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Development of the Calculation Method for Barge Hull

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Declaration:
Hereby I declare that this doctoral thesis, my original investigation and achievement, submitted for the doctoral degree at Tallinn University of Technology, has not been submitted for any academic degree.

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The personal contribution of the author

The contribution of the author to the papers above is as follows:

For papers 2, 3, 4, and 5 Dmitri Gornostajev is the main author of the paper. He is responsible for the literature overview, analysis, development of the theory and calculation. He had a major role in writing. The author participated in development of the theory. The author has made calculations.

SYMBOLS

x,y,z - Rectangular coordinates

r,φ - Polar coordinates

rx, ry - Radius of curvature of the middle of a plate

h - Thickness of a plate

q(x,y) - intensity of a continuously distributed load

D - Flexural rigidity of the plate

$w(x,y)$ - Deflection of the plate

γ - Weight per unit volume

$\sigma_x, \sigma_y, \sigma_z$ - Normal components of stress

$\tau_{xy}, \tau_{xz}, \tau_{yz}$ - Shearing stress components in rectangular coordinates

$\varepsilon_x, \varepsilon_y, \varepsilon_z$ - Unit elongations

$\gamma_{xy}, \gamma_{xz}, \gamma_{yz}$ - Shearing strain components in rectangular coordinates

E - Modulus of elasticity in tension and compression

G - Modulus of elasticity in shear

μ - Poisson's ratio

M_x, M_y, M_z - Bending moments per unit length

M_{xy} - Twisting moment per unit length

V_x, V_y - Vertical reactions in the plate

Q_x, Q_y - Shearing forces

N_x, N_y - Normal forces

INTRODUCTION

Background

Today's fish farming industry is growing very fast due to increased fish consumption in the world. The market in Norway is very stable and has been growing very well. Fish farmer companies are investing increasingly in their business. Those developments are required to open more slots in the fjords, to establish new fish feeding centers, to fulfill the requirement of the consumers. Due to limitations in the law, farmers need to place fish farms farther away from the shore. The requirement to be taken into consideration is the load capacity of the barge and store room size. Those parameters are very important because of remote location of the barges from the shore and limitation of availability service ships who supply the feed for the fish. In other words, the barges need to be lighter in weight and have more capacity to meet the strength requirements.

A barge like any other vessel is located on the water and some loads are forced on the hull of the barge. Figure 1.1 shows the hull of the barge with loads on it.

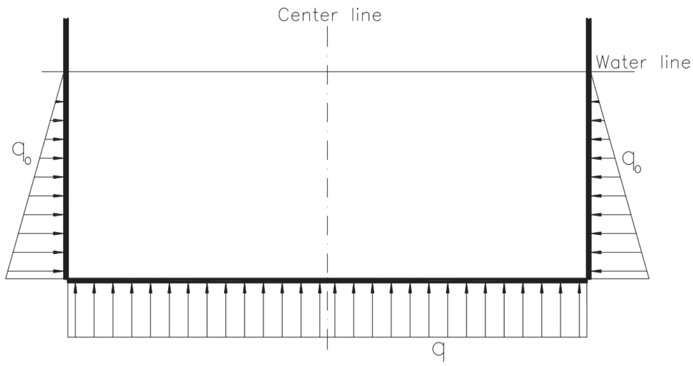


Figure 1.1. Scheme of loads on the hull of the barge in cross-section.

As shown in the figure, the uniformly distributed load is located on the bottom plate and the hydrostatic pressure is placed on the sides of the barge.

As soon as the barge is in calm water, the dynamic forces are not present. Only static loads will be taken into account in our calculations. In fish feeding barges the weight is a very important parameter. The lower the weight of the barge the more feed it can carry and the more money the owner can save. Reduction of the weight is a common problem in the fish feeding industry and it is a challenge for engineers. Thus, one of the possibilities to reduce the weight is to use thinner materials on the hull of the barge. At the same time, the strength requirements must be satisfied. In order to achieve this, plates with variable thicknesses can be used. In places at minimum load thinner steel can be implemented, which reduces the weight. However, a method is to be developed to calculate plates with variable thicknesses. In this work a method of calculation of plates with variable thicknesses will be developed.

Main objectives of the thesis

The main aim is to develop a method of calculation of plate thickness for the hull of the barges and ships using plates with variable thicknesses that were not used before. To achieve this, it is necessary to investigate the existing methods of calculations and to compare them with the new method.

The objectives are as follows:

1. To analyze the existing methods of calculations of plates and to compare them with the real solution.
2. To propose a new method of the calculation of rectangular plates with variable thicknesses.
3. To find the curve of deflection for results obtained by the new method.
4. To make an experiment with rectangular plates with constant and variable thicknesses under a distributed load.
5. To provide calculations by the FEM method.
6. To compare results achieved by the new method and those from the experiment with FEM calculations and find the deviation.

The study required the following steps to be taken:

- To provide calculations of plates by Navier, Levi, collocation, Kantorovits-Vlassov, grid and FEM methods for the plate under distributed loads and hydrostatic pressure.
- To compare the above methods with the real solution in order to define the method most suitable for the calculation of a plate for a hull.
- To introduce the new method of calculation of plates with variable thicknesses.
- Using a program obtain the results and find the curve of deflection. In order to solve the equations and obtain the numerical results of deflection, the MATLAB program will be used.
- To make an experiment for rectangular plates with variable and constant thickness under a distributed load.
- To compare the results and draw a conclusion from the results achieved by experiment, new method and FEM.

1. OVERVIEW OF THE VESSEL TYPES

Until the 20th century, generally, ships were all-purpose cargo vessels, with very little specialization (with the exception of tank vessels which first appeared in the 1880s). All cargoes were carried in general purpose holds, or on deck. Modern commercial vessels are designed and built to carry specific cargo types. The names given to the various vessel types reflect the type of cargo for which they are designed and built to carry. For example, a "bulk carrier" is specially designed to carry cargo "in bulk" and the hatch cover and hold design is focused on the carriage of raw dry cargo goods, such as coal, grain, iron ore and bauxite, which are simply poured into cavernous holds, then grabbed and bulldozed out at the port of discharge.

Tankers carry liquid cargo in tanks. The most obvious example is the well-known oil tanker, but even within this generic type, each tanker is specially designed to carry a particular type of liquid cargo, not just crude oil. Other liquid cargoes would include petroleum products, chemicals and yes, even wine. Two recent hybrid tanker designs carry Liquefied Natural Gas (LNG) and Liquefied Petroleum Gas (LPG), both of which need to be kept under pressure and at low temperature to maintain the cargo in a liquefied state. A further hybrid is the Floating Production, Storage and Offloading unit (FPSO), which is usually a large tanker (maybe a converted old VLCC, but now brand new specialized FPSOs are being built) specifically designed for the oil industry, working offshore where an onshore facility to process and store offshore oil is deemed impractical [1].

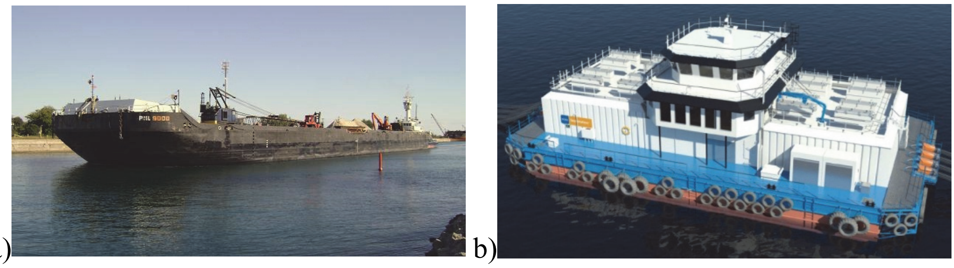
All ships are categorized to acquire better understanding of their usage. The main characteristic feature is the purpose of the ship. Also, vessels are divided by their place of the usage type of the main engine, hull material, and architectural type and by other aspects. Civil ships by their purpose are divided to transportation, commercial and technical types. The transportation vessels are the main types of vessels used on sea and on rivers. Figure 1.1 shows some ship types used on the sea.

a) 

b) 

Figure 1.1. a) Cruise vessel [2], b) Container ship [2].

Some vessels as barges have no to move by their own force and have to be towed by other vessels such as tugboats. Figure 1.2 shows some types of barges.



a) b)

Figure 1.2. a) Transportation barge [2], b) Fish feeding barge [3].

There are various types of vessels for different purposes. This study concentrates on fish feeding barges, the most useful and fast developing type of vessels in the fish feeding industry. Focus will be on the forces having impact on the hull of the barge.

Nowadays the fish farming industry is growing very fast due to increased fish consumption in the world. The market in Norway is very stable and has been growing very well. Fish farmer companies are investing increasingly in their business. Those developments require that more slots be opened in the fjords, new fish feeding centers be established to fulfill the requirements of the consumers. Due to limitations in the law, farmers need to place fish farms farther from the shore. The requirement to be taken into consideration is the load capacity of the barge and the store room size. Those parameters are very important because of offshore location of the barge and limitations of availability of service ships who supply the feed for the fish. In other words, the barges need to be lighter in weight and higher in capacity to meet strength requirements.

First, the construction of the fish feeding barge is introduced. Figures 1.3 and 1.4 show the typical construction of the fish feeding barge.

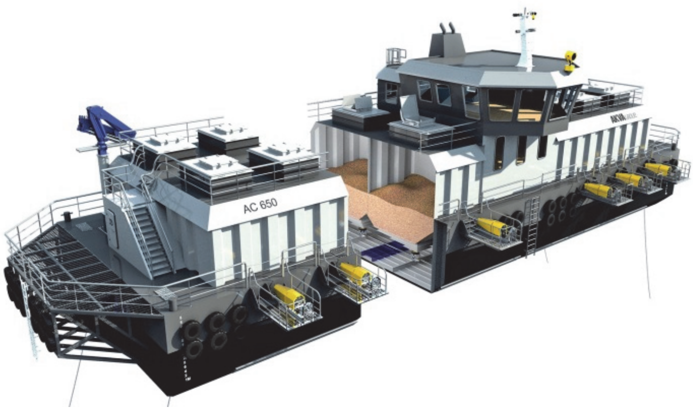


Figure 1.3. Fish feeding barge crossed in silo section [4].

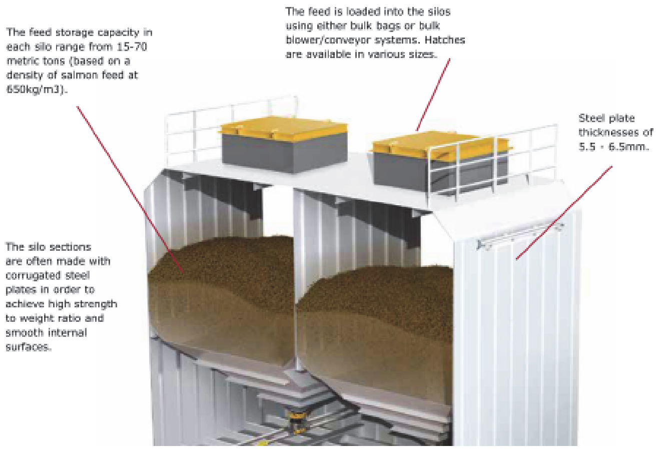


Figure 1.4. Cross-section of a fish feeding barge [4].

Each barge consists of a hull, superstructure, control room, engine room, technical room, living room, etc. The purpose of the barge is to feed the fish located in the cages, as shown in Figure 1.5.

Figure 1.5. Cages with fish supply fed by the fish feeding barge [4].

Conclusion

The amount of fish feeding barges is growing very fast and the market of the fish industry is growing. Today it is very important to have barges with larger capacity and lower weight. For that reason, the weight needs to be increased in order to increase the capacity. To achieve this, more accurate methods of calculations have to be developed. Today's methods for barge hull calculations do not consider the weight of the barge. They are concentrated only on the strength considerations. Thus, a new method for hull calculations is required to achieve the goals set in order to fulfill the market and client requirements, such as larger capacity and lower weight of the barge. Still another very important aspect is the price of the barge. If the weight is reduced, the price of the barge will be cheaper and the demand for barges can be increased, which has a positive effect on the fish farming business. In order to achieve this goal, a plate thickness has to be decreased and keep strength requirements. To achieve this new method of calculation has to be developed. Many different methods of plate calculations are available. In this thesis most of them will be described, analyzed and compared. The methods discussed here are: Navier, Levi, collocation, Kantorovits-Vlassov, grid, and the FEM methods. This study has to prove that the new method has more advantages for the calculation of the plate and for future use in barge hull calculation. In order to propose a new method, existing methods will be analyzed.

2. OVERVIEW AND ANALISYS OF THE METHODS FOR CALCULATION OF PLATES

Thin plates are initially flat structural members bounded by two parallel planes called faces, and a cylindrical surface called an edge or a boundary. The generators of the cylindrical surface are perpendicular to the plane faces. The distance between the plane faces is called the thickness of the plate. It will be assumed that the plate thickness is small compared with other characteristic dimensions of the faces (length, width, diameter, etc.). Geometrically, plates are bounded either by straight or curved boundaries. The static or dynamic loads carried by plates are predominantly perpendicular to the plate faces [5-10].

The theoretical tasks of the calculations of plate bending even for simple shape plates and continuous thickness face some mathematical problems, which in most cases are solved by approximation methods or by using numerical methods. The mathematical problems are growing if the plate has variable stiffness. For those cases theoretical solutions are provided in general for round and rectangular plates with linear thickness changing [11-18].

Real solutions [19] of plate calculations can be provided only in some particular cases, mainly for plates with constant thickness, easy shape and with special boundary conditions. Variational methods of calculations are a more efficient instrument when defining deflections in more difficult cases. One of the few tasks of plate calculations with real solution that can be reached relatively easily is the task where a rectangular plate is randomly loaded with two sides freely supported [20-22].

2.1. Main equations and relations for plate calculations

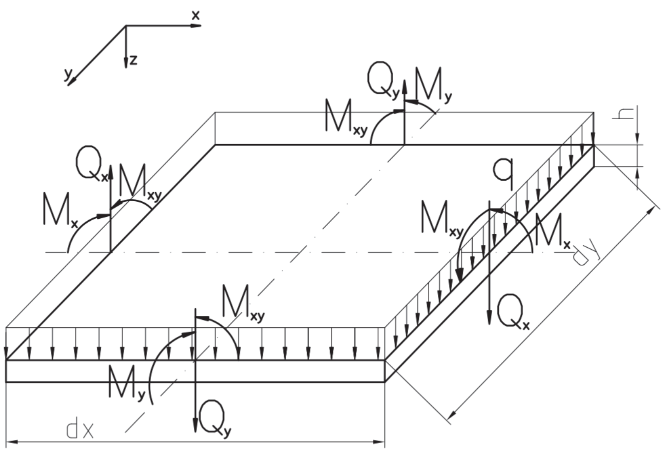


Figure 2.1. Plate under a distributed load [20].

At the beginning we have to obtain main equations and relations for an isotropic plate [30]. Having done all the calculations, we obtain the main deferential equation for an isotropic plate with constant thickness, (Eqs. (2.1) - (2.6) [20-22]):

$$\nabla^2\nabla^2 w = \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D}\,. \tag{2.1}$$

Shearing forces are defined by the following equations:

$$Q_x = -D\left[\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right) + (1-\mu)\frac{\partial}{\partial y}\frac{\partial^2 w}{\partial x \partial y}\right] = -D\frac{\partial}{\partial x}\nabla^2 w\,,$$

$$Q_x = -D\left(\frac{\partial^3 w(x,y)}{\partial x^3} + \frac{\partial^3 w(x,y)}{\partial x \partial y^2}\right), \tag{2.2}$$

$$Q_y = -D\left(\frac{\partial^3 w(x,y)}{\partial y^3} + \frac{\partial^3 w(x,y)}{\partial x^2 \partial y}\right),$$

$$Q_y = -D\frac{\partial}{\partial y}\nabla w \tag{2.3}$$

Bending moments are defined by the equations:

$$M_x = -\frac{Eh^3}{(1-\mu^2)}\left(\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right) = -D\left(\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right), \tag{2.4}$$

$$M_y = -\frac{Eh^3}{12(1-\mu^2)}\left(\frac{\partial^2 w}{\partial y^2} + \mu\frac{\partial^2 w}{\partial x^2}\right) = -D\left(\frac{\partial^2 w}{\partial y^2} + \mu\frac{\partial^2 w}{\partial x^2}\right), \tag{2.5}$$

$$M_{xy} = -\frac{Eh^3}{12(1-\mu^2)}(1-\mu)\frac{\partial^2 w}{\partial y^2} = -D(1-\mu)\frac{\partial^2 w}{\partial x \partial y}\,, \tag{2.6}$$

where,

$\frac{Eh^3}{12(1-\mu^2)} = D$ - flexural rigidity of the plate.

To solve Eq. (2.1), different approximation methods described below will be used.

2.2. Navier' method for a simply supported plate

Navier' method [30] is the method of calculation with approximation. The plate is landed simply supported, as shown in Figure 2.2. (Eqs. (2.7) - (2.8) [20-22]):

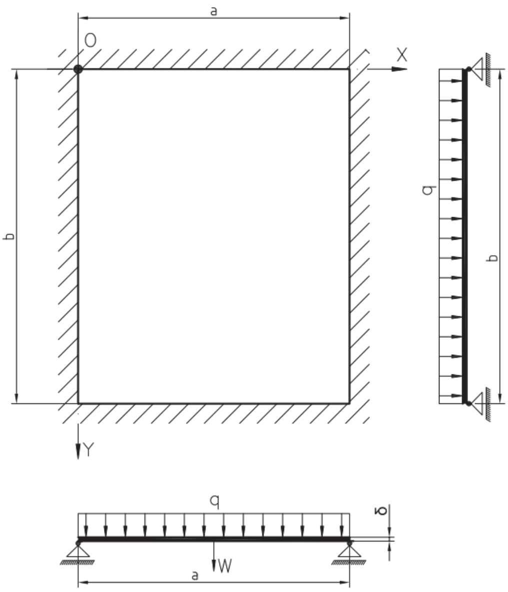


Figure 2.2. Plate simply supported under a distributed load [20].

The boundary conditions for this method are:

$$(W)_{x=a} = 0\,;\left(\frac{\partial^2 w}{\partial x^2}\right)_{x=a} = 0\,.$$

The solution can be found by the trigonometrical row below:

$$W(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn} \sin\frac{m\pi x}{a} \sin\frac{m\pi y}{b}\,. \tag{2.7}$$

After all calculations, the deflection of plate is defined as follows:

$$W = \frac{16qa^4}{\pi^6 D} \sum_{m=1,3,5...}^{\infty} \sum_{n=1,3,5...}^{\infty} \frac{1}{mn\left[m^2 + \left(\frac{n}{\beta}\right)^2\right]^2} \sin\frac{m\pi x}{a} \sin\frac{m\pi y}{b}\,. \tag{2.8}$$

In the same way, bending moments M_x, M_y of shearing forces Q_x, Q_y can be found by Eqs. (2.2)-(2.5).

For our case the scheme of the plate loaded by the hydrostatic pressure is shown in Figure 2.3.

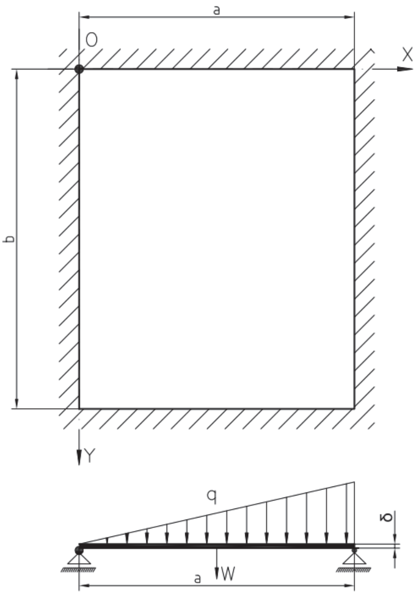


Figure 2.3 Plate simply supported under a load of the hydrostatic pressure [20].

In our case the distributed load is changing as follows:

$$q = \frac{q_0}{b} y , \tag{2.9}$$

where

q_0 - pressure in the depth of 3 meters under water.

For our case, the deflection of the plate is defined as:

$$w(x, y) = \frac{8q_0}{\pi^6 D} \sum_{m=1,2,3,..}^{\infty} \sum_{n=1,3,5,..}^{\infty} \frac{(-1)^{m+1}}{mn\left[\left(m / a^2\right) + \left(n / b\right)^2\right]^2} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} =$$

$$= \frac{8q_0 a^4}{\pi^6 D} \sum_{m=1,2,3,..}^{\infty} \sum_{n=1,3,5,..}^{\infty} \frac{(-1)^{m+1}}{mn\left[m^2 + \left(n / \beta\right)^2\right]^2} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} , \tag{2.10}$$

where,

$\beta = b / a$.

The bending moments M_x, M_y and the torque moment M_{xy} are found by Eqs. (2.4), (2.5) and (2.6).

Shearing forces Q_x and Q_y are found by substituting Eqs. (2.2) and (2.3).

A disadvantage of the Navier' method is that the solution can be found only if the plate is simply supported. For most cases, this method does not suit for barge calculations.

Also, the deviation for the shearing forces is up to 18 % from the real solution, for vertical reactions up to 15 % from the real solution. The most accurate solutions by the Navier' method can be found for deflections close to 0 and for bending moments up to 2 % [20-22].

Figure 2.4 shows the deviation between the Navier' method and the real solution for a distributed load.

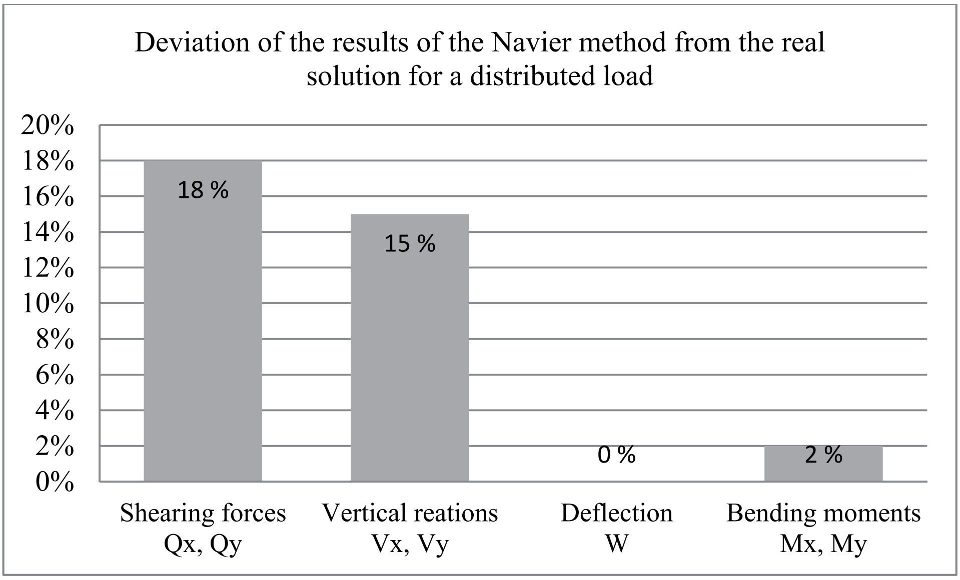


Figure 2.4. Deviation between the Navier' method and the real solution for a distributed load.

As can be seen from Figure 2.4, deviation for the deflection W is zero, so it means that it matches exactly a real solution. Thus, this method suits for the calculations of deflection. I our case the deflection is the distance to which a structural element is displaced under a load [20]. For bending moments M the deviation is small, only 2 %. This method is also suitable for calculations of bending moments. The deviation for shearing forces Q and vertical reactions V is up to 18 % as compared with a real solution, which is large.

Figure 2.5 shows the deviation between the Navier' method and the real solution for the hydrostatic pressure.

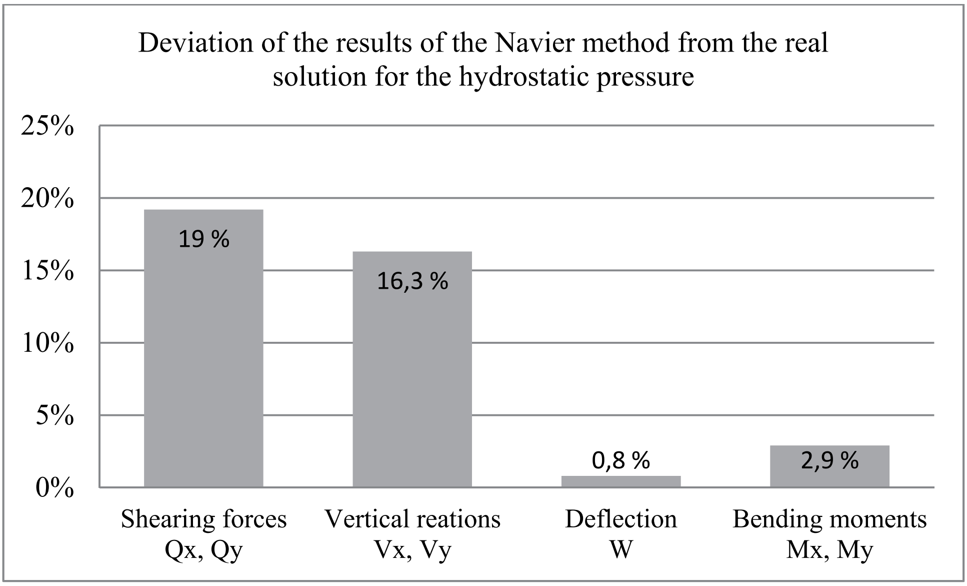


Figure 2.5. Deviation between the Navier method and the real solution for the hydrostatic pressure.

We can see that the deviation for the hydrostatic pressure is on the same level as that for a distributed load. The maximum difference is only 0,8 %, so we do not have to take that into consideration.

In conclusion, the Navier' method has no deviation from the real solution for the deflection *W,* for the bending moments *M* the deviation is only 2,9 %. This method cannot be used in plate calculations for hulls because the boundary conditions are not suit the case. We have to take into consideration the deviation from the real solution for shearing forces *Q* and vertical reactions *V* is 19 % and 16,3 %, accordingly.

2.3. Levy solution for rectangular plates with two opposite edges simply supported and two edges built in under hydrostatic pressure

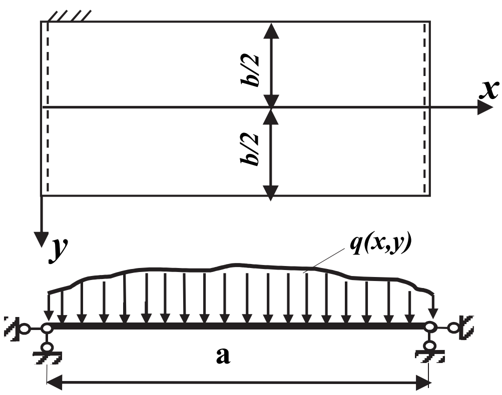


Figure 2.6. Plate with two sides simply supported and two sides built in [20].

If the bending plates have two opposite sides $x = 0, x = a$ simply supported and two opposite edges $y = \pm b/2$ built in (Figure 2.6), M. Levy [20-22] suggested a solution in the form a series:

$$w(x, y) = \sum_{m=1}^{\infty} Y_m(y) \sin \frac{m\pi x}{a}, \tag{2.11}$$

where $Y_m(y)$ is a function of y only.

Hence, each term of series (2.11) satisfies the boundary conditions $w(x, y) = 0$ and $\frac{\partial^2 w(x, y)}{\partial x^2} = 0$ at these two sides.

Further, $Y_m(y)$ should be determined in a form to satisfy the boundary conditions on the sides $y = \pm b/2$ and also the equation of the deflection surface.

We represent a distributed load q in the form of a trigonometric series of Fourier:

$$q(x, y) = \sum_{m=1}^{\infty} K_m(y) \sin \frac{m\pi x}{a}, \tag{2.12}$$

where,

$$K_m(y) = \frac{2}{a} \int_0^a q(x,y) \sin \frac{m\pi x}{a} dx. \tag{2.13}$$

The differential equation for the deflection surface of the plate is described by the biharmonic equation (2.1).

Differentiating the expressions (2.1) and substituting them with the expressions (2.11) in Eq. (2.12), we obtain an equation:

$$\begin{aligned} &\sum_{m=1}^{\infty} \left[\alpha_m^4 Y_m(y) + \alpha_m^2 Y_m''(y) + Y_m^{IY}(y)\right] \sin \frac{m\pi x}{a} = \\ &= \frac{1}{D} \sum_{m=1}^{\infty} K_m(y) \sin \frac{m\pi x}{a} \end{aligned}, \tag{2.14}$$

where,

$$\alpha_m = \frac{m\pi}{a}.$$

This equation can satisfy all the values of x only if the function $Y_m(y)$ satisfies the equation

$$\alpha_m^4 Y_m - 2\alpha_m^2 Y_m'' + Y_m^{IY} = K_m / D, \tag{2.15}$$

where the function $Y_m(y)$ consists of a general integral $Y_m^o(y)$ and a particular integral $Y_m^e(y)$

$$Y_m(y) = Y_m^o(y) + Y_m^e(y). \tag{2.16}$$

The general integral Y_m^o of this equation is

$$\begin{aligned} Y_m^o(y) = A_m\, sh\alpha_m y + B_m\, ch\alpha_m y + \\ + C_m\, y \cdot sh\alpha_m y + D_m\, y \cdot ch\alpha_m y \end{aligned}, \tag{2.17}$$

which has to satisfy all the boundary conditions of the plate.

A particular solution $Y_m^e(y)$ represents the deflection of a strip under the hydrostatic pressure $q = q_o x / a$ (Figure 2.7). It satisfies the differential equation (2.1) and the boundary conditions at the edges x = 0 and $x = a$.

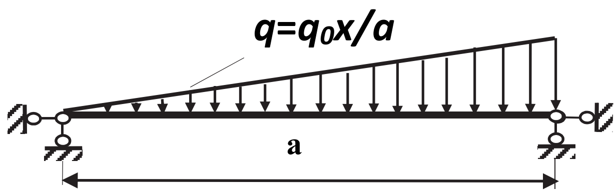


Figure 2.7. Plate with two sides simply supported and two sides built under hydrostatic pressure [20].

According to Eq. (2.14), coefficients $K_m(y)$ of the trigonometric series of Fourier (2.13) are

$$K_m(y) = \frac{2}{a}\int_0^a g\ \sin\frac{m\pi x}{a}dx = -\frac{2q_o}{am\pi}\left[a(-1)^m - 0\right] = \frac{2q_o}{m\pi}(-1)^{m+1}. \tag{2.18}$$

As a result, the load by the hydrostatic pressure in the form of a trigonometric series of Fourier is

$$q(x,y) = \sum_{m=1}^{\infty}\frac{2q_o}{m\pi}(-1)^{m+1}\sin\frac{m\pi x}{a} \tag{2.19}$$

Substituting the expression (2.18) in Eq. (2.15), we obtain the particular integral Y_m^e (y) in the form

$$Y_m^e(y) = \frac{2q_o a^4}{m^5 \pi^5 D}(-1)^{m+1}, \tag{2.20}$$

and

$$w_1(x,y) = Y_m^e(y)\sin\frac{m\pi x}{a} = \frac{2q_o a^4}{m^5\pi^5 D}(-1)^{m+1}\sin\frac{m\pi x}{a}. \tag{2.21}$$

represents the deflection of a strip parallel to the x-axis under the distributed load q.

The deflection surface Eq. (2.11) is represented by the following expression:

$$w(x,y) = \sum_{m=1}^{\infty}\left[B_m\, ch\alpha_m y + C_m\, y\cdot sh\alpha_m y + \frac{2q_o}{m\pi}(-1)^{m+1}\right]\sin\frac{m\pi x}{a}, \tag{2.22}$$

which satisfies Eq. (2.11) and also the boundary conditions at the edges $x = 0$ and $x = a$. Next, we have to adjust the constants of integration B_m and C_m such that that they will satisfy the boundary conditions

$$w(x,y) = 0,\ \frac{\partial w(x,y)}{\partial x} = 0, \tag{2.23}$$

on the sides $y = \pm b/2$.

Substituting the expression (2.22) in the boundary conditions (2.23) and using the notation

$$u_m = \alpha_m \frac{b}{2} = \frac{m\pi b}{2a}, \tag{2.24}$$

Substituting these values of the constants of integration in Eq. (2.22), we obtain the deflection surface of the plate that satisfies differential equation (2.1) and the boundary conditions in the following form:

$$w(x,y) = \frac{2q_o a^4}{\pi^5 D}\sum_{m=1}^{\infty}\frac{(-1)^{m+1}}{m^5}\left[1 - \frac{u_m ch u_m + sh u_m}{u_m + sh u_m ch u_m} ch\frac{m\pi y}{a} + \right.$$

$$\left. + \frac{u_m sh u_m}{u_m + sh u_m ch u_m}\cdot\frac{2}{b}y\cdot sh\frac{m\pi y}{a}\right]\sin\frac{m\pi x}{a}, \tag{2.25}$$

from which the deflection at any point can be calculated by using the tables of hyperbolic functions [3].

In case $m = 1$, we obtain the deflection of the plate at any point in the form

$$w(x,y) \approx \frac{2q_o a^4}{\pi^5 D}\begin{pmatrix} 1-\dfrac{u\,chu+shu}{u+shu\cdot chu}ch\dfrac{\pi y}{a}+ \\ +\dfrac{u\,shu}{u+shu\cdot chu}\dfrac{2y}{b}sh\dfrac{\pi y}{a} \end{pmatrix}\sin\frac{\pi x}{a}, \tag{2.26}$$

where

$$u = \pi b / 2a. \tag{2.27}$$

The deflection at the center of the plate is

$$w(x,y) \approx 0.00087\frac{q_o a^4}{D}. \tag{2.28}$$

which is one-half of the deflection of a uniformly loaded plate as required.

The bending moments M_x, M_y and the torque moment M_{xy} are found by Eqs. (2.4), (2.5) and (2.6).

Shearing forces Q_x and Q_y are found by substituting expressions (2.2) and (2.3).

A disadvantage of the Levy method is that the solution can be found only if the plate is two sides free landed and two sides fixed. Thus, the boundary conditions are limited.

The deviation for the shearing forces is up to 17 % from the real solution, for the vertical reactions up to 15 % from the real solution. The most accurate solutions by the Levy method can be found for the deflection around zero and for the bending moment up to 2 % [20-22].

Figure 2.8 shows the deviation between the Levy method and the real solution for a distributed load.

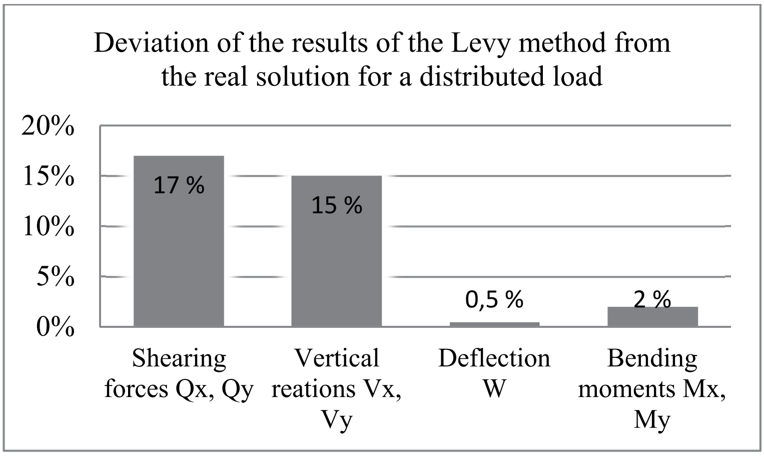


Figure 2.8. Deviation between the Levy method and the real solution for a distributed load.

As Figure 2.8 shows, the deviation for the deflection *W* is only 0,5 %. Thus, this method can be used for the calculations of deflection. For the bending moments *M* the deviation is small, only 2 %. This method is also suitable for the calculations of bending moments. The deviation for shearing forces *Q* and vertical reactions *V* is up to 17 % as compared with the real solution, which is quite large.

Figure 2.9 shows the deviation between the Levy method and the real solution for the hydrostatic pressure.

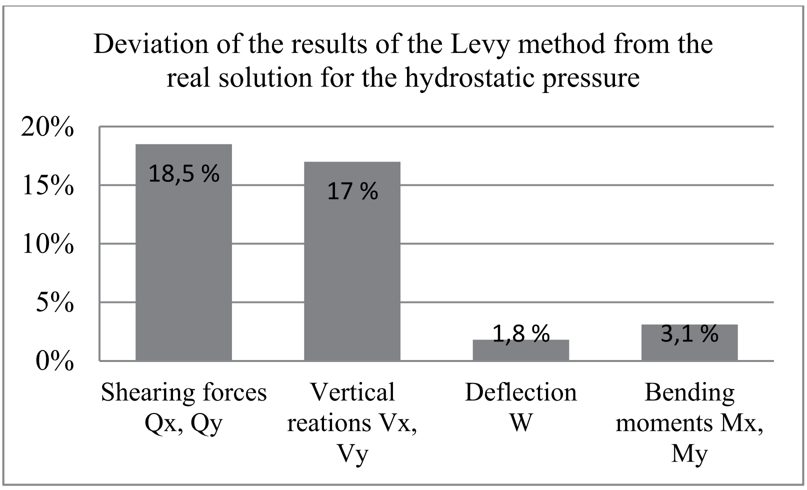


Figure 2.9. Deviation between the Levy method and the real solution for the hydrostatic pressure.

Figure 2.9 shows that the deviation for the hydrostatic pressure is on the same level as that for the distributed load. The maximum difference is only 1,8 %, which can be neglected.

In conclusion, the Levy method has a small deviation from the real solution for deflection *W* and for bending moments *M* and it can be used in the plate calculations for hulls if the boundary conditions suit the case. We have to take into consideration the deviation from the real solution for shearing forces *Q* and vertical reactions *V* due to the deviation of up to 18,5 %.

2.4. Collocation method of the four-side fixed plate

The collocation method [20-22] is used when the plate is fixed with four sides as shown in Figure 2.10. (Eqs. (2.29) - (2.34) [20-22]):

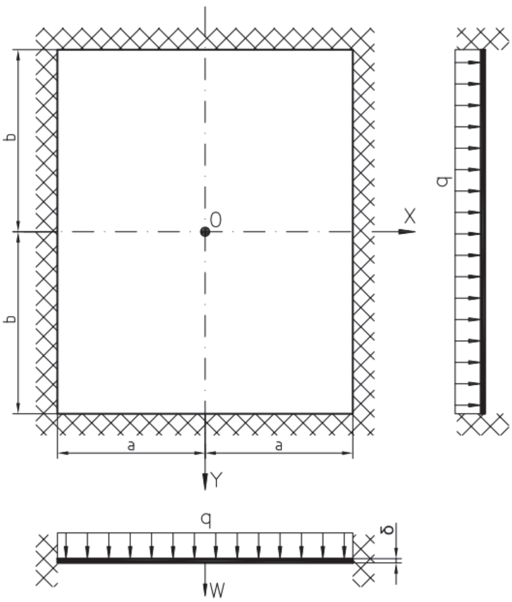


Figure 2.10. Four side fixed plate with a distributed load [20].

Approximation of the plate's elastic equation is defined by

$$w(x, y) = \sum_m \sum_n C_{mn} w_{mn}(x, y), \quad (2.29)$$

where $w(x, y)$ is chosen to satisfy boundary conditions in order to describe the plate shape quite accurately.

The differential equation of plate bending is defined by

$$\nabla^2 \nabla^2 w = \frac{q}{D}. \quad (2.30)$$

Using the polynomial and the approximation method we obtain the equation

$$w = C_{11}(1-\xi^2)^2(1-\eta^2)^2 + C_{12}(1-\xi^2)^2(1-\eta^2)^2 + \\ + C_{21}\xi^2(1-\xi^2)^2(1-\eta^2)^2 + C_{22}\xi^2(1-\xi^2)^2\eta^2(1-\eta^2)^2, \quad (2.31)$$

where

$$\xi = \frac{x}{a};\ \eta = \frac{y}{b}.$$

The boundary conditions are:

if $\xi = \pm 1$, the $\Delta x = 0$; $w = 0$, and if $\eta = \pm 1$, the $\Delta y = 0$; $w = 0$. (2.32)

Equation (2.31) satisfies the boundary conditions (2.32)

For a rectangular plate where $a = b$ chosen collocation $(0;0);(0;0,5);(0,5;0);(0,5;0,5)$ we can define the equation system to obtain the parameter C_{mn} and after solving the system of equations, we have:

$$C_{11} = \frac{0{,}0201qa^4}{D} \\ C_{12} = C_{21}\frac{0{,}0051qa^4}{D}. \quad (2.33) \\ C_{22} = \frac{0{,}0048qa^4}{D}$$

The deflection in the middle of the plate with $\mu = 0{,}3$ is defined by:

$$w = C_{11} = \frac{0{,}0201qa^4}{D} = \frac{0{,}0138q(2a)^4}{Eh^3}. \quad (2.34)$$

For our case the scheme of the plate shown in Figure 2.11 is loaded by the hydrostatic pressure.

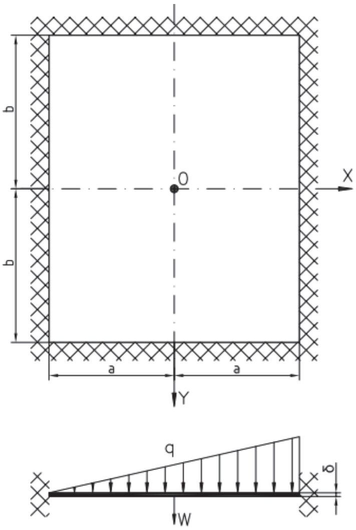


Figure 2.11. Plate fixed at four sides under hydrostatic pressure.

In our case the equation of plate bending is defined by

$$\begin{aligned} w = {} & C_{11}(\xi^4 - 6\xi^2 + 4\xi + 1)^2(1-\eta^2)^2 + \\ & + C_{12}(\xi^4 - 6\xi^2 + 4\xi + 1)^2(1-\eta^2)^2 + \\ & + C_{21}(\frac{\xi^2}{2} - \xi)(\xi^4 - 6\xi^2 + 4\xi + 1)^2(1-\eta^2)^2 + \\ & + C_{22}(\frac{\xi^2}{2} - \xi)(\xi^4 - 6\xi^2 + 4\xi + 1)^2\eta^2(1-\eta^2)^2 \end{aligned} \quad . \tag{2.35}$$

For a rectangular plate where $a=b$ are chosen collocation points $(0;0);(0;0,5);(0,5;0);(0,5;0,5)$,we can define the equation system in order to obtain the parameter C_{mn} .

In our case the distributed loads are defined by

$$q = \frac{q_0}{2}\left(1+\xi\right) \Rightarrow \frac{q_0}{2}\left(1+\frac{x}{a}\right). \tag{2.36}$$

For collocation points $(0;0);(0;0,5);(0,5;0);(0,5;0,5)$ we can define the distributed loads as follows:

in points $(0;0)$ and $(0;0,5)$: $q = \dfrac{q_0}{2}$, (2.37)

in points $(0,5;0)$ and $(0,5;0,5)$: $q = \dfrac{3q_0}{4}$. (2.38)

From the solving of the system of equations we obtain the equation:

$$C_{11} = 0{,}0028\frac{q_0 a^4}{D},\quad C_{12} = -0{,}0008\frac{q_0 a^4}{D}, \tag{2.39}$$

$$C_{21} = 0{,}000013\frac{q_0 a^4}{D},\quad C_{22} = -0{,}0005\frac{q_0 a^4}{D}.$$

The deflection in the middle of the plate with $\mu = 0{,}3$ is defined by

$$w = C_{11} = 0{,}0028\frac{q_0 a^4}{D} = 0{,}0028\frac{q_0 a^4}{Eh^3}. \tag{2.40}$$

The deviation for shearing forces is up to 11% from the real solution, for vertical reactions up to 11 % from the real solution. The solutions found by the collocation method and the real solution match and the deviation is zero for the deflection and for the bending moment up to 2 % [30].

Figure 2.12 shows the deviation between the collocation method and the real solution for a distributed load.

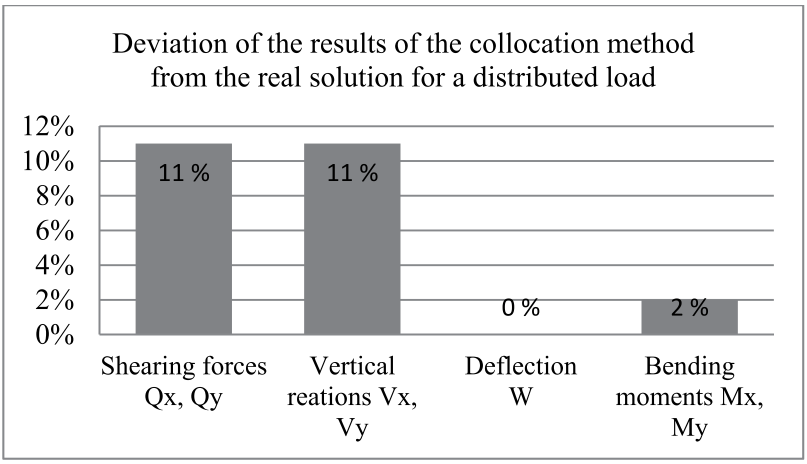


Figure 2.12. Deviation between the collocation method and the real solution for a distributed load.

As shown by Figure 2.12, the deviation for the deflection *W* is zero. Thus, it matches exactly the real solution and the method can be used for calculations of deflection. For bending moments *M* the deviation is small, accounting for 2 %. This method is also suitable for calculations of bending moments. The deviation for shearing forces *Q* and vertical reactions *V* is up to 11 % as compared with the real solution, which appears not so large.

Figure 2.13 shows the deviation between the collocation method and the real solution for the hydrostatic pressure.

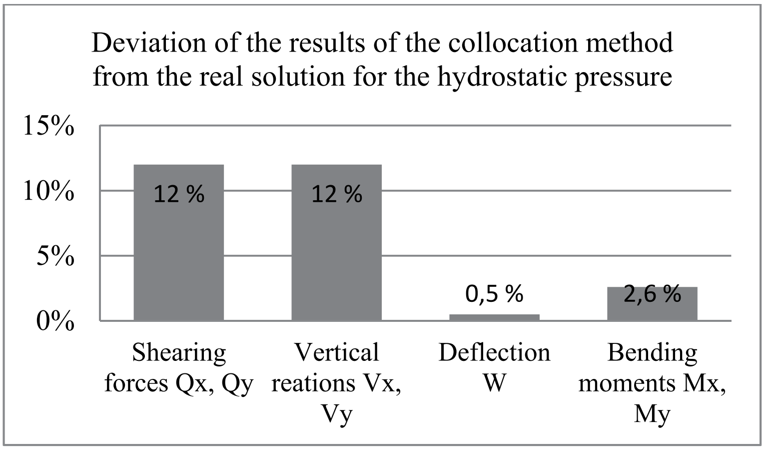


Figure 2.13. Deviation between the collocation method and the real solution for the hydrostatic pressure.

As can be seen, the deviation for the hydrostatic pressure is on the same level as for the distributed load. The difference is very small, so we can neglect it.

In conclusion, the collocation method has a small deviation from the real solution for the deflection *W* and for bending moments *M* and can be used in the plate calculations for hulls. A minor deviation of up to 12 % from the real solution for shearing forces *Q* and vertical reactions *V* was found.

2.5. Kantorovits-Vlassov method for a four side fixed plate

Kantorovits-Vlassov [20-22] method is one of the approximations methods.

The plate fixed on four sides with the relation $\frac{b}{a} = 1$ with a distributed load is shown in Figure 2.14.

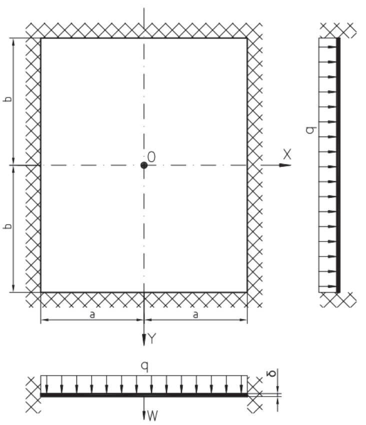


Figure 2.14. Plate fixed on four sides under a distributed load [30].

The equation for deflection is defined by

$$w = Y(y)(1-\xi^2)^2 . \tag{2.41}$$

To satisfy the boundary conditions on the sides $x = \pm a$, the differential equation of deflection is defined by

$$\nabla^2\nabla^2 w - \frac{q}{D} = 0 . \tag{2.42}$$

To take the differential, we can obtain the equation

$$\int_{-1}^{1}(\nabla^2\nabla^2 w - \frac{q}{D})(1-\xi^2)^2 d\xi = 0 . \tag{2.43}$$

After all the calculations we obtain:

$$\begin{aligned} Y_0 = C_1 ch\frac{2{,}075y}{a}\cos\frac{1{,}143y}{a} + C_2 ch\frac{2{,}075y}{a}\sin\frac{1{,}143y}{a} + \\ + C_3 sh\frac{2{,}075y}{a}\cos\frac{1{,}143y}{a} + C_4 sh\frac{2{,}075y}{a}\sin\frac{1{,}143y}{a} \end{aligned} . \tag{2.44}$$

Use to the symmetric matter, the $Y(y)$ is the even function of the coefficients $C_2 = C_3 = 0$. In that case Eq. (2.4) is as follows:

$$Y = C_1 ch\frac{2{,}075y}{a}\cos\frac{1{,}143y}{a} + C_4 sh\frac{2{,}075y}{a}\sin\frac{1{,}143y}{a} + \frac{qa^4}{24D} . \tag{2.45}$$

Having in mind that $y = b$ or $y = -b$, we can find the coefficients C_1 and C_4;

$$C_1 = -\frac{0{,}014qa^4}{D},\ C_4 = -\frac{0{,}004qa^4}{D} .$$

Therefore,

$$w = \begin{pmatrix} -0{,}014ch\frac{2{,}075y}{a}\cos 1{,}143\frac{y}{a} - \\ -0{,}004sh\frac{2{,}075y}{a}\sin 1{,}143\frac{y}{a} + \frac{1}{24} \end{pmatrix} \cdot \left(1 - 2\frac{x^2}{a^2} + \frac{x^4}{a^4}\right)\frac{qa^4}{D} . \tag{2.46}$$

The deflection in the middle of the plate is defined by

$$(w)_{\substack{x=0\\y=0}} = 0{,}028\frac{qa^4}{D} = 0{,}019\frac{q(2a)^4}{Eh^3} . \tag{2.47}$$

The deviation for shearing forces is up to 14 % from the real solution, for the vertical reactions up to 15 % from the real solution. The most accurate solutions by the Kantorovits-Vlassov method can be found for zero deflection and for the bending moment only 0.5 % [20].

Figure 2.15 shows the deviation between the Kantorovits-Vlassov method and the real solution for a distributed load.

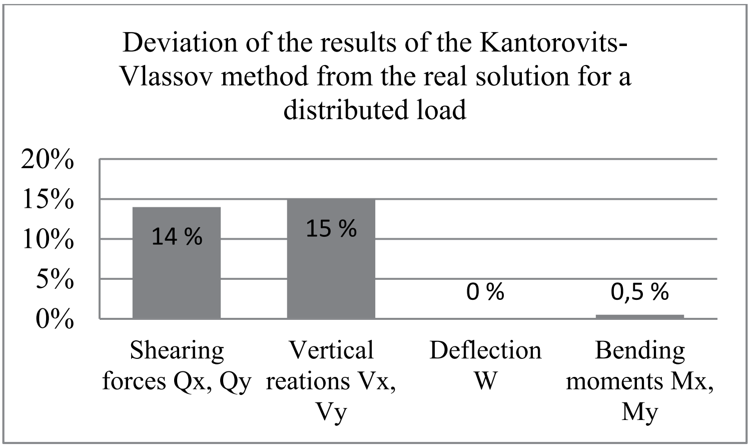


Figure 2.15. Deviation between the Kantorovits-Vlassov method and the real solution for a distributed load.

As shown in Figure 2.15, the deviation for the deflection *W* is zero. Thus, it matches exactly the real solution and this method can be used for the calculations of deflection. For bending moments *M,* the deviation is very small, only 0,5 %. This method is also suitable for calculations of bending moments. However, the deviation for shearing forces *Q* and vertical reactions *V* is relatively large - up to 15 % as compared to the real solution. Figure 2.16 shows the deviation between the Kantorovits-Vlassov method and the real solution for the hydrostatic pressure.

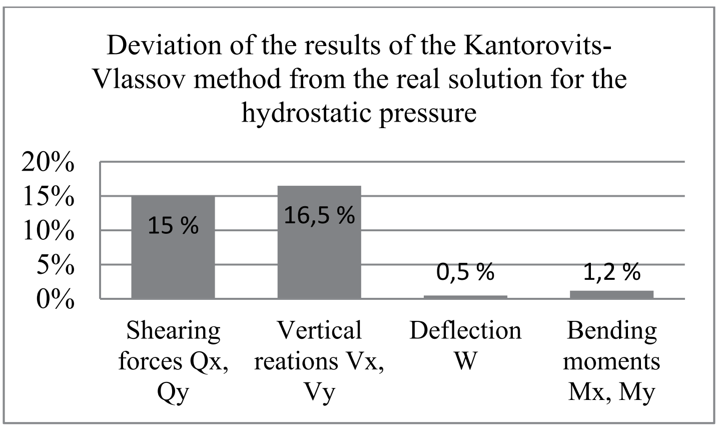


Figure 2.16. Deviation between the Kantorovits-Vlassov method and the real solution for the hydrostatic pressure.

We can see that the deviation for the hydrostatic pressure is on the same level as that for a distributed load. As there is only a slight difference, we can neglect that.

In conclusion, the Kantorovits-Vlassov method has a small deviation from the real solution for the deflection *W* and for bending moments *M* and can be used in plate calculations for hulls considering the deviation from the real solution for shearing forces *Q* and vertical reactions *V* that amounts up to 16,5 %.

2.6. Grid method for the calculation of a plate

The differential method can be extended by the partial derivative equation, so- called grid method [23-29]. The accuracy of this method is low but it can be increased by using a more rapid mesh. Using this method, different shape plates and different boundary conditions can be calculated. (Eqs. (2.48) - (2.55) [20-22]):

Let us divide the plate into a mesh, as shown in Figure 2.17; the step in the direction of axis X is Δ_x, in the direction of axis Y is Δ_y. Mesh points around the point CC are as shown in the figure.

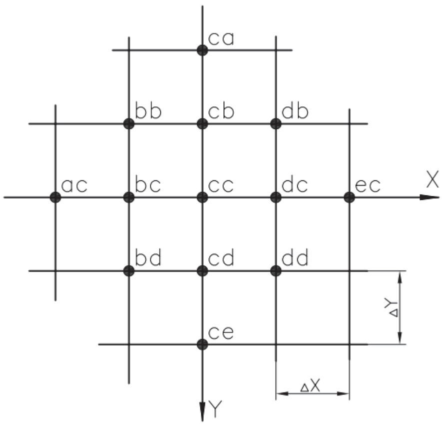


Figure 2.17. Plate divided by a mesh [20].

The first derivative in the point CC is defined by

$$\left(\frac{\partial w}{\partial x}\right)_{cc} = \frac{-w_{bc} + w_{dc}}{2\Delta_x}. \tag{2.48}$$

The derivative with two variables is defined by

$$\left(\frac{\partial^4 w}{\partial x^2 \partial y^2}\right)_{cc} = \left[\frac{\partial^2}{\partial y^2}\left(\frac{\partial^2 w}{\partial x^2}\right)\right]_{cc} = \frac{\left(\frac{\partial^2 w}{\partial x^2}\right)_{cb} - 2\left(\frac{\partial^2 w}{\partial x^2}\right)_{cc} + \left(\frac{\partial^2 w}{\partial x^2}\right)_{cd}}{\Delta_{y^2}}. \tag{2.49}$$

After some calculations we obtain a harmonized equation:

$$\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}=\frac{q}{D}. \tag{2.50}$$

Under the consideration that

$$\left(\frac{\Delta_y}{\Delta_x}\right)^2=\alpha. \tag{2.51}$$

For a plate fixed in all four edges and with the boundary condition $w_{cc}=0$ we obtain:

$$\left(\frac{\partial w}{\partial x}\right)_{cc}=\frac{-w_{bc}+w_{dc}}{2\Delta_x}=0. \tag{2.52}$$

After simplification we have:

$$w_{dc}=w_{bc}. \tag{2.53}$$

For a plate free landed and under the boundary condition $w_{cc}=0$ we obtain:

$$\left(\frac{\partial^2 w}{\partial x^2}\right)_{cc}=\frac{w_{bc}-2w_{cc}+w_{dc}}{\Delta_{x^2}}. \tag{2.54}$$

Taking into consideration the boundary condition $w_{cc}=0$ we have:

$$w_{dc}=-w_{bc}. \tag{2.55}$$

The deviation for the shearing forces is up to 22 % from the real solution, for the vertical reactions up to 20 % from the real solution. The most accurate solutions by the grid method can be found for a deflection around 0,7 % and for a bending moment up to 4,6 % [20].

Figure 2.18 demonstrates the deviation between the grid method [30-37] and the real solution for a distributed load.

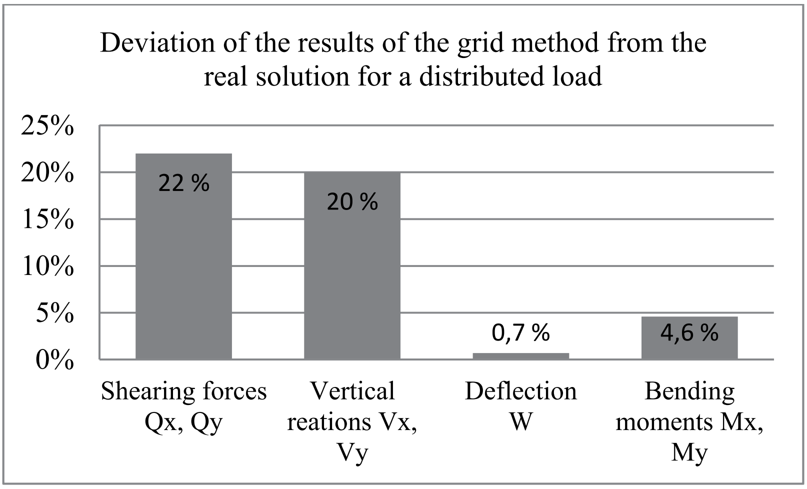


Figure 2.18. Deviation between the grid method and the real solution for a distributed load.

As seen from Figure 2.18, the deviation for the deflection W is very small. Thus, this method can be used for the calculations of the deflection. However, it should be taken into account that the deviation for the shearing forces Q and the vertical reactions V is large, up to 22 % from the real solution.

Figure 2.19 shows the deviation between the grid method and the real solution for the hydrostatic pressure.

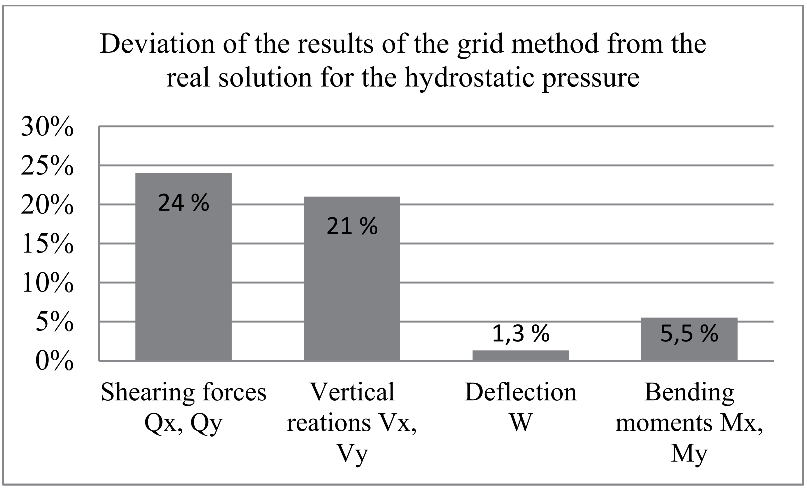


Figure 2.19. Deviation between the grid method and the real solution for the hydrostatic pressure.

As can be seen from Figure 2.19, the deviation for the deflection is small. Thus, this method can be used to calculate the deflection. However, it should be taken into account that the deviation for the shearing forces and the vertical reactions is large, accounting for up to 24 % from the real solution.

In conclusion, the grid method has a small deviation from the real solution for the deflection W and can be used in the plate calculations for hulls at the same time taking into account the major deviation from the real solution for the shearing forces Q and the vertical reactions V.

2.7. Plate calculations by FEM

Calculations for the plate can be provided by the integration of the biharmonic equation, which consists of the derivative of the plate deflection perpendicular to the plate plane. Under finite elements, we can define the deflection w by using an equation that has a matrix $[N]$ and an angular displacement $\{\delta\}^e$. It is used for rectangular and triangular elements in the plate calculations. In each node n the movement has three components, such as the deflection that is perpendicular to the plate plane w in the direction of axis z, rotation angle $(\theta_x)_n$ around the axis x and the rotation angle $(\theta_y)_n$ around the axis y. (Eqs. (2.56) - (2.65) [38-48]):

Figure 2.20 illustrates triangle and rectangular elements.

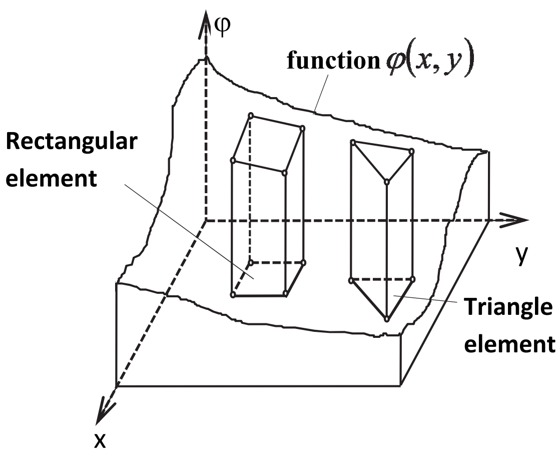


Figure 2.20. Triangle and rectangular elements in the FEM [20].

For each element we have twelve parameters, so for w we have a polynomial as follows:

$$\begin{aligned} w = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 xy + \alpha_6 y^2 + \alpha_7 x^3 + \\ + \alpha_8 x^2 y + \alpha_9 xy^2 + \alpha_{10} y^3 + \alpha_{11} x^3 y + \alpha_{12} xy^3 \end{aligned} . \quad (2.56)$$

Constants $\alpha_1, \alpha_2, ..., \alpha_{12}$ are defined from the system of equations.

In the matrix we can write that as follows:

$$\{\delta\}^e = [C]\{\alpha\}. \tag{2.58}$$

where

$$\{\alpha\} = [C]^{-1}\{\delta\}^e. \tag{2.59}$$

The deflections inside the element are defined by

$$\{w\} = [N]\{\delta\}^e = [P][C]^{-1}\{\delta\}^e. \tag{2.60}$$

The matrix $[P]$ is the function of coordinates *x* and *y*. If we place the center of the coordinate system in the center of the gravity of the rectangular element, Eq. (2.60) can be defined by

$$\{w\} = [N_i, N_j, N_l, N_k]\{\delta\}^e. \tag{2.61}$$

We determine the matrix of deformation by the second derivative of the deflection and the stresses will be defined by using the bending and torque moments. Taking into consideration Eqs. (2.56), (2.57) and (2.58) we have:

$$\{\varepsilon\} = [\Omega]\{\alpha\} = [\Omega][C]^{-1}\{\delta\}^e = [B]\{\delta\}^e. \tag{2.62}$$

The matrix of elasticity is a part of the equation of tension:

$$\{\sigma\} = [D][\varepsilon]. \tag{2.63}$$

The matrix of stiffness is defined by

$$[k]^e = \iint [B]^T [D][B] dxdy. \tag{2.64}$$

Taking into consideration that the matrix $[C]$ does not depend on x and y, we obtain:

$$[k]^e = \{[C]^{-1}\}^T [C]^{-1} \iint [\Omega]^T [D][\Omega] dxdy. \tag{2.65}$$

The deviation for the shearing forces is up to 14 % from the real solution, for the vertical reactions up to 15 % from the real solution. For the deflection, the deviation by the FEM method is around 7 % and for the bending moment up to 6,5 %.

Figure 2.21 shows the deviation between the FEM method [49-60] and the real solution for a distributed load.

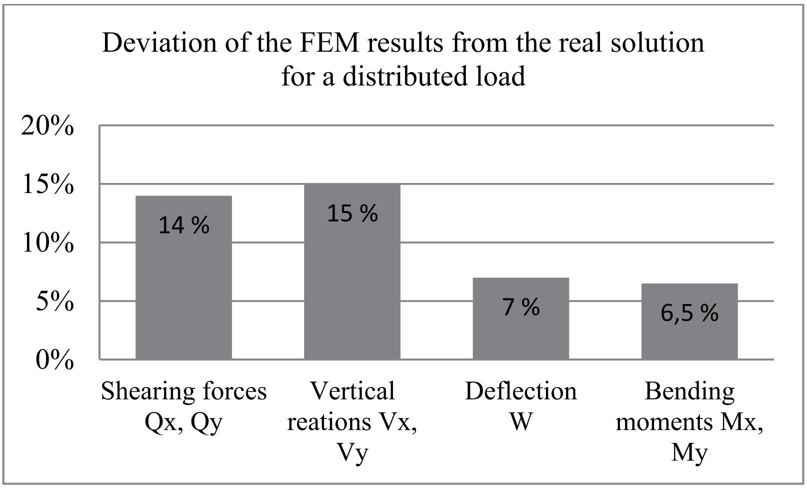


Figure 2.21. Deviation between the FEM method and the real solution for a distributed load.

As demonstrated by the figure, the deviation from the real solution is quite large and the deviation is also great.

Figure 2.22 illustrates the deviation between the FEM method and the real solution for the hydrostatic pressure.

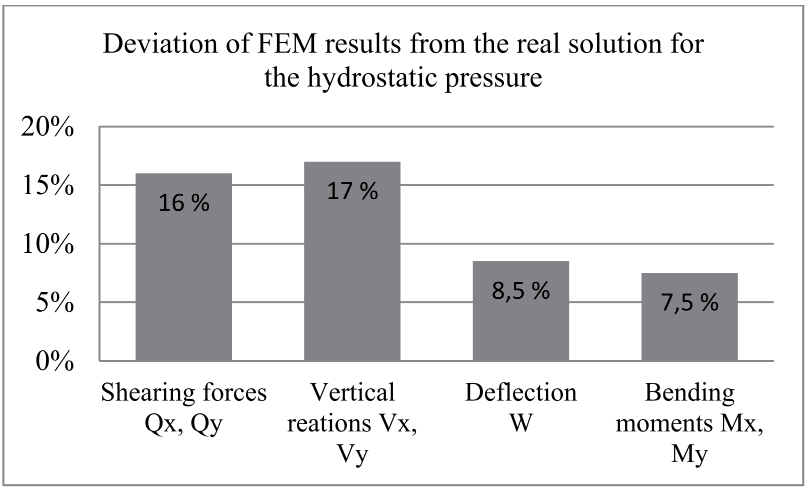


Figure 2.22. Deviation between the FEM method and the real solution for the hydrostatic pressure.

As shown by the figure, the deviation is larger for the results of the hydrostatic pressure than for the distributed load.

The advantage [61-65] of the FEM method is:

- the possibility to include any shape into the FE.

The disadvantages of the FEM method are:

- results depend on the choice of the mesh by the operator,
- evaluation of the accuracy of the results is complicated.

In conclusion, the FEM method has a large deviation from the real solution for the deflection *W*, moments *M*, shearing forces *Q* and vertical reactions *V*. But keeping in mind that others method are not suitable for plate calculation FEM method can be used for plate calculation in comparison with other methods. FEM method will be used as a method with which we compare our results.

2.8. Analysis of methods for plate calculation

In order to determine which of the methods described above suits best for plate calculation and is closer to the real solution, comparison of the deviations of all the methods is essential. The data of all deviations for plates loaded by a distributed load are shown in Table 2.1. Resulting from the table, the most suitable method for plate calculation for a ship hull structure can be determined.

Table 2.1. Deviations of all methods from the real solution for a distributed load.

Parameters	Method					
	Navier'	Levy	Collocation	K-Vlassov	Grid	FEM
Shearing forces *Qx, Qy*	18 %	17 %	11 %	14 %	22 %	14 %
Vertical reations *Vx, Vy*	15 %	15 %	11 %	15 %	20 %	15 %
Deflection *W*	0 %	0,5 %	0 %	0 %	0,7 %	7 %
Bending moments *Mx, My*	2 %	2 %	2 %	0,5 %	4,6 %	6,5 %

For better visualization and understanding, all the results are shown in Figure 2.23 for plates loaded by a distributed load.

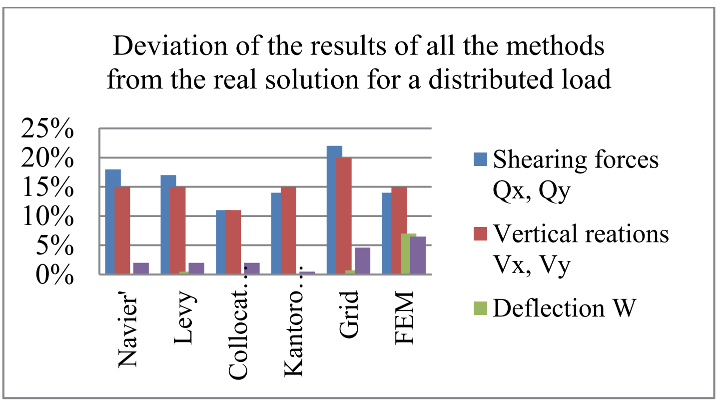


Figure 2.23. Deviation of all the methods and the real solution for a distributed load.

Figure 2.23 demonstrates that the Navier', the collocation and the Kantorovits-Vlassov method have the smallest deviation from the real solution. The deviation for the deflection *W* is zero, which means that the solution exact in terms of the real solution for calculations with certain boundary conditions. Next in ranking are the Levy and the grid method with deviations for the deflection *W* 0,5 % and 0,7 %, accordingly. The FEM method appears to have the largest deviation, accounting for 7 % [66-71]. In summary, all the methods can be used for plate calculation taking into account the boundary conditions and deviations for bending moments, shearing forces and vertical reactions, if necessary. If we take all the parameters into consideration, we can conclude that the most suitable method for plate calculation is the FEM method, the deviations of which are not the smallest in terms of all the parameters, such as the deflection *W*, the bending moments *M*, the shearing forces *Q* and the vertical reactions *V*. However, taking into consideration the boundary conditions, it is most suitable for hull calculations.

Table 2.2 presents the results of deviations for the plates loaded by the hydrostatic pressure.

Table 2.2. Deviations of all the methods and the real solution for the hydrostatic pressure.

Parameters	Method					
	Navier'	Levy	Collocation	K-Vlassov	Grid	FEM
Shearing forces *Qx, Qy*	19 %	18,5 %	12 %	15 %	24 %	16 %
Vertical reations *Vx, Vy*	16,3 %	17 %	12 %	16,5 %	21 %	17 %
Deflection *W*	0.8 %	1,8 %	0,5 %	0,5 %	1,3 %	8,5 %
Bending moments *Mx, My*	2,9 %	3,1 %	2,6 %	1,2 %	5,5 %	7,5 %

For better visualization and understanding, all the results are illustrated in Figure 2.24 for the plates loaded by the hydrostatic pressure.

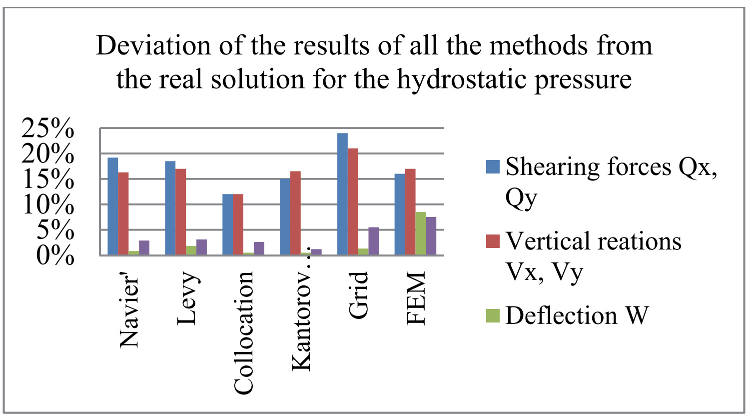


Figure 2.24. Deviation of all the methods and the real solution for a distributed load.

Figure 2.24 demonstrates that the collocation, the Kantorovits-Vlassov and the Navier' method have the smallest deviation from the real solution. The deviation for the deflection *W* is variable from 0,5 % to 0,8 %, which is a very small deviation from the real solution. Next in ranking are the grid and the Levy method with deviations for the deflection *W* 1,3 % and 1,8 %, respectively. The FEM method has the largest deviation, accounting for a deflection of 8,5 %. In summary, all the methods except FEM cannot be used for plate calculation loaded by the hydrostatic pressure taking account of the boundary conditions. If we take all the parameters into consideration, we can conclude that the most suitable method for plate calculation loaded by the hydrostatic pressure is the FEM method, the deviations of which are not the smallest on most of the parameters, such as the deflection *W*, the shearing forces *Q* and the vertical reactions *V*. However, taking into consideration that the FEM method does not depend on the boundary conditions, it suits best for comparison of our results.

Conclusion

Many different methods exist for plate calculations. Most common of them were described above in the thesis. Most of them depend on the boundary conditions and cannot be used for the calculation of plates with variable thicknesses. In order to choose the best method the boundary conditions and deviations from the real solution must be taken into consideration.

Resulting from the analysis, we can conclude that the Navier' method cannot be used for the hull calculation because the Navier 'method is valid only if the plate is simply supported with all four sides. However, in the ship building it is impossible to find a plate simply supported, which makes the Navier' method unsuitable for the hull calculation. Another disadvantage of the Navier' method is that the plates must have very simple shape. It is impossible to calculate more complicated plate shapes by the Navier' method. Neither can this method be used for the calculation of plates with variable thicknesses.

The Levi method can theoretically be used for the hull calculations because its boundary conditions match ship calculations. The Levi method and the Navier method have the same disadvantage that lies in the plate shape, which has to be very simple. In other cases this method cannot be used.

The Kantorovits-Vlassov and the collocation method cannot be used in the hull calculations because of their boundary conditions. Those methods suit when the plate is four sides fixed. They are unsuitable for our calculations.

The grid method is suitable for the hull calculations due to its boundary conditions. But in terms of its accuracy and deviation from the real solution for the shearing forces *Q* and the vertical reactions *V*, the grid method is unsuitable and cannot provide good results.

The FEM method does not depend on the boundary conditions and thus can be used for the hull calculation. Also, the FEM can be used for plates with variable thicknesses. The deviation is neither small nor large. The disadvantage of the FEM method is that it depends on the operator. Thus, the probability of wrong results is also significant. Also, the more frequent mesh takes too much time for the calculations if the shape of the plate is complicated. For that reason the FEM method is not appropriate for hull calculations. However, but if the shape of the plate is simple, we can use this method as an alternative for the hull calculation.

In summary, of all the methods, the FEM is most suitable for the plate calculation loaded by a distributed load and the hydrostatic pressure. In conclusion, none of the methods is completely (100%) suitable for use in the hull calculations and will not enable most accurate results to be achieved. All the methods have their own limitations, which makes them unsuitable for the hull calculations. Thus, a new method has to be developed that would be independent of the boundary conditions, the shape of the plate and would provide good calculation results.

3. NEW METHOD OF CALCULATION OF PLATES

To increase stiffness and ensure a more optimal distribution of the stress, the plates can have variable thicknesses. Those constructions are widely used in shipbuilding for most of the hull elements of the ships. In plate calculations by variational methods it is necessary to determine the basic set functions for unknown variables which satisfy the boundary conditions on the edge of the construction. The task is to develop the theory and methods of plates with variable thicknesses. For that purpose calculation algorithms are required, which can expand the area of plate calculation tasks and improve existing solutions. The considerations above define the topicality of the thesis of the present work and its goals.

In particular, the calculations are complex if the elements consist of variable thicknesses. Composite structures include beams, plates with stepwise changing stiffness as well as a shells consisting of elements of various shapes. Composite structures are typically calculated by their decomposition into individual elements, within each of which the stiffness and geometric characteristics change monotonously. For each of the elements obtained, a solution must be known in advance. To ensure neighboring conjugation sites on the displacements and the internal forces, a system of algebraic equations must be set up with unknowns, where $N \cdot n$ is the order of the differential equation, and N is the number of elements.

However, if we use some properties of generalized functions, we need to set up a system of algebraic equations containing only n unknowns.

G.Aryassov and coauthors [64-70] proposed a method based on the properties of generalized functions for shell plates and beams with variable thicknesses. The method provides increased accuracy and decreased time for building the algorithm.

Through a generalized function, the above method of calculation enables comparison with other methods used for the same purpose of plate calculations.

In this thesis the method of generalized function is modified and generalized for the calculation of rectangular plates with under the hydrostatic pressure.

3.1. Calculation of plates with variable thicknesses by the generalized functions

Taking into account the method of additional partial solutions for the properties of generalized functions, in this thesis the author develops a method of rectangular plates with variable stiffness using generalized functions. This can be defined as a private case of a method of generalized solutions [64].

As was described above, to calculate constructions with variable thicknesses need a system of algebraic equations is required that consists of $N \cdot n$ unknown, where n is the degree of differential equation and N is the quantity of elements.

But taking into account some properties of generalized functions, to solve this task, it is necessary to develop a system of algebraic equations only with n unknown.

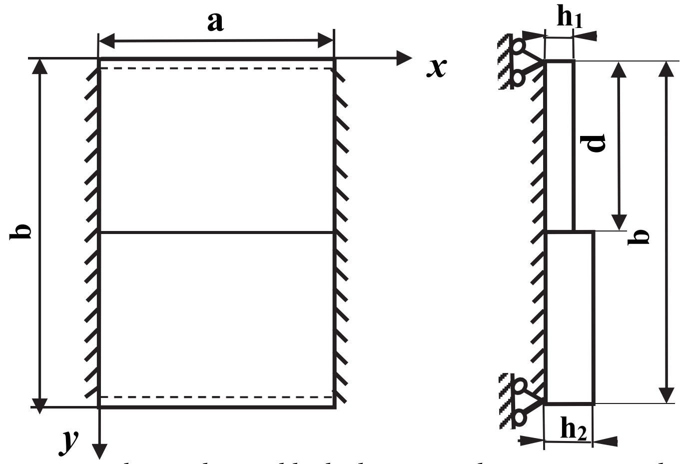


Figure 3.1. Rectangular plate with variable thicknesses with two opposite edges freely supported and two edges built in.

Figure 3.1 shows a rectangular plate with variable thicknesses at two opposite edges freely supported $y = 0, y = b$ and two edges built in $x = 0,\ x = a$. This corresponds to the boundary conditions of our experiment.

We note that in calculations of barges the boundary conditions of a barge hull element supported by stiffeners are following: at two opposite edges simply supported and two edges built in.

However, in general the method of solution does not depend on the boundary conditions.

The differential equation for the deflection surface $w(x, y)$ of $i-$section the plate (Fig. 3.1) is described by the biharmonic equation [20]:

$$\nabla^2\nabla^2 w(x, y) = \frac{\partial^4 w(x, y)}{\partial x^4} + 2\frac{\partial^4 w(x, y)}{\partial x^2 \partial y^2} + \frac{\partial^4 w(x, y)}{\partial y^4} = \frac{q(x, y)}{D_i}, \tag{3.1}$$

$(i = 1,2,...N)$,

where D_i - is the flexural rigidity of the plate.

The solution is in the form

$$w(x, y) = Y(y)X(x), \tag{3.2}$$

where

$Y(y)$ and $X(x)$ are functions of y and x respectively.

Taking into consideration that stiffness is changing uneven only in one direction along the y axis, the function $X(x)$ can be defined as the polynomial below:

$$X(x) = (1 - x^2/a^2)^2\,, \tag{3.3}$$

and the deflection surface $w(x, y)$ will be

$$w(x, y) = Y(y)(1 - x^2/a^2)^2\,. \tag{3.4}$$

Equation (3.2) satisfies the boundary conditions $w(x, y) = 0$ and

$\dfrac{\partial w(x, y)}{\partial x} = 0$ at these two sides $x = 0,\ x = a$.

Polynomial (3.3) can be accepted as more generalized [D. Gornostajev publications 1-5], which means that the view of the polynomial does not affect the algorithm of the solution which is calculated by generalized functions.

Secondly, it is very important in our case that the polynomial $X(x)$, which defines the deflection of the plate along the short side of the plate a (Fig. 3.1), does not affect the accuracy of the solution because of the relation of the plate sides $b/a \geq 3$ [20, 63].

The task is to determine $Y(y)$ in such a form that would satisfy the boundary conditions on the sides $y = 0, y = b$ and also the equation of the deflection surface.

For this we use the principle of virtual displacements as follows:

$$\begin{aligned}&\int_0^a (\nabla^2\nabla^2 w(x, y) - \frac{q}{D_i})X(x)dx = \\ &= \int_0^a (\nabla^2\nabla^2 w(x, y) - \frac{q}{D_i})(1 - x^2/a^2)^2\,dx = 0\end{aligned}\ . \tag{3.5}$$

Let us place $w(x, y)$ from Eq. (3.4) to Eq. (3.5), we obtain

$$\int_0^a \begin{pmatrix} \dfrac{24}{a^4}Y(y) + \dfrac{8}{a^2}(-1 + 3\dfrac{x^2}{a^2})Y''(y) + \\ + (1 - \dfrac{x^2}{a^2})^2 Y^{IV}(y) - \dfrac{q}{D_i} \end{pmatrix} (1 - \frac{x^2}{a^2})^2\,dx = 0\,. \tag{3.6}$$

After integration of Eq. (3.6) we have

$$\frac{8}{21}Y^{IV}(y)-\frac{16}{7a^2}Y''(y)+\frac{24}{a^4}Y(y)=\frac{q}{D_i}, \tag{3.7}$$
$(i=1,2,...N)$.

We assume that the structure consists of two sections only (Fig. 3.1). Then, instead of Eq. (3.7), we consider the following auxiliary differential equation:

$$\frac{8}{21}Y^{IV}(y)-\frac{16}{7a^2}Y''(y)+\frac{24}{a^4}Y(y)=\frac{q}{D_i}+B_0(d)\cdot\delta(y-d)+$$
$$+B_1(d)\cdot\delta'(y-d)+B_2(d)\cdot\delta''(y-d)+B_3(d)\cdot\delta'''(y-d),\quad (i=1,2) \quad , (3.8)$$

where $\delta(y-d)$ - is delta-function,

$B_0(d)$, $B_1(d)$, $B_2(d)$ and $B_3(d)$ - are unknown coefficients,

d - is the coordinate of the point at which stiffening behavior changes abruptly or design changes rigidity.

We will seek a special solution of the differential equation (3.8) in this form:

$$\tilde{Y}_r(y)=\{[1-\eta(y-d)][Z_0(y)+Z_q(y)]+\eta(y-d)V_q(y)\}, \tag{3.9}$$

where $\eta(y-d)$ - is the unit function,

$Z_0(y)$ - is the general solution of the corresponding homogeneous differential equation,

$Z_q(y)$, $V_q(y)$ - are partial solutions for the first and second sections corresponding to the action on the structure of the normal pressure q.

Consistently differentiating the expression (3.9) four times and taking into account the filtering property of the delta function, we obtain

$$\tilde{Y}_r'(y)=[1-\eta(y-d)][Z_0'(y)+Z_q'(y)-V_q'(y)]-\delta(y-d)[Z_0(y)+Z_q(y)-V_q(y)]$$

$$\tilde{Y}_r''(y)=[1-\eta(y-d)][Z_0''(y)+Z_q''(y)-V_q''(y)]-$$
$$-2\delta(y-d)[Z_0'(y)+Z_q'(y)-V_q'(y)]-\delta'(y-d)[Z_0(y)+Z_q(y)-V_q(y)],$$

$$\tilde{Y}_r'''(y)=[1-\eta(y-d)][Z_0'''(y)+Z_q'''(y)-V_q'''(y)]-3\delta(y-d)[Z_0''(y)y+Z_q''(y)-V_q''(y)]-$$
$$-3\delta'(y-d)[Z_0'(y)+Z_q'(y)-V_q'(y)]-\delta''(y-d)[Z_0(y)+Z_q(y)-V_q(y)],$$

$$\widetilde{Y}_r^{IY}(y)=[1-\eta(y-d)][Z_0^{IY}(y)y+Z_q^{IY}(y)-V_q^{IY}(x)]-$$
$$-4\delta(y-d)[Z_0'''(y)+Z_q'''(y)-V_q''(y)]-6\delta'(y-d)\left[Z_0''(y)+Z_q''(y)-V_q''(y)\right]-, \quad (3.10)$$
$$-4\delta''(y-d)[Z_0'(y)+Z_q'(y)-V_q'(y)]-\delta'''(y-d)[Z_0(y)+Z_q(y)-V_q(y)]$$

Substituting the values $\widetilde{Y}_r^{IY}(y)$, $\widetilde{Y}_r''(y)$ and $\widetilde{Y}_r(y)$ in Eq. (3.8) and comparing the coefficients of the delta-function and its derivatives on the left and right sides of Eq. (3.8), we will have

$$B_0(d)=\left\{B_3'''(y)-\frac{12}{a^2}B_3'(y)\right\}\Big|_{y=d};\; B_1(d)=\left\{4B_3''(y)-\frac{6}{a^2}B_3(y)\right\}\Big|_{y=d};$$
$$B_2(d)=4B_3'(y)\big|_{y=d};\; B_3(d)=\frac{8}{21}[V_q(y)-Z_0(y)-Z_q(y)]_{x=d}. \quad (3.11)$$

Taking into account that Eq. (3.8) is the differential equation of the fourth degree, the unknown coefficients $B_0(d)$, $B_1(d), B_2(d)$ and $B_3(d)$ for this equation must satisfy the relations, where four integration constants C_j are included

$$B_0(d)=\left\{B_3'''(y)-\frac{12}{a^2}B_3'(y)\right\}\Big|_{y=d};\; B_1(d)=\left\{4B_3''(y)-\frac{6}{a^2}B_3(y)\right\}\Big|_{y=d};$$
$$B_2(d)=4B_3'(y)\big|_{y=d};\; B_3(d)=\sum_{j=1}^{4}\frac{8}{21}C_j[V_q(y)-Z_{0j}(y)-Z_q(y)]_{y=d}, \quad (3.12)$$

where C_j - integration constants determined from the boundary conditions, and $Z_{0j}(y)$ - particular linearly independent solutions of the homogeneous differential equation

$$\frac{8}{21}Y^{IV}(y)-\frac{16}{7a^2}Y''(y)+\frac{24}{a^4}Y(y)=\frac{q}{D_i}. \quad (3.13)$$

From the homogeneous differential equation (3.13), we have the particular solutions $Z_{0j}(y)$.

$$Z_{01}(y)=ch\frac{2{,}343y}{a}\cos\frac{1{,}575y}{a};\; Z_{02}(y)=ch\frac{2{,}343y}{a}\sin\frac{1{,}575y}{a};$$

$$Z_{03}(y)=sh\frac{2{,}343y}{a}\cos\frac{1{,}575y}{a};\; Z_{04}(y)=sh\frac{2{,}343y}{a}\sin\frac{1{,}575y}{a}. \quad (3.14)$$

Then the function

$$\overline{Y}(y) = \sum_{j=1}^{4} C_j \{[1-\eta(y-d)]Z_{0j}(x)\} + \\ + \{[1-\eta(y-d)]Z_q(x) + \eta(y-d)V_q(x)\}, \tag{3.15}$$

is the general solution of the differential equation (3.8).

It follows that the general solution of the initial differential equation (3.7) in the case of two sections is determined by the formula

$$Y(y) = \overline{Y}(y) - \widetilde{Y}_r(y), \tag{3.16}$$

where $\widetilde{Y}_r(y)$ is the special particular solution of the non-homogeneous differential equation (3.8) on the condition that the coefficients $B_0(d)$, $B_1(d)$, $B_2(d)$ and $B_3(d)$ satisfy the relations (3.12).

As a result of all the calculations we attain the main equation for the deflection by multiplying Eqs. (3.4) and (3.16):

$$w(x,y) = \left(\overline{Y}(y) - \widetilde{Y}_r(y)\right)(1 - x^2/a^2)^2 \tag{3.16*}$$

Using Eq. (3.16*) we obtain the numerical results of the deflection of the plate.

3.1.1. Particular solution $\widetilde{Y}_r(y)$ in the case of a distributed load

A particular solution $\widetilde{Y}_r(y)$ can be found by the method of variation of arbitrary constants.

In this case we have

$$\widetilde{Y}_r(y) = \frac{\eta(x-d)}{2}\Bigg\{-Z_{01}(y)\left[\frac{a^3 \cdot B_0(d)}{4\sqrt{108}} Z_{02}(y) + \frac{a^2 \cdot B_1(d)}{12} Z_{03}(y) - \right.$$

$$\left. - \frac{a \cdot B_2(d)}{\sqrt{6}} Z_{03}(y) - B_3(d) \cdot Z_{01}(y)\right] +$$

$$+ Z_{02}(y)\left[\frac{a^3 \cdot B_0(d)}{4\sqrt{108}} Z_{03}(y) + \frac{a^2 \cdot B_1(d)}{12} Z_{01}(y) + \right.$$

$$\left. + \frac{a \cdot B_2(d)}{\sqrt{6}} Z_{04}(y) + B_3(d) \cdot Z_{02}(y)\right] -$$

$$-Z_{02}(y)\left[\frac{a^3B_0(d)}{4\sqrt{108}}Z_{04}(y)-\frac{a^2\cdot B_1(d)}{12}Z_{01}(y)+\right]\Bigg\}$$

$$+\frac{a\cdot B_2(d)}{\sqrt{6}}Z_{03}(y)+B_3(d)\cdot Z_{02}(y)\Bigg]\Bigg\}, \qquad (3.17)$$

where

$Z_{01}(y), Z_{02}(y), Z_{03}(y)$ and $Z_{04}(y)$ are the particular solutions of the homogeneous differential equation (3.13), which are determined by the expressions (3.14)

and the coefficients $B_0(d)$, $B_1(d), B_2(d)$ and $B_3(d)$,

$$B_0(d)=\frac{2\sqrt{108}}{a^3}\left[C_2Z_{03}(d)-C_1Z_{04}(d)+C_3Z_{03}(d)+C_4Z_{04}(d)\right],$$

$$B_1(d)=\frac{24}{a^2}\left[C_2Z_{01}(d)-C_1Z_{02}(d)+C_3Z_{02}(d)+C_4Z_{01}(d)\right],$$

$$B_2(d)=\frac{\sqrt{6}}{a}\left[C_1Z_{03}(d)+C_2Z_{04}(d)+C_3Z_{04}(d)-C_4Z_{03}(d)\right],$$

$$B_3(d)=C_1\left[Z_{01}(d)-Z_{02}(d)\right]+C_2\left[Z_{02}(d)-Z_{03}(d)\right]+$$

$$+C_3\left[Z_{03}(d)-Z_{04}(d)\right]+C_4\left[Z_{04}(d)-Z_{01}(d)\right]+\left(\frac{a^4}{36D_2}+\frac{a^4}{36D_1}\right)\frac{q}{4}. \qquad (3.18)$$

Thus, the general solution of the non-homogeneous differential equation (3.1) can be found by the following formula, taking into account Eqs. (3.15), (3.16) and (3.17)

$$Y(y)=C_1[1-\eta(y-d)]Z_{01}(y)+C_2[1-\eta(y-d)]Z_{02}(y)+$$

$$+C_3[1-\eta(y-d)]Z_{03}(y)+C_4[1-\eta(y-d)]Z_{04}(y)+$$

$$+\left\{[1-\eta(y-d)]\frac{a^4}{36D_1}+\eta(y-d)\frac{a^4}{36D_2}\right\}\frac{q}{4}-\tilde{y}_r(x). \qquad (3.19)$$

If the construction contains several sections with different stiffness characteristics, the general solution can be written as follows:

$$Y(y)=\sum_{l=1}^{\alpha} C_1\{[\eta(y-d_{l-1})-\eta(y-d_l)]Z_{01}(y)+$$
$$+C_2[\eta(y-d_{l-1})-\eta(y-d_l)]Z_{02}(y)+$$
$$+C_3[\eta(y-d_{l-1})-\eta(y-d_l)]Z_{03}(y)+ \qquad , \qquad (3.20)$$
$$+C_4[\eta(y-d_{l-1})-\eta(y-d_l)]+[\eta(y-d_{l-1})-\eta(y-d_l)]\frac{a^4 q}{36 D_l}\Big\}-$$
$$-\widetilde{Y}_r(y)$$

where $\eta(y-d_0)=1$, $\eta(y-d_{l-1})=0$. (3.20*)

d_{l-1} $(l=1,2,...,\alpha)$ are the coordinates of the points in which the stiffness characteristics of the construction are changed.

The particular solution $\widetilde{Y}_r(y)$, which is included in Eq. (3.20), takes the form

$$\widetilde{Y}_r(y)=\sum_{l}^{\alpha-1}\widetilde{Y}_{rl}(y), \qquad (3.21)$$

where $\widetilde{Y}_{rl}(y)$ is the partial solution, which can be calculated by Eq. (3.17) with the replacement d for d_l.

Equation (3.20) shows that the unknown included in this formula are arbitrary constants C_1, C_2, C_3 and C_4 which can be calculated from the boundary conditions of the problem.

3.1.2. Particular solution $\widetilde{Y}_r(y)$ in the case of hydrostatic pressure

Similarly, we can find the particular solution $\widetilde{Y}_r(y)$ for the hydrostatic pressure.

In this case the differential equation (3.8) will be

$$\frac{8}{21}Y^{IV}(y)-\frac{16}{7a^2}Y''(y)+\frac{24}{a^4}Y(y)=$$
$$\frac{q_0\cdot y}{D_i\cdot b}+B_0(d)\cdot\delta(y-d)+B_1(d)\cdot\delta'(y-d)+ \qquad . \qquad (3.22)$$
$$B_2(d)\cdot\delta''(y-d)+B_3(d)\cdot\delta'''(y-d), \quad (i=1,2)$$

Similarly to the case of the uniformly distributed load, the particular solution $\widetilde{Y}_r(y)$ for the hydrostatic pressure can be found by the method of variation of arbitrary constants.

In this case we have

$$\widetilde{Y}_r(y)=\frac{\eta(y-d)}{2}\left\{-Z_{01}(y)\left[\frac{a^3\cdot B_0(d)}{4\sqrt{108}}Z_{02}(y)+\frac{a^2\cdot B_1(d)}{12}Z_{03}(y)-\right.\right.$$

$$\left.-\frac{a\cdot B_2(d)}{\sqrt{6}}Z_{03}(y)-y\cdot B_3(d)\cdot Z_{01}(y)\right]+$$

$$+Z_{02}(y)\left[\frac{a^3\cdot B_0(d)}{4\sqrt{108}}Z_{03}(y)+\frac{a^2\cdot B_1(d)}{12}Z_{01}(y)+\right.$$

$$\left.+\frac{a\cdot B_2(d)}{\sqrt{6}}Z_{04}(y)+y\cdot B_3(d)\cdot Z_{02}(y)\right]-$$

$$-Z_{02}(y)\left[\frac{a^3B_0(d)}{4\sqrt{108}}Z_{04}(y)-\frac{a^2\cdot B_1(d)}{12}Z_{01}(y)+\right]\Bigg\}$$

$$\left.\left.+\frac{a\cdot B_2(d)}{\sqrt{6}}Z_{03}(y)+y\cdot B_3(d)\cdot Z_{02}(y)\right]\right\} \quad . \qquad (3.23)$$

where

$Z_{01}(y), Z_{02}(y), Z_{03}(y)$ and $Z_{04}(y)$ are the particular solutions of the homogeneous differential equation (3.13), which are determined by Eq. (3.14)

and the coefficients $B_0(d)$, $B_1(d), B_2(d)$ and $B_3(d)$,

$$B_0(d)=\frac{2\sqrt{108}}{a^3}\left[C_2Z_{03}(d)-C_1Z_{04}(d)+C_3Z_{03}(d)+C_4Z_{04}(d)\right],$$

$$B_1(d)=\frac{24}{a^2}\left[C_2Z_{01}(d)-C_1Z_{02}(d)+C_3Z_{02}(d)+C_4Z_{01}(d)\right],$$

$$B_2(d)=\frac{\sqrt{6}}{a}\left[C_1Z_{03}(d)+C_2Z_{04}(d)+C_3Z_{04}(d)-C_4Z_{03}(d)\right], \qquad (3.24)$$

$$B_3(d)=C_1\left[Z_{01}(d)-Z_{02}(d)\right]+C_2\left[Z_{02}(d)-Z_{03}(d)\right]+$$

$$+C_3[Z_{03}(d)-Z_{04}(d)]+C_4[Z_{04}(d)-Z_{01}(d)]$$

$$+\left(\frac{a^4}{72D_1}\cdot\frac{d}{b}+\frac{a^4}{48D_2}+\right)\frac{q_0}{4}$$

Thus, the general solution of the non-homogeneous differential equation (3.1) for the hydrostatic pressure, taking into account Eqs. (3.15), (3.16) and (3.23), can be found by the next formula:

$$Y(y)=C_1[1-\eta(y-d)]Z_{01}(y)+C_2[1-\eta(y-d)]Z_{02}(y)+$$

$$+C_3[1-\eta(y-d)]Z_{03}(y)+C_4[1-\eta(y-d)]Z_{04}(y)+$$

$$+\left\{[1-\eta(y-d)]\frac{a^4}{72D_1}\cdot\frac{d}{b}+\eta(y-d)\frac{a^4}{48D_2}\right\}\frac{q_0}{4}-\tilde{Y}_r(y). \qquad (3.25)$$

The obtained results can be generalized to any type of a composite construction of beams and plates. The number of arbitrary constants of integration will be equal to the order of the differential equations corresponding to the considered type of constructions. These constants are determined from the appropriate boundary conditions of the problem.

3.2. Results of calculations

In order to solve the equations and obtain the numerical results of deflection, it is a program is needed. In our case, the MATLAB program will be used. The results will be presented by the curve of deflection for all our cases: for the plate with constant thickness loaded by a distributed load and by the hydrostatic pressure and for the plate with variable thicknesses loaded by a distributed load and by the hydrostatic pressure. The load will be 6 kPa, 12 kPa, 18 kPa and 24 kPa for the distributed load. The same load will be used for calculations under the hydrostatic pressure. The first calculations will be made for a plate with a constant thickness of 8 mm with the dimensions of 180 mm in width and 400 mm in length loaded by a distributed load of 6 kPa. Steel grade NVA with yield stress 235 N/mm^2 will be used. After obtaining all the results, we will compare these with the FEM results to find the deviation. The comparison is needed to understand how accurate the results of the new method are. Thus, the correctness of the new theory can be verified and further use of the new method for calculations can be confirmed. Figure 3.2 shows the curve of deflection for the plate with a constant thickness of 8 mm loaded by a distributed load of 24 kPa.

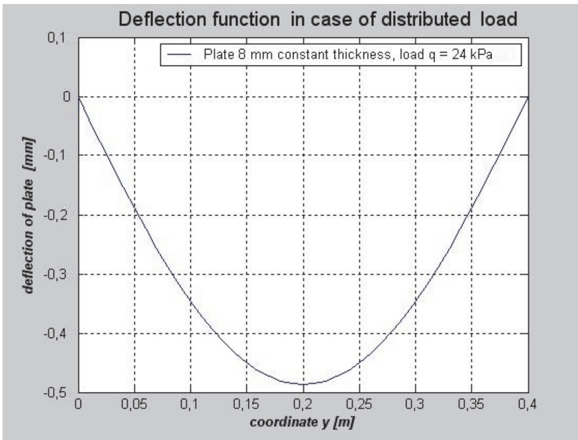


Figure 3.2. Curve of deflection for the plate with a constant thickness of 8 mm loaded by a distributed load of 24 kPa.

Figure 3.2 demonstrates that the maximum deflection is in the middle area, accounting for 0.49 mm. That result will be compared with the FEM results. In the same way we can obtain deflections for other loads. All the deflections for the plate with constant thickness under the distributed load are shown in Table 3.1.

Table 3.1. Deflection for the 8 mm plate with constant thickness under a distributed load.

8 mm plate with constant thickness, distributed load					
Load, kPa	0	6	12	18	24
Deflection, mm	0	0,11	0,24	0,37	0,49

We obtain all the deflections for the 8 mm plate with constant thickness under the distributed load. Next, the same plate will be loaded by the hydrostatic pressure. Figure 3.3 shows the curve of deflection for the 8 mm plate with constant thickness under the hydrostatic pressure of 24 kPa.

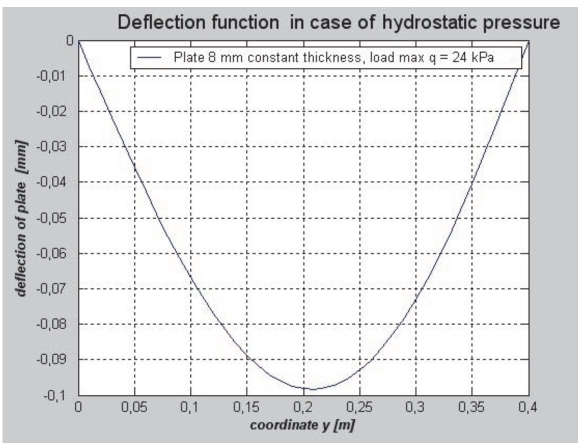


Figure 3.3. Curve of deflection for a 8 mm plate with constant thickness under the hydrostatic pressure of 24 kPa.

Figure 3.3 demonstrates that the maximum deflection is in the middle area, accounting for 0.099 mm. All the deflections for the plate with constant thickness loaded by the hydrostatic pressure are shown in Table 3.2.

Table 3.2. Deflection for the 8 mm plate with constant thickness under the hydrostatic pressure.

8 mm plate with constant thickness, hydrostatic pressure					
Load, kPa	0	6	12	18	24
Deflection, mm	0	0,022	0,049	0,072	0,099

We can obtain all the deflections for the 8 mm plate with constant thickness under the hydrostatic pressure. Next, a plate with variable thicknesses of 6 mm and 8 mm will be loaded by a distributed load. Figure 3.4 shows the curve of deflection for plates of variable thicknesses 6 mm and 8 mm under the distributed load of 24 kPa.

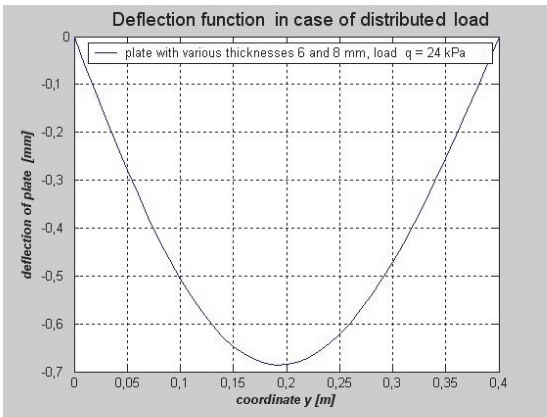


Figure 3.4. Curve of deflection for a plate with variable thicknesses of 6 mm and 8 mm under the distributed load of 24 kPa.

Figure 3.4 demonstrates that the maximum deflection is in the middle area, accounting for 0.69 mm. All the deflections for the plate with variable thicknesses under a distributed load are shown in Table 3.3.

Table 3.3. Deflection for the plate with variable thicknesses of 6 mm and 8 mm under a distributed load.

Plate of 6 mm and 8 mm variable thicknesses, distributed load					
Load, kPa	0	6	12	18	24
Deflection, mm	0	0,16	0,32	0,51	0,69

We can obtain all the deflections for the plate with variable thicknesses of 6 mm and 8 mm under a distributed load. Next, the plate with variable thicknesses of 6 mm and

8 mm will be loaded by the hydrostatic pressure. Figure 3.5 shows the curve of deflection for the plate with variable thicknesses of 6 mm and 8 mm loaded by the hydrostatic pressure 24 kPa.

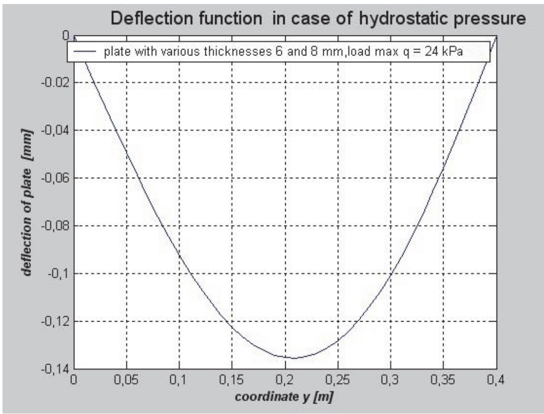


Figure 3.5. Curve of deflection for the plate with variable thicknesses of 6 mm and 8 mm loaded by the hydrostatic pressure 24 kPa.

Figure 3.5 demonstrates that the maximum deflection is in the middle area, accounting for 0.133 mm. All the deflections for the plate with constant thickness loaded by the hydrostatic pressure are shown in Table 3.3.

Table 3.4. Deflection for the plate with variable thicknesses of 6 mm and 8 mm loaded by the hydrostatic pressure.

Plate of 6 mm and 8 mm variable thicknesses, hydrostatic pressure					
Load, kPa	0	6	12	18	24
Deflection, mm	0	0,031	0,062	0,093	0,133

All the results from the tables will be used below in the comparisons with the experimental and FEM results.

Conclusion

A new method was developed in order to provide improved results of hull calculations to be used in the future. The method was developed for the calculations of a plate with variable thicknesses. Current methods except for FEM are limited and enable only calculation of plates with constant thickness. Moreover, all of them depend on the boundary conditions and are very sensitive to the shape of plate, i.e. the shape of plate has to be very simple.

The new method does not require that the construction be divided into separate homogeneous parts and support the performance of kinematic and static conditions of the matching points of neighboring parts of the construction. This method can be used under different boundary conditions. The new method does not depend on the boundary conditions like other methods do.

As a result, we obtained a general number of algebraic equations used to define the solutions that do not depend on the number of elements composing the whole construction, but rather are equal to the order of the differential equation “n”.

The proposed method enables the calculation of rectangular plates with variable thicknesses.

The method can be used for the calculation of plates with any quantity of plate parts with variable thicknesses.

The load is not limited, which means that we can use the point force and the distributed load.

The advantage of the method is that it approaches the FEM especially in places where the stress is changing significantly, for instance, where the plate thickness is changing. It means that the FEM cannot provide sufficient accuracy especially in places where the thickness of the plate is changing significantly.

The new method allows complicated mathematical modelling of the plate, including the factors that were not in use before and that cannot be done in the FEM because of an established algorithm in the FEM.

The method allows solving the dynamic tasks of plate calculations, for instance, to take into account load impact of waves, hull vibrations on the waves and other aspects.

The curve of deflection was found in order to find deflections in any point of the plate.

Common methods for the calculations of constructions enable us to transfer a task to solving a system of algebraic equations; the number of those is proportional to the number of the constructions. Moreover, those methods are used to calculate plates with constant thickness.

In order to prove the accuracy of the new method, calculations have to be compared with the FEM results.

4. ANALYSIS OF THE EXPERIMENTAL RESULTS

4.1. Experiment on the plate with constant and variable thicknesses under a distributed load

In order to check and prove that our new method of calculation is correct and can be used in the future, a comparison with experimental results is required [71-73]. For the experiment a machine shown in Figure 4.1 was chosen. The experiment was made in the certified laboratory in Elme TKS. The laboratory and equipment have all necessary certificates and the laboratory has been accredited.

Figure 4.1.Experimental setup: 1- force dynamometer, 2 – fixed table 3 – pressure shaft.

The experimental scheme for the plate with variable thicknesses is shown in Figure 4.2.

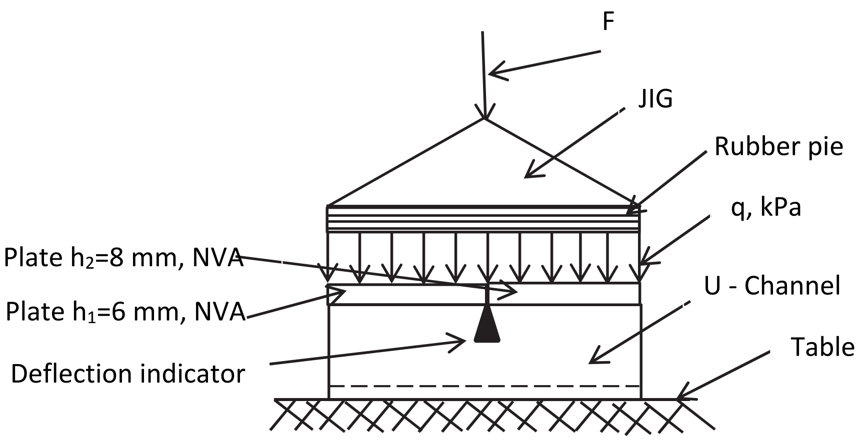


Figure 4.2. Principal scheme of the experiment.

The experiment was carried out for two types of plates: a plate with variable thicknesses of 6 and 8 mm, and a plate with constant thickness of 8 mm. Steel grade NVA with yield stress 235 N/mm^2 was used. The aim was to show the size of the deflection on the plate under a distributed load on the real plate. The main challenge was to convert a single force to a distributed load. For this reason a special jig was designed and produced. Another aim was to imitate a distributed load on the whole surface of the plate. Therefore, a rubber pie was produced and connected to the jig. Medium soft rubber was used, which in our opinion suits perfectly to transfer a load on the whole surface of the plate. It enables us to imitate a maximum distributed load in the real situation. The plate with variable thicknesses was fixed at two sides to the U-channel in order to imitate a real hull side plate. U-channel with the plate was fixed to the table to avoid any movements during the pressure work. The deflection was measured by the clock indicator ИЧ-10 [74], which has an accuracy of +/-0.01 mm.

In our experiment three attempts of each 6 kPa load were made, with a total of 12 measurements of the deflection in order to achieve more accurate results. In the first experiment a plate with a constant thickness of 8 mm was used. The dimensions of the plate were 180 mm in width and 400 mm in length. The plate was fixed at two sides and two sides were free landed. This connection imitates most realistically the connection of plates in the hull of barge or ship. The dimensions of the plate were chosen due to the limited area of the stand of the setup. Figure 4.3 shows the 8 mm plate welded to the U-channel installed on the stand with the rubber pie, the sensor and the jig.

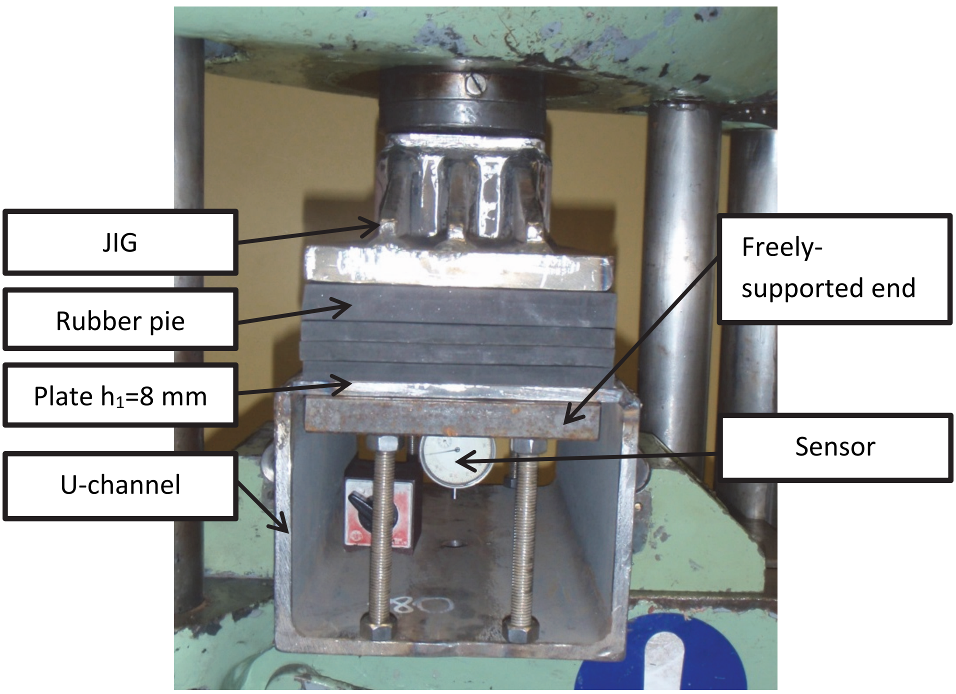


Figure 4.3. 8 mm plate welded to the U-channel installed on the stand with the rubber pie and the jig.

The results of the experiment are presented in the table. The deflections of all the 12 attempts with the plate with constant thickness are shown in Table 4.1.

Table 4.1. Deflection of the plate with constant thickness under a distributed load.

8 mm plate with constant thickness				
	Deflection, mm			
Attempt	Load 6 kPa	Load 12 kPa	Load 18 kPa	Load 24 kPa
1	0,12	0,26	0,37	0,47
2	0,14	0,27	0,37	0,48
3	0,14	0,27	0,38	0,46

Next, experiments were made for the plate with variable thicknesses with dimensions 6x180x200 mm and 8x180x200 mm welded together and welded to the U-channel. The plate with variable thicknesses welded to the U-channel installed on the stand with the rubber pie and the jig is shown in Figure 4.4.

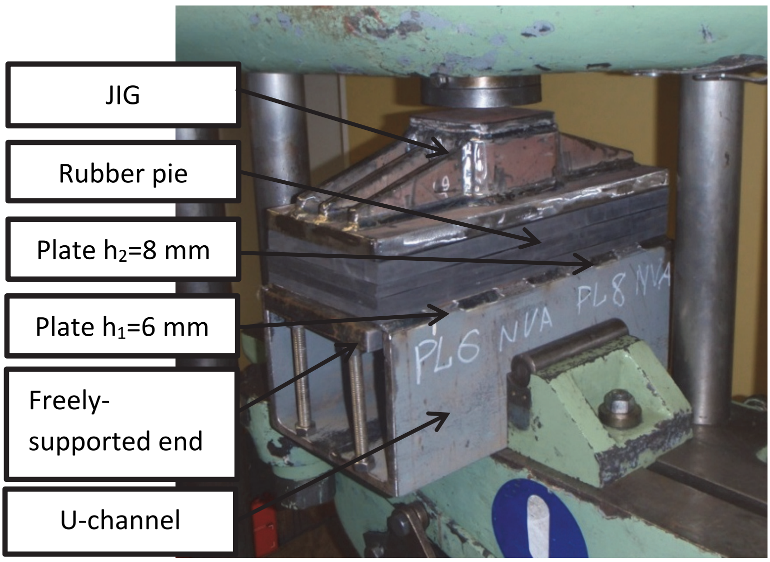


Figure 4.4. Plate with variable thicknesses of 6 mm and 8 mm welded to the U-channel installed on the table with the rubber pie, the hinge and the jig.

Similarly to the first experiment, 12 measurements were taken at the same load with step, each 6 kPa. The results of the measurements are presented in Table 4.2.

Table 4.2. Deflection of the plate with variable thicknesses under a distributed load.

6-8 mm plate with variable thicknesses				
	Deflection, mm			
Attempt	Load 6 kPa	Load 12 kPa	Load 18 kPa	Load 24 kPa
1	0,15	0,29	0,44	0,59
2	0,15	0,30	0,44	0,58
3	0,16	0,29	0,43	0,57

The results are very close to those obtained by our equations, which means that the equations developed are correct and can be used in the plate calculations. Still, the results obtained so far are not sufficient to use plates with variable thicknesses in the ships because the strength is still unknown. In order to make sure that the strength is within the allowed limits, the FEM calculations need to be done.

4.2. Results by the FEM

In order to check the accuracy of the new method and the experimental results, the FEM calculations were done by the Solid Works Cosmos program. In the calculation the same data were used as in the experiment and in the new method calculations. Two calculations for the plate with variable thicknesses of 6 and 8 mm and for the plate with a constant thickness of 8 mm were made.

First, calculations were made for the plate with a constant thickness of 8 mm with dimensions 180 mm in width and 400 mm in length under a distributed load. The stress simulation and results are shown in Figure 4.5.

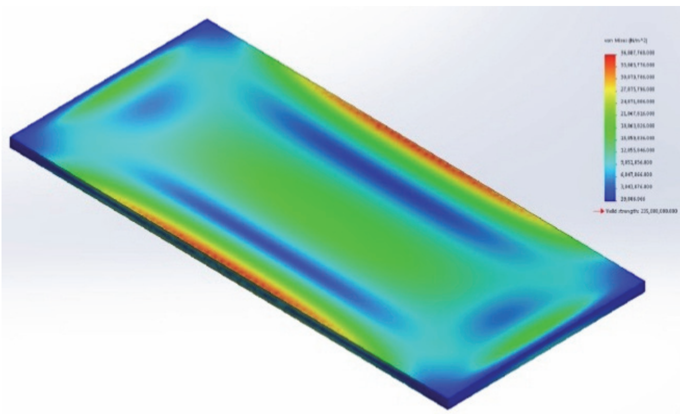


Figure 4.5. Stress simulation and the results for the plate with a constant thickness of 8 mm under the distributed load of 24 kPa.

We can see that the maximum stress is 36 N/mm^2, which is less than the yield strength for the NVA steel, i.e. 235 N/mm^2. So the allowable stress $[\sigma] = \frac{235}{1,5} = 157$ N/mm^2.Thus, the strength requirement is satisfied. The deflection for the plate with constant thickness is shown in Figure 4.6.

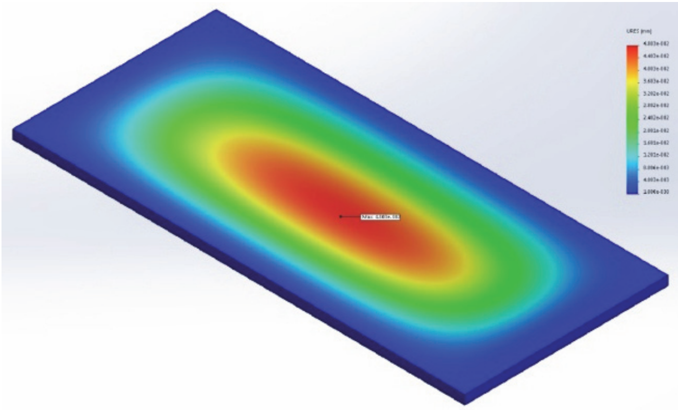


Figure 4.6. Deflection for the plate with a constant thickness of 8 mm under the distributed load of 24 kPa.

The results demonstrate that the maximum deflection is 0,47 mm, which is very close to the results of our new method and the experiments. In conclusion, the experiment proved successful for the plates with constant thickness.

Next, the same calculations were made for the plate loaded by the hydrostatic pressure. The stress simulation and the results are shown in Figure 4.7.

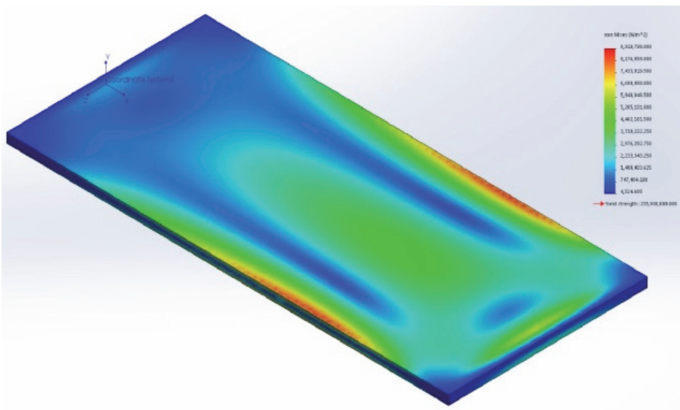


Figure 4.7. Stress simulation and the results for the plate with a constant thickness of 8 mm loaded by the hydrostatic pressure of 24 kPa.

We can see that the maximum stress is 9 N/mm^2, which is less than the yield stress for the NVA steel, i.e. 235 N/mm^2. So the yield stress $[\sigma] = \frac{235}{1,5} = 157$ N/mm^2. Thus, the strength requirement is satisfied.

The deflection for the plate of 8 mm loaded by the hydrostatic pressure is shown in Figure 4.8.

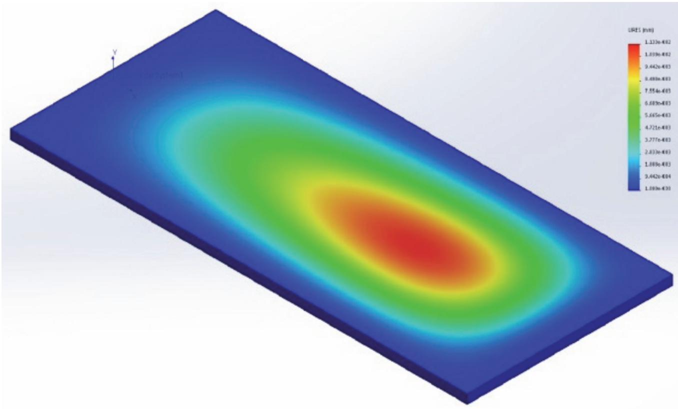


Figure 4.8. Deflection for the plate with a constant thickness of 8 mm loaded by the hydrostatic pressure of 24 kPa.

The maximum deflection is 0,09 mm. It occurs in the area where the maximum load is concentrated.

The same calculation is required for the plate with variable thicknesses of 6 mm and 8 mm, first loaded by a distributed load. The stress simulation and the results are shown in Figure 4.9.

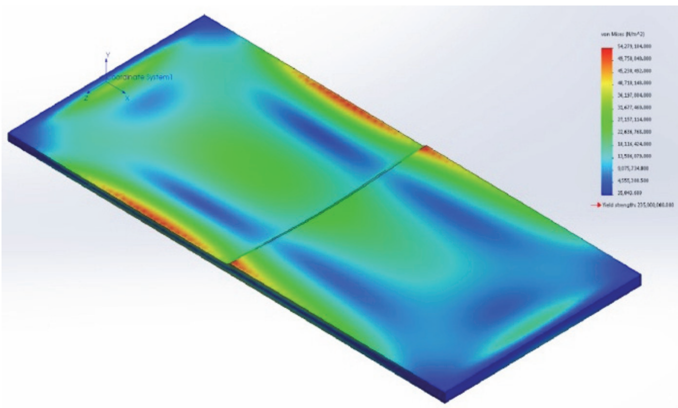


Figure 4.9. Stress simulation and results for the plate with variable thicknesses of 6 and 8 mm under the distributed load of 24 kPa.

We can see that the maximum stress is 54 N/mm^2, which is less than the yield strength for the NVA steel, i.e. 235 N/mm^2 and the allowable stress $[\sigma] = \frac{235}{1,5} = 157$ N/mm^2. From this result, we can make a conclusion that the experiment was successful and the strength is within the allowable limit. The minimum safety factor is 2,9, which means that at the same strength plates with lower thickness and lower weight can be used. That finding is very important and useful for our work. We know that the hydrostatic pressure is loaded on the hull. Due to the difficulties to make the experiment using the hydrostatic pressure we should prove that the strength is kept using a distributed load which allows us to state that the new method calculations are made correctly for the hydrostatic pressure as well. In addition, we can obtain a deflection by the FEM and compare the result with the new method calculations and experimental results. In the calculation the same data will be used for the load and the dimensions of the plates. The results will be compared with the results achieved by the experiment and by the new method calculations. Analysis and conclusions will be made. The deflection for the plate with variable thicknesses is shown in Figure 4.10.

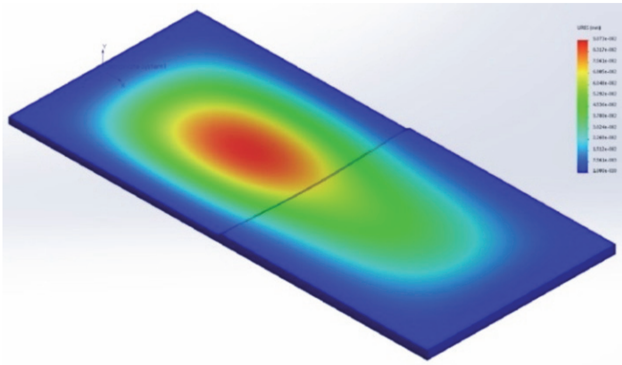


Figure 4.10. Deflection for the plate with variable thicknesses of 6 and 8 mm under the distributed load of 24 kPa.

We can see that the maximum deflection is equal to 0,9 mm and it is concentrated where the maximum pressure occurs in the middle of the 6 mm plate. To compare with experimental results, the deflection needs to be taken from the middle point of the plate. The deflection in the middle of the plate is equal to 0,7 mm. We can see that the deflection achieved by the experiment and the deflection obtained by the FEM are very similar. In conclusion, the experiment is successful for the plate with variable thicknesses.

All the considerations above prove that our equations are correct for the distributed load and match the results of the experiment and in the FEM calculations we can provide the same calculations using the hydrostatic pressure for the same plates and conditions. The results of stress for the plate with variable thicknesses of 6 mm and 8 mm are shown in Figure 4.11.

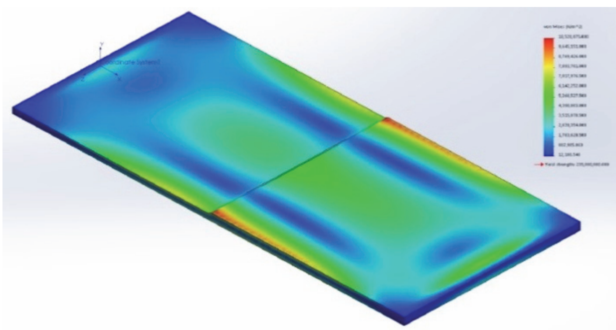


Figure 4.11. Stress simulation and results for the plate with variable thicknesses of 6 mm and 8 mm loaded by the hydrostatic pressure of 24 kPa.

We can see that the maximum stress is 10,5 N/mm^2, which is much less than the yield strength of 235 N/mm² for the NVA steel. So the allowable stress $[\sigma] = \frac{235}{1,5} = 157$ N/mm^2. That means that at loading by the hydrostatic pressure we can use thinner plate thickness and at the same time keep the strength. The minimum safety factor is 15. As a result, we have decreased the weight of the plate, which is a very good result regarding the weight of barges and ships. The real numbers of savings in weights is engineer task and can be found later. We also need to obtain the deflection for this plate. The next step is to find the deflection for the plate with variable thicknesses of 6 mm an 8 mm loaded by the hydrostatic pressure, as shown in Figure 4.12.

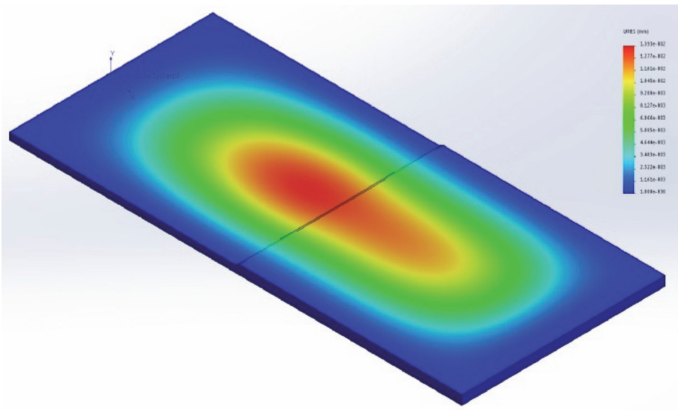


Figure 4.12. Deflection for the plate with variable thicknesses of 6 and 8 mm loaded by the hydrostatic pressure of 24 kPa.

As shown in the figure, the maximum deflection is equal to 0,14 mm and it occurs in the place where the maximum load is concentrated in the 6 mm plate. To compare with the experimental results, the deflection needs to be taken from the middle point of the plate. The deflection in the middle of the plate is equal to 0,13 mm, which means that using a plate with lower thickness satisfies all the strength requirements and enables us to use plates with variable thicknesses in the ship and barge building.

Resulting from the above, we can conclude that the provided method of plate calculation is correct and accurate. The results achieved by this method are similar to the results of the experiment and to the calculations made by the FEM. This method enables saving some % of steel, which can reduce the cost of the barge. The real numbers of savings in weights is engineer task and can be found later. The algorithms of the proposed method can be used in the program calculations in the future. The method can be developed further with regard to the dynamic forces and heating due to welding.

4.3. Analysis of the results

The results from all the calculations have to be compared. First, let us compare the experimental and the FEM results. Figure 4.13 shows the deflections of the plate with a constant thickness of 8 mm under a distributed load obtained by the experimental and the FEM method.

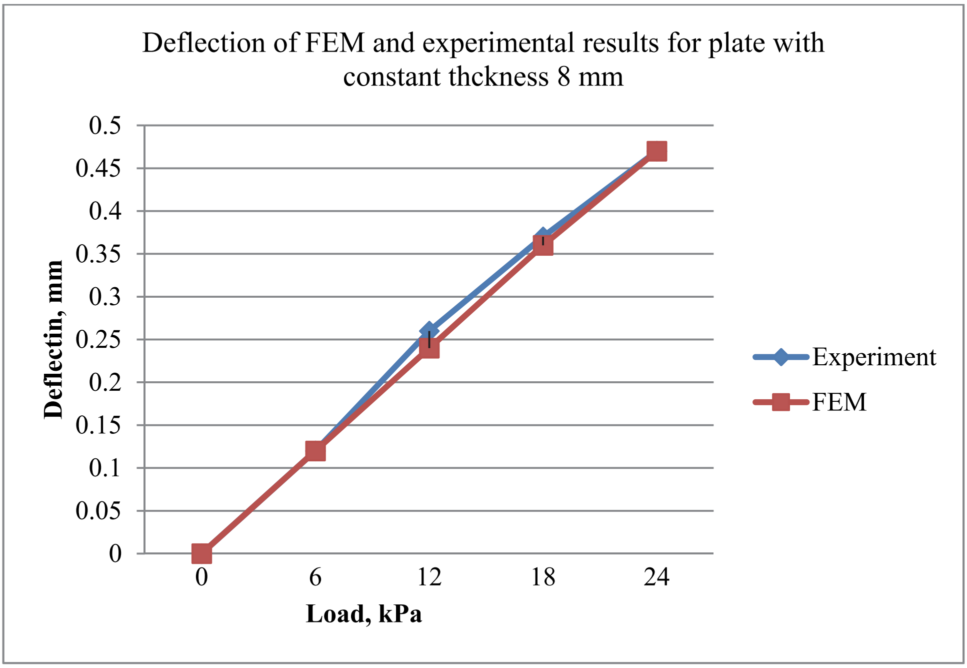


Figure 4.13. Deflection for the plate with a constant thickness of 8 mm under a distributed load from the experimental and the FEM method.

As the chart shows, the results are very similar, which satisfies us.

In order to simplify it, the results of the deflections achieved for the plate with a constant thickness of 8 mm are shown in Table 4.3.

Table 4.3. Deflection for the plate with a constant thickness of 8 mm under a distributed load at different methods.

8 mm plate of constant thickness, distributed load					
	Deflection, mm				
Method	No load	Load 6 kPa	Load 12 kPa	Load 18 kPa	Load 24 kPa
New method	0	0,11	0,24	0,37	0,49
Experiment	0	0,12	0,26	0,37	0,47
FEM	0	0,12	0,24	0,36	0,47

The chart in Figure 4.14 presents a comparison of the results.

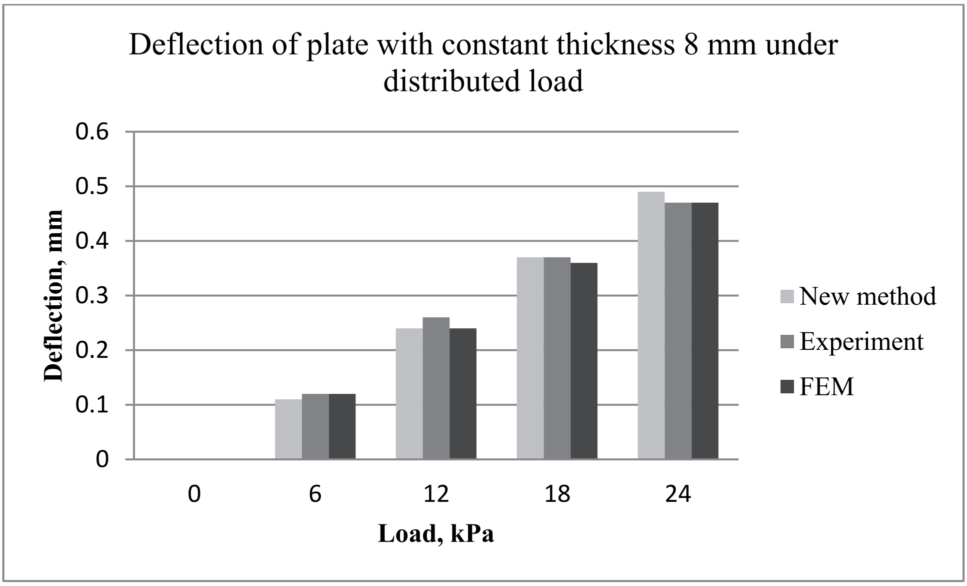


Figure 4.14. Deflection for the plate with a constant thickness of 8 mm under a distributed load at different methods.

The FEM is known to provide more accurate results, so we need to find the deviation of the new method and the experimental calculations as compared to the results of the FEM. The deviation of the new method and the experimental results for the plate with a constant thickness of 8 mm loaded by a distributed load is shown in Table 4.4.

Table 4.4. Deviation of the new method and the experimental results compared with the FEM results for the plate with a constant thickness of 8 mm under a distributed load.

8 mm plate with constant thickness, distributed load					
	Deviation, %				
Method	No load	Load 6 kPa	Load 12 kPa	Load 18 kPa	Load 24 kPa
New method	0	8	0	3	4
Experiment	0	0	8	3	0

The table demonstrates that the deviation is very little with a maximum of 8 % for the experimental results and 8 % for the new method results. The results of the new method are very close to the FEM result, which confirms that our theory proves correct and can be used for plate calculations.

The procedure for the plate with variable thicknesses will be the same as with the plate with constant thickness. Figure 4.15 shows the deflections of the plate with variable thicknesses of 6 mm and 8 mm under a distributed load found by the experimental and the FEM method.

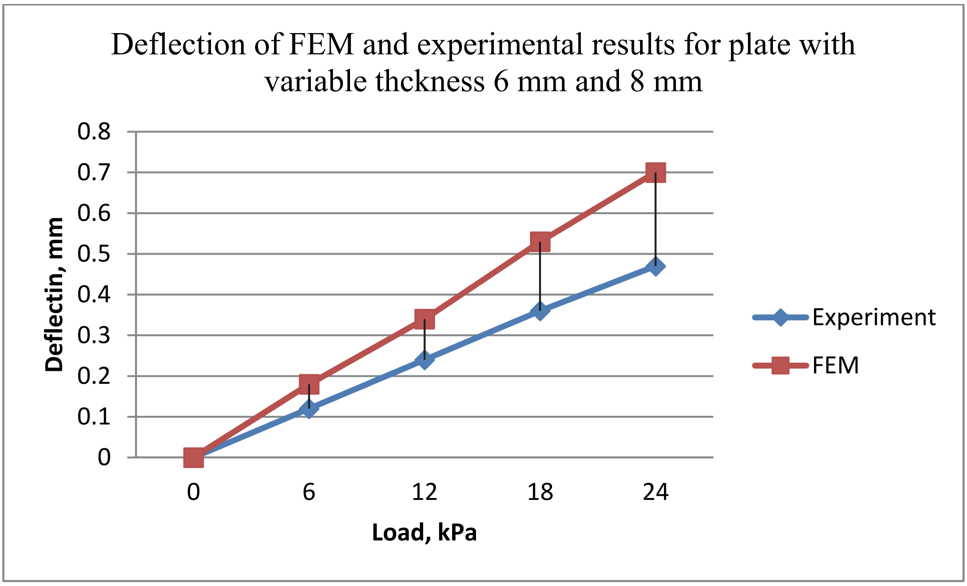


Figure 4.15. Deflection for the plate with variable thicknesses of 6 and 8 mm under a distributed load from the experimental and the FEM method.

As we can see in the chart, the results are very similar, which satisfies us.

In the same way we can obtain the results of deflection found by all the methods for the plate with variable thickness under a distributed load. The results are presented in Table 4.5.

Table 4.5. Deflection for the plate with variable thicknesses of 6 mm and 8 mm under a distributed load obtained by different methods.

Plate of 6 mm and 8 mm variable thicknesses, distributed load					
	Deflection, mm				
Method	No load	Load 6 kPa	Load 12 kPa	Load 18 kPa	Load 24 kPa
New method	0	0,16	0,32	0,51	0,69
Experiment	0	0,15	0,29	0,44	0,59
FEM	0	0,18	0,34	0,53	0,70

The deflection of the plate with variable thickness 6 mm and 8 mm is shown in Figure 4.16.

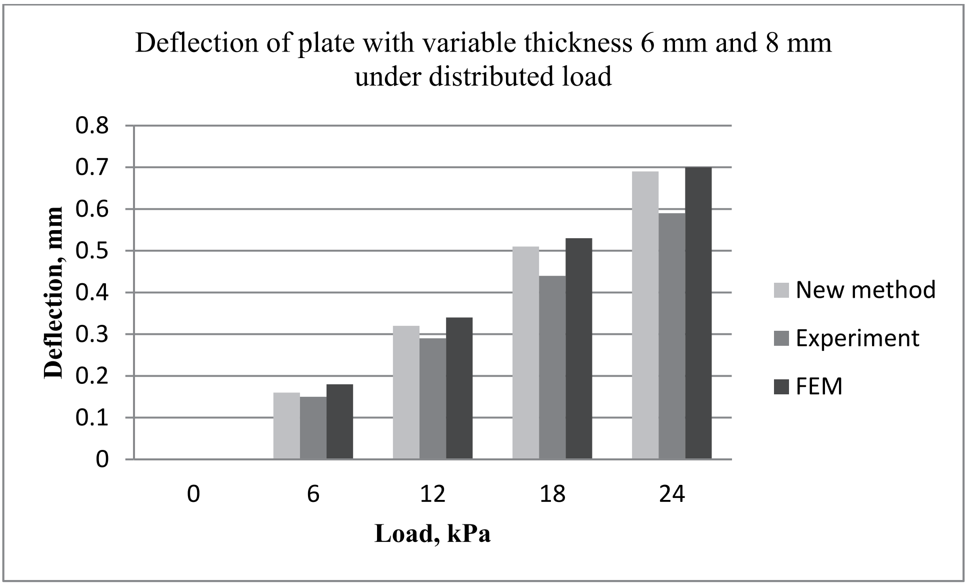


Figure 4.16. Deflection for the plate with variable thicknesses of 6 mm and 8 mm loaded under a distributed load obtained by different methods.

Similarly to the plate with constant thickness, we will find deviations for the plate with variable thicknesses by the new method and the experimental results as compared with the FEM calculations that are most accurate. The deviation for the plate with variable thicknesses is shown in Table 4.6.

Table 4.6. Deviation of the new method and the experimental results compared with the FEM results for the plate with variable thicknesses of 6 mm and 8 mm under a distributed load.

6 mm and 8 mm plate of variable thicknesses, distributed load					
	Deviation, %				
Method	No load	Load 6 kPa	Load 12 kPa	Load 18 kPa	Load 24 kPa
New method	0	11	6	4	1
Experiment	0	17	15	17	16

As we can see from the table, the deviation is insignificant, the maximum accounting for 17 % for the experimental results and 11 % for the new method results. That means that the results of the new method are accurate when the load is large. The results of the new method are closer to the FEM result, which proves that the theory is correct and can be used for plate calculations.

Now we need to do the same comparison for the plates loaded by the hydrostatic pressure. We cannot compare the experimental results because of data missing for the experimental results, as was described above. For the hydrostatic pressure, we compare only the results of the new method and the FEM results.

Figure 4.17 shows the deflections of the plate with a constant thickness of 8 mm loaded by the hydrostatic pressure made by the new calculation method and by the FEM methods.

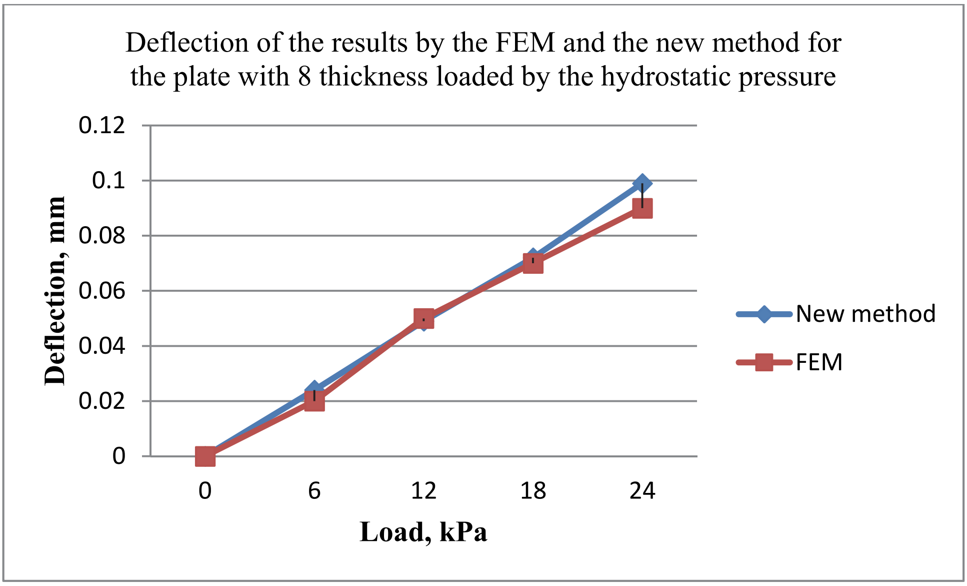


Figure 4.17. Deflection for the plate with 8 mm constant thickness loaded by the hydrostatic pressure from the new method and the FEM method.

We can see that the results are very similar. In order to simplify it, the results for the deflections achieved for the plate with 8 mm constant thickness loaded by the hydrostatic pressure are shown in Table 4.7.

Table 4.7. Deflection for the plate with 8 mm constant thickness loaded by the hydrostatic pressure by different methods.

8 mm plate with constant thickness, hydrostatic pressure.					
	Deflection, mm				
Method	No load	Load 6 kPa	Load 12 kPa	Load 18 kPa	Load 24 kPa
New method	0	0,022	0,049	0,072	0,099
FEM	0	0,020	0,050	0,070	0,090

The chart in Figure 4.18 compares the results from the new method and those from the FEM.

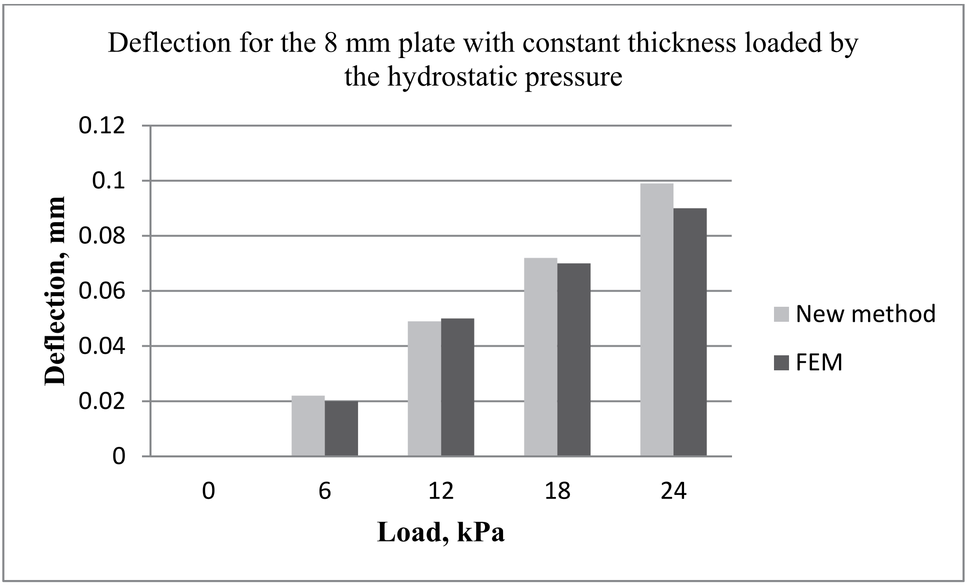


Figure 4.18. Deflection for the 8 mm plate with constant thickness loaded by the hydrostatic pressure at different methods.

As was described earlier, the FEM calculation provides more accurate results, so to find the deviation of the calculations by the new method, we have to compare the results with those from the FEM. The deviation of the results by the new method for the 8 mm plate with constant thickness loaded by the hydrostatic pressure is shown in Table 4.8.

Table 4.8. Deviation of the results from the new method compared with those from the FEM for the 8 mm plate with constant thickness loaded by the hydrostatic pressure.

8 mm plate with constant thickness, hydrostatic pressure					
	Deviation, %				
Method	No load	Load 6 kPa	Load 12 kPa	Load 18 kPa	Load 24 kPa
New method	0	9	2	3	9

As we can see from the table, the deviation is high for a minimum load, accounting for 9 %, which is a good accuracy.

The same procedure as we used for the constant plate, will be applied to the plate with variable thicknesses. Figure 4.19 shows the deflections of the plate with variable thicknesses of 6 mm and 8 mm loaded by the hydrostatic pressure found by the FEM method.

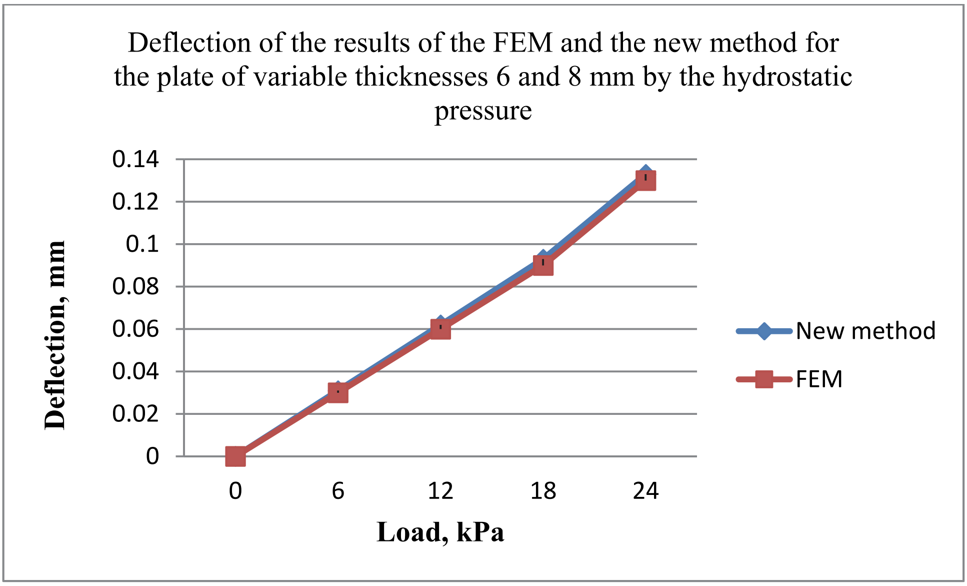


Figure 4.19. Deflection for the plate with variable thicknesses of 6 and 8 mm loaded by the hydrostatic pressure from the new method and the FEM method.

As we can see in the chart, the results are very similar, which is good for our purpose.

In the same way we can obtain the results of the deflection achieved by all the methods for the plate with variable thicknesses under a distributed load. The results are presented in Table 4.9.

Table 4.9. Deflection for the plate with variable thicknesses of 6 and 8 mm loaded by the hydrostatic pressure at different methods.

Plate 6 and 8 mm variable thicknesses, hydrostatic pressure					
	Deflection, mm				
Method	No load	Load 6 kPa	Load 12 kPa	Load 18 kPa	Load 24 kPa
New method	0	0,031	0,062	0,093	0,133
FEM	0	0,030	0,060	0,090	0,130

The deflection of the plate with variable thicknesses of 6 and 8 mm is shown in Figure 4.20.

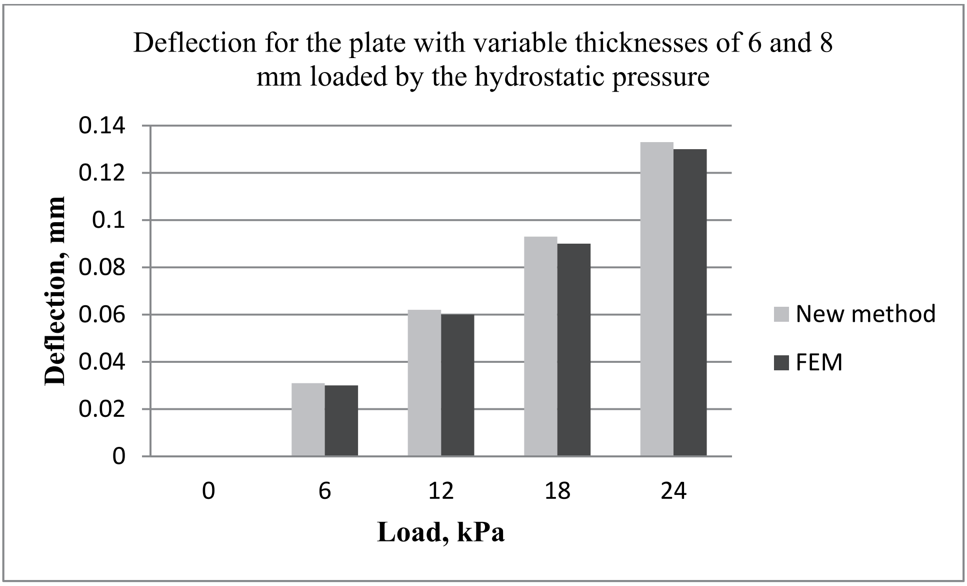


Figure 4.20. Deflection for the plate with variable thicknesses of 6 and 8 mm loaded by the hydrostatic pressure at different methods.

With a similar procedure, we will find the deviation of the new method and the FEM calculations, for as is known, the FEM results are most accurate. The deviation is shown in Table 4.10.

Table 4.10. Deviation of the results of the new method compared with the FEM results for the plate with variable thicknesses of 6 and 8 mm loaded by the hydrostatic pressure.

6 and 8 mm plate with variable thicknesses, hydrostatic pressure					
	Deviation, %				
Method	No load	Load 6 kPa	Load 12 kPa	Load 18 kPa	Load 24 kPa
New method	0	3	3	3	2

As we can see from the table, the deviation is high for the minimum load, accounting for 3 % and it is falling to 2 % when the load is increasing to the maximum level, which means that the results are accurate. The results of the new method are very close to those of the FEM result, which proves that the correctness of our theory and thus the method can be used for plate calculations. Thus, the main aim of this thesis has been achieved. The main conclusions are drawn in the final chapter of this thesis.

CONCLUSIONS

Scientific Results

The current thesis proposes an improved method of calculation for plates with variable thicknesses. In this method, it is not necessary to divide the construction into separate homogeneous parts and support the performance of kinematic and static conditions of the matching points of the neighboring parts of the construction. This method can be used under different boundary conditions. The new method is independent of the boundary conditions, which makes it different from other methods. The proposed method enables the calculations of rectangular plates with variable thicknesses. A substantial advantage of the method of generalized function is that it can be compared with other methods, which are used to solve the tasks of the theory of plates.

The results achieved show that the calculations with the new method are correct and can be used for plates with variable thicknesses. The maximum deviation was in the range from 0 to 11% for the results from the new method as compared with the FEM results. The strength requirements are also satisfied. As a result, the weight was decreased, which is very good to take into consideration the total weight of the barge or the ship. It enables saving money, increases the capacity of the barge and reduces the weight of the barge or the ship. The real numbers of savings in weights is engineer task and can be found later in optimization [75-77] process. Using the MATLAB program, the curve of deflection was found and is presented. Using that curve, a deflection in any point can be found.

Chapter 4 describes the experiment with two plates: first, with an 8 mm plate of constant thickness and second, a 6 and 8 mm plate with variable thicknesses. All those plates were loaded by a distributed load and the deflection was measured. The results show that the results from the new method and those from the experiments are quite similar, which is a proof that the proposed method is correct and can be used to calculate plates with variable thicknesses. To ensure validity, the calculations were made by the Solid Works Cosmos program. The results achieved by different methods were compared and they all were found to match. That is another proof for the correctness of the proposed method and validity for calculations for plates with variable thicknesses.

The main conclusions are as follows:

1. A new method for the calculation of plates with variable thicknesses was developed.
2. The results of the new method are close to those of the FEM. The deviation is in the range from 0% to 11 % to compare with FEM calculations.
3. The experimental results are close to the results of the new method. The maximum deviation is in the range from 0% to 17 %.

4. The new method enables the weight of the plates on the barge to be decreased by using plates with variable thicknesses. At the same time, the strength requirements are satisfied.
5. The method can be used as a basis for the development of an algorithm for program calculations of plate thickness.
6. Implementation of the new method could give an economical effect on the cost of the barge or another object.

Novelty

The new method has an advantage of avoiding the practice of dividing the construction into separate homogeneous parts and supporting performance of kinematic and static conditions of the matching points of the neighboring parts of the construction. This method can be used under different boundary conditions. In contrast to other methods, the method is independent of the boundary conditions.

As a result, we obtained a general number of algebraic equations which are used to define the solutions, independent of the number of elements that compose the whole construction, but equal to the order of the differential equation "*n*".

Common methods for the calculation of constructions enable us to transfer the task to solving the system of algebraic equations, the number of those being proportional to the number of the construction. Also, those methods are used to calculate plates with constant thickness.

The proposed method enables the calculation of rectangular plates with variable thicknesses.

The new method can be used to calculate plates with any quantity of plate parts with variable thicknesses.

The load is not limited, which means that we can use the point force and the distributed load.

The new method has an advantage related to the FEM, especially in places where the stress is changing significantly as, for instance, where the plate thickness is changing. Thus, the FEM cannot provide sufficient accuracy especially in places where the thickness of the plate is changing significantly.

The new method allows complicated mathematical modeling of the plate and includes factors which were not in use before, those that cannot be done in the FEM because of established algorithm in the FEM.

The new method allows moving towards the dynamic tasks of plate calculations, for instance, to consider load impact of waves, hull vibrations on the waves and other aspects.

The existing methods were used to calculate plates loaded by the hydrostatic pressure.

Future work

During the research, several ideas and problems emerged that require further investigation.

- In the future it is needed to make an experiment using an imitation of hydrostatic pressure. The research and development of this experiment can be conducted in the future.
- The calculations were made under static forces. In the future the dynamic forces [78] have to be taken into consideration.
- The influence of welding can be taken into consideration in the future.
- Calculations of plates reinforced by beams can be done in the future.
- Real numbers of weight decreasing can be found in the future.
- In-depth analysis of economic aspects has to be done in the future.

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I took on this work to prove myself that everything is possible, you just have to wish and believe in yourself.

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ABSTRACT

Today's fish farming industry is growing very fast due to increased fish consumption. The market in Norway is very stable and has been growing very well. Fish farmer companies are investing increasingly in their business. The developments require that more slots be opened in the fjords, new fish feeding centres be established, the requirements of the consumers be satisfied. Due to limitations in the legislation, farmers need to place fish farms farther offshore. The requirement to be taken into consideration is the load capacity of the barge and storeroom size. Those parameters are very important because of offshore locations of the barges and limitation of availability service ships that supply the feed for the fish. In other words, the barges need to be lighter in weight and higher in capacity, at the same time keeping strength requirements.

As soon as the barge is located in calm waters, the dynamic forces are not presented. Only static loads will be taken into account in our calculations. A very important parameter in fish feeding barges is their weight. The lighter the barges the more feed they can carry the more money their owners can save. The reduction of the weight is a common problem in the fish feeding industry and it is a challenge for engineers. One of the possibilities to reduce the weight is to use thinner materials on the hull of the barge. At the same time, the strength requirements are to be met.

The thesis proposes a new method of calculation for plates with variable thicknesses. Using that method, it is not required to divide the construction into separate homogeneous parts and support the performance of cinematic and static conditions of the matching points of the neighbouring parts of the construction. Moreover, the method can be used under different boundary conditions. In contrast to other methods, the new method is independent of the boundary conditions. The proposed method enables us to calculate rectangular plates with variable thicknesses.

The thesis consists of four parts: an overview of vessel types, an overview and analysis of existing methods of plate calculation, a new method of plate calculation, practical experiments and conclusions. The introduction presents the objectives of the thesis and describes its structure.

Chapter 1 describes the variety of vessel types and their use. Also, the fish-feeding barges are described.

Chapter 2 describes the methods of calculation of the plates, such as Navier, Levi, collocation, Kantorovits-Vlassov, grid and FEM methods. Each method is briefly introduced with formulas of calculation of plates under a distributed load and the hydrostatic pressure. The analysis of each method is performed. The results of each method are compared with the real solution and the resulting conclusions state which method is more suitable for the hull barge calculation.

Chapter 3 proposes the new method of the calculation of plates and provides equations for the new method and the results.

To give higher stiffness and more optimal distribution of the stress, the plates can have variable thicknesses. Those constructions are widely used in shipbuilding for most of the hull elements of the ships. In plate calculations by the variation methods it is needed to shift focus from basic set functions to unknown variables that would satisfy the boundary conditions on the edge of the construction. At the fixed contour of the plate different from free landed, the system of functions used for calculations is missing. Thus, a theory and methods of plates with variable thicknesses are required. The topicality of this task is in the development of calculation algorithms, which can expand the area of plate calculation tasks and improve existing solutions. In particular, the calculations are complex in the case of elements consisting of variable thicknesses. Among the composite structures are beams, plates with stepwise changing stiffness as well as shells consisting of elements of variable shapes. Typically, composite structures are calculated by decomposition into individual elements within each of which the stiffness and geometric characteristics change monotonously. For each of the elements obtained, the solution must be known in advance. To ensure neighbouring conjugation sites on the displacements and internal forces, a system of algebraic equations is required with unknowns, where $N \cdot n$ is the order of differential equation, and N the number of elements. However, if we use some properties of generalized functions in solving this problem, we have to set up a system of algebraic equations containing only n unknowns. In this thesis a method of generalized function is proposed to calculate the rectangular plates with variable thickness under the uniformly distributed load and under the hydrostatic pressure.

The final chapter describes the experiment for the plates with variable thicknesses under a distributed load. The results are compared with those from the new method and the calculations are made by the Solid Works Cosmos program. The comparison shows that the proposed method provides good results and can be used in the calculations for plates with variable thicknesses.

The content of this doctoral thesis is summarized in the conclusion, which outlines the main achievements and future-oriented ideas.

KOKKUVÕTE

Kaasaegse kalamajanduse kasv on väga kiire, kuna maailmas kala tarbimine kasvab. Norra turg on väga stabiilne ja kasvab järgmise paari aasta jooksul väga hästi. Kalakasvatusfirmad investeerivad oma ärisse üha suuremaid summasid. Need faktid on olulised selleks, et tarbijate soovide täitmiseks avada fjordides üha uusi kohti ja rajada uusi kalade söötmise keskusi. Piirangute tõttu seadusandluses peavad kalakasvatajad rajama kalakasvandusi kaldast üha kaugemale. Nõudmised, millega peab arvestama, on pargase kandevõime ja ladustamisruumi suurus. Need parameetrid on äärmiselt olulised, sest pargased asuvad kaldast kaugel ja teeninduslaevade, mis toovad kalade söötmiseks söödajahu, kättesaadavus on piiratud. Teisisõnu peavad pargased olema kergemad ja mahutama rohkem, kuid peavad säilitama nõuded tugevusele.

Niipea, kui pargas on paigutatud rahulikesse vetesse, ei mõju talle dünaamilised jõud. Oma arvutustes arvestame ainult staatiliste koormustega. Kalade söötmise pargaste väga oluliseks parameetriks on nende kaal. Mida kergem on pargas, seda rohkem sööta suudab see mahutada ja seda rohkem raha omanik säästab. Kaalu vähendamine on kalasööda tööstuses üldiseks probleemiks ning selle saavutamine on väljakutseks inseneridele. Kaalu vähendamise üheks võimaluseks on kasutada pargase kere juures õhemat materjali. Kuid seda saab rakendada vaid tugevusnõudeid järgides.

See väitekiri pakub erineva paksusega plaatidele välja uue arvutusmeetodi. Seda meetodit kasutades ei pea konstruktsiooni jagama eraldiseisvateks homogeenseteks osadeks ning see toetab konstruktsiooni kõrvutiasuvate osade kokkupuutepunktide kinemaatiliste ja staatiliste tingimuste rakendamist. Seda meetodit saab kasutada erinevate piirtingimustega. Erinevalt teistest meetoditest ei sõltu meie meetod piirtingimustest. Soovitatud meetod annab võimaluse arvutada erineva paksusega kandilisi plaate.

Väitekiri koosneb neljast osast – laevatüüpide ülevaatest, olemasolevate plaadiarvutusmeetodite ülevaatest ja analüüsist, plaatide uue arvutusmeetodist, praktilistest katsetest ja järeldustest. Sissejuhatus tutvustab väitekirja eesmärke ja kirjeldab ülesehitust.

Peatükk “Laevatüüpide ülevaade” kirjeldab erinevaid laevatüüpe ja nende kasutuseesmärke. Kirjeldatud on ka kalasöödapargast ja esitatud on pargase lühikirjeldus.

Peatükk “Olemasolevate plaatide arvutusmeetodite ülevaade ja analüüs” kirjeldab plaatide arvutusmeetodeid Navier, Levi, collocation, Kantorovits-Vlassov, grid ja FEM. Iga meetodi juurde on lisatud lühike sissejuhatus ning arvutusvalemid plaatidele, millele on rakendatud jaotuskoormus ja hüdrauliline surve. Lõpus on esitatud iga meetodi lõpp-tulemuste võrdlus koos täpse lahendusega ja nende omavaheline võrdlus ning on tehtud järeldused, milline meetod on pargase kere arvutuseks parim.

Kolmandas peatükis on plaatide uue arvutusmeetodi ettepanek ning esitatud on teoreetilised võrrandid ja meetodi tulemused.

Parema tugevuse ja optimaalsema pinge jagunemise jaoks võivad plaadid ja kered olla erineva paksusega. Sarnaseid konstruktsioone kasutatakse laialdaselt laevaehituses laevakere enamike elementide juures. Plaatide ja kerede erinevate arvutusmeetodite korral on vaja paika panna põhilised baasfunktsioonid tundmatutele muutujatele, mis on konstruktsiooni servades rahuldavateks piirtingimusteks. Juhul, kui fikseeritud plaadi kontuurid erinevad vabalt asetatust, siis arvutuseks kasutatud funktsioonide süsteem puudub. Tegelik ülesanne on arendada erineva paksusega plaatide ja kerede teooriat ning meetodeid. Tegelik ülesanne on arendada arvutuse algoritme, mis laiendaks plaatide arvutusülesannete ala ning parandaks olemasolevaid lahendusi. Eriti keeruliseks lähevad arvutused juhul, kui elemendid on erineva paksusega. Komposiitstruktuuri hulka kuuluvad talad, plaadid ja kered, mille tugevus muutub astmeliselt, kui ka kere, mis koosneb eri kujuga elementidest. Üldjuhul tehakse komposiitstruktuuride arvutusi jagades need üksikuteks osadeks, kus tugevus ja geomeetrilised näitajad muutuvad monotoonselt. Sel moel saadud lahendused peavad iga elemendi jaoks olema ette teada. Kindlustamaks, et naaberkonjugatsioonid oleksid nihutus- ja sisemistes jõududes paigas, tuleb paika panna tundmatutega algebravõrrandite süsteem, kus $N \cdot n$ – on esimese järgu diferentsiaalvõrrand ja N – osade arv. Siiski, kui me kasutame üldiste funktsioonide mõningaid omadusi selle probleemi lahendamiseks, tuleb paika panna algebravõrrandite süsteem, mis sisaldab ainult n tundmatuid. Selles väitekirjas üldistav funktsioon on pakutud ja üldistab kandiliste plaatide arvutusi rakendades hüdraulilist survet.

Viimases peatükis viiakse erineva paksusega plaatidele läbi katse jagatud koormuse all. Saavutatud tulemusi võrreldakse soovitatud meetodi teoreetiliste arvutustega ja arvutustega, mis tehtud Solid Works Cosmos programmiga. Võrdlus näitab, et soovitatud meetodil on häid tulemusi ja seda võib kasutada erineva paksusega plaatide plaadiarvutusteks.

Selle doktoritöö sisu on kokku võetud lõppjärelduses, mis toob välja peamised saavutused ja tulevikule orienteeritud ideed.

ELULOOKIRJELDUS

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2. Hariduskäik

Õppeasutus (nimetus lõpetamise ajal)	Lõpetamise aeg	Haridus (eriala/kraad)
Tallinna Tehnikaülikool	2004	Mehhatroonika, bakalaureusekraad
Tallinna Tehnikaülikool	2007	Mehhatroonika, magistrikraad

3. Keelteoskus (alg-, kesk- või kõrgtase)

Keel	Tase
Vene	emakeel
Eesti	kõrgtase
Inglise	kõrgtase

4. Teenistuskäik

Töötamise aeg	Tööandja nimetus	Ametikoht
01.05.2013 –…	OÜ Marketex Marine	Vanem Projektijuht
2010 – 30.04.2013	BLRT Marketex OÜ	Vanem Projektijuht
2009 – 2010	BLRT Marketex OÜ	Projektijuht
2008 – 2009	BLRT Marketex OÜ	Tehnika osakonna juhataja
2007 – 2008	BLRT Marketex OÜ	Iseneer-konstruktor

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2. Education

Institution	Graduation year	Education (field of study/degree)
Tallinn University of Technology	2004	B. Sc. In Mechatronics
Tallinn University of Technology	2007	M. Sc. In Mechatronics

3. Language competence/skills

Language	Level
Russian	native
Estonian	very good
English	very good

4. Professional Employment

Working period	Organization	Position
01.05.2013 –…	OÜ Marketex Marine	Senior Project Manager
2010 – 30.04.2013	BLRT Marketex OÜ	Senior Project Manager
2009 – 2010	BLRT Marketex OÜ	Project Manager
2008 – 2009	BLRT Marketex OÜ	Head of technical department
2007 – 2008	BLRT Marketex OÜ	Engineer-designer

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