$ and $\overline{\mathbf{C}} = \mathbf{I}_K \otimes \mathbf{I}_{M^2}$. For the Consensus algorithm [33], we take $\mathbf{A}_1 = \mathbf{A}_2 = \mathbf{C} = \mathbf{I}_K$, $\mathbf{A}_0 \neq \mathbf{I}_K$ according to the network topology and thus we have $\overline{\mathbf{A}}_0 = \mathbf{A}_0^T \otimes \mathbf{I}_{M^2}$ and $\overline{\mathbf{A}}_1 = \overline{\mathbf{A}}_2 = \mathbf{I}_K \otimes \mathbf{I}_{M^2}$. Note, that the proposed Kronecker extension retains the stochastic property of the extended matrix and due to the transpose, the matrices $\overline{\mathbf{A}}_1$ and $\overline{\mathbf{A}}_2$ are now right stochastic and $\overline{\mathbf{C}}$ is left stochastic.

For studying the performance of the LMS algorithms, we first need to evaluate the moments - mean and covariance of the stacked estimates $\hat{\mathbf{p}}(n)$ and we provide the corresponding recursions for evaluating these moments.

1) Mean of estimates: Let us denote the conditional expectation of the observation vector as $\mathrm{E}[\mathbf{d}(n)|H_i]$, where $i = 0, 1$. We specify these values in the Section IV-B1.

Proposition 2. *The general recursion (27), can be expressed as*

$$\begin{aligned}\mathrm{E}[\hat{\boldsymbol{p}}(n+1)|H_i] = \overline{\boldsymbol{A}}_2 \left(\overline{\boldsymbol{A}}_0 - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\boldsymbol{A}}_1 \mathrm{E}[\hat{\boldsymbol{p}}(n)|H_i] \\ + \overline{\boldsymbol{A}}_2 \overline{\boldsymbol{\mathcal{M}}} \overline{\boldsymbol{C}} E[\boldsymbol{d}(n)|H_i],\end{aligned} \tag{28}$$

for $i = 0, 1$, where the initial value for the mean vector is given as $\mathrm{E}[\hat{\boldsymbol{p}}(0)|H_i]$, $i = 0, 1$.

After iterating we see, that the mean recursion can be given in the following equivalent form

$$\begin{aligned}\mathrm{E}[\hat{\mathbf{p}}(n)|H_i] = \left[\overline{\mathbf{A}}_2 \left(\overline{\mathbf{A}}_0 - \overline{\mathcal{M}}\right) \overline{A\mathbf{1}}\right]^n \hat{\mathbf{p}}(0) \\ + \left[\sum_{i=0}^{n-1} \left[\overline{\mathbf{A}}_2 \left(\overline{\mathbf{A}}_0 - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1\right]^i\right] \\ \times \overline{\mathbf{A}}_2 \overline{\boldsymbol{\mathcal{M}}} \overline{\mathbf{C}} E[\mathbf{d}(n)|H_i].\end{aligned} \tag{29}$$

For the asymptotic analysis of the mean recursion (29), we need to analyse the asymptotic behavior of $\left[\overline{\mathbf{A}}_2 \left(\overline{\mathbf{A}}_0 - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1\right]^n$ and the limit of the geometric series $\sum_{i=0}^{n-1} \left[\overline{\mathbf{A}}_2 \left(\overline{\mathbf{A}}_0 - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1\right]^i$, when $n \to \infty$.

According to [52, Theorem 5.6.12], the convergence $\lim_{n\to\infty} [\overline{\mathbf{A}}_2 \left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1]^n \to 0$ happens if and only if the spectral radius of the matrix $\overline{\mathbf{A}}_2 \left(\overline{\mathbf{A}}_0 - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1$ satisfies

$$\rho\left(\overline{\mathbf{A}}_2 \left(\overline{\mathbf{A}}_0 - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1\right) < 1. \tag{30}$$

As also noted in [33], the stability of the consensus algorithm is dependent not only on the selection of step sizes but also on the estimation exchange topology $\mathbf{A}_0$. This fact limits the usage of consensus algorithm in practice.

For the diffusion LMS based algorithms, the choice of step sizes in the $\overline{\boldsymbol{\mathcal{M}}}$ of the block diagonal matrix $\left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right)$ should guarantee that the stability condition (30) holds, given the left stochastic matrices $\mathbf{A}_1$ and $\mathbf{A}_2$ and by considering the proposed Kronecker extensions. It was shown in [45, Lemma D.6], that by using the block maximum norm, denoted as $\|.\|_{b,\infty}$, then for the matrix of type $\overline{\mathbf{A}}_2 \left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1$, it holds that

$$\begin{aligned}\rho\left(\overline{\mathbf{A}}_2 \left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1\right) &\leq \|\overline{\mathbf{A}}_2 \left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1\|_{b,\infty} \\ &\leq \|\overline{\mathbf{A}}_2\|_{b,\infty} \| \left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right) \|_{b,\infty} \|\overline{\mathbf{A}}_1\|_{b,\infty} \\ &= \| \left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right) \|_{b,\infty} \\ &= \rho\left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right).\end{aligned} \tag{31}$$

Since the matrix $\left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right)$ is diagonal we impose to have that

$$\rho\left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right) = \max_k |1 - \bar{\mu}_k| < 1, \tag{32}$$

where the $\bar{\mu}_k$, $k = 1, \dots, KM^2$ are the diagonal elements of $\overline{\boldsymbol{\mathcal{M}}}$. Thus based on (32), the sufficient condition for the (30) to hold (i.e to make the power component in the (29) to zero) is to select every $\bar{\mu}_k$ in $\overline{\boldsymbol{\mathcal{M}}}$ so that the diagonal matrix $\left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right)$ is stable - i.e all the eigenvalues of $\left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right)$ are inside the unit circle. Since $\overline{\boldsymbol{\mathcal{M}}} = \boldsymbol{\mathcal{M}} \otimes \mathbf{I}_{M^2}$, the step size condition (32)

applies for the diagonal elements μ_k of the $K \times K$ diagonal matrix $\boldsymbol{\mathcal{M}}$ directly. Thus for every $k = 1 \dots K$ we should have

$$0 < \mu_k < 2. \tag{33}$$

The CR system designer can choose the step size(s) of the nodes (freely) in the range (33), by taking into account the CR system design considerations (which are however out of the scope of the paper). Usually the step sizes are taken quite small to get more precise estimates (and thus better detection performance) i.e $\mu_k \ll 2$, but with the cost of longer convergence time of the adaptive estimations. We illustrate the effect of convergence in Section V.

Next we analyse the convergence condition of the second component on the RHS of (29). Based on the result of [52, Corollary 5.6.16] the geometric series $S_n = \sum_{i=0}^{n-1} \left[\overline{\mathbf{A}}_2 \left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1\right]^i$ is generated by the matrix $\left[\overline{\mathbf{A}}_2 \left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1\right]$ and converges if for a matrix norm it holds that $\|\overline{\mathbf{A}}_2 \left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1\| < 1$. This condition guarantees that $\left[\mathbf{I} - \left[\overline{\mathbf{A}}_2 \left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1\right]\right]$ is invertible. Since from (31) we have $\rho\left(\overline{\mathbf{A}}_2 \left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1\right) \leq \| \left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right) \|_{b,\infty} = \rho\left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right)$, then the sufficient condition for the convergence of the series is given by (32). Hence when the condition (32) is satisfied, then as $n \to \infty$ the geometric series converges to

$$S_n = \left[\mathbf{I} - \left[\overline{\mathbf{A}}_2 \left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1\right]\right]^{-1}. \tag{34}$$

Thus by noting the mean of $\hat{\mathbf{p}}(n)$ in steady state and under both hypothesis H_i, $i = 0, 1$ as $\mathrm{E}\left[\hat{\mathbf{p}}(\infty)|H_i\right]$, we have that

$$\begin{aligned}\mathrm{E}\left[\hat{\mathbf{p}}(\infty)|H_i\right] = &\left[\mathbf{I} - \left[\overline{\mathbf{A}}_2 \left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1\right]\right]^{-1} \\ &\times \overline{\mathbf{A}}_2 \overline{\boldsymbol{\mathcal{M}}\mathbf{C}}\, \mathrm{E}\left[\mathbf{d}(n)|H_i\right],\end{aligned} \tag{35}$$

where the conditional expectations of observations $\mathrm{E}\left[\mathbf{d}(n)|H_i\right]$ are given in the Section IV-B1.

The steady state result (35) is asymptotically biased. Let us note, that the mean error (or bias) in steady state is given as

$$\overline{\mathrm{E}\left[\hat{\mathbf{p}}(\infty)|H_i\right]} = \|(\mathbf{1}_K \otimes \mathbf{p}^o|H_i) - \mathrm{E}\left[\hat{\mathbf{p}}(\infty)|H_i\right]\|^2, \tag{36}$$

for, $i = 0, 1$, where $\mathbf{p}^o|H_i$ denotes the optimal solution (15) and $\mathrm{E}\left[\hat{\mathbf{p}}(\infty)|H_1\right]$ follows from (35). Since the global solution (15) follows the Pareto model, we refer in this paper to the generic result [49, Th. 3] for characterizing the bias term, such as (36). The referred theorem determines that under certain conditions (for example when we have the same step-sizes and a doubly-stochastic matrix $\mathbf{A}$), a lower step-size makes the bias term also lower - i.e the estimates are closer to the optimal solution. Thus in practice, when very low step-size values are used, the bias term can be ignored.

2) Covariance of estimates: Let us denote the conditional covariance of the estimates under the hypothesis H_i, $i = 0, 1$ as $\mathrm{Cov}\left[\hat{\mathbf{p}}(n+1)|H_i\right]$. Similarly let $\mathrm{Cov}\left[\mathbf{d}(n)|H_i\right]$ denote the conditional covariance of the observations.

Proposition 3. *By using recursions (27), (28), the definition of covariance and by considering the fact that $\hat{\boldsymbol{p}}(n)|H_i$ is independent of the stacked observation vector $\boldsymbol{d}(n)|H_i$, it can be shown that the covariance recursion is*

$$\begin{aligned}\mathrm{Cov}\left[\hat{\boldsymbol{p}}(n+1)|H_i\right] = &\overline{\boldsymbol{A}}_2 \left(\overline{\boldsymbol{A}}_0 - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\boldsymbol{A}}_1 \mathrm{Cov}\left[\hat{\boldsymbol{p}}(n)|H_i\right] \\ &\times \overline{\boldsymbol{A}}_1^T \left(\overline{\boldsymbol{A}}_0^T - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\boldsymbol{A}}_2^T \\ &+ \overline{\boldsymbol{A}}_2 \overline{\boldsymbol{\mathcal{M}}\boldsymbol{C}}\, \mathrm{Cov}\left[\boldsymbol{d}(n)|H_i\right] \overline{\boldsymbol{C}}^T \overline{\boldsymbol{\mathcal{M}}\boldsymbol{A}}_2^T.\end{aligned} \tag{37}$$

where initial estimate of covariance matrix is noted by $Cov\left[\hat{\boldsymbol{p}}(0)|H_i\right]$, $i = 0, 1$.

The covariance matrix of the observations, $Cov\left[\mathbf{d}(n)|H_i\right]$, is constant over time n and we provide the values in the Section IV-B1. Note that (37) is in the form of a discrete time algebraic Riccati equation (DARE). Thus the covariance results in steady state (i.e the solution to DARE), can be found by using standard procedures, such as [53, App. E].

Finally we note, that according to the theory of adaptive filtering it is generically known that a smaller step size causes lower co-variance of an adaptive estimate in steady state [51] and this leads to better detection result.

B. Statistical modeling of adaptive CM estimates

In this section we first find the theoretical moments for the rank one (Hermitian) observations $\mathbf{d}_{R,k}(n)$, which are then transformed to real domain for the spatio-temporal moment recursions of CM estimate $\hat{\mathbf{p}}_k(n)$, described in the previous subsection. Then we describe the statistical modelling of adaptive CM estimates. Thirdly we propose two methods for approximating the adaptive CM estimates by a Wishart distribution.

1) Moments of rank one observations: First we summarize the generic and known results about the moments of $M \times M$ NCW and CCCW matrices $\hat{\mathbf{R}}_k$, based on [54].

When a $M \times M$ matrix $\hat{\mathbf{R}}_k$ follows a NCW distribution with a DoF parameter $\bar{N}$, a noise population covariance matrix $\bar{\boldsymbol{\Sigma}}_{v,k}$ and a non-centrality matrix $\bar{\boldsymbol{\Omega}}_k = \left[\bar{\boldsymbol{\Sigma}}_{v,k}\right]^{-1} \bar{\boldsymbol{T}}_k$, where $\bar{\boldsymbol{T}}_k = \bar{\mathbf{E}}_k \bar{\mathbf{E}}_k^H$ and where the non-zero column k of $M \times N$ mean matrix $\bar{\mathbf{E}}_k$ is $\mathrm{E}\left[\mathbf{y}_k(n)\right]$, i.e $\hat{\mathbf{R}}_k \sim CW_M(\bar{N}, \bar{\boldsymbol{\Sigma}}_{v,k}, \bar{\boldsymbol{\Omega}})$, then the first and vectorized second moments are given as

$$\begin{aligned}\mathrm{E}\left[\hat{\mathbf{R}}_k\right] &= \bar{N}\bar{\boldsymbol{\Sigma}}_{v,k} + \bar{\boldsymbol{T}}_k, \\ \mathrm{Cov}\left[\mathrm{vec}(\hat{\mathbf{R}}_k)\right] &= (\bar{\boldsymbol{\Sigma}}_{v,k}^T \otimes \bar{\boldsymbol{T}}_k) + (\bar{\boldsymbol{T}}_k^T \otimes \bar{\boldsymbol{\Sigma}}_{v,k}) \\ &\quad + \bar{N}(\bar{\boldsymbol{\Sigma}}_{v,k}^T \otimes \bar{\boldsymbol{\Sigma}}_{v,k}).\end{aligned} \tag{38}$$

As a special case, when the matrix $\hat{\mathbf{R}}_k$ follows a CCCW distribution with a population covariance matrix $\bar{\boldsymbol{\Sigma}}_k$, i.e $\hat{\mathbf{R}}_k \sim CW_M\left(\bar{N}, \bar{\boldsymbol{\Sigma}}_k\right)$, then the matrix $\bar{\boldsymbol{T}}_k$ equals zero and we get

$$\begin{aligned}\mathrm{E}\left[\hat{\mathbf{R}}_k\right] &= \bar{N}\bar{\boldsymbol{\Sigma}}_k, \\ \mathrm{Cov}\left[\mathrm{vec}(\hat{\mathbf{R}}_k)\right] &= \bar{N}(\bar{\boldsymbol{\Sigma}}_k^T \otimes \bar{\boldsymbol{\Sigma}}_k).\end{aligned} \tag{39}$$

These results in [54] are based on the characteristic functions of the corresponding Wishart distributions and apply for $\bar{N} \geq 1$. We note that $\bar{\boldsymbol{\Sigma}}_k^T = \bar{\boldsymbol{\Sigma}}_k^c$ for a Hermitian matrix and then (39) also follows from [55] and [56]. Thus the moments of

$\mathbf{d}_{R,k}(n)$ can be found by using the results (38) and (39) with $\bar{N}=1$, $\hat{\mathbf{R}}_k=\mathbf{y}_k(n)\mathbf{y}_k(n)^H=\mathbf{D}_{R,k}(n)$, $\bar{\mathbf{\Sigma}}_{v,k}=\sigma_v^2\mathbf{I}_M$, $\bar{\mathbf{\Sigma}}_k=\mathbf{R}_{s,k}+\sigma_v^2\mathbf{I}_{M^2}$, $\mathbf{R}_{s,k}=\mathrm{E}\left[|\alpha_k|^2\right]\mathbf{\Sigma}_s$ and where in NCW case $\bar{\boldsymbol{T}}_k=\mathrm{E}\left[|\alpha_k|^2\right]\mathbf{m}_s\mathbf{m}_s^H$.

Based on the signal model (2) and on the AS 1, obviously under H_0 we have that

$$E\left[\mathbf{d}_{R,k}(n)|H_0\right]=\mathrm{vec}\left[\sigma_v^2\mathbf{I}_M\right]. \tag{40}$$

Under H_1 the mean at node k is given as

$$E\left[\mathbf{d}_{R,k}(n)|H_1\right]=\mathrm{vec}\left[\mathbf{R}_{s,k}+\sigma_v^2\mathbf{I}_M\right]. \tag{41}$$

Given the network size K, the stacked $KM^2\times 1$ vector $E\left[\mathbf{d}_R(n)|H_i\right]$ over $k=1\ldots K$ and for $i=0,1$ can be formed based on the results (40) and (41) respectively.

Due to the AS 1, the k,k ($k\in K$) diagonal block of the $KM^2\times KM^2$ network-wise covariance matrix $\mathrm{Cov}\left[\mathbf{d}_R(n)|H_0\right]$ is given as

$$\mathrm{Cov}\left[\mathbf{d}_{R,k}(n)|H_0\right]=\sigma_v^4\mathbf{I}_{M^2}, \tag{42}$$

while the off-diagonal blocks are zeros, since the observation noise is not correlated over the CR nodes.

The $KM^2\times KM^2$ network-wise $\mathrm{Cov}\left[\mathbf{d}_R(n)|H_1\right]$ is constructed as follows. Firstly, when $\mathbf{m}_s=0$ and $\mathbf{\Sigma}_s\neq 0$ (i.e Case 2 type) it can be verified, that the $k,j\in K$ blocks of the $\mathrm{Cov}\left[\mathbf{d}_R(n)|H_1\right]$ are given as

$$\mathrm{Cov}\left[\mathbf{d}_{R(k,j)}(n)|H_1\right]=\begin{cases}\left[\left(\bar{\mathbf{\Sigma}}_k\right)^c\otimes\bar{\mathbf{\Sigma}}_k\right], & k=j\\ \left[\left(\mathbf{R}_{s,k,j}\right)^c\otimes\mathbf{R}_{s,k,j}\right], & k\neq j\end{cases} \tag{43}$$

where $\bar{\mathbf{\Sigma}}_k=\mathrm{E}\left[|\alpha_k|^2\right]\mathbf{\Sigma}_s+\sigma_v^2\mathbf{I}_{M^2}$ and where for $k\neq j$ $\mathbf{R}_{s,k,j}=\mathrm{E}\left[\mathbf{y}_k(n)\mathbf{y}_j(n)^H\right]=\mathrm{E}\left[\alpha_k\alpha_j^c\right]\mathbf{\Sigma}_s$, since due to (AS 1) in this case the observations $\mathbf{y}_k(n)$, $\mathbf{y}_j(n)$ are zero mean Gaussian vectors with independent noise processes. Secondly, when $\mathbf{m}_s\neq 0$ and $\mathbf{\Sigma}_s=0$ (i.e Case 1 type) and $k=j$, then the k,k on-diagonal block of $\mathrm{Cov}\left[\mathbf{d}_R(n)|H_1\right]$ is given as

$$\begin{aligned}\mathrm{Cov}\left[\mathbf{d}_{R(k,k)}(n)|H_1\right]=&\left[\left(\sigma_v^2\mathbf{I}_{M^2}\right)^T\otimes\sigma_v^2\mathbf{I}_{M^2}\right]\\&+\left[\left(\mathrm{E}\left[|\alpha_k|^2\right]\mathbf{m}_s\mathbf{m}_s^H\right)^T\otimes\sigma_v^2\mathbf{I}_{M^2}\right]\\&+\left[\left(\sigma_v^2\mathbf{I}_M^2\right)^T\otimes\left(\mathrm{E}\left[|\alpha_k|^2\right]\mathbf{m}_s\mathbf{m}_s^H\right)\right].\end{aligned} \tag{44}$$

When $k\neq j$, then due to (AS 1) the observation noise is not correlated over the CR nodes and it can be verified, that for the k,j off-diagonal blocks, $\mathrm{Cov}\left[\mathbf{d}_{R(k,j)}(n)|H_1\right]=0$. Given the network size K, the network-wise covariance matrix $\mathrm{Cov}\left[\mathbf{d}_R(n)|H_1\right]$ can be composed by using (43) and (44) respectively.

Finally the moments of the real observations (as the inputs for the moment recursions of the estimates $\hat{\mathbf{p}}_k(n)$, provided in the previous subsection) can be given for $i=0,1$ as

$$E\left[\mathbf{d}(n)|H_i\right]=\left[\mathbf{T}^{-1}\otimes\mathbf{I}_{M^2}\right]E\left[\mathbf{d}_R(n)|H_i\right], \tag{45}$$

and

$$\begin{aligned}\mathrm{Cov}\left[\mathbf{d}(n)|H_i\right]=&\left[\mathbf{T}^{-1}\otimes\mathbf{I}_{M^2}\right]\\&\times\mathrm{Cov}\left[\mathbf{d}_R(n)|H_i\right]\left[\left(\mathbf{T}^H\right)^{-1}\otimes\mathbf{I}_{M^2}\right].\end{aligned} \tag{46}$$

2) Distributions of the adaptive estimates: To study the detection performance of the proposed distributed, adaptive LE detector, we need to specify the conditional distributions for the detection test statistics - the LE of

$$\hat{\mathbf{R}}_k(n)=\mathrm{vec}^{-1}\left[\mathbf{T}\hat{\mathbf{p}}_k(n)\right] \tag{47}$$

under both detection hypothesis. As summarized in (7), when the estimate $\hat{\mathbf{R}}_k(n)$ is obtained by using the linear, equal weighting based method (6) in a non-distributed and non-cooperative manner, then according to the definition of Wishart matrices [16, Chapter 2], $\mathbf{R}_k(n)$ follows a Wishart distribution. Based on the literature, several results exist for the distributions of the LE of Wishart distributed matrices under both detection hypotheses.

The non-asymptotic cumulative distribution function (CDF) model of the LE of a NCW distributed CM matrix is more complicated for practical and numerical evaluation, compared to the corresponding model of a CCCW distribution. Thus often a NCW distribution is approximated by a CCCW distribution, where the non-centrality part of the NCW distribution is incorporated into the population covariance matrix parameter of the CCCW distribution [42], [12], [57].

When the estimate $\hat{\mathbf{R}}_k(n)$ is obtained by using the exponential type of averaging (as used in LMS type of algorithms), then due to different weights at every $n\in N$, it can be seen, that a sum of non-equally weighted Wishart matrices over N is not Wishart distributed [42, Theorem 3.3.1, 3.5.2]. Based on (27) it is easy to verify, that the adaptive CM estimate $\hat{\mathbf{R}}_k(n)$ is an average over non-equally weighted vectorized observation matrices. At iteration step n, at node k the elements of the vectors $\hat{\mathbf{p}}_k(n)$ are weighted equally and fused without changing or mixing the order of the elements of $\hat{\mathbf{p}}_k(n)$. The Hermitian property of the estimated CMs is not affected. Thus we need to seek generic CC(C)W approximations for studying the conditional CDFs of LE of adaptively estimated CMs.

3) Total and General variance approximations: We propose the usage of two methods for approximating the adaptive CM estimates $\hat{\mathbf{R}}_k(n)$ (47) by conditional approximate CC(C)W distributions. Thus based on (39) and we assume that

$$\hat{\mathbf{R}}_k(n)|H_i\sim CW_M\left(\bar{N}_i,\bar{\mathbf{\Sigma}}_{k,i}\right), \tag{48}$$

for $i=0,1$, and where $\sim$ denotes an approximate distribution, $\bar{N}_i$ is the approximating DoF and $\bar{\mathbf{\Sigma}}_{k,i}$ is the approximating population covariance matrix parameter of the corresponding CC(C)W distribution. As shown at next, the values for $\bar{N}_i$ and $\bar{\mathbf{\Sigma}}_{k,i}$ are found by matching the mean and trace or determinant of moments of $\hat{\mathbf{R}}_k(n)|H_i$ with the corresponding moments of the devectorized adaptive estimate $\mathrm{vec}^{-1}\left[\mathbf{T}\hat{\mathbf{p}}_k(n)\right]$ under both detection hypothesis.

Proposition 4. *For the approximation (48), $\bar{\mathbf{\Sigma}}_{k,i}$ is found as*

$$\bar{\mathbf{\Sigma}}_{k,i}=\frac{1}{\bar{N}_i}\mathrm{E}\left[\hat{\boldsymbol{R}}_k(n)|H_i\right] \tag{49}$$

and $\bar{N}_i$ can be found using the Total Variance (TV) or General Variance (GV) method, respectively, as

$$\bar{N}_{TV,i} = \left\lceil \frac{\mathrm{Tr}\left[\mathrm{E}\left[\hat{\boldsymbol{R}}_k(n)|H_i\right]^c \otimes \mathrm{E}\left[\hat{\boldsymbol{R}}_k(n)|H_i\right]\right]}{\mathrm{Tr}\left[\boldsymbol{T}\,\mathrm{Cov}\left[\boldsymbol{p}_k(n)|H_i\right]\boldsymbol{T}^H\right]} \right\rceil \tag{50}$$

or

$$\bar{N}_{GV,i} = \left\lceil \sqrt[M^2]{\frac{\det\left[\mathrm{E}\left[\hat{\boldsymbol{R}}_k(n)|H_i\right]^c \otimes \mathrm{E}\left[\hat{\boldsymbol{R}}_k(n)|H_i\right]\right]}{\det\left[\boldsymbol{T}\,\mathrm{Cov}\left[\boldsymbol{p}_k(n)|H_i\right]\boldsymbol{T}^H\right]}} \right\rceil, \tag{51}$$

where $\mathrm{E}\left[\hat{\boldsymbol{R}}_k(n)|H_i\right] = \mathrm{vec}^{-1}\left[\boldsymbol{T}\,\mathrm{E}\left[\boldsymbol{p}_k(n)|H_i\right]\right]$ *for* $i = 0, 1$.

These results are found as follows. Firstly we insert the $\bar{\boldsymbol{\Sigma}}_{k,i} = \mathrm{E}\left[\hat{\mathbf{R}}_k(n)|H_i\right]/\bar{N}_i$ from the first equation of (39) into the RHS of the second equation of (39) and we have that

$$\mathrm{Cov}\left[\mathrm{vec}(\hat{\mathbf{R}}_k(n)|H_i)\right] = \frac{1}{\bar{N}_i}\left[\mathrm{E}\left[\hat{\mathbf{R}}_k(n)|H_i\right]^c \otimes \mathrm{E}\left[\hat{\mathbf{R}}_k(n)|H_i\right]\right]. \tag{52}$$

Based on (28) or (35) and the first equation of (39), we equalize the means of matrices $\hat{\mathbf{R}}_k(n)|H_i$ and $\mathrm{vec}^{-1}[\mathbf{T}\mathbf{p}_k(n)|H_i]$ and get (49). For the DoF, $\bar{N}_i$, to use in the approximation, we adapt the idea proposed in [40], [41] and equalize the total variances (i.e the traces of corresponding covariance matrices) of the matrices $\hat{\mathbf{R}}_k(n)|H_i$ and $\mathrm{vec}^{-1}[\mathbf{T}\mathbf{p}_k(n)|H_i]$. Thus based on (52) we require that $\mathrm{Tr}\left[\mathrm{Cov}\left[\mathrm{vec}(\hat{\mathbf{R}}_k(n)|H_i)\right]\right] = \mathrm{Tr}\left[\mathbf{T}\,\mathrm{Cov}\left[\mathbf{p}_k(n)|H_i\right]\mathbf{T}^H\right]$ for $i = 0, 1$. By solving for $\bar{N}_i$ we have the total variance (TV) type of DoF approximation as given by (50). An alternative for finding the approximation for $\bar{N}_i$ is to equalize the determinants of both matrices [42]. Thus based on (52), we require that $\det\left[\mathrm{Cov}\left[\mathrm{vec}(\hat{\mathbf{R}}_k(n)|H_i)\right]\right] = \det\left[\mathbf{T}\,\mathrm{Cov}\left[\mathbf{p}_k(n)|H_i\right]\mathbf{T}^H\right]$. Similarly, by solving for $\bar{N}_i$ the general variance (GV) type of DoF approximation is given by (51).

Obviously the total variance method takes into account only the variances of the elements of the corresponding matrices, while the general variance method includes also the covariances of the elements of the corresponding matrices into the approximation of parameter $\bar{N}_i$. AS observed, by using the proposed TV or GV procedures under hypothesis H_1, a NCW matrix is approximated by the CCCW distribution, by matching the moments of NCW matrix into the CCCW model. This is a desired effect, as we explain in the next section. Based on these results we can proceed with the detection performance analysis.

It can be verified, that under H_0 the DoF value approximations (50) and (51) are, via the moment analysis of the adaptive estimate $\mathbf{p}_k(n)$, dependant on the step size parameter μ_k and on the full network topology. Since the same noise power value σ_v^2 is present both in the mean and covariance formulas of the adaptive estimate $\mathbf{p}_k(n)$, then a change in the $\sigma_{v,k}^2$ value does not affect the DoF value under H_0. However under H_1 both the DoF approximations are additionally dependant on the noise power value $\sigma_{v,k}^2$. This effect is illustrated in Section V.

Since under H_0, the DoF parameter does not affect the threshold calculation, then a robust detector can also be applied in Algorithm 1, by changing the detection module accordingly. We give an example with the MME detector in Section V. On the other hand, since under H_1 the DoF parameter is affected by the uncertainty in the noise power value, then this effect possibly makes the formula of the theoretical detection performance of a robust detector inaccurate as well, but that robust detector can still be used.

C. Detection Performance Analysis

In this section we provide formulas for studying the probability of false alarm (P_{FA}) and probability of detection (P_D) of the proposed, adaptive LE detector. For this, we need to evaluate the conditional CDFs of the LE of adaptive CM estimate $\hat{\mathbf{R}}_k(n)$ (48) under both detection hypotheses and under the assumption that $\hat{\mathbf{R}}_k(n)$ is approximated by a CC(C)W distribution as proposed in Section IV-B3. The resulting detection performance of LE detector is dependent on the performance of the underlying adaptive, distributed CM estimation. Let the eigenvalues of $\bar{\boldsymbol{\Sigma}}_{k,i}$ in (48) be denoted in non-increasing order as $\nu_{1,i} \geq \nu_{2,i} \geq \cdots \geq \nu_{M,i}$.

1) LE under H_0 Hypothesis: Based on [12], [58], the $\hat{\mathbf{R}}_k(n)|H_0$ (48) is assumed to follow the CCW distribution and the eigenvalues of $\bar{\boldsymbol{\Sigma}}_{k,0}$ are $\nu_{1,0} = \cdots = \nu_{M,0} = \sigma_v^2/\bar{N}_0$. The $P_{FA,e}$, based on the non-asymptotic CDF model of the $\hat{\mathbf{R}}_k(n)|H_0$, is given by

$$\begin{aligned} F_{H_0,e}(x) &= |\det(\hat{\mathbf{A}})| \\ P_{FA,e}(\gamma_{LE,k,e}) &= 1 - F_{H_0,e}(\gamma_{LE,k,e}) \end{aligned} \tag{53}$$

where the $M \times M$ matrix $\hat{\mathbf{A}}_{i,j} = \binom{\bar{N}_0 - j - i - 1}{i-1}\gamma_R(\bar{N}_0 + i - j, \frac{x}{\nu_{1,0}})$, for $i, j = 1, \ldots, M$ and where $\gamma_R(k, u) = \frac{1}{\Gamma(k)}\int_0^u x^{k-1}e^{-x}dx$ is the regularized incomplete Gamma function. The (ideal) detection threshold $\gamma_{LE,k,e}$, based on the non-asymptotic model is expressed as

$$\gamma_{LE,k,e} = F_{H_0,e}^{-1}(1 - P_{FA,e}) \tag{54}$$

and can be evaluated in terms of a numerical inversion of the exact CDF formula at a desired $P_{FA,e}$ value. An asymptotic CDF based on the Gaussian approximation of Tracy-Widom distribution is proposed in [12]. When $\bar{N}_0 \to \infty$, $M \to \infty$ and $M/\bar{N}_0 \in (0, 1)$, the approximate CDF under H_0 can be given as

$$\begin{aligned} F_{H_0,g}(x) &= \Phi\left(\frac{x - \mathrm{E}[\lambda_1]|H_0}{\sqrt{\mathrm{Var}[\lambda_1]|H_0}}\right), \\ \mathrm{E}[\lambda_1]|H_0 &= \nu_1\left(a_{LE} + (b_{LE}(-1.7711))\right), \\ \mathrm{Var}[\lambda_1]|H_0 &= (\nu_1 b_{LE})^2(0.8132), \\ a_{LE} &= (\sqrt{M} + \sqrt{\bar{N}_0})^2, \\ b_{LE} &= (\sqrt{M} + \sqrt{\bar{N}_0})(\frac{1}{M} + \frac{1}{\bar{N}_0})^{1/3}. \end{aligned} \tag{55}$$

This leads to the $P_{FA,g}$ formula

$$P_{FA,g}(\gamma_{LE,k,e}) = Q\left(\frac{\gamma_{LE,k,g} - \mathrm{E}[\lambda_1]|H_0}{\sqrt{Var[\lambda_1]|H_0}}\right), \tag{56}$$

where Q is the complementary distribution function of the standard Gaussian and to the threshold formula is

$$\gamma_{LE,k,g} = \mathrm{E}[\lambda_1]|H_0 + \sqrt{\mathrm{Var}[\lambda_1]|H_0}Q^{-1}(P_{FA,g}). \quad (57)$$

As seen in Section IV, the calculation of the threshold of the LE detector at node k and time index n requires knowledge of the moments of adaptive CM estimates (present at the reference node k) under hypothesis H_0 i.e $\hat{\mathbf{R}}_k(n)|H_0$. Thus based on the values of step sizes, the noise power, the desired P_{FA}, the provided moment recursions and the distribution parameter approximations models for the $\hat{\mathbf{R}}_k(n)|H_0$ in Section IV can be applied, to evaluate the detection threshold at node k and at time instant n. As seen, CR nodes need to know the noise power value(s) to evaluate the moments of $\hat{\mathbf{R}}_k(n)|H_0$. In practice every node k needs to calculate its own threshold by using the provided procedure. While the threshold at node k can be updated iteratively based on the exact moments of $\hat{\mathbf{R}}_k(n)|H_0$, the steady state moments are preferred in practice.

2) LE under H_1 Hypothesis: Next we obtain a common model for the non-asymptotic CDF$|H_1$ of the LE of adaptively estimated CM matrix. As explained in Section IV-B2, we approximate the NCW matrix by a CCCW matrix by matching the moments of the matrices. In Section V we show this approximation works quite well.

Thus we assume the $\hat{\mathbf{R}}_k(n)|H_1$ is distributed by a CCCW distribution. The CDF of the LE of a CCCW matrix $\hat{\mathbf{R}}_k(n)|H_1$ is given by [59] as follows

$$F_{H_1,e}(x) = K_{CC}\left|\left\{\nu_i^{\bar{N}_1-M+j}\bar{\Gamma}\left(\bar{N}_1-M+j,\frac{x}{\nu_{i,1}}\right)\right\}_{i,j}\right|,$$
$$K_{CC} = \left[\prod_{i=1}^{M}(\bar{N}_1-i)!\prod_{j=1}^{M}(M-i)!\right]^{-1}\prod_{k=1}^{M}(k-1)! \quad (58)$$

for $i,j = 1,\ldots,M$ and where $\bar{\Gamma}(k,u) = \int_0^u x^{k-1}e^{-x}dx$ is the lower incomplete gamma function [60, 8.350].

This result follows from [61, Eq. 1] by integrating the joint PDF of ordered eigenvalues of a CCCW matrix, by using [61, Corollary 2]. It should be emphasized, that as explained in [61, Chapter II. B], when some of the eigenvalues of $\bar{\mathbf{\Sigma}}_{k,1}$ are coincident, then [62, Lemma. 2] needs to be used to study the limit [61, Eq. 3].

In case of the matrix dimension is $M = 2$, the eigenvalues of the population covariance matrix are naturally not coincident under H_1 (i.e $\nu_{1,1} > \nu_{2,1}$). It can be shown, that when $M = 2$, the following simplified version of (58) can be used to evaluate the CDF numerically

$$\begin{aligned} F_{H_1,e}(x) &= \frac{\bar{D}}{\left(\frac{1}{\bar{a}}-\frac{1}{\bar{b}}\right)\bar{a}\bar{b}}, \\ \bar{a} &= \nu_{1,1}\bar{N}_1, \\ \bar{b} &= \nu_{2,1}\bar{N}_1, \\ \bar{D} &= \bar{b}\gamma_R(\bar{N}_1-1,\frac{x}{\nu_{1,1}})\gamma_R(\bar{N}_1,\frac{x}{\nu_{2,1}}) \\ &\quad -\bar{a}\gamma_R(\bar{N}_1-1,\frac{x}{\nu_{2,1}})\gamma_R(\bar{N}_1,\frac{x}{\nu_{1,1}}), \end{aligned} \quad (59)$$

where $\gamma_R(k,u)$ is the regularized incomplete gamma function.

Finally the probability of detection of the LE of a CCW matrix under H_1 using the exact CDF model is

$$P_{D,e}(\gamma_{LE,k,e}) = 1 - F_{H_1,e}(\gamma_{LE,k,e}). \quad (60)$$

As earlier, we observe that the channel gain values and the noise power value are required to complete the chain of approximations for the theoretical detection performance analysis.

V. Simulation results

In this numerical simulation Section we investigate the detection performance of the ATC type of distributed, adaptive LE detection algorithm. We describe the exact signal model, used in the simulations and then investigate the probability of false alarm (P_{FA}) and the probability of detection P_D of the proposed algorithms.

A. Simulation model

The channel gains in the following simulations are assumed to be constant over N and M dimension and are sampled for the CR node $k \in K$ as $\alpha_k \sim CN(0,1)$. We assume there is only one PU signal present in the CR network i.e $\mathbf{s}(n) = s(n)\mathbf{1}$, where $s(n) \sim CN(0,P_s)$ and $P_s = 1$. Using the same examples as in [46], we use for Case 1: $\mathbf{m}_s = s\mathbf{1}$, $\mathbf{\Sigma}_s = 0$, where s is a complex signal realization, and for Case 2: $\mathbf{m}_s = 0$ and $\mathbf{\Sigma}_s = P_s\mathbf{1}\mathbf{1}^H$. Obviously rank($\mathbf{1}\mathbf{1}^H$)=1. Also in (43) and (44) we have $\mathbf{R}_{s,k} = |\alpha_k|^2P_s\mathbf{1}\mathbf{1}^H$, $\mathbf{R}_{s,k,j} = \alpha_k\alpha_j^cP_s\mathbf{1}\mathbf{1}^H$ and $\bar{\boldsymbol{T}}_k = |\alpha_k|^2P_s\mathbf{1}\mathbf{1}^H$.

When the CR nodes do not cooperate, the local correlation matrix $\mathbf{R}_k$ (4) is given as follows

$$\mathbf{R}_k = \left[|\alpha_k|^2\frac{\mathrm{E}\left[\|\mathbf{s}\|^2\right]}{N}\right]\mathbf{1}\mathbf{1}^H + \sigma_{v,k}^2\mathbf{I}_M. \quad (61)$$

For Case 1 we assume $|s|^2 = P_s$, where s is a complex signal realization. Then we get $\mathrm{E}\left[\|\mathbf{s}\|^2\right] = NP_s$ for both Case 1 and Case 2. The first moment of the rank one input for these two cases is given as

$$E\left[\mathbf{d}_{R,k}(n)|H_1\right] = \mathrm{vec}\left[|\alpha_k|^2P_s\mathbf{1}\mathbf{1}^H + \sigma_v^2\mathbf{I}_M\right]. \quad (62)$$

1) Network topology selection: To improve the communication link failure resistance in the CR network, but to keep the need for processing the data from neighbor nodes minimal, we propose to select the diffusion topology of the estimates in the CR network, i.e the $\mathbf{A}$ matrix, as a combination of the local ($\mathbf{A},\mathbf{C} = \mathbf{I}$) and ring-around ($\mathbf{A} = \mathbf{A}_{\mathrm{ring}}^T, \mathbf{C} = \mathbf{I}$) topologies [1, Eq. 11]. Thus at time instant n, at every node k two $M^2 \times 1$ estimates: the local estimate $\hat{\mathbf{p}}_k(n)$ and the estimate $\hat{\mathbf{p}}_{(k-1)\mathrm{mod}K}(n)$ from node $(k-1)\mathrm{mod}K$ are fused together using equal, constant weight 0.5. Therefore, in the subsequent sections we assume, that $\mathbf{C} = \mathbf{I}$, the matrix $\mathbf{A}$ is in such case doubly stochastic (i.e we have additionally $\mathbf{A}\mathbf{1} = \mathbf{1}$) and all the conditions for selecting elements $a_{l,k}$ and $c_{l,k}$, as listed in the Section III-C, are satisfied.

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For example when $K = 3$ and by keeping the same notation and conditions for the elements of matrix $\mathbf{A}$, the ring around and diffusion topologies are given as follows

$$\mathbf{A}_{\text{ring}}^{T} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad \mathbf{A}_{\text{diff}}^{T} = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix}. \qquad (63)$$

A schematic view of the proposed diffusion and incremental steps for the ATC type of algorithm with $K = 2$ is illustrated in Fig. 1.

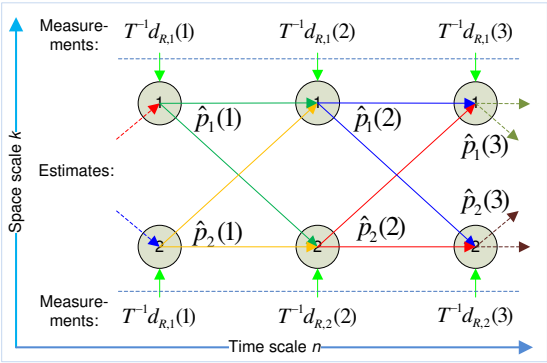

Fig. 1. Proposed diffusion method

In the next sections we select the dimension of the estimated matrix as $M = 2$ and use (11) and (59). The step size of the algorithms in all the simulations is selected to be $\mu = 0.001$ for all the nodes, unless stated otherwise. Given the step-size value, all the nodes in the network receive $N = 7000$ $[2 \times 1]$ vector-samples to get converged adaptive CM estimates at the last iteration/sample. These CM estimates are used in the simulations to obtain the LE observations. A system designer can choose other values for μ and N (depending on the system requirements).

In Fig. 2 we illustrate the change of the LE of adaptively estimated CM with respect to the threshold (54). We set the noise power to one. After the initialization, the algorithm first tracks and then converges to the steady state level of LE under the H_1 hypothesis. At time instant 7001 the PU signal switches off, the algorithm adapts and convergences to the H_0 level of the LE value.

2) DoF values under noise uncertainty: In Fig. 3 we illustrate of the effect of the noise power uncertainty to the TV based DoF approximation under Case 2 and H_1. The network sizes are $K = 1, 3, 10, 30$ and the results are taken from the last node in the network. The horizontal axis represents the (network averaged) SNR, which is changed by scaling the noise power value σ_v^2. We use the noise perturbation model [14, Eq. 8] and denote the $\bar{\alpha}$ as the noise uncertainty factor. Two noise value perturbations are added to the non-perturbed case 0 dB ($\bar{\alpha} = 1$): -1 dB ($\bar{\alpha} = 0.796$) and 2 dB ($\bar{\alpha} = 1.585$). As we see, in case of σ_v^2 is inaccurate, then the TV approximated DoF$|H_1$ values are shifted in accordance to

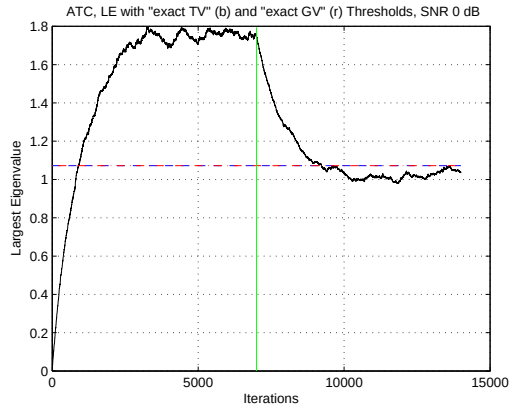

Fig. 2. LE Adaptive Principle

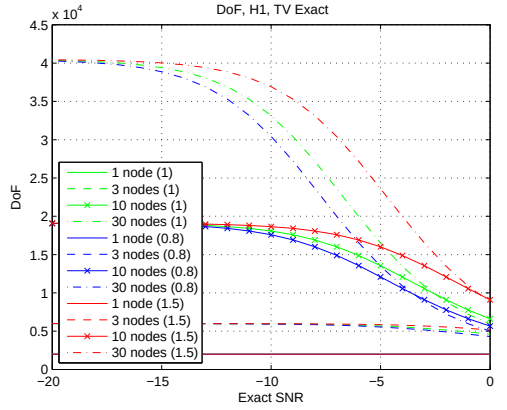

Fig. 3. ATC, DoF $|H_1$ values with perturbations 0 dB, -1 dB and 2 dB

the value of $\bar{\alpha}$. For GV based DoF$|H_1$ values, the results are very similar.

Next we investigate the performance of the proposed LE algorithms by studying the P_{FA} in case of PU signal is missing and the P_D, when the PU signal is present. Both the P_{FA} and P_D based on adaptive CM estimates are estimated using the Monte Carlo (MC) method [9]. To have an equal comparison between the node sets in one plot, we take all the reference results from the last node in the network. Obviously, based on the global estimation model (9), when we have more nodes in the network, then the CM estimates at every node have been better averaged over the channel gain values of the nodes in the CR network.

B. Probability of false alarm

We start the investigation of the proposed algorithms by studying the P_{FA}. Under the detection hypothesis H_0 we assume $\sigma_v^2 = 1$. We select 21 threshold points in the range of σ_v^2 and determine the LE realizations of adaptive CMs

estimates. Then we estimate the P_{FA} over 1000 experiments at every threshold point. The estimated P_{FA} is denoted as **Experiments** in the Fig. 4. We compare the estimated P_{FA} with the theoretical P_{FA} models when the Total variance (TV) or the General variance (GV) method are used for determining an approximately equivalent CCW matrix. The results using (55) are denoted as **Th. TV** and **Th. GV** respectively. Similarly the results using (53) are denoted as **Th. Exact TV** and **Th. Exact GV** respectively. Finally, based on the moments of the adaptive CM estimates, we generate the approximate CCW matrices (by using Cholesky decomposition method), and study the P_{FA} performance based on those matrices in addition (denoted as **Wishart TV** and **Wishart GV** respectively). The P_{FA} versus threshold results are given in Fig. 4 for the ATC algorithm. We note that the performance of the TV and GV methods are

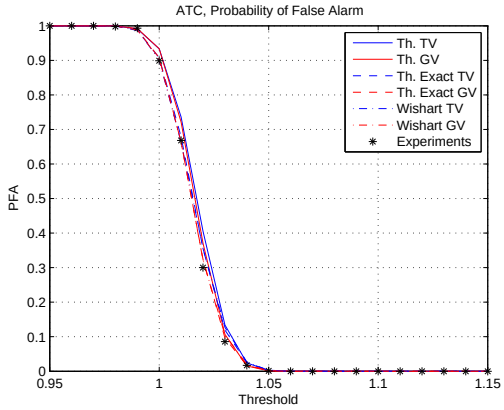

Fig. 4. P_{FA} versus threshold using ATC

almost equal and the TV/GV approximations are sufficient for studying the P_{FA} of the adaptive CM estimates. We see a good match between the estimated P_{FA} and the theoretical P_{FA} models are achieved. The Gaussian approximate P_{FA} model (which is easier to use in numerical analyses compared to non-asymptotic P_{FA} model), follows the estimated P_{FA} results quite well and can therefore be used to characterize the P_{FA} of the adaptive estimates. Therefore by knowing the noise power value, the theoretical Gaussian approximate P_{FA} model can be also used for deriving the detection threshold, when we fix a desired P_{FA} value.

C. Probability of detection

Next we investigate the probability of detection under different noise power conditions using the proposed distributed and adaptive LE detection algorithms with signal models Case 1 or 2. In Case 1 we select one complex PU signal realization, while in Case 2 we set $P_s = 1$ for all the simulations. We note, that the performance of the moment estimation framework of adaptively estimated CMs is well illustrated by the P_D versus SNR analysis. In the comparison of algorithms we use the same individual channel gains of the nodes in all the simulations performed under hypothesis H_1. We set the desired $P_{FA} = 10^{-2}$ for all the nodes. The thresholds of the LE detectors at nodes $k \in K$ are calculated using (54) with both the TV and GV approximation. Simulations studies showed, that the performance of the Gaussian CDF$|H_1$ based threshold (57) is almost equal to the performance of the non-asymptotic threshold (54) and thus not shown in this paper.

In the following simulations we compare the performance of 4 different network sizes: $K = 1, 3, 10, 30$ nodes, while the comparable results are taken from the last node in the set. The P_D is estimated over 1000 experiments on a given noise power value. We compare the MC estimated P_D results (based on the adaptively estimated CMs and denoted as **Ad. Exp.** in the figures) with the non-asymptotic theoretical model (60) (denoted as **Theory**) and with the P_D results based on approximately equivalent CCW matrices (denoted as **W. Exp.**). These latter matrices are generated based on the respective moments under H_1. For the signal model Case 1, the P_D/SNR results are given in Fig. 5 when the TV approximation is used and in Fig. 6 when the GV approximation is used for the ATC algorithm. Similarly for the signal model Case 2, the P_D versus SNR results are given in Fig. 7 when TV approximation is used and in Fig. 8 when GV approximation is used, respectively for the ATC algorithm.

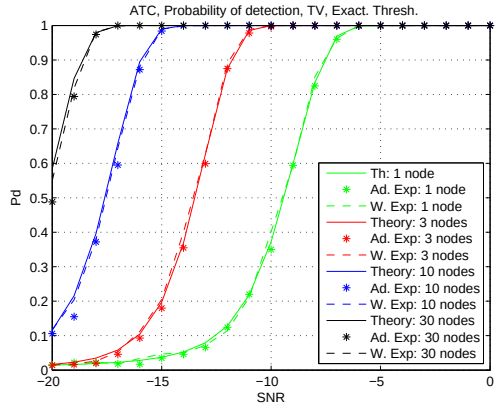

Fig. 5. Probability of detection, ATC, TV, Case 1

For comparison, the MC estimated P_D/SNR performance of the MME detector [14] under Case 2 is shown additionally in Fig. 7 and Fig. 8 (where denoted as **MME. Exp.**). The threshold of the MME detector is calculated by using [14, Eq. 29], where in our case $L = 1$ and $N_s = \bar{N}_{TV,0}$ or $N_s = \bar{N}_{GV,0}$. Based on the discussion in Section IV-B3, it is obvious, that since the noise value perturbations are not affecting the threshold of the MME detector, then the corresponding MC based P_D/SNR performance is not affected as well. In Fig. 9 we show a comparison of P_D/SNR performance of the LE detector by using the FC based algorithm in Table 1, TV approximation based exact threshold, and Case 2 model only. In such case the observations of every CR nodes are available for all the CR nodes in the CR network and the CR networks can (independently and adaptively) estimate the

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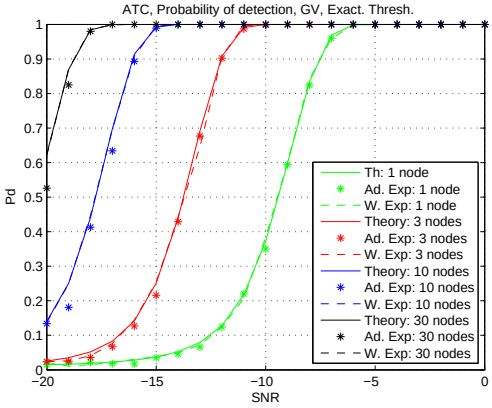

Fig. 6. Probability of detection, ATC, GV, Case 1

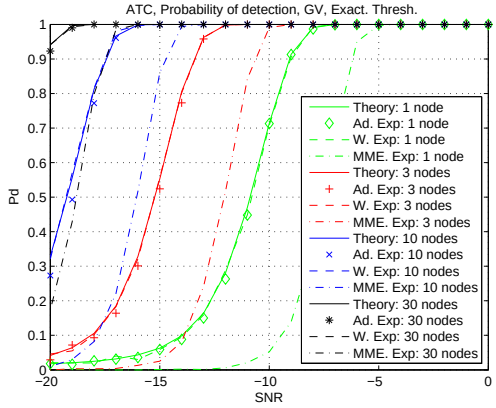

Fig. 8. Probability of detection, ATC, GV, Case 2

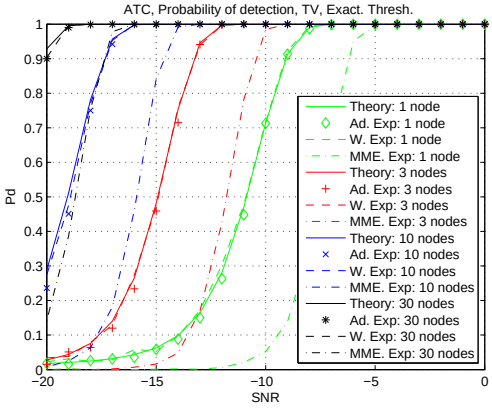

Fig. 7. Probability of detection, ATC, TV, Case 2

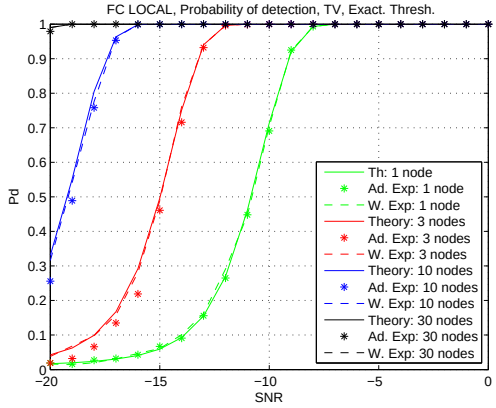

Fig. 9. Probability of detection, FC, TV, Case 2

CM. In Fig. 10 we provide similar comparison of the P_D/SNR performance of the LE detection scheme in Fig. 10, by using the consensus algorithm ([33]), TV approximation based exact threshold and Case 2 model only and we select $\mathbf{A}_0 = \mathbf{A}_{\text{diff}}^T$.

We note that the non-asymptotic theoretical P_D model describes the detection performance of adaptively estimated CMs well, also in the low SNR regime. The performance of TV and GV methods is almost equal and thus the TV approximation is computationally less demanding method for the numerical performance analysis of the LE detector. The Case 1 signal model is well approximated by the signal model of Case 2 (CCCW), via the TV and GV based mean and DoF parameter matching.

We observe, that as the number of nodes in the network increases, the point where the P_D starts to decrease from one, moves to the left. In case of one node in the CR network (or in case of the non-cooperating nodes) the P_D is highly dependent on the channel constant of that node. As the number of nodes increases, more channel gain realizations are involved in the network-averaged CM estimation process and thus the P_D results are more equalized over the nodes.

It can be seen, that the LE detector performs better than the MME in terms of perfect detection ($P_D = 1$) in the low SNR region and in case of non-perturbed noise power values.

The detection performance of LE detector, when the FC based diffusion LMS algorithm is used, is slightly better, compared with the case of ATC type of LMS. The difference is however not significant. So that in ATC case, where only two exchanges of estimates are allowed for a CR node at time instant n, we can save energy in terms of processing less data at a node k. Also in case of ATC we are not limited to the specific network topology. The detection performance of LE detector, when the consensus algorithm is used, is very similar to the case of the ATC algorithm. As argued in Section IV, the usage of ATC type algorithm is less limited by the estimate exchange topology, while this is not the case with the consensus algorithm.

Additionally we note that, in [1] we showed with scalar estimates ($M = 1$) in Case 2, that when there are more nodes in the network, then the ATC performs slightly better compared

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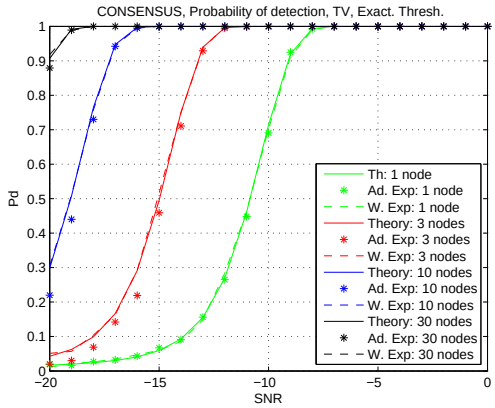

Fig. 10. Probability of detection, Consensus, TV, Case 2

to the CTA type of algorithm. While ATC fuses more data than CTA [30], the difference of detection performance with CTA is rather small and thus we also skip these comparisons in this paper. We also observed in [1], [2], [3] that for $K > 30$, P_D does not improve significantly any more.

For illustrating the closeness of the detection results of different CR nodes, we use the theoretical results and plot the P_D/SNR performances of all the CR nodes in the network of size K, in Fig. 11, by using the ATC algorithm, the TV based exact threshold and the Case 2 model. The four groups of P_D/SNR results from right to the left in Fig. 11 correspond to the network of sizes $K = 1, 3, 10, 30$ accordingly, i.e the leftmost group shows the P_D/SNR results of all the 30 nodes in the CR network. It can be seen, that the detection

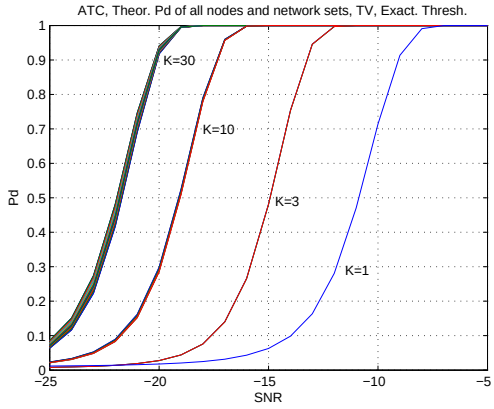

Fig. 11. Probability of detection, ATC, TV, Case 2

performances of the CR nodes in the CR network are quite close to each other. In practice we are more concerned about the point, where the P_D starts to decrease from 1. In case of 30 nodes in the network, the deviation slightly increases, but is still sufficiently close.

We observe, that the non-asymptotic CDF models, the TV/GV approximations and CCCW based approximation of NCW type of CMs are usable for studying the performance of the LE detection of adaptively estimated CMs - for determining the threshold and for evaluating the theoretical P_D of the LE detector. When the nodes cooperate in estimating the network-wise CM (while nodes are able to communicate directly only with limited subset of neighbor nodes) then the resulting LE detection performance is equalized and stabilized over the individual CR nodes. We note that other distributed eigenvalue based detection schemes can be studied in similar manner by using the proposed framework in this paper.

VI. Conclusions

In this paper we studied distributed and adaptive diffusion LMS based LE detection algorithms, which are applicable in CR networks, for detecting the presence of a PU signal. We proposed a network-wise CM estimation model, and derived ATC and CTA type of diffusion based LE detection algorithms. We proposed a general framework for analyzing the performance of the diffusion LMS based LE detection schemes. In our simulation study we demonstrated that the proposed framework and the approximations used for studying the detection performance of the proposed distributed and adaptive LE detection schemes provided matching results between the theory and simulations. The proposed algorithms are able to learn the statistical changes in the LE in real time.

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APPENDIX F

Publication P6.

A. Ainomäe, T. Trump, and Y. L. Moullec, "SNR weighted distributed largest eigenvalue based spectrum sensing," in Submitted to 16th Biennial Baltic Electronics Conference (BEC 2018)*, Tallinn, Estonia, 2018, ETIS 3.1.*

Distributed Adaptive Largest Eigenvalue Detection with SNR weighted observations

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***Abstract*—Cognitive radio (CR) systems should be able detect the presence of a primary user (PU) signal by sensing the spectrum area of interest. Due to radiowave propagation effects like fading and shadowing, spectrum sensing is often complicated, because the PU signal can be attenuated in a particular area. Strategies that can sense the spectrum in a cooperative manner have the potential to enhance the primary user signal detection. In this paper we explore a distributed spectrum sensing approach that exploits the largest eigenvalue of correlation matrices (CMs) that are adaptively estimated; local signal to noise ratio (SNR) values are used to assign weights to the input observations. More specifically, CR nodes exchange also observations with a subset of neighbouring nodes and combine the neighbouring observations based on the locally estimated SNR values. We propose a mean vector estimation mechanism that is based on combine and adapt least mean square diffusion and that does not require a fusion center (FC). We analyse the resulting detection performance and verify the theoretical findings through simulations.**

***Index Terms*—Cognitive radio, distributed estimation, distributed detection, diffusion LMS, spectrum sensing, SNR estimation, subspase detection.**

I. Introduction

Radio spectrum is a scare resource. It has been found that even if the licensed radio spectrum becomes nominally more crowded there is significant underutilization of the resource [1]. Cognitive radio (CR) technology has been proposed to provide an opportunistic access for cognitive radio systems to the licensed spectrum areas [2], [3]. In CR context it is highly desirable to detect the PU user and identify free spectrum opportunities as rapidly as possible and create no disturbances for the (licenced) PU communication. Distributed, adaptive network learning methods can be used to track the changes in the statistical information of the observations received by the CR nodes in real time, to enhance the detection of PU signals.

Three main types of classic detection schemes for spectrum sensing in CR networks have been considered in the literature: the Matched filter detector (MFD) [4], the Energy Detector (ED) [4], [5], and the Cyclostationary detector [6]. A second large group of detectors for CR networks are based on the properties of an estimated signal correlation matrix [7], [8], [9]. The Largest Eigenvalue (LE) method [7] uses *a priori* knowledge about the additive noise power to determine the detection threshold.

Distributed, adaptive estimation and detection research area has gained an increasing interest over the last decade and many algorithms have been proposed in the literature. For example, Least mean square (LMS) based estimation schemes were studied in [10], [11], [12], where good properties of these algorithms were shown. On the other hand optimal, distributed MFD, based on diffusion type LMS estimation schemes were studied in [13]. In [14], [15] and [16] we proposed and analysed diffusion LMS based energy detectors in a CR network and in [17] and [18] we studied diffusion LMS based Largest Eigenvalue (LE) detectors.

Various signal to noise ratio (SNR) estimation methods have been proposed and studied in the literature, for example [19], [20], [21]. In the PU signal detection context, SNR estimation methods can provide additional information about the quality of the input observations and that knowledge can be used for enhancing the distributed estimation process and thus the overall detection results of the main spectrum sensing method.

In this paper we explore a distributed spectrum sensing approach that exploits the largest eigenvalue of correlation matrices that are adaptively estimated. No FC unit (as a potential single point of failure) is used. Compared to the solutions in our papers [17] and [18], in this paper, we study additionally the local observation exchange and combination strategy, which is based on the local SNR estimates and is adapted to the context of binary hypothesis testing. We show, that when the PU signal is present and when the local SNR estimates are available, then the network-wise PU signal detection performance can be slightly improved, compared to the standard case with no observation exchange, studied in our paper [17].

We assume that the CM of the PU signal is of low rank. On the other hand, the CR network operates without prior information about the PU signal's waveform and the secondary nodes' channel gains. We assume that while the PU signal may be absent for a time period, the radio channel properties under the detection hypothesis H_1 do not change over the time of interest and that long time statistics are usable in enhancing the overall PU signal detection performance. For example, the classical TV White space model [3, Chapt. 1.2.4] could be considered, where the on/off working patterns of the PUs (i.e. TV transmitters) are quite static, the power of the PUs is constant and where the CR nodes have fixed positions in the nearby space. In the distributed CR network, we assume that every node acts as an independent detector in

terms of detection decision making based on the available CM estimates.

II. Distributed Adaptive Largest Eigenvalue Detection

A. Signal model and assumptions

Let us follow the same signal model as in [17], where

$$\begin{aligned} H_0 &: \mathbf{y}_k(n) = \mathbf{v}_k(n), \\ H_1 &: \mathbf{y}_k(n) = \alpha_k \mathbf{s}(n) + \mathbf{v}_k(n) \end{aligned} \tag{1}$$

and the detection hypothesis is denoted by H_i, $i = 0, 1$, the CR node index by $k = 1, 2, ..., K$, and the sample discrete time index by $n = 1, 2, ...N$. The noise $\mathbf{v}_k(n)$ and channel gains α_k at node k are assumed to be statistically independent. The PU signal follows $\mathbf{s}(n) \sim CN_M(0, \mathbf{\Sigma}_s)$. The noise follows $\mathbf{v}_k(n) \sim CN_M\left(0, \sigma_v^2 \mathbf{I}_M\right)$ and is assumed to be independently and identically distributed, uncorrelated in time and space. The theoretical $M \times M$ dimensional CM $\mathbf{R}_k$ at every node is given as

$$\mathbf{R}_k = \mathrm{E}\left[\mathbf{y}_k(n)\mathbf{y}_k(n)^H\right] = |\alpha_k|^2 \mathbf{R}_{s,k} + \sigma_{v,k}^2 \mathbf{I}_M. \tag{2}$$

The noise power σ_v^2 is assumed to be known a priori and to be identical at every node. Secondly, we assume, that $\mathbf{R}_{s,k}$ has a low rank. Thirdly, we assume, that when H_1 is present, the PU signal power and the channel constants do not change over the time of interest (of slow fading channel). Fourthly, internal communication channels between the CR nodes are assumed to be error free and the communication capacity is not limited.

For summarizing the LE detection method, let the eigenvalues of the estimate $\hat{\mathbf{R}}_k(n)$ of CM $\mathbf{R}_k$ be denoted in non-increasing order as $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M$. Every node k detects the presence of a PU signal by determining the largest eigenvalue λ_1 of $\hat{\mathbf{R}}_k(n)$ as follows

$$\lambda_1\left[\hat{\mathbf{R}}_k(n)\right] \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_{LE}. \tag{3}$$

Here threshold γ_{LE} is given by (14).

For improving the detection performance of the LE detector, we introduce a second parameter - local SNR of the received observations at node k. The theoretical form is given as

$$\mathrm{SNR}_k = \frac{\mathrm{Tr}\left[|\alpha_k|^2 \mathbf{R}_{s,k}\right]}{\mathrm{Tr}\left[\sigma_v^2 \mathbf{I}_M\right]}. \tag{4}$$

In the SNR estimation phase, each node k estimates locally the SNR of the received signal, denoted as $\widehat{\mathrm{SNR}}_k$, based on the locally estimated sample covariance matrix (SCM). The SCM $\hat{\mathbf{R}}_k$ at node $k \in K$ and over the N_{SNR} samples can be estimated linearly as

$$\hat{\mathbf{R}}_k(n) = \frac{1}{N_{SNR}} \sum_{k=n-N_{SNR}}^{n} \left[\mathbf{y}_k(n)\mathbf{y}_k(n)^H\right]. \tag{5}$$

Local SNR can be estimated separately from the (cooperative) CM estimation phase for the LE detection (3) and it can be considered as a "goodness measure" of the received observations in CR network. Similarly to the standard Maximum ratio combining (MRC) method, i.e. [22], local SNR estimates can be used for weighting up the observations with strong PU signal in the neighbourhood of CR nodes (and for weighting down the available neighbouring observations flows, where PU signal is more strongly attenuated). Since the LE detector requires knowledge of the σ_v^2, then according to (1) and the assumptions, the estimation of the local SNR reduces to the estimation of the (attenuated) PU signal power at node k. Thus in this paper we propose the usage of the following simple SNR estimation method

$$\widehat{\mathrm{SNR}}_k(n) = \frac{\mathrm{Tr}\left[\hat{\mathbf{R}}_k(n)\right]}{M\sigma_{v,k}^2} - 1. \tag{6}$$

For avoiding negative and zero SNR values in the upcoming calculations, when the PU signal is very weak or no signal subspace is present, we assign that if $\widehat{\mathrm{SNR}}_k(n) < 0.0001 \Rightarrow \widehat{\mathrm{SNR}}_k(n) = 0.0001$.

Compared, for example, to the more sophisticated Minimum Description Length (MDL) and Akaike Information Criterion (AIC) criterion based SNR estimation methods [19], [20], [21], (6) is computationally simpler and based on experiments, requires significantly less samples N_{SNR} in SCM to detect the PU signal. Once the local SNR estimates are obtained, these corresponding results could be deterministically used in the distributed adaptive CM estimation phase for the LE detection.

B. Adaptive, Distributed LE detection with SNR weighted observations

The first part of this section summarizes the adaptive, distributed CTA type of Diffusion LMS based CM estimation algorithm, which was derived in [17, Chapter II]. The second part focuses on the usage of local SNR estimates for observation weighting and exchange in the mentioned Diffusion estimation strategy.

It was shown in [17] that when CR nodes cooperate in the estimation of $\hat{\mathbf{R}}_k(n)$ in (3) by means of the system internal communication links, then the detection performance (of the PU signal(s)) at every node k can be enhanced. In this paper we continuously assume that 1) K nodes in the CR network can rely only on the subset of global information that is available to them and 2) that the CR network topology is assumed to be fixed over the sensing time and strongly connected.

Let us denote $\mathcal{N}_k$ as the neighbourhood group of node $k \in K$, i.e. $\mathcal{N}_k$ and μ_k be a positive step size of node k. We introduce the $K \times K$ matrix $\mathbf{C}_{\mathrm{SNR}}$ with non-negative elements as follows

$$c_{l,k} = 0 \quad \text{if} \quad l \notin \mathcal{N}_k, \quad \mathbf{C}_{\mathrm{SNR}}\mathbf{1} = \mathbf{1}. \tag{7}$$

For simplicity we have dropped the time index n in $\mathbf{C}_{\mathrm{SNR}}$. Similarly, let us have constant $K \times K$ matrix $\mathbf{A}_{\mathrm{diff}}^T$ as

$$a_{l,k} = 0 \quad \text{if} \quad l \notin \mathcal{N}_k, \quad \mathbf{1}^T\mathbf{A}_{\mathrm{diff}}^T = \mathbf{1}^T. \tag{8}$$

The coefficients $c_{l,k}$ and $a_{l,k}$ define respectively how the measurements $\mathbf{d}_l(n)$ and estimates $\hat{\mathbf{p}}_l(n)$ are available for the

node k in the CR network (unidirectionally). Let μ_k be positive step size of node k.

Similarly, as in [17], for keeping the dimension of the estimated vector minimal in the adaptive recursions, we decompose the observation at node k at time instant n as $\mathbf{d}_{R,k}(n) = \mathrm{vec}\left[\mathbf{y}_k(n)\mathbf{y}_k(n)^H\right] = \mathbf{T}\mathbf{d}_k(n)$, where $M^2 \times M^2$ dimensional constant, the complex invertible matrix $\mathbf{T}$ is given in [23, Eq. 11]) and $M^2 \times 1$ vector $\mathbf{d}_k(n)$ is real valued. Thus, $\mathbf{d}_k(n) = \mathbf{T}^{-1}\,\mathrm{vec}\left[\mathbf{y}_k(n)\mathbf{y}_k(n)^H\right]$. By denoting the $M^2 \times 1$ dimensional estimate of the real valued $\mathrm{E}\left[\mathbf{d}_k(n)\right]$ as $\hat{\mathbf{p}}_k(n)$, then with the help of $\mathbf{T}$, we can re-define $\hat{\mathbf{R}}_k(n) = \mathrm{vec}^{-1}\left[\mathbf{T}\mathbf{p}_k(n)\right]$.

We skip the derivation details of the adaptive, fully distributed CTA diffusion type of LMS based LE detection algorithm and show the result in Algorithm 2.

One of the disadvantage of (5) is that it is not adaptive and requires significant amout of memory, since all the N_{SNR} samples have to be present for estimating $\hat{\mathbf{R}}_k(n)$. Since (6) does not use N_{SNR} directly for SNR estimation, we propose to also use local exponential averaging based adaptive estimation method for calculating the CM for local SNR estimation. In the light of previously showed decompositions, let us denote the real time adaptive local CM estimate (for local SNR estimaton) at node k at time instant n as $\hat{\mathbf{w}}_k(n)$, while $\hat{\mathbf{R}}_k(n) = \mathrm{vec}^{-1}\left[\mathbf{T}\mathbf{w}_k(n)\right]$. Let the step sizes for all the nodes k be equal and denoted as μ_{SNR}. The step-size μ_{SNR} needs to be selected so that converged estimates are achieved after the expected number of samples N_{SNR} (which determines also the accuracy of the estimates), while the algorithm can continue running after N_{SNR} have been processed.

Each node k communicates the signal observations and $\widehat{\mathrm{SNR}}_k$ value in real time to the neighbouring nodes, which are connected to the node k. For defining the network connections in $\mathbf{C}_{\mathrm{SNR}}$ (7) we introduce a non-negative matrix $\mathbf{C}_{\mathrm{O}}^T$ with the element of $c_{O,l,k}$, which are formed as follows

$$c_{O,l,k} = \begin{cases} 1, & l \in \mathcal{N}_k \\ 0, & \text{otherwise,} \end{cases} \tag{9}$$

to define if node l is connected to node k. The combination weights $\mathbf{C}_{\mathrm{SNR}}$ at each node $k \in K$ are formed as follows

$$c_{l,k} = \frac{c_{O,l,k}\widehat{\mathrm{SNR}}_k}{\sum_{k=1}^{K}\left[c_{O,l,k}\widehat{\mathrm{SNR}}_k\right]}, \tag{10}$$

We note that the rows of $\mathbf{C}_{\mathrm{SNR}}$ are normalized to 1, which is useful in PU signal detection context. In case of detection hypothesis H_1 is present and sufficiently accurate SNR estimates are available, then the observations with higher SNRs are slightly weighted up in the observation exchange in the neighbourhood of the nodes and observations. In such a way, the observations with higher channel gains are more dominating in the adaptive estimation algorithm and this property can enhance the overall detection performance of the LE detector in Algorithm 2. On the other hand, when H_0 or in case a weak PU signal is present, the SNR estimates are set equal to 0.0001. If the local SNR estimates are not available at every node, then it is easy to verify that equal weights $c_{l,k}$ (10) are obtained in the neighbourhood of the CR nodes. In such a way, the existing threshold determination solution (for the LE detector) under H_0, as presented in [17], can still be used. After $\mathbf{C}_{\mathrm{SNR}}$ is formed, then this matrix can be used in the adaptive CTA type of LMS based LE detection method, proposed in [17], summarized in Algorithm 2.

The local SNR estimation steps are given in Algorithm 1.

Algorithm 1 Local SNR Estimation

1. Local CM estimation:
Start with $\hat{\mathbf{w}}_k(0) = \mathbf{w}(0)$ for every k .
for every time instant $n \geq 1$ **do**
  for every node $k = 1, ..., K$ **do**
    $\hat{\mathbf{w}}_k(n+1) = \hat{\mathbf{w}}_k(n) + \mu_{SNR}\left[\mathbf{d}_k(n) - \hat{\mathbf{w}}_k(n)\right]$
  end for
end for
2. SNR estimation:
for every node $k = 1, ..., K$ **do**
  $\hat{\mathbf{R}}_k(n+1) = \mathrm{vec}^{-1}\left[\mathbf{T}\mathbf{w}_k(n+1)\right] \Rightarrow \widehat{\mathrm{SNR}}_k$ (6)
  if $\widehat{\mathrm{SNR}}_k \leq 0.0001$ **set** $\widehat{\mathrm{SNR}}_k = 0.0001$
  $c_{l,k} = \frac{c_{O,l,k}\widehat{\mathrm{SNR}}_k}{\sum_{k=1}^{K}\left[c_{O,l,k}\widehat{\mathrm{SNR}}_k\right]}$ (10)
end for

Algorithm 2 CTA type of LE Detection with SNR weighted observations [17]

Start with $\hat{\mathbf{p}}_k(0) = \mathbf{p}(0)$ for every k .
Given non-negative real coefficients $a_{l,k}, c_{l,k}$
for every time instant $n \geq 1$ **do**
  for every node $k = 1, ..., K$ **do**
    1. CTA type of CM estimation recursions:
    $\hat{\boldsymbol{\psi}}_k(n) = \sum_{l \in N_k} a_{l,k}\hat{\mathbf{p}}_l(n)$.
    $\hat{\mathbf{p}}_k(n+1) = \hat{\boldsymbol{\psi}}_k(n)$
      $+\mu_k \sum_{l \in N_k} c_{l,k}\left[\mathbf{d}_l(n) - \hat{\boldsymbol{\psi}}_k(n)\right]$
    2. LE detection decision:
    $H_0 : \lambda_1\left[\mathrm{vec}^{-1}\left[\mathbf{T}\hat{\mathbf{p}}_k(n+1)\right]\right] < \gamma_k$ or
    $H_1 : \lambda_1\left[\mathrm{vec}^{-1}\left[\mathbf{T}\hat{\mathbf{p}}_k(n+1)\right]\right] > \gamma_k$.
    (Refer to (14) for selecting the γ_k).
  end for
end for

The algorithms have been presented separately and in principle the system designer can study these two algorithms in different combinations (also in separate time scales), depending on the system requirements and noise properties.

III. Theoretical Detection Performance

In this section we summarize the steps needed to set the detection threshold of the LE detector with SNR weighted observation exchange and to evaluate the theoretical detection performance (for verifying the Monte-Carlo based simulation results). The performance analysis of the proposed algorithm can in general be performed based on the same framework, that was developed in [17] and [23]. Thus, we skip the details and shortly summarize the main steps. For the theoretical

performance analysis of the LE detector, we assume that the channel gains are known. The analysis is divided into three parts.

Firstly, the moments of the adaptive CM estimates of Algorithm 2 are studied. As shown in [17], $K \times K$ matrices $\mathbf{A}_1 = \mathbf{A}_{\text{diff}}^T$, $\mathbf{A}_2$, $\mathbf{C}_{\text{SNR}}$ and $\boldsymbol{\mathcal{M}} = \text{diag}\{\mu_1, \ldots, \mu_K\}$ are in CR network extended to $KM^2 \times KM^2$ matrices for the CTA type of algorithm as follows: $\overline{\mathbf{A}}_1 = \mathbf{A}_{\text{diff}}^T \otimes \mathbf{I}_{M^2}$, $\overline{\mathbf{A}}_2 = \mathbf{I}_K \otimes \mathbf{I}_{M^2}$, $\overline{\mathbf{C}}(n) = \mathbf{C}_{\text{SNR}} \otimes \mathbf{I}_{M^2}$ and $\overline{\boldsymbol{\mathcal{M}}} = \boldsymbol{\mathcal{M}} \otimes \mathbf{I}_{M^2}$. Let us note that with respect to Chapt IV and for simplifying the analysis, matrix $\mathbf{C}_{\text{SNR}}$ (7) is considered as constant and deterministic. Thus, the CTA based estimation recursion can be given as

$$\begin{aligned}\hat{\mathbf{p}}(n+1)|H_i = \overline{\mathbf{A}}_2 \left(\mathbf{I} - \overline{\boldsymbol{\mathcal{M}}}\right) \overline{\mathbf{A}}_1 \hat{\mathbf{p}}(n)|H_i \\ + \overline{\mathbf{A}}_2 \overline{\boldsymbol{\mathcal{M}}} \overline{\mathbf{C}} \mathbf{d}(n)|H_i. \end{aligned} \tag{11}$$

For determining the threshold of the LE detector (under H_0) and for studying the theoretical detection performance (under H_1), the moments $\text{E}[\hat{\mathbf{p}}(n+1)|H_i]$ and $\text{Cov}[\hat{\mathbf{p}}(n+1)|H_i]$ for $i = 0, 1$ need to be determined [17].

Secondly, the adaptively estimated $\mathbf{T}\hat{\mathbf{p}}(n)|H_i$ is approximated by Complex Central (Correlated) Wishart distributions (CC(C)W) as

$$\mathbf{T}\hat{\mathbf{p}}(n)|H_i = \hat{\mathbf{R}}_k(n)|H_i \sim CW_M\left(\bar{N}_{TV,i}, \bar{\boldsymbol{\Sigma}}_{k,i}\right), \tag{12}$$

for being able to find the conditional CDFs of LE of adaptively estimated CMs. The values for $\bar{N}_{TV,i}$ (for approximating the DoF parameter) and $\bar{\boldsymbol{\Sigma}}_{k,i}$ (for approximating the population covariance matrix parameter) can be found based on the Total Variance (TV) approximation method, shown in [17, Chapt. 3].

Thirdly, since under H_0, $\hat{\mathbf{R}}_k(n)|H_0$ is assumed to follow the CCW distribution, then $P_{FA,e}$ of the largest eigenvalue of $\hat{\mathbf{R}}_k(n)|H_0$ is given as

$$P_{FA,e}(\gamma_{LE,e}) = 1 - F_{H_0,e}(\gamma_{LE,e}) \tag{13}$$

where the CDF$|H_0$, denoted as $F_{H_0,e}(x)$, is given in [17, Chapt. 3].

The detection threshold $\gamma_{LE,e}$, based on the non-asymptotic model is given as

$$\gamma_{LE,e} = F_{H_0,e}^{-1}(1 - P_{FA,e}) \tag{14}$$

and numerical inversion method can be used to determine the exact CDF formula at a desired $P_{FA,e}$ value .

Under H_1, the $\hat{\mathbf{R}}_k(n)|H_1$ is assumed to be distributed by a CCCW distribution. The P_D formula, based on the non-asymptotic CDF$|H_1$ of the LE of a CCCW matrix $\hat{\mathbf{R}}_k(n)|H_1$ is given as

$$P_{D,e}(\gamma_{LE,e}) = 1 - F_{H_1,e}(\gamma_{LE,e}), \tag{15}$$

where the CDF$|H_1$, denoted as $F_{H_1,e}(x)$, is also given in [17, Chapt. 3].

IV. Simulation results

In the numerical simulation section we investigate the probability of detection P_D of the CTA type of distributed, adaptive LE detection algorithm together with the SNR weighted observations. The algorithm performance is presented in terms of the P_D versus (network averaged) SNR analysis; in the SNR, the noise power value changes. In this example we consider a rather ideal use-case, which on the other hand illustrates well the achieved LE detection performance gain – we assume H_1 is known to be present during the SNR estimation and thus good local SNR estimates have been obtained over longer time. The channel gains are assumed to be constant over the simulation time and are sampled as $\alpha_k \sim CN(0,1)$. We assume to have one PU signal $\mathbf{s}(n) = s(n)\mathbf{1}$, $s(n) \sim CN(0,1)$ and $\boldsymbol{\Sigma}_s = \mathbf{1}\mathbf{1}^H$.

For the local SNR estimation step, we select $N_{SNR} = 50000$ and $\mu_{SNR} = 0.00015$ for getting an estimate also in the highest noise power region of interest, as seen in Fig. 1. The matrix $\mathbf{C}_{\text{SNR}}$ is constructed based on the local SNR estimate realizations at every SNR point and used in the LE detection algorithm for exchanging and weighting the measurements in the CR network.

For the adaptive LE detection, we select the following simulation parameters: $M = 2$, $N = 7000$, $\boldsymbol{\mathcal{M}} = \mu\mathbf{1}_K$, $\mu = 0.001$ and $P_{FA} = 10^{-2}$ for all the nodes. The thresholds of the LE detectors at nodes $k \in K$ are found by using (14) with the TV approximation. We select the diffusion topology of the estimates in the CR network similarly as in [16, Eq. 11]. For example, when $K = 3$ and by keeping the same notation and conditions for the elements of matrix $\mathbf{A}$, then the diffusion topology is given as follows

$$\mathbf{A}_{\text{diff}}^T = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix}, \tag{16}$$

while $\overline{\mathbf{A}}_1 = \mathbf{A}_{\text{diff}}^T \otimes \mathbf{I}_{M^2}$.

In the following simulations we compare the performance of 3 different network sizes: $K = 1, 3$ and 10 nodes. All the results are taken from the last node in the set. The P_D versus SNR results are given in Fig. 1 when TV approximation is used for the CTA algorithm. The Monte Carlo (M-C) estimated P_D results, based on the adaptively estimated CMs with local SNR based weighting of observations, are denoted as **Ad. Obs. Ex.** in the figure. These Monte Carlo based results are compared with the non-asymptotic theoretical model (15) (denoted as **Th. Obs. Ex.**). In addition, for reference we plot the The Monte Carlo estimated P_D results, based on the observation weighting with the theoretical SNR values (4), denoted as **Th. SNR Ex.**. Finally we add the M-C based P_D results based on adaptively estimated CMs with equal observations exchange $\mathbf{C}_{\text{SNR}} = \mathbf{A}_{\text{diff}}^T$ for the reference (denoted as **Ad. Eq. Ex**).

We note that in [16] we found that for the case of $M = 1$, an equal weighing of the observations does not improve significantly the resulting detection performance. We see that when CR nodes in addition to sharing the estimates share also their

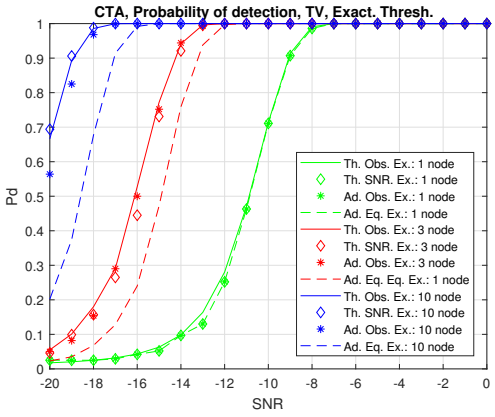

Fig. 1. Probability of detection, CTA, TV, SNR Weighted Obs.

observations, while these observations in the neighbourhood are weighted not equally, but based on the locally estimated SNR values, then an observable increase in the P_D is seen. As the number of nodes in the network increases, the point where the P_D starts to decrease from 1 is moved to the left about 1.5-2dB. In general, the values of N_{SNR} and μ_{SNR} need to be determined experimentally, based on the PU activity patterns, CR system requirements and the noise power conditions. I.e. in a low SNR region, $N_S NR \gg 1$ and usually more samples have to be collected to get a reasonably accurate local SNR estimates, while less samples need to be processed in a high SNR region.

We see that when the SNR estimates with sufficient accuracy are available, then the SNR based observation weighting solution can be used for improving the performance of the adaptive, distributed LE detection algorithm. When the nodes cooperate in estimating the network-wise CM (while nodes are able to communicate directly only with limited subset of neighbour nodes) then the resulting LE detection performance is improved in the CR network.

V. Conclusions

In this paper we studied a distributed and adaptive, CTA diffusion LMS based LE detection algorithm, with uses local SNR estimates for additional observation exchange between the CR nodes for PU signal detection. We analysed the performance of the proposed diffusion LMS based LE detection scheme and verified the theoretical results with the simulations. With the proposed algorithm PU signal detection performance is enhanced in CR network.

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