

DOCTORAL THESIS

Efficient Ultrasonic Measurements Based on Signal Processing

Chengxiang Peng

TALLINN UNIVERSITY OF TECHNOLOGY
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Declaration:

Hereby I declare that this doctoral thesis, my original investigation and achievement, submitted for the doctoral degree at Tallinn University of Technology has not been submitted for doctoral or equivalent academic degree.

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CHENGXIANG PENG



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List of Publications

The present Ph.D. thesis is based on the following publications that are referred to in the text by Roman numbers.

- I Chengxiang Peng, Paul Annus, Marek Rist, Raul Land, and Madis Ratassepp. Fractional Fourier transform-based signal separation for ultrasonic guided wave inspection of plates. *Sensors*, 24(23):7564, November 2024.
- II Chengxiang Peng, Paul Annus, Marek Rist, Raul Land, and Madis Ratassepp. Convolution of Barker and mutually orthogonal Golay complementary codes for ultrasonic testing. *Sensors*, 25(13):5007, August 2025.
- III Chengxiang Peng, Paul Annus, Marek Rist, Raul Land, and Madis Ratassepp. Accelerating ultrasonic guided-wave measurements via SNR enhancement using coded excitation: an experimental investigation. *Applied Sciences*, 16(6):2762, March 2026.

Other publications during the PhD study:

Chengxiang Peng, Paul Annus, Marek Rist, Raul Land, and Madis Ratassepp. Investigation of signal separation for ultrasonic nondestructive evaluation. *Baltic Electronics Conference*, October 2022.

Chengxiang Peng, Paul Annus, Marek Rist, Raul Land, and Madis Ratassepp. Flexible data acquisition system with custom front-end for ultrasonic NDT research. *Baltic Electronics Conference*, October 2024.

Author's Contributions to the Publications

- I The author conceptualized the study and developed the algorithm for ultrasonic signal separation. The author contributed to the numerical simulations and experiments, analysis of the results, and preparation of the manuscript.
- II The author conceptualized the study and developed the convolved coding algorithm for ultrasonic signals. The author contributed to the numerical simulations and experiments, signal processing, and the preparation of the manuscript.
- III The author conceptualized the study and proposed the algorithm of time-efficient ultrasonic measurements. The author contributed to the experiments, the processing of signals, and the preparation of the manuscript.

Abbreviations

AM	Amplitude Modulation
AWGN	Additive White Gaussian Noise
BCGC	Barker Convolved Golay Code
BMOGCC	Barker Convolved Mutually Orthogonal Golay Complementary Code
CTFT	Continuous Time Fourier Transform
DAQ	Data Acquisition
DFT	Discrete Fourier Transform
DTFT	Discrete Time Fourier Transform
FFT	Fast Fourier Transform
FM	Frequency Modulation
FrFT	Fractional Fourier Transform
FWHM	Full Width at Half Maximum
LFM	Linear Frequency Modulation
MG	Magnitude Gain
MME	Maximum Magnitude Error
MOGCC	Mutually Orthogonal Golay Complementary Code
NDT	Nondestructive Testing
PM	Phase Modulation
PSL	Peak Sidelobe Level
RMS	Root Mean Square
RMSE	Root Mean Square Error
SNR	Signal-to-Noise Ratio
STFT	Short-Time Fourier Transform
UGWs	Ultrasonic Guided Waves
UT	Ultrasonic Testing
WVD	Wigner-Ville Distribution

1 Introduction

1.1 Background

Engineering structures are widely employed across various industries, including construction, shipping, and aviation. The integrity of these structures is critical to public safety and economic sustainability. However, factors such as corrosion, impact damage, fatigue, and material aging can degrade structural integrity and result in severe consequences, including property damage, loss of life, and even catastrophic failures. An example is the 2010 San Bruno pipeline explosion in California, USA, where the rupture of a high-pressure natural gas transmission pipeline resulted in substantial loss of life and extensive property damage [1]. Such incidents highlight the importance of reliable structural integrity assessment. Nondestructive testing (NDT), which enables the inspection of engineering structures without causing damage or interrupting their operation, is an essential tool to ensure their safety, reliability and integrity [2].

Among various NDT techniques, ultrasonic testing (UT) is one of the most important and widely adopted methods owing to its high testing efficiency and ability to detect a wide range of defects [3–5]. Compared with visual inspection, which is often labor-intensive and relies heavily on inspector experience, UT is more efficient and objective [6]. Unlike radiographic testing, UT does not involve ionizing radiation, making it safer for both operators and the surrounding environment [7]. In addition, whereas vision-based inspection techniques are generally limited to surface-visible defects, UT can detect and characterize internal defects within a structure [8]. UT involves the generation, transmission, and reception of ultrasonic waves, and the structural integrity is subsequently assessed by analyzing the received signals.

The performance of UT depends not only on the inspection accuracy but also on the measurement efficiency. In many UT applications, repeated signal acquisitions and multiple measurement locations are commonly employed to obtain reliable inspections, resulting in prolonged measurement times. As the size of the inspected structure increases, or in high-throughput manufacturing applications that require near real-time quality assessment [9], measurement time becomes a major factor limiting measurement efficiency. This limitation is particularly significant in industrial applications such as aircraft maintenance, pipeline inspection, wind turbine monitoring, and manufacturing quality control, where prolonged inspections increase operational downtime, reduce inspection throughput, and raise labour and maintenance costs [10–13]. Consequently, improving measurement efficiency has become increasingly important for the practical deployment of UT. Moreover, faster measurement enables more frequent inspections, facilitates the inspection of large structures within limited maintenance windows, supports automated and robotic inspection systems, and provides the temporal capability required for near real-time quality assessment in high-throughput manufacturing. Therefore, enhancing measurement efficiency has become an important research direction for advancing the practicality and capability of modern UT systems.

1.2 Research Problems

To improve the efficiency of UT, various approaches have been explored. An intuitive approach is to employ an array transducer consisting of multiple transducer elements. As illustrated in Figure 1, pitch-catch inspections using a transmitter–receiver pair require transducers to be mechanically repositioned to cover the whole inspection area, resulting in lengthy acquisition times. In contrast, array transducers enable inspection using multiple transmitter–receiver pairs from fixed locations, thereby eliminating repeated mechan-

ical transducer repositioning and significantly improving measurement efficiency. Another increasingly adopted approach is the use of ultrasonic guided waves (UGWs). Compared with ultrasonic bulk waves, UGWs can propagate over much longer distances, enabling a substantially larger inspection area to be covered from a single measurement location and consequently reducing the number of required signal acquisitions [14].

Although existing approaches have significantly improved the efficiency of UT, there remains considerable potential for improvements. Even when guided waves and array transducers are employed, measurements relying on sequential transmissions still require sufficient time delays between successive transmissions to prevent signal overlap (Figure 1b) and repeated signal acquisitions for temporal averaging to obtain an adequate signal-to-noise ratio (SNR).

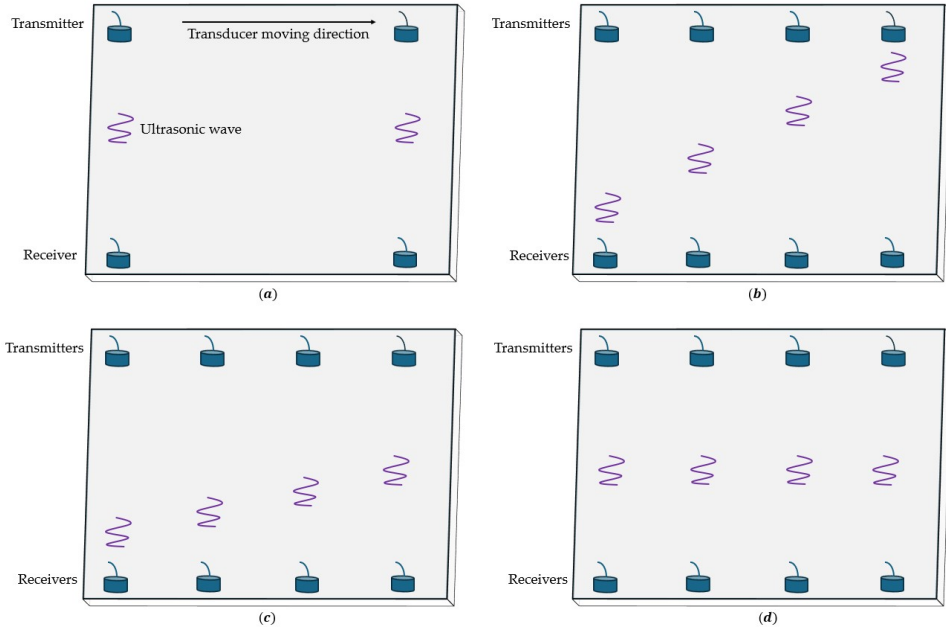


Figure 1: A schematic comparison of different UT approaches. (a) A transmitter-receiver pair requiring mechanical scanning. (b) An array transducer with sufficient transmission time delay. (c) An array transducer with reduced transmission time delay. (d) An array transducer with parallel transmission.

To further improve the measurement efficiency of UT, three approaches are investigated in this thesis: (1) reducing the time delay between successive transmissions (**publication I**), (2) reducing the number of signal acquisitions required for temporal averaging (**publication III**), and (3) reducing the number of transmission events through parallel transmission (**publication II**). Based on these three approaches, three corresponding research problems are formulated.

Research problem 1: need for signal separation caused by reduced time delays.

Array transducers have become increasingly popular in UT due to several advantages, particularly their enhanced flexibility, which reduces the need for transducer reconfiguration during inspections [15]. Conventionally, signals are acquired by exciting each transducer sequentially with a sufficient time delay (Figure 1b). To reduce the overall mea-

surement time, one approach is to shorten the delay (Figure 1c). However, decreasing the delay can cause the received signals to overlap in the time domain, making individual responses difficult to distinguish within the measured signal. This complicates signal interpretation and may render it unsuitable for subsequent signal analysis.

Various methods have been proposed to separate overlapped signals. One straightforward approach is to employ high-frequency broadband excitation signals with short time durations, thereby reducing the likelihood of signal overlap in the time domain. In this case, signals may overlap in the frequency domain while remaining temporally orthogonal, allowing them to be separated using time windowing. However, the use of high-frequency signals is often impractical in many applications because signal attenuation increases significantly with frequency, thereby reducing SNR. Another straightforward approach is to employ frequency-division-based techniques, which separate overlapped signals in the frequency domain by exploiting the orthogonality of signals occupying distinct frequency bands [16]. However, these methods generally require different excitations to be transmitted by different transducers. Many UT applications prefer identical excitations for all transducers to ensure consistent interpretation of the received signals. This requirement limits the applicability of frequency-division-based techniques [17, 18]. Therefore, reducing the time delay (Figure 1c) causes ultrasonic signals to overlap in both the time and frequency domains (Figure 2), such that neither time windowing nor frequency-division-based methods can achieve reliable signal separation. Therefore, one of the key motivations of this thesis is to investigate signal separation approaches capable of separating overlapping ultrasonic signals caused by reduced delays, thereby enabling shorter delays and improving measurement efficiency.

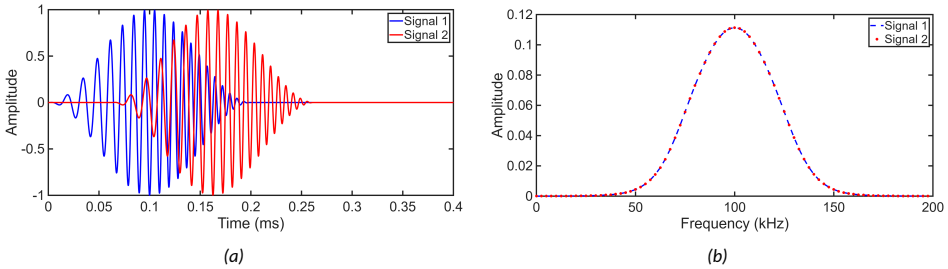


Figure 2: Overlapping linear chirps. (a) Time domain signals. (b) Frequency spectrum.

Research problem 2: need for temporal averaging.

Ultrasonic waves are often subject to significant attenuation due to long distance propagation and contamination by noise from various sources, such as electronic instrumentation and environmental disturbances, resulting in a reduced SNR. One direct method for improving the SNR is to increase the excitation voltage. Nevertheless, high excitation voltages can be undesirable because of safety concerns, particularly in hazardous or explosive environments, as well as limitations imposed by measurement hardware systems [19, 20]. Temporal averaging offers an alternative solution by averaging received signals from repeated signal acquisitions, thereby improving the SNR through coherent signal enhancement and uncorrelated noise suppression. This method can be incorporated into existing UT systems without additional hardware or complex algorithms. However, achieving a substantial SNR improvement often requires a large number of averages, that is, repeated signal acquisitions, which considerably decreases measurement efficiency (Figure 3). This motivates the investigation of approaches that reduce the number of repeated signal ac-

quisitions for temporal averaging while maintaining an adequate SNR, thereby improving the measurement efficiency of UT.

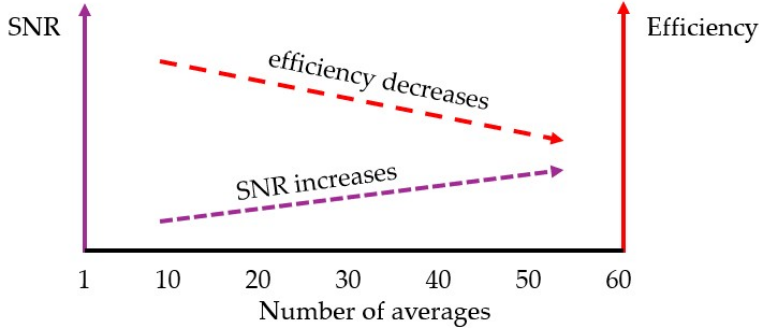


Figure 3: Trade-off between SNR improvement and measurement efficiency.

Research problem 3: need for parallel transmission in convolved coding schemes.

Unlike sequential transmission, which excites transducers one by one, parallel transmission simultaneously excites multiple transducers, thereby reducing the number of transmission events and improving measurement efficiency. As illustrated in Figures 1b and d, if four transducers are excited simultaneously, the number of transmission events can theoretically be reduced to a quarter of that required for sequential transmission.

However, parallel transmission inevitably causes signals from multiple transducers to overlap at the receiver, making reliable signal separation essential. Nevertheless, signal separation in parallel transmission can be achieved by exploiting the orthogonality between coded excitations.

Recently, convolved codes have attracted increasing attention because of their superior magnitude enhancement capability. However, existing convolved codes do not satisfy the orthogonality requirement for parallel transmission [20, 21]. Consequently, they cannot be directly employed for parallel transmission. This motivates the development of convolved coding schemes capable of supporting parallel transmission while preserving the advantages of convolved codes.

1.3 Research Objectives

The overall objective of this thesis is to improve the efficiency of UT while maintaining reliable UT performance.

To address the dual-domain signal overlap caused by reduced transmission delays, several signal separation techniques have been proposed, including matching pursuit, the Wigner-Ville distribution (WVD), and the fractional Fourier transform (FrFT) [22–27]. Among them, FrFT is particularly attractive because it provides superior energy concentration for chirp-like signals, enabling effective separation of overlapping signals in the fractional Fourier domain. Compared with matching pursuit, it avoids iterative dictionary searches and therefore has lower computational complexity. In addition, unlike the WVD, the FrFT is free from cross-term interference. Furthermore, since linear chirps are widely employed in ultrasonic testing for pulse compression, the FrFT provides a compact representation of these signals and is therefore well suited for separating overlapping linear chirps. Although the FrFT has been successfully applied in optics, medical ultrasonics, and NDT [27–29], to the best of the author’s knowledge, its application to separate overlapping signals caused by reduced time delays in array transducer measurements has not yet

been reported. Accordingly, the **first objective** of this thesis is to develop and evaluate a FrFT-based signal separation method for separating overlapping ultrasonic signals under reduced delays, thereby enabling more efficient ultrasonic measurements. The proposed method will be validated through both numerical simulations and laboratory experiments under different delay settings.

To reduce the need for temporal averaging, coded excitations are investigated in this thesis. Unlike temporal averaging, coded excitations improve the SNR by increasing the transmitted signal energy through a longer signal duration. Consequently, for a given SNR requirement, fewer temporal averages are required than with conventional excitation methods, such as the single-cycle pulse [20]. Although coded excitations have demonstrated considerable potential for improving SNR, systematic experimental validation and quantification of the relationship between coded excitations and temporal averaging remain limited. Therefore, the **second objective** of this thesis is to investigate the effectiveness of coded excitations in reducing the need for temporal averaging while maintaining an adequate SNR. Specifically, widely used Barker and Golay codes [20] will be experimentally evaluated under different excitation voltages and averaging numbers to quantify the extent to which coded excitations can reduce the need for temporal averaging, with comparisons made against conventional excitations.

The **third objective** of this thesis is to develop a convolved coding scheme for parallel transmission. Mutually orthogonal Golay complementary codes (MOGCC), which have demonstrated to be effective for parallel transmission in medical ultrasonics [30], are adopted to provide the required orthogonality. Barker codes are then convolved with MOGCC to combine the SNR enhancement capability of convolved coding with the orthogonality required for signal separation. The proposed coding scheme will be validated through both numerical simulations and laboratory experiments.

1.4 Research Hypothesis

The research hypotheses of this thesis are:

1. It is hypothesized that the FrFT can reliably separate overlapping ultrasonic signals caused by reduced transmission delays, thereby improving the overall measurement efficiency.
2. It is hypothesized that coded excitations can achieve a given SNR with significantly fewer temporal averages than conventional excitations, thereby reducing the number of repeated signal acquisitions and improving measurement efficiency.
3. It is hypothesized that the proposed convolved coding scheme can enable parallel transmission while preserving the magnitude gain achieved by conventional convolved coding, thereby reducing the number of transmission events and improving measurement efficiency.

1.5 Research Contributions

This thesis makes several contributions to improving the measurement efficiency of UT, by extending the application of the FrFT and coded excitations to practical measurement scenarios. The findings and methodologies presented in this thesis have the potential to stimulate further research and development in efficient UT techniques. More specifically, its contribution lies in:

1. In **publication I**, a FrFT-based signal separation method is proposed for separating overlapping ultrasonic signals. By demonstrating the effectiveness of the FrFT-based method through both finite-element simulations and experiments conducted on an aluminum plate, this study provides a new approach to improving the time efficiency of UT in scenarios where reduced transmission time delays lead to signal overlap.
2. In **publication III**, the effectiveness of coded excitations in reducing the number of temporal averages is experimentally demonstrated. Compared with conventional excitations, coded excitations achieve a given SNR with fewer averages, thereby improving the measurement efficiency. This work provides practical guidelines for reducing the reliance on temporal averaging using coded excitations.
3. In **publication II**, a novel convolved coding scheme is proposed to make convolved coding compatible with parallel transmission. The proposed coding scheme combines the magnitude enhancement capability of convolved coding with the orthogonality required for parallel transmission. Its effectiveness is demonstrated through numerical simulations and experiments conducted on a steel block, providing a practical solution for parallel transmission in UT.

1.6 Thesis Structure

The overall structure of this thesis is illustrated in Figure 4, which summarizes the relationships among the three research objectives and the corresponding publications. The thesis is organized around the common objective of improving the measurement efficiency of UT. To achieve this objective, three approaches are investigated: reducing the time delay between successive transmissions, reducing the number of temporal averages, and reducing the number of transmission events through parallel transmission. These three approaches correspond to **publications I, III, and II**, respectively.

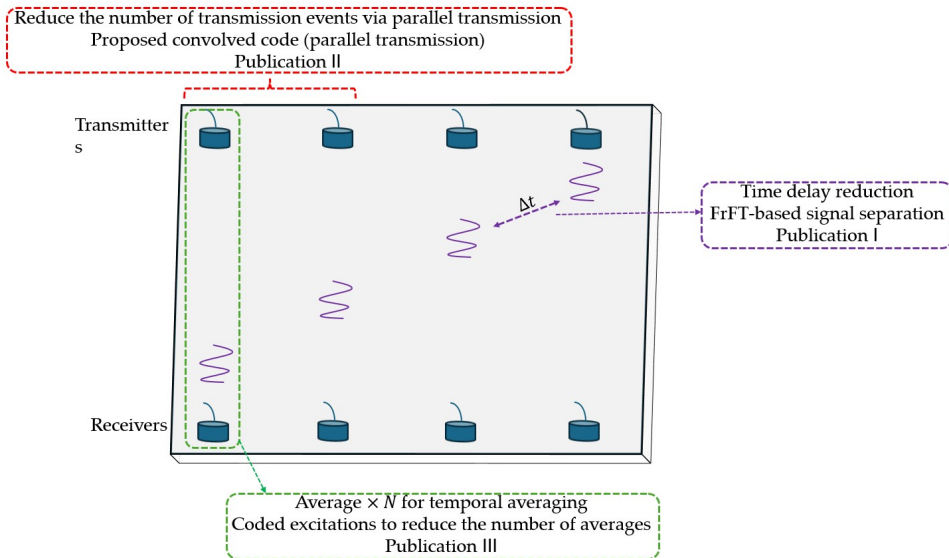


Figure 4: Conceptual framework of the thesis, illustrating the three approaches investigated to improve the measurement efficiency of UT.

The remainder of this thesis is organized as follows. Chapter 2 introduces the background knowledge and theoretical foundations relevant to this thesis. Chapter 3 presents the proposed methods for improving the measurement efficiency. Chapter 4 describes the experimental investigations and discusses the corresponding results. Finally, Chapter 5 concludes this thesis and discusses directions for future work.

2 Theory

Following the research problems and objectives presented in chapter 1, this chapter introduces the theoretical background required for the proposed methods. It begins with the fundamental principles of ultrasonic bulk waves and guided waves, followed by signal representation and coded excitation techniques. Signal representation forms the theoretical foundation for analyzing ultrasonic signals in different domains and is essential for understanding the FrFT-based signal separation method presented in the subsequent chapters, while coded excitation techniques provide the theoretical basis for the coded excitation methods developed later in this thesis.

2.1 Bulk Waves

Ultrasonic bulk waves refer to ultrasonic waves that propagate in the bulk of the medium without encountering boundaries, such as in an infinite medium [1]. According to the relationship between the particle vibration direction and the wave propagation direction, bulk waves can generally be classified into two types: longitudinal waves and shear waves. In longitudinal waves, the particle motion is parallel to the propagation direction, whereas in shear waves, the particle motion is perpendicular to the propagation direction. Longitudinal waves are capable of propagating in solids, liquids, and gases due to their compressional nature. Since shear deformation requires resistance to tangential stress, shear waves can only propagate in solid materials.

An important characteristic of bulk waves is that, in isotropic elastic solids, their velocities are determined by the material properties of the medium and are independent of wave frequency; therefore, bulk waves are considered non-dispersive. The velocities of longitudinal and shear waves in solid media are respectively given by [14]:

$$\begin{aligned} c_1 &= \sqrt{\frac{E(1-\nu)}{\rho(1+\nu)(1-2\nu)}}, \\ c_2 &= \sqrt{\frac{E}{2\rho(1+\nu)}}, \end{aligned} \tag{1}$$

where c_1 , c_2 are longitudinal wave velocity and shear wave velocity, respectively; E is the material's Young's modulus; ν is the Poisson's ratio; and ρ denotes the material density. For a given material, the velocities of longitudinal and shear waves are constant. In general, longitudinal waves propagate faster than shear waves.

Bulk waves are widely used in UT owing to their relatively simple propagation characteristics and well-established inspection methodologies. One common application based on bulk waves is thickness measurement, where the material thickness is typically calculated by the time-of-flight method [31]. Another important application of bulk waves is defect detection in materials. In bulk wave testing, the ultrasonic wave energy is mainly concentrated below the transducer, meaning that only a localized region can be examined at a single measurement point. Consequently, inspection of large structures requires point-by-point scanning through mechanical movement of the transducer. Such scanning processes not only increase inspection time but also require highly accurate mechanical positioning systems. Array transducers could be used to alleviate this issue; however, they require more complex measurement circuitry and multiple transducer elements. Figure 5 illustrates a schematic diagram (cross-sectional view) of bulk wave testing for a plate, showing that complete inspection of the plate requires the transducer to be scanned across the entire surface. However, even when array transducers are employed, measurements

based on sequential transmission still require multiple transmission events and sufficient delays between transmissions, leaving considerable potential for improving measurement efficiency through advanced signal processing methods.



Figure 5: Schematic diagram of ultrasonic bulk wave testing.

2.2 Guided Waves

Ultrasonic guided waves are elastic waves that propagate in waveguide structures, such as plates and pipes. Unlike bulk waves, which are generally assumed to propagate in effectively infinite media, guided waves are strongly influenced by the geometry and boundary conditions of the waveguide. Because the wave energy is confined within the waveguide, guided waves can propagate over relatively long distances, making them particularly suitable for the inspection of large structures.

Guided waves exhibit two important characteristics: multimodality and dispersion. Multiple wave modes may coexist at a given frequency, and each mode possesses distinct propagation characteristics, including phase velocity, group velocity, and displacement distribution. In isotropic plate-like structures, guided waves are commonly referred to as Lamb waves and are classified into symmetric (S) and antisymmetric (A) modes according to their displacement distributions with respect to the plate mid-plane. The propagation of these modes is governed by the Rayleigh–Lamb frequency equations [14], which are expressed as follows:

$$\frac{\tan(qh)}{\tan(ph)} = \frac{-4k^2 pq}{(q^2 - k^2)^2}, \quad (2)$$

$$\frac{\tan(qh)}{\tan(ph)} = \frac{(q^2 - k^2)^2}{-4k^2 pq}, \quad (3)$$

where k is the wavenumber, h is the half of the plate thickness, $p = \sqrt{\frac{\omega^2}{c_1^2} - k^2}$, $q = \sqrt{\frac{\omega^2}{c_2^2} - k^2}$, and $k = \frac{2\pi}{\lambda}$. By solving Equations 2 and 3, the relationship between the wave number and frequency can be obtained. Subsequently, dispersion curves describing wave velocities as functions of frequency can be plotted. The phase velocity c_p and group velocity c_g can be calculated as follows:

$$\begin{aligned} c_p &= \frac{2\pi f}{k}, \\ c_g &= 2\pi \frac{df}{dk}. \end{aligned} \quad (4)$$

Equation 4 indicates that the phase velocity of guided waves depends on frequency, demonstrating their dispersive nature. The phase and group velocities are generally different, and both vary with frequency for a given structure. An example of the dispersion curves

for a 5 mm thick aluminum plate is shown in Figure 6, illustrating the multimodal and dispersive characteristics of guided waves.

Unlike bulk waves, which are limited to inspecting a localized region from a single measurement location, guided waves can propagate over much longer distances, enabling the inspection of much larger areas from the same measurement location. Consequently, guided wave testing generally offers higher measurement efficiency than bulk wave testing for large structures. A schematic diagram of guided wave testing is shown in Figure 7. As illustrated in Figure 7, guided waves enable the detection of defects over long propagation distances without mechanically moving the transducer.

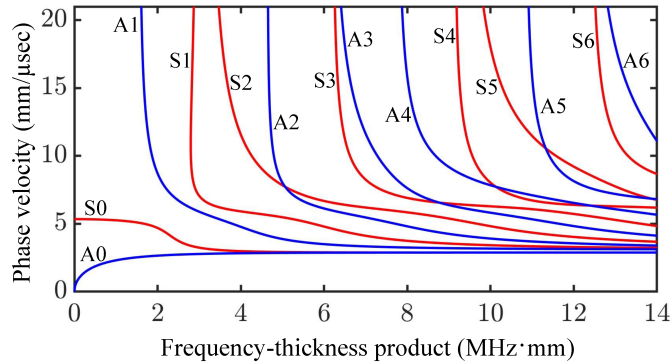


Figure 6: Dispersion curves for a 5 mm thick aluminum plate [3].

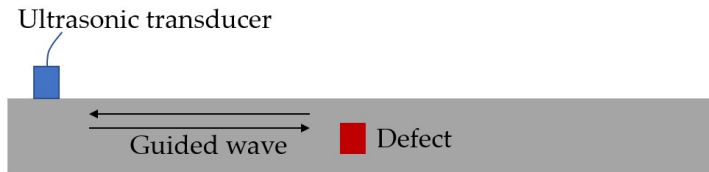


Figure 7: Schematic diagram of ultrasonic guided wave testing.

Owing to their high measurement efficiency, UGWs are widely used in UT, particularly for the inspection of pipelines, plate structures, and railway tracks. For example, guided wave tomography has been employed for the inspection of pipe bends [32], structural health monitoring of plate structures has been achieved using UGWs [33], and the detection of internal defects in rail heads has also been investigated using UGWs [34]. However, the multimodal and dispersive characteristics of UGWs often result in waveform distortion. In addition, mode conversion may occur at defects or structural discontinuities, further increasing the complexity of signal interpretation and analysis. Therefore, low-frequency narrowband excitation signals are commonly employed in practical applications. Lower frequencies generally excite fewer propagating modes than higher frequencies, while narrowband excitation reduces dispersion-induced waveform distortion.

2.3 Signal Representation

2.3.1 Signals in the Time Domain

The time domain representation is the most fundamental and intuitive form of signal representation, in which a signal is expressed as a function of time. In this domain, the am-

plitude variation of a signal can be directly observed with respect to time. A continuous time signal is generally expressed as $s(t)$, where t denotes time. In practical applications, continuous time signals need to be sampled and converted into discrete time signals. The sampling process should satisfy the Nyquist-Shannon sampling theorem [35]. The discrete time signal can be written as $s[n]$ where n is the discrete index.

In UT, ultrasonic waves received by transducers are converted into electrical signals in the time domain. Time domain signals preserve the original waveform information and directly reflect the propagation characteristics of ultrasonic waves within the inspected structure. Important signal features, including arrival time, echo amplitude, pulse duration, and attenuation, can be directly extracted from time domain signals.

Many signal processing techniques in the time domain, including temporal averaging, time windowing, and correlation-based techniques, are widely used in UT. Since these techniques do not require transformations into other signal domains, they can often be implemented efficiently in UT systems. Despite these advantages, processing signals in the time domain also has limitations. The frequency content of a signal cannot be explicitly observed, making it difficult to analyze frequency-dependent wave phenomena. Furthermore, when multiple signals overlap in the time domain, reliable signal separation becomes difficult using simple time windowing. These limitations can be alleviated by representing signals in the frequency domain and the time-frequency domain, which are introduced in the following subsections.

2.3.2 Signals in the Frequency Domain

The frequency domain representation describes a signal in terms of its spectral components as a function of frequency. Unlike time domain signals, which focus on amplitude variations with respect to time, frequency domain signals reveal how the signal energy is distributed across different frequency components. This representation is particularly useful for analyzing the frequency characteristics of ultrasonic signals. Fourier analysis establishes the relationship between the time domain and the frequency domain. The principal transformations in Fourier analysis include the continuous time Fourier transform (CTFT), the discrete time Fourier transform (DTFT), and the discrete Fourier transform (DFT). The CTFT and DFT are expressed as follows [35]:

$$S(f) = \int_{-\infty}^{\infty} s(t) e^{-j2\pi ft} dt, \quad (5)$$

$$S[k] = \sum_{n=0}^{N-1} s[n] e^{-j\frac{2\pi}{N} kn}, \quad (6)$$

where f is the frequency, j is the imaginary unit, $S(f)$ represents the continuous spectrum, k is the discrete frequency index, N is the total number of sampling points, and $S(k)$ denotes the discrete spectrum. The commonly used fast Fourier transform (FFT) is an efficient algorithm for computing the DFT.

Frequency domain analysis plays an important role in UT because many signal characteristics, such as centre frequency and bandwidth, are more readily observed in the frequency domain. In UGWs, different wave modes exhibit different propagation characteristics over different frequency ranges, and their propagation velocities vary with frequency. In practical UT applications, frequency domain analysis is widely used for spectral analysis, such as band-pass filtering to suppress unwanted frequency components. One major limitation of frequency domain analysis is that the temporal information of a signal is lost after Fourier transformation, meaning that the exact occurrence time of individual frequency components cannot be determined. This limitation becomes particularly

restrictive for non-stationary ultrasonic signals whose spectral characteristics vary with time. In such cases, time-frequency representations provide a more suitable framework by simultaneously representing signals in both the time and frequency domains.

2.3.3 Signals in the Time-Frequency Domain

Signals represented in the time-frequency domain simultaneously illustrate signal characteristics in both the time and frequency domains. Unlike the time domain or the frequency domain representations, the time-frequency represented signal not only reveal which frequency components are present, but also indicates when these frequency components occur. Time-frequency analysis is particularly suitable for analyzing non-stationary signals whose spectral characteristics vary with time. In UT, time-frequency analysis is especially valuable because practical ultrasonic signals may overlap in both the time and frequency domains. Under such circumstances, time domain or frequency domain analysis alone is often insufficient to effectively distinguish and characterize the signals.

Various methods have been developed for time-frequency analysis. A classical method is the short-time Fourier transform (STFT), which divides a time domain signal into a sequence of short segments using a window function and then performs the Fourier transform on each segment individually [35]. In addition to the STFT, time-frequency analysis methods can be formulated within the framework of atomic decomposition. Representative examples include the wavelet transform, based on scaled and shifted wavelet atoms, and Gabor decomposition, based on shifted Gaussian-windowed complex exponential atoms [36, 37].

Another important category of time-frequency analysis methods is based on energy distribution. Unlike atomic decomposition methods, which represent a signal using a set of basis atoms, energy distribution methods distribute signal energy over the joint time-frequency plane. Cohen's class distributions are among the most widely used time-frequency energy distributions and are defined as a class of distributions that are covariant under translations in both time and frequency [38]. In UT, the Wigner-Ville distribution (WVD), which belongs to Cohen's class, is often employed because it provides high time and frequency resolution with very limited spread in the time-frequency plane [39, 40]. However, the WVD also generates artificial interference terms (cross-terms), which may obscure the true signal components and limit its performance in practical applications [41].

Regardless of the specific time-frequency analysis method employed, time-frequency representations are fundamentally constrained by the Fourier uncertainty principle, which states that the product of a signal's time spread and frequency spread has a lower bound. The Fourier uncertainty principle can be expressed as follows [42]:

$$\Delta_t \Delta_f \geq \frac{1}{2}, \quad (7)$$

where Δ_t and Δ_f are the standard deviations of time and frequency of a signal. According to the Fourier uncertainty principle, it is impossible to simultaneously localize a signal with arbitrarily high resolution in both the time and frequency domains. In other words, achieving higher resolution in the time domain results in lower resolution in the frequency domain, and vice versa. This limitation is particularly relevant to ultrasonic measurement efficiency. Pursuing extremely short-duration excitation signals improves time-domain separability but requires a broad frequency bandwidth, which may exceed the usable bandwidth of practical ultrasonic transducers. Conversely, pursuing extremely narrow-band excitation signals inevitably increases the signal duration, resulting in reduced measurement efficiency.

Compared with time domain and frequency domain representations, time-frequency representations generally require higher computational complexity and greater memory requirements. Nevertheless, because they simultaneously provide temporal and spectral information, time-frequency representations have become an important tool for analyzing non-stationary ultrasonic signals in modern ultrasonic signal processing. Figure 8 shows the representation of two overlapping signals in the time domain with their corresponding time-frequency representation obtained using the WVD. The joint time-frequency information enables signals that overlap in both the time and frequency domains to become more distinguishable, thereby providing an effective basis for subsequent signal separation. As the FrFT is a time-frequency analysis method, the concepts presented in the preceding discussion provide the theoretical background for understanding the FrFT-based signal separation method presented in Chapter 3.

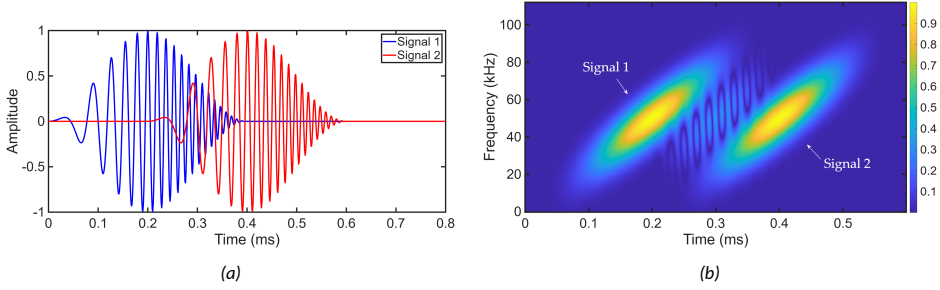


Figure 8: Signal representations. (a) Time domain representation. (b) Time-frequency representation obtained with the WVD.

2.4 Coded Excitation

Coded excitation is a signal coding technique widely used in UT to improve the SNR. Instead of transmitting a short signal, such as a five-cycle sinusoidal waveform, the system transmits a longer coded waveform, which is subsequently processed using pulse compression. This process, also referred to as decoding, compresses the coded waveform into a narrow pulse, thereby increasing the peak amplitude while preserving temporal resolution. A schematic diagram of a UT system utilizing coded excitation is shown in Figure 9. As illustrated in Figure 9, the decoded signal exhibits a significantly higher peak amplitude than the received uncoded waveform.

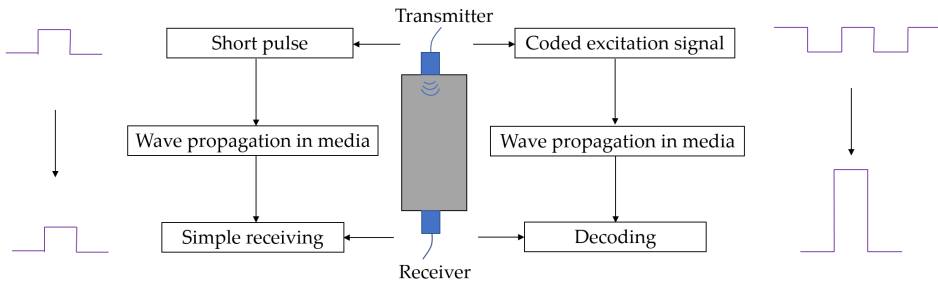


Figure 9: Schematic diagram of a UT system employing coded excitation [16].

2.4.1 The Matched Filter

In ultrasonic signal processing, pulse compression is generally implemented using matched filtering. Since the signals received by ultrasonic transducers are generally contaminated by noise, filtering is required before further signal analysis. Assuming that the noise is additive white Gaussian noise (AWGN), the matched filter is the optimal linear filter to maximize the SNR [43]. A schematic diagram of the matched-filter-based ultrasonic signal processing is shown in Figure 10.

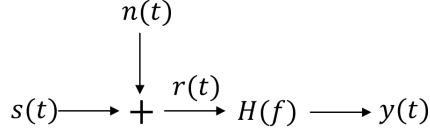


Figure 10: Matched-filter-based ultrasonic signal processing.

In Figure 10, $s(t)$ is the noise-free ultrasonic signal, $n(t)$ represents the AWGN, and $r(t) = s(t) + n(t)$ is the received signal. $H(w)$ denotes the frequency response of the filter and $y(t)$ is the filter output. The SNR, defined as the ratio of the instantaneous signal power to the average noise power, satisfies the following inequality:

$$SNR \leq \frac{2E}{N_0}, \quad (8)$$

where E is the energy of the $s(t)$ and N_0 is the noise spectral density. When the impulse response of the filter satisfies $h(t) = ks(\tau_d - t)$, the SNR after matched filtering reaches its maximum value of $\frac{2E}{N_0}$, where k and τ_d are the gain factor and the time delay, respectively [43]. In this case, $h(t)$ is referred to as the matched filter.

The maximum achievable SNR is determined by the energy of the excitation signal and the noise power density. When the noise cannot be eliminated, the SNR can only be improved by increasing the transmitted signal energy. The signal energy can be increased either by increasing the amplitude of the excitation signal or by extending its duration. This observation explains why coded excitation can effectively improve the SNR.

The increment in SNR can be described by the SNR gain before and after matched filtering as [43]

$$GSNR = \frac{SNR_a}{SNR_b} = TB, \quad (9)$$

where SNR_a and SNR_b denote the SNR before and after matched filtering, respectively. Equation 9 indicates that a larger TB leads to a higher SNR gain. However, in practice, the achievable TB is constrained by factors such as the bandwidth of the ultrasonic transducer and the allowable excitation duration. Therefore, the signal duration and bandwidth should be jointly considered in the design of coded excitations to maximize the TB , thereby improving the SNR.

2.4.2 Frequency Modulation

Based on the modulation type, coded excitation techniques can generally be classified into frequency modulation (FM), phase modulation (PM), and amplitude modulation (AM) [18,19,20]. FM signals exhibit a varying instantaneous frequency, which provides enhanced pulse compression performance. Among various FM techniques, linear frequency modulation (LFM), in which the instantaneous frequency varies linearly with time, is one of the simplest and most widely used FM methods in UT. The LFM signal, commonly known as

the chirp $s(t)$, can be defined as

$$s(t) = a(t)e^{j2\pi(f_0t + \frac{B}{2T}t^2)}, \quad -\frac{T}{2} \leq t \leq \frac{T}{2}, \quad (10)$$

where f_0 denotes the center frequency of the chirp, T is the signal duration, B represents the bandwidth, and $a(t)$ is generally assumed to be a rectangular window with unit amplitude. The corresponding real-valued chirp can also be written as $s(t) = a(t)\cos(f_0t + \frac{B}{2T}t^2)$. The instantaneous frequency of $s(t)$ is given by

$$f_i = \frac{d(f_0t + \frac{B}{2T}t^2)}{dt} = f_0 + \frac{B}{T}t, \quad (11)$$

where $\frac{B}{T}$ is the chirp rate that determines the rate of frequency variation. Consequently, the chirp sweeps over the frequency range $[f_0 - \frac{B}{2}, f_0 + \frac{B}{2}]$. Assuming that $a(t) = 1$, the matched filter output of the chirp can be written as [43]

$$R(\tau) = Te^{-j2\pi f_0\tau} \frac{\sin[\pi B\tau(1 - \frac{|\tau|}{T})]}{B\pi\tau}, \quad (12)$$

where $R(\tau)$ denotes the matched filter output and τ is the time delay. According to Equation 12, $R(\tau)$ exhibits a sinc-like shape. A comparison of a chirp and two sinusoidal signals after matched filtering is shown in Figure 11. Increasing the duration of a sinusoidal signal increases the transmitted signal energy but also significantly degrades the temporal resolution after pulse compression, as can be observed by comparing Figures 11c and f. In contrast, a chirp with the same duration preserves the temporal resolution. Moreover, because the chirp has a much larger TB the sinusoidal signals, it provides a higher SNR gain according to Equation (9).

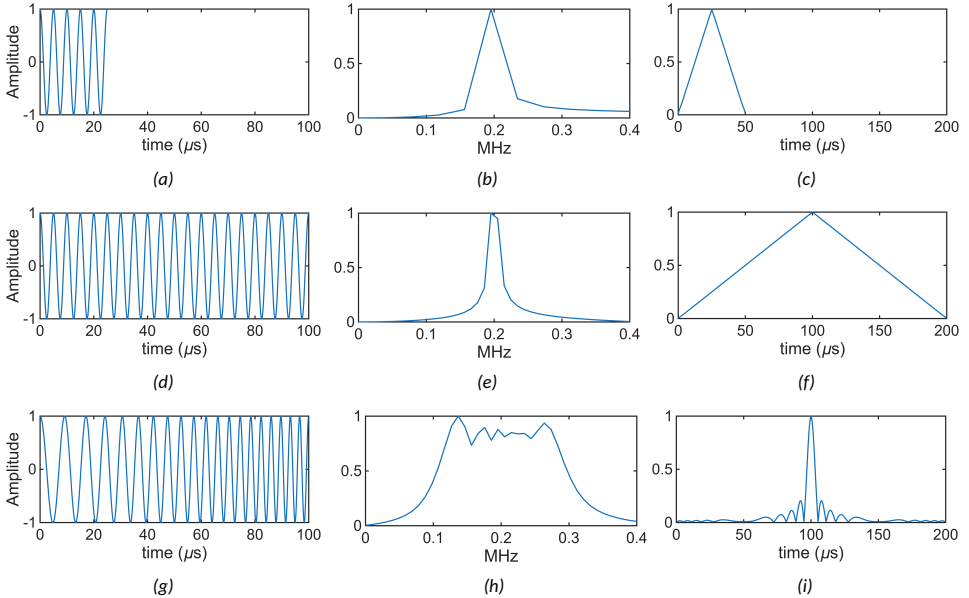


Figure 11: Comparison of a chirp and two sinusoidal signals after matched filtering. (a)–(c) show the time domain waveform, frequency spectrum, and matched filter output of a 5-cycle sinusoidal signal with a frequency of 200 kHz. (d)–(f) show the corresponding results for a 20-cycle sinusoidal signal with the same frequency. (g)–(i) show the corresponding results for a chirp with a center frequency of 200 kHz and a bandwidth of 200 kHz. All amplitudes are normalized.

2.4.3 Phase Modulation

PM maintains a constant carrier wave amplitude and frequency, while achieving modulation through controlled phase shifts. These phase shifts enable low sidelobe levels after pulse compression. Various PM codes have been applied in UT, among which Barker and Golay codes are the most widely adopted. Barker codes possess several desirable properties. As binary phase codes, they require less complex hardware implementation than multiphase codes. Furthermore, they exhibit the lowest sidelobe levels among all known binary sequences of the same length [20]. A Barker code is a binary sequence composed of bits -1 and 1 . Known Barker codes exist for only seven sequence lengths, with the maximum sequence length limited to 13. Table 1 summarizes all known Barker codes, where the codes of lengths 2 and 4 each have two different bit sequences. For a Barker code B_N of length N , composed of bit $b_1, b_2, \dots, b_N$, its autocorrelation, C_B is defined as [44]:

$$C_B(m) = \sum_{n=1}^N b_n b_{n+m} = \begin{cases} N, & m = 0 \\ 0 \text{ or } \pm 1, & 0 < m < N. \\ 0, & m \geq N \end{cases} \quad (13)$$

Equation 13 shows that the peak of the autocorrelation equals the code length N , whereas the sidelobe magnitude is limited to 1. This demonstrates the excellent sidelobe suppression capability of Barker codes. Furthermore, the autocorrelation peak increases with the code length, indicating that longer Barker codes produce higher peak amplitudes. Since the longest known Barker code has a length of 13, the maximum achievable autocorrelation peak is therefore limited to 13. Figure 12 presents the 5-bit Barker code and its corresponding autocorrelation. It can be observed that the autocorrelation peak equals the code length of 5, while the sidelobe magnitude is 1.

Table 1: Barker codes.

Code Length	Code Bits
2	{1,1} or {1,-1}
3	{1,1,-1}
4	{1,1,1,-1} or {1,1,-1,1}
5	{1,1,1,-1,1}
7	{1,1,1,-1,-1,1,-1}
11	{1,1,1,-1,-1,-1,1,-1,1,-1,1}
13	{1,1,1,1,1,-1,-1,1,1,-1,1,1,1}

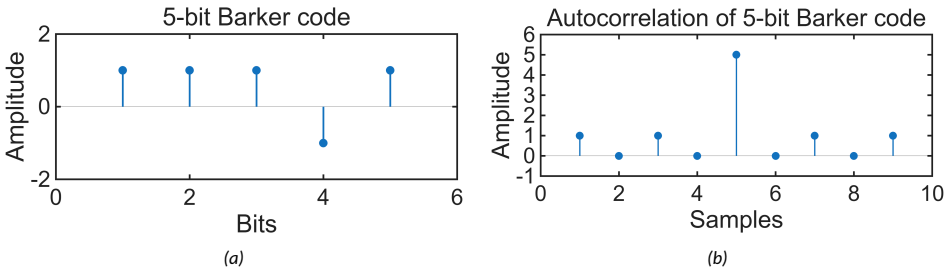


Figure 12: 5-bit Barker code and its autocorrelation. (a) 5-bit Barker code. (b) Autocorrelation.

Although Barker codes exhibit relatively low sidelobe levels, sidelobes remain unavoid-

able. In contrast, Golay codes achieve zero sidelobes while maintaining a high autocorrelation peak. Golay codes consist of a pair of binary sequences consisting of 1 and -1 . The autocorrelations of the two sequences have identical peak values, whereas their sidelobes have equal magnitudes but opposite signs. Consequently, when the two autocorrelations are summed, the sidelobes cancel each other completely while the peaks add constructively [23]. For a Golay code pair consisting of two sequences GA_N and GB_N , where GA_N is composed of the bits $Ga_1, Ga_2, \dots, Ga_N$ and GB_N is composed of bits $Gb_1, Gb_2, \dots, Gb_N$, their autocorrelations are given by

$$\begin{aligned} C_{GA}(m) &= \sum_{n=1}^N Ga_n Ga_{n+m}, \\ C_{GB}(m) &= \sum_{n=1}^N Gb_n Gb_{n+m}, \end{aligned} \quad (14)$$

where C_{GA} and C_{GB} are autocorrelations of GA_N and GB_N respectively. Summing C_{GA} and C_{GB} yields

$$C_G(m) = C_{GA}(m) + C_{GB}(m) = \begin{cases} 2N, & m = 0 \\ 0, & m \neq 0. \end{cases} \quad (15)$$

Equation 15 shows that the peak value of C_G is $2N$ for a Golay code pair of length N , while sidelobes are eliminated. However, this sidelobe-free performance requires two transmissions, reducing the measurement efficiency by a factor of two. Table 2 lists examples of length-8 and length-16 Golay codes. It should be noted that multiple Golay code pairs may exist for the same length. Figure 13 illustrates GA_8 , GB_8 , and their autocorrelations. The sum of autocorrelation shows a peak value of 16, which is twice the length of the code, while all sidelobes are canceled.

Table 2: Examples of 8-bit and 16-bit Golay codes.

Code Notation	Code Bits
GA_8	{1,1,1,-1,1,1,-1,1}
GB_8	{1,1,1,-1,-1,1,1,-1}
GA_{16}	{1,1,1,-1,1,1,-1,1,1,1,-1,-1,-1,-1,1,1}
GB_{16}	{1,1,1,-1,1,1,-1,1,-1,-1,1,1,1,1,-1,1}

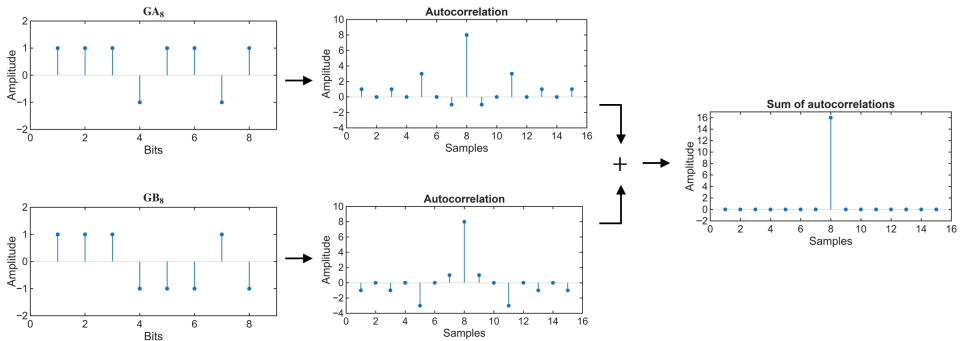


Figure 13: Sidelobe cancellation using GA_8 and GB_8 [44].

In practical UT, the selected code is used to modulate a carrier wave to generate the coded excitation. This process, referred to as coding, can be expressed as

$$s(t) = \sum_{i=1}^N c_i w(t - iT), \quad 0 \leq t \leq NT, \quad (16)$$

where $s(t)$ is the coded excitation, c_i is the i -th code bit, and $w(t)$ is the carrier wave with duration T . After transmission and reception by the ultrasonic transducer, the coded excitation is decoded using the matched filter or correlation to improve the SNR. The coding and decoding processes for the 7-bit Barker code are illustrated in Figure 14.

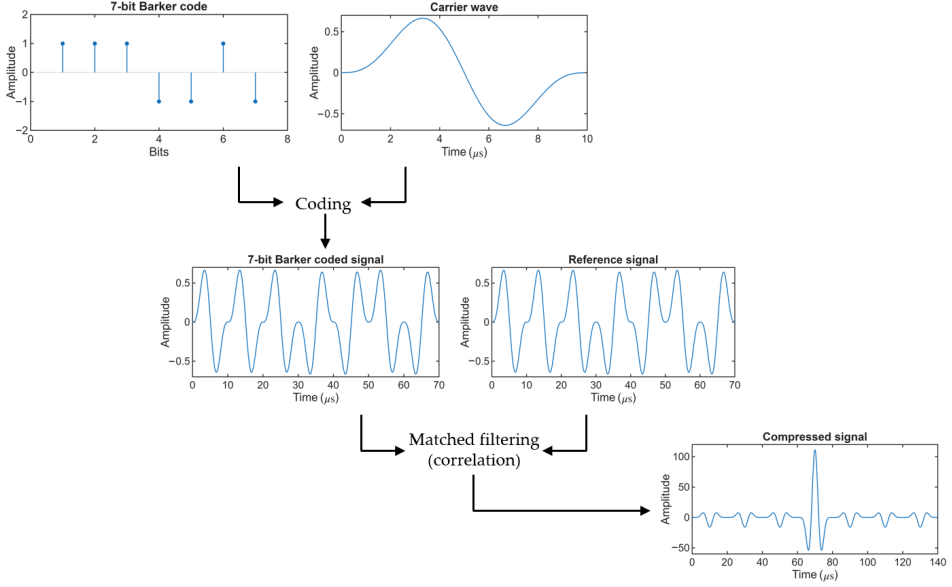


Figure 14: Coding and decoding using the 7-length Barker code [44].

2.4.4 Evaluation Criteria

Several metrics can be used to evaluate the performance of coded excitations, such as peak sidelobe level (PSL), SNR, SNR gain, integrated sidelobe level, and temporal resolution. Among these, PSL, temporal resolution, and SNR are the most commonly adopted. The PSL characterizes the capability of coded excitations to suppress self-induced interference after pulse compression. It can be defined as the logarithmic ratio between the amplitude of the mainlobe and the maximum amplitude of sidelobes [45]:

$$PSL = 20 \log_{10} \frac{A_m}{A_s}, \quad (17)$$

where A_m is the mainlobe amplitude, and A_s is the maximum amplitude of sidelobes. Temporal resolution is commonly evaluated by the full width at half maximum (FWHM) of the decoded signal envelope [46]. The SNR can be defined as the ratio between the mainlobe amplitude of the decoded signal and the root mean square (RMS) of the noise-only region in decoded signal [47]:

$$SNR = 20 \log_{10} \frac{A_m}{A_{rms}}, \quad (18)$$

where A_{rms} denotes the RMS value of the noise. In addition to PSL, temporal resolution, and SNR, magnitude gain (MG) can also be used to evaluate the coded excitation performance [20]:

$$MG = 20 \log_{10} \frac{A_m}{A_p}, \quad (19)$$

where A_p is the peak amplitude of the matched filter output of the carrier wave. Figure 15 compares the decoded outputs of a Hanning-windowed 2.25 MHz sinusoidal pulse and a 5-bit Barker-coded excitation generated using the same pulse as the carrier wave, illustrating the mainlobe, sidelobes, and temporal resolution.

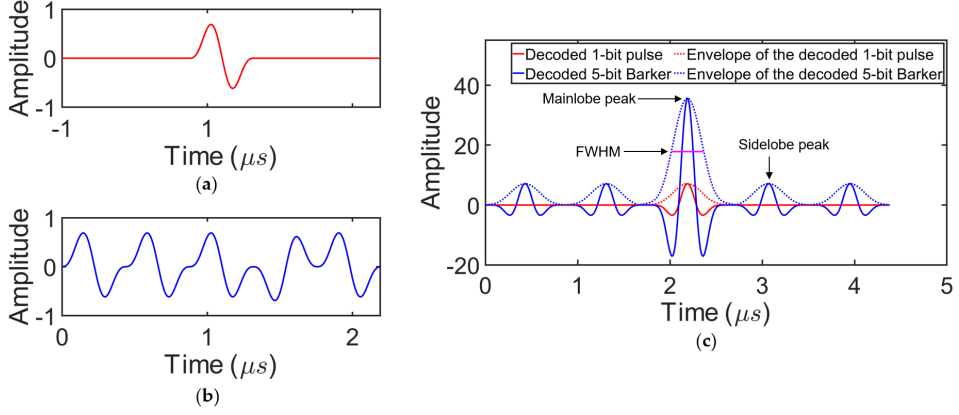


Figure 15: Comparison of a 1-bit pulse and a Barker-coded excitation [44]. (a) shows the 1-bit carrier wave. (b) shows the 5-bit Barker coded signal. (c) represents the decoded signals.

3 Methods

This chapter presents the methods developed for the three approaches investigated in this thesis to improve the efficiency of UT. The first subsection introduces the FrFT-based method for separating overlapping ultrasonic signals caused by reduced transmission time delays. The second subsection presents the use of coded excitations to reduce the number of repeated signal acquisitions required for temporal averaging. The final subsection introduces a novel convolved excitation coding technique for parallel transmission.

3.1 Signal Separation using the FrFT

3.1.1 Signal Overlap under the Reduced Time Delay

As discussed in Chapter 1, reducing the time delay between successive transmissions improves the efficiency of array transducer inspections but can cause overlap between the received ultrasonic signals. Figure 1 illustrates the conventional sequential transmission strategy and the reduced-delay transmission strategy. In the conventional approach (Figure 1b), a transmission delay greater than the duration of the transmitted signal is used to prevent interference between adjacent transmissions. By reducing the transmission delay to a shorter interval (Figure 1c), the overall measurement time can be reduced. However, reducing the delay results in overlap between adjacent received signals, making subsequent signal interpretation and analysis more challenging.

Due to the requirement that transmitted signals should be identical in many applications, they consequently share the same frequency spectrum [3, 32]. As a result, the received signals overlap in both the time and frequency domains. In such cases, conventional time windowing or frequency filtering techniques are insufficient for effective signal separation. In contrast, the FrFT represents signals in the fractional Fourier domain, enabling effective separation of chirps overlapping simultaneously in both the time and frequency domains [48].

3.1.2 Related Works

The concept of the fractional Fourier transform (FrFT), a generalized Fourier transform, was first proposed in 1920s; however, it was not until the 1980s that it gained widespread attention, largely due to the significant contributions of Namias [49, 50]. For practical engineering applications, the FrFT must be discretized, analogous to the use of the FFT is used for efficient computation for the Fourier transform. The development of efficient digital calculation algorithms for the FrFT by researchers such as Ozaktas and Pei has paved the way for broad engineering applications [51–54]. Since then, the FrFT has been successfully applied in a wide range of fields. For example, Iwai et al. used the FrFT for the fingerprints image analysis [55]. Subramaniam et al. proposed an FrFT-based filtering scheme for elastogram denoising [56]. Norman et al. compensated the dispersion in optical coherence tomography using the FrFT [57].

Recent studies demonstrate that the FrFT is an effective tool for separating overlapping ultrasonic signals. Harput et al. demonstrated its use in human tooth imaging, where it effectively separated overlapping ultrasonic signals to improve image clarity [58]. Cowell et al. employed the FrFT to separate ultrasonic echoes created by transmission of a chirp signal in a steel plate [59]. Amir et al. used the FrFT to isolate ultrasonic echo signals in simulations performed using the software Field II [60]. Zheng et al. achieved signal separation in ultrasonic range systems using the FrFT [61]. Pachauri investigated the separation of synthetic ultrasonic signals using the short-time FrFT [62]. In addition, Li et al. employed the FrFT to monitor elbow erosion of pipes by separating useful signal compo-

nents from noise [63].

Despite these advancements, existing studies on the application of the FrFT for ultrasonic signal separation have mainly focused on single-transducer configurations or simulations. The application of the FrFT to array transducer inspection systems under reduced transmission delays has not been systematically investigated.

3.1.3 Principles of the FrFT

The FrFT can be defined in several equivalent forms, such as a linear integral transform and differential equation solutions [64, 65]. The definition using the linear integral transform is expressed as

$$X_p(u) = A_\alpha \int_{-\infty}^{+\infty} s(t) \exp[j\pi \cot \alpha (t^2 + u^2) - 2ut \csc \alpha] dt, \quad (20)$$

where

$$A_\alpha = \frac{\exp[\frac{-j\pi \operatorname{sgn}(\sin \alpha)}{4} + \frac{j\alpha}{2}]}{|\sin \alpha|^{\frac{1}{2}}} \quad (21)$$

and

$$\operatorname{sgn}(t) = \begin{cases} -1, & t < 0 \\ 0, & t = 0. \\ 1, & t > 0 \end{cases} \quad (22)$$

In Equation 20, $s(t)$ is the time domain signal, j is the imaginary unit, and u is the fractional domain variable. The parameter α denotes the rotation angle of the FrFT, and the corresponding fractional power is defined as $p = \frac{2\alpha}{\pi}$. Since α ranges from $-\pi$ to π , p ranges from -2 to 2 with 4 as the period. By comparing Equations 20 and 5, it can be observed that a set of chirps with a chirp rate of $\cot \alpha$ serves as the kernel of the FrFT, analogous to the $\exp(-2j\pi ft)$ of the CTFT. Unlike the CTFT, the FrFT includes an additional parameter α , which rotates the signal representation between the time domain and the frequency domain. When $\alpha = \frac{\pi}{2}$, the FrFT becomes the CTFT. In the following, the FrFT with the rotation angle α is denoted as Fr^α . The FrFT possesses several important properties [66, 67]:

- $Fr^0[f(t)] = s(t)$.
- $Fr^{\frac{\pi}{2}}[s(t)]$ equals the CTFT of $s(t)$.
- $Fr^{-\frac{\pi}{2}}[s(t)]$ equals the inverse CTFT of $s(t)$.
- Linearity: $Fr^\alpha[\sum_{k=1}^N a_k s_k(t)] = \sum_{k=1}^N a_k \{Fr^\alpha[s_k(t)]\}$.
- Additivity: $Fr^{\alpha+\beta}[s(t)] = Fr^\alpha\{Fr^\beta[s(t)]\}$.
- Rotation invariance: the WVD of $Fr^\alpha[s(t)]$ equals the WVD of $s(t)$ with the axis rotated by α .

With the rotation invariance property, it is known that the FrFT does not change the energy distribution of a signal in the time-frequency domain. This property provides a geometric interpretation of the FrFT as a rotation in the time-frequency plane, as illustrated in Figure 16a, where the fractional domain lies between the time domain and the frequency domain. For a linear chirp signal, the signal energy becomes maximally concentrated at an optimal rotation angle in the fractional domain, as shown in Figure 16b.

This optimal angle α_o is given by $-\arctan \frac{T}{B}$, where T and B are the time duration and bandwidth of the chirp, respectively.

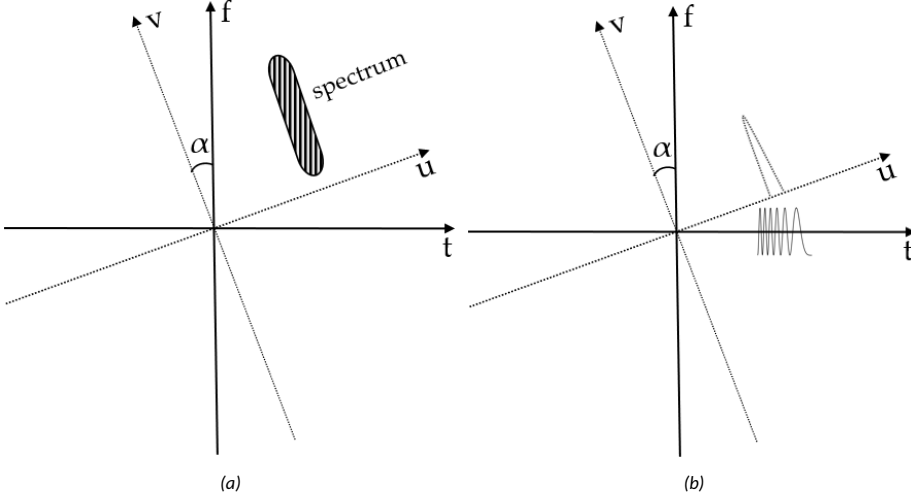


Figure 16: Illustration of the FrFT. (a) Rotation invariance: the location of spectral components remains unchanged with the axis rotation. (b) Chirp compression: a long chirp signal is compressed as a narrow peak by the FrFT [68].

The availability of fast digital algorithms is crucial for the practical application of the FrFT. Among various fast algorithms, Ozaktas' algorithm is one of the most widely used due to its high accuracy and low computational complexity [59]. The preliminary step of Ozaktas' algorithm is dimensional normalization, which equalizes the support lengths of the FrFT in the fractional domain regardless of the rotation angles, thereby significantly reducing the computational complexity. Next, Equation 20 is decomposed into a set of simpler equations. Finally, the decomposed equations are discretized using Shannon's interpolation formula to enable numerical computation. For a digital signal with N sampling points, the computational complexity of the algorithm is $O(N \log N)$, which is comparable to that of the FFT. The optimal angle α_{od} for a digital chirp using Ozaktas' algorithm is given by:

$$\alpha_{od} = -\arctan \frac{f_s T^2}{BN}, \quad (23)$$

where f_s is the sampling frequency for the digital chirp.

3.1.4 Procedure of the FrFT-Based Signal Separation

Using the properties of the FrFT described above, signals overlapping in both the time and frequency domains can be separated. It should be noted that successful separation requires the overlapping signals to be distinguishable in the time-frequency plane, even though they overlap in both the time and frequency domains. Therefore, for two signals with identical spectra, a time delay is required to enable their separation. As illustrated in Figure 17, the separation procedure consists of three main steps:

- Transform overlapping signals into the fractional domain, with a rotation angle α . As shown in Figure 17, the signal A and B overlap in both the time and frequency domains but become distinguishable in the fractional domain.

- Separate the FrFT transformed signals using a window function. In this work, a rectangular window is used to isolate each signal in the fractional domain.
- Transform the separated fractional domain signals back to the time domain. This transformation is performed by the inverse FrFT, which corresponds to the FrFT with an angle equal to the negative of that used in the first step, $-\alpha$. The inverse FrFT is expressed as

$$s(t) = Fr^{\alpha-\alpha}[s(t)] = Fr^{-\alpha}\{Fr^{\alpha}[s(t)]\}. \quad (24)$$

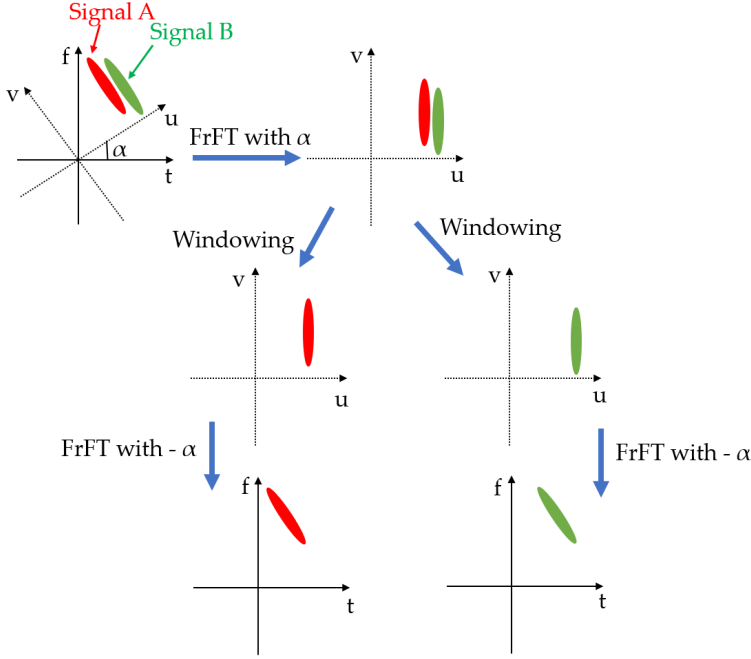


Figure 17: Schematic diagram of signal separation using the FrFT [68].

3.2 Number of Temporal Averages Reduction using Coded Excitations

3.2.1 Repeated Measurements Caused by Temporal Averaging

In UT, measured signals are often contaminated by noise, making subsequent signal analysis difficult or even impossible. A widely used approach for extracting useful signal components or improving the SNR is to perform repeated measurements and average the acquired signals. This process is commonly referred to as temporal averaging or signal averaging. Temporal averaging typically relies on the following assumptions [69]:

- The signal and noise are uncorrelated.
- The signal remains unchanged throughout repeated measurements.
- The noise is random with zero mean.

Although these assumptions may only be approximately satisfied in practice, temporal averaging has been shown to remain effective under minor deviations of these assumptions.

Consider a signal s contaminated by AWGN, from which N repeated measurements are acquired. The averaged signal $\bar{s}$ is given by

$$\bar{s} = \frac{\sum_{n=1}^N s_n}{N}, \quad (25)$$

where s_n is the signal acquired from the n -th measurement. If the variance of noise in s is σ^2 , the variance of noise in $\bar{s}$ is $\frac{\sigma^2}{N}$. Since the noise is assumed to be zero-mean, its RMS value is equal to the square root of its variance. Based on the definition of SNR in Equation 18, the SNR from a single measurement is

$$SNR_s = 20 \log_{10} \frac{A_m}{\sigma}, \quad (26)$$

and the SNR from the averaged signal is

$$SNR_{\bar{s}} = 20 \log_{10} \frac{\sqrt{N} A_m}{\sigma}, \quad (27)$$

where A_m is the peak amplitude of the signal. Comparing the above equations shows that temporal averaging improves the SNR by $20 \log_{10}(\sqrt{N})$. However, this improvement comes at the cost of an increased signal acquisition time because repeated signal acquisitions are required. For weak signals, the number of required averages can become prohibitively large, thereby significantly reducing the measurement efficiency.

Unlike temporal averaging, coded excitations improve the SNR by increasing the transmitted signal energy and subsequently applying pulse compression, thereby reducing the number of required averages to achieve a given SNR. Figure 18 illustrates this concept by comparing conventional temporal averaging with coded excitations in measurements. It should be noted that coded excitations are generally longer than conventional pulse excitations, and therefore increase the signal acquisition time of an individual measurement. However, increasing the code length generally provides greater SNR enhancement. Consequently, practical coded excitation design requires a trade-off between code length and SNR improvement.

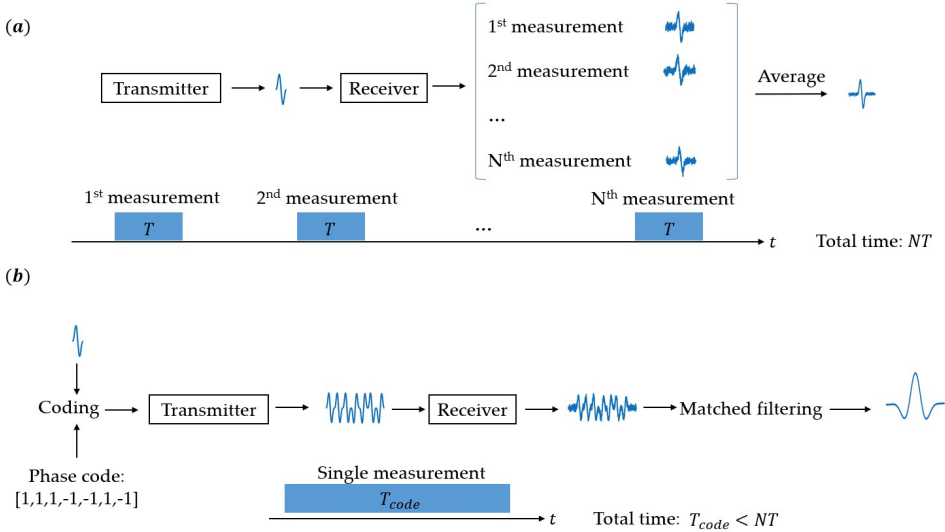


Figure 18: Comparison between temporal averaging and coded excitations. (a) Temporal averaging. (b) Measurement using coded excitations.

3.2.2 Related Works

Originally developed for radar applications, coded excitations were later adopted in ultrasonic systems. The earliest application of coded excitations in ultrasound can be traced back to Takeuchi's work in 1979, where the technique was introduced for medical ultrasound systems [70]. Several studies have explored the potential of coded excitations to reduce the number of required averages. Song et al. explored the application of Golay codes to accelerate bone ultrasonic thickness measurements and demonstrated that the overall measurement time could be reduced from minutes to seconds [71]. Nevertheless, only the 16-bit Golay code was investigated. Xia et al. evaluated the enhancement of SNR provided by a 13-bit Barker code and compared it with that achieved by signal averaging in ultrasonic thickness measurements [72]. The 13-bit Barker code demonstrated superior performance. Nevertheless, only a single Barker code was investigated and the number of temporal averages was kept constant throughout the study. Zhang et al. compared the SNR achieved using an 8-bit Golay code with that obtained from a single-cycle pulse averaged over a fixed number of measurements in airborne ultrasonic inspection [73]. Although coded excitations improved the SNR without reducing the measurement efficiency, only a single Golay code length and a fixed number of averages were considered.

Despite these studies, existing investigations have generally been limited to a single code type, fixed code lengths, or a fixed number of temporal averages. Consequently, the influence of code type, code length, excitation voltage, and the number of temporal averages on SNR improvement remains unclear. This study therefore systematically investigates these factors through comprehensive experimental measurements and quantifies the extent to which coded excitations can reduce the need for temporal averaging.

3.3 Parallel Transmission using Convolved Coded Excitations

3.3.1 Parallel Transmission using Coded Excitations

For a UT system consisting of multiple transducers, sequential transmission significantly limits measurement efficiency, particularly as the number of transducer increases. In contrast, parallel transmission substantially improves measurement efficiency by allowing all transducers to transmit simultaneously. However, simultaneous transmission causes severe signal overlap, making reliable signal separation a major challenge.

In parallel transmission, the transmission delay is zero. Consequently, transmitted signals are no longer distinguishable in the time-frequency plane, rendering FrFT-based signal separation ineffective. Therefore, an alternative separation strategy is required. Although coded excitations are primarily employed to improve the SNR, the orthogonality between different coding sequences also enables the separation of signals transmitted simultaneously from multiple transducers, making coded excitations suitable for parallel transmission.

3.3.2 Related Works

Golay codes and Barker codes are among the most widely used codes in UT owing to their favorable correlation properties, which enable efficient pulse compression [74–76]. Nevertheless, the maximum length of Barker codes is limited to 13, and the length of Golay codes is not arbitrary selectable. Furthermore, for Golay codes, a twofold increase in compressed signal amplitude requires a corresponding doubling of the code length. These limitations reduce the flexibility in selecting appropriate code lengths to satisfy different SNR requirements in practical measurements. To overcome these limitations, convolved codes have recently been proposed. By convolving existing codes, they provide greater flexibility in code length while achieving higher MG. Kim et al. first proposed the Barker-

convolved Golay code (BCGC) for medical ultrasound imaging [77]. However, the proposed BCGC was evaluated only through simulations and was constructed exclusively using the 3-bit Barker code. Zeng et al. subsequently performed a comprehensive theoretical investigation considering all available Barker code lengths [20] and later evaluated the BCGC for pipeline inspection [21]. Nevertheless, BCGC are not suitable for parallel transmission.

Reliable parallel transmission requires orthogonal or near-orthogonal coding sequences to enable signal separation. Mutually orthogonal Golay complementary codes (MOGCC) provide an orthogonal coding scheme by enabling simultaneous transmission from two transducers while preserving the SNR enhancement capability of conventional Golay codes [78]. For example, Tasinkevych et al. successfully applied MOGCC in medical ultrasound, achieving improved imaging efficiency [30]. However, although MOGCC improve measurement efficiency through parallel transmission, their SNR performance remains inferior to that of BCGC with comparable code lengths. Therefore, this work develops a convolved coding scheme that combines the high MG of convolved coding with the orthogonality of MOGCC, thereby enabling parallel transmission while retaining a high MG.

3.3.3 Barker Convolved Mutually Orthogonal Golay Complementary Code

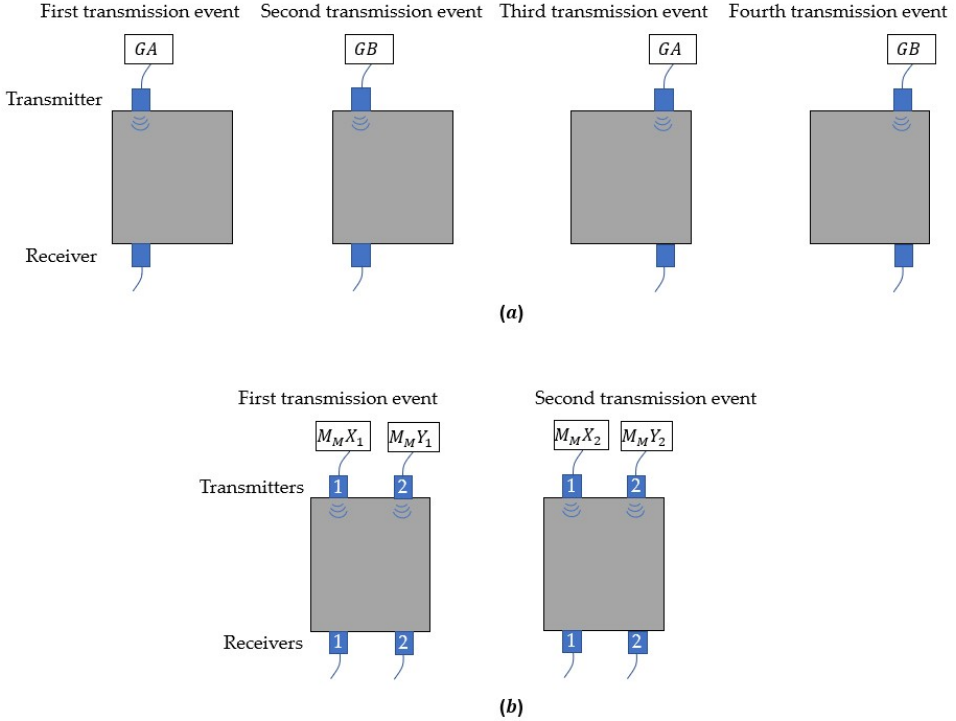


Figure 19: Comparison of conventional Golay codes and BMOGCC for measurements at two measurement locations. (a) Conventional Golay codes requiring four transmission events. (b) BMOGCC requiring two transmission events through parallel transmission.

The Barker convolved mutually orthogonal complementary Golay code (BMOGCC) is proposed in this thesis by convolving the Barker code with the MOGCC. When conventional Golay codes consisting of two sequences, GA and GB , are employed, each transducer requires two transmission events to achieve zero sidelobes. Consequently, four trans-

mission events are required to complete the inspection at two measurement locations, as illustrated in Figure 19a. In contrast, BMOGCC consists of four sequences $B_N M_M X_1$, $B_N M_M X_2$, $B_N M_M Y_1$, and $B_N M_M Y_2$, enabling the simultaneous transmission of two transducers and thereby improving the measurement efficiency by a factor of two (Figure 19b). As a result, each receiver receives overlapping signals originating from both transmitters. The orthogonality property inherited from the MOGCC enables these overlapping signals to be separated into distinct components corresponding to each transmitter. As an example, the procedure for separating the signal transmitted from transmitter 1 at receiver 1 is illustrated in Figure 20. The process consists of five steps, beginning with the generation of the BMOGCC followed by four decoding steps.

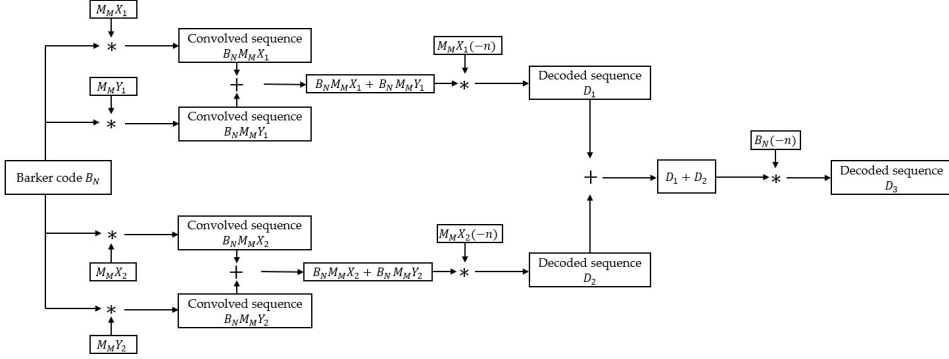


Figure 20: Generation and decoding of BMOGCC for separating the signal transmitted from transmitter 1 at receiver 1.

Step 1: Each of the four MOGCC sequences, $M_M X_1$, $M_M X_2$, $M_M Y_1$, and $M_M Y_2$ is convolved with the Barker code B_N . The resulting sequences, namely $B_N M_M X_1$, $B_N M_M X_2$, $B_N M_M Y_1$, and $B_N M_M Y_2$, are given by

$$\begin{aligned} B_N M_M X_1 &= M_M X_1 * B_N, B_N M_M X_2 = M_M X_2 * B_N, \\ B_N M_M Y_1 &= M_M Y_1 * B_N, B_N M_M Y_2 = M_M Y_2 * B_N. \end{aligned} \quad (28)$$

The resulting BMOGCC has a code length $N_{BM} = N + N - 1$, where N and M are the lengths of the Barker code and MOGCC, respectively.

Step 2: The received signals corresponding to the first transmission event $B_N M_M X_1$ and $B_N M_M Y_1$ are summed. Similarly, the received signals corresponding to the second transmission event $B_N M_M X_2$ and $B_N M_M Y_2$ are summed.

Step 3: Two summations obtained in the previous step are convolved with $M_M X_1(-n)$ and $M_M X_2(-n)$, which are time-reversed versions of $M_M X_1$ and $M_M X_2$, respectively, yielding the decoded sequences D_1 and D_2 , as follows:

$$\begin{aligned} D_1 &= (B_N M_M X_1 + B_N M_M Y_1) * M_M X_1(-n) = B_N * (M_M X_1 + M_M Y_1) * M_M X_1(-n), \\ D_2 &= (B_N M_M X_2 + B_N M_M Y_2) * M_M X_2(-n) = B_N * (M_M X_2 + M_M Y_2) * M_M X_2(-n). \end{aligned} \quad (29)$$

Step 4: The decoded sequences D_1 and D_2 are summed.

Step 5: The summed sequence from Step 4 is convolved with the time-reversed Barker

code $B_N(-n)$ to obtain the final decoded sequence D_3 :

$$\begin{aligned}
D_3 &= (D_1 + D_2) * B_N(-n) \\
&= B_N * [(M_M X_1 + M_M Y_1) * M_M X_1(-n) + (M_M X_2 + M_M Y_2) * M_M X_2(-n)] \\
&\quad * B_N(-n) \\
&= B_N * B_N(-n) * [M_M X_1 * M_X(-n) + M_M X_2(-n) * M_M X_2 \\
&\quad + M_M Y_1 * M_M X_1(-n) + M_M Y_2(-n) * M_M X_2] \\
&= B_N * B_N(-n) * [M_M X_1 * M_X(-n) + M_M X_2(-n) * M_M X_2] \\
&= C_B * C_{XY}.
\end{aligned} \tag{30}$$

Equation 30 is obtained by exploiting the orthogonality property of the MOGCC, $M_M Y_1 * M_M X_1(-n) + M_M Y_2(-n) * M_M X_2 = 0$ [84]. $C_B = B_N * B_N(-n)$ is the autocorrelation of the Barker code. C_{XY} denotes the decoded result of the MOGCC, given by

$$\begin{aligned}
C_{XY}(m) &= (M_M X_1 + M_M Y_1) * M_M X_1(-n) + (M_M X_2 + M_M Y_2) * M_M X_2(-n) \\
&= M_M X_1 * M_X(-n) + M_M X_2(-n) * M_M X_2 \\
&= \begin{cases} 2M, & m = 0 \\ 0, & m \neq 0, \end{cases}
\end{aligned} \tag{31}$$

where m is the correlation delay.

The decoded result D_3 of the BMOGCC equal to the convolution of decoded results of the Barker code and the MOGCC. Consequently, the proposed BMOGCC achieves a higher pulse compression peak than either the Barker code or the MOGCC alone. Moreover, its pulse compression peak is identical to that of the BCGC, which is constructed by convolving Barker codes with Golay codes [82]. Furthermore, the proposed BMOGCC preserves the parallel transmission capability of the MOGCC, enabling inspections to be completed in half the transmission events required by either the BCGC or conventional Golay codes.

Figure 21 presents an example of the BMOGCC decoding process using $B_7 M_4$. The code $B_7 M_4$ is constructed by convolving the 7-bit Barker code B_7 with the 4-bit MOGCC listed in Table 3. The resulting $B_7 M_4$ consists of four sequences $B_7 M_4 X_1$, $B_7 M_4 X_2$, $B_7 M_4 Y_1$, and $B_7 M_4 Y_2$. The decoded sequence D_3 of the 10-bit $B_7 M_4$ exhibits a peak amplitude of 56, which is significantly higher than the corresponding peaks of 13 and 32 obtained with the 13-bit Barker code and the 16-bit MOGCC, respectively. These results demonstrate that the proposed BMOGCC provides a substantially higher compressed peak than either Barker codes or MOGCC with a shorter code length while retaining signal separability required for parallel transmission.

Table 3: Examples of 4-bit and 8-bit MOGCC.

Coded Sequence Notation	Code Bit Value
$M_4 X_1$	1, -1, -1, -1
$M_4 Y_1$	-1, 1, -1, -1
$M_4 X_2$	-1, -1, 1, -1
$M_4 Y_2$	1, 1, 1, -1
$M_8 X_1$	1, 1, 1, -1, 1, 1, -1, 1
$M_8 Y_1$	1, -1, 1, 1, 1, -1, -1, -1
$M_8 X_2$	1, 1, 1, -1, -1, -1, 1, -1
$M_8 Y_2$	1, -1, 1, 1, -1, 1, 1, 1

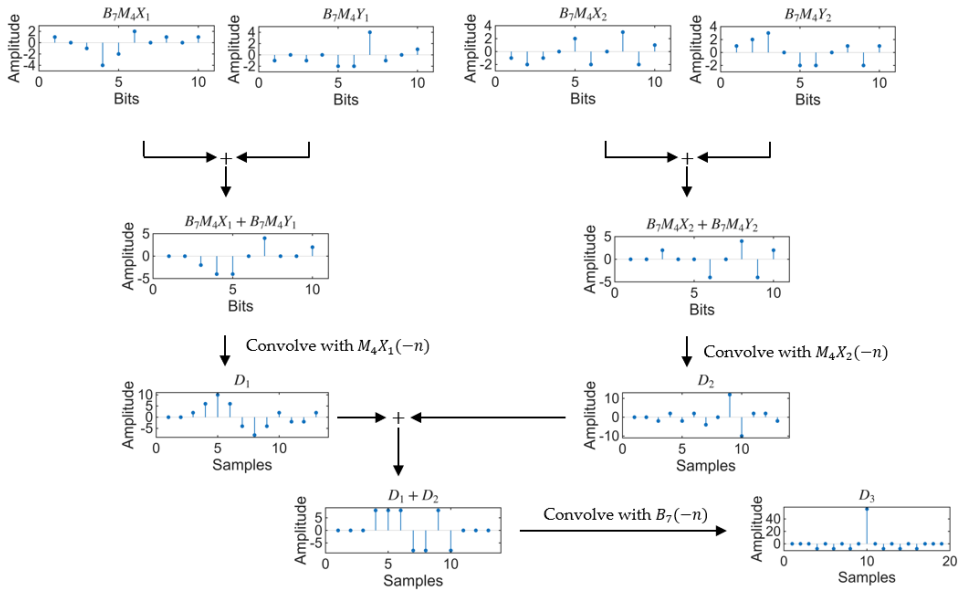


Figure 21: Decoding for B_7M_4 .

4 Experiments and Results

This chapter presents the experimental validation of the three methods proposed in this thesis. The first subsection investigates FrFT-based signal separation for overlapping ultrasonic signals in plate inspections. The second subsection evaluates the effectiveness of coded excitations in reducing the need for temporal averaging. The final subsection validates the capability of the proposed BMOGCC for parallel transmission in bulk-wave inspections.

4.1 FrFT-Based Signal Separation for UGWs Inspection of a Plate

4.1.1 Numerical Simulations

Numerical simulations of UGWs propagation in plates were carried out using the software Abaqus [79], as demonstrated in Figure 22. A three-dimensional finite element model was developed to represent a $1000 \text{ mm} \times 1000 \text{ mm} \times 2 \text{ mm}$ aluminum plate containing a centrally located circular hole with a diameter of 11 mm. The model was meshed using the C3D8R element type with element size of 1 mm. Generation of the A_0 mode UGWs was achieved by applying out-of-plane forces at two source nodes on the plate surface. The resulting out-of-plane displacements at two receiving nodes positioned on the surface of the plate were recorded as the received signals. To minimize interference from boundary reflections, both the source and receiving nodes were positioned with enough distance from the plate boundaries.

Two linearly increasing chirps, generated according to Equation 10, were used as excitations. The first excitation $chirp_{50}$ had a bandwidth 50 kHz, a center frequency of 100 kHz, a duration of 0.2 ms, and was windowed by a Hanning function. The second excitation $chirp_{100}$ retained the same center frequency, duration, and windowing function, but its bandwidth was increased to 100 kHz to evaluate the influence of excitation bandwidth on separation performance. The center frequency was selected to match the nominal frequency of the transducer used in the experiments, ensuring consistency between simulations and experimental measurements. Since the excitation frequency falls within the dispersive region of the plate's dispersion curves (Figure 23), waveform distortion occurs during wave propagation. Together with the scattering introduced by the circular hole, these effects provide a realistic simulation environment for evaluating the capability of the FrFT-based method to separate overlapping ultrasonic signals.

After establishing the finite element model and selecting the excitations, simulations were performed using transmission delays shorter than half of the excitation length between the two source nodes. The reduced transmission delays caused the received signals from the two source nodes to overlap in both the time and frequency domains. Different transmission delays produced different degrees of signal overlap, allowing the performance of the FrFT-based method to be evaluated under varying overlap conditions. The transmission delay settings used in each simulation are summarized in Table 4. Simulations 1–3 employed $chirp_{50}$ with three different transmission delays, whereas simulations 4–5 employed $chirp_{100}$ with two different transmission delays to investigate the influence of signal bandwidth on the separation performance. Simulations 1–5 were all conducted on the plate containing the circular hole. To provide reference signals for comparison with the separated signals, each source was excited individually to obtain the original non-overlapping signals. Furthermore, to examine the effect of dispersion on the separation performance, two $chirp_{50}$ were delayed and summed according to the signal locations observed in simulation 6, and the same separation procedure was subsequently applied to the resulting overlapping signals.

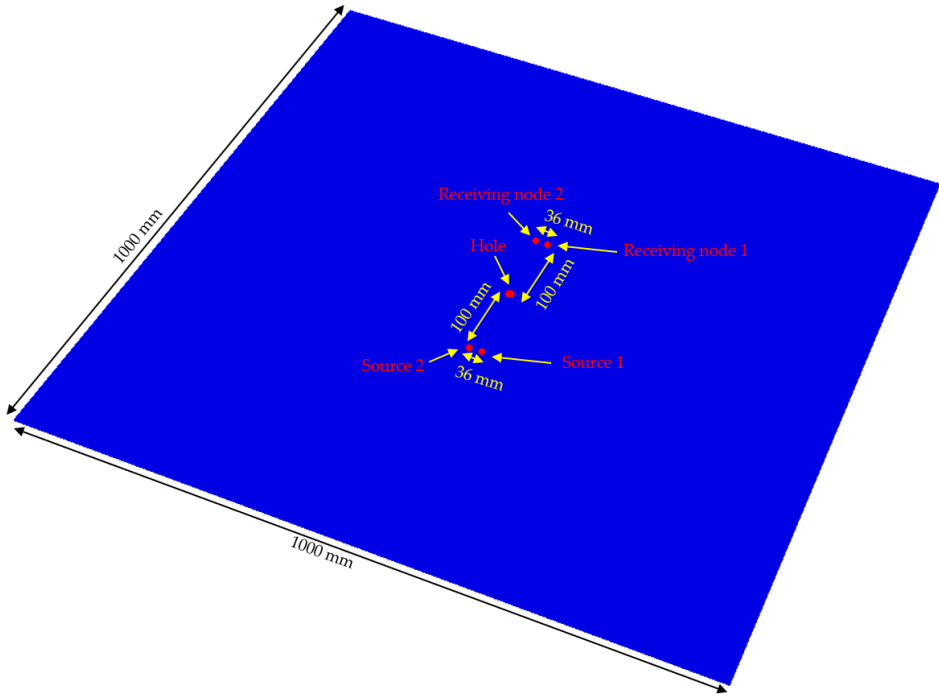


Figure 22: Numerical simulations performed in Abaqus: model of the aluminum plate.

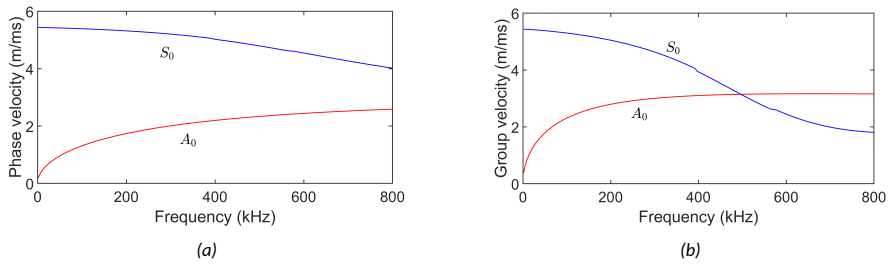


Figure 23: Dispersion curves of A_0 and S_0 modes in a 2 mm thick aluminum plate [68]. (a) Phase velocity. (b) Group velocity.

Table 4: Time delay settings.

Hole Presence	Simulation No.	Excitation Signal	Source 1	Source 2
Yes	1	<i>chirp</i> ₅₀	No delay	100 μ s delay
Yes	2	<i>chirp</i> ₅₀	No delay	80 μ s delay
Yes	3	<i>chirp</i> ₅₀	No delay	60 μ s delay
Yes	4	<i>chirp</i> ₁₀₀	No delay	80 μ s delay
Yes	5	<i>chirp</i> ₁₀₀	No delay	60 μ s delay
No	6	<i>chirp</i> ₅₀	No delay	60 μ s delay

The overlapping signals received at receiving node 1 were separated using the FrFT.

Here, S_{11} denotes the signal received at receiving point 1 when only source 1 was excited, while S_{21} denotes the signal received at receiving node 1 when only source 2 was excited. Figure 24 presents the detailed separation results for simulations 1–3. The columns correspond to simulations 1–3, arranged sequentially from left to right. The first row shows the original reference signals obtained by exciting the two sources individually. The second row presents the overlapped signals obtained by summing the two reference signals. The signals were transformed into the fractional domain using the optimal rotation angle calculated from Equation 23, where the signal components were compressed into distinct peaks, as shown in the third row. As the delay decreases, the overlap between the compressed peaks becomes more pronounced. The individual peaks were isolated using rectangular windows, whose boundaries are indicated by the black dashed lines. Finally, the windowed signals were transformed back into the time domain using the inverse FrFT. The separated S_{21} are shown in the fourth row together with the corresponding reference signals for comparison. Additional details about the separation results are provided in [68].

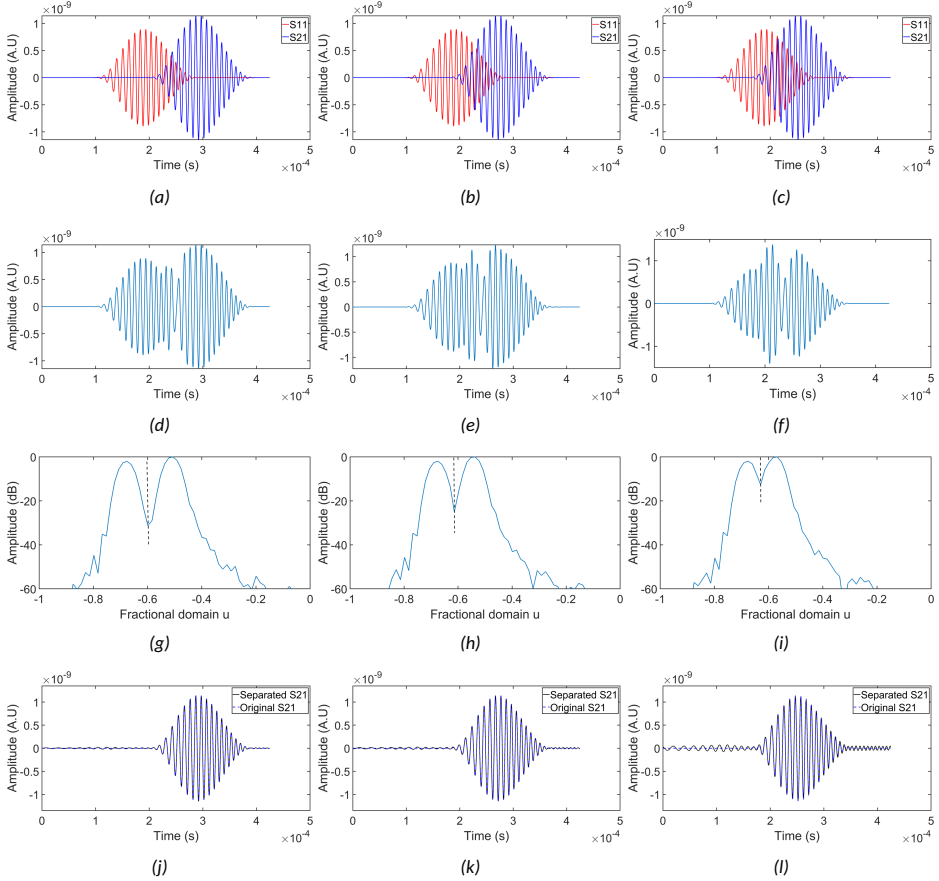


Figure 24: Separation of overlapping signals collected at receiving node 1 using $chirp_{50}$ (simulations 1–3) [68]. The first row (a–c) shows the original signals from each source. The second row (d–f) represents the summation of these signals, resulting in overlap. The third row (g–i) displays the FrFT of the overlapping signals. In the fourth row (j–l), the separated signal from source 2 is compared with its original reference signal.

4.1.2 Experimental Validation

To validate the FrFT-based signal separation method under practical conditions, laboratory experiments were conducted. The experimental setup, shown in Figure 25, consisted of a 1000 mm $\times$ 1000 mm $\times$ 2 mm aluminum plate with a centrally located hole of 11 mm diameter, two ultrasonic transducers, a laser vibrometer, and a data acquisition (DAQ) box. The ultrasonic transducers (S0208, ACS Ltd., Saarbrücken, Germany), with a center frequency of 100 kHz and a bandwidth of 50 kHz were attached to the plate using coupling gel to ensure efficient acoustic coupling. Vibrations at the plate surface were measured using a VFX-I-160 laser vibrometer (Polytec GmbH, Waldbronn, Germany). The DAQ box has functions such as signal generation, signal amplification, and signal averaging, was connected to a computer and controlled through LabVIEW-based software for data acquisition [80].

The experimental geometry (Figure 25b) was consistent with the Abaqus simulation model (Figure 22). Two transducers were used to generate A_0 mode waves, which propagated along the plate to two receiving points on the plate surface. The $chirp_{50}$ was employed to validate the FrFT-based signal separation, whereas the $chirp_{100}$ used in numerical simulations was not considered because its bandwidth exceeded that of the transducers. The delay settings in experiments 1–3 were selected to match those used in simulations 1–3. Unlike the simulations, temporal averaging was performed during the experimental measurements to improve the SNR. Specifically, the signals acquired by the laser vibrometer were averaged over 100 repeated measurements to reduce the influence of mechanical vibrations, ambient disturbances, and DAQ system noise.

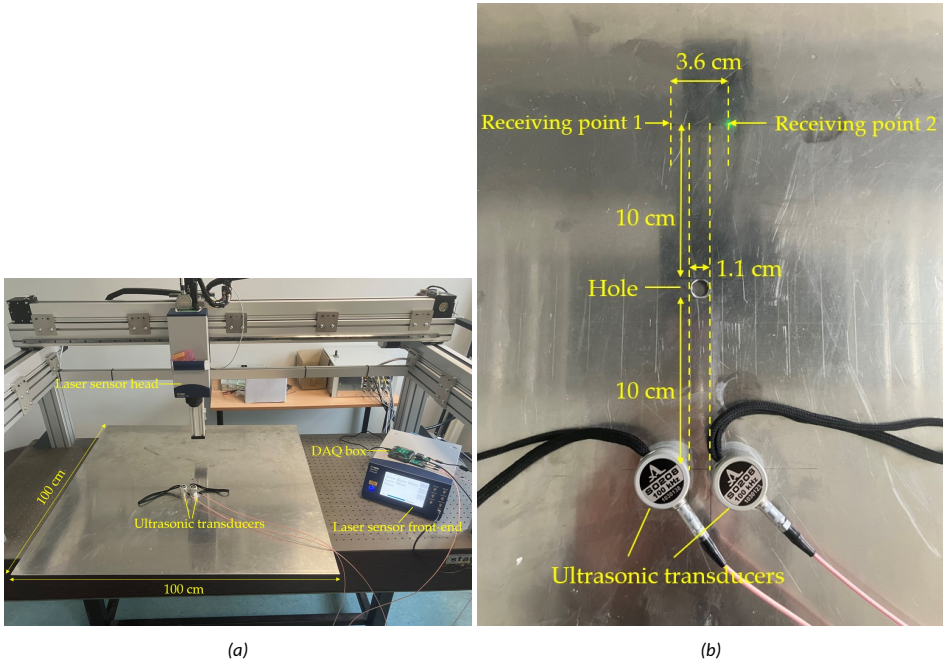


Figure 25: Experimental setup for FrFT validation [68]. (a) Experimental setup including an aluminum plate, ultrasonic transducers, a laser vibrometer, and a DAQ system. (b) Geometry of the transducers, the hole, and the receiving positions.

After the overlapping signals were acquired at the receiving points, they were sepa-

rated using the same FrFT-based procedure as that employed in the numerical simulations. The corresponding results are presented in Figure 26. The four rows show the original reference signals, the overlapping signals, the transformed signals in the fractional domain, and the separated S21 signals together with their corresponding reference signals, respectively.

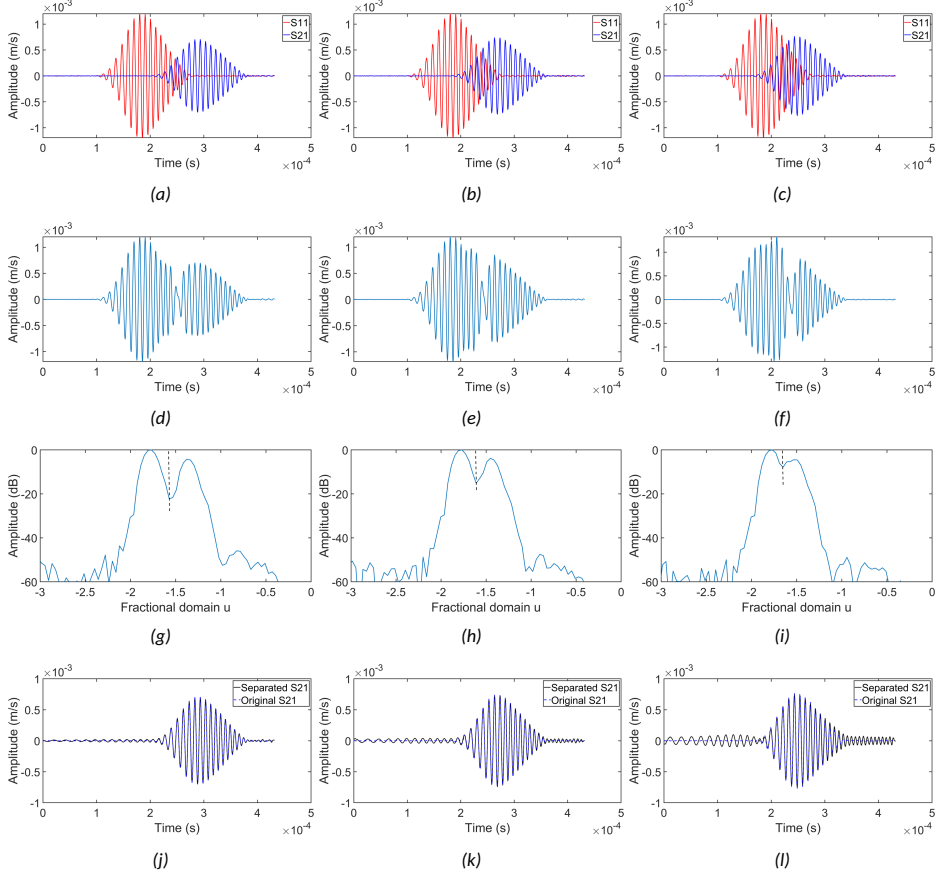


Figure 26: Experimental separation results for signals collected at receiving point 1 [68]. The first row (a–c) displays the original signals from each source. The second row (d–f) shows the summation of these signals. The third row (g–i) represents the FrFT of the overlapping signals. In the fourth row (j–l), the separated signal from source 2 is compared with its original reference signal.

4.1.3 Results Analysis

Results of simulations 1–3 (Figure 24), all conducted using $chirp_{50}$, showed that reducing the time delay increased the overlap between the compressed peaks in the fractional domain. Accurate separations were achieved for delays of 100 μ s and 80 μ s. When the delay was reduced to 60 μ s, the separation performance degraded significantly, with noticeable deviations between the separated signal and the reference signal observed at the sidelobes because of harmonic-like artifacts. Results of simulations 4–5, conducted using $chirp_{100}$, exhibited a similar trend, confirming that a shorter time delay resulted in degraded separation performance. Additionally, at the time delay of 60 μ s, the separation performance achieved using $chirp_{100}$ was superior to that achieved using $chirp_{50}$, indi-

cating that a larger signal bandwidth improves the separation capability of the FrFT under severe signal overlap.

Comparison between simulation 6 (without a hole) and simulation 3 (with a hole) showed that the presence of the hole had little influence on the separation performance. This is because the time delay between the scattered waves and the corresponding incident waves was much smaller than the time delay between the signals generated by the two source nodes. Consequently, the scattered waves did not significantly affect the separation of the two overlapping source signals. Furthermore, comparison between the dispersive signals in simulation 6 and the corresponding non-dispersive signals showed that the wave dispersion accumulated over the relatively short propagation distance was limited and therefore had a negligible influence on the separation performance.

Experimental results also showed that the separation performance degraded as the time delay decreased, with a significant deterioration observed at a delay of $60 \mu\text{s}$. Comparison between Figures 24 and 26 shows that the experimental results exhibited poorer separation performance than the corresponding simulation results under the same delay.

To quantitatively evaluate the separation performance, the maximum magnitude error (MME) and the root mean square error (RMSE) were used. The MME is defined as the maximum relative magnitude error between the reference signal and the separated signal,

$$MME = \frac{\max|s_o(n) - s_s(n)|}{\max|s_o(n)|}, \quad (32)$$

where $s_o(n)$ and $s_s(n)$ denote the reference signal and the separated signal, respectively. RMSE is defined as the square root of the mean squared difference between the reference signal and the separated signal,

$$MME = \sqrt{\frac{\sum_{n=1}^N [s_o(n) - s_s(n)]^2}{N}}, \quad (33)$$

where N is the number of discrete signal sampling points. Figure 27 summarizes the MME and RMSE obtained for simulations 1–5. For both $chirp_{100}$ and $chirp_{50}$, the MME and RMSE increased as the delay decreased, indicating that shorter delays resulted in poorer separation performance. Although $chirp_{100}$ exhibited a significantly lower MME at the $60 \mu\text{s}$ delay, it produced a slightly higher MME at the $80 \mu\text{s}$ delay. This is because effective separation was achieved for both excitations at the $80 \mu\text{s}$ delay, and the resulting MME was dominated by numerical errors rather than by limitations in the separation process itself. Moreover, $chirp_{100}$ consistently yielded lower RMSE values than $chirp_{50}$ at all delays, indicating superior separation performance. This is attributed to the larger bandwidth of $chirp_{100}$, which produces greater energy concentration in the fractional domain and thereby facilitates more effective separation of the overlapping signals (Figure 28).

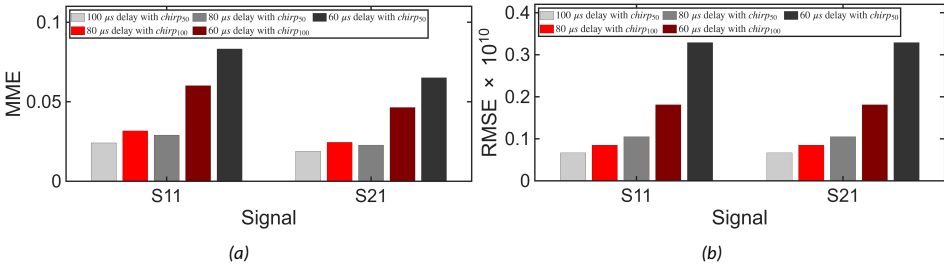


Figure 27: FrFT-based separation errors for simulations 1–5. (a) MME. (b) RMSE.

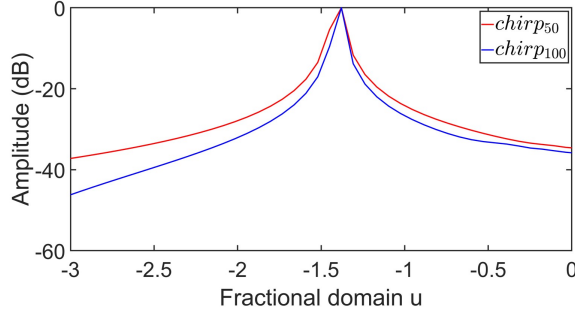


Figure 28: Two excitations $chirp_{100}$ and $chirp_{50}$ in the fractional Fourier domain [68].

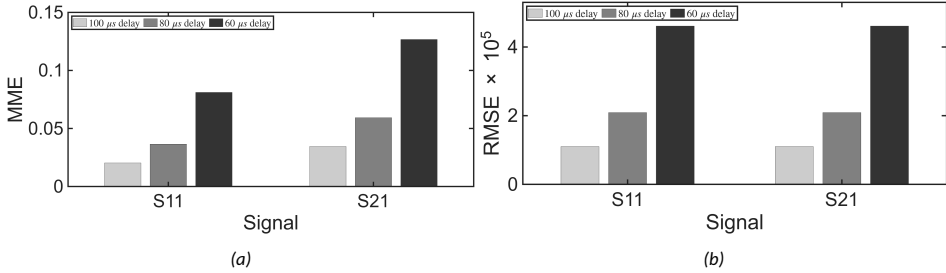


Figure 29: FrFT-based separation errors for experiments. (a) MME. (b) RMSE.

Figure 29 shows that both MME and RMSE for the experiments increase as the time delay decreases. The comparison of Figures 27 and 29 shows that the experimental MME values were generally higher than those obtained from the simulations, particularly at the 60 μ s delay. The experimental RMSE values were several orders of magnitude larger than those obtained from the simulations. One of the primary reasons for this is the influence of the transducers on the transmitted excitations. The actual transmitted excitations differed from the designed excitations, resulting in a mismatch between the practical transmitted excitations and the designed excitations used by the FrFT-based separation method, thereby increasing the separation errors.

Both the simulations and experiments demonstrated accurate signal separation at delays of 100 μ s and 80 μ s, whereas a noticeable degradation in separation performance was observed at a delay of 60 μ s due to the increased overlap between the signals, which constitutes a major source of error in FrFT-based signal separation. Overall, the experimental results showed good agreement with the numerical simulations, confirming the effectiveness of the proposed FrFT-based signal separation method. Under the investigated conditions, accurate signal separation was maintained with a time delay corresponding to approximately 40% of the excitation length. In conventional ultrasonic measurements, the transmission delay is typically selected to be longer than the excitation duration due to factors such as the transducer response and wave dispersion, which increase the duration of the received signals. Therefore, reducing the transmission delay directly shortens the waiting time between successive transmissions, thereby improving the efficiency of sequential ultrasonic measurements.

An additional practical advantage arises in DAQ systems employing short signal acquisition windows. In sequential transmission, if the signal acquisition window is sufficiently long, multiple transmissions can be accommodated within a single acquisition

cycle. However, when a shorter acquisition window is required, for example to separate the direct wave from nearby reflections, conventional transmission delays may prevent successive transmissions from being accommodated within the same acquisition cycle. Consequently, an additional acquisition cycle is required, increasing the overall measurement time. Since the duration of a signal acquisition cycle is often considerably longer than the signal acquisition window itself, depending on the DAQ system and the measurement setup, avoiding additional acquisition cycles can result in a substantial reduction in measurement time. Figure 30 illustrates how reducing the transmission delay enables successive transmissions to be accommodated within a single acquisition cycle under a short signal acquisition window, whereas conventional sequential transmission requires multiple acquisition cycles. By reducing the transmission delay while maintaining signal separability, the proposed method makes it possible for successive transmissions to be accommodated within a single acquisition cycle under such conditions, thereby further improving the acquisition efficiency.

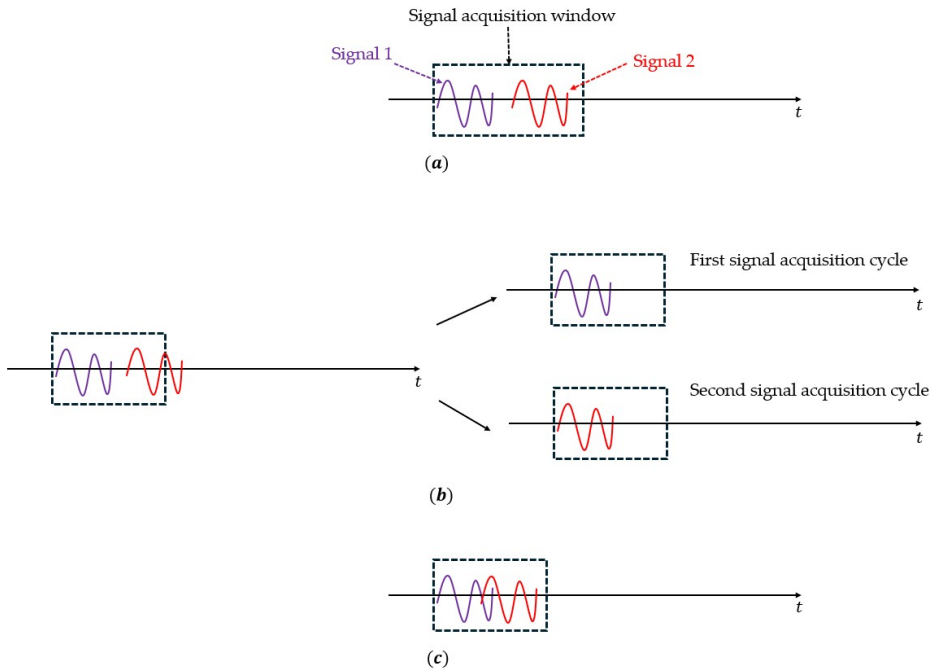


Figure 30: Effect of the signal acquisition window on sequential transmission. (a) Conventional acquisition with a sufficiently long signal acquisition window. (b) A short signal acquisition window requires two acquisition cycles when a conventional transmission delay is used. (c) Reduced delay enables both transmissions to be accommodated within a single acquisition cycle.

Key findings:

- The FrFT-based method enables reliable separation of overlapping ultrasonic signals under reduced time delays, demonstrating its suitability for accelerating measurements based on sequential transmissions.
- The separation performance is primarily governed by the degree of signal overlap, while increasing the excitation bandwidth improves robustness under severe overlap conditions.

- For the investigated plate, defect scattering and the accumulated wave dispersion have a minor influence on the separation performance.
- The experimental results are in good agreement with the numerical simulation results, confirming the practical feasibility of the proposed method.
- Under the investigated conditions, accurate signal separation was maintained with a transmission delay of only approximately 40% of the excitation duration. This reduction in transmission delay not only shortens the waiting time between successive transmissions but also makes it possible for successive transmissions to be accommodated within a single acquisition cycle in DAQ systems employing short signal acquisition windows, thereby improving the acquisition efficiency.

4.2 Temporal Averaging Reduction in UGWs Inspection of a Plate

4.2.1 Experimental Validation

To evaluate the effectiveness of coded excitations in reducing the need for temporal averaging, laboratory experiments were conducted. The experimental setup, shown in Figure 31, consisted of a 2000 mm $\times$ 2000 mm $\times$ 2 mm aluminum plate, a piezoelectric transducer (S0208, ACS Ltd., Saarbrücken, Germany) with a nominal frequency 100 kHz, a VFX-I-160 laser vibrometer, and a DAQ system consisting of a homemade amplifier and a commercial DAQ box (Analog Discovery 3, Digilent Inc. Austin, TX, USA). The transducer was coupled to the plate using coupling gel and a 100 g weight was placed on top of the transducer to ensure efficient coupling. The DAQ box was connected to a computer and controlled using custom LabVIEW-based software (LabVIEW 2014) to configure the excitation voltage, triggering, and the number of temporal averages. A pitch-catch measurement configuration was used throughout the experiments. To minimize complex multimodal wave propagation, 100 kHz A_0 mode UGWs were generated by applying an out-of-plane force to the plate surface using the transducer. The out-of-plane displacements at the receiving point were measured by the laser vibrometer. As shown in Figure 31, the transducer and laser measurement point were positioned sufficiently far from the plate boundaries to prevent boundary reflections from interfering with the direct waves propagating between them.

The excitations used in the experiments included two Barker coded excitations, a pair of 16-bit Golay coded excitations and a one-cycle pulse, defined as a Hanning-windowed 100 kHz sine wave with a duration of one cycle. The one-cycle pulse was used as the carrier wave to construct coded excitations. Figure 32 shows the one-cycle pulse together with the corresponding 13-bit Barker coded excitation generated using the one-cycle pulse as the carrier waveform. The experimental parameters, including the excitations, voltages, and temporal averaging numbers, are summarized in Table 5. Experiments were conducted for all combinations of these parameters. The voltage was limited to 10 V by the DAQ system. As this voltage level is practically low, noise became more pronounced, making the experiments suitable for evaluating the performance of coded excitations [20]. The required number of averages depends on the specific measurement. For example, an averaging number of 50 was used in the work [81]. In this thesis, the number of temporal averages varied from 1 to 40 because further averaging beyond 40 measurements provided negligible improvement in SNR. By quantifying the relationship among excitation type, excitation voltage, temporal averaging number, and SNR, the effectiveness of coded excitations in reducing the need for temporal averaging can be systematically evaluated.

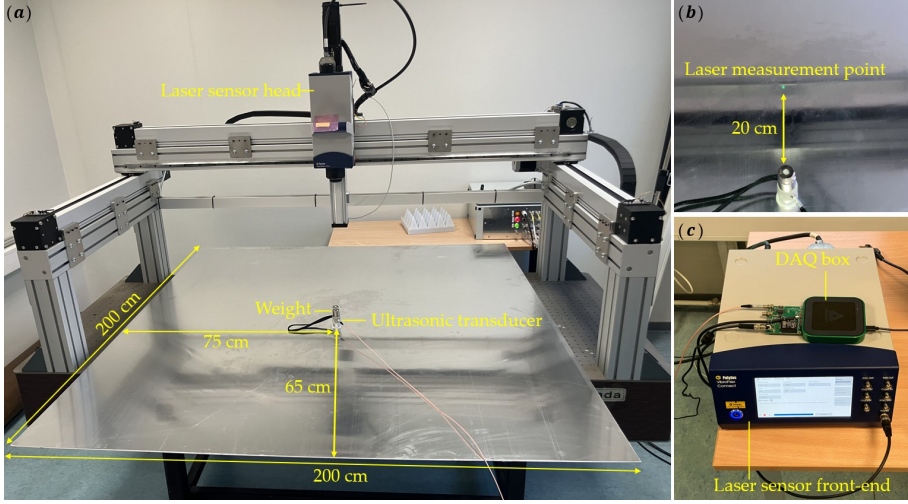


Figure 31: Experiment setup for evaluating coded excitations in UGWs measurements [82]. (a) Experiment platform. (b) Geometry of the transducer and the laser measurement point. (c) DAQ box and the laser front-end.

Table 5: Experimental excitation parameters and averaging numbers.

Excitations	Excitation voltage (V)	Averaging number
one-cycle pulse, B_7 , B_{13} , G_{16}	0.5, 1, 2, 5, 10	1, 5, 10, 20, 30, 40

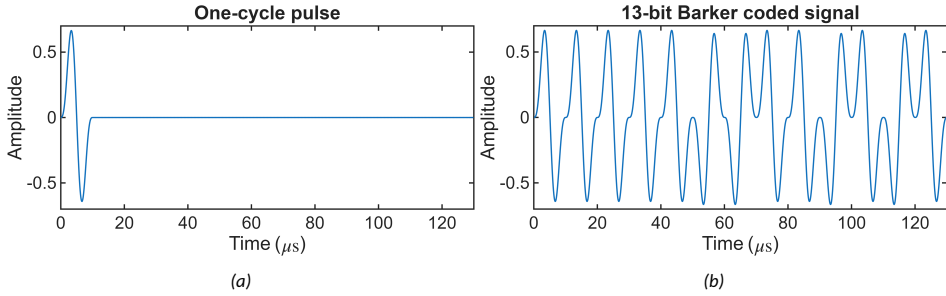


Figure 32: Representative excitations used in the temporal averaging experiments. [82]. (a) One-cycle pulse. (b) 13-bit Barker coded signal.

The received signals were first windowed in the time domain to remove boundary reflections and then filtering using an 8 kHz high-pass filter to suppress low-frequency noise. The filtered signals were subsequently decoded, after which their envelopes were extracted and normalized. The SNR was then calculated from the normalized envelopes according to Equation 18. Figure 33 presents the representative decoded signals obtained at an excitation voltage of 2 V. The figure contains four subfigures, each corresponding to a different excitation. Within each subfigure, three representative signals obtained with averaging numbers of 1, 10, and 30 are shown from top to bottom, respectively. More experimental results are provided in [82].

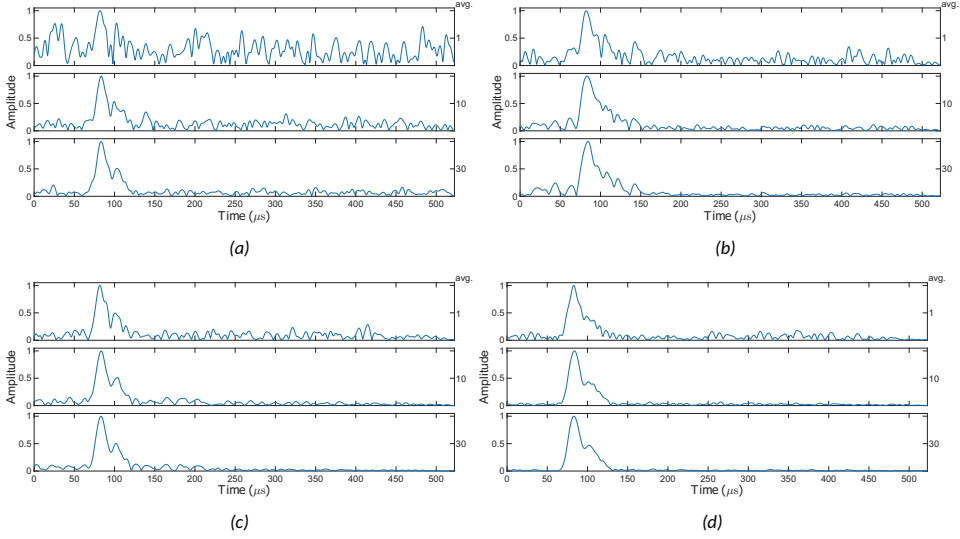


Figure 33: Decoded signal envelopes obtained under a 2 V excitation voltage [82]. (a-d) represent the signals correspond to the one-cycle pulse, B_7 , B_{13} , and G_{16} , respectively.

4.2.2 Results Analysis

The results show that the noise level decreases as the number of averages increases, confirming the effectiveness of temporal averaging in improving the SNR. This improvement is pronounced when comparing the signals obtained after ten temporal averages with those obtained without averaging. It is observed that coded excitations require fewer temporal averages than the one-cycle pulse to achieve a comparable SNR. At an excitation voltage of 0.5 V, a clearly identifiable mainlobe is obtained using coded excitations after temporal averaging, whereas this is not achievable with the one-cycle pulse regardless of the number of averages. At 1 V, coded excitations with 10 averages yield a lower noise level than the one-cycle pulse with 30 averages. At 2 V, coded excitations without temporal averaging achieve a noise level comparable to that obtained with the one-cycle pulse with 10 averages. Furthermore, at excitation voltages of 5 V and 10 V, coded excitations without temporal averaging provide substantially higher SNR than the one-cycle pulse without averaging. In terms of sidelobe performance, B_7 exhibits higher sidelobe levels than B_{13} , as expected from theory. For G_{16} , the sidelobes are substantially suppressed due to its sidelobe cancellation property. Nevertheless, perfect sidelobe cancellation is not achieved in practice due to waveform distortions introduced by the transducer response and propagation dispersion. Overall, these results qualitatively demonstrate that coded excitations provide SNR performance comparable to or better than that of the one-cycle pulse while requiring substantially fewer temporal averages.

To quantitatively evaluate the effectiveness of coded excitations in reducing number of averages, SNR was calculated under a series of excitation voltages, as shown in Figure 34. For all excitation types, the SNR generally increases with the number of temporal averages. Moreover, coded excitations consistently achieve higher SNR than the one-cycle pulse under identical excitation voltages and average numbers. The measured SNR follows the order $G_{16} > B_{13} > B_7 > \text{one-cycle pulse}$, which agrees well with the theory. The SNR improvement is not strictly linear with the number of temporal averages because of the random nature of noise.

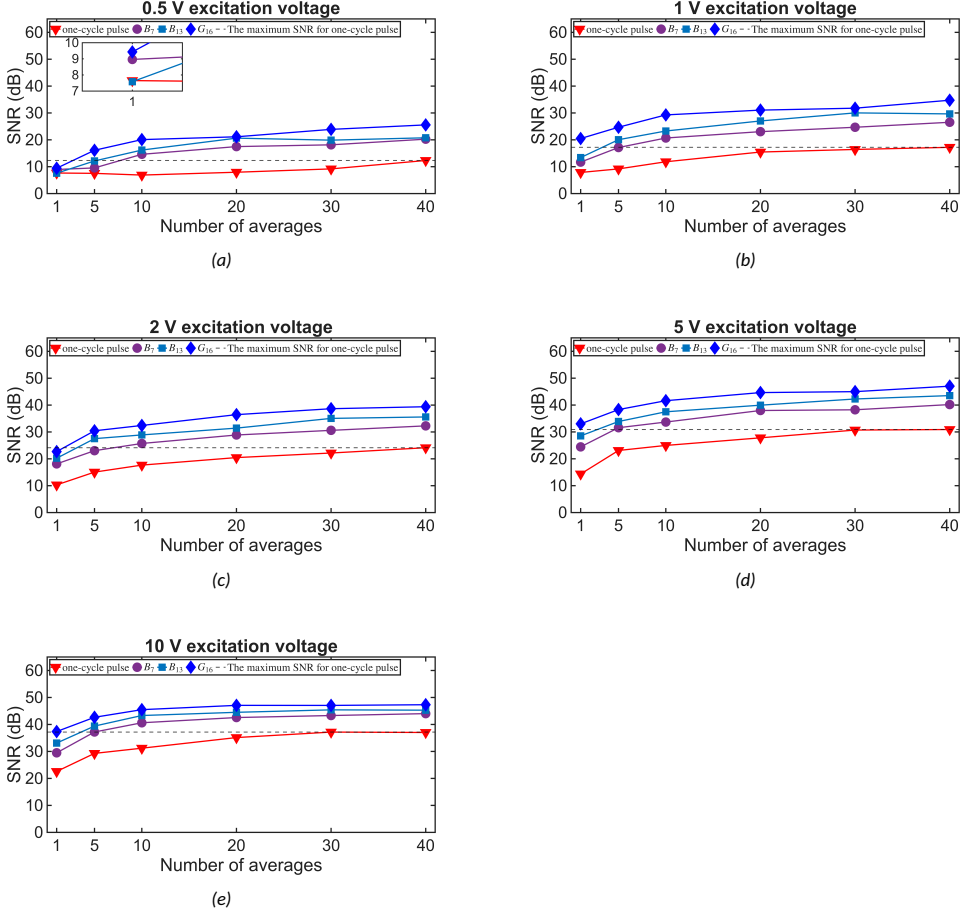


Figure 34: SNR for different excitations under various voltages [82]. (a-e) represent the SNR obtained using the 0.5 V, 1 V, 2 V, 5 V, and 10 V voltage, respectively. The horizontal dashed line indicates the maximum SNR achieved by the one-cycle pulse.

To further quantify the improvement in measurement efficiency, the minimum number of averages required for coded excitations to achieve the maximum SNR obtained by the one-cycle pulse was determined. The one-cycle pulse reached its maximum SNR using 40 temporal averages. Coded excitations achieved comparable SNR levels with significantly fewer averages. Table 6 summarizes the minimum number of averages required for the coded excitations to achieve the maximum SNR obtained by the one-cycle pulse under the same excitation voltage. As listed in Table 6, G_{16} without temporal averaging or with only five averages achieved a higher SNR than the one-cycle pulse with 40 averages, depending on the voltage. For B_{13} , five averages are sufficient to obtain a comparable SNR achieved by the one-cycle pulse with 40 averages, except at 0.5 V, where ten averages are required. For B_7 , five averages are sufficient to achieve the maximum SNR of on-cycle pulse at excitation voltages of 1 V, 5 V, and 10 V, whereas ten averages are required at 0.5 V and 2 V. It should be noted that, each average of G_{16} , requires two transmission events; consequently, its signal acquisition efficiency is reduced by a factor of two compared with single-transmission excitations. Overall, the investigated coded excitations reduced the required number of temporal averages from 40 to as few as one, while maintaining com-

parable or higher SNR. These results quantitatively demonstrate that coded excitations can substantially reduce the number of repeated signal acquisitions required for ultrasonic measurements, thereby significantly improving the efficiency of UT.

Table 6: Minimum number of averages required by coded excitations.

Voltage (V)	Excitation	No. of Averages	SNR (dB)
0.5	one-cycle pulse	40	12.3
0.5	B_7	10	14.6
0.5	B_{13}	10	16.2
0.5	G_{16}	5	16.1
1	one-cycle pulse	40	17.2
1	B_7	5	17.2
1	B_{13}	5	20.0
1	G_{16}	1	20.5
2	one-cycle pulse	40	24.1
2	B_7	10	25.7
2	B_{13}	5	27.5
2	G_{16}	5	30.4
5	one-cycle pulse	40	30.9
5	B_7	5	31.6
5	B_{13}	5	33.9
5	G_{16}	1	33.0
10	one-cycle pulse	40	37.0
10	B_7	5	37.2
10	B_{13}	5	39.4
10	G_{16}	1	37.4

From a physical perspective, the SNR improvement provided by coded excitations originates from the increased signal energy. Pulse compression concentrates this energy into a narrow mainlobe, thereby improving the SNR. Unlike the one-cycle pulse, which requires higher excitation voltages to increase the transmitted signal energy, coded excitations can achieve higher energy without increasing the excitation voltage, thereby providing a safer operating condition. This advantage is obtained at the expense of longer excitation durations, and in general, longer codes produce higher SNR. It should be noted that the duration of the coded excitations is very small compared with that of a complete acquisition cycle in the present experiments. The excitation duration was less than 1 ms, whereas the fixed signal acquisition window was 10 ms and a complete acquisition cycle required approximately 300 ms because sufficient time was needed for the hardware to prepare for the next signal acquisition. Therefore, the duration of the coded excitations has a small impact on the measurement efficiency, whereas reducing the number of signal acquisition cycles provides a substantial improvement in measurement efficiency. Overall, the results demonstrate that coded excitations are particularly advantageous for low-voltage and time-constrained ultrasonic measurements, where extensive temporal averaging is impractical.

Key findings:

- Coded excitations consistently achieved comparable or higher SNR than the one-cycle pulse, while requiring substantially fewer temporal averages, demonstrating

their effectiveness in reducing repeated signal acquisitions for temporal averaging.

- Increasing the code length generally improved the achievable SNR, following the order $G_{16} > B_{13} > B_7 > \text{one-cycle pulse}$, which is consistent with the theoretical pulse-compression gain.
- Under the investigated conditions, the required number of temporal averages was reduced from 40 for the one-cycle pulse to as few as one for coded excitations.
- The proposed coded excitation strategy is particularly advantageous for low-voltage and time-constrained ultrasonic measurements, where extensive temporal averaging is impractical.

4.3 BMOGCC for Bulk Wave Inspection of a Steel Block

4.3.1 Numerical Simulations

Numerical simulations were conducted using the software Abaqus. A two-dimensional model was established consisting of a steel block of 75 mm $\times$ 140 mm, two excitation source nodes, two receiver nodes, as shown in Figure 35. The steel has a Young's modulus of 195 GPa, a Poisson ratio of 0.3, and a density of 7910 kg/m³. The model was meshed using CPE4R elements. To minimize reflections from the model boundaries, absorbing boundaries were implemented by meshing the outer boundary with a layer of CINPE4 infinite elements. Vertical forces at the sources were applied as excitations, and vertical displacements were recorded at the receivers.

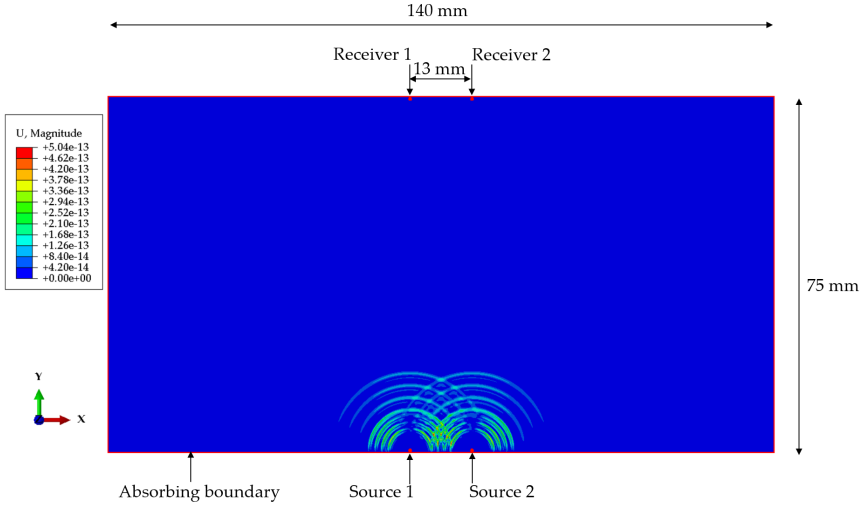


Figure 35: Steel block model in Abaqus, showing the displacement field for simulation 5 at 3 μ s [44].

The 1-bit pulse, which was a Hanning-windowed sinusoidal wave with a center frequency of 2.25 MHz, was used as the carrier wave to construct the coded excitations. The center frequency was selected to match the nominal frequency of the transducers used in laboratory experiments. In Abaqus, the upper frequency limit of a simulation is constrained by the element size, and smaller elements are required to accurately simulate higher-frequency wave propagation [83]. Coded excitations may contain frequency components several times higher than the center frequency, making the use of a sufficiently small element size essential to ensure the convergence of the solution. In this model, an

element size of 0.025 mm was adopted. A larger element size, such as 0.05 mm, leads to non-convergent solutions. The 1-bit pulse, Barker codes, MOGCC, and the proposed BMOGCC were used as excitations, with the 1-bit pulse serving as the reference excitation for calculating the MG. For simulations involving two excitation sources, the excitations were transmitted simultaneously. Table 7 lists the excitations for all simulations. Figure 35 illustrates the displacement field for simulation 5. The faster propagating longitudinal waves were selected for subsequent signal analysis, whereas the slower shear waves were removed through time windowing.

Table 7: Coded excitations used in the steel block model.

Simulation No.	Code	Source 1	Source 2
1	-	1-bit pulse	-
2	Barker	B_5	-
3	Barker	B_7	-
4	Barker	B_{11}	-
5	M4	M_4X_1	M_4Y_1
6	M4	M_4X_2	M_4Y_2
7	M8	M_8X_1	M_8Y_1
8	M8	M_8X_2	M_8Y_2
9	B5M4	$B_5M_4X_1$	$B_5M_4Y_1$
10	B5M4	$B_5M_4X_2$	$B_5M_4Y_2$
11	B5M8	$B_5M_8X_1$	$B_5M_8Y_1$
12	B5M8	$B_5M_8X_2$	$B_5M_8Y_2$
13	B7M4	$B_7M_4X_1$	$B_7M_4Y_1$
14	B7M4	$B_7M_4X_2$	$B_7M_4Y_2$
15	B7M8	$B_7M_8X_1$	$B_7M_8Y_1$
16	B7M8	$B_7M_8X_2$	$B_7M_8Y_2$

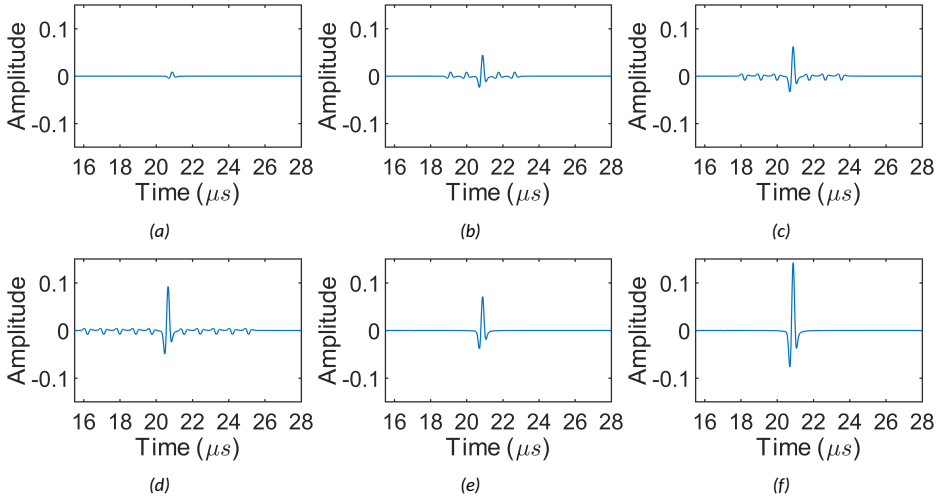


Figure 36: Decoded signals for simulations 1-8 [44]. (a-f) show the decoded signals for the 1-bit pulse (simulation 1), 5-bit Barker (simulation 2), 7-bit Barker (simulation 3), 11-bit Barker (simulation 4), M4 (simulations 5-6), and M8 (simulations 7-8), respectively.

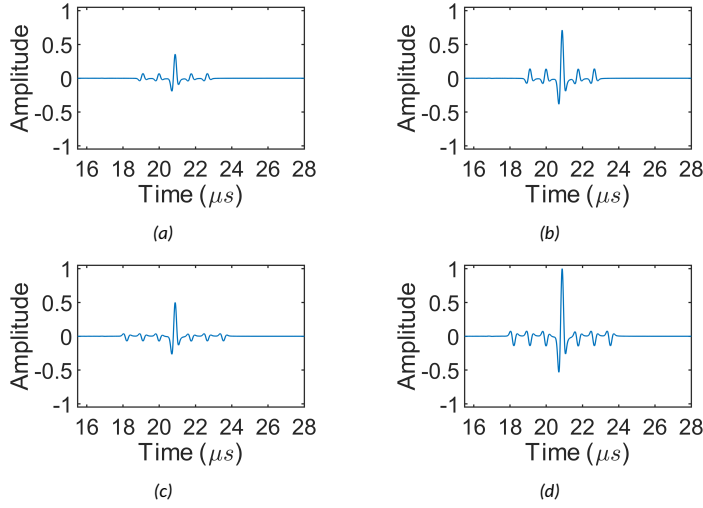


Figure 37: Decoded signals for simulations 9-16 [44]. (a-d) show the decoded signals for B5M4 (simulations 9-10), B5M8 (simulations 11-12), B7M4 (simulations 13-14), and B7M8 (simulations 15-16), respectively.

Figure 36 presents the decoded signals obtained using the 1-bit pulse, Barker codes, and MOGCC. Figure 37 presents the decoded signals obtained using BMOGCC. For MOGCC and BMOGCC, only the decoded signals from receiver 1 are presented, since the decoded signals at both receivers were symmetric. To facilitate comparison, all decoded signals were normalized with respect to the maximum peak amplitude observed among all simulations.

4.3.2 Experimental Validation

To further evaluate the performance of the proposed BMOGCC under practical conditions, laboratory experiments were conducted. The experimental setup, illustrated in Figure 38, consisted of a steel block, two ultrasonic transducers, and a DAQ system. The steel block had dimensions of 140 mm $\times$ 74 mm $\times$ 100 mm. Two M1036 transducers (Olympus, Hamburg, Germany) with a center frequency of 2.25 MHz were mounted on the steel block using magnetic holders. To ensure efficient acoustic coupling between the transducers and the steel surface, ultrasonic coupling gel was applied. The DAQ system incorporated an HS805 signal generator (TiePie, Sneek, The Netherlands) with an 8-bit amplitude resolution. The signal generator was connected to a computer and controlled using TiePie software. It generated excitations, which were applied to the transducers. Ultrasonic waves propagated through the steel block were measured by the receiving transducer and converted back into electrical signals. These signals were then digitized by the DAQ system and transferred to the computer for subsequent signal processing.

A pitch-catch configuration was used, with the transmitting and receiving transducers positioned on opposite sides of the steel block. The transducer locations (Figure 38) were selected to match those used in the numerical model (Figure 35). To maintain consistency with the Abaqus model, the experimental setup included two transmission and two receiving locations, which would normally require a total of four transducers. However, only two suitable transducers were available. An equivalent measurement approach was employed, whereby one transducer was sequentially positioned at the two transmission locations, and the signals acquired at the same receiving location were subsequently

superimposed. This approach reproduced the response that would have been obtained if all transducers had operated simultaneously. The same excitations were employed in both numerical simulations and experiments. Accordingly, the excitation configurations for experiments 1–16 were identical to those used in simulations 1–16.

Only the decoded signals from receiving location 1 are presented, as the decoded signals at two locations were symmetric. Figure 39 presents the decoded signals obtained using the 1-bit pulse, Barker codes, and MOGCC, whereas Figure 40 presents the decoded signals obtained using the proposed BMOGCC. To facilitate comparison, all decoded signals were normalized using the same procedure as that adopted in the numerical simulations.

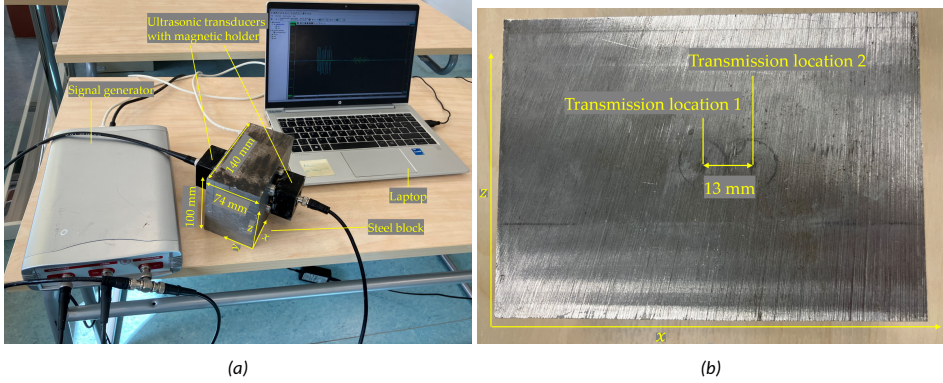


Figure 38: Ultrasonic measurements on a steel block [44]. (a) Experimental setup. (b) Transmission locations on the steel block with a spacing of 13 mm; two receiving locations were positioned at the corresponding locations on the opposite side of the block.

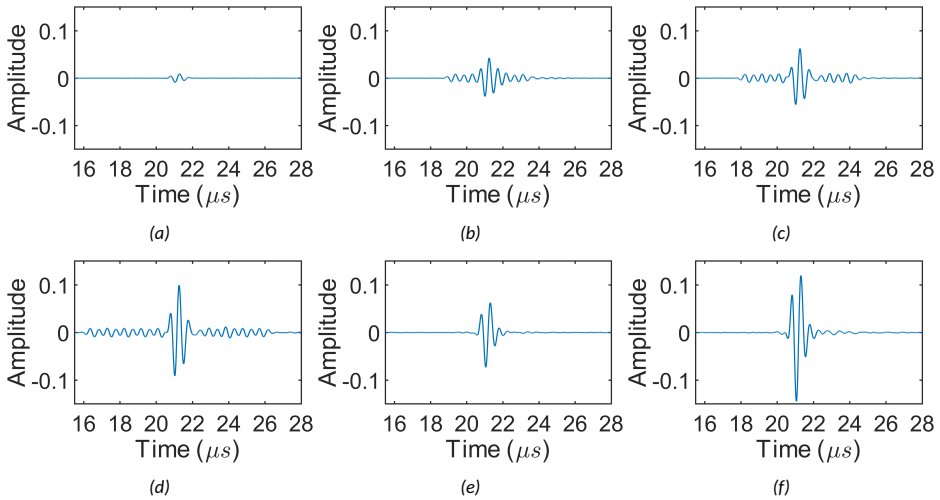


Figure 39: Decoded signals for experiments 1–8 [44]. (a-f) show the decoded signals of the 1-bit pulse (experiment 1), 5-bit Barker (experiment 2), 7-bit Barker (experiment 3), 11-bit Barker (experiment 4), M4 (experiments 5–6), and M8 (experiments 7–8), respectively.

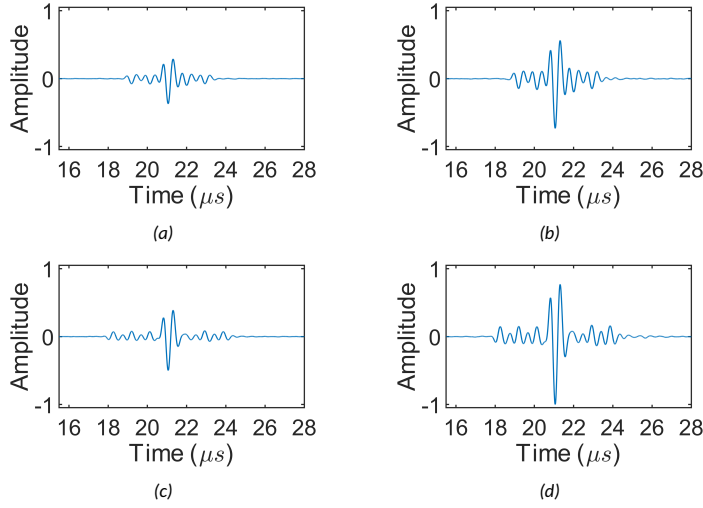


Figure 40: Decoded signals for experiments 9–16 [44]. (a-d) present the decoded signals for B5M4 (experiments 9–10), B5M8 (experiments 11–12), B7M4 (experiments 13–14), and B7M8 (experiments 15–16), respectively.

4.3.3 Results Analysis

Simulation results showed that the amplitudes of decoded signals depend on the coding scheme. As shown in Figure 36, the amplitude of the decoded signals increased with Barker code length, and sidelobes are observed in all decoded signals obtained using the Barker codes. In contrast, the decoded signals obtained using MOGCC exhibited sidelobe-free characteristics. Figure 37 shows that BMOGCC achieved a higher decoded signal amplitude than both MOGCC and the Barker codes. This result agrees well with the theoretical derivation in Equation 31, which shows that the decoded BMOGCC is obtained by convolving the decoded Barker code with the decoded MOGCC.

The experimentally obtained decoded signals were in good agreement with the simulated signals. As shown in Figure 39, the decoded peak amplitude increased with Barker code length while maintaining relatively low sidelobe levels. Moreover, M8 achieved a higher peak amplitude than M4 because of its longer code length. Figure 40 further shows that the proposed BMOGCC produced higher decoded peak amplitudes than both MOGCC and the Barker codes. Compared with the simulation results, the sidelobes observed experimentally were less uniform because the transducer impulse response and other system characteristics distorted the transmitted waveforms.

To quantitatively evaluate the code performance, two key metrics, PSL and MG, were employed. Figure 41 presents the MG values obtained from simulations and experiments, both of which are in good agreement with the theoretical predictions. Notably, BMOGCC exhibited significantly higher MG values than either the Barker codes or MOGCC, reaching the highest value for B7M8. As predicted by the theoretical analysis, the MG of each BMOGCC was approximately equal to the sum of the MG values of its constituent Barker code and MOGCC. For example, in the experiments, B7M4, B7, and M4 achieved MG values of 34.90 dB, 16.84 dB, and 18.19 dB, respectively, with the MG value of B7M4 differing from the sum of those of B7 and M4 by only 0.13 dB. Besides, the 8-bit B5M4 and 10-bit B7M4 achieved higher MG values than the 11-bit Barker code, demonstrating that BMOGCC can provide greater decoded signal amplitude even with shorter code lengths. This characteristic is particularly advantageous in applications requiring short excitations,

for example when the DAQ memory is limited or boundary reflections interfere with the valid signals [84]. Overall, these results demonstrate that BMOGCC provides higher MG than conventional Barker codes and MOGCC while maintaining signal separability for parallel transmission.

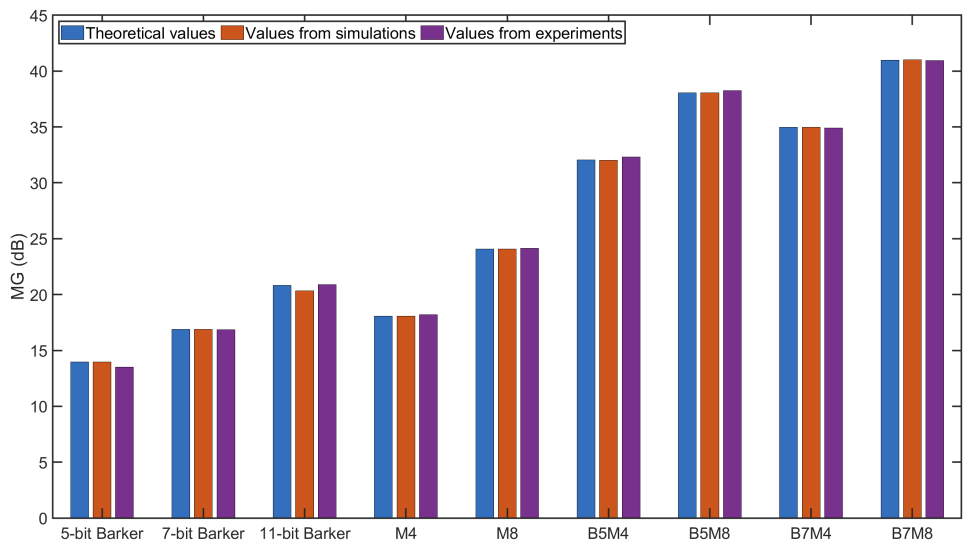


Figure 41: MG of different coded excitations [44].

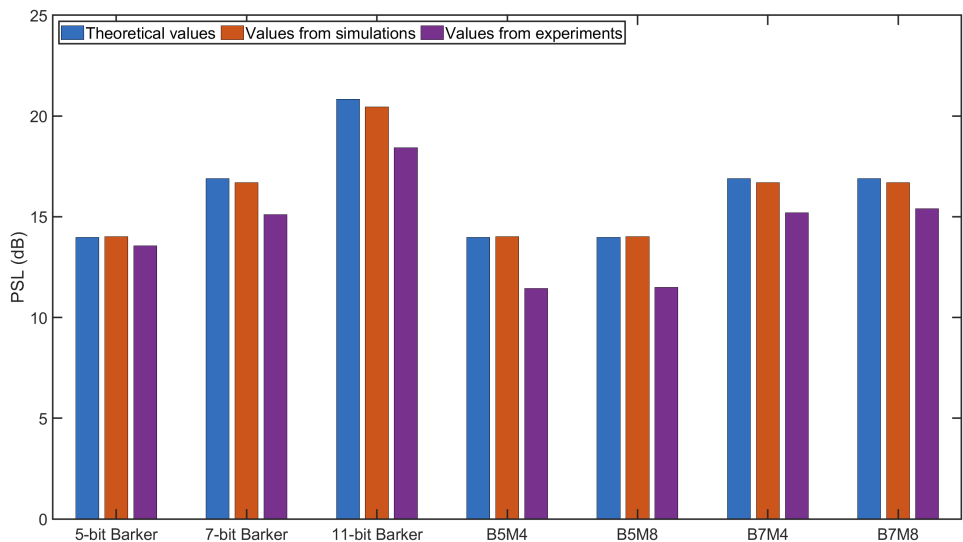


Figure 42: PSL of different coded excitations [44].

The theoretical, simulated, and experimental PSL values are compared in Figure 42. The simulated results agree well with the theoretical predictions, with the maximum deviation being only 0.37 dB for the 11-bit Barker code. Furthermore, B5M4 and B5M8 exhibited PSL values nearly identical to that of the 5-bit Barker code, whereas B7M4 and B7M8 exhibited PSL values nearly identical to that of the 7-bit Barker code. These results are con-

sistent with Equation 31, which predicts that the PSL of a BMOGCC is determined by that of its constituent Barker code. The experimentally measured PSL values were generally lower than those obtained in the simulations, with the largest difference of 2.40 dB observed for the 11-bit Barker code. This difference is attributed primarily to the impulse response of the transducers, which distorted the transmitted waveforms and consequently altered the sidelobe characteristics. Nevertheless, the experimental and simulation results show good overall agreement. These findings confirm the theoretical prediction that BMOGCC retains the sidelobe performance of its constituent Barker code.

In addition to PSL and MG, the temporal resolution was evaluated using the FWHM of the decoded signal envelope [85]. The FWHM values obtained from both the simulations and experiments are listed in Table 8. All decoded signals in the simulations exhibited an identical FWHM of $0.35 \mu\text{s}$. In the experiments, the FWHM values ranged from $0.67 \mu\text{s}$ to $0.89 \mu\text{s}$ and were generally larger than those obtained in the simulations, primarily due to the transducer impulse response. Nevertheless, the proposed BMOGCC achieved FWHM values comparable to those of the 1-bit pulse, Barker codes, and MOGCC, demonstrating that the increased MG was achieved without sacrificing the temporal resolution.

Table 8: Temporal resolution for different excitations.

Excitation	FWHM of Simulations (μs)	FWHM of Experiments (μs)
1-bit Pulse	0.35	0.75
5-bit Barker	0.35	0.89
7-bit Barker	0.35	0.78
11-bit Barker	0.35	0.78
M4	0.35	0.70
M8	0.35	0.69
B5M4	0.35	0.68
B5M8	0.35	0.67
B7M4	0.35	0.67
B7M8	0.35	0.68

Overall, the simulation and experimental results confirm the effectiveness of the proposed BMOGCC for UT. The proposed BMOGCC retains the low sidelobe levels of Barker codes while achieving substantially higher decoded peaks than either Barker codes or MOGCC, even with shorter code lengths, thereby providing greater flexibility in code design. Furthermore, it preserves temporal resolution and the signal separability required for parallel transmission. Compared with the recently proposed BCGC, the proposed BMOGCC doubles the measurement efficiency.

Key findings:

- The proposed BMOGCC achieves substantially higher decoded peaks than both the Barker codes and MOGCC at comparable code lengths.
- BMOGCC retains the PSL of its constituent Barker code.
- The proposed BMOGCC maintains temporal resolution comparable to that of the one-bit pulse, indicating that the increased decoded peak is achieved without degrading the temporal resolution.
- Compared with the Barker codes and MOGCC, BMOGCC provides greater flexibility in code-length selection by achieving higher MG with shorter code lengths.

- Compared with BCGC, the proposed BMOGCC retains the signal separability of MOGCC, thereby enabling parallel transmission and doubling the measurement efficiency.

5 Conclusions and Future Work

This thesis presents a comprehensive framework for improving the measurement efficiency of UT through advanced signal processing techniques, a topic of significant interest in the field. The proposed framework addresses three factors contributing to measurement efficiency, namely the time delay between successive transmissions, the need for temporal averaging, and the inability of existing coded excitations to support parallel transmission.

The successful implementation of this framework relies on three complementary approaches. Firstly, a signal separation method based on the FrFT is developed to separate overlapping chirps, enabling shorter transmission delays between successive transmissions. Secondly, coded excitations are employed to reduce the required number of temporal averages for a given SNR. Finally, BMOGCC is proposed to enable parallel transmission while preserving signal separability, thereby reducing the number of transmission events required for ultrasonic measurements. Together, these approaches address complementary bottlenecks in the measurement efficiency of UT.

The methods presented in this thesis have not previously been investigated in such a systematic manner for improving the measurement efficiency of UT, making this thesis unique.

5.1 Summary of Results

The first work of this thesis focused on reducing the time delay between successive transmissions by separating overlapping signals. The use of transmission delays shorter than the excitation length can cause signal overlap, making reliable signal separation essential. **Publication I** developed an FrFT-based signal separation method to separate overlapping chirp signals caused by reduced transmission delays. Numerical simulations and experimental investigations demonstrated that the proposed method successfully separated overlapping signals, enabling the use of shorter transmission delays. Additionally, in applications with limited signal acquisition window lengths, where two temporally separated signals cannot be accommodated, the proposed method enables overlapping signals to be separated within a single acquisition window, thereby reducing the number of required signal acquisitions.

The second work addressed the need for repeated signal acquisitions for temporal averaging. In **publication III**, the relationship between coded excitation and temporal averaging was systematically investigated to evaluate the trade-off between the SNR and the number of temporal averages. This work experimentally demonstrated that coded excitations substantially reduce the number of temporal averages required to achieve a given SNR, providing practical guidance for reducing the reliance on temporal averaging in ultrasonic measurements.

The third work focused on improving the measurement efficiency by enabling parallel transmission. Existing BCGC only supports sequential transmission, limiting its measurement efficiency. Consequently, **publication II** proposed BMOGCC, which is constructed by convolving Barker codes with MOGCC. The proposed BMOGCC retains the high MG of BCGC while inheriting the parallel transmission capability of MOGCC. Simulation and experimental results demonstrated that BMOGCC doubles the measurement efficiency compared with BCGC while achieving substantially higher decoded peaks than either Barker codes or MOGCC despite using shorter code lengths. This work extended convolved coding from sequential to parallel transmission while preserving the advantages of BCGC.

Together, these three studies demonstrate that measurement efficiency of UT can be

improved through three complementary strategies: reducing transmission delay, reducing the need for temporal averaging, and enabling parallel transmission. Collectively, the proposed methods establish a comprehensive framework for improving the measurement efficiency and contribute to advancement of efficient UT technologies and provide a foundation for future high-speed, automated, and intelligent NDT systems.

5.2 Limitations

Although the proposed methods have demonstrated their effectiveness through numerical simulations and experimental investigations, several limitations should be acknowledged. The FrFT-based signal separation method is primarily applicable to chirps, since its performance relies on the sparse representation of chirps.

The proposed BMOGCC was developed and validated for parallel transmission from two transducers. Extending the coding scheme to support parallel transmission from a larger number of transducers remains to be investigated.

The relationship between coded excitations and temporal averaging was investigated under laboratory conditions. The influence of practical industrial environments on this relationship remains to be investigated. Further validation under representative industrial environments is therefore desirable.

5.3 Key Conclusions Presented for Defense

- Neither conventional time domain windowing nor frequency domain filtering can effectively separate ultrasonic chirps that overlap in both the time and frequency domains. The FrFT overcomes this limitation by enabling reliable signal separation, allowing transmission delays shorter than the excitation length and improving ultrasonic measurement efficiency.
- Coded excitations reduce the need for temporal averaging to achieve a given SNR, providing an effective approach to improving ultrasonic measurement efficiency.
- Compared with BCGC, BMOGCC doubles the measurement efficiency by enabling parallel transmission, while extending the convolved coding from sequential to parallel transmission.
- Reducing transmission time delay, reducing the need for temporal averaging, and enabling parallel transmission constitute three complementary strategies for improving the measurement efficiency of UT without requiring modifications to ultrasonic hardware.

5.4 Future Work

Future work could extend the methods developed in this thesis to more complex engineering structures. The proposed methods were validated on relatively simple structures, namely aluminum plates and steel blocks. Their applicability to more complex structures, such as pipes, pressure vessels, and other curved components, remains to be investigated. Extending these methods to practical engineering structures would broaden their applicability and facilitate their adoption in a wider range of industrial UT applications.

The FrFT represents only one of many available time-frequency analysis methods for ultrasonic signal separation. Future research could investigate other time-frequency methods, such as the Born–Jordan transform [86], to further improve the separation of overlapping ultrasonic signals under more challenging measurement conditions. Comparative

studies among different time-frequency methods may also provide a better understanding of their respective advantages and limitations for ultrasonic signal separation.

The proposed BMOGCC currently supports parallel transmission from two transducers. Future work could investigate coded excitation schemes capable of supporting parallel transmission from multiple transducers. In particular, the recently proposed Hadamard aperiodic interval codes [87] provide a promising direction for enabling parallel transmission from multiple transducers.

Future work could also investigate the implementation of the proposed algorithms on FPGA or GPU platforms for real-time ultrasonic inspection. Furthermore, integrating the proposed techniques into autonomous robotic inspection systems and digital twin frameworks for continuous structural health monitoring would further advance high-speed, automated, and intelligent UT.

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Abstract

Efficient Ultrasonic Measurements based on Signal Processing

Ultrasonic testing (UT) is one of the most widely adopted nondestructive testing (NDT) techniques for structural integrity assessment. However, the efficiency of UT is often limited by the need for sufficiently long transmission delays to avoid signal overlap, repeated signal acquisitions for temporal averaging to achieve an adequate signal-to-noise ratio (SNR), and sequential transmission strategies. These limitations increase measurement time and restrict the applicability of UT in large-scale structures and high-throughput industrial inspections. Accordingly, this thesis investigates signal processing techniques for improving the measurement efficiency of UT by addressing these limitations.

The research presented in this thesis is structured around three key components: developing a signal separation method based on the fractional Fourier transform (FrFT) to separate overlapping ultrasonic signals caused by reduced transmission delays, investigating the effectiveness of coded excitations in reducing the required number of temporal averages while maintaining a given SNR, and developing Barker convolved mutually orthogonal Golay complementary codes (BMOGCC) to enable parallel transmission. The proposed methods were validated through numerical simulations and experimental investigations using guided-wave measurements on aluminum plates and bulk-wave measurements on steel blocks, demonstrating their effectiveness under practical ultrasonic measurement conditions.

Key findings include the successful separation of overlapping chirps using the FrFT under reduced transmission delays, thereby enabling shorter transmission delays. The coded excitation approach reduced the number of temporal averages required to achieve a given SNR from 40 repeated acquisitions to only one or a few acquisitions, thereby substantially reducing the overall acquisition time. Furthermore, the proposed BMOGCC retained the high magnitude gain of Barker convolved Golay codes (BCGC) while inheriting the parallel transmission capability of mutually orthogonal Golay complementary codes (MOGCC), thereby halving the number of required transmission events compared with BCGC. The findings demonstrate that the three proposed approaches provide complementary strategies for improving ultrasonic measurement efficiency by addressing its key limitations.

This thesis makes contributions to the field by systematically investigating three approaches for improving the measurement efficiency of UT. The proposed methodologies establish a comprehensive framework for improving the measurement efficiency while maintaining signal reliability. The proposed methodologies and findings have the potential to drive further research and development toward more efficient UT.

Kokkuvõte

Signaalitöötlusel põhinevad efektiivsed ultrahelimõõtmised

Ultrahelitestimine (UT) on üks enim kasutatavaid mittepurustava katsetamise meetodeid konstruktsioonide terviklikkuse hindamiseks. UT meetodi efektiivsust piirab sageli vajadus kasutada piisavalt pikki edastusviivitusi signaalide kattumise vältimiseks, teha signaali ja müra suhte parandamiseks korduvaid mõõtmisi ajalise keskmistamise abil ning rakendada järjestikust signaaliedastust. Need piirangud pikendavad mõõtmisaega ning vähendavad UT rakendatavust suuremõõtmeliste konstruktsioonide kontrollimisel ja suure läbilaskevõimega tööstuslikes inspekteerimissüsteemides. Käesolevas doktoritöös arendatakse signaalitöötlusmeetodeid, mille eesmärk on suurendada ultrahelimõõtmiste efektiivsust nende piirangute ületamise kaudu.

Doktoritöö koosneb kolmest põhisuunast. Esmalt töötatakse välja murdarvulisel Fourier' teisendusel (fractional Fourier transform, FrFT) põhinev signaalide eraldamise meetod, mis võimaldab lahutada lühendatud edastusviivituste tõttu kattuvaid ultrahelisignaale. Teiseks uuritakse kodeeritud ergastuse ja ajalise keskmistamise vahelist seost eesmärgiga vähendada korduvate mõõtmiste arvu, säilitades seejuures etteantud signaali ja müra suhte. Kolmandaks töötatakse välja Barkeri ja vastastikku ortogonaalsete Golay komplementaarkoodide konvolutsioonil põhinev kodeerimisskeem (Barker Convolved Mutually Orthogonal Golay Complementary Codes, BMOGCC), mis võimaldab kasutada paralleelset signaaliedastust. Väljatöötatud meetodeid valideeriti nii arvutuslike simulatsioonide kui ka eksperimentaalsete uuringutega, kasutades juhitud lainetel põhinevaid mõõtmisi alumiiniumplaatidel ja ruumilainetel põhinevaid mõõtmisi terasplokkidel. Tulemused kinnitavad meetodite tõhusust praktilistes ultrahelimõõtmise tingimustes.

Olulisemate tulemustena näidati, et FrFT võimaldab edukalt eraldada kattuvaid lineaarse sagedusmodulatsiooniga ultrahelisignaale lühendatud edastusviivituste korral, võimaldades seeläbi kasutada lühemaid edastusviivitusi ja parandada mõõtmiste efektiivsust. Kodeeritud ergastuse kasutamine võimaldas vähendada etteantud signaali ja müra suhte saavutamiseks vajalike ajaliste keskmistamiste arvu 40 kordusmõõtmiselt ühe või mõne mõõtmiseni, parandades märkimisväärselt mõõtmiste efektiivsust. Lisaks säilitas väljatöötatud BMOGCC Barkeri ja Golay koodide konvolutsioonil põhinevate koodide (BCGC) suure amplituudivõime, omandades samal ajal vastastikku ortogonaalsete Golay komplementaarkoodide (MOGCC) paralleelse signaaliedastuse võimekuse. Tulemused näitavad, et kõik kolm välja pakutud lähenemist täiendavad üksteist ning pakuvad tõhusaid võimalusi ultrahelimõõtmiste efektiivsuse suurendamiseks.

Doktoritöö panustab ultraheliuuringute valdkonda kolme ultrahelimõõtmiste efektiivsuse suurendamisele suunatud lähenemise süstemaatilise uurimisega. Väljatöötatud meetodid moodustavad tervikliku raamistiku ultrahelimõõtmiste efektiivsuse parandamiseks, säilitades samal ajal mõõtetulemuste usaldusväärsuse. Käesolevas töös esitatud meetodid ja tulemused loovad eeldused edasiseks teadus- ja arendustegevuseks veelgi tõhusamate ultraheliuuringute meetodite väljatöötamisel.

Appendix 1

I

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Article

Fractional Fourier Transform-Based Signal Separation for Ultrasonic Guided Wave Inspection of Plates

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Abstract: Detecting defects in plates is crucial across various industries due to safety risks. While ultrasonic bulk waves offer point-by-point inspections, they are time-consuming and limited in coverage. In contrast, guided waves enable the rapid inspection of larger areas. Array transducers are typically used for more efficient coverage, but conventional excitation methods require sufficient time delays between the excitation of array elements that prolong inspection time, necessitating data acquisition time optimization. Reducing time delays can lead to signal overlapping, complicating signal separation. Conventional frequency domain or time-domain filtering methods often yield unsatisfactory separation results due to the signal overlapping in both domains. This study focuses on the application of the Fractional Fourier Transform (FrFT) for separating overlapping ultrasonic signals, leveraging the FrFT's ability to distinguish signals that overlap in both the time and frequency domains. Numerical simulations and experiments were conducted to investigate the FrFT's separation performance for guided waves inspection with array transducers. Results showed that a smaller time delay worsened separation, while using a chirp signal with a broader bandwidth improved separation for signals of fixed duration. Additionally, the effect of signal dispersion on the results was minimal. The findings confirm that the FrFT can effectively separate overlapping signals, enhancing time efficiency in guided wave inspections using array transducers.



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Keywords: ultrasonic guided waves; array transducer; signal separation; fractional Fourier transform; time efficiency

1. Introduction

Plates are extensively used in various industries, such as construction, aviation, and shipping [1–3]. However, defects can form in these metal plates during or after manufacturing. The occurrence of defects can be attributed to various reasons, such as rolling processes or large mechanical impact [4,5]. Different types of defects including holes, cracks, and corrosion are commonly observed in plates [6]. These defects lead to reduced structure integrity, increasing the risk of structural failure, which can, in turn, result in property damage, loss of life, and even catastrophic incidents. For example, the collapse of the I-35W Mississippi River bridge was caused by a defective metal plate, demonstrating that even minor defects can lead to catastrophic failures [7]. Ensuring the proper functioning and safety monitoring of structures incorporating plates is essential, making it crucial to develop reliable methods to detect and quantify these defects.

Numerous techniques have been developed for plate inspection, including radiography, visual inspection, and ultrasonic bulk wave testing. Radiography uses X-rays to penetrate the inspected objects, offering high accuracy and the ability to detect the internal defects beneath the object's surface. However, X-ray inspection systems pose a risk of radiation exposure to users [8]. Visual inspection performed by humans in industries is simple but not time-efficient and can be tedious, leading to the popularity of camera-based inspection [9]. Camera-based inspection is widely used due to its high accuracy,

convenience, and low labor requirements; however, it is generally restricted to surface defects [10]. Ultrasonic bulk wave inspection, while safer than radiography and requiring less complex image processing compared to camera-based methods, involves inspecting objects point by point, which demands high-performance mechanical control systems and is time-consuming [11].

Ultrasonic guided waves (UGWs) have become a promising tool for non-destructive evaluation due to their remarkable advantages. UGW-based inspection methods are cost-effective, capable of detecting both surface and internal defects, and able to inspect large areas without the need to move transducers point by point [12]. Consequently, UGW-based methods are widely employed for both inspecting and long-term monitoring of plates. The use of UGWs in plate inspection has been investigated by numerous researchers. For instance, Cho et al. studied crack growth in plates using UGWs, while Sharma et al. investigated the defects for submerged plates using UGWs [13,14]. UGW inspection techniques can be categorized based on signal reception methods: pulse–echo techniques, which collect reflected signals, and pitch–catch techniques, which collect transmission-through signals [15,16]. Additionally, UGW inspection techniques can be classified according to the number of transducers used: array techniques that utilize multiple transducers and non-array techniques that employ only one or two transducers [17,18].

UGW inspection using a single transducer or a pair of transducers can only scan a limited area, making it inefficient for covering large areas, as shown in Figure 1a. In contrast, an array transducer enables rapid data collection through multiple transducers without needing to move them, facilitating more accurate and efficient inspection over large areas [19]. Conventionally, transducers in an array are excited individually, often with one transducer repeatedly excited multiple times to improve the signal-to-noise ratio through signal averaging [20]. Additionally, a sufficient time delay TD greater than the excitation duration between successive excitations is necessary to ensure that received signals are not interfered with by adjacent excitations, as shown in Figure 1b. However, this approach reduces data acquisition efficiency. To improve the time efficiency of conventional array transducer inspections, the time delay between successive excitations can be shortened to less than the excitation duration, as shown in Figure 1c. By comparing Figure 1b and Figure 1c, the signals emitted by the four sensors on the right in Figure 1c arrive earlier than those in Figure 1b due to the reduced time delay. In conventional array transducer inspection, if the number of signal averaging and transducers in an array is denoted as P and Q , the total time delay is $P \times Q \times TD$. With the reduced time delay td , the total time delay becomes $P \times Q \times td$, which can significantly reduce time costs, particularly when the array contains a large number of transducers or requires extensive signal averaging. However, reducing the time delay may cause received signals to overlap, making them challenging to identify within a measured signal. This can complicate signal interpretation, rendering it unsuitable for further data processing.

Various methods have been employed to separate ultrasonic signals. For signals that overlap only in the frequency domain, appropriate frequency filtering can effectively separate them. Similarly, if signals overlap only in the time domain, time-domain windowing can be sufficient for separation. The most challenging case arises when signals overlap in both time and frequency domains, where simple time windowing or frequency filtering alone are inadequate. Eliminating overlap in both domains requires the signal duration to be as short as possible while maintaining a narrow bandwidth. However, according to the uncertainty principle, achieving both a short duration and a narrow bandwidth simultaneously is impossible [21]. Several studies have attempted to address this challenge of dual-domain overlap. High-frequency broadband signals with short durations are often recommended to reduce time-domain overlap, yet high-frequency signals are often impractical due to significant attenuation [22,23]. Frequency-division-based techniques minimize overlap in the frequency domain; however, they often require identical transmitted signals across applications to allow for a consistent interpretation of the received signals, limiting the applicability of frequency-division-based techniques [24–27]. Matching

pursuit methods approximate overlapping signals from a dictionary of atom signals to enable separation in both domains, though constructing an efficient dictionary remains a practical challenge [28–30]. Signal coding techniques, such as Barker and Golay codes, help compress signals into narrow peaks, improving signal component localization, but may not always guarantee complete signal separation [31,32].

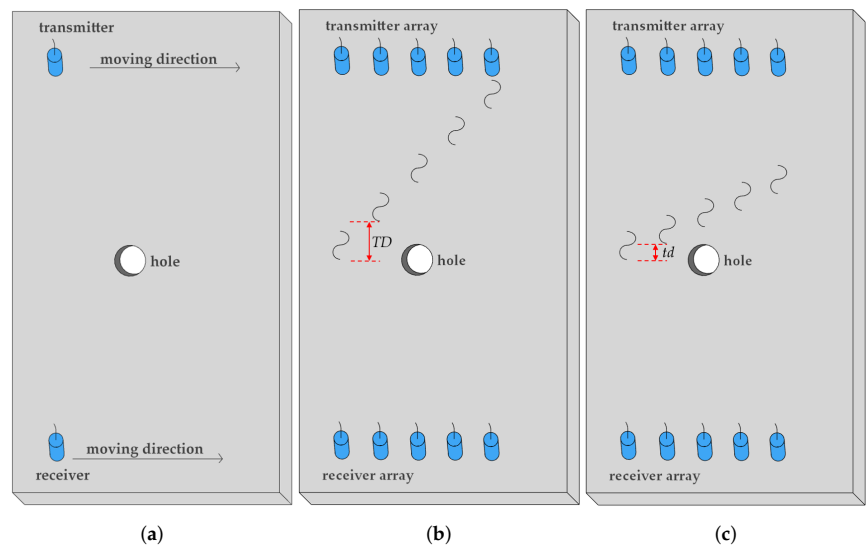


Figure 1. Schematic of UGW inspections. (a) Single transmitter–receiver inspection requiring transducer movement for larger areas. (b) Array transducer inspection with sufficient time delay. (c) Array transducer inspection with reduced time delay.

The limitations of traditional separation methods can be addressed by the Fractional Fourier Transform (FrFT), a generalized Fourier Transform (FT) that has gained increasing attention for ultrasonic signal separation in recent years. Although the concept of the FrFT first appeared in the mathematical literature in the 1920s, it gained significant traction in the 1980s with contributions from Namias [33,34]. Digital computation algorithms for the FrFT, developed by researchers like Ozaktas and Pei have further facilitated its widespread application [35–41]. Similar to the FT, which decomposes signals into a series of sinusoids, the FrFT represents signals as a decomposition of a series of linear chirps, enabling sharp compression of linear chirp signals. This property allows the FrFT to effectively separate linear chirp signals that overlap in both the time and frequency domains. Recent research has successfully applied the FrFT for ultrasonic signal separation. For instance, Cowell et al. utilized the FrFT to separate echoes generated by a single linear chirp transmitted through a steel plate [42]. Harput et al. explored the FrFT for human tooth imaging, separating overlapping chirp signals for enhanced image clarity [43]. Pachauri applied the short-time FrFT to separate synthetic ultrasonic composite signals [44]. Additionally, Li et al. proposed an FrFT-based method for monitoring pipe elbow erosion by isolating useful signals from interfering signals [45]. Despite these advancements, most FrFT-based ultrasonic signal separation techniques have been limited to single-transducer inspection. Its application in array transducer inspection, along with the effects of chirp parameters and signal dispersion, remain underexplored areas for further research.

The main aim of this paper is to investigate the application of the FrFT in separating overlapping signals for UGW array transducer inspection. To evaluate the effectiveness of the FrFT, we simulated the UGW propagation of the A_0 mode in an aluminum plate with a hole using finite element (FE) modeling, and conducted corresponding laboratory experiments. Instead of driving each transducer individually, we excited all transducers simultaneously with a reduced time delay to enhance time efficiency. In addition, the

performance of the FrFT under different delay settings was compared and analyzed. This paper is organized as follows: Section 2 covers the fundamentals of the FrFT, including its calculation, key properties, and mechanism for separating signals that overlap in both the time and frequency domains. Section 3 validates the separation performance of the FrFT through numerical simulations. Section 4 demonstrates the FrFT's separation performance based on experimental data. Section 5 discusses the results, and Section 6 provides the conclusions.

2. Fractional Fourier Transform Theory and Methods for Signal Separation

2.1. Principles of the FrFT

The FrFT has been defined and interpreted in various ways, including as a linear integral transform, a rotation of the time–frequency plane, and as the solutions of differential equations[46,47]. A common definition of the FrFT involves a linear integral transform, expressed as follows:

$$X_p(u) = A_\alpha \int_{-\infty}^{+\infty} f(t) \exp[j\pi \cot \alpha (t^2 + u^2) - 2ut \csc \alpha] dt, \quad (1)$$

where

$$A_\alpha = \frac{\exp[-\frac{j\pi \operatorname{sgn}(\sin \alpha)}{4} + \frac{j\alpha}{2}]}{|\sin \alpha|^{\frac{1}{2}}} \quad (2)$$

and

$$\operatorname{sgn}(t) = \begin{cases} -1, & t < 0 \\ 0, & t = 0. \\ 1, & t > 0 \end{cases} \quad (3)$$

Here, $f(t)$ represents the time-domain function to be transformed, j is the imaginary unit, and u is the fractional domain variable, functioning as the time in a time-domain signal or the frequency of the FT of a signal. The parameter α represents the rotation angle of the FrFT, and the fractional power given by $p = \frac{2\alpha}{\pi}$. The range of α is $[-\pi, +\pi]$ with a period of 2π , making p range within $[-2, 2]$ with a period of 4. The function $\operatorname{sgn}(t)$ denotes the sign function.

The chirp signal, characterized by a frequency that varies with time, is widely used in ultrasonic testing due to its frequency-sweeping nature [48]. A common example is the linear chirp signal, where frequency changes linearly over time, defined as

$$c(t) = \exp[2j\pi(f_c t - \frac{B}{2}t + \frac{B}{2T}t^2)], 0 \leq t \leq T, \quad (4)$$

where T is the time duration, B is the sweeping bandwidth, and f_c is the center frequency. The swept frequency range is $(f_c - \frac{B}{2}, f_c + \frac{B}{2})$, and the chirp rate is $\frac{B}{T}$. The instantaneous frequency of a linear chirp is defined as

$$f_i(t) = \frac{d}{dt}(f_c t - \frac{B}{2}t + \frac{B}{2T}t^2) = f_c - \frac{B}{2} + \frac{B}{T}t. \quad (5)$$

By comparing Equations (1) and (4), it is clear that the kernel of the FrFT consists of a set of linear chirps with a chirp rate of $\cot \alpha$, similar to the FT, $\exp(-2j\pi ft)$. Unlike the FT, the FrFT includes an additional parameter α , and the FrFT becomes the FT when $\alpha = \frac{\pi}{2}$. In the following, Fr^α denotes the FrFT with angle α . Some key properties of the FrFT are given as follows [49,50]:

- $Fr^0 f(t) = f(t)$;
- $Fr^{\frac{\pi}{2}} f(t)$ is equal to the FT of $f(t)$;
- $Fr^{-\frac{\pi}{2}} f(t)$ is equal to the inverse FT of $f(t)$;
- Linearity: $Fr^\alpha[\sum_{k=1}^n b_k f_k(t)] = \sum_{k=1}^n b_k \{Fr^\alpha[f_k(t)]\}$;

- Index additivity: $Fr^{\alpha+\beta}[f(t)] = Fr^{\alpha}\{Fr^{\beta}[f(t)]\}$;
- Rotation invariance: the Wigner–Ville Distribution (WVD) of $Fr^{\alpha}[f(t)]$ is identical to the WVD of $f(t)$ with the axis rotated by α .

The rotation invariance property indicates that the FrFT preserves a signal's time-frequency distribution. With this, the FrFT can be viewed as a rotation of the time–frequency plane (Figure 2a), where the fractional domain is a domain between the time domain and frequency domain. Additionally, Figure 2b shows that a linear chirp signal can be sharply compressed at an optimal angle in the fractional domain. This optimal angle α_o is calculated as $-\arctan(\frac{T}{B})$, with the corresponding fractional power $\frac{-2}{\pi} \arctan(\frac{T}{B})$ [51].

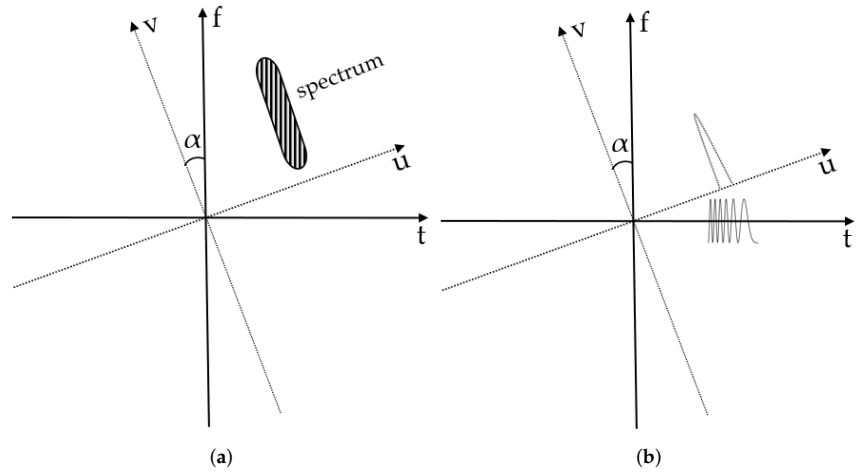


Figure 2. Schematic diagram of the FrFT. (a) Rotation invariance: the location of spectral components is unaffected by the rotation angle. (b) Chirp compression: the transformation of a long chirp signal into a narrow peak.

2.2. Calculation of the FrFT

Similar to how the Fast Fourier Transform (FFT) significantly advanced the development of the FT, the fast calculation algorithm for the FrFT is crucial for its successful application in signal processing. However, unlike the FFT, which strictly adheres to the properties of the FT, current fast digital FrFT calculation algorithms do not fully conform to all FrFT properties of the FrFT [49]. Despite this limitation, some algorithms effectively approximate these properties for practical use. In this study, we employed Ozaktas' fast FrFT calculation algorithm due to its high accuracy and low calculation complexity [35].

The preliminary step of Ozaktas' algorithm involves dimension normalization, which equalizes the support for the FrFT regardless of the rotation angle. To achieve this normalization, a scaling factor $S = \sqrt{\frac{\Delta t}{\Delta f}}$ is used to rescale the signal, where Δt and Δf represent the signal's time length and bandwidth, respectively. Following rescaling, the time and frequency coordinates are transformed into new coordinates as $t_{new} = \frac{t}{S}$ and $f_{new} = fS$, keeping the same signal length $\Delta x = \sqrt{\Delta f \Delta t}$ in both t_{new} and f_{new} . The next step is to decompose Equation (1) into simpler equations and perform discretization using Shannon's interpolation formula. For a digital signal with N sampling points, the computational complexity of the algorithm is $O(N \log N)$, which matches that of the FFT. Due to dimension normalization and discretization, the optimal angle α_{od} for the digitized signal, aimed at achieving the best compression of the linear chirp signal $c(t)$ in the fractional domain, is modified as

$$\alpha_{od} = -\arctan\left(\frac{f_s^2 T}{BN}\right), \quad (6)$$

where f_s is the sampling frequency.

2.3. The FrFT Separation Process for Overlapping Signals

The separation of overlapping linear chirp signals using the FrFT relies on its properties of linearity and rotation invariance. Linearity ensures that the FrFT of a sum of signals equals the sum of the FrFT of each individual signal. Rotation invariance enables the sharp compression of a linear chirp by rotating the axis to a suitable angle. Utilizing these properties, overlapping linear chirp signals in both the time and frequency domains can be decomposed into separated, compressed signals in the fractional domain, as illustrated in Figure 3.

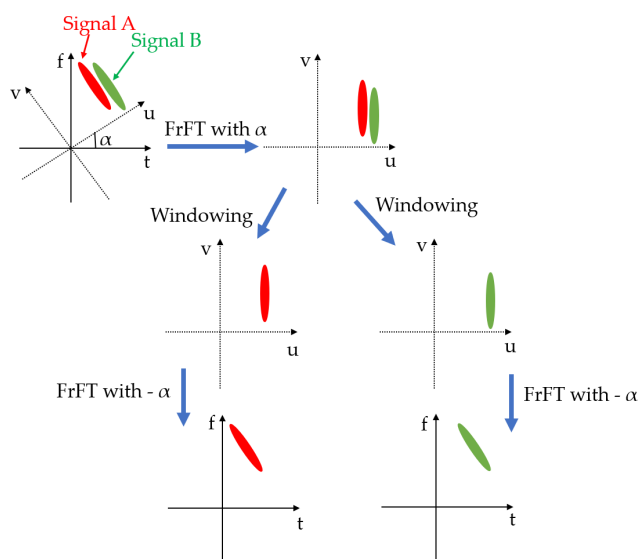


Figure 3. Flow chart of the three-step FrFT separation.

The separation process involves three main steps:

- **Transformation to the Fractional Domain:** the overlapping signals are transformed into the fractional domain, with the selection of a suitable rotation angle α being critical. Effective separation depends on choosing an optimal angle determined by Equation (6).
- **Peak Separation:** in the fractional domain, each individual signal appears as a distinct peak. These peaks are separated using a window function, typically a rectangular window, to minimize distortion.
- **Restoration to the Original Time Domain:** once separated in the fractional domain, the signals are transformed back to the time domain via the inverse FrFT. To achieve signal restoration, the inverse FrFT is employed. Based on the index additivity property, the inverse FrFT can be obtained simply by applying the FrFT with the opposite angle $-\alpha$, as shown in

$$f(t) = Fr^{\alpha-\alpha}[f(t)] = Fr^{-\alpha}\{Fr^{\alpha}[f(t)]\}. \quad (7)$$

Therefore, unlike the inverse FT, the inverse FrFT is simply the FrFT with the opposite angle. A key requirement for separation is a time delay between the overlapping signals, ensuring that the signals are distinguishable in the time–frequency plane even though they overlap in both the time and frequency domains. Figure 4 illustrates the flow of signal separation using the FrFT. Figure 5 provides an example of how a synthetic overlapping signal is separated as two individual signals, where a linear chirp signal is overlapped by another chirp with a time delay.

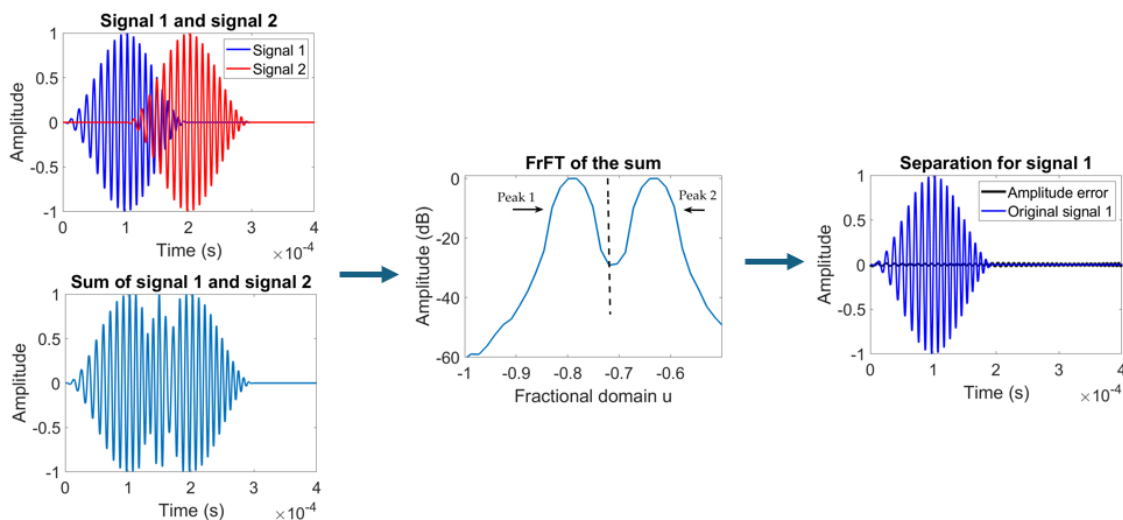


Figure 4. Separation of overlapping linear chirps. The two chirps share a bandwidth of 50 kHz and a center frequency of 100 kHz, with signal 2 delayed by 0.1 ms. Peak 1 corresponds to signal 1, while peak 2 corresponds to signal 2. The boundary between the peaks is marked with the black dashed line. The amplitude error represents the difference between the separated signal and the original signal.

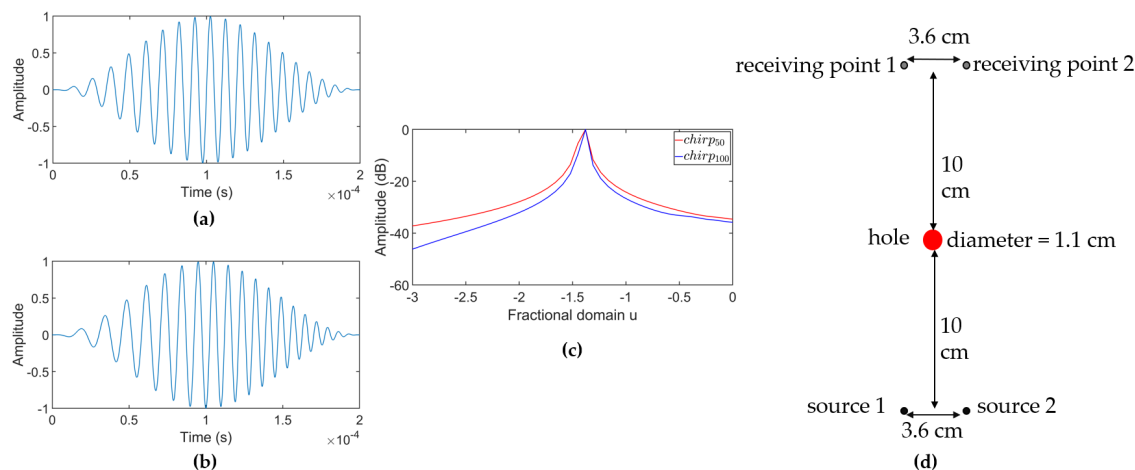


Figure 5. (a) $chirp_{50}$. (b) $chirp_{100}$. (c) The FrFT of $chirp_{100}$ and $chirp_{50}$. (d) Geometry of the simulation setup.

3. Numerical Simulations

3.1. Finite Element Modeling

Simulations were performed in ABAQUS software (Version 6.9) [52] to evaluate the FrFT's effectiveness in separating signals. A three-dimensional (3D) finite element model was created, consisting of an aluminum plate of size $1000 \text{ mm} \times 1000 \text{ mm} \times 2 \text{ mm}$, with an 11 mm diameter hole located centrally. The model was meshed with element type C3D8R with a size of 1 mm. The A_0 mode was generated by applying an out-of-plane force at two source nodes on the plate surface, while out-of-plane displacements were measured at two receiving nodes. Two different increasing linear chirp signals were used for excitation: the first, $chirp_{50}$ (Figure 5a), is a 20-cycle Hanning-windowed chirp with a 100 kHz center

frequency and 50 kHz bandwidth, matching the bandwidth and center frequency of the experimental ultrasonic transducer. The 20-cycle length balances high compression in the fractional domain with minimal plate boundary reflections, as shorter chirps offer less compression, and longer chirps risk interference from boundary reflections. The second signal, $chirp_{100}$ (Figure 5b), retains these parameters but has a 100 kHz bandwidth to evaluate the influence of bandwidth on FrFT performance. The FrFT of $chirp_{100}$ and $chirp_{50}$ is shown in Figure 5c. Dispersion curves for this 2 mm aluminum plate, showing phase and group velocities as functions of frequency, are displayed in Figure 6. Since the excitation signal frequency lies in a dispersive region of the dispersion curve, the signal undergoes shape changes due to dispersion. Both the hole and dispersion effects allow for an assessment of their impact on the FrFT’s separation performance.

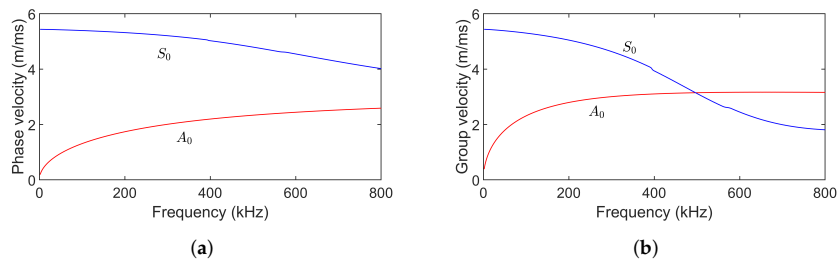


Figure 6. Dispersion curves of Lamb waves in a 2 mm thick aluminum plate. (a) Phase velocity. (b) Group velocity.

Simulations were performed with reduced time delays (see Figure 5d), covering two groups: one with $chirp_{50}$ as the excitation signal and another with $chirp_{100}$. Table 1 lists the time delay settings. Due to the reduced delay, the received signals overlapped in time and were separated using the FrFT.

Table 1. Delay setting for simulations.

Group No.	Simulation No.	Receiving Point No.	Excitation Signal	Source 1	Source 2
1	1	1	$chirp_{50}$	No delay	100 μ s delay
	2	1	$chirp_{50}$	No delay	80 μ s delay
	3	1	$chirp_{50}$	No delay	60 μ s delay
2	4	1	$chirp_{100}$	No delay	80 μ s delay
	5	1	$chirp_{100}$	No delay	60 μ s delay

To assess separation performance, each source was also excited individually, allowing the collection of original, non-overlapping signals for reference. Additionally, simulation 6 used the same delay as simulation 3 but was conducted on a plate without a hole, examining the hole’s impact on separation. To further study dispersion’s influence on separation, two $chirp_{50}$ signals were delayed and summed based on signal locations of simulation 6, and separation was performed on this overlapping summation signal.

3.2. Results of Signal Separation

The signal collected at receiving point 1 when only source 1 is excited was labeled S11, with S21 following the same naming convention. Figure 7 illustrates the detailed separation results for simulation group 1, with each column representing the separation process for simulations 1 to 3 in sequence from left to right. The first-row figures show the original signals from individual sources with different delays. The second row displays the summations of two overlapping signals, showing the complexity of signal interpretation. By applying Equation (6) and using known chirp parameters, the optimal rotation angle was determined. Transforming the signals into the fractional domain using this angle

achieved signal compression, as seen in the third-row figures. In this domain, signal components appear as distinct peaks, with smaller delays resulting in greater peak overlap. The peaks were isolated by windowing in the fractional domain, with peak boundaries indicated by black dashed lines. The windowed signals were then transformed to the time domain using the inverse FrFT. The separated signals are compared to the original signals in the fourth and fifth rows, with the fourth row corresponding to S11, and the fifth to S21.

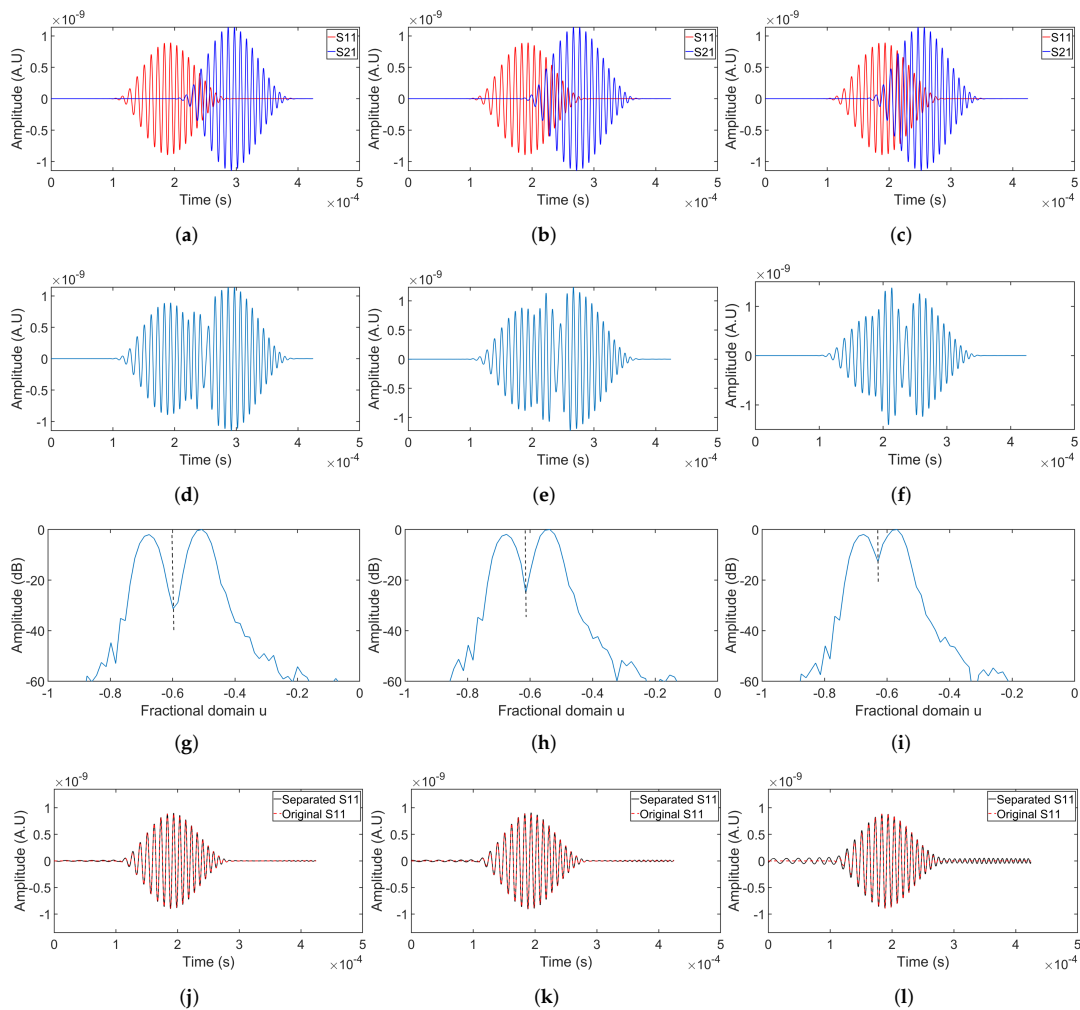


Figure 7. Cont.

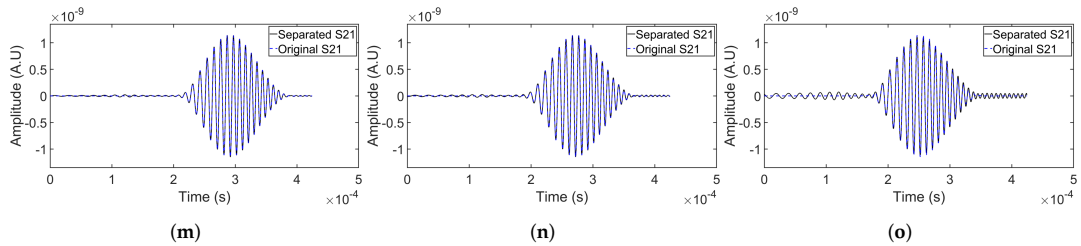


Figure 7. Separation of overlapping signals collected at receiving point 1 using $chirp_{50}$ (simulations 1–3). The first row (a–c) shows the original signals from each source. The second row (d–f) represents the summation of these signals, creating overlap. The third row (g–i) displays the FrFT of the overlapping signals. In the fourth row (j–l), the separated signal from source 1 is compared with its original signal. The fifth row (m–o) compares the original signal from source 2 with its separated version.

Figure 8 presents the separation results for simulations 4 and 5, where $chirp_{100}$ was used. Separation results for simulation 6, which is without a hole, as well as for the summation of two overlapping $chirp_{50}$ without dispersion, are shown in Figure 9.

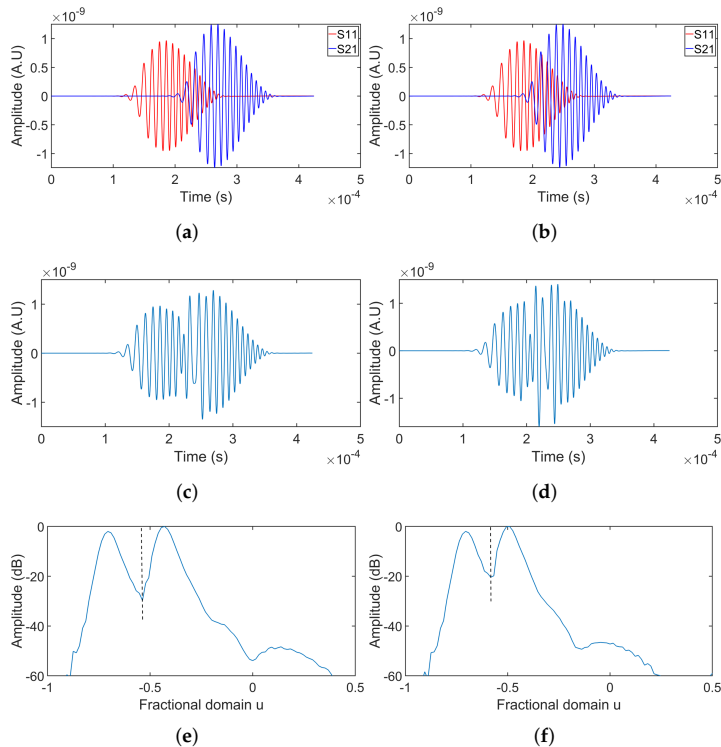


Figure 8. Cont.

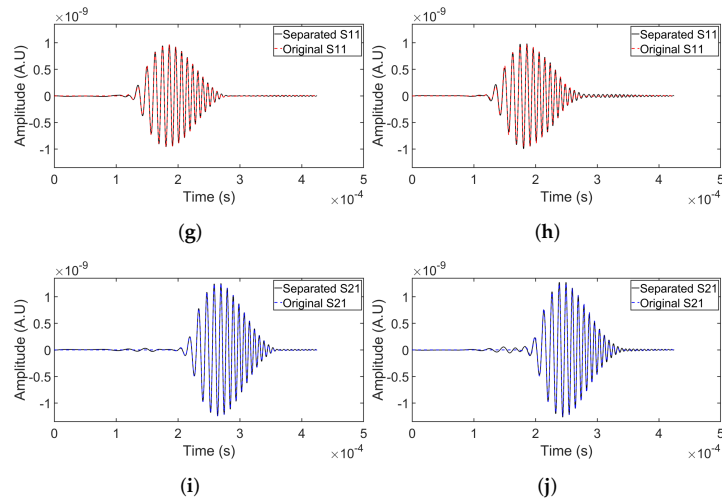


Figure 8. Separation of overlapping signals using *chirp*₁₀₀ at receiving point 1. The first column shows the separation results for simulation 4, and the results for simulation 5 are shown in the second column. The first row (a,b) shows the original signals from each source. The second row (c,d) represents the summation of these signals. The third row (e,f) displays the FrFT of the overlapping signals. In the fourth row (g,h), the separated signal from source 1 is compared with its original signal. The fifth row (i,j) compares the original signal from source 2 with its separated version.

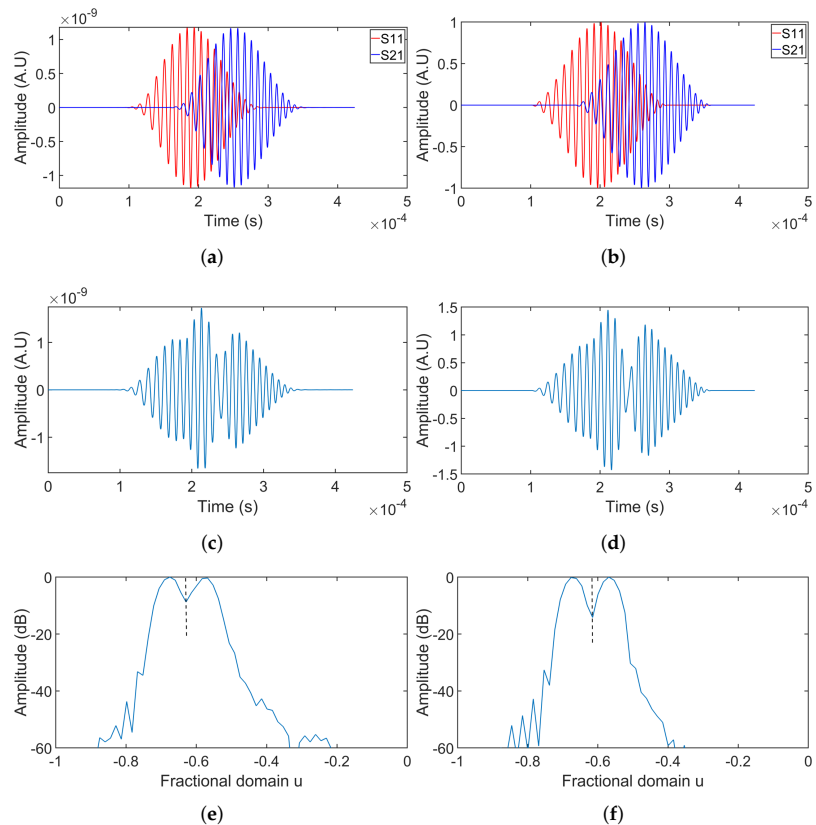


Figure 9. Cont.

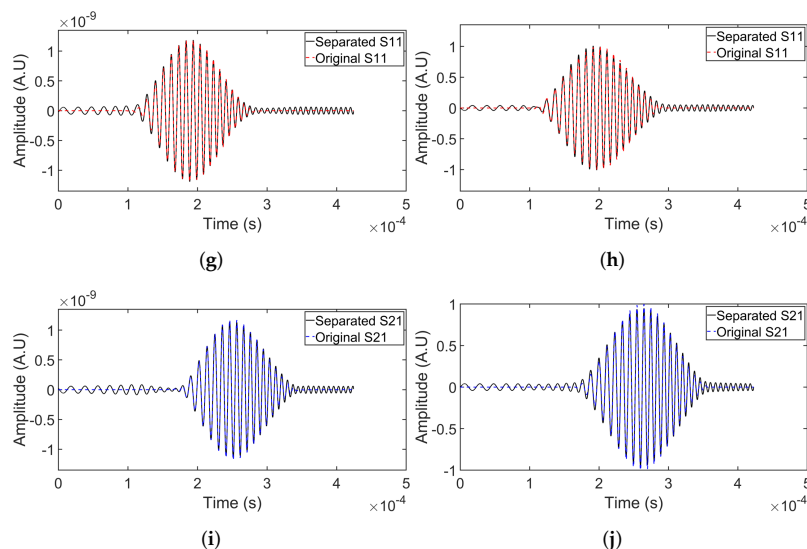


Figure 9. Separation results for simulation 6 and summation of two overlapping $chirp_{50}$. The separation results for simulation 6 are shown in the first column. The separation results for summation of two overlapping $chirp_{50}$ are shown in the second column. The first row (a,b) shows the original signals from each source. The second row (c,d) represents the summation of these signals. The third row (e,f) displays the FrFT of the overlapping signals. In the fourth row (g,h), the separated signal from source 1 is compared with its original signal. The fifth row (i,j) compares the original signal from source 2 with its separated version.

4. Experimental Measurements

4.1. Experimental Setup

To further evaluate the FrFT's effectiveness, experimental measurements were performed to ensure that factors such as transducer response and data acquisition (DAQ) did not significantly impact the separation performance. The experimental setup, shown in Figure 10a, included a 1000 mm $\times$ 1000 mm $\times$ 2 mm aluminum plate with an 11 mm diameter central hole, two ultrasonic transducers, a laser vibrometer, and a DAQ system. The transducers (S0208, ACS Ltd., Saarbrücken, Germany) with a center frequency of 100 kHz and a bandwidth of 50 kHz were coupled to the plate using a coupling agent for optimal contact. Vibrations at the receiving points on the plate surface were measured using a Polytec VFX-I-160 laser vibrometer. The DAQ box, equipped with functions such as signal amplification and averaging, connected to the PC via USB and was controlled by LabVIEW-based software to set the excitation signals, time delay, sampling rate, amplification, and signal averaging [53].

The experiments focused on A_0 mode propagation for two excitation sources and two receiving points. The $chirp_{50}$ signal was chosen to validate FrFT separation, while $chirp_{100}$, used in the simulations, was excluded due to the transducers' bandwidth limitations. Delay settings of experiments 1–3 matched simulations 1–3. The positioning of the hole, transducers, and receiving points in Figure 10b mirrored the simulated setup in Figure 5d. Signal averaging was used in the experiments, with signals collected by the laser vibrometer averaged 100 times to minimize mechanical, ambient, and DAQ system noise.

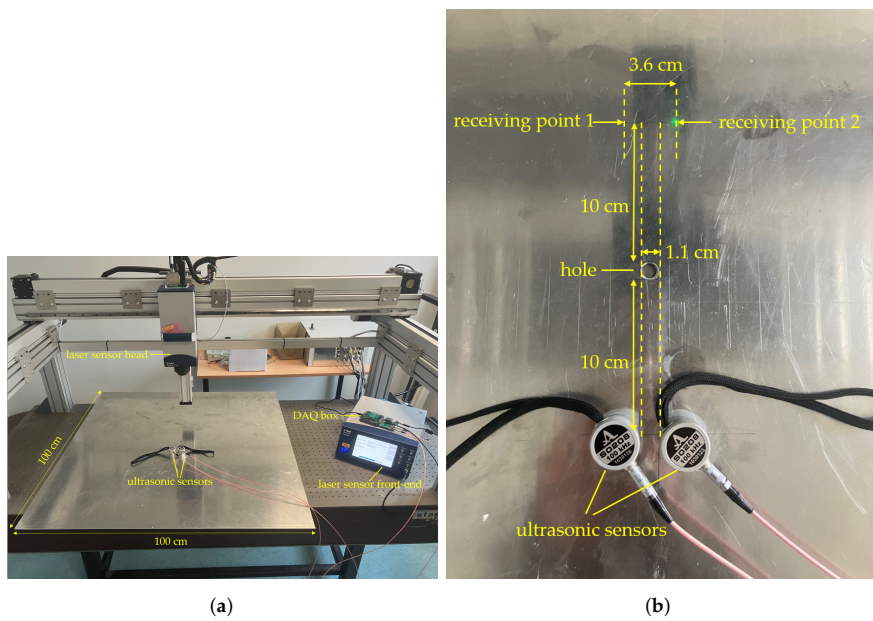


Figure 10. Experimental setup. (a) Measurement setup including ultrasonic sensors, laser vibrometer, and DAQ system. (b) Geometry of the sensors, hole, and receiving positions.

4.2. Results of Signal Separation

Figure 11 illustrates the detailed separation results for experiments, with each column representing the separation process for experiments 1–3 in sequence from left to right. The separation process follows the same flow as in the simulations: the rows from first to fifth correspond to the original signals, the summation of the overlapping signals, the transformed signals in the fractional domain, and the separated signals, respectively.

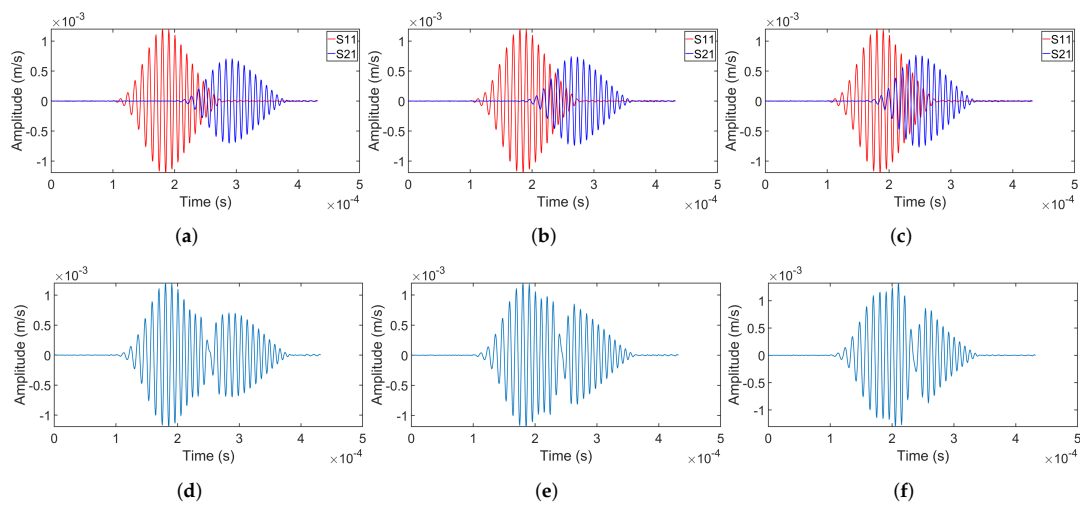


Figure 11. Cont.

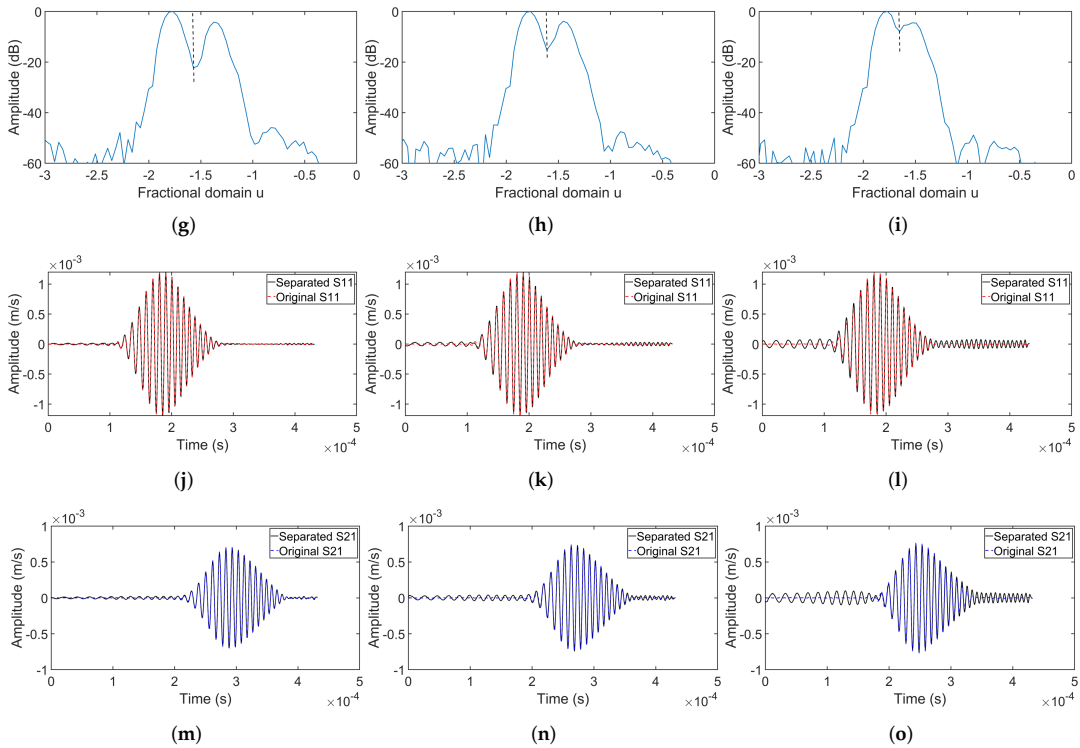


Figure 11. Separation of overlapping signals collected at receiving point 1 (experiments 1–3). The first row (a–c) displays the original signals from each source. The second row (d–f) shows the summation of these signals. The third row (g–i) represents the FrFT of the overlapping signals. In the fourth row (j–l), the separated signal from source 1 is compared with its original signal. The fifth row (m–o) compares the original signal from source 2 with its separated version.

5. Discussions

Observations from simulations 1–3 (Figure 7) indicated that overlap in the fractional domain increased as the time delay decreased, adversely affecting separation performance. The fourth and fifth rows of Figure 7 demonstrated that harmonic-like artifacts at the sidelobes significantly contribute to separation errors. As the delay increased, the peaks in the fractional domain moved further apart, reducing harmonic-like artifacts when applying the rectangular window to separate the overlapping signals. Acceptable separation was achieved for two delays: 100 μ s and 80 μ s. However, at the 60 μ s delay, the separation performance deteriorated rapidly as the difference between the separated signal and the original signal became noticeable at the sidelobes.

Simulations 4–5 (Figure 8) revealed similar trends, confirming that a larger time delay resulted in improved separation performance. Additionally, separation performance using *chirp*₁₀₀ outperformed that of *chirp*₅₀ under the same time delay.

Figure 9 indicated that the presence of the hole did not significantly impact the FrFT separation performance, as the shape of the signal scattered from the hole was similar to that of the incoming signal. Comparing the dispersive signals shown in the first column with the non-dispersive signals in the figures of the second column, it is evident that the dispersion was mild due to the short wave propagation distance, posing no apparent influence on the separation.

Experiments (Figure 11) also showed that a larger time delay yielded a better separation, and, at the 60 μ s delay, the separation performance decreased rapidly. Comparing Figures 7 and 11, it can be observed that experimental measurements demonstrated lower

separation accuracy than simulations due to greater separation errors in the sidelobes, particularly at the 60 μ s delay.

To quantitatively evaluate the signal separation performance, we employed the maximum magnitude error (MME) and the root mean square error (RMSE) to calculate the error between the original signal and separated signal. MME is defined as the relative magnitude difference among all discrete time points as

$$MME = \frac{\max |S_o(i) - S_s(i)|}{\max |S_o(i)|}, \quad (8)$$

where $S_o(i)$ and $S_s(i)$ represent the discrete original and separated signals, respectively. RMSE is defined as the square root of the squared difference between the original signal and separated signal:

$$RMSE = \sqrt{\frac{\sum_{i=1}^N [S_o(i) - S_s(i)]^2}{N}}, \quad (9)$$

where N is the number of discrete time points. Figure 12 shows the MME and RMSE for simulations 1–3, revealing that both metrics increased as the delay decreased, indicating that a shorter delay resulted in poorer separation. Figure 13 compares the separation errors between $chirp_{100}$ and $chirp_{50}$. While $chirp_{100}$ exhibited lower MME at the 60 μ s delay, it had slightly higher MME at the 80 μ s delay. At the 80 μ s delay, signals were effectively separated using either $chirp_{100}$ or $chirp_{50}$, with numerical error contributing significantly to the MME. On the other hand, $chirp_{100}$ had smaller RMSE than $chirp_{50}$ at all delays. $chirp_{100}$ favored separation over $chirp_{50}$, which can be intuitively explained by the greater compression of $chirp_{100}$ in the fractional domain. As shown in Figure 5c, the FrFT of $chirp_{50}$ had a larger main lobe width than that of $chirp_{100}$. Figures 12 and 13 collectively demonstrate that smaller time delays lead to greater MME and RMSE. This is due to the increased signal overlap, which becomes a major source of error in FrFT-based signal separation.

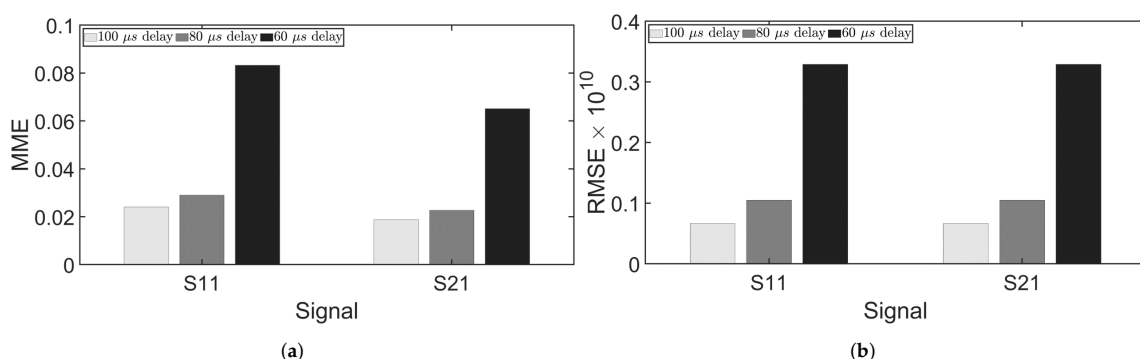


Figure 12. Signal separation errors for simulations 1–3. (a) MME. (b) RMSE.

Experimental measurement errors, depicted in Figure 14, demonstrated that both RMSE and MME increased as the time delay decreased. Notably, separation errors at 60 μ s became evident, though the main lobe of the signal remained well separated. By eliminating the signal before and after the main lobe, ideal separation results could still be obtained. It can be observed from Figures 12a and 14a that MME values from experiments were greater than those from simulations. Table 2 lists the MME and RMSE from simulations 1–3 and experiments 1–3 where $chirp_{50}$ was used as the excitation signal. Nonetheless, both the experiments and simulations have shown accurate separation at delays of 100 μ s and 80 μ s, but accuracy dropped at 60 μ s. Overall, the observations from simulations agreed well with the experimental findings, confirming that signal separation can be achieved using the FrFT with an appropriate delay. This method allows for reduced time delays between excitations, facilitating more time-efficient inspections.

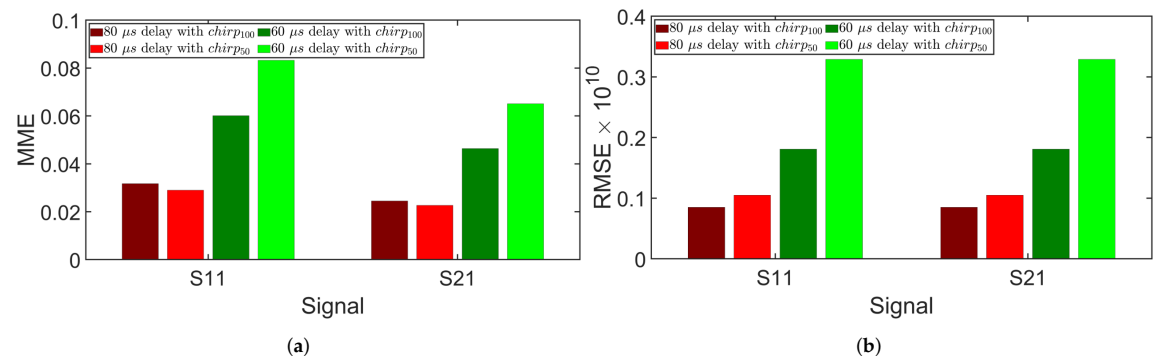


Figure 13. Comparison of signal separation errors using chirp signals with different bandwidths (simulations 4–5). (a) MME. (b) RMSE.

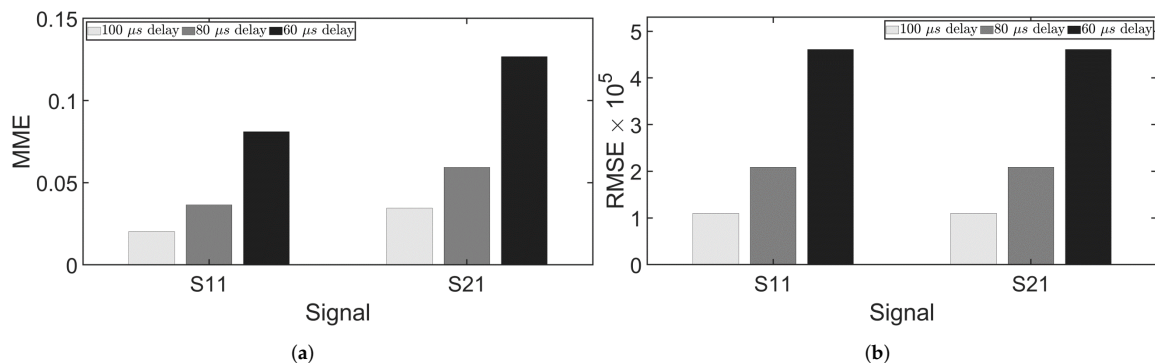


Figure 14. Signal separation errors for experiments. (a) MME. (b) RMSE.

Table 2. Maximum values of MME and RMSE.

	Sim. No. 1	Sim. No. 2	Sim. No. 3	Exp. No. 1	Exp. No. 2	Exp. No. 3
Time Delay	60 μ s	80 μ s	100 μ s	60 μ s	80 μ s	100 μ s
MME	8.3%	2.9%	2.4%	12.7%	5.9%	3.5%
RMSE	3.3×10^{-11}	1.0×10^{-11}	6.7×10^{-12}	4.6×10^{-5}	2.1×10^{-5}	1.1×10^{-5}

Sim. No.: simulation number. Exp. No.: experiment number.

6. Conclusions

This study presented an efficient method aimed at separating overlapping signals caused by the reduced time delay between excitations, thereby improving the time efficiency of conventional array-transducer-guided wave inspections for plates. The proposed method is based on the FrFT, which enables the separation of signals in the rotated time–frequency domain using an optimal rotation angle. Unlike traditional time or frequency windowing, the FrFT allows for the separation of signals that overlap in both time and frequency domains—a challenge often encountered in practical ultrasonic measurements. The effectiveness of the FrFT was validated through 3D FE simulations and experiments involving varying time delays and bandwidths, demonstrating good agreement between simulation and experimental results. Successful separations were achieved with a minimum time delay of 80 μ s, while contamination of the separation process became evident at the 60 μ s delay. In addition, using *chirp*₁₀₀ proved to be more advantageous than *chirp*₅₀ for signal separation. Overall, it was confirmed that larger time delays and wider signal bandwidths enhance separation accuracy, and the FrFT exhibited robustness against dispersion and the presence of holes in simulations.

One limitation of the proposed signal separation method is that it is currently limited to linear chirp signals due to the inherent properties of the FrFT, which restricts its broader application in certain scenarios. Another limitation of this study is the use of only two excitation sources and two receiving points. Future work should explore the application of the FrFT-based separation method with array transducers having a larger number of transducers. Additionally, while the dispersion observed in this study was mild, future research should address scenarios involving severe dispersion. Furthermore, the influence of various types and sizes of defects on the performance of FrFT separation will also be investigated.

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Appendix 2

II

Chengxiang Peng, Paul Annus, Marek Rist, Raul Land, and Madis Ratassepp. Convolution of Barker and mutually orthogonal Golay complementary codes for ultrasonic testing. *Sensors*, 25(13):5007, August 2025.

Article

Convolution of Barker and Mutually Orthogonal Golay Complementary Codes for Ultrasonic Testing

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Abstract

Ultrasonic testing (UT) is a vital nondestructive testing (NDT) technique used to evaluate the integrity of materials and structures. However, conventional excitation signals often suffer from significant attenuation in highly attenuative materials, resulting in low signal energy and poor signal interpretation. Coded excitation techniques, such as the Barker code and the complementary Golay code (CGC), have been used to enhance signal energy and signal-to-noise ratio. Yet, Barker codes are limited by short sequence lengths, while CGC requires two transmission events, reducing time efficiency. This paper proposes a novel excitation method: the Barker-convolved mutually orthogonal Golay complementary code (BMOGCC). By convolving the Barker code with the mutually orthogonal Golay complementary code (MOGCC), BMOGCC combines the advantages of both, including flexibility in code length, improved signal amplitude, low sidelobe levels, and enhanced time efficiency. Performance was evaluated using numerical simulations and laboratory experiments, with key indices including the peak sidelobe level (PSL), mainlobe gain (MG), and temporal resolution. The results show that BMOGCC achieves a significantly higher MG than either the Barker code or MOGCC alone while maintaining a low PSL and preserving the temporal resolution. These findings demonstrate that BMOGCC is effective and efficient for coding excitation signals in ultrasonic testing, offering improved signal quality and measurement time efficiency.



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Keywords: nondestructive testing; ultrasonic testing; coded signals; time efficiency

1. Introduction

Nondestructive testing (NDT) enables the inspection of materials, components, and structures without causing damage or interrupting their operation and is essential to ensure the safety and integrity of civil structures such as metal plates and pipelines [1–3]. Ultrasonic testing (UT), a critical NDT technique, is widely used in a number of fields, such as civil engineering, aerospace, and the energy industry, due to its high efficiency and ability to detect various types of defects [4–6]. UT involves the generation, transmission, and reception of ultrasonic waves that interact with defects in the material. The health condition of the material is then assessed by processing the received signals. Consequently, the quality of the received signals is vital to the reliability of UT.

Ultrasonic signals are vulnerable to attenuation and noise contamination in practical measurement environments. For example, signals can be heavily distorted due to long-distance propagation, such as in rail inspections [7]. In UT, signal energy is a valuable

resource. High energy enables ultrasonic waves to propagate over longer distances, allowing the detection of material defects located farther from the ultrasonic transducer. One approach to increasing the signal energy under adverse measurement conditions is to raise the signal voltage. However, high voltage is often prohibited for various reasons, such as in flammable environments and due to limitations of data acquisition systems [8]. Another approach to increasing the signal energy is to extend the signal duration. However, simply extending the duration degrades the temporal resolution and creates strong sidelobes [9].

Coded signals employ extended coded pulses to increase signal energy without raising pulse voltage, thereby enhancing measurement reliability and safety. For a signal received from a UT system using a coded excitation signal, a decoding process commonly known as pulse compression is needed to compress the long coded signal into a narrow peak, thus enhancing signal amplitude without sacrificing temporal resolution [10,11]. These advantages make coded excitation signals widely used in UT, particularly for testing highly attenuated materials and performing long-distance inspections. Figure 1 presents a UT system using a coded excitation signal.

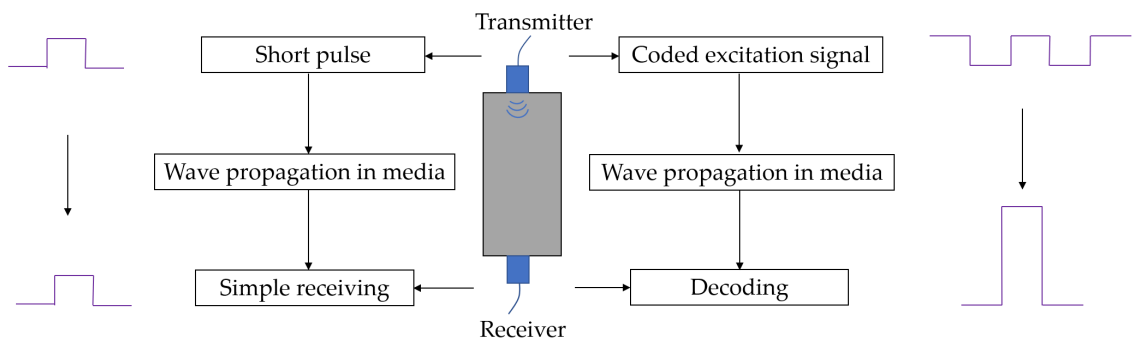


Figure 1. Schematic diagram of a UT system using a coded excitation signal.

Coded signals were initially used in radar systems and later introduced into ultrasonic systems. In 1979, Takeuchi was the first to propose their application in medical ultrasound systems, emphasizing the benefits of spreading signal energy over time and highlighting the time-bandwidth product for improving the signal-to-noise ratio (SNR) [12]. Based on the modulation type of the carrier wave used to construct coded signals, coding methods can be categorized into frequency modulation (FM), amplitude modulation (AM), and phase modulation (PM). FM signals have a varying instantaneous frequency, resulting in better pulse compression. Numerous FM techniques, including linear FM, have been applied in UT. Linear FM is the simplest case, in which the instantaneous frequency changes linearly over time. Gan et al. demonstrated an SNR improvement in measurements of metal plates using linear FM [13]. AM encodes the signal by modulating the amplitude of the carrier wave. Eckersley et al. investigated the effectiveness of AM for imaging at low acoustic power [14]. In PM, the carrier wave retains constant amplitude and frequency, while modulation occurs through controlled phase shifts. These phase shifts facilitate more effective pulse compression by suppressing sidelobes. Various PM codes have been employed in UT, with Barker codes and the complementary Golay code (CGC) among the most commonly used. Barker codes are valued for their simplicity and relatively low sidelobe levels. For example, Jiang et al. used Barker codes to improve the SNR in the inspection of highly attenuated wood materials [15]. Using Barker codes, Xia et al. achieved greater effectiveness than average filtering in measuring material thickness [16]. CGC is capable of eliminating sidelobes, which are inevitable in Barker codes. Garcia-Rodriguez et al. employed CGC in air-coupled ultrasonic Lamb-wave systems to achieve more precise

time-of-flight calculations [17]. Germano et al. applied CGC in the ultrasonic total focusing method, obtaining greater penetration depth and improved image quality [18].

However, Barker codes are limited to only seven unique sequences, with a maximum length of 13. This restricts their effectiveness in enhancing signal amplitude and SNR, and limits the flexibility to choose a suitable code length for practical measurements [19,20]. Although CGC shows excellent performance in sidelobe suppression and signal amplitude enhancement, it requires two transmission events, thereby halving the time efficiency of measurements [21,22]. Recent studies have attempted to address the limitations of Barker codes and CGC. Kim et al. proposed the Barker-convolved Golay code (BCGC), formed by convolving Barker codes with CGC, which expands code length flexibility and increases signal amplitude [23]. However, the study used only the 3-bit Barker code to construct the BCGC. Zeng et al. conducted a more comprehensive theoretical study covering all Barker code lengths [19]. Later, Zeng et al. explored the performance of BCGC for pipeline inspection [24]. BCGC provides more options in code length and improves signal amplitude but still suffers from the same low time efficiency as CGC. Yang et al. developed multivariate codes (MC), constructed from multiple convolutions of Barker codes and CGC, reducing the number of transmissions to one [25]. However, to achieve the same peak sidelobe level, MC requires nearly double the code length of BCGC, again compromising time efficiency. Although still requiring two transmission events, the mutually orthogonal Golay complementary code (MOGCC) enables simultaneous transmission from two transducers, thus achieving twice the data acquisition efficiency compared to CGC [26,27]. Although MOGCC addresses the time efficiency problem, its improvement in signal amplitude remains equivalent to that of CGC and is still inferior to BCGC. Therefore, the limitations of Barker codes and CGC have only been partially addressed, and more research is needed to achieve performance improvement.

In this paper, the Barker-convolved mutually orthogonal Golay complementary code (BMOGCC) is proposed for UT. BMOGCC combines the advantages of Barker codes and MOGCC, offering improved performance compared to conventional Barker codes, CGC, and recent developments like MOGCC, BCGC, and MC. Specifically, BMOGCC provides a higher signal amplitude than Barker codes, CGC, and MOGCC. In addition, BMOGCC achieves better time efficiency than the recently developed BCGC and MC. Moreover, BMOGCC offers more flexible code lengths to suit different testing requirements. To evaluate the effectiveness of BMOGCC, ultrasonic wave propagation in a steel block was simulated using two-dimensional (2D) finite element (FE) modeling, and corresponding laboratory experiments were conducted. The performance of BMOGCC was analyzed and compared with that of Barker codes and MOGCC. The rest of this paper is organized as follows. Section 2 introduces Barker codes and MOGCC, presents the derivation of BMOGCC, and defines the performance evaluation criteria. Section 3 validates the performance of BMOGCC through numerical simulations. Section 4 demonstrates the performance of BMOGCC on the basis of experimental data. Section 5 discusses the results, and Section 6 concludes this paper.

2. Methodology

2.1. Barker Codes

BMOGCC is formed by convolving the Barker code with MOGCC, thereby inheriting certain properties of Barker codes. Barker codes are widely used binary phase sequences due to their low sidelobe level after decoding. For a Barker-coded sequence B of length N_B

composed of $B(1), B(2), B(3), \dots, B(N)$, its autocorrelation, namely the decoded sequence C_B , is as follows [28]:

$$C_B(j) = \sum_{i=1}^{N_B} B(i)B(i+j) = \begin{cases} N_B, & j = 0 \\ 0 \text{ or } \pm 1, & 0 < j < N_B \\ 0, & j \geq N_B \end{cases} \quad (1)$$

Equation (1) shows that the autocorrelation peak value is equal to the coded sequence length N_B , and the sidelobe level is limited to 1, demonstrating the effectiveness of Barker codes in pulse compression. It is also evident that a longer sequence results in better compression performance. However, the maximum length of Barker codes is 13, which limits the maximum value of C_B to 13. Table 1 lists Barker codes of different lengths.

Table 1. Barker codes.

Coded Sequence Notation	N_B	Code Bit Value
B_2	2	1, 1 or 1, −1
B_3	3	1, 1, −1
B_4	4	1, 1, 1, −1 or 1, 1, −1, 1
B_5	5	1, 1, 1, −1, 1
B_7	7	1, 1, 1, −1, −1, 1, −1
B_{11}	11	1, 1, 1, −1, −1, −1, 1, −1, −1, 1, −1
B_{13}	13	1, 1, 1, 1, 1, −1, −1, 1, 1, −1, 1, −1, 1

2.2. MOGCC

When using CGC, a transducer requires two transmissions to achieve zero sidelobes, resulting in low time efficiency. This issue becomes more serious when inspecting large areas that require transducer relocation or the use of multiple transducers. For example, the CGC consisting of two coded sequences GA and GB requires four transmission events to complete inspection at two locations, as illustrated in Figure 2. MOGCC, which consists of two pairs of mutually orthogonal complementary sequences, doubles time efficiency by enabling simultaneous transmission from two transducers, with each still transmitting twice [26], resulting in two transmission events in total. Due to the orthogonality between the pairs, the received signal can be separated into components corresponding to each individual transmitter. For the MOGCC composed of two pairs MX and MY with length N_M , where MX consists of two sequences MX_1 and MX_2 , and MY consists of MY_1 and MY_2 , the combined autocorrelations are given as

$$C_X(j) = C_{X1}(j) + C_{X2}(j) = MX_1 * MX_1(-i) + MX_2 * MX_2(-i) = 2N_M\delta(j), \quad (2)$$

$$C_Y(j) = C_{Y1}(j) + C_{Y2}(j) = MY_1 * MY_1(-i) + MY_2 * MY_2(-i) = 2N_M\delta(j), \quad (3)$$

where C_X and C_Y denote the combined autocorrelations of MX and MY , respectively. C_{X1} , C_{X2} , C_{Y1} , and C_{Y2} are autocorrelations for MX_1 , MX_2 , MY_1 , and MY_2 , respectively. The symbol $*$ denotes the convolution operation, and $MY_1(-i)$ is the time inverse of MY_1 . Furthermore, the sum of the cross-correlations between the two pairs is given by

$$MX_1 * MY_1(-i) + MX_2 * MY_2(-i) = MY_1 * MX_1(-i) + MY_2 * MX_2(-i) = 0. \quad (4)$$

Using Equations (2)–(4), MOGCC can be applied in a UT system that includes two transmitters and two receivers, where MX_1 and MX_2 are two transmitted sequences for one transmitter, and MY_1 and MY_2 for another. Figure 3 illustrates a UT system using

MOGCC, where MX_1 and MY_1 are sent simultaneously in the first transmission event, and MX_2 and MY_2 are sent simultaneously in the second transmission event.

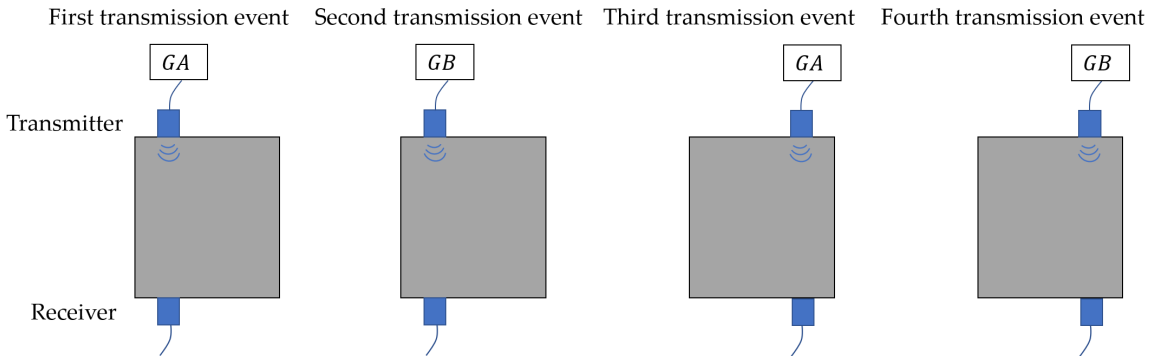


Figure 2. Use of the complementary Golay code (CGC) for ultrasonic testing.

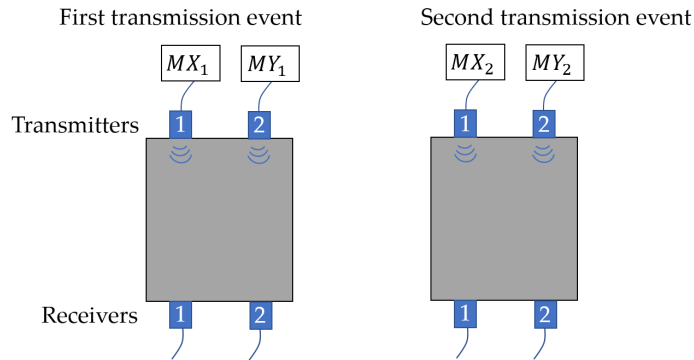


Figure 3. Implementation of MOGCC in a UT system.

Although the receivers capture overlapping signals from both transmitters, the signals can be separated so that receiver 1 extracts only the signal from transmitter 1, and receiver 2 from transmitter 2. Taking receiver 1 as an example, to extract the signal originating from transmitter 1, we have

$$\begin{aligned}
 C_{XY} &= (MX_1 + MY_1) * MX_1(-i) + (MX_2 + MY_2) * MX_2(-i) \\
 &= MX_1 * MX_1(-i) + MX_2 * MX_2(-i) + MY_1 * MX_1(-i) + MY_2 * MX_2(-i) \\
 &= MX_1 * MX_1(-i) + MX_2 * MX_2(-i) \\
 &= 2N_M \delta(j),
 \end{aligned} \tag{5}$$

where C_{XY} is the separation result, and also the decoded result, in receiver 1. The decoded sequence C_{XY} has a peak amplitude of $2N_M \delta(j)$ and zero sidelobes, illustrating the effectiveness of MOGCC in pulse compression. In addition, the separation result for receiver 2 is also $2N_M \delta(j)$. Moreover, CGC and MOGCC yield the same peak amplitude C_{XY} after decoding, provided that they have the same code length. Comparing Figures 2 and 3, it is evident that MOGCC requires only two transmission events to complete inspections at two locations, which is half the time required by CGC. Table 2 lists the 4-bit MOGCC (M4) and the 8-bit MOGCC (M8), where M4 consists of $M4X_1$, $M4X_2$, $M4Y_1$, and $M4Y_2$, and M8 consists of $M8X_1$, $M8X_2$, $M8Y_1$, and $M8Y_2$.

Table 2. MOGCC.

Coded Sequence Notation	Code Bit Value
$M4X_1$	1, −1, −1, −1
$M4Y_1$	−1, 1, −1, −1
$M4X_2$	−1, −1, 1, −1
$M4Y_2$	1, 1, 1, −1
$M8X_1$	1, 1, 1, −1, 1, 1, −1, 1
$M8Y_1$	1, −1, 1, 1, 1, −1, −1, −1
$M8X_2$	1, 1, 1, −1, −1, −1, 1, −1
$M8Y_2$	1, −1, 1, 1, −1, 1, 1, 1

2.3. BMOGCC

BMOGCC is formed by convolving the Barker code and MOGCC, thereby retaining the advantageous properties of both. The process of generating and decoding BMOGCC is illustrated in Figure 4.

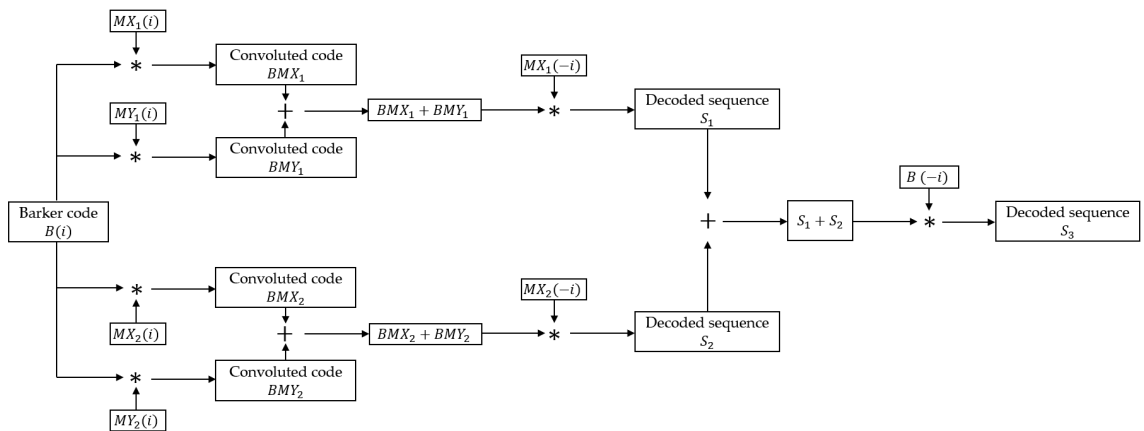


Figure 4. Flow of generating and decoding BMOGCC.

The process consists of five steps. The first step involves the generation of BMOGCC, while the remaining four steps are dedicated to decoding.

Step 1: The four MOGCC sequences MX_1 , MX_2 , MY_1 , and MY_2 are each convolved with the Barker code. The resulting sequences, denoted as BMX_1 , BMX_2 , BMY_1 , and BMY_2 , make up the BMOGCC. These codes are calculated as follows:

$$BMX_1 = MX_1 * B, \quad BMX_2 = MX_2 * B, \quad BMY_1 = MY_1 * B, \quad BMY_2 = MY_2 * B. \quad (6)$$

Step 2: A summation is performed between BMX_1 and BMY_1 , and separately between BMX_2 and BMY_2 .

Step 3: Each summation result from Step 2 is convolved with the time-reversed versions of MX_1 and MX_2 , respectively, yielding the decoded sequences S_1 and S_2 . The convolution operations are written as

$$\begin{aligned} S_1 &= (BMX_1 + BMY_1) * MX_1(-i) = B(i) * (MX_1 + MY_1) * MX_1(-i), \\ S_2 &= (BMX_2 + BMY_2) * MX_2(-i) = B(i) * (MX_2 + MY_2) * MX_2(-i). \end{aligned} \quad (7)$$

Step 4: The decoded sequences S_1 and S_2 are summed.

Step 5: The sum of S_1 and S_2 is convolved with the time-reversed Barker code $B(-i)$, resulting in the final decoded sequence S_3 , which is written as

$$\begin{aligned}
 S_3 &= (S_1 + S_2) * B(-i) \\
 &= B(i) * [(MX_1 + MY_1) * MX_1(-i) + (MX_2 + MY_2) * MX_2(-i)] * B(-i) \\
 &= B(i) * B(-i) * (MX_1 * MX_1(-i) + MX_2(-i) * MX_2) \\
 &= C_B * C_{XY}.
 \end{aligned} \tag{8}$$

From the steps above, it is clear that the length of BMOGCC is $N_{BM} = N_B + N_M - 1$. Equation (8) demonstrates that the decoded sequence S_3 is the convolution of the decoded Barker code and MOGCC. Therefore, BMOGCC achieves a higher compression peak than either the Barker code or MOGCC alone and reaches the same compression peak as BCGC, which is formed by convolving the Barker code with CGC [23]. Moreover, BMOGCC offers the same time efficiency as MOGCC, which is twice that of BCGC and CGC.

Figure 5 presents the decoding results of B7M4, which is the convolution of the 7-bit Barker code and the M4 from Table 2. The four sequences that make up B7M4 are denoted as $B7M4X_1$, $B7M4X_2$, $B7M4Y_1$, and $B7M4Y_2$. The decoded sequence S_3 of the 10-bit B7M4 clearly shows a main peak amplitude of 56, which is significantly higher than that of the 13-bit Barker code or the 16-bit MOGCC. This highlights the superior signal-amplitude enhancement capability of BMOGCC compared to both the Barker code and MOGCC.

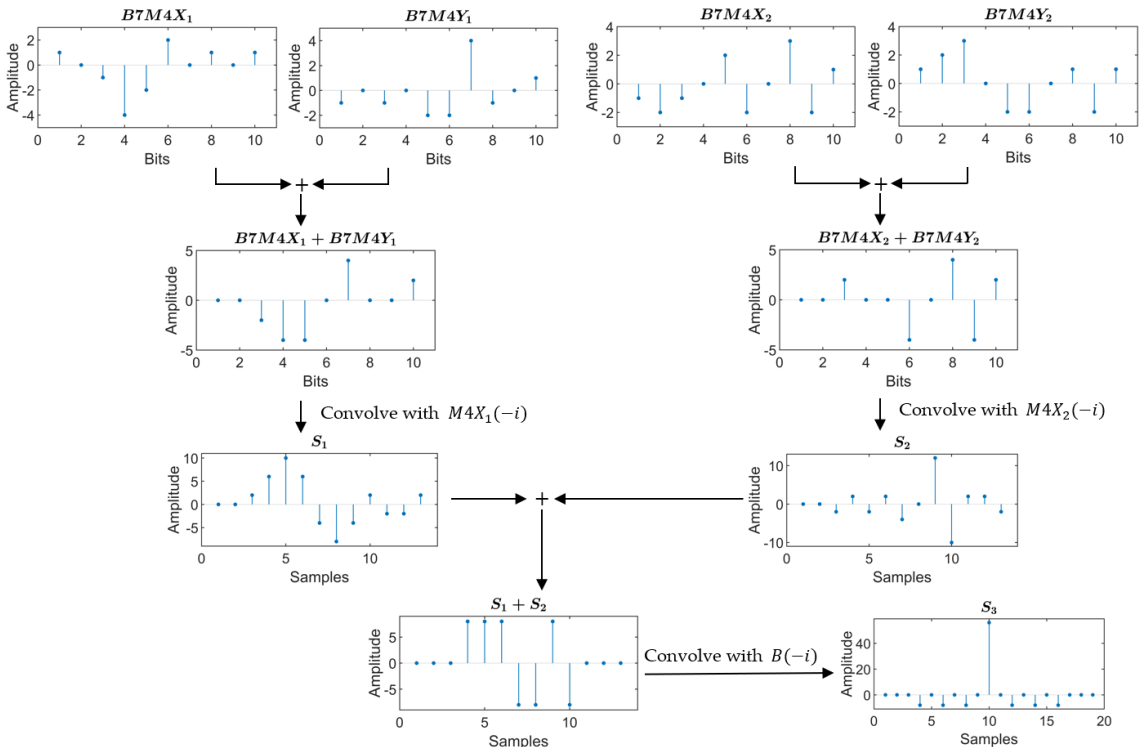


Figure 5. Decoding process for B7M4.

2.4. Performance Evaluation of Codes

In practical measurements, the carrier wave is modulated by the code to generate the coded excitation signal, where each bit of the code modulates the carrier wave. The modulated excitation signal S_c is defined as

$$S_c = \sum_{i=1}^N c(i)w(t - i \cdot T), 0 \leq t \leq T \quad (9)$$

where $c(i)$ is the i -th bit of the code and $w(t)$ is the carrier wave.

The performance of coded excitation signals, or equivalently, of codes themselves, is evaluated using several indices, including the peak sidelobe level (PSL), integrated sidelobe level, mainlobe gain (MG), and mainlobe level. Definitions may vary across studies [10,19,24–26,29]. Among these indices, the PSL and MG are two of the most critical for evaluating code performance. The PSL is defined as the logarithmic ratio of the mainlobe peak to the maximum sidelobe peak of the decoded signal [29]:

$$PSL = 20 \log_{10} \left(\frac{A_m}{A_s} \right), \quad (10)$$

where A_m is the amplitude of the mainlobe peak and A_s is the maximum sidelobe amplitude. The PSL quantifies the code's ability to suppress self-noise introduced in the decoding process. The MG is the logarithmic ratio of the mainlobe peak amplitude of the decoded signal obtained using a coded excitation signal to that obtained using a 1-bit pulse (unmodulated carrier wave) [24]:

$$MG = 20 \log_{10} \left(\frac{A_m}{A_p} \right), \quad (11)$$

where A_p is the peak amplitude of the compressed 1-bit pulse. The MG represents the ability of a code to enhance signal amplitude. Table 3 lists the theoretical PSL and MG values. Note that the PSL is not defined for CGC and MOGCC because they ideally produce zero sidelobes.

Table 3. PSL and MG values of codes.

Code	Length	PSL	MG
Barker	N_B	$20 \log_{10}(N_B)$	$20 \log_{10}(N_B)$
CGC	N_C	-	$20 \log_{10}(2N_C)$
MOGCC	N_M	-	$20 \log_{10}(2N_M)$
BMOGCC	$N_B + N_M - 1$	$20 \log_{10} N_B$	$20 \log_{10}(2N_B N_M)$

In addition to the PSL and MG, the temporal resolution, estimated by the full width at half maximum (FWHM) of the decoded signal envelope, is important in certain NDT imaging methods, such as the total focusing method [30,31]. Figure 6c shows the decoded signals for a 1-bit pulse and the 5-bit Barker-coded signal, in which the mainlobe, sidelobes, and temporal resolution are clearly characterized. The 1-bit pulse is a 2.25 MHz sine wave windowed by a Hanning window and is shown in Figure 6a. The 5-bit Barker-coded excitation signal generated using this 1-bit pulse as the carrier wave is shown in Figure 6b.

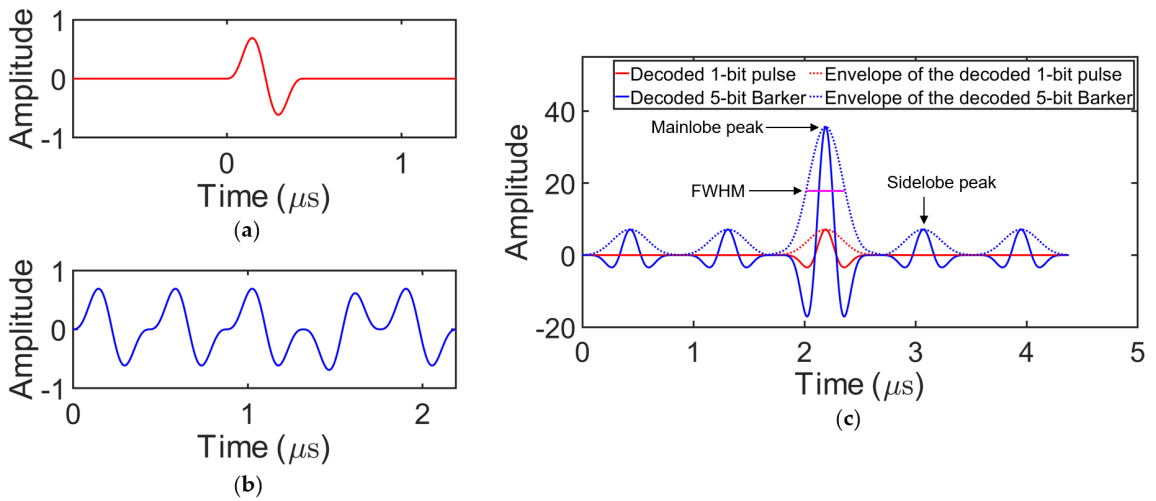


Figure 6. (a) 1-bit pulse. (b) 5-bit Barker-coded excitation signal. (c) Decoded results.

3. Numerical Simulations

3.1. Finite Element Modeling

Simulations were conducted using the software ABAQUS version 2021 [32] to evaluate the performance of the coded signals. A 2D finite element model (Figure 7) was developed, consisting of a steel plate of size 75 mm $\times$ 140 mm, with two parallel excitation source nodes and two parallel receiver nodes aligned accordingly. The vertical distance between the source nodes and receiver nodes was 74 mm. The material properties of the steel used in the simulations are listed in Table 4. The model was meshed using the CPE4R element type. To prevent reflections at the model boundaries, a layer of CINPE4 elements was applied outside the model perimeter. The excitation was applied as a vertical force at the source nodes, and the vertical displacements were recorded at the receiver nodes.

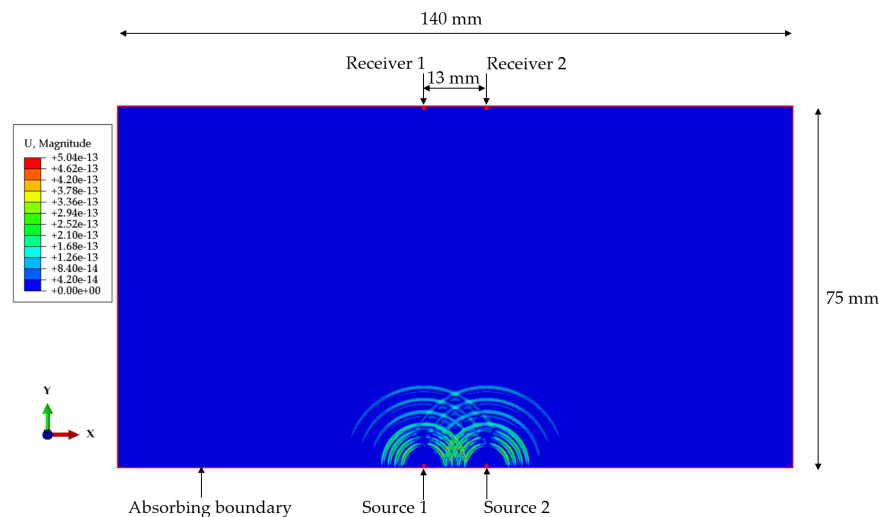


Figure 7. Geometry of the simulation model and the displacement of the propagating waves from two sources, shown at 3 μs .

The 1-bit pulse used to construct the coded excitation signals is shown in Figure 6a, with a center frequency of 2.25 MHz, matching that of the ultrasonic transducers used in the experiments. In ABAQUS, the maximum frequency that can be simulated is limited by the element size; higher frequencies require smaller elements [33]. Coded excitation signals may contain high-frequency components that are several times the center frequency because of the phase modulation. To ensure that the simulations were convergent, an element size of 0.025 mm was used, which corresponds to approximately 102 elements per wavelength. If a larger element size, such as 0.05 mm, is used, the resulting wave may fail to converge, leading to incorrect signals for further analysis.

Table 4. Steel properties.

Young’s Modulus	Poisson Ratio	Density
195 GPa	0.3	7910 kg/m ³

A series of simulations was performed using different excitation signals to evaluate the performance of BMOGCC. Specifically, codes B5M4, B5M8, B7M4, and B7M8 were evaluated. For comparison, three Barker codes (from Table 1), as well as M4 and M8 (from Table 2), were tested. In addition, the 1-bit pulse was included as a reference excitation signal for calculating the mainlobe gain. Since both BMOGCC and MOGCC require two transmission events, two separate simulations were conducted for each of these codes. For simulations involving two sources, the excitations were applied simultaneously. Table 5 lists the excitation signals applied to each source across all simulations. Figure 7 also illustrates the displacement field for simulation 5 as an example, where the faster-propagating compression wave was used in the decoding, while the slower-propagating shear wave was filtered out by a time window.

Table 5. Excitation signals for sources.

Simulation No.	Code	Source 1	Source 2
1	-	1-bit pulse	-
2	Barker	B5	-
3	Barker	B7	-
4	Barker	B11	-
5	M4	MX ₁ 4	MY ₁ 4
6	M4	MX ₂ 4	MY ₂ 4
7	M8	MX ₁ 8	MY ₁ 8
8	M8	MX ₂ 8	MY ₂ 8
9	B5M4	B5MX ₁ 4	B5MY ₁ 4
10	B5M4	B5MX ₂ 4	B5MY ₂ 4
11	B5M8	B5MX ₁ 8	B5MY ₁ 8
12	B5M8	B5MX ₂ 8	B5MY ₂ 8
13	B7M4	B7MX ₁ 4	B7MY ₁ 4
14	B7M4	B7MX ₂ 4	B7MY ₂ 4
15	B7M8	B7MX ₁ 8	B7MY ₁ 8
16	B7M8	B7MX ₂ 8	B7MY ₂ 8

3.2. Results of Decoding

Figure 8 presents the decoded signals for the 1-bit pulse (simulation 1) and the three Barker codes (simulations 2–4), where only source 1 was excited. In simulations involving MOGCC or BMOGCC, only the decoded signals from receiver 1 are shown, as the decoded signals from both receivers were symmetric. Figure 9 shows the decoded signals for MOGCC (simulations 5–8), with each decoded signal based on the signals received from

two transmission events. Figure 10 displays the decoded signals for BMOGCC (simulations 9–16), including B5M4, B5M8, B7M4, and B7M8. The amplitude of the decoded signals was normalized to the maximum amplitude value of all decoded signals to clearly illustrate the amplitude difference among the codes.

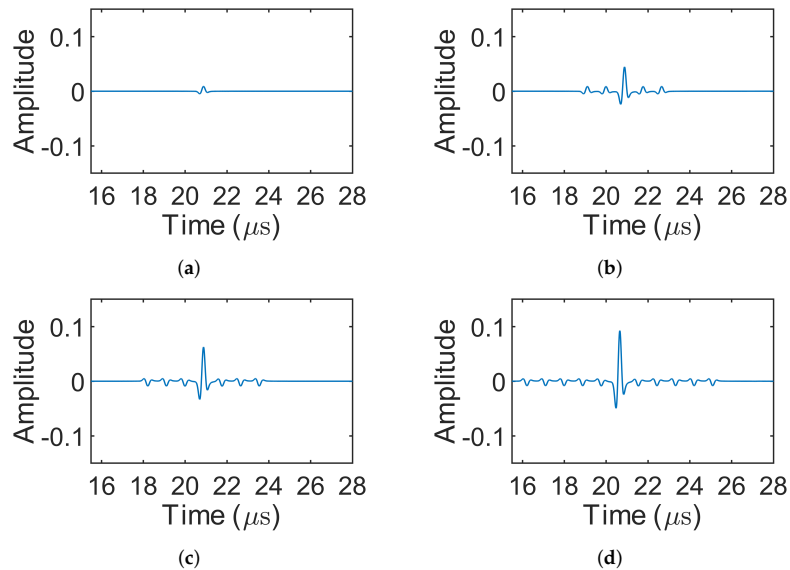


Figure 8. Decoded signals for the 1-bit pulse and the Barker codes (simulations 1–4). (a–d) 1-bit pulse, 5-bit Barker, 7-bit Barker, and 11-bit Barker, respectively.

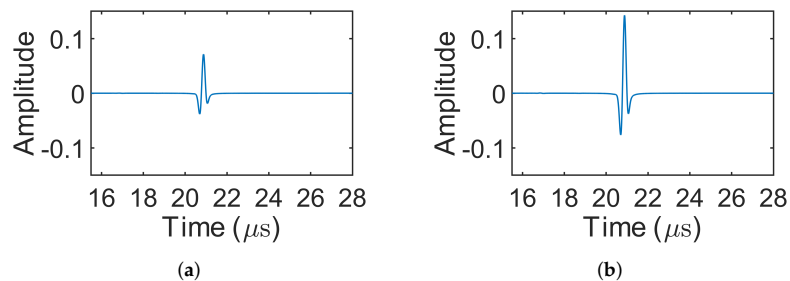


Figure 9. Decoded signals for MOGCC (simulations 5–8). (a,b) M4 (simulations 5–6) and M8 (simulations 7–8), respectively.

It can be observed in Figures 8–10 that the signal amplitudes varied depending on the code type, reflecting the distinct properties of each coding scheme. As shown in Figure 8, the amplitude of the decoded signals increased progressively with the length of the Barker code, and visually uniform sidelobes were present due to the inherent correlation characteristics of these codes. Figure 9 shows that MOGCC produced no sidelobes and that M8 exhibited a higher amplitude than M4, attributed to its longer code length. In contrast, Figure 10 demonstrates that BMOGCC achieved a markedly higher decoded amplitude compared to both MOGCC and the Barker codes. This result aligns with Equation (8), which expresses the BMOGCC-decoded signal as the convolution of the MOGCC- and Barker-code-decoded signals.

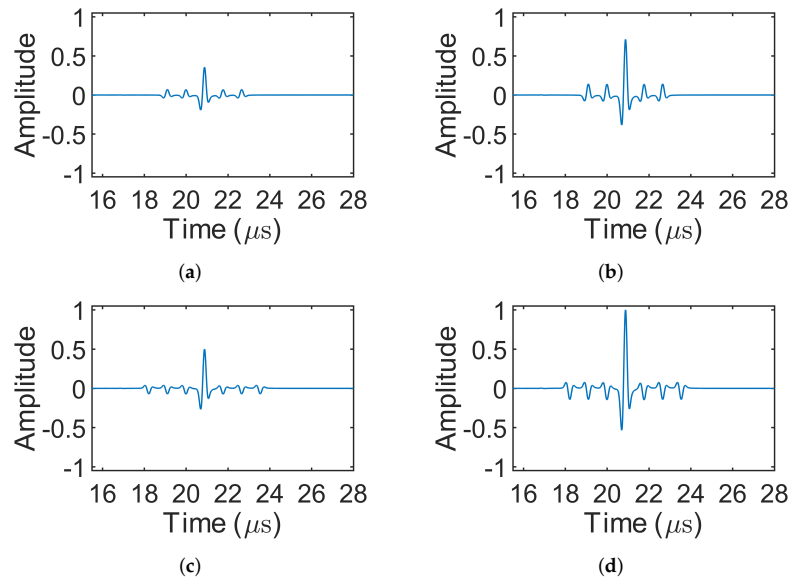


Figure 10. Decoded signals for BMOGCC (simulations 9–16). (a–d) B5M4 (simulations 9–10), B5M8 (simulations 11–12), B7M4 (simulations 13–14), and B7M8 (simulations 15–16), respectively.

4. Experimental Measurements

4.1. Experimental Setup

To further evaluate the performance of the coded signals, experimental measurements were carried out to ensure that factors such as the transducer response and data acquisition (DAQ) system characteristics did not significantly impact the results. The experimental setup, shown in Figure 11a, included a 140 mm × 74 mm × 100 mm steel block, two ultrasonic transducers, and a DAQ system. The transducers (M1036, Olympus, Hamburg, Germany) had a center frequency of 2.25 MHz and were attached to the steel block using magnetic holders. Ultrasonic coupling gel was applied to ensure good acoustic contact. The DAQ system included a signal generator (HS805, TiePie, Sneek, The Netherlands) with an 8-bit amplitude resolution. It was connected to a computer via USB and operated using the TiePie oscilloscope software version 1.33.2.0/0.5.6.2, which allowed configuration of the excitation signals, voltage, and sampling rate.

A through-transmission scheme was used to capture the ultrasonic compression waves, with the transducers placed on opposite sides of the steel block. The transmission and reception locations mirrored the simulation setup shown in Figure 7. As in the simulations, the setup included two transmission and two receiving locations, requiring four transducers in total. The distance between the two transmission locations was 13 mm, which is equal to the diameter of the ultrasonic transducers (Figure 11b). However, due to the availability of only two suitable transducers, a practical workaround was adopted: one transducer was moved from one transmission location to the other, and the signals received at the same receiving location were added. This approach effectively replicated the signal that would have been obtained if both transmitters had operated simultaneously.

The excitation signals used in the experiments were identical to those used in the simulations, resulting in a total of 16 experimental trials. The settings of the excitation signals for experiments 1–16 were identical to those of simulations 1–16.

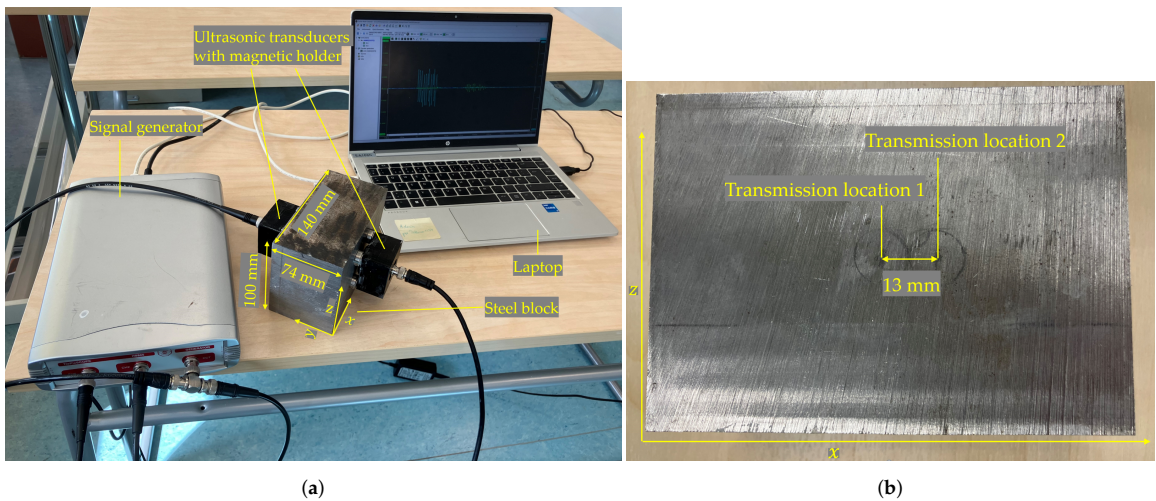


Figure 11. Experimental setup. (a) Measurement setup including ultrasonic transducers, a signal generator, and a laptop. (b) Transmission locations on the steel block.

4.2. Results of Decoding

As in the simulations, only the decoded signals from receiving location 1 are presented, since the decoded signals from both locations were symmetric. Figure 12 shows the decoded signals for the 1-bit pulse (experiment 1) and the Barker codes (experiments 2–4). Figure 13 displays the decoded signals for MOGCC (experiments 5–8). Figure 14 presents the decoded signals for BMOGCC (experiments 9–16), including B5M4, B5M8, B7M4, and B7M8. The decoded signals were normalized using the same method as in the simulations, dividing by the global maximum amplitude, to facilitate comparison across different codes.

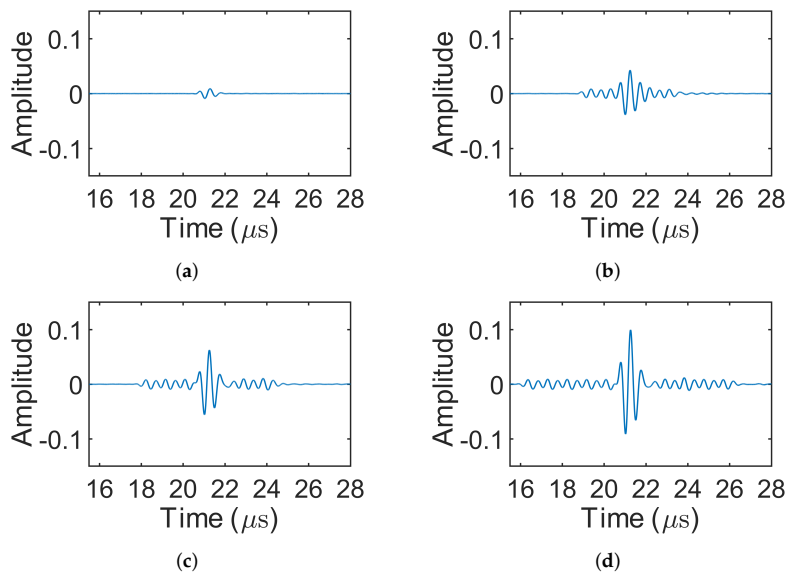


Figure 12. Decoded signals for experiments 1–4. (a–d) 1-bit pulse, 5-bit Barker, 7-bit Barker, and 11-bit Barker, respectively.

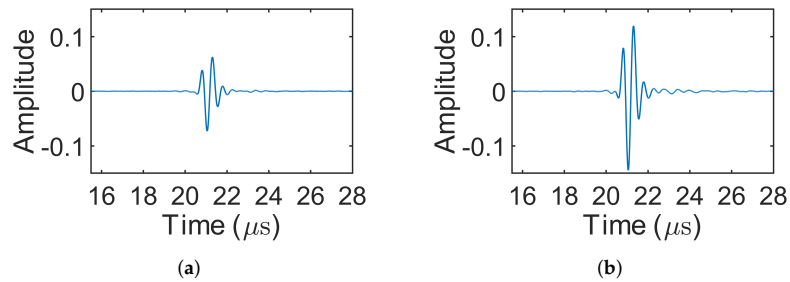


Figure 13. Decoded signals for MOGCC (experiments 5–8). (a,b) Decoded signal of M4 (experiments 5–6) and M8 (experiments 7–8), respectively.

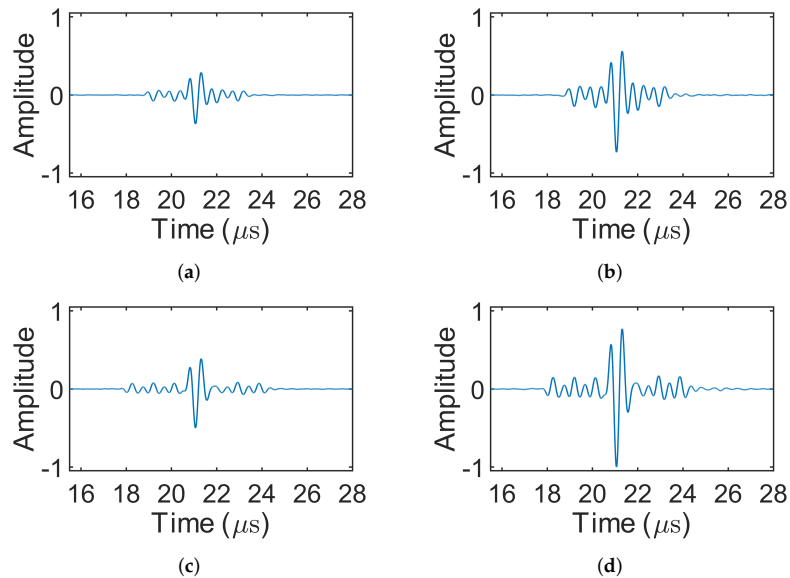


Figure 14. Decoded signals for BMOGCC (experiments 9–16). (a–d) B5M4 (experiments 9–10), B5M8 (experiments 11–12), B7M4 (experiments 13–14), and B7M8 (experiments 15–16), respectively.

The amplitudes of the decoded signals were found to be consistent with the simulation results. Figure 12 shows that the amplitude increased with the length of the Barker codes, accompanied by low-level sidelobes. As depicted in Figure 13, M8 yielded a higher amplitude than M4, owing to its doubled code length. Figure 14 demonstrates that BMOGCC produced a higher decoded signal amplitude than both MOGCC and the Barker codes. However, the sidelobes were less uniform than those seen in the simulations, likely due to factors such as modulation by the transducer's impulse response and increased noise resulting from the limited amplitude resolution of the DAQ system.

5. Discussion

By comparing Figures 8–10 and 12–14, it is evident that all coded signals yielded higher decoded mainlobe amplitudes than the 1-bit pulse, with BMOGCC yielding the highest amplitudes among all the codes evaluated. The experimental results (Figures 12–14) showed that the mainlobe amplitudes of the decoded signals were approximately consistent with those of the corresponding simulation results. These observations confirm the capability of BMOGCC to significantly enhance the mainlobe amplitude.

Two key indicators, the peak sidelobe level (PSL) and the mainlobe gain (MG), were calculated and compared to evaluate the code performance, based on the decoded signals from Figures 8–10 and 12–14. The MG values from both the simulations and experiments are shown in Figure 15, along with the corresponding theoretical values based on Table 2. The measured MG values closely matched the theoretical predictions. Notably, BMOGCC achieved significantly higher MG values than either the Barker codes or MOGCC, reaching maximum values for B7M8. Moreover, the MG value of each BMOGCC was approximately equal to the sum of the MG values of the corresponding Barker and MOGCC in its construction, as expected. For instance, in the experiments, the MG values of B7M4, B7, and M4 were 34.90 dB, 16.84 dB, and 18.19 dB, respectively. The difference between the MG of B7M4 and the sum of the MG values of B7 and M4 was only 0.13 dB. Significantly, the 8-bit B5M4 and 10-bit B7M4 outperformed the 11-bit Barker code, demonstrating that BMOGCC achieves higher signal amplitude even with shorter code lengths. This is particularly advantageous in scenarios where shorter excitation signals are needed, for example, when DAQ memory is limited or boundary reflections interfere with useful signals [34]. These results further validate the superior performance of BMOGCC in enhancing the mainlobe amplitude.

The PSL values obtained from both the simulations and experiments are shown in Figure 16, along with the theoretical values from Table 2. The PSL values from the simulations closely matched the theoretical expectations, with a maximum deviation of only 0.37 dB observed for the 11-bit Barker code. In the simulations, the PSL values of B5M4 and B5M8 were nearly identical to those of the 5-bit Barker code, while the PSL values of B7M4 and B7M8 were similar to those of the 7-bit Barker code. In the experimental results, the PSL values were lower than those of the simulations, with the largest discrepancy being 2.40 dB for the 11-bit Barker code. This variation was likely caused by the impulse response of the transducers and the quantization noise resulting from the low amplitude resolution of the DAQ system, both of which influenced the shape of the excitation signals and affected the PSL. Despite this, the PSL values observed in the experiments and simulations were generally consistent, confirming that BMOGCC provides sidelobe suppression comparable to that of the Barker code used in its formation.

The temporal resolution, estimated by the full width at half maximum (FWHM) of the decoded signal envelope, is shown in Table 6 for both the simulations and experiments. All decoded signals from simulations had a constant FWHM of 0.35 μ s, regardless of the code type. The experimental FWHM values were higher and varied between 0.67 μ s and 0.89 μ s, mainly due to the transducer impulse response. Importantly, the results showed that BMOGCC maintained comparable temporal resolutions to those of the 1-bit pulse and other codes, confirming that it does not degrade the temporal resolution.

Table 6. Temporal resolution for different excitation signals.

Excitation Signal	FWHM of Simulations (μ s)	FWHM of Experiments (μ s)
1-bit Pulse	0.35	0.75
5-bit Barker	0.35	0.89
7-bit Barker	0.35	0.78
11-bit Barker	0.35	0.78
M4	0.35	0.70
M8	0.35	0.69
B5M4	0.35	0.68
B5M8	0.35	0.67
B7M4	0.35	0.67
B7M8	0.35	0.68

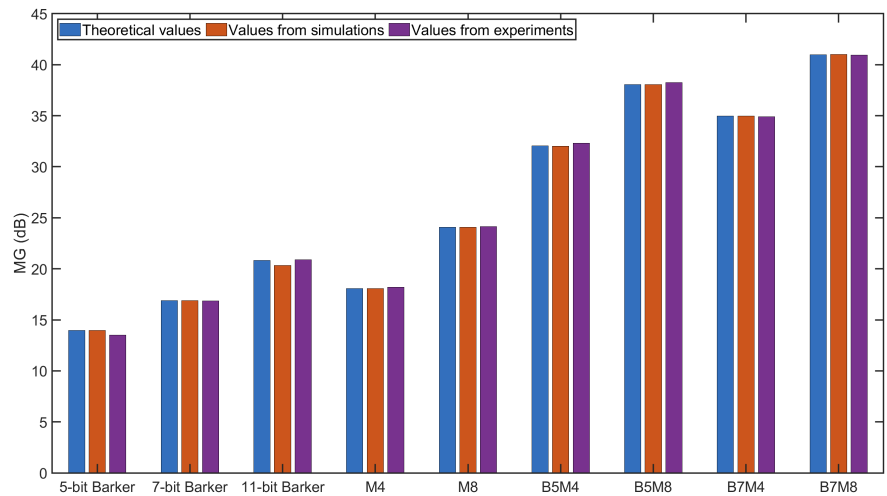


Figure 15. MG values of different codes.

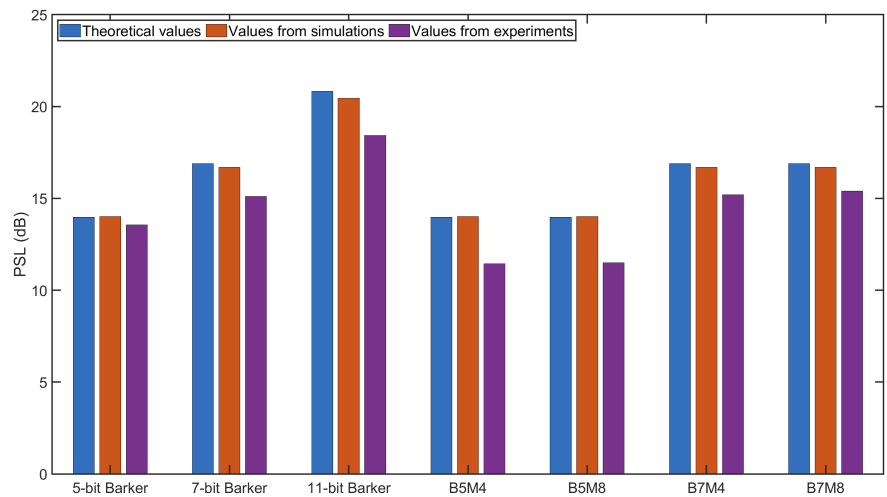


Figure 16. PSL values of different codes.

Overall, the results from both the simulations and experiments confirm the effectiveness of BMOGCC as the excitation signal for ultrasonic measurements.

- BMOGCC retains the low sidelobe levels of Barker codes and the double time efficiency of MOGCC, offering a key advantage over CGC and BCGC, which require two sequential transmissions.
- BMOGCC also provides a higher mainlobe amplitude than both Barker codes and MOGCC, even at shorter lengths, allowing more flexibility in signal design.
- Finally, BMOGCC preserves the temporal resolution, making it suitable for imaging applications where resolution is critical.

6. Conclusions

This study presented a novel coded excitation method, the Barker-convolved mutually orthogonal Golay complementary code (BMOGCC), designed to improve both time efficiency and signal quality in ultrasonic NDT. The proposed method is based on convolving the Barker code with MOGCC, combining the beneficial properties of both.

BMOGCC retains the low sidelobe level of the Barker code and the simultaneous transmission capability of MOGCC, thereby doubling the time efficiency compared to traditional complementary Golay codes. In contrast to the Barker code, BMOGCC provides a greater signal amplitude and allows for flexible code-length selection, enhancing adaptability in practical measurements. The method was validated through both numerical simulations and laboratory experiments involving BMOGCC, the Barker code, MOGCC, and the 1-bit pulse. The results showed strong agreement between the simulations and experimental data. The findings demonstrated that the PSL of BMOGCC was comparable to that of the Barker codes used in its construction, confirming effective sidelobe suppression; BMOGCC achieved a significantly higher mainlobe amplitude than the other tested codes, even when using shorter sequences, indicating enhanced signal energy; and BMOGCC maintained the temporal resolution without any degradation compared to other coded excitation signals, ensuring its suitability for high-resolution ultrasonic imaging applications. Overall, BMOGCC shows excellent performance and provides a practical advancement in the field of coded excitation for ultrasonic testing, particularly in applications requiring high energy or time-efficient measurements.

One limitation of the proposed coded excitation method is that only two excitation signals can be transmitted simultaneously due to the inherent structure of BMOGCC codes. This constraint limits its broader applicability in array transducer systems with a larger number of elements. Future work should focus on developing convolved codes that enable the simultaneous transmission of multiple excitation signals. Moreover, the present study concentrated on ultrasonic compression waves, which are non-dispersive. Future research should be extended to investigate dispersive wave modes. Additionally, the influence of transducer characteristics on the performance of coded excitation signals warrants further detailed investigation.

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Appendix 3

III

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Article

Accelerating Ultrasonic Guided-Wave Measurements via SNR Enhancement Using Coded Excitation: An Experimental Investigation

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Abstract

Conventional excitation signals used in ultrasonic measurements, such as the one-cycle pulse, produce waveforms that experience significant attenuation and dispersion during propagation in highly attenuative materials, resulting in a low signal-to-noise ratio (SNR) and unreliable signal interpretation. Coded excitation is a well-established technique for improving the SNR; however, its practical benefit for ultrasonic guided-wave measurements under low-voltage and limited averaging conditions has not been systematically quantified. This paper presents an experimental investigation of coded excitations for accelerating ultrasonic guided-wave data acquisition through SNR improvement. A one-cycle pulse is compared with Barker-coded and complementary Golay-coded excitations over a wide range of excitation voltages (0.5–10 V) and averaging numbers (1–40). Guided waves are generated using piezoelectric excitation and measured using laser Doppler vibrometry, ensuring repeatable and coupling-independent measurements. The results show that the SNR achieved with Barker-coded excitations using fewer than ten averages is comparable to that obtained with a one-cycle pulse using forty averages. The 16-bit complementary Golay codes achieve a comparable SNR while requiring fewer than five averages. These findings demonstrate that coded excitations can significantly reduce the number of data acquisition cycles in guided-wave measurement, offering a practical pathway toward faster and more energy-efficient ultrasonic measurement systems.

Keywords: ultrasonic measurement; guided waves; coded excitation; signal-to-noise ratio; data acquisition efficiency



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1. Introduction

Nondestructive testing (NDT) refers to a set of inspection methods used to evaluate the properties of materials, components, and structures without causing destruction or interfering with their in-service operation, making it essential for monitoring the structural health and integrity of civil structures such as pipelines and storage tanks [1–3]. Among various NDT methods, ultrasound is widely used across various fields, such as infrastructure, aerospace, and the automation industry, due to its low cost, high efficiency, and ability to detect a broad range of defects [4–6]. Ultrasonic measurement relies on the transmission of ultrasonic waves through a material and reflections from defects or material interfaces, where characteristics such as wave velocity provide valuable information about the mate-

rial's condition [7]. By analyzing the characteristics of the ultrasonic waves, the condition of the material can be assessed.

Ultrasonic guided waves (UGWs) have emerged as a promising tool due to their unique advantages. Unlike ultrasonic bulk waves, which may require time-consuming point-by-point transducer scanning and rely on complex, high-precision mechanical positioning systems, UGWs enable inspection of large areas without the need for mechanical scanning. As a result, UGWs are becoming increasingly popular in ultrasonic measurements, especially for long-range measurements. Numerous researchers have investigated the application of UGW-based measurements. For example, Pasadas et al. studied the localization of fiber breaks in carbon-fiber-reinforced plastic structures using UGWs, while Li et al. investigated damage detection in pipes using UGWs [8,9].

One critical issue associated with guided waves is that they suffer from significant attenuation due to long-distance propagation and noise introduced by electronic instrumentation, transducer coupling variations, environmental disturbances, and other factors. Consequently, the signal-to-noise ratio (SNR) is probably insufficient for reliable signal interpretation. One straightforward approach to improve the SNR is to increase the excitation signal voltage. However, the use of high excitation voltages (above 50 Volts) is restricted for several reasons, such as safety risks in explosive atmospheres and the hardware limitations of data acquisition systems [10–12]. Another approach to increase the SNR is temporal averaging, a technique that can be readily integrated into existing measurement systems without additional hardware or algorithmic complexity, in which signals acquired from multiple repeated measurements are averaged. Since the true signal is coherent across repetitions while the noise is largely uncorrelated, temporal averaging can enhance the SNR by suppressing uncorrelated random noise. However, temporal averaging increases the data acquisition duration, which is unfavorable for real-time applications, such as those in the aviation industry [13].

Coded excitation is a technique that transmits a long coded signal and subsequently compresses the received signal into a high-intensity temporally narrow signal by utilizing the correlation properties of the coded signal [14,15]. Coded excitation enhances the SNR by increasing signal energy through an extended time duration without requiring high excitation voltage, as the high-intensity temporally narrow signal can be achieved by pulse compression. Consequently, coded excitation has been widely used in ultrasonic measurements, particularly for measuring highly attenuative materials and conducting long-range measurements. As a result of its SNR enhancement capability, coded excitation requires fewer averages to achieve a sufficient SNR than conventional excitations, such as the single-cycle pulse [16]. Therefore, coded excitation provides a promising strategy to maintain a high SNR with reduced data acquisition cycles, which directly enhances data acquisition efficiency and allows for lower excitation voltages in practical UGW-based measurements. Figure 1 presents a schematic of an ultrasonic measurement system employing the coded signal.

One of the most common coding methods is phase modulation (PM). In PM, the modulation is achieved by varying the phase of the carrier wave in a predefined manner. The designed phase variation results in better pulse compression and, consequently, a higher SNR. Barker codes are valued for their simple construction and for yielding the lowest sidelobes among all binary codes at the same length [17]. For example, Han et al. proposed the use of Barker codes in the detection of inclined cracks [18]. Complementary Golay codes (CGCs) provide ideal pulse compression with a single high-amplitude peak, enabling SNR enhancement without sidelobes, which is impossible for Barker codes. Wang et al. employed CGCs in ultrasonic air-coupled systems to achieve more accurate defect detection in wood materials [19].

Research on ultrasonic coded excitation has mainly focused on SNR improvement, whereas studies on its use for reducing the number of averages are still limited. Recently, several studies have begun exploring the potential of coded excitation to reduce the required number of averages. Song et al. investigated the use of CGCs with a 4-bit carrier wave to accelerate ultrasonic measurements on bone phantoms [20]. However, only 16-bit CGCs were evaluated in their measurements. Xia et al. explored the performance of Barker codes in speeding up the data acquisition efficiency compared to the single-pulse excitation method [21]. The 13-bit Barker code shows strong potential for reducing the required number of averages; however, a limitation of the study is that other Barker code lengths were not examined. Zhang et al. compared the SNR between CGCs and single-pulse excitation under varying sensor contact forces in ultrasonic airborne inspection [22]. However, only a single fixed number of averages was used, and only 8-bit CGCs were evaluated. These studies indicate that, although coded excitation shows potential for reducing the number of averages, the existing research is limited by the use of single code types such as Barker codes or CGCs, fixed code lengths, or a fixed number of averages. The quantitative relationships among code type, excitation voltage, and the number of averages in UGW-based UT remain insufficiently explored through experimental studies. Consequently, more comprehensive investigations are necessary to further explore the application of coded excitation in reducing the number of averages.

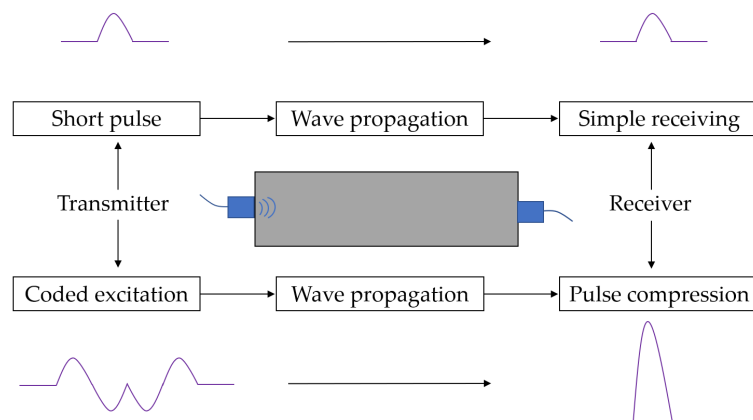


Figure 1. Schematic diagram of an ultrasonic measurement system using a short pulse and coded excitation.

An experimental study was conducted to demonstrate the feasibility of using coded excitation to improve data acquisition efficiency by reducing the number of required data acquisition cycles. Coded signals offer improved SNR performance compared to the conventional one-cycle pulse. This improvement enables the required SNR to be achieved with fewer averages, thus enhancing the overall time efficiency of the data acquisition. To evaluate the time efficiency of coded excitation, laboratory experiments on guided-wave propagation in an aluminum plate were performed. Specifically, two Barker codes of different lengths and the 16-bit CGC, along with a one-cycle pulse, were employed as excitation signals under varying excitation voltages and numbers of averages. The time performance of coded excitation signals was evaluated and compared with that of the one-cycle pulse based on the SNR as a function of the number of averages. It should be noted that the present study evaluates coded excitation in terms of the received guided-wave SNR and data acquisition efficiency, rather than directly demonstrating defect detection or imaging performance.

The rest of this paper is organized as follows: Section 2 introduces Barker codes and CGCs, as well as the coding and pulse compression processes, followed by the performance

evaluation indicators used in this study. Section 3 describes the measurement setup and presents the experimental data. Section 4 discusses the results, and Section 5 concludes the paper.

2. Methodology

2.1. Barker Codes

Barker codes are widely used binary phase codes due to several advantages. Compared with multiphase codes, such as convoluted codes, Barker codes are biphasic and therefore impose lower requirements on hardware implementation [23,24]. Compared with CGCs, Barker codes operate as single-transmission codes and do not require double transmission, resulting in higher time efficiency. Furthermore, their sidelobe levels are the lowest among all known binary sequences of the same length.

For a length N Barker code B_N composed of bits $b_1, b_2, b_3, \dots, b_N$, with each bit taking values of -1 or 1 , its autocorrelation C_B is defined as [24]:

$$C_B(j) = \sum_{i=1}^N b_i b_{i+j} = \begin{cases} N, & j = 0 \\ 0 \text{ or } \pm 1, & 0 < j < N. \\ 0, & j \geq N \end{cases} \tag{1}$$

Equation (1) indicates that the peak value of the autocorrelation is equal to the code length N , while the sidelobe levels are bounded by 1, demonstrating the effectiveness of Barker codes in sidelobe suppression. Additionally, the autocorrelation peak grows as the code length increases. However, Barker codes exist for only seven different lengths, and the maximum length is 13, which limits the peak value to 13. Table 1 lists all available Barker codes, with B_2 and B_4 each having two different bit sequences. Figure 2 shows the 7-bit Barker code and its autocorrelation, where we can see that the peak value equals the code length 7 and the magnitude of the sidelobes is 1, in good agreement with the theoretical result given in Equation (1).

Table 1. Barker codes.

Code Notation	N	Code Bit Value
B_2	2	(1, 1) or (1, −1)
B_3	3	1, 1, −1
B_4	4	(1, 1, 1, −1) or (1, 1, −1, 1)
B_5	5	1, 1, 1, −1, 1
B_7	7	1, 1, 1, −1, −1, 1, −1
B_{11}	11	1, 1, 1, −1, −1, −1, 1, −1, −1, 1, −1
B_{13}	13	1, 1, 1, 1, 1, −1, −1, 1, 1, −1, 1, −1, 1

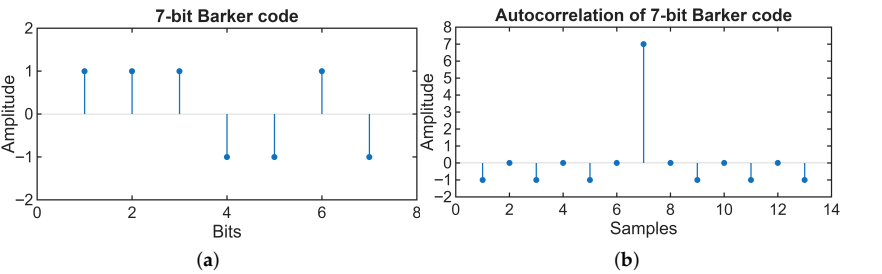


Figure 2. The 7-bit Barker code and its autocorrelation. (a) The 7-bit Barker code. (b) Autocorrelation.

2.2. Complementary Golay Codes (CGCs)

One limitation of Barker codes is the presence of sidelobes in their autocorrelations. CGCs eliminate sidelobes while maintaining a high autocorrelation peak. CGCs consist of a pair of complementary binary codes with the same length. The autocorrelations of the two complementary codes have identical peak values, while their sidelobes have equal magnitudes but opposite signs. By summing the two autocorrelations, the sidelobes cancel each other, whereas the main peaks add constructively [25]. For the CGC composed of two complementary length N codes GA_N and GB_N , GA_N consists of bits $Ga_1, Ga_2, Ga_3, \dots, Ga_N$, and GB_N consists of bits $Gb_1, Gb_2, Gb_3, \dots, Gb_N$, with each bit taking values of 1 or -1 . Their autocorrelations are defined as

$$CG_A(j) = \sum_{i=1}^N Ga_i Ga_{i+j}, \quad (2)$$

$$CG_B(j) = \sum_{i=1}^N Gb_i Gb_{i+j}, \quad (3)$$

where CG_A and CG_B denote the autocorrelations of GA_N and GB_N respectively. Thus, the sum of the autocorrelations is then given by

$$CG(j) = CG_A(j) + CG_B(j) = \begin{cases} 2N, & j = 0 \\ 0, & j \neq 0. \end{cases} \quad (4)$$

Equation (4) shows that for a length- N CGC, the autocorrelation peak value is $2N$ and no sidelobes exist. Additionally, it demonstrates that the peak value is proportional to the code length. However, to eliminate sidelobes and achieve a high peak value, two transmissions are required. This requirement reduces the data acquisition rate by a factor of two. Consequently, CGCs are not well suited for the inspection of fast-moving targets. Table 2 lists the length-8 CGC G_8 and the length-16 CGC G_{16} , where G_8 consists of GA_8 and GB_8 , and G_{16} consists of GA_{16} and GB_{16} . It should be noted that multiple CGCs exist for the same code length. For example, there are 48 different CGCs of length 8 and 384 different CGCs of length 16 [26]. Figure 3 shows the length-8 CGCs from Table 2 and their autocorrelations. The peak value of the summed autocorrelations equals 16, which is twice the code length, and the sidelobe level is zero. These results correspond well with Equation (4).

Table 2. CGCs.

Code Notation	Code Bit Value
GA_8	1, 1, 1, -1 , 1, 1, -1 , 1
GB_8	1, 1, 1, -1 , -1 , -1 , 1, -1
GA_{16}	1, 1, 1, -1 , 1, 1, -1 , 1, 1, 1, -1 , -1 , -1 , 1, -1
GB_{16}	1, 1, 1, -1 , 1, 1, -1 , -1 , -1 , -1 , 1, 1, 1, 1, -1 , 1

2.3. Coding and Pulse Compression in Practical Ultrasonic Measurements

In practice, codes cannot be transmitted directly by ultrasonic transducers. Instead, a code is first modulated onto a carrier wave to generate a coded excitation signal, with each bit of the code modulating the phase of the carrier wave. The modulation process, referred to as coding, is defined as

$$S = \sum_{i=1}^N c(i)w(t - i \cdot T), \quad 0 \leq t \leq T \quad (5)$$

where S is the coded excitation signal, $c(i)$ denotes the i -th bit of the code, $w(t)$ represents the carrier wave, and T is the period of $w(t)$.

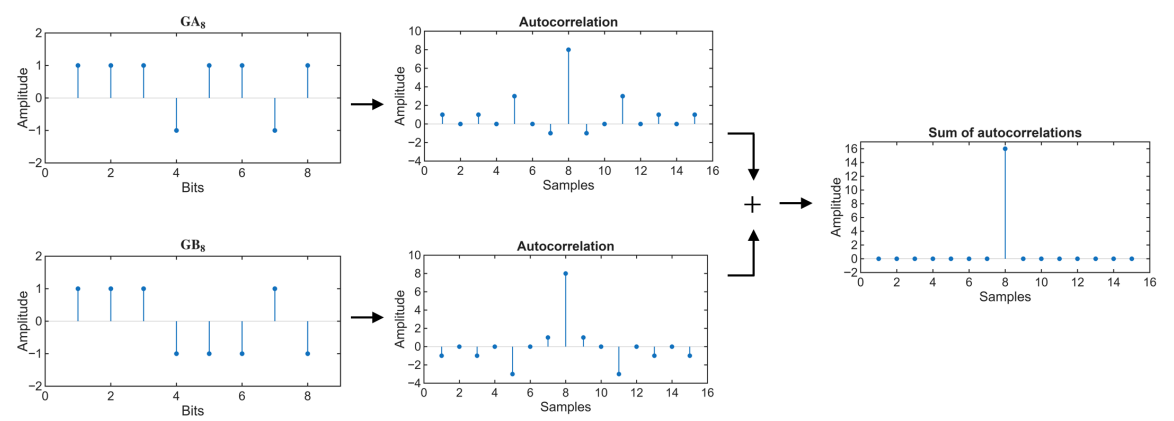


Figure 3. Sidelobe cancellation using the length-8 CGC.

After coding, the resulting coded excitation signals are transmitted into the medium by the ultrasonic transducers and are subsequently received by the receiving transducers after propagating through the medium. Temporal averaging may be applied to the received signals to reduce noise, depending on the required SNR. After temporal averaging, pulse compression is performed for further signal analysis. Pulse compression, also referred to as decoding, compresses the long coded signals into short-duration pulses with high peak amplitudes, significantly improving the SNR. Decoding is commonly achieved by applying a matched filter or correlation operation to compress the received signals, where the reference signal is generally the coded excitation signal itself [23]. Once decoding is completed, the compressed signals are used for further signal processing to extract valuable information such as the signal peak location. The flow of coding and pulse compression is presented in Figure 4, and an example of coding and compressing the 7-bit Barker code is shown in Figure 5.

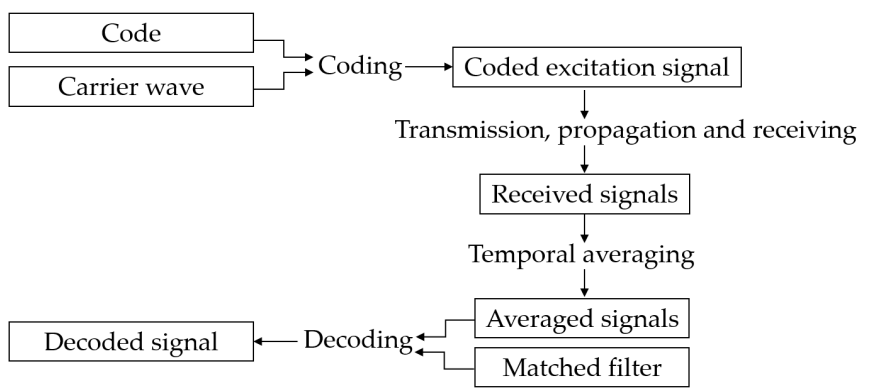


Figure 4. Coded excitation signal processing flow.

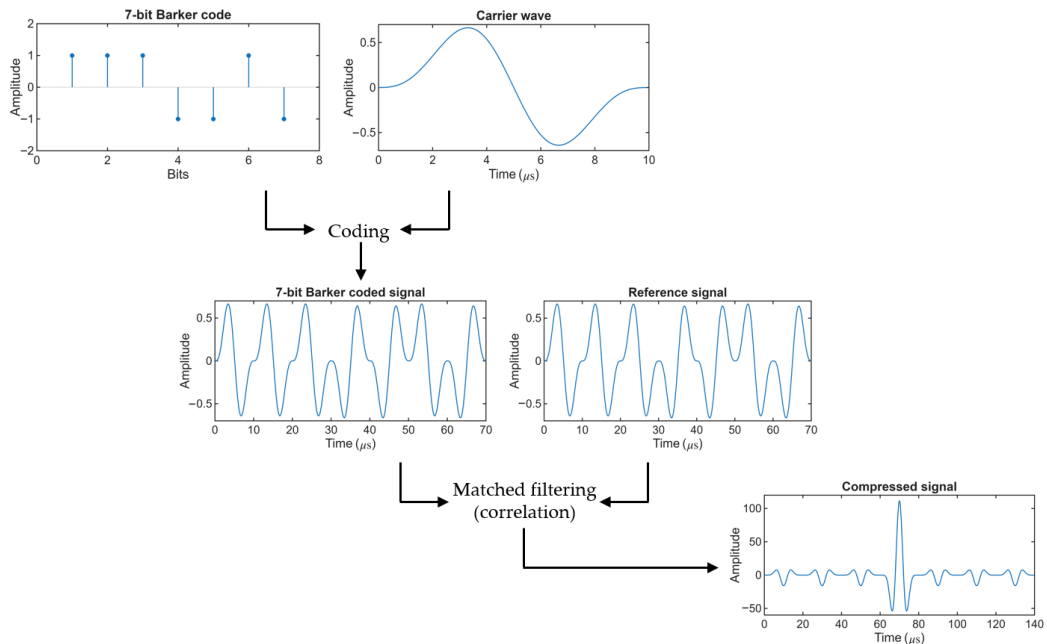


Figure 5. Coding and pulse compression using the 7-bit Barker code.

2.4. Performance Evaluation of Coded Excitation Signals

The performance of coded excitation signals can be evaluated using several parameters, including the SNR, peak sidelobe level, and SNR gain (SNRG). Definitions of these evaluation parameters may vary among different research works [10,23,24,27–29]. Among these parameters, the SNR is one of the most critical metrics. The SNR is the ratio between the peak value of the compressed signal mainlobe to the root mean square (RMS) of a signal region containing only noise [29]:

$$SNR = 20 \log_{10} \left(\frac{A_m}{A_{rms}} \right), \quad (6)$$

where A_m and A_{rms} denote the peak of the signal mainlobe and the RMS of the noise, respectively. In addition to the SNR, SNRG quantifies the improvement in the SNR achieved by coded excitation relative to uncoded excitation and is defined, under the assumptions of constant bandwidth and additive white Gaussian noise, as [28]

$$SNRG = 20 \log_{10} \left(\frac{SNR_a}{SNR_b} \right) = \zeta HNT, \quad (7)$$

where SNR_a and SNR_b are the SNR values after and before matched filtering, respectively. Here, N is the code length, T is the duration of the carrier wave, H denotes the signal bandwidth, and ζ is the number of transmissions; for CGCs, $\zeta = 2$, whereas for Barker codes, $\zeta = 1$. Equation (7) indicates that the SNRG is proportional to ζN [29].

3. Experimental Measurements

3.1. Measurement Setup

To evaluate the effectiveness of coded excitations in reducing the need for temporal averaging, laboratory experiments were conducted. For the experimental measurements, we used the measurement setup shown in Figure 6. The measurement setup includes a

2000 mm $\times$ 2000 mm $\times$ 2 mm aluminum plate, a piezoelectric transducer, a laser vibrometer, and a data acquisition (DAQ) box consisting of a homemade amplifier and a data acquisition device (Analog Discovery 3, Digilent Inc. Austin, TX, USA). The aluminum plate dimensions were selected to avoid wave reflections from the plate boundaries, which could be effectively excluded by time gating. The transducer (S0208, ACS Ltd., Saarbrücken, Germany), with a nominal frequency of 100 kHz, was coupled to the plate using ultrasonic gel and a 100 g weight placed on top to ensure stable and repeatable acoustic coupling. The VFX-I-160 laser vibrometer (Polytec GmbH, Waldbronn, Germany) was employed to perform non-contact measurements of vibrations on the plate surface. The DAQ box, providing functions such as signal amplification and temporal averaging, was connected to a computer and operated via custom LabVIEW-based software (LabVIEW version: 2014) to configure the excitation voltages, triggering, and the number of averages [30].

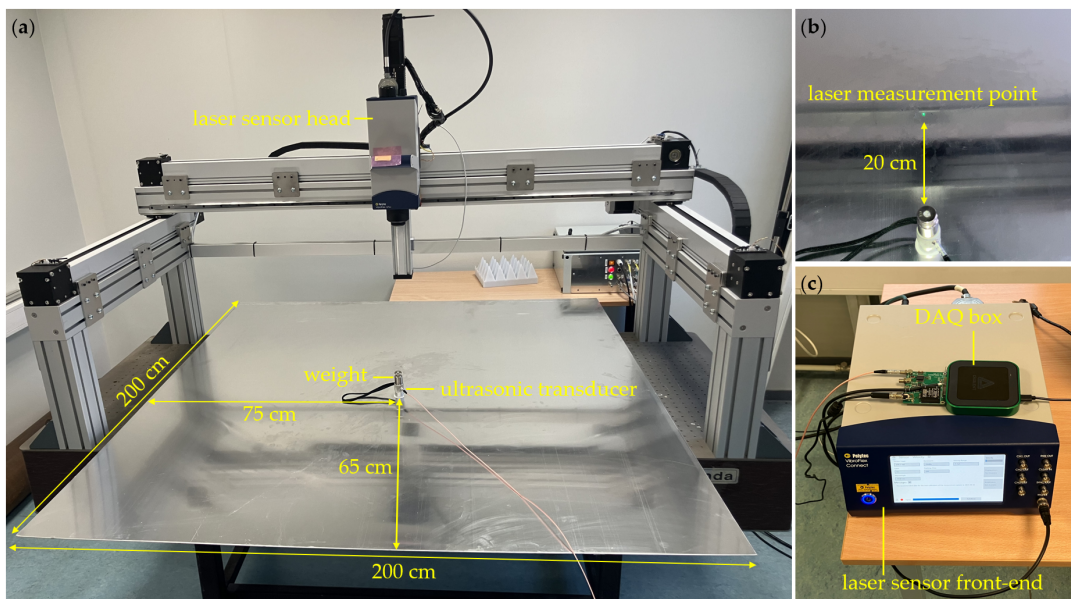


Figure 6. Measurement setup. (a) Measurement platform with a transducer, a laser vibrometer, and an aluminum plate. (b) Locations of the transducer and the laser measurement point. (c) DAQ box and laser front-end.

A pitch-catch scheme was used to capture UGWs. To avoid complex multimode propagation, 100 kHz A_0 -mode UGWs were generated by applying an out-of-plane force to the plate surface using the piezoelectric transducer. The out-of-plane displacements at the receiving measurement point were measured using the laser vibrometer. Dispersion curves for this 2 mm aluminum plate are shown in Figure 7. As the A_0 -mode wave is in a dispersive region, as shown in the dispersion curve, the received signals experience waveform distortion. The received signals were first processed by the laser vibrometer front-end unit and then recorded on a computer via the DAQ box for further analysis. As shown in Figure 6a,b, the distance between the transducer and the laser measurement point was 20 cm, and both the transducer and laser measurement locations were positioned sufficiently far from the plate boundaries. This ensured that boundary reflections did not interfere with the directly propagating waves between the transducer and the laser measurement point.

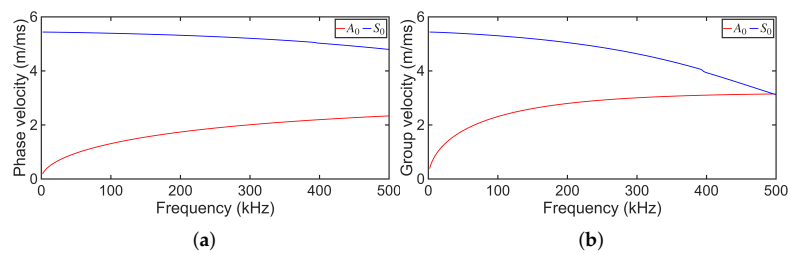


Figure 7. Dispersion curves for the A_0 and S_0 modes in a 2 mm thick aluminum plate. (a) Phase velocity. (b) Group velocity.

The excitation signals used in the experiments included a one-cycle pulse, defined as a Hanning-windowed sine wave with a duration of one cycle; two Barker-coded signals of different lengths; and a pair of 16-bit CGC-coded signals. The frequency of the one-cycle pulse was 100 kHz, which matched the transducer’s nominal frequency. The coded excitation signals were generated according to Equation (5), where the carrier wave was identical to the one-cycle pulse. The one-cycle pulse and the 13-bit Barker-coded signal are shown in Figure 8. Experiments were performed using different excitation voltages and different numbers of averages. For each excitation voltage, four different excitation signals were tested using six averaging numbers, resulting in 24 received signals per voltage. The same experimental design was applied to all five excitation voltages. Table 3 presents some of the experimental parameters, and all combinations of the listed parameters were tested. The excitation voltage was restricted to 10 V due to the limitations of the DAQ system, which is comparable to many commonly used DAQ devices, such as HS5 (TiePie engineering, Koperslagersstraat, The Netherlands) [31]. In practice, this voltage level is considered low, leading to increased noise influence and thus providing a suitable condition for assessing the performance of coded excitations [23]. The choice of the averaging number depends on the specific measurement setup; for instance, an averaging number of 50 has been reported in the literature [32]. In this study, averaging numbers ranging from 1 to 40 were investigated to evaluate the performance of coded excitations. This range was selected because increasing the averaging number beyond 40 leads to a negligible improvement in SNR. By comparing the SNR values obtained with different numbers of averaging under a series of voltage levels, the effectiveness of coded excitation signals in reducing the required number of averages can be evaluated.

Table 3. Some of the experimental parameters.

Excitation signals	one-cycle pulse, B_7 , B_{13} , G_{16}
Excitation voltage (V)	0.5, 1, 2, 5, 10
Averaging number	1, 5, 10, 20, 30, 40

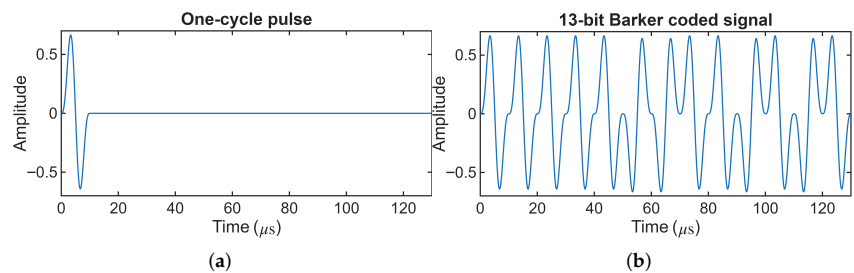


Figure 8. Excitation signals. (a) The one-cycle pulse. (b) The 13-bit Barker-coded signal.

3.2. Results of Decoding

All received signals were time-windowed to gate the boundary-reflected waves and were then filtered using the DAQ system’s built-in 8 kHz high-pass filter to remove low-frequency noise before further processing. After high-pass filtering, the received signals were decoded and time-shifted to compensate for the time lag introduced by the matched filtering, followed by envelope extraction. Finally, the envelope was normalized. Using the normalized envelope, the SNR was computed according to Equation (6), where A_m was obtained as the peak amplitude of the envelope mainlobe, and A_{rms} was evaluated from the noise-only region after 250 μs . A schematic diagram illustrating the practical calculation of the SNR using a 7-bit Barker code with 40 temporal averages at 5 V is shown in Figure 9.

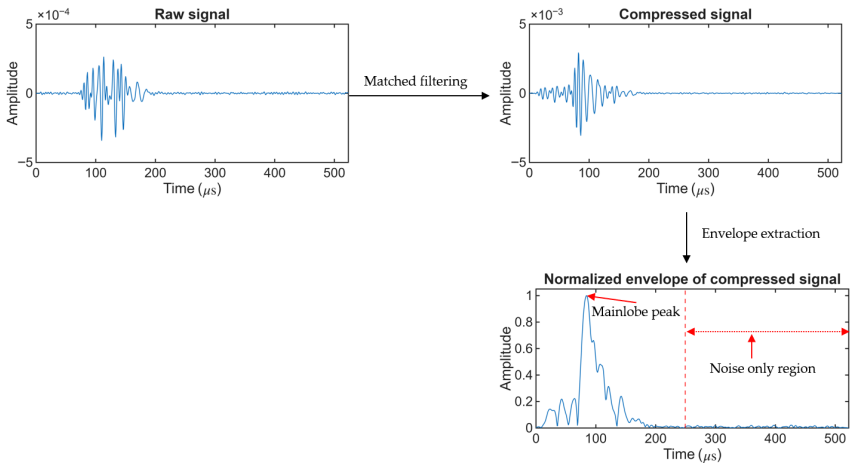


Figure 9. Schematic of the signal processing procedure for SNR calculation with a 7-bit Barker code, 40 temporal averages, and 5 V excitation voltage.

Figures 10–14 present the compressed signal envelopes obtained under excitation voltages of 0.5, 1, 2, 5, and 10 V, respectively. Each figure consists of four subfigures, labeled (a)–(d), corresponding to different excitation signals. Within each subfigure, three representative signals corresponding to averaging numbers of 1, 10, and 30 are shown from top to bottom, respectively.

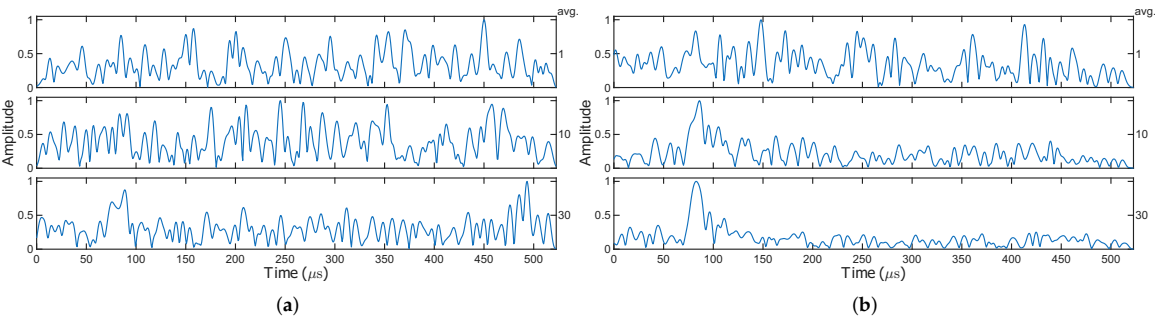


Figure 10. Cont.

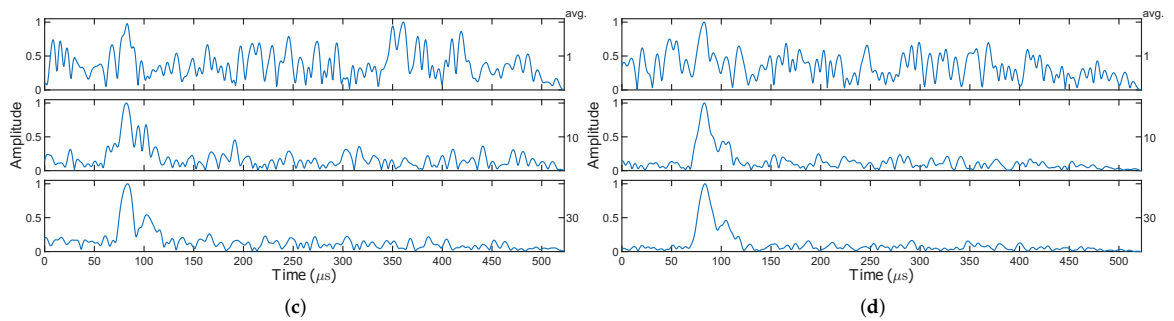


Figure 10. Resulting signal envelopes for a 0.5 V excitation voltage. (a–d) The signals obtained using the one-cycle pulse, B_7 , B_{13} , and G_{16} , respectively. The number of averages is labeled next to each signal.

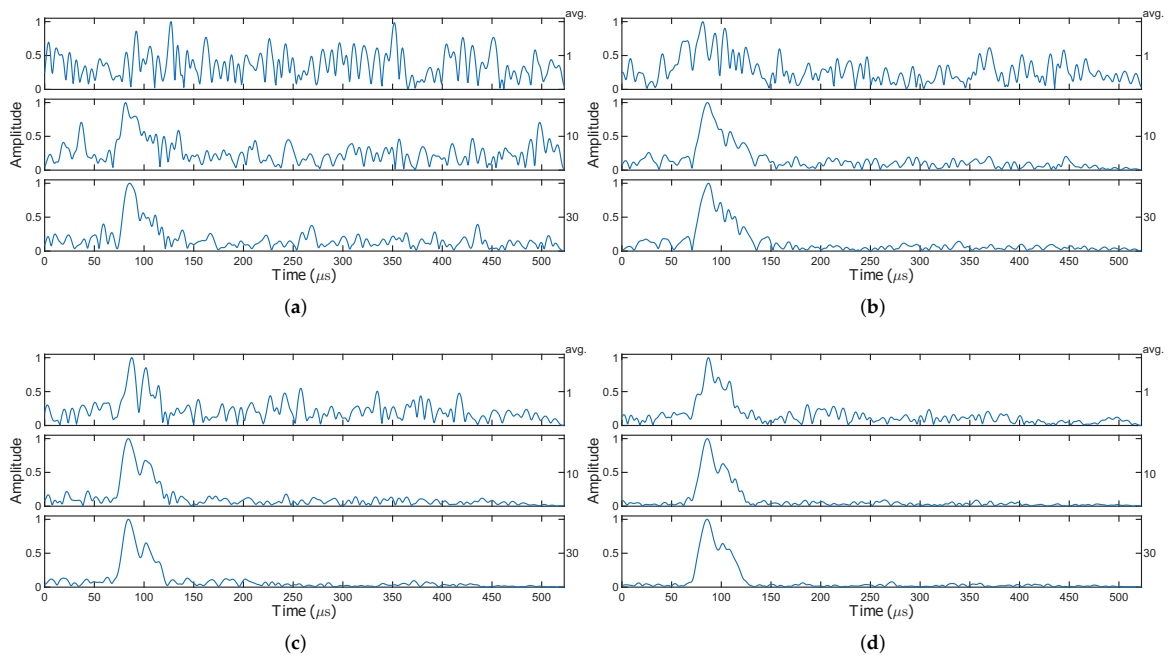


Figure 11. Resulting signal envelopes for a 1 V excitation voltage. (a–d) The signals obtained using the one-cycle pulse, B_7 , B_{13} , and G_{16} , respectively.

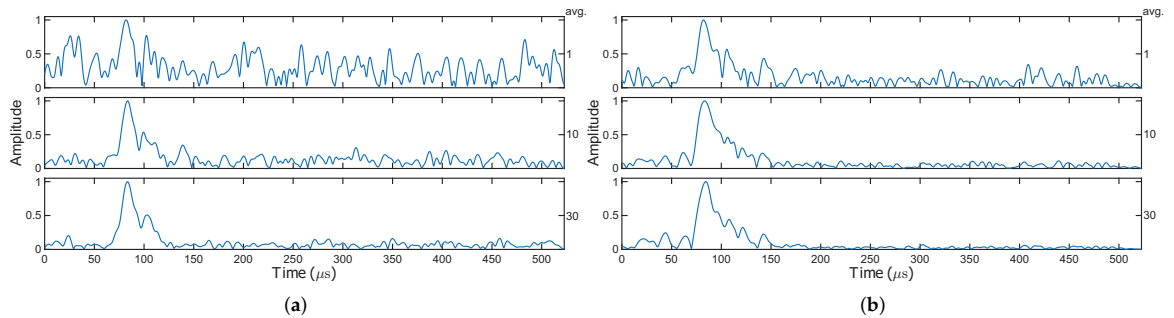


Figure 12. Cont.

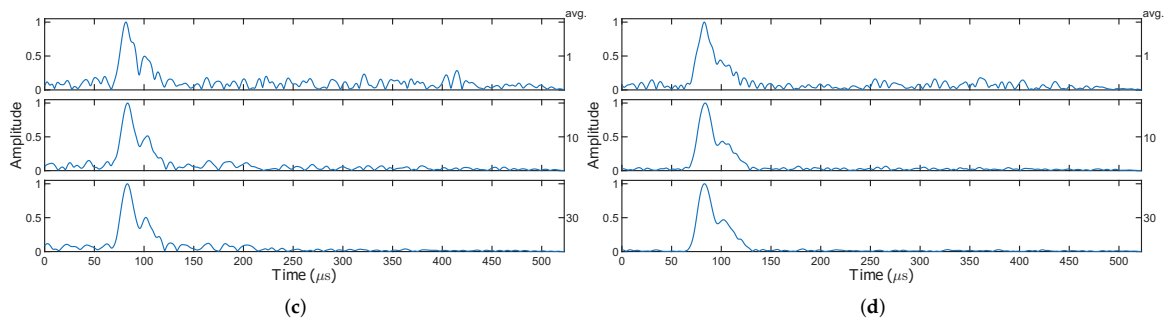


Figure 12. Resulting signal envelopes for a 2 V excitation voltage. (a–d) The signals obtained using the one-cycle pulse, B_7 , B_{13} , and G_{16} , respectively.

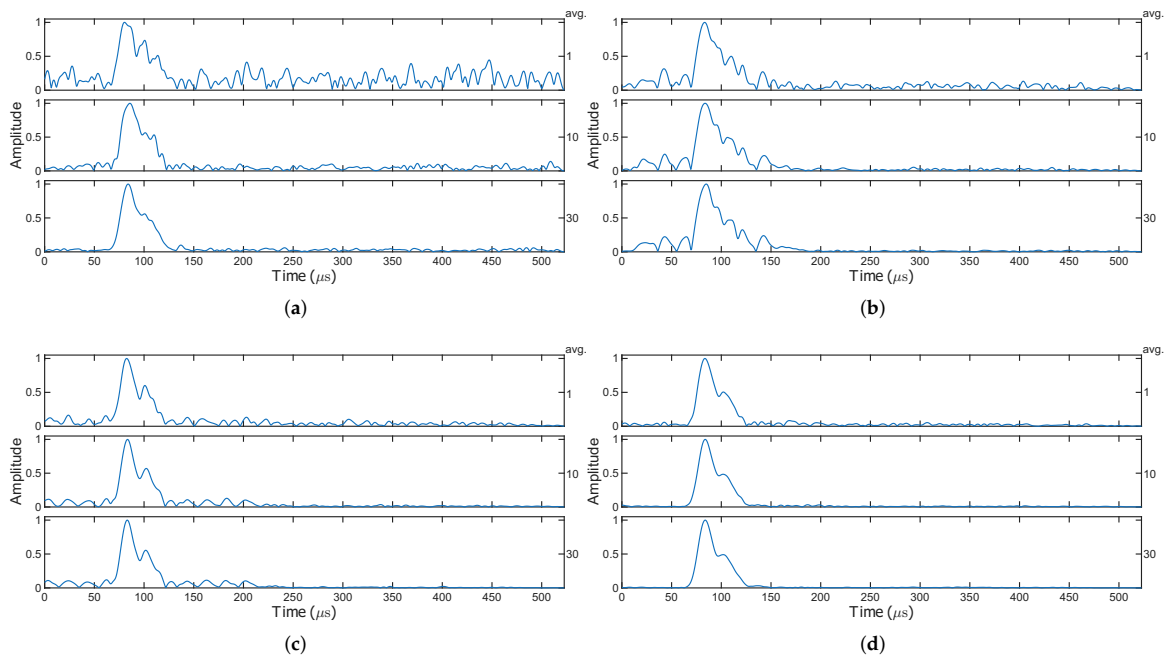


Figure 13. Resulting signal envelopes for a 5 V excitation voltage. (a–d) The signals obtained using the one-cycle pulse, B_7 , B_{13} , and G_{16} , respectively.

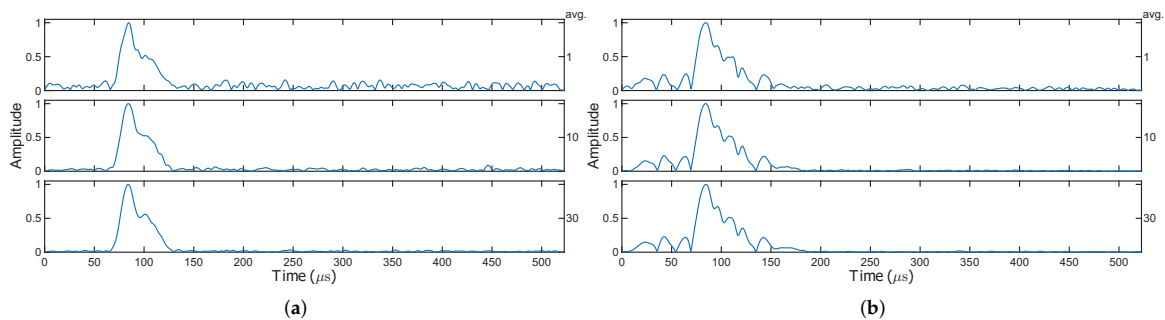


Figure 14. Cont.

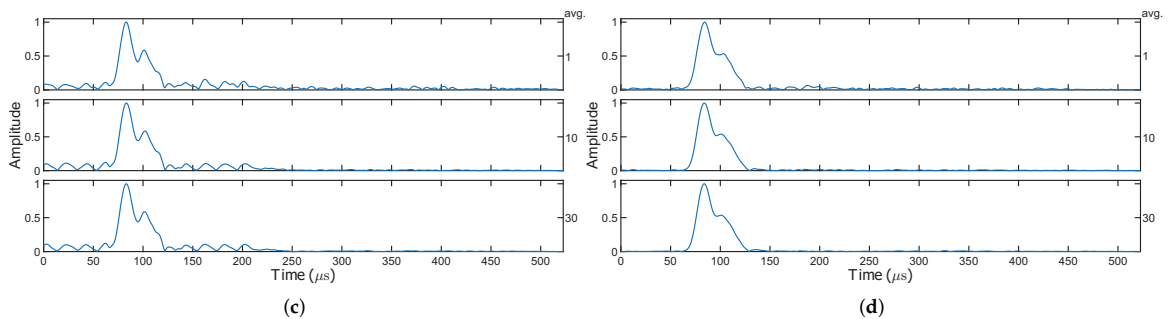


Figure 14. Resulting signal envelopes for a 10 V excitation voltage. (a–d) The signals obtained using the one-cycle pulse, B_7 , B_{13} , and G_{16} , respectively.

4. Discussion

Figures 10–14 show that the noise level decreases as the number of averages increases across a series of excitation voltages and different excitation signals, confirming the effectiveness of temporal averaging in noise reduction. This effect is particularly evident when comparing the signals obtained with ten averages to those without temporal averaging, as shown in the representative results of Figures 10–12 at excitation voltages 0.5 V, 1 V and 2 V, respectively.

It is observed that, to achieve a relatively low-noise signal, coded excitations require fewer temporal averages than the one-cycle pulse. At an excitation voltage of 0.5 V, the one-cycle pulse in Figure 10a fails to yield reliable results with a sufficient SNR and a clean mainlobe. By contrast, the coded excitations in Figure 10b–d achieve an obviously lower noise level and a clean mainlobe with temporal averaging. As shown in Figure 11, at an excitation voltage of 1 V, the noise level obtained using coded excitations with only 10 averages is already lower than that of the one-cycle pulse with 30 averages. Figure 12 shows that, at an excitation voltage of 2 V, coded excitations achieve a comparable noise level without temporal averaging, compared to the one-cycle pulse with 10 averages. As shown in Figures 13 and 14, at excitation voltages of 5 V and 10 V, respectively, the SNR obtained using coded excitations without temporal averaging is significantly higher than that of the one-cycle pulse without averaging.

In addition to noise characteristics, B_7 exhibits higher sidelobe levels than B_{13} , which is consistent with theoretical expectations. For G_{16} , the sidelobes are significantly suppressed thanks to the complementary sidelobe cancellation property. However, perfect sidelobe cancellation is not achieved, as shown in Figure 11d. This occurs because the double transmission required by CGCs makes perfect sidelobe cancellation difficult to achieve in practice, particularly in the presence of guided-wave dispersion. Moreover, if the wave propagation conditions change between the two transmissions, for example, due to slight motion of the transducer or variations in the measurement setup, the complementary condition can be violated. In such cases, sidelobe cancellation becomes less effective and residual sidelobes remain in the compressed signal. Overall, these observations indicate that coded excitations can achieve comparable SNR levels with substantially fewer temporal averages, thereby improving data acquisition efficiency.

To quantify the effectiveness of coded excitation in reducing the number of averages, the SNR was evaluated and compared based on the results shown in Figures 10–14. The SNR values of the one-cycle pulse, B_7 , B_{13} , and G_{16} under a series of excitation voltages are compared and presented in Figure 15, where subfigures a–e correspond to the results in Figures 10–14, respectively. From Figure 15b–e, it can be observed that the SNR generally

increases with the number of temporal averages, regardless of the type of excitations. Figure 15 also demonstrates that coded excitations achieve a higher SNR than the one-cycle pulse under the same excitation voltage and number of averages. The SNR values follow the order $G_{16} > B_{13} > B_7 > \text{one-cycle pulse}$, which agrees well with Equation (7). In addition, the increase in SNR is not linear with respect to the number of averages. This behavior can be attributed to random variations in noise.

In this experiment, the one-cycle pulse achieves its maximum SNR when averaged 40 times. In contrast, the coded excitations reach comparable SNR levels with substantially fewer averages, as shown in Figure 15. The maximum SNR obtained by the one-cycle pulse is indicated by a horizontal dashed line in Figure 15 to clearly illustrate how many averages are required for the other excitations to reach the same SNR level. Table 4 summarizes the minimum number of averages required for coded excitations to achieve the maximum SNR obtained by the one-cycle pulse under the same excitation voltage. As shown in Table 4, the G_{16} code can achieve a higher SNR than the one-cycle pulse with 40 averages either without averaging or with only five averages, depending on the excitation voltage. For B_{13} , five averages are sufficient to exceed the SNR of the one-cycle pulse with 40 averages, except at an excitation voltage of 0.5 V, where ten averages are required. For B_7 , five averages are sufficient at excitation voltages of 1 V, 5 V, and 10 V, whereas ten averages are required at 0.5 V and 2 V. It should be noted that for G_{16} , the time efficiency is reduced by a factor of two, since each temporal average requires two data acquisition cycles. Overall, these results indicate that coded excitations can achieve comparable SNR levels with fewer than ten averages compared with the 40 averages required by the one-cycle pulse, thereby significantly reducing the number of data acquisition cycles.

From a physical perspective, the SNR improvement achieved by coded excitation arises from the increased transmitted signal energy distributed over a longer time duration. During pulse compression, this energy is coherently compressed into a narrow mainlobe. In contrast to the one-cycle pulse, which introduces only limited energy into the structure, coded signals improve robustness against noise without requiring higher excitation voltages. Longer codes generally provide higher SNRs due to their increased transmitted energy, but this comes at the cost of longer signal duration. This explains why G_{16} consistently achieves the highest SNR. It should be noted that, in this study, the duration of the coded signals is relatively short compared with the duration of a full data acquisition cycle. Specifically, the excitation signal duration is less than 1 ms, and the fixed data acquisition window is 10 ms. In contrast, a complete acquisition cycle takes approximately 300 ms, because sufficient time is required for reflected waves from plate boundaries to decay, and the hardware system requires additional time to prepare for the next data acquisition. Therefore, the duration of the coded signal does not significantly affect the measurement time efficiency; instead, the measurement time is mainly determined by the number of data acquisition cycles. However, if substantially longer codes are used—for example, HAPI codes that may contain tens of thousands of bits—the increased signal duration would have a pronounced impact on measurement time efficiency [33].

From a practical standpoint, these results indicate that coded excitation is particularly advantageous in low-voltage and time-constrained applications, such as real-time monitoring or automated inspection systems, where extensive temporal averaging is impractical. However, the benefits of coded excitation can be limited by guided-wave dispersion. Dispersion causes waveform distortion during propagation, whereas the matched filter used for pulse compression is designed based on the undistorted excitation signal. This mismatch between the distorted received signal and the reference signal used in matched filtering reduces the effectiveness of pulse compression and consequently degrades the achievable SNR.

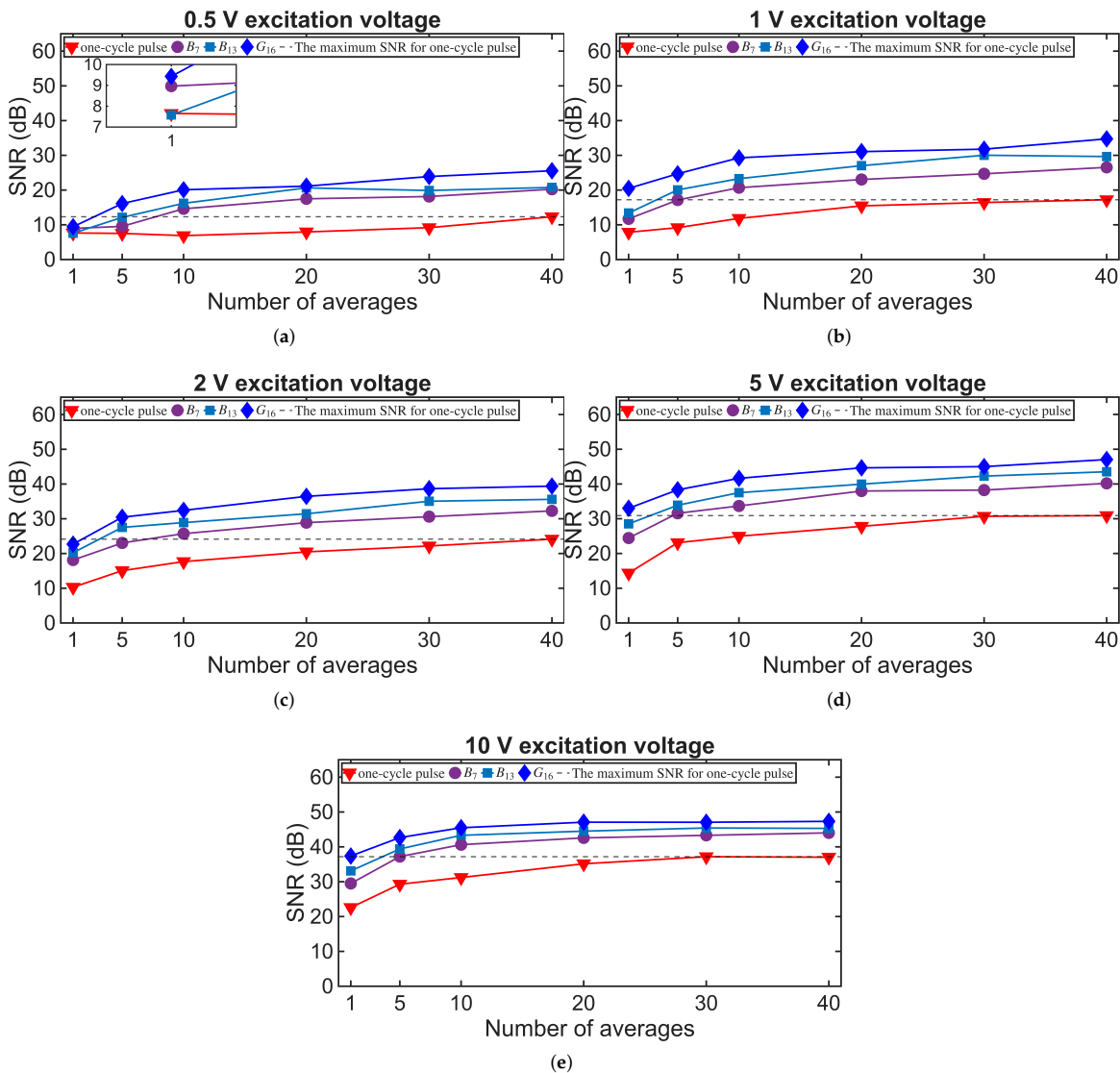


Figure 15. SNRs for different excitation signals at various excitation voltages. (a–e) The SNRs obtained using the 0.5 V, 1 V, 2 V, 5 V, and 10 V excitation voltages, respectively.

In this study, the excitation signal was centered at 100 kHz. As shown in Figure 7, the group velocity of the A_0 mode varies around this frequency, which leads to waveform distortion during propagation. If a higher excitation center frequency, such as 200 kHz, is employed while maintaining a similar bandwidth, the dispersion effect becomes less pronounced. Conversely, under more demanding conditions, such as longer propagation distances, waveform distortion becomes increasingly severe and may further reduce the achievable SNR. This effect can be mitigated by applying dispersion compensation techniques.

In practice, the choice of carrier waveform should be tailored to the dispersion characteristics of the inspected structure in order to further improvement the achievable SNR. Dispersion effects can also be reduced by using carrier waves with narrower bandwidths than that of a one-cycle pulse, such as a five-cycle Hanning-windowed pulse, which is

commonly used as a conventional excitation signal. However, in this study, only a one-cycle pulse was used as the conventional baseline excitation. The primary reason is that if a multi-cycle pulse were used as the conventional excitation, the coded excitations would also need to adopt the same multi-cycle pulse as the carrier wave to ensure a fair comparison and minimize the influence of bandwidth differences. Employing a multi-cycle carrier wave would significantly increase the duration of the coded signal, which could lead to overlap between the directly propagating wave and boundary-reflected waves, thereby compromising the accuracy of SNR estimation. Figure 16 illustrates this effect by comparing the spectra of the 100 kHz one-cycle and five-cycle Hanning-windowed pulses together with two 13-bit Barker-coded signals that employ these pulses as their respective carrier waves. As shown in Figure 16, the carrier waveform strongly influences the bandwidth of the coded signals.

Table 4. Minimum averages for coded excitations to reach the maximum SNR of the one-cycle pulse.

Voltage (V)	Excitation	No. of Averages	SNR (dB)
0.5	one-cycle pulse	40	12.3
0.5	B_7	10	14.6
0.5	B_{13}	10	16.2
0.5	G_{16}	5	16.1
1	one-cycle pulse	40	17.2
1	B_7	5	17.2
1	B_{13}	5	20.0
1	G_{16}	1	20.5
2	one-cycle pulse	40	24.1
2	B_7	10	25.7
2	B_{13}	5	27.5
2	G_{16}	5	30.4
5	one-cycle pulse	40	30.9
5	B_7	5	31.6
5	B_{13}	5	33.9
5	G_{16}	1	33.0
10	one-cycle pulse	40	37.0
10	B_7	5	37.2
10	B_{13}	5	39.4
10	G_{16}	1	37.4

In this study, the use of coded excitations is validated for an aluminum plate using the A_0 guided-wave mode under laboratory conditions. However, the methodology presented here can be extended to more complex structures and other guided-wave modes because the underlying signal processing procedures remain the same.

One limitation when applying coded excitations in complex structures, such as those containing rivets, fasteners, or stringers, is the presence of interference waves reflected from boundaries or interfaces. In such cases, the excitation frequency and code length must be carefully selected to avoid signal overlap, ensuring that the compressed mainlobes of different wave packets remain distinguishable.

Ultrasonic guided waves are inherently multimodal, and the simultaneous generation of multiple modes may further increase the likelihood of signal overlap. Nevertheless, coded excitation techniques remain applicable provided that signals from different modes can be effectively separated during signal processing.

In addition, the experiments in this study were conducted in a laboratory environment using a laser Doppler vibrometer to measure the propagated waves. For practical

field measurements, conventional ultrasonic transducers can serve as receiving sensors, providing a more convenient and portable solution for on-site inspections.

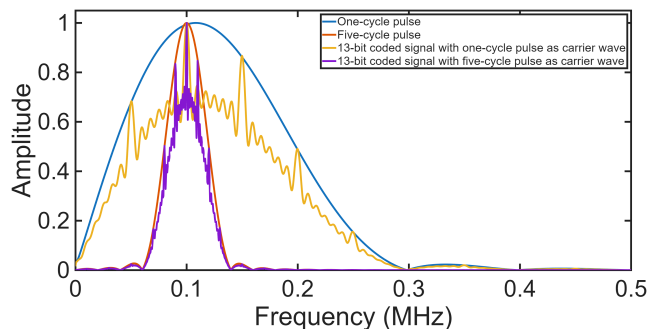


Figure 16. Spectrum of coded signals and their carrier waves.

5. Conclusions

This study experimentally investigated the use of coded excitation to improve the data acquisition efficiency of ultrasonic guided-wave measurements by increasing the achievable SNR and thereby reducing the required number of temporal averages. Barker codes and CGCs were evaluated and compared with a conventional one-cycle pulse under varying excitation voltages and averaging numbers.

The results demonstrate that coded excitation enables a substantial reduction in the number of averages required to achieve a given SNR. Compared with the one-cycle pulse, both Barker codes and CGCs achieved comparable or higher SNRs with significantly fewer data acquisition cycles. Among the tested codes, longer codes generally provided higher SNRs, while CGCs exhibited superior sidelobe suppression at the cost of double transmissions.

These findings confirm that coded excitation is a practical and effective strategy for improving the efficiency of ultrasonic guided-wave measurements. Future work will focus on mitigating dispersion-related degradation, investigating how dispersion limits the usable code length, and extending the approach to more complex structures and inspection scenarios.

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