

TALLINN UNIVERSITY OF TECHNOLOGY  
Tallinn School of Economics and Business Administration  
Department of Finance and Economics  
Chair of Finance and Banking

Matis Tomiste

**CORRELATION MODELLING AND FORECASTING OF  
FINANCIAL DATA**

Master's Thesis

Supervisor: Professor Karsten Staehr

Tallinn 2014

## TABLE OF CONTENTS

ABSTRACT .....	4
ABBREVIATIONS .....	5
INTRODUCTION .....	6
1. SIX STYLIZED FACTS ABOUT CORRELATION DYNAMICS .....	8
1.1. Data .....	11
1.2. Time trend .....	12
1.3. Mean reversal .....	14
1.4. Correlation Breaks .....	15
1.5. Correlation Asymmetry .....	19
1.6. Persistence .....	19
1.7. Outliers .....	20
1.8. Conclusions .....	22
2. REVIEW OF CORRELATION MODELS .....	23
2.1. Notation .....	23
2.2. Some technical considerations .....	24
2.3. Naïve correlation models .....	25
2.3.1. Simple Moving Average .....	25
2.3.2. RiskMetrics EWMA .....	26
2.4. Models of conditional covariance matrix .....	27
2.4.1. VEC Model .....	27
2.4.2. BEKK Model .....	29
2.4.3. Orthogonal GARCH .....	30
2.5. Models of conditional variances and covariances .....	32
2.5.1. Constant Conditional Correlation .....	32
2.5.2. Dynamic Conditional Correlation .....	33
2.5.3. Asymmetric Dynamic Conditional Correlation .....	34

2.6. Conclusions.....	35
3. CORRELATION MODEL FORECASTING PERFORMANCE. A LITERATURE REVIEW.....	36
3.1. Loss Functions .....	36
3.1.1. Statistical loss functions.....	37
3.1.2. Choosing a proxy for statistical loss functions.....	39
3.1.3. Economic loss functions.....	40
3.2. Procedures for model comparison .....	41
3.3. Other considerations .....	42
3.4. Empirical results .....	43
3.4.1. Model forecasting performance in smaller dimensions .....	44
3.4.2. Model forecasting performance in larger dimensions.....	48
3.5. Conclusions.....	50
4. CORRELATION MODEL FORECASTING PERFORMANCE IN EMERGING MARKETS.....	52
4.1. Competing models.....	53
4.2. Data.....	54
4.3. Forecast Evaluation.....	59
4.4. Equity Sample Results .....	60
4.5. Currency Sample Results.....	64
4.6. Conclusions.....	67
SUMMARY .....	69
RESÜMEE .....	72
BIBLIOGRAPHY .....	75
APPENDICES.....	78
Appendix 1. Selected features of empirical surveys.....	78
Appendix 2. WIG and PX in-sample correlation dynamics (initial training set).....	80
Appendix 3. WIG and PX one day ahead correlation forecast dynamics.....	81
Appendix 4. EURCHF and EURPLN in-sample correlation dynamics (initial training set) ....	83
Appendix 5. EURCHF and EURPLN one day ahead out-of-sample correlation dynamics.....	84

## **ABSTRACT**

The importance of correlation modelling has long been recognized as one of the cornerstones of modern portfolio risk management. Though univariate volatility modelling has been in focus of academic research for over thirty years, multivariate volatility modelling has just recently been getting more academic attention. The importance of correlations in portfolio risk management is however hard to overestimate, as their dynamics will define the ultimate costs and benefits of diversification. This thesis seeks to explain the general dynamics of time-varying correlations, introduces the most prominent correlation models and evaluates their out-of-sample forecasting performance with an objective to find a superior model that could be used in real life applications. By using one day ahead correlation forecasts, each model is tested for high and low volatility periods on samples from developed markets and emerging markets, where developed market results are based on previous academic research and emerging market results on the calculations of the author. Furthermore, as different asset types can experience different dynamics, separate samples are created for equity, currency and multi asset portfolios. The evaluation criteria used to find the best performing model is based on two factors. First, which model is able to predict the outcome of the next day most precisely? Second, what is the cost, measured in computing hours, of using certain model? General results are mixed as no single best model is found that would unconditionally outperform others in all market and asset type conditions. There are some preference towards asymmetric version of dynamic conditional correlation model (ADCC) in developed market setting and towards constant conditional correlation model (CCC) in emerging market setting. Furthermore, more sophisticated models, including the dynamic versions of conditional correlation models show unrealistically high computational time cost to be used in real life applications.

Keywords: correlation modelling, multivariate GARCH, multivariate volatility, loss function, model confidence set, forecasting

## **ABBREVIATIONS**

ADCC – Asymmetric Dynamic Conditional Correlation

BEKK - Baba-Engle-Kraft-Kroner correlation

BIP-cDCC – Bounded Innovation Propagation cDCC

BIP-DCC – Bounded Innovation Propagation DCC

CCC – Constant Conditional Correlation

cDCC – Aielli version of DCC

DBEKK – Diagonal BEKK

DCC – Dynamic Conditional Correlation

DECO – Dynamic Equicorrelation

DVEC – Diagonal VEC

EWMA – Exponentially Weighted Moving Average correlation

EWMA<sup>FKO</sup> – Fleming-Kirby-Ostdiek version of EWMA

EWMA<sup>RM</sup> – RiskMetrics version of EWMA

FBEKK – Full BEKK

FVEC – Full VEC

GARCH – Generalized Autoregressive Conditional Heteroskedasticity

GJR-GARCH - Glosten-Jagannathan-Runkle GARCH

MCS – Model Confidence Set

MGARCH – Multivariate GARCH

O-GARCH – Orthogonal GARCH

SBEKK – Scalar BEKK

SMA – Simple Moving Average correlation

SVEC – Scalar VEC

VEC – Vectorized correlation

## INTRODUCTION

During the financial crisis of 2007-08 volatilities of global asset returns surged to levels not seen in recent history. One of the consequences of this increased risk perception was the substantial strengthening of co-movements between various asset returns. Latter posed a problem for global banks, insurance companies and money managers as these institutions had historically relied on correlations as a tool to diversify risk in their asset portfolios. Now when both volatilities and correlations reached new highs, their total asset portfolio volatility levels often reached the levels that violated both internal as well as external regulatory risk limits. Violations in turn forced them to unload the riskier assets to already oversold markets putting even more pressure on global asset prices and risk levels. Such regulation driven forced selling was and still can be a source of systematic risk in the global financial system.

Since 1952 when Markowitz published an article on portfolio selection (Markowitz 1952), correlation estimates have been considered as one of the cornerstones of portfolio risk management. Since then, global regulatory environment has embraced various risk systems (value-at-risk and expected shortfall being the most prominent) all relying on correlations as risk diversifiers. Being ones objective the minimization of portfolio risk given a target return, or return maximization given a target risk, all decisions rely on three types of variables: expected returns and variances of underlying instruments and their respective expected correlations. While variance modelling has been in focus of academic research since 1982 when Engle published his seminal paper on autoregressive conditional heteroskedasticity (Engle 1982), modelling of correlations has been getting somewhat less focus. This might be due to two reasons. First, the importance of correlations compared to univariate volatility in risk models is somewhat less significant. Second, correlation modelling suffers from the so called „dimensionality curse“. The latter means that most of the existing correlation models are not even usable with portfolios above 100 instruments.

This thesis seeks to explain the general dynamics of time-varying correlations, introduces the most prominent correlation models and evaluates their out-of-sample forecasting performance with an objective to find a superior model that could be used in real life applications. Further emphasis will be put on the usage of underlying data. This means that correlation model should perform equally well in various market conditions (i.e. in calm and turbulent markets) as well as not be dependent on the underlying asset type (i.e. equities, fixed income, currencies, etc.) nor be influenced by market type (i.e. developed market or emerging market). In addition, modelling efficiency represented by computational time cost of various correlation models will be investigated. Looking at correlation model forecasting performance together with its estimation efficiency could potentially provide some useful insights about correlation models' integration possibilities into real-life risk systems.

The thesis proceeds as follows. In Chapter 1 some well-known facts about correlation dynamics will be introduced. In Chapter 2, theoretical concepts of correlation modelling together with formal definitions of eight correlation models will be given. Chapter 3 will be dedicated to the literature review on the forecasting ability of these correlation models in the context of developed markets. In Chapter 4, empirical analysis will be carried out to evaluate forecasting performance of nine correlation models in the context of emerging markets.

# 1. SIX STYLIZED FACTS ABOUT CORRELATION DYNAMICS

In this chapter six well known facts about correlation dynamics will be presented. These so called “stylized facts” will then serve as requirements against which theoretical correlation models will be benchmarked in Chapter 2.

Consider the time series of three equity market indices. Figure 1.1 plots index level and daily log return information from three countries (S&P 500 of United States, DAX of Germany and HSI of Hong Kong). Based on the charts in Figure 1.1, couple of observations can be made. Firstly, in general stock market indices move in tandem, indicating that there should be a positive correlation between different equity market indices. Furthermore, looking at the return charts (charts 2 to 4), one can also notice that the co-movements have been more synchronized in the end of time series than in the beginning of it. This in turn could be an indication of positive relationship between correlation and time. Indeed, investigating seven major country stock indices over the period of 1960-1990, Longin and Solnik (1995) found evidence of increasing correlations over this period.

Secondly, even though in short term the correlation can fluctuate substantially, it still has a tendency to revert back to its long term (increasing) average value. In fact, the correlation time trend and the fluctuations around this time trend are rather similar to the long and short term growth dynamics in economic growth. Namely, when describing economic growth dynamics as a combination of long term steady growth trend and a short term business cycle driven growth dynamics, a parallel can be drawn in which increasing correlations serve as long term steady trend dynamics (in case of correlations, this means increasing integration of global economy) and a more hectic short term correlation dynamics (mainly driven by business cycles). From the correlation modelling perspective, this means that the correlation model should be able to assign dif-



ferent weights to different historical return observations as well as to be able to incorporate a more static long term average correlation term.

Thirdly, it is a well-known fact that stock market returns are heteroskedastic (see return charts for illustrative evidence). As correlations depend on volatilities, this could very well mean that also correlations can change (break) during more volatile periods. Using information from the stock market crashes of 1987 (Black Monday), 1989 (Russian Crisis), 2001 (dot-com bubble burst) and 2008 (subprime mortgage crisis) Sandoval et al. (2010) showed that correlations between stock market indices<sup>1</sup> do increase significantly during great crashes.

Fourthly, as the downside deviation of returns tends to be much higher than the upside deviation (see return charts), it could also mean that there is some asymmetry in correlation dynamics. Longin and Solnik (2001) showed by using monthly equity index returns from 1959 to 1996 for United States, United Kingdom, France, Germany and Japan that empirical correlations tend to increase during down-markets and decrease during up-markets.

For the modelling perspective, facts three and four mean that the model should be flexible enough to take into account both the possible correlation breaks as well as asymmetry.

Fifth fact relates to the persistence in correlation dynamics. Again turning back to our return charts, it is evident that returns tend to cluster, meaning that high absolute returns are likely to be followed by more high absolute returns and vice versa. If persistence in correlation dynamics exists, then from the correlation modelling perspective it is important also to take account of the lagged correlation terms.

The sixth fact is also evident from daily index level and return charts, as it involves outliers, or sudden jumps in prices. Boudt et al. (2013) explain that such outliers are often caused by one-off events such as news announcements. As an example, consider a news announcement involving one asset but not the other. In such case, correlation will tend to zero. When however, the announcement involves both assets, the correlation can tend to either plus one or minus one depending on the announcement symmetry (both positive/negative or opposite). Furthermore, same problems can be found in an index framework. Boudt et al. (2010) point to a October 19, 1987 market crash as a clear outlier that biases correlation estimate. Latter is also observable in Figure

---

<sup>1</sup> Sandoval et al. (2010) used stock market indices from US, UK, Germany, Spain, Sweden, Brazil, Mexico, Japan, Hong Kong, Malaysia, South Korea and Australia.

1.1. As all three equity indices experience drastic fall on that day, the correlation coefficients between indices tend to plus one. The existence of outliers therefore indicate that it is important to control for outliers whenever modelling correlations.

In the next sections, I will try to find illustrative evidence for the six (stylized) facts:

- 1) time trend;
- 2) mean reversal;
- 3) correlation breakdown;
- 4) correlation asymmetry;
- 5) correlation persistence; and
- 6) outliers.

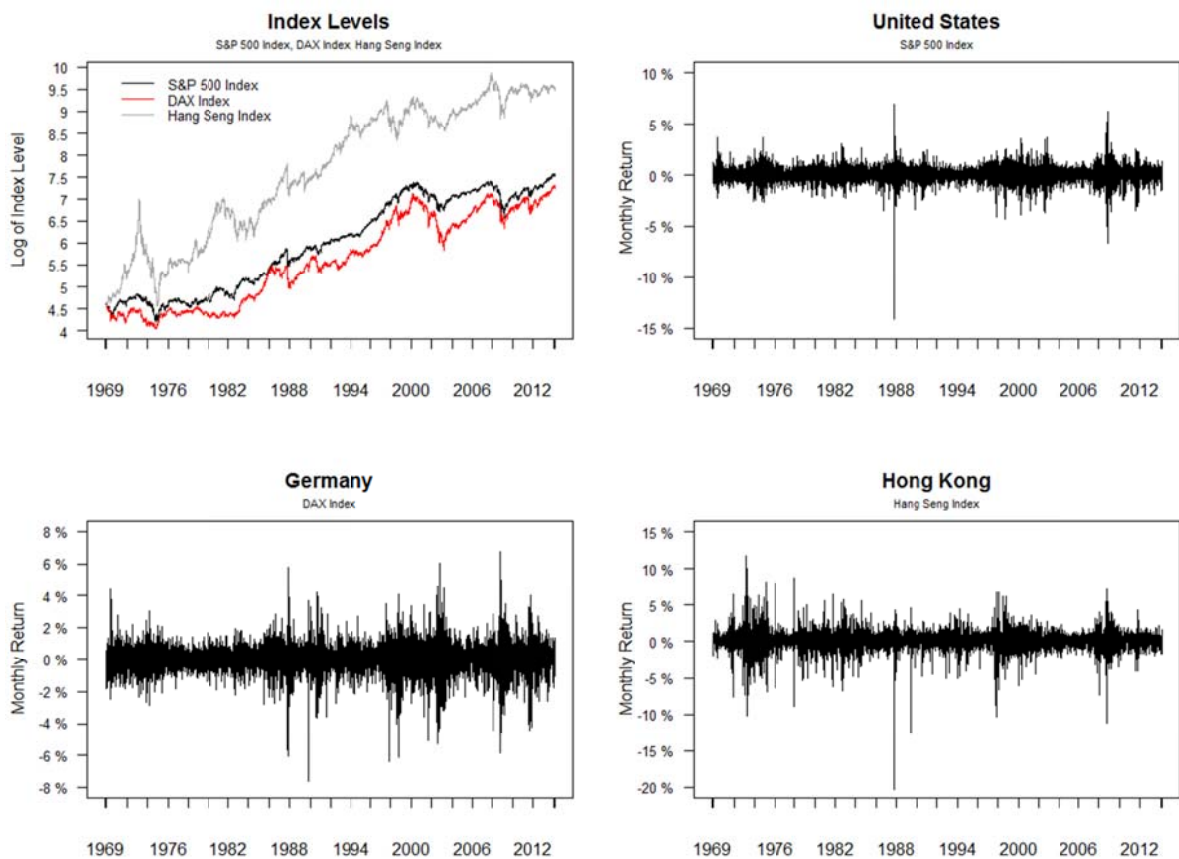


Figure 1.1. United States, German and Hong Kong stock market index log level and return developments from November 21, 1969 to February 21, 2014

## 1.1. Data

This section will introduce the data sample that is used throughout this chapter. In the presentation of the first three stylized facts (time trend, breakdown and asymmetry), daily close-to-close equity index data from Bloomberg database is used. Equity indices observed are grouped into two samples (see Table 1.1). The first, smaller sample is constructed so that it would include as many geographic regions as well as historical observations as possible. In so doing, the sample includes price information starting from November 21, 1969 to February 21, 2014. The second, larger sample includes more indices, but because of lack of historical observations in case of some indices, the observation period length is from January 23, 1992 to February 21, 2014.

Table 1.1. Sample data

Index name	Region	Country	Symbol	Small sample	Large sample
Standard and Poor's 500 Index	North America	United States	spx	Yes	Yes
S&P/Toronto SE Comp Index	North America	Canada	sptsx	No	Yes
Ibovespa Index	Latin America	Brazil	ibov	No	Yes
FTSE 100 Index	Europe	United Kingdom	ukx	No	Yes
CAC 40 Index	Europe	France	cac	No	Yes
DAX Index	Europe	Germany	dax	Yes	Yes
IBEX 35 Index	Europe	Spain	ibex	No	Yes
AEX Index	Europe	Netherlands	aex	No	Yes
OMX Stockholm 30 Index	Europe	Sweden	omx	No	Yes
Swiss Market Index	Europe	Switzerland	smi	No	Yes
Nikkei-225 Stock Average	Asia	Japan	nky	No	Yes
Hang Seng Index	Asia	Hong Kong	hsi	Yes	Yes

Source: Author's compilation

To address the data synchronization problem arising from sample geographical and time-zone diversity, 2-day rolling averages of log returns have been used [see for example Forbes and Rigobon (2002), Hon et al. (2004) and Kotkatvuori-Örnberg et al. (2013)]. Furthermore, for verification purposes, both local currency as well as US dollar subsamples are constructed.

For the presentation of the last two stylized facts (i.e. persistence and outliers), high-frequency data from Bloomberg database is used. Sample is constructed based on the selection of

German DAX index constituents (see Table 1.2) and spans from August 14, 2013 to February 25, 2014. Data is organized into 5 minute intervals including total of 13 586 observations per component (101 observations per trading day).

Table 1.2. Selected DAX index components

Component	Symbol	Currency	Trading hours
E.ON	eaon	EUR	10:00 – 18:30
Deutsche Bank	dbk	EUR	10:00 – 18:30
Commerzbank	cbk	EUR	10:00 – 18:30
Infineon Technologies	ifx	EUR	10:00 – 18:30
Daimler	dai	EUR	10:00 – 18:30
Deutsche Lufthansa	lha	EUR	10:00 – 18:30
Deutsche Telekom	dte	EUR	10:00 – 18:30
SAP	sap	EUR	10:00 – 18:30

Source: Author's compilation

The analysis is performed with statistical computing software R (source code is available upon request) and econometrics software Gretl. When not stated otherwise, all correlations and volatilities presented in the current chapter are unconditional, meaning that they are simple averages of past observations without conditioning on any information set<sup>2</sup>.

## 1.2. Time trend

To model correlation time trend, two samples are used as defined in Table 1.1. Small and large sample 1-year rolling mean correlations are calculated in local currency and US dollars. The results are presented in Figure 1.2.

Both small and large sample indicate upward trending correlations in time. Furthermore, the smaller sample which includes stock market indices from United States, Germany and Hong Kong and starts from the year 1969 provides especially strong evidence of the time trend. When

---

<sup>2</sup> Unconditional volatility is calculated as  $\sigma_x = \sqrt{\frac{1}{T-1} \sum_{t=1}^T (x_t - \bar{x})^2}$  and unconditional correlation as  $\rho_{xy} = \frac{\sum_{t=1}^T (x_t - \bar{x})(y_t - \bar{y})}{(T-1)s_x s_y}$ , where  $\bar{x}$  and  $\bar{y}$  are simple historical averages of assets  $x$  and  $y$  returns respectively.

for example in the 1970s, the sample correlation was on average around 0.1, then by the 2000s average correlation between US, German and Hong Kong stock markets had already risen to 0.5.

As there is no significant difference between local currency and US dollar based results [the results also confirmed by Hon et al. (2004)], then the future analysis will be based only on local currency returns.

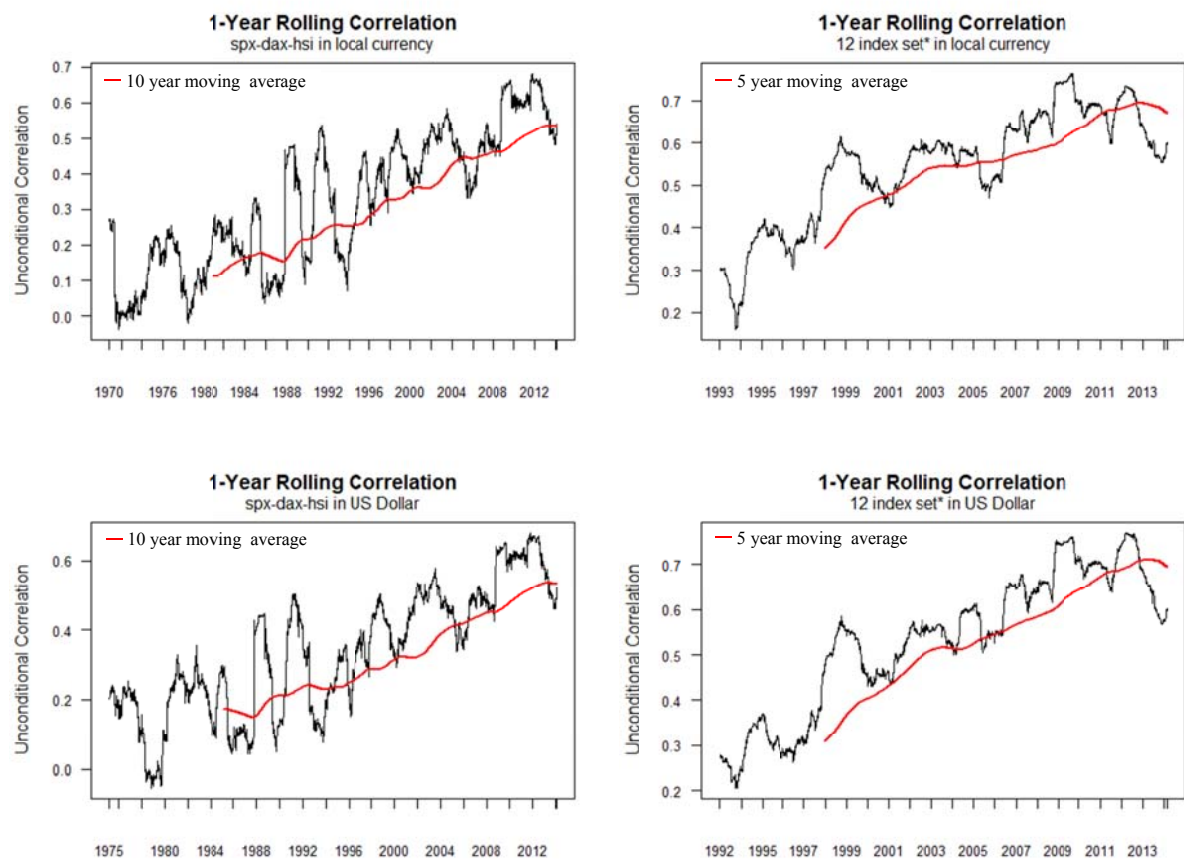


Figure 1.2. Correlation time trend

### 1.3. Mean reversal

Using data starting from the 1970s, it was shown in the last section that long term correlations between global equity markets have been increasing. Nevertheless, when measuring correlation dynamics using shorter time periods, another interesting dynamics arises. Figure 1.3 plots 30 day rolling average correlations between years 2000 and 2007 for both small and large sample of equity indices. Red line on both charts represents a 5 year moving average of rolling 30 day correlations. It is well seen that in case of US, Germany and Hong Kong, the 30 day rolling correlation fluctuates around average correlation of approximately 0.4. In case of the larger sample, the moving average of rolling correlations fluctuates around a steadily increasing average correlation. However, in both cases it appears that short term correlations move back to some long term (moving) average correlation. From the correlation modelling standpoint it is therefore important to consider incorporating some long term average correlation as an intercept to a more dynamic correlation component.

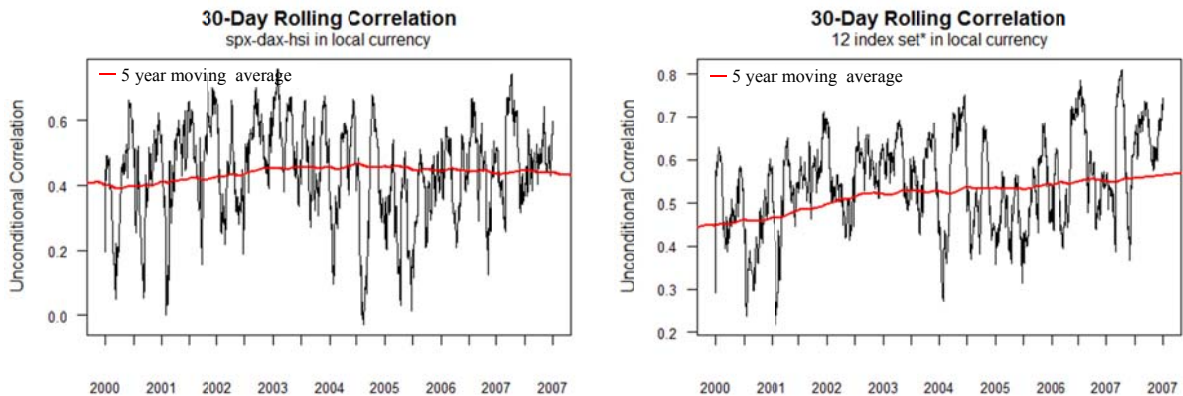


Figure 1.3. Correlation mean reversal

## 1.4. Correlation Breaks

In order to investigate possible breaks in correlations, again two samples from Table 1.1 were used.

To find out whether correlations differ in various market conditions, local currency two day rolling log returns (adjusted returns) were allocated into 6 groups. First group includes all adjusted returns of sample indices for periods when S&P 500 index adjusted return was less than minus 2 standard deviations from its mean. Second group includes adjusted returns conditional on S&P 500 adjusted return falling between -2 and -1 standard deviations from its mean and so on [Group 3: (-1,0); Group 4: (0,+1); Group 5: (+1,+2); Group 5: (+2,+∞)]. Conditional mean correlations were thereafter calculated based on the adjusted returns in each group.

The results are provided in Figure 1.4. “Conditional Volatility” charts give information on the average absolute return in each group and “Conditional Mean Correlation” charts provide information on the average correlation in those groups.

As can be seen, the correlations tend to increase with volatility. Hence we can conclude that there exists at least illustrative evidence that correlations do increase significantly in more volatile periods.

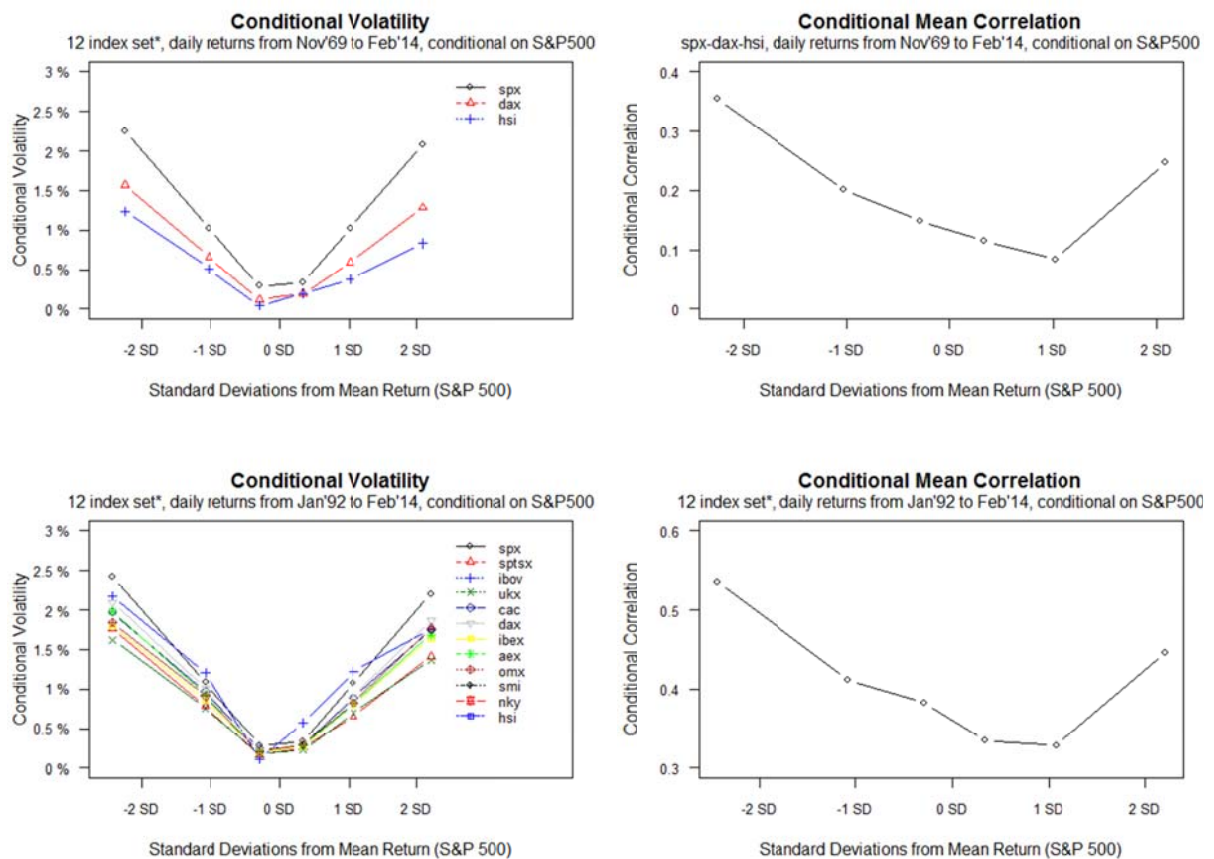


Figure 1.4. Correlation breakdown

Having shown that empirical correlations do increase in more volatile periods, the fact that is also supported by many empirical surveys<sup>3</sup>, there is still no universal agreement on whether the increase is only empirical or also theoretical. To clarify, Boyer et al. (1999) provide a following theorem.

Theorem: Consider a pair of bivariate normal random variables  $x$  and  $y$  with variances  $\sigma_x^2$  and  $\sigma_y^2$ , respectively, and covariance  $\sigma_{xy}$ . Put  $\rho = \sigma_{xy}/(\sigma_x\sigma_y)$ , the unconditional correlation between  $x$  and  $y$ . Consider any event  $x \in A$ , where  $A \subset \mathbb{R}$  such that  $0 < \Pr(A) < 1$ . The conditional correlation  $\rho_A$  between  $x$  and  $y$ , conditional on the event  $x \in A$ , is equal to:

<sup>3</sup> See for example Longin and Solnik 1995, Longin and Solnik 2001, Chua et al. 2009, Loretan and English 2000 among others.



$$\rho_A = \rho \left( \rho^2 + (1 - \rho^2) \frac{Var(x)}{Var(x | x \in A)} \right)^{-\frac{1}{2}} \quad (1.1)$$

The most important observation that can be made from equation (1.1) is that when  $\rho \neq 0$  and  $Var(x | x \in A) \geq Var(x)$  then  $|\rho_A| \geq |\rho|$ , indicating that when volatilities temporarily increase, then correlations increase as well (in absolute value) even though the full sample correlation remains the same.

In general, the theoretical correlations can be shown to be upward sloping in more volatile periods as demonstrated by Boyer et al. (1999) and Loretan et al. (2000) or downward sloping as demonstrated by Longin and Solnik (2001) and Chua et al. (2009). In the end it all depends on conditioning of sub-samples: when sub-sampling is done based on absolute values then normal correlations<sup>4</sup> increase and when it is done on signed values then normal correlations decrease. Furthermore, Campbell et al. (2008) demonstrate that when replacing the normal distribution with Student-*t* distribution (which is more appropriate for fat-tailed return distributions), then theoretical correlations increase even in case the conditioning is done over the signed values. To illustrate the above effects, author used Monte Carlo simulation to generate 1 million standard normally and Student-*t* distributed random observations for two variables (altogether 2 million observations for both distributions). The variable observations were thereafter correlated with unconditional correlation of 0.5 using Cholesky decomposition. The results for normal correlations with conditioning on absolute and signed values as well as Student-*t* correlations with 5 degrees of freedom and conditioning on signed values are provided in Figure 1.5.

There are two observations one can make from Figure 1.5. First, theoretical correlation moves in the same direction as theoretical volatility, i.e. increases when theoretical volatility rises and decreases when it falls. Second, under more relevant Student-*t* distribution with 5 degrees of freedom theoretical correlation is increasing.

In conclusion it can be argued that based on the simulation as well as more recent research results, correlation breaks are not only empirical but also theoretical phenomenon.

---

<sup>4</sup> Normal correlations are correlations for jointly normally distributed variables.

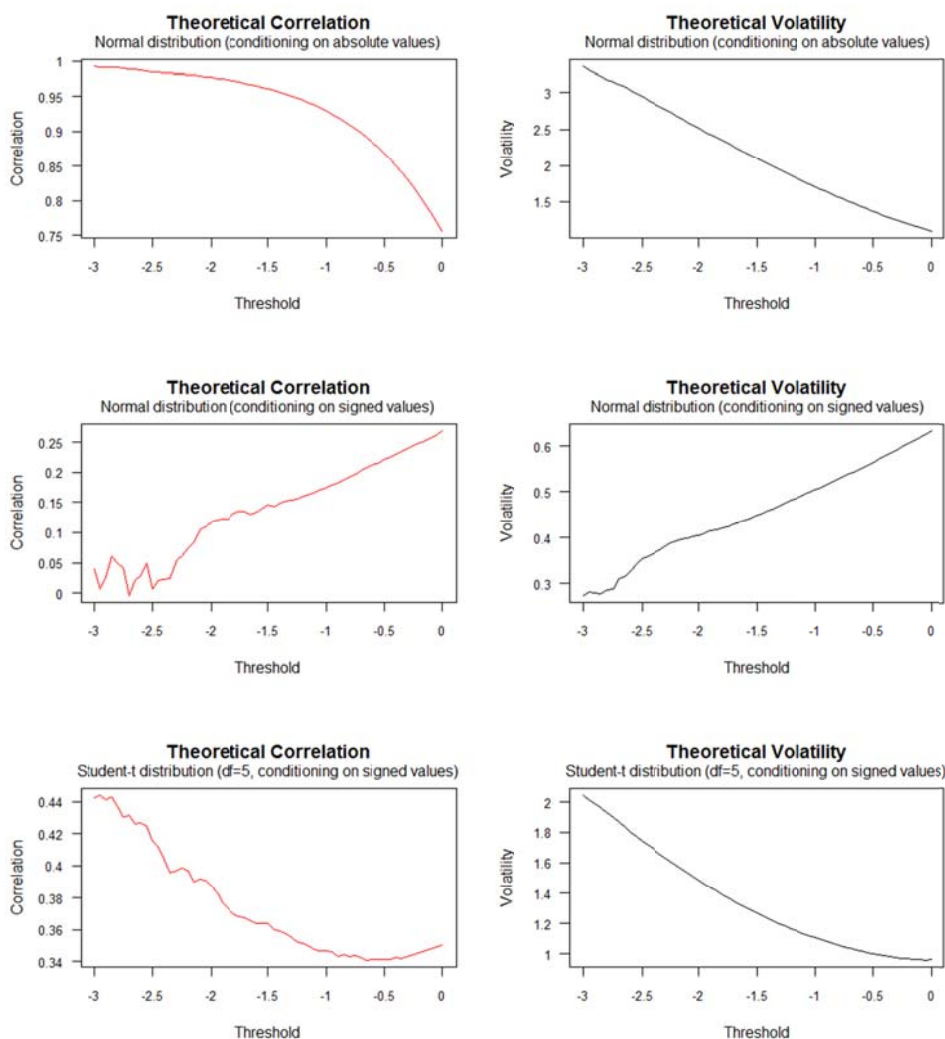


Figure 1.5. Theoretical correlation and volatility dependence on conditioning parameter  $A$ . First two charts plot correlation and volatility dynamics for jointly normally distributed standard random variables with unconditional correlation of 0.5, in case conditioning is done on the absolute returns of both variables. Third and fourth chart plot correlation and volatility dynamics for jointly normally distributed standard random variables with unconditional correlation of 0.5, in case conditioning is done on the signed returns of both variables. Last two charts plot correlation and volatility dynamics for jointly Student- $t$  distributed (with 5 degrees of freedom) standard random variables with unconditional correlation of 0.5, in case conditioning is done on the signed returns of both variables.

## 1.5. Correlation Asymmetry

For correlation asymmetry analysis, the same data and results can be used as in case of correlation breakdowns. Namely, investigating Figure 1.4 one can see some evidence that negative return periods do bring about larger correlation increases than positive return periods. As mentioned in the introductory section to this chapter, Longin and Solnik (2001) as well as Chua et al. (2009) both using global equity index data have found similar evidence of asymmetry in correlation dynamics.

## 1.6. Persistence

To illustrate the possible persistence in correlation dynamics, high frequency data was used for instruments defined in Table 1.2. Based on 5-minute log return data spanning from August 14, 2013 to February 25, 2014, realized correlation coefficients were calculated in accordance with Andersen et al. (2003). According to Andersen et al. (2003), realized correlations can be approximated for each day using sufficient amount of return observations from that day<sup>5</sup>. Data synchronisation issue was addressed by using instruments from the same stock exchange together with the constraint on liquidity (i.e. only the most liquid instruments were included in the sample).

Based on sample data, 135 realized correlations were calculated for each of the randomly selected 7 instrument pairs from Table 1.2 (see table Table 1.3). Table 1.3 provides results from partial autocorrelation analysis of realized correlation time series. There is strong evidence of persistence (represented by partial autocorrelation) of at least one day lag. In most cases, persistence also exists for the 2 day lag.

---

<sup>5</sup> Andersen et al. (2003) used 30 minute intervals in 24 hour foreign exchange markets to minimize market micro-structure noise. Due to XETRA exchange shorter opening hours, the reason that the analysis is only illustrative and only the most liquid stocks were selected, 5 minute intervals was considered as sufficient for current analysis.

Table 1.3. Partial autocorrelation coefficients and the significance of the first three lags

Component	1-day	2-day	3-day
Commerzbank - Infineon Technologies	0.2307 ***	0.2193 **	0.0428
Daimler - Commerzbank	0.2695 ***	0.2436 ***	0.0400
Deutsche Bank - Daimler	0.3677 ***	0.1725 **	0.2069 **
Deutsche Telekom - Deutsche Lufthansa	0.3007 ***	0.2717 ***	0.0585
Infineon Technologies - E.ON	0.4122 ***	0.2384 ***	0.1441 *
Deutsche Lufthansa - Deutsche Bank	0.3780 ***	0.0963	0.0962
SAP - Deutsche Telekom	0.3467 ***	0.2614 ***	0.0930

\*\*\* statistically significant at the 1% level; \*\* st. significant at the 5% level; \* st. significant at the 10% level

Source: Author's compilation

## 1.7. Outliers

Similarly to the analysis of persistence related stylized fact, the illustration of outlier effect entails using high frequency data and realized correlations. As mentioned in the introductory section of the current chapter, outliers caused by one-off events (such as news announcements) can potentially cause a bias in the correlation estimate.

To illustrate the point, 5-minute log returns of two stocks (E.ON and Deutsche Bank) trading on a German stock exchange (XETRA) have been used from August 14, 2013 to February 25, 2014. The results are presented in Figure 1.6, Figure 1.8 and Figure 1.8.

The forth chart in Figure 1.6 („Realized Correlation“) plots daily realized correlations against 5-minute E.ON-Deutsche Bank log return differential. Consider for example the observation circled in red. The log return differential on October 11<sup>th</sup> exceeds 1.5 percent consequently pushing the correlation down towards zero.

More in depth analysis (see Figure 1.8 and Figure 1.8) provides further evidence on outlier effect. Figure 1.8 (October 1<sup>st</sup> and October 2<sup>nd</sup>) provide log return frequency distributions for „normal“ periods (i.e. without outliers). The realized correlations calculated for these days are 0.45 and 0.46 respectively. Alternatively, Figure 1.8 provides frequency distributions for „non-normal“ or „outlier“ days. The information is for the October 7<sup>th</sup> and October 11<sup>th</sup> (mentioned above) with outliers circled in red. As can be seen, the realized correlation drops from around 0.45 to around 0.

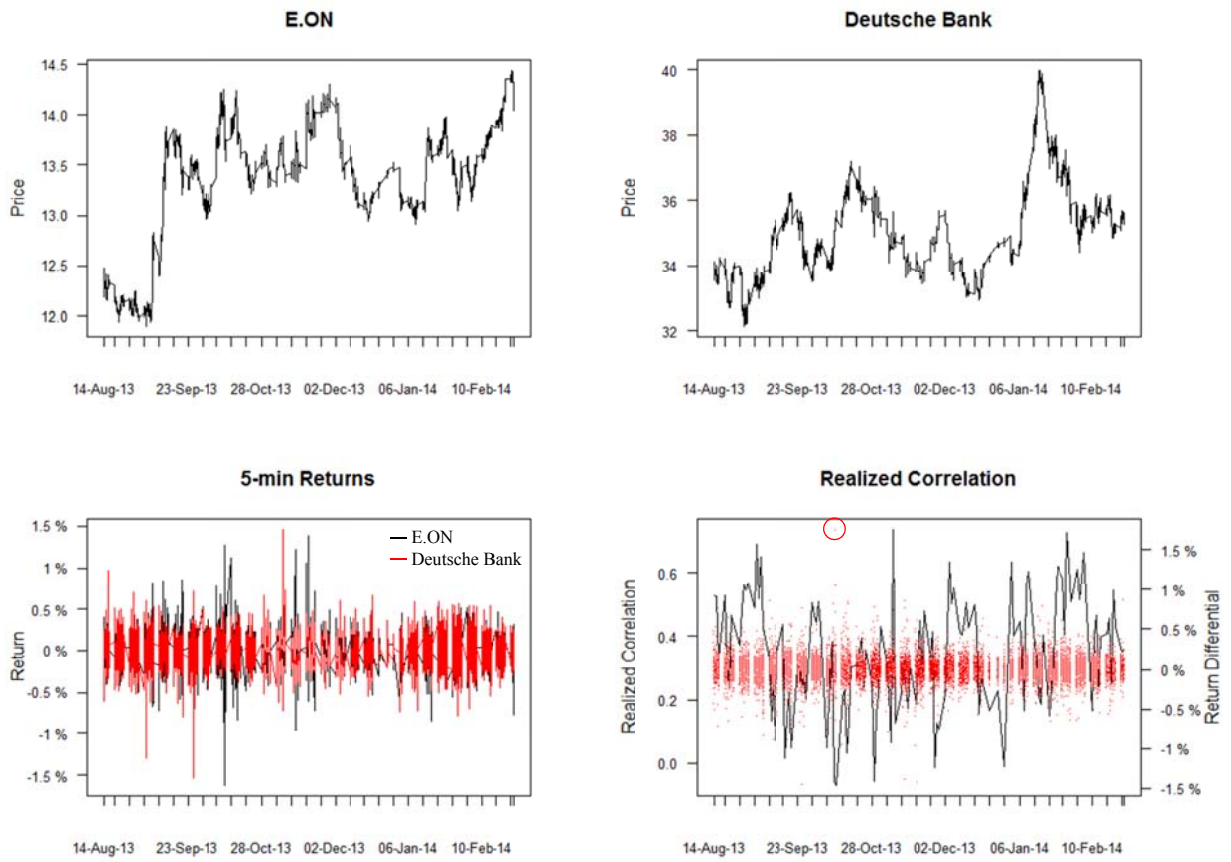


Figure 1.6. Outliers: realized correlation and return differential

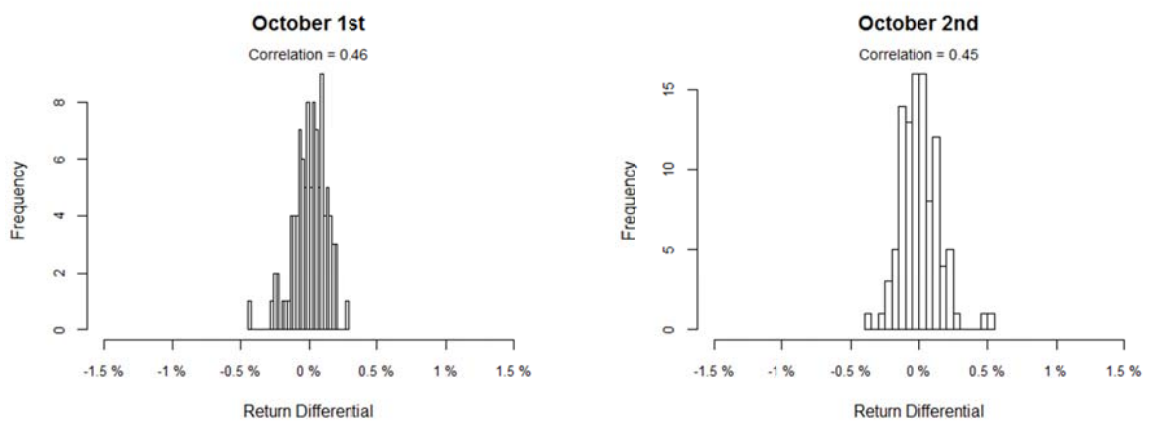


Figure 1.7. Outlier sources: days without outliers

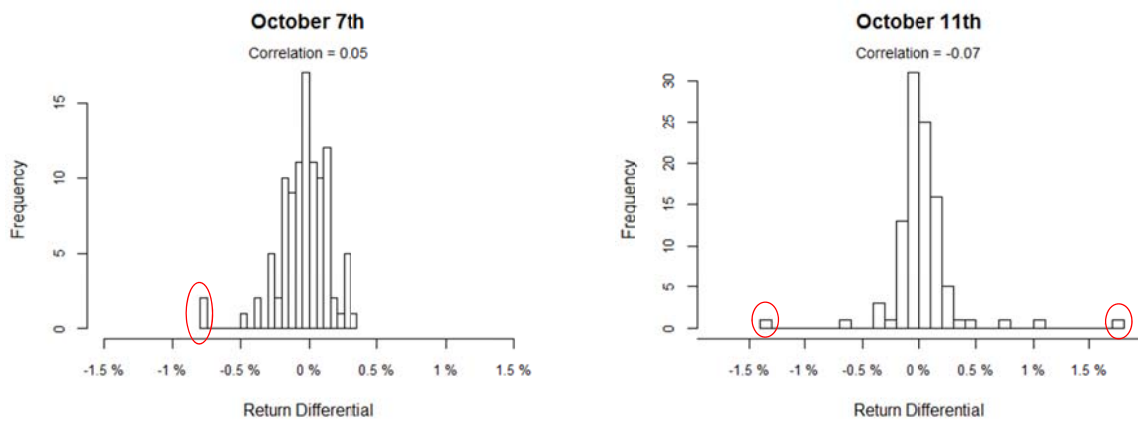


Figure 1.8. Outlier sources: days with outliers

Figures 1.6 through 1.8 illustrate what effect one-off events (i.e. outliers) can potentially have to correlation estimate. When not properly accounted for, such events will push correlations toward zero when once-off events affect only one asset and towards +1/-1 when the effect is simultaneously similar/opposite.

## 1.8. Conclusions

In the current chapter, six stylized facts about correlation dynamics were discussed. Facts considered were time trend in correlations, mean reversal, correlation breakdown, correlation asymmetry, correlation persistence and outliers. Based on illustrative evidence, support was found for the increasing correlations between global equity returns as well as for the mean reversal. Additionally, based on conditional correlation analysis, correlations do appear to break when markets experience turbulence. However, as it was shown with Monte Carlo simulation, this break is also evident in theoretical correlations. Some support was found that negative shocks will cause correlations to increase more than similar positive shocks. Investigating intraday high frequency data, support was also found for the persistence in correlation dynamics as well as the existence of outlier problem.

Next chapter will be dedicated to the introduction of various existing correlation models as well as to the investigation of how well these models meet the aforementioned requirements.

## 2. REVIEW OF CORRELATION MODELS

In Chapter 1 six stylized facts were introduced about correlation dynamics. In this chapter, theoretical concepts of correlation modelling together with formal definitions of eight correlation models will be given. The chapter proceeds as follows. In the first subsection, some notational aspects will be discussed that will be used throughout the rest of the thesis. In the second subsection, two technical considerations will be introduced that are of significant importance when modelling correlations. Subsections three through five will then provide theoretical definitions of the eight, in authors opinion most prominent, correlation models. In the last subsection, some concluding remarks regarding theoretical concepts of correlation modelling will be made.

### 2.1. Notation

As per Silvennoinen and Terasvirta (2008) and Bouwens et al. (2006) we can set the model up as follows. Consider a stochastic vector process  $\{ \mathbf{r}_t \}$  with dimension  $N \times 1$  such that  $E\mathbf{r}_t = 0$ . Let  $\mathcal{F}_{t-1}$  denote the information set generated by the observed series  $\{ \mathbf{r}_t \}$  up to and including time  $t - 1$ . We assume that  $\mathbf{r}_t$  is conditionally heteroskedastic:

$$\mathbf{r}_t = \mathbf{H}_t^{1/2} \boldsymbol{\eta}_t \quad (2.1)$$

given the information set  $\mathcal{F}_{t-1}$ , where the  $N \times N$  matrix  $\mathbf{H}_t = [h_{ij,t}]$  is the conditional covariance matrix of  $\mathbf{r}_t$  and  $\boldsymbol{\eta}_t$  is an iid vector error process such that  $E\boldsymbol{\eta}_t \boldsymbol{\eta}_t' = \mathbf{I}$ .  $\mathbf{H}_t^{1/2}$  is any  $N \times N$  positive definite matrix such that  $\mathbf{H}_t$  is the conditional variance matrix of  $\mathbf{r}_t$ , e.g.  $\mathbf{H}_t^{1/2}$  may be obtained by Cholesky factorization of  $\mathbf{H}_t$ . This defines the standard multivariate generalized autoregressive conditional heteroskedasticity (MGARCH) framework, in which there is no linear dependence structure in  $\{ \mathbf{r}_t \}$ .

In the following subsections a review of correlation models with different specifications of  $\mathbf{H}_t$  will be provided. Based on their specification, three nonmutually exclusive classifications of correlation models are provided: (i) naïve correlation models; (ii) models of conditional covariance matrix; and (iii) models of conditional variances and covariances. First category includes models such as Simple Moving Average correlation and RiskMetrics version of Exponentially Weighted Moving Average correlation. VEC<sup>6</sup>, BEKK<sup>7</sup> and Orthogonal GARCH models are in the second category. The last category contains constant and dynamic conditional correlation models.

## 2.2. Some technical considerations

Before the introduction of various model specifications, there are two additional technical considerations that need to be explained. Firstly, the covariance matrix of a correlation model needs to be positive semidefinite. When it is not the case and covariance matrix ends up being negative definite, the resulting aggregated asset portfolio variance will be negative. Negative variance however is not an economically realistic result.

There are couple of reasons that might result in covariance matrix not to be positive semidefinite. Firstly, when the number of historical observations is less than the number of assets. The most probable situation where it could happen is with MGARCH type models where the covariance matrix is estimated with decaying weights. Namely, when those weights are decaying too fast, it could make the number of effective historical observations too small for the covariance matrix to be positive semidefinite. Second reason for covariance matrix not to be positive semidefinite is when the asset returns are strongly linearly correlated with each other. (Jorion 2006)

Another technical consideration when modelling correlations is the dimensionality problem. Consider for example a portfolio of 100 assets. In order to estimate the covariance matrix, altogether 5 050 covariance and variance terms need to be estimated. Using for example VEC

---

<sup>6</sup> Named after *vech*(·) operation used in the model.

<sup>7</sup> Named after its authors Baba-Engle-Kraft-Kroner.



type MGARCH model for the estimation, further 51 010 050 model parameters need to be estimated. Latter makes the whole process infeasible for medium to large systems. So in order to make system feasible, some simplifying assumptions are often made for modelling purposes.

## 2.3. Naïve correlation models

Before the introduction of more sophisticated MGARCH type models, two so called naïve correlation estimation models will be introduced in this subsection. The models include equally weighted rolling average correlation (Simple Moving Average) and RiskMetrics version of Exponentially Weighted Moving Average correlation. The reason for describing the two aforementioned models separately is because of their relative simplicity compared to most MGARCH models and their wide use by industry practitioners.

### 2.3.1. Simple Moving Average

In bivariate case Simple Moving Average correlation (SMA) can be defined as follows (Engle 2002):

$$\rho_{12,t} = \frac{\mathbf{r}_{1,t-1}\mathbf{r}'_{2,t-1}}{\sqrt{(\mathbf{r}_{1,t-1}\mathbf{r}'_{1,t-1})(\mathbf{r}_{2,t-1}\mathbf{r}'_{2,t-1})}} \quad (2.2)$$

where  $\rho_{12,t}$  is the conditional correlation estimate made for time  $t$  on  $t-1$ .  $\mathbf{r}_{1,t-1}$  is a  $1 \times k$  column vector for asset 1 daily returns up to  $t-1$  so that  $\mathbf{r}_{1,t-1} = (r_{1,t-k-1}, \dots, r_{1,t-1})$  and  $\mathbf{r}_{2,t-1}$  is a  $1 \times k$  column vector for asset 2 daily returns up to  $t-1$  so that  $\mathbf{r}_{2,t-1} = (r_{2,t-k-1}, \dots, r_{2,t-1})$ .

In (2.2), the length of the rolling window  $k$  determines the degree of memory that is used in estimation. For example, assuming  $k = 252$  (1 year) and  $t = 1\ 008$  (4 years), correlation estimate will give an equal weight to all daily returns in year 4 and zero weight to all returns in the first three years.

The aforementioned feature is also the biggest critique against using the SMA to estimate correlation. Namely, by choosing a smaller  $k$  value to incorporate heteroskedasticity, the model

will disregard potentially important more distant historical observations. Alternatively, by choosing a high  $k$ , the model might average out the important heteroskedastic conditions of current state.

Coming back to the six stylized facts introduced in Chapter 1, considering a commonly used  $k$  value of 252 (one year), the model is incapable of handling neither correlation breaks, nor mean reversion or time trend. Furthermore, even though correlation estimate from SMA model is persistent, it most probably is persistent on a wrong level. Under no setting in (2.2) can we incorporate asymmetry nor can we avoid outliers. On the positive side, SMA estimated covariance matrix is guaranteed to be positive semi-definite and the model does not suffer from dimensionality problem.

### 2.3.2. RiskMetrics EWMA

RiskMetrics version of Exponentially Weighted Moving Average correlation (RiskMetrics EWMA) developed by RiskMetrics<sup>TM</sup> (RiskMetrics 1996), uses a decay factor  $\lambda$  to assign weight to historical observations. The weighting is done so that the importance of historical observations declines exponentially as a function of  $\lambda$ .

For  $N$  assets we can define exponentially smoothed correlation measure as follows (Andersen et al. 2007):

$$\mathbf{H}_t = \lambda \mathbf{H}_{t-1} + (1 - \lambda) \mathbf{r}_{t-1} \mathbf{r}'_{t-1} \quad (2.3)$$

where  $\mathbf{H}_t$  is the conditional covariance matrix estimated at  $t-1$  for time  $t$  and  $\mathbf{r}_{t-1}$  is an  $N \times 1$  row vector of daily returns for  $N$  assets. From (2.3) conditional correlation matrix  $\mathbf{R}_t$  can be derived as follows:

$$\mathbf{R}_t = (\mathbf{I} \odot \mathbf{H}_t)^{-1/2} \mathbf{H}_t (\mathbf{I} \odot \mathbf{H}_t)^{-1/2} \quad (2.4)$$

where  $\odot$  denotes Hadamard product (i.e. matrix elementwise product) and  $\mathbf{I}$  is a  $N \times N$  identity matrix with ones on the main diagonal and zeros everywhere else. As equation (2.4) can be used

whenever we need to transform conditional covariance matrix  $\mathbf{H}_t$  to a conditional correlation matrix  $\mathbf{R}_t$ , only  $\mathbf{H}_t$  will be formally defined in the following subsections.

RiskMetrics EWMA has two important advantages over the equally weighted model. First, correlation reacts faster to shocks in the market as recent data carry more weight than data in the distant past. Second, the model does not have a fixed observation termination point in the past. This means that the model is capable of incorporating all of the observable history into its estimate (although using standard decay factor of 0.94 is already very discriminating against the more distant history<sup>8</sup>). Furthermore, due to its simple structure, covariance matrix of RiskMetrics EWMA model is easily estimated, and provided that the decay factor is not too low and  $N$  is not too high, model is also guaranteed to be positive semidefinite (Andersen et al. 2007).

On the negative side, as the model imposes the same degree of smoothness on all elements of the estimated covariance matrix, the result can be potentially biased. This is confirmed by various studies which have found that the optimal decay factor varies both across assets and among asset classes. Moreover, similarity to SMA, RiskMetrics EWMA ignores correlation mean-reverting (Andersen et al. 2007) and time trend properties as well as asymmetry and possible outliers.

## 2.4. Models of conditional covariance matrix

### 2.4.1. VEC Model

VEC-GARCH model of Bollerslev, Engle, and Wooldridge (1988), is a generalization of the univariate GARCH model. Every conditional variance and covariance is a function of all lagged conditional variances and covariances, as well as lagged squared returns and cross-products of returns (Silvennoinen, Teräsvirta 2008).

The VEC(1,1) model is defined as follows (Bollerslev et al. 1988):

$$\text{vech}(\mathbf{H}_t) = \mathbf{c} + \mathbf{A} \text{vech}(\mathbf{r}_{t-1} \mathbf{r}'_{t-1}) + \mathbf{B} \text{vech}(\mathbf{H}_{t-1}) \quad (2.5)$$

---

<sup>8</sup> For example, when using annual daily return observations, the last six month of data will have a combined weight of 99.96% in the model.

where  $\mathbf{c}$  is an  $N(N+1)/2 \times 1$  vector, and  $\mathbf{A}$  and  $\mathbf{B}$  are  $N(N+1)/2 \times N(N+1)/2$  parameter matrices.  $\text{vech}(\cdot)$  operator converts the unique lower triangular elements of a symmetric matrix into a  $N(N+1)/2 \times 1$  column vector.

The generality of the VEC model is an advantage in the sense that the model is very flexible by allowing mean-reversion (intercept vector  $\mathbf{c}$ ) as well as correlation breaks and persistence. However such flexibility also brings disadvantages. One is that there exist only sufficient, rather restrictive, conditions for  $\mathbf{H}_t$  to be positive definite for all  $t$  (Silvennoinen, Teräsvirta 2008).

Another disadvantage of this model is that the dimensionality of the model increases exponentially. In fact the number of parameters is defined as  $(p+q)(N(N+1)/2)^2 + N(N+1)/2$ , where  $p$  and  $q$  are VEC lagged terms, i.e.  $\text{VEC}(p,q)$ <sup>9</sup>. In our  $\text{VEC}(1,1)$  specification, having a 100 asset portfolio ( $N=100$ ) results in 51 010 050 parameters to be estimated.

The dimensionality problem can be alleviated somewhat by assuming a diagonal matrix for both  $\mathbf{A}$  and  $\mathbf{B}$  (diagonal VEC, or DVEC) where each element  $\mathbf{h}_{ij,t}$  depends only on its own lag and on the previous value of  $\mathbf{r}_{t-1}\mathbf{r}'_{t-1}$  (Bauwens et al. 2006). Number of parameters to be estimated in DVEC decreases then to  $N(N+5)/2$  (e.g. for  $N=100$  number of parameters will be 5 250).

The DVEC(1,1) is defined as follows (Bauwens et al. 2006):

$$\mathbf{H}_t = \mathbf{C}^\circ + \mathbf{A}^\circ \odot (\mathbf{r}_{t-1}\mathbf{r}'_{t-1}) + \mathbf{B}^\circ \odot \mathbf{H}_{t-1} \quad (2.6)$$

where  $\mathbf{C}^\circ$ ,  $\mathbf{A}^\circ$ ,  $\mathbf{B}^\circ$  are symmetric  $N \times N$  matrices so that  $\mathbf{A} = \text{diag}[\text{vech}(\mathbf{A}^\circ)]$ ,  $\mathbf{B} = \text{diag}[\text{vech}(\mathbf{B}^\circ)]$  and  $\mathbf{c} = \text{diag}[\text{vech}(\mathbf{C}^\circ)]$ . As long as  $\mathbf{C}^\circ$ ,  $\mathbf{A}^\circ$ ,  $\mathbf{B}^\circ$  as well as initial covariance matrix  $\mathbf{H}_0$  are positive definite, then so is  $\mathbf{H}_t$  for all  $t$  (Bauwens et al. 2006).

Nevertheless, even in DVEC setting there are too many parameters to be jointly estimated, which is computationally infeasible in systems of medium and large size (Andersen et al. 2007). An even simpler version of VEC is called scalar VEC, or SVEC. SVEC model constrain  $\mathbf{A}^\circ$ ,  $\mathbf{B}^\circ$  matrices to be rank one matrices, or a positive scalar times the matrix of ones (Bauwens et al. 2006). The SVEC(1,1) can then be written as in (2.7).

---

<sup>9</sup> In equations (2.5)  $p=q=1$ , i.e.  $\text{VEC}(1,1)$ .

$$vech(\mathbf{H}_t) = \mathbf{c} + a(\mathbf{r}_{t-1}\mathbf{r}'_{t-1}) + b\mathbf{H}_{t-1} \quad (2.7)$$

As can be seen, RiskMetrics EWMA of (2.3) is a particular case of SVEC model where  $c = 0$ ,  $a = 1 - \lambda$  and  $b = \lambda$ . Latter demonstrates well one of the previously stated negative aspects of RiskMetrics EWMA model. Namely by setting  $c = 0$ , RiskMetrics EWMA model is not mean reverting.

In conclusion, when benchmarking VEC, DVEC and SVEC models against stylized facts of Chapter 1, all models can handle mean-reversion, correlation breaks and persistence (though on different levels, VEC being the most flexible). Nevertheless, none of the specifications of (2.5), (2.6) and (2.7) can handle time trend, asymmetry nor outliers.

#### 2.4.2. BEKK Model

Because it is difficult to guarantee the positivity of  $\mathbf{H}_t$  in the VEC representation without imposing strong restrictions on the parameters, Engle and Kroner (1995) propose a new parameterization for  $\mathbf{H}_t$  that guarantees its positivity, i.e. the Baba-Engle-Kraft-Kroner (or BEKK) model (Bauwens et al. 2006).

BEKK (1,1, $K$ ) model is (Engle, Kroner 1995):

$$\mathbf{H}_t = \mathbf{C}^{*'}\mathbf{C}^* + \sum_{k=1}^K \mathbf{A}_k^{*'}\mathbf{r}_{t-1}\mathbf{r}'_{t-1}\mathbf{A}_k^* + \sum_{k=1}^K \mathbf{B}_k^{*'}\mathbf{H}_{t-1}\mathbf{B}_k^* \quad (2.8)$$

where  $\mathbf{C}^*$ ,  $\mathbf{A}_k^*$  and  $\mathbf{B}_k^*$  are  $N \times N$  parameter matrices,  $\mathbf{C}^*$  is lower triangular and the  $\mathbf{H}_t$  summation limit  $K$  determines the generality of the process.

The number of parameters in the BEKK(1,1,1) model is  $N(5N + 1)/2$  (e.g. if  $N = 100$  then there are altogether 25 050 parameters to be estimated). To reduce this number, and consequently to reduce the generality, one can impose a diagonal BEKK model in which  $\mathbf{A}$  and  $\mathbf{B}$  in (2.8) are diagonal matrices. This model is also a DVEC model but it is less general, although it is guaranteed to be positive definite while the DVEC is not. (Bauwens et al. 2006)

The most restricted version of the diagonal BEKK model is the scalar BEKK with  $\mathbf{A} = a\mathbf{I}$  and  $\mathbf{B} = b\mathbf{I}$  where  $a$  and  $b$  are scalars (Silvennoinen, Teräsvirta 2008).

With regard to meeting the requirements established in Chapter 1, the outcome is the same as for VEC type models, namely all BEKK specifications (though on various levels) are able to handle mean reversion, correlation breaks and persistence.

### 2.4.3. Orthogonal GARCH

Orthogonal GARCH (O-GARCH) model of Alexander and Chibumba (1997) is a generalization of the factor GARCH model introduced by Engle, Ng and Rothschild (1990) to a multi-factor model with orthogonal factors. The O-GARCH model allows  $N \times N$  GARCH covariance matrices to be generated from  $m \leq N$  univariate GARCH models, where the  $m$  univariate models are based on principal components that are linearly independent of each other (Alexander 2001). Furthermore, Alexander (2001) notes that since only the univariate GARCH models are used, it does not really matter if  $m$  is significantly smaller than  $N$ .

With regards to the advantages of O-GARCH model, Alexander (2001) states that there are at least three major advantages in using principle components in covariance matrix estimation. First, in a highly correlated system, only a few principal components are required to represent the system variation to a very high degree of accuracy. Second, the covariance matrix that is constructed using the principal components method is guaranteed to be positive semidefinite. Third, O-GARCH method gives one the option of cutting out any 'noise' in the data that would otherwise make correlation estimates unstable.

The procedure to calculate conditional covariance matrix with O-GARCH model is to first construct unconditionally uncorrelated linear combinations of the series  $r$ . Then, as a second step, estimate univariate GARCH models for some or all of these series, and in a third and final step, to construct full covariance matrix by assuming the conditional correlations are all zero. (Engle 2002)

Thus the  $m \times m$  diagonal matrix of variances of the principal components is a time-varying matrix denoted  $D_t$  and the time-varying covariance matrix  $H_t$  of the original system is approximated by (Alexander 2001):

$$H_t = AD_tA' \quad (2.9)$$

where  $\mathbf{A}$  is the  $N \times m$  matrix of re-scaled factor weights and  $\mathbf{D}_t$  is a diagonal matrix of variances of the principal components estimated using a GARCH univariate model. Factor weights in (2.9) are eigenvalues of standardized  $\mathbf{\Gamma}^{N \times N} = \mathbf{r}_{t-1}^{(s)} \mathbf{r}_{t-1}^{(s) \prime}$ , where  $\mathbf{\Gamma}$  has zero mean and unit variance<sup>10</sup>, i.e.  $\mathbf{\Gamma}^{N \times N} \mathbf{W}^{N \times N} = \mathbf{W}^{N \times N} \mathbf{\Lambda}^{N \times N}$  where  $\mathbf{W}$  represents eigenvectors corresponding to  $\mathbf{\Gamma}$ , and  $\mathbf{\Lambda}$  is a diagonal matrix of eigenvalues of  $\mathbf{\Gamma}$ . Matrix of principal components  $\mathbf{P}^{N \times N}$  is thereafter calculated by multiplying the original return matrix  $\mathbf{\Gamma}$  and its eigenvalues based re-ordered matrix of eigenvectors  $\mathbf{W}^O$  so that  $\mathbf{P} = \mathbf{\Gamma} \mathbf{W}^O$ . Dimension of  $\mathbf{A}$  will be then set according to the variances of principal components. Namely, as the sum of variances of principal components equals the sum of individual  $N$  return series in  $\mathbf{\Gamma}$  (Tuckman, Serrat 2011) and knowing that we have standardized our variances to one, then the variance contribution is defined as the eigenvalue of  $i^{th}$  principal component divided by  $N$ .  $m$  will then be set according to the sum of first  $m$  principal component variances that explains sufficient amount of total variance of the system. The final matrix  $\mathbf{A}$  is then obtained by multiplying each factor weight by the corresponding standard deviation.

According to Alexander (2001) equation (2.9) will give a positive semidefinite matrix at every point in time for any size of  $m$ . On a negative side, Bauwens et al. (2006) argue that as the conditional variance matrix has reduced rank when  $m < N$ , it might cause problems for applications and for diagnostic tests which depend on the inverse of  $\mathbf{H}_t$ .

It was already established that O-GARCH meets the technical requirements of positivity and low dimensionality, but how does O-GARCH deal with the stylized facts introduced in Chapter 1? As with VEC and BEKK models, O-GARCH can handle mean-reversion, correlation breaks and persistence, but falls again short in coping with time-trend and asymmetry.

---

<sup>10</sup> Standardization of  $\mathbf{r}_{t-1}$  to get  $\mathbf{r}_{t-1}^{(s)}$  is done by subtracting sample mean from  $\mathbf{r}_{t-1}$  and dividing the result by  $\sqrt{T}$  times the sample standard deviation.

## 2.5. Models of conditional variances and covariances

### 2.5.1. Constant Conditional Correlation

Bollerslev (1990) proposes a class of MGARCH models in which the conditional correlations are constant and thus the conditional covariances are proportional to the product of the corresponding conditional standard deviations (Constant Conditional Correlation, or CCC model). This restriction greatly reduces the number of unknown parameters and thus simplifies the estimation (Bauwens et al. 2006).

The CCC model is defined as (Bollerslev 1990):

$$\mathbf{H}_t = \mathbf{D}_t \mathbf{R} \mathbf{D}_t \quad (2.10)$$

where  $\mathbf{D}_t$  denotes the  $N \times N$  stochastic diagonal matrix with elements  $\sqrt{h_{iit}}, \dots, \sqrt{h_{NNt}}$  and  $\mathbf{R}$  is a positive definite  $N \times N$  time invariant matrix of correlations  $[\rho_{ij}]$  where  $\rho_{ii} = 1, i = 1, \dots, N$ . Each element  $\sqrt{h_{iit}}$  in  $\mathbf{D}_t$  is estimated via GARCH(1,1) specification<sup>11</sup>:

$$h_{iit} = \omega_i + \alpha_i r_{i,t-1}^2 + \beta_i h_{i,t-1} \quad (2.11)$$

Under GARCH(1,1) specification, the CCC model contains  $N(N+5)/2$  parameters. According to Bauwens et al. (2006), when all  $N$  conditional variances in (2.10) are positive and  $\mathbf{R}$  is a positive definite matrix, then  $\mathbf{H}_t$  is guaranteed to be positive definite as well.

Even though the assumption of constant conditional correlation can be too restrictive, the model has some good features that none of the previous models had. Namely, the model has separated volatility dynamics from correlation dynamics. Latter gives us the flexibility to use vast amount of valuable knowledge regarding univariate volatility dynamics. For instance, we can now incorporate volatility asymmetry (for example by using GJR-GARCH<sup>12</sup> type model instead of standard GARCH model). Nevertheless, considering that the conditional correlation in CCC

---

<sup>11</sup> The model can also be extended to GARCH( $p, q$ ) specification (see Silvennoinen, Teräsvirta 2008).

<sup>12</sup> GJR-GARCH model after the names of its authors (Glosten, Jagannathan and Runkle) introduces different weighting schemes for negative and positive shocks taking into account the leverage effect (Ali 2013).



model is by definition constant, the model is not fulfilling none of the correlation dynamics requirements established in Chapter 1.

### 2.5.2. Dynamic Conditional Correlation

Engle (2002) proposes an estimator called Dynamic Conditional Correlation model or DCC. The DCC model of Engle differs from CCC model in allowing  $\mathbf{R}$  to be time varying giving a model (Engle 2002):

$$\mathbf{H}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t \quad (2.12)$$

where

$$\mathbf{R}_t = (\mathbf{I} \odot \mathbf{Q}_t)^{-1/2} \mathbf{Q}_t (\mathbf{I} \odot \mathbf{Q}_t)^{-1/2} \quad (2.13)$$

$$\mathbf{Q}_t = (1 - \alpha - \beta) \mathbf{S} + \alpha \boldsymbol{\varepsilon}_{t-1} \boldsymbol{\varepsilon}'_{t-1} + \beta \mathbf{Q}_{t-1} \quad (2.14)$$

with  $\boldsymbol{\varepsilon}_t$  defined as the vector of scaled residuals (i.e.  $\varepsilon_{i,t} = \frac{r_{i,t}}{\sqrt{h_{i,t}}}$ ) and  $\mathbf{S}$  is set to the unconditional covariance matrix. Because  $\alpha$  and  $\beta$  are scalars, all conditional correlations obey the same dynamics (Jorion 2006). This is necessary to ensure that  $\mathbf{R}_t$  is positive definite.

If the conditional variances are specified as GARCH(1,1) models then DCC model contains  $(N+1)(N+4)/2$  parameters (Bauwens et al. 2006).

DCC model is designed to allow for two-stage estimation of the conditional covariance matrix  $\mathbf{H}_t$ : in the first stage univariate volatility models are fitted for each of the assets and estimates of  $h_{i,t}$  are obtained; in the second stage asset returns, transformed by their estimated standard deviations resulting from the first stage are used to estimate the parameters of the conditional correlation.

Even though two-stage estimation is far more efficient than the to one-stage estimation of VEC and BEKK models, Engle and Kelly (2012) argue that the estimation of DCC model parameters becomes increasingly cumbersome as the size of the system grows. In fact Engle and Kelly (2012) also note that the DCC model of Engle has only been successfully applied to up to 100 assets.

When benchmarking DCC model against the six stylized facts, it can be shown that the model satisfies the mean-reverting properties as well as it can handle correlation breaks and persistence. Also, by using asymmetric univariate GARCH model (such as GJR-GARCH) in the first estimation phase, it is possible to add asymmetry into volatility estimate (but not into correlation estimate). Neither time trend nor outliers are handled by DCC model.

### 2.5.3. Asymmetric Dynamic Conditional Correlation

As explained in Chapter 1, conditional estimates of the second moments of certain assets often exhibit asymmetric phenomenon, where volatilities as well as correlations increase more after a negative shocks than after positive shocks of the same magnitude. Furthermore, in the last subsection it was argued that even though DCC model is capable of handling asymmetric univariate volatility dynamics, it is still not able to handle asymmetry in correlation dynamics.

An Asymmetric Dynamic Conditional Correlation (or ADCC) extends the DCC model by accounting for asymmetries in the correlation dynamics through the additional term  $\gamma(\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}'_{t-1} \odot \mathbf{1}_{\varepsilon_{t-1}<0}\mathbf{1}'_{\varepsilon_{t-1}<0})$  in (2.14) where  $\mathbf{1}_{\varepsilon_{t-1}<0}$  is a vector of dimension  $N$  such that  $[\mathbf{1}_{\varepsilon_{t-1}<0}]_i = 1$  if  $\varepsilon_{i,t-1} < 0$  and 0 otherwise (Laurent et al. 2010). ADCC model can then be defined as follows:

$$\mathbf{Q}_t = (1 - \alpha - \beta)\mathbf{S} + \alpha\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}'_{t-1} + \gamma(\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}'_{t-1} \odot \mathbf{1}_{\varepsilon_{t-1}<0}\mathbf{1}'_{\varepsilon_{t-1}<0}) + \beta\mathbf{Q}_{t-1} \quad (2.15)$$

Conditional correlation matrix  $\mathbf{R}_t$  and conditional covariance matrix  $\mathbf{H}_t$  is then calculated according to equation (2.13) and (2.12) respectively.

Additionally to satisfying the properties of mean-reversion and persistence together with the ability to handle correlation breaks, the ADCC model is also capable of handling asymmetry in correlation dynamics. Latter makes ADCC model out of the models introduced the most flexible with respect to stylized facts established in Chapter 1.

## 2.6. Conclusions

In this chapter, formal definitions of eight correlation models were given. Based on model complexity and estimation routine, the models were allocated into three categories: naïve models, models of conditional covariance matrix and the models of conditional variances and covariances. Within the naïve models class, Simple Moving Average and RiskMetrics Exponentially Weighted Moving Average correlation models were introduced. It was argued that even though naïve models are easy to use and the resulting covariance matrix is guaranteed to be positive semidefinite, they lack flexibility needed to incorporate most of the requirements established in Chapter 1. The second class of models, models of conditional covariance matrix, included VEC, BEKK and Orthogonal GARCH. These models are far more flexible than the naïve models, but at the expense of increased dimensionality and the fact that the resulting covariance matrix might not always be positive semidefinite (VEC, BEKK). In the third class, models such as Constant Conditional Correlation (CCC), Dynamic Conditional Correlation (DCC) and Asymmetric Dynamic Conditional Correlation (ADCC) models were introduced. Compared to VEC, BEKK and O-GARCH models in which the estimation takes place in one step, in CCC, DCC and ADCC the estimation is done in two stages, where in the first stage conditional volatilities are estimated and in the second stage, based on the newly standardized residuals, conditional correlations are estimated. The two-step estimation procedure both simplifies the estimation routine as well as guarantees the positiveness of estimated covariance matrix.

In addition to the eight correlation models introduced, there are many other correlation models that were not formally defined in this chapter. One such model, that will also be mentioned in Chapter 3, is BIP-cDCC. BIP-cDCC model of Boudt et al. (2013) is an extension of the DCC model with the ability to handle outliers. Additionally, there are Copula GARCH type models, multivariate versions of stochastic volatility models and realized volatility models. With regard to the latter, one multivariate realized volatility model was already introduced in Chapter 1. More will be said about this family of models in Chapter 3.

### **3. CORRELATION MODEL FORECASTING PERFORMANCE. A LITERATURE REVIEW**

In the previous chapter, various multivariate volatility models were introduced. It was shown that multivariate volatility models can range from relatively easy configurations to highly complex ones. This chapter will be dedicated to the literature review on the forecasting ability of the aforementioned models, with the goal to clarify which models are superior in their predictive ability as well as to find out whether this increased predictive ability is accompanied by unreasonable computational cost. Multivariate volatility model forecasting performance in this chapter will be evaluated based on its covariance forecast  $\mathbf{H}_t$ . This means that the results will be influenced by covariances as well as volatilities. However, since all correlation models introduced in Chapter 2 were multivariate models with correlations derived directly from  $\mathbf{H}_t$  estimate, then by using covariance matrix we are able to assess the model performance correctly. Further evaluation on correlation forecast performance can then be made by controlling for how the univariate models have been specified<sup>13</sup>. Before going into the empirical findings, certain methodological aspects will be touched upon to make sure that the results can be considered as robust and comparable.

#### **3.1. Loss Functions**

It is well known that in case of correlation we are dealing with an unobservable variable. This makes it difficult to benchmark forecasting model result against realized outcome. In order to overcome this difficulty, two sets of so called loss functions have been proposed in the litera-

---

<sup>13</sup> For example, in case of models with conditional variances and covariances (models such as CCC, DCC and Asymmetric DCC) we can assess correlation estimates by using the same specification to model univariate volatility [this is done in Chapter 4, where the univariate volatility is modelled with the GARCH(1,1) specification].

ture. First set of loss functions are the ones that try to evaluate the goodness of correlation model covariance forecasts directly. This class of functions are called direct or statistical loss functions and include measures such as mean squared error, quasi-likelihood, forbenius distance and others. Second set of loss functions include the ones that try to evaluate the goodness of the models via some underlying economic consideration indirectly. This class of functions are referred to as indirect or economic loss functions and include functions based on (global) minimum variance portfolio, value at risk, tracking error, utility from the returns to the minimum variance portfolio and others.

Various empirical research papers have used many different statistical and economic loss functions to assess the relative predictive ability of various multivariate volatility models. It is therefore critical to make sure that the results provided by those various loss functions are comparable and robust. This section will provide an overview of the most widely used robust statistical and economic loss functions. The list draws on the research by Patton and Sheppard (2007), Clements et al. (2009) and Laurent et al. (2013).

### 3.1.1. Statistical loss functions

Given forecasting errors  $\widehat{\Sigma}_t - \mathbf{H}_t$ , the family of consistent loss functions is defined as follows (Laurent et al. 2013):

$$L_t(\widehat{\Sigma}_t, \mathbf{H}_t) = \text{vech}(\widehat{\Sigma}_t - \mathbf{H}_t)' \widehat{\Lambda} \text{vech}(\widehat{\Sigma}_t - \mathbf{H}_t) \quad (3.1)$$

where  $\mathbf{H}_t$  is the covariance forecast,  $\widehat{\Sigma}_t$  is an observable covariance proxy for true unobservable conditional covariance matrix  $\Sigma_t$  and  $\widehat{\Lambda}$  is a matrix of weights that defines the relative importance of the forecasting errors in  $\widehat{\Sigma}_t - \mathbf{H}_t$ . As a reminder,  $\text{vech}(\cdot)$  is an operator that converts the unique lower triangular elements of a symmetric matrix into a  $N(N+1)/2 \times 1$  column vector.

Based on the aforementioned quadratic form, mean squared error (MSE) loss function is defined as follows (Clements et al. 2009):

$$L_t^{MSE}(\widehat{\Sigma}_t, \mathbf{H}_t) = \frac{1}{N^2} \text{vech}(\widehat{\Sigma}_t - \mathbf{H}_t)' \text{vech}(\widehat{\Sigma}_t - \mathbf{H}_t) \quad (3.2)$$

Laurent et al. (2013) propose two additional distance measures that are based on forecasting error  $\widehat{\Sigma}_t - \mathbf{H}_t$  and weight matrix  $\widehat{\Lambda}$ .

First, Euclidean distance where  $\widehat{\Lambda} = \mathbf{I}_N$  is a general form of MSE. Only difference is that MSE is a mean loss per matrix element, whereas Euclidean loss is the sum of errors across whole error matrix.

$$L_t^{Euclidean}(\widehat{\Sigma}_t, \mathbf{H}_t) = \text{vech}(\widehat{\Sigma}_t - \mathbf{H}_t)' \text{vech}(\widehat{\Sigma}_t - \mathbf{H}_t) \quad (3.3)$$

It should be noted that the use of MSE and Euclidean loss functions will give exactly the same ranking in the evaluation of covariance forecast performance.

Second loss functions proposed by Laurent et al. (2013) is the Frobenius loss function.

$$L_t^{Frobenius}(\widehat{\Sigma}_t, \mathbf{H}_t) = \text{Tr}[(\mathbf{H}_t - \widehat{\Sigma}_t)'(\mathbf{H}_t - \widehat{\Sigma}_t)] \quad (3.4)$$

where  $\text{Tr}(\cdot)$  is a trace operator that sums up all the main diagonal elements.

Frobenius loss function in (3.4) is a result of setting  $\widehat{\Lambda} = \text{diag}(\text{vech}(\mathbf{V}))$ , where  $\mathbf{V}$  is a symmetric matrix with 1's on its main diagonal and 2's everywhere else. This allows to assign double weights on covariance forecast errors.

Alternative robust loss function specification from (3.1) is a quasi-likelihood (QLIKE) function as defined in (3.5).

$$L_t^{QLIKE}(\mathbf{H}_t) = \log|\mathbf{H}_t| - \boldsymbol{\varepsilon}_t' \mathbf{H}_t^{-1} \boldsymbol{\varepsilon}_t \quad (3.5)$$

where  $\boldsymbol{\varepsilon}_t$  is an  $N \times 1$  vector of  $N$  asset disturbances, so that  $\mathbf{r}_t = \boldsymbol{\mu}_t + \boldsymbol{\varepsilon}_t$  and  $\boldsymbol{\varepsilon}_t \sim \mathbf{F}(0, \boldsymbol{\Sigma}_t)$ . In Chapter 2 the assumption was made that the expected return  $\mathbf{r}_t$  is zero. As in the real world it is not always the case (i.e. asset mean return  $\boldsymbol{\mu}_t \neq 0$ ), zero-mean residuals  $\boldsymbol{\varepsilon}_t$  are used in (3.5)<sup>14</sup>. As

---

<sup>14</sup> In Chapter 4, AR(1) model will be used to generate zero mean residuals.

can be seen, quasi-likelihood function is not a distance measure. Nevertheless, it still allows different forecasts of  $\Sigma_t$  to be compared (Clements et al. 2009). In fact, according to Clements et al. (2009), likelihood based statistical measures outperform the distance based functions (in their case QLIKE outperformed MSE).

There are other statistical loss functions that are considered robust (such as Stein function that also accounts for asymmetry, Mahalanobis distance, etc.). Nevertheless, as the focus of this thesis is on correlation model evaluation and the aforementioned loss functions are somewhat less widely used in empirical research, the introduction of those models will remain out of scope.

### 3.1.2. Choosing a proxy for statistical loss functions

Equations (3.1) through (3.4) used an observable covariance proxy  $\hat{\Sigma}_t$  against which losses were measured. Covariance proxy  $\hat{\Sigma}_t$  was used instead of the true covariance matrix  $\Sigma_t$  because the true covariance matrix is unobservable. For this reason, various covariance proxies have been proposed in the literature, all derived from the intraday return observations. The following will give an overview on various covariance proxies that have been proposed in the literature.

Firstly, Andersen et al. (2003) argue that intraday returns sampled with 30 minute frequency can be used as a proxy for realized covariance. Realized Covariance (RCov) is then defined as the sum of the outer products of those intraday returns. Furthermore, they argue that when the sampling frequency is less than 30 minutes an increased microstructure noise might start to compromise the estimation of RCov. Laurent et al. (2013) argue however that when the quality of covariance proxy deteriorates (sampling frequency gets smaller), inferior correlation models might start to outperform models that would be otherwise preferred when more accurate proxy would be used. As a result sampling frequency between 5 and 20 minutes is suggested by Laurents et al. (2013) as the best compromise between loss of accuracy and noise caused by microstructure frictions.

In addition to realized covariance proxy of Andersen et al. (2003), various other proxies have been proposed. Firstly, in order to overcome the outlyingness problem described in Chapter 1, Berndorff-Nielsen and Sheppard (2004) proposed Realized BiPower Covariation (RBPCov)

which is a combination of conditionally normal component with time-varying covariance matrix and a jump component. Despite solving most of the outlier related bias, Boudt et al. (2011) point to some weaknesses in RBPCov. Namely, they pointed to the upward bias that is caused mainly by co-jumps in continuous returns. Also, they note that RBPCov matrices are not always positive semidefinite and correlation coefficients derived from covariance coefficients might therefore not lie between -1 and 1. To overcome latter deficiencies, Boudt et al. (2011) proposed Realized Outlyingness Weighted Covariation (ROWCov) measure that downweights returns of large outlyingness. Outlyingness itself in ROWCov is defined as having extreme value relative to its neighbouring values. According to Boudt et al. (2011) the measure is both more efficient and more robust to jumps than RBPCov.

### 3.1.3. Economic loss functions

Building on the Engle and Colacito (2006) and Patton and Sheppard (2007), Clements et al. (2009) propose two robust economic loss functions that are both based on minimum variance portfolio.

$$L_t^{MVP}(\mathbf{H}_t) = L_t^{GMVP}(\mathbf{H}_t) = \frac{1}{T} \sum_{t=1}^T \mathbf{w}_t' \mathbf{r}_t \mathbf{r}_t' \mathbf{w}_t \quad (3.6)$$

where weights for minimum variance portfolio loss function ( $L_t^{MVP}$ ) are defined in (3.7) and for global minimum variance portfolio loss function ( $L_t^{GMVP}$ ) in (3.8).

$$\mathbf{w}_t^{MVP} = \frac{\mathbf{H}_t^{-1} \hat{\boldsymbol{\mu}}_t}{\hat{\boldsymbol{\mu}}_t' \mathbf{H}_t^{-1} \hat{\boldsymbol{\mu}}_t} \boldsymbol{\mu}_0 \quad (3.7)$$

$$\mathbf{w}_t^{GMVP} = \frac{\mathbf{H}_t^{-1} \boldsymbol{\iota}}{\boldsymbol{\iota}' \mathbf{H}_t^{-1} \boldsymbol{\iota}} \quad (3.8)$$

where  $\boldsymbol{\mu}_0$  in (3.7) is the target return for the portfolio so that  $\boldsymbol{\mu}_0 = \mathbf{w}_t' \hat{\boldsymbol{\mu}}_t$ ,  $\boldsymbol{\iota}$  in (3.8) is an  $N \times 1$  unit vector,  $\mathbf{H}_t$  is a covariance estimate and  $\hat{\boldsymbol{\mu}}_t$  is an  $N \times 1$  vector of expected asset returns. As



can be seen, the only difference between  $\mathbf{w}_t^{MVP}$  and  $\mathbf{w}_t^{GMVP}$  is that the latter does not require any assumptions regarding  $\hat{\boldsymbol{\mu}}_t$ .

Lack of the aforementioned requirement might also be the reason why in empirical literature loss functions based on the global minimum variance portfolio are more widely used than the ones based on minimum variance portfolio. Engle and Sheppard (2008) for example used GMVP based loss function together with Diebold-Moreno test to compare the performance of covariance models in a setting of 50 sector indices belonging to a S&P 500 total market index. Clements et al. (2012) used GMVP based loss functions together with to assess the relative performance of covariance models in the context of larger dimensions (up to 200 instrument sample).

It is worthwhile to note that economic loss functions have one distinct advantage over most statistical loss functions. Namely, as the size of the covariance matrix increases, comparisons based on statistical loss functions that use intraday realized covariance as a proxy might become infeasible. Latter is due to the reason that when  $N$  gets bigger than the intraday sampling frequency  $T$ , covariance proxy might itself become negative-definite. Fortunately there are also some loss functions among the class of statistical loss functions that do not suffer from this problem (such as QLIKE).

### **3.2. Procedures for model comparison**

Once the losses have been properly calculated, the relative ranking of models can be established. Again there are various methods to pinpoint the models with superior predictive ability. Using the classification of Clements et al. (2009), the first such class of methods are for model pairwise comparison. This class includes methods such as Diebold-Mariano test and West test. In both tests, correlation models are evaluated in pairs, so effectively we need to perform more than one test to find out which models are superior to others (given that we have more than two models to compare). Second class of methods are able to compare more than two models at the time, but require a benchmark model against which all other models are evaluated. Reality Check of White and Superior Predictive Ability is among the ones belonging to this class of methods. On the positive side, the number of tests in this class of methods decreases to one. The negative as-

pect however is that we are left with a subjective choice on which correlation model to use as a benchmark. Lastly, Model Confidence Set (MCS) which itself is a modified version of Superior Predictive Ability test, can evaluate many models at one time and does not require a benchmark. Under MCS, the process starts with the full set of candidate models. MCS process then sequentially drops inferior models one by one until the null hypothesis that all the remaining models have equal predictive ability cannot be rejected (Laurent et al. 2010).

Due to its generality, MCS is by far the most widely used method in the empirical research on relative performance of MGARCH models.

### **3.3. Other considerations**

Some further considerations need to be made before making conclusions on empirical results of MGARCH model performances. Firstly, are the results based on in-sample or out-of-sample observations? If the first is true, the most flexible model almost always wins<sup>15</sup>. Good in-sample performance will not however guarantee good out-of sample performance as it might only be due to model overfitting. Secondly, as both volatilities and correlations are influenced by return heteroskedasticity, it could mean that the correlation model might have different forecasting powers under different states of market volatility. Therefore, it would be useful to divide the full observation sample into low and high volatility subsamples. Thirdly, type of asset as well as liquidity might influence covariance model out-of-sample performance. Fourthly, as shown in the last chapter, most MGARCH models are difficult to use in larger dimensions. Considering the aforementioned issues, subsequent analyses will be carried out based on out-of-sample results for various asset types and in both high/low volatility as well as small/large dimensional settings.

---

<sup>15</sup> In the context of correlation models, the flexibility can be defined as the number of estimated parameters included in the the model. For example, with BEKK type models introduced in Chapter 2, full BEKK model was the most flexible, followed by diagonal BEKK and then by scalar BEKK as the least flexible model. Also, as none of the loss functions defined in Subsection 3.1.1 are able to punish the unnecessary flexibility, then added flexibility is always rewarded in case of in-sample model evaluation.

### 3.4. Empirical results

Even though there are various studies that evaluate correlation model in-sample performance<sup>16</sup>, there are only a handful of studies that deal with the evaluation of correlation model out-of-sample performance. Furthermore, as the area of correlation model out-of-sample performance evaluation has just recently been receiving focus, then all the empirical studies used in the following section ended up being published within the last five years<sup>17</sup>. Selection of empirical surveys used in this chapter was further limited to include sections that follow covariance model evaluation best practices established in sections 3.1 through 3.3 (see Table 3.1 for further details).

Table 3.1. Robust out-of-sample model performance evaluation requirements for loss functions, covariance proxies and estimation sampling

Category	Method	Abbreviation
Loss function	Mean Squared Error as in equation (3.2)	MSE
	Euclidean distance as in equation (3.3)	Euclidean
	Frobenius distance as in equation (3.4)	Frobenius
	Quasi-likelihood function as in equation (3.5)	QLIKE
	Minimum variance portfolio as in equations (3.6) and (3.7)	MVP
	Global minimum variance portfolio as in equations (3.6) and (3.8)	GMVP
Covariance proxies	Realized covariance of Andersen et al. (2003) with sampling frequency of 30 minutes or higher	RCov
	Realized bipower covariance of Berndorff-Nielsen and Sheppard (2004) with sampling frequency of 30 minutes or higher	RBPCov
	Realized outlyingness weighted covariance of Boudt et al. (2011) with sampling frequency of 30 minutes or higher	ROWCov
Performance evaluation	Out-of-sample	
Market state	Low volatility sub-sample; High volatility sub-sample	
Dimension	Small sample: 1 to 10 instruments, Large sample: 50 to 200 instruments	
Asset types	Equities, Currencies, Multi asset	

Source: Authors compilation

<sup>16</sup> Among others, Engle (2002) evaluates DCC model in-sample performance against SMA, EWMA, O-GARCH, Scalar BEKK and Diagonal BEKK using analytical functions (constant, sine, step and ramp functions); Engle and Colacito (2006) evaluate in-sample performance of Scalar BEKK, Diagonal BEKK, O-GARCH, DCC and Asymmetrical DCC using minimum variance portfolio based loss function and Diebold-Mariano test.

<sup>17</sup> The lack of research in this field was also confirmed by the studies that were included in the empirical literature review section.

The survey is divided into two parts. In the first part, overview will be given about the correlation model relative performance in a smaller dimensional setting. The second part will provide an overview on correlation model relative performance in larger dimensions.

The separation into small and large samples is important for two reasons. Firstly, in larger dimensions some correlation models simply become time wise too costly to use. Secondly, positive-definiteness of realized covariance proxies cannot be assured when the number of assets increases above the sampling frequency of the covariance proxy. Latter in turn limits the loss functions that can be used in the assessment of model relative performance in case of larger samples. Based on the available groupings in different empirical surveys, small sample will be limited to 10 or less instruments. Large sample spans from 50 to 200 instruments.

Full list of surveys included in the empirical literature review together with relevant configurations is provided in Appendix 1.

### 3.4.1. Model forecasting performance in smaller dimensions

In this section overview will be given on covariance model performances in a smaller dimensional setting. There were altogether six empirical studies that provided results for samples of less than or equal to 10 instruments (see Table 3.2 for complete list). Five of the studies also provided separate results for low volatility and high volatility periods. Even though Laurent et al. (2013) provided results only for the full sample, this full sample was classified as a low volatility sample [the results for EURUSD and JPYUSD sample volatilities almost exactly matched those of Boudt et al. (2013) low volatility sample volatilities]. List of surveys used together with respective low and high volatility period definitions as well as covered sample sizes are presented in Table 3.2.

Table 3.2. Empirical studies on covariance model relative performance in smaller samples

Survey	Low volatility period	High volatility period	Sample size
Laurent et al. 2010	Apr 2003 – Jul 2007	Jan 1999 – Mar 2003 Aug 2007 – Dec 2008	10
Caporin and McAleer, 2012	2006	Apr 2008 – Mar 2009	5-10
Clements et al. 2012	2003-2007	2001 – 2002 2008 – 2009	5,10

Boudt, et al. 2013	2004 - 2006	2006 - 2009	2
Laurent et al. 2013	Jan 2002 – Sep 2004	na	3
Clements et al. 2009	Before Financial crisis	Financial crisis of 2008	5

Source: Authors compilation

### 3.4.1.1. Low volatility period

Out of the five studies for small low volatility samples three covered equities, one currencies and one was a mix of different asset types. Summary results for low volatility periods are presented in Table 3.3.

Table 3.3. Small low volatility sample one day ahead forecast results for selected covariance models

	Laurent et al. 2010	Caporin and McAleer 2012	Clements et al. 2012	Boudt et al. 2013	Clements et al. 2009
Loss Function	MSE	MSE	GMVP	$\sum_{1 \leq i, j \leq N}  \sigma_{i,j,t} - h_{i,j,t} $	QLIKE
Proxy	RCov <sub>5min</sub>	RBPCov <sub>5min</sub>		ROWCov <sub>30min</sub>	
EWMA <sup>RM</sup>	out	out		2	out
EWMA <sup>FKO</sup>			10		5
SBEKK	10			out	
DBEKK	out			out	
BEKK		out			
CCC	10	5, 6, 7, 8, 9, 10		out	out
DCC	10	5, 6, 7, 8, 9, 10	5	out	5
BIP-DCC				2	
cDCC		7, 8, 9, 10		out	
BIP-cDCC				2	
ADCC	10	7, 8, 9, 10			5
OGARCH	10	5, 9, 10			
DECO	10		out		

The result cells in Table 3.3 can be interpreted as follows: when cell is empty, given row model was not included in the study; when 'out' is displayed, the model was not included in the final model confidence set ( $\alpha=25\%$ ); when number(s) are displayed, number(s) indicate instrument samples that were included in the final model confidence set ( $\alpha=25\%$ ). Source: Authors compilation based on Laurent et al. 2010, Caporin and McAleer 2012, Clements et al. 2012, Boudt et al. 2013 and Clements et al. 2009

As Laurent et al. (2013) was the only study that did not provide results at 25% significance level, their results were omitted from Table 3.3. Nevertheless, at 10% significance level and using 5 minute RCov as a covariance proxy in equation (3.4), authors were able to narrow model confidence set down to only CCC model (with symmetric and asymmetric univariate volatility dynamics).

Based on results in Table 3.3 and of Laurent et al. (2013) couple conclusions can be drawn. Firstly, simple models such as RiskMetrics version of EWMA ( $EWMA^{RM}$ ) and different BEKK type models tend to underperform DCC type models under low volatility conditions. Only „simple“ model that does well is the Fleming-Kirby-Ostdiek version of EWMA ( $EWMA^{FKO}$ )<sup>18</sup>. The finding that  $EWMA^{FKO}$  strongly outperforms  $EWMA^{RM}$  is somewhat interesting as former is an extension of latter. For direct comparison, we can analyze results from Laurent et al. (2010) and Clements et al. (2012)<sup>19</sup>. Laurent et al. (2010) found DCC and DECO models to be superior to  $EWMA^{RM}$ , whereas Clements et al. 2012 found  $EWMA^{FKO}$  to be equally good to DCC and superior to DECO. As  $EWMA^{RM}$  and  $EWMA^{FKO}$  are similar in all respects other than the value of decay factor, it could mean that with careful selection of decay factor, simple EWMA methods can potentially be as good as more sophisticated dynamic conditional correlation models.

There are also some interesting differences between equity and other asset class model performances. Based on the their results, Laurent et al. (2010) state that over low volatility periods assumption of constant conditional correlation and symmetry cannot be rejected. This is also confirmed by Caporin and McAleer (2012) for equities (see Table 3.3) and Laurent et al. (2013) for currencies. Nevertheless, the hypothesis of constant conditional correlation in low volatility setting is rejected by Boudt et al. (2013) for currencies as well as by Clements et al. (2009) for multi asset class portfolio. The hypothesis of symmetry however tends to hold for all asset classes.

Considering the results for different asset classes, DCC type models tend to outperform simple models over low volatility periods with the exception of  $EWMA^{FKO}$ . Nevertheless, the results are still quite mixed as the same models receive dissimilar rankings in different studies.

---

<sup>18</sup> Fleming-Kirby-Ostdiek version of EWMA is an extension of RiskMetrics EWMA model defined as  $\mathbf{H}_t = \exp(-\alpha)\mathbf{H}_{t-1} + \alpha\exp(-\alpha)\mathbf{r}_{t-1}\mathbf{r}'_{t-1}$ , where  $\exp(-\alpha)$  is a rate of decay similar to  $\lambda$  in RiskMetrics EWMA (Clements et al. 2009).

<sup>19</sup> Both used US equity return samples during the period of 2003 – 2007.

### 3.4.1.2. High volatility period

Summary results for small high volatility period relative model out-of-sample performance are provided in Table 3.4.

Table 3.4. Small high volatility sample one day ahead forecast results for selected covariance models.

	Laurent et al. 2010	Caporin and McAleer, 2012	Clements, et al. 2012	Boudt et al. 2013	Clements et al. 2009
Loss Function	MSE	QLIKE	GMVP	$\sum_{1 \leq i, j \leq N}  \sigma_{i,j,t} - h_{i,j,t} $	QLIKE
Proxy	RCov <sub>5min</sub>			ROWCov <sub>30min</sub>	
EWMA <sup>RM</sup>	10	out		out	out
EWMA <sup>FKO</sup>			5, 10   5, 10		out
SBEKK	out			out	
DBEKK	out			out	
BEKK		out			
CCC	10	out		out	out
DCC	10	5, 6, 7	out   5	out	out
BIP-DCC				out	
cDCC		5, 6, 7, 8, 9, 10		out	
<b>BIP-cDCC</b>				2	
<b>ADCC</b>	10	5, 6, 7, 8, 9, 10			5
OGARCH	10	out			
DECO	10		out   out		

The result cells in Table 3.4 can be interpreted as follows: when cell is empty, given row model was not included in the study; when ‘out’ is displayed, the model was not included in the final model confidence set ( $\alpha=25\%$ ); when number(s) are displayed, number(s) indicate instrument samples that were included in the final model confidence set ( $\alpha=25\%$ ); when ‘|’ is used, two separate high volatility sample results have been reported. Source: Authors compilation based on Laurent et al. 2010, Caporin and McAleer 2012, Clements et al. 2012, Boudt et al. 2013 and Clements et al. 2009.

Similarly to low volatility sample results, DCC type specifications tend to outperform other models under high volatility conditions. Additionally, all studies that included ADCC as a competing model, ended up having ADCC also in a final model confidence set on 25% significance level. ADCC performed equally well in equity and in multi asset samples whereas outlier adjusted version of cDCC (BIP-cDCC) was the best performer in currency samples. As the

aforementioned model has not been used in other studies, it is hard however to assess its relative performance under other asset type settings.

While the results for the hypothesis of constant conditional correlation were mixed during low volatility periods, the hypothesis of constant conditional correlation is rejected for all asset classes during high volatility periods. Furthermore, contrary to the low volatility period result in which ADCC model performance was not statistically superior to its symmetric counterpart DCC, it is evident that asymmetry improves model out-of-sample performances during high volatility periods<sup>20</sup>. This is in agreement with the asymmetry related stylized fact introduced in Chapter 1, according to which both correlations and volatilities tend to increase more during down-markets (when volatility is high) than during the up-markets (when volatility is relatively low). We can therefore conclude that the hypothesis of covariance symmetry can be rejected during high volatility periods.

#### **3.4.1.3. Overall conclusions for smaller dimensions**

Based on low and high volatility period empirical survey results, the following conclusions can be drawn for smaller samples. Firstly, DCC type models tend to outperform the rest across different volatility periods as well as asset type specifications. Secondly, assumption of constant conditional correlation can be rejected during high volatility periods, whereas results for low volatility period are somewhat mixed. Thirdly, the assumption of symmetry can be rejected during high volatility periods, but not during low volatility periods. Fourthly, there is no significant difference in relative model performances between various asset types.

#### **3.4.2. Model forecasting performance in larger dimensions**

In this section overview will be given on correlation model performances in a larger dimensional setting. There were two empirical studies that provided results for samples of more

---

<sup>20</sup> In addition to ADCC outperformance in multi asset sample of Clements et al. (2009), both DCC model in Laurent et al. (2009) and DCC/cDCC models in Caporin and McAleer (2012) had been modelled with asymmetric univariate GARCH models.



than or equal to 50 instruments<sup>21</sup>. One of the studies provided separate results for both low and high volatility periods whereas the other gave formal results only for the latter. List of surveys used, together with respective low and high volatility period definitions as well as covered sample sizes are presented in Table 3.5.

Table 3.5. Empirical studies on covariance model relative performance in larger samples

Survey	Low volatility period	High volatility period	Sample size
Caporin and McAleer, 2012	na	Apr 2008 – Mar 2009	50, 60, 70, 80, 89
Clements et al. 2012	2003-2008	2001 – 2002 2008 – 2009	50, 100, 200

Source: Authors compilation

Summary results for both low and high volatility periods are presented in Table 3.6.

Table 3.6. Large low volatility sample one day ahead forecast results for selected covariance models.

	Low volatility period	High volatility period	
	Clements et al. 2012	Caporin and McAleer 2012	Clements et al. 2012
Loss Function	GMVP	QLIKE	GMVP
EWMA <sup>RM</sup>		out	
<b>EWMA<sup>FKO</sup></b>	50, 100, 200		50,100,200   50,100,200
SBEKK			
DBEKK			
BEKK		out	
CCC		out	
DCC	out	out	out   200
BIP-DCC			
cDCC		out	
BIP-cDCC			
<b>ADCC</b>		50, 60, 70, 80, 89	
OGARCH		out	
DECO	out		out   out

The result cells in Table 3.6 can be interpreted as follows: when cell is empty, given row model was not included in the study; when ‘out’ is displayed, the model was not included in the final model confidence set ( $\alpha=25\%$ ); when number(s) are displayed, number(s) indicate instrument

<sup>21</sup> The number of studies that could have been used in this section was limited for two reasons. First, as previously explained, the number of studies dedicated to correlation model out-of-sample performance evaluation is itself limited and second, dimensionality problem in correlation modelling has made it extremely hard to carry out large scale modelling exercises (DCC model with 200 instruments took Clements et al. (2012) 287 hours to estimate).

samples that were included in the final model confidence set ( $\alpha=25\%$ ). Source: Authors compilation based on Caporin and McAleer 2012, Clements et al. 2012

Based on QLIKE loss function and 25% significance level two correlation models outperform others: ADCC with GJR-GARCH univariate dynamics (GJR-ADCC) and Fleming-Kirby-Ostdiek version of EWMA (EWMA<sup>FKO</sup>). Unfortunately, as GJR-ADCC and EWMA<sup>FKO</sup> did not directly compete with each other, the relative performance of these models cannot be evaluated. Furthermore, as correlation modelling literature defines large dimension normally starting from 100 instruments, then in this case EWMA<sup>FKO</sup> is strongly preferred. This is also confirmed by Clements et al. (2012) calculation cost results. Namely, the CPU time<sup>22</sup> for their 2 029 forecasts for 200 instrument sample took 01:17 (HH:MM) for EWMA<sup>FKO</sup>, 08:32 for DECO and 13:18 for DCC model. Furthermore the estimation (2 000 observations) for the same 200 instrument sample took DCC model 287 hours. This clearly shows the drawbacks of DCC type models: they become very expensive to use in case of larger dimensions.

### 3.5. Conclusions

The objective of the chapter was to provide a literature review on covariance model out-of-sample performance. Empirical literature covering various asset types, volatility states and sample sizes was used. In addition, further empirical survey filtering was carried out in order to control for robustness of empirical results via proper loss function and covariance proxy assumptions. Based on remaining empirical findings, the following conclusions were drawn. Firstly, in case of smaller samples DCC type of models are preferred. Secondly, during more volatile periods asymmetric version of DCC model should further improve model out-of-sample performance. Thirdly, in a larger samples DCC type models become computationally too costly to use with no additional performance gain relative to simple EWMA model of Fleming-Kirby-Ostdiek.

---

<sup>22</sup> Central Processing Unit. Clements et al. (2012) computer specification: 12 core 2.66GHz 64bit Inter Xeon processor.

It is also worth to note, that in general EWMA model of Fleming-Kirby-Ostdiek works relatively well in most conditions and is strongly preferred to its class member RiskMetrics EWMA<sup>23</sup>.

Some additional side-observations were made based on the selected empirical results. Firstly, during low volatility periods one cannot make strong conclusions on the effect of constant correlation. However, during the more volatile periods it was observed that the hypothesis of constant correlation is indeed rejected for all asset classes by all empirical results. Secondly, it was observed that covariance asymmetry becomes relevant only during the more volatile periods.

---

<sup>23</sup> Though it was also mentioned that this outperformance might only be due to better decay factor selection.

## **4. CORRELATION MODEL FORECASTING PERFORMANCE IN EMERGING MARKETS**

So far the thesis has researched and surveyed correlation dynamics and its modelling tools as follows. Firstly, in Chapter 1 some more important stylized facts about correlation dynamics were presented. These stylized facts were then used as theoretical requirements against which various correlation models were benchmarked in Chapter 2. In Chapter 3, using the existing empirical literature and the sample of correlation models introduced in Chapter 2, out-of-sample performance of these models was surveyed.

In this chapter, the one day ahead out-of-sample forecasting performance of nine correlation models will be evaluated in the context of emerging market equity and currency samples. The following research differs from the previous empirical research presented in Chapter 3 in three important ways. Firstly, contrary to the surveys introduced in the last chapter that all used developed market asset samples (in fact, all equity samples were built only from US domiciled assets), in this chapter more volatile emerging market data samples will be used<sup>24</sup>. Secondly, correlation model out-of-sample forecasting performance will be evaluated in case of two different asset classes during the same period. Previous studies have only concentrated on either equity sample, currency sample or a multi asset sample, but none have analyzed correlation model out-of-sample performance using various asset classes concurrently in the same market environment. Thirdly, currency pairs were chosen so that the correlation between pairs would on average be negative. Both Boudt et al. (2013) and Laurent et al. (2013) used developed market currencies that experience largely same correlation dynamics in various market states. In this study, two cur-

---

<sup>24</sup> The countries that will be included in the dataset are Poland and Czech Republic as emerging market countries and Switzerland as developed market country. Even though economic classification of Poland and Czech Republic (by GDP per capita) can be different from the classification into emerging markets, using equity index provider MSCI country classifications, both Poland and Czech Republic fall under emerging markets category. The rationale in doing so is to follow asset allocation principles of global asset managers that tend to use index provider asset class classifications.

rencies were chosen on the basis of them to behave differently in various market states, i.e. on average move in opposite directions.

In all other respects, similar methods and procedures were used wherever possible as in the literature surveys presented in Chapter 3.

#### **4.1. Competing models**

The selection of correlation models that were included in model performance evaluation sample was based on two criteria. First, at least one of the models from each group of correlation models introduced in Chapter 2<sup>25</sup> needed to be included in the evaluation sample. From the naïve correlation model group, both Simple Moving Average and RiskMetrics Exponentially Weighted Moving Average models were included. From the group of models with conditional covariance matrix scalar VECH, Full BEKK, Diagonal BEKK and Scalar BEKK were included. Finally, from the group of models with conditional variances and covariances, CCC (Constant Conditional Correlation), DCC (Dynamic Conditional Correlation) and ADCC (Asymmetric Dynamic Conditional Correlation) models were included. Second criteria in choosing the models to be included into the evaluation sample was the condition that the resulting covariance matrix must be positive semidefinite. As explained in Chapter 2 all the aforementioned models will provide a positive semi-definite covariance matrix. Furthermore, as most of the models included in the model evaluation sample were also surveyed in Chapter 3, further analysis about model relative performance in developed versus emerging markets is made possible. Full names of correlation models together with their abbreviations and model specifications are provided in Table 4.1.

The procedure for parameter estimation was as follows. In the first step, AR(1) equations were estimated to extract conditional means of each return series. AR(1) equation residuals were then used to estimate all conditional volatilities, conditional correlations and model parameters.

---

<sup>25</sup> Correlation model groups introduced in Chapter 2 were as follows: (i) naïve correlation models; (ii) models of conditional covariance matrix; and (iii) models of conditional variances and covariances.

Furthermore, univariate GARCH(1,1)<sup>26</sup> specification was used to model volatility dynamics in all conditional correlation type models (CCC, DCC, ADCC).

Table 4.1. Correlation models included in out-of-sample performance evaluation

Model Name	Formulation	Abbreviation
Simple Moving Average (252-day rolling)	Equation 2.2 ( $k=252$ )	SMA
Risk Metrics version of Exponentially Weighted Moving Avg.	Equation 2.4 ( $\lambda=0.94$ )	EWMA
Scalar VECH	Equation 2.12 with variance targeting <sup>27</sup>	SVECH
Full BEKK	Equation 2.13	FBEKK
Diagonal BEKK	Equation 2.13 $\mathbf{A}$ and $\mathbf{B}$ are diagonal matrices	DBEKK
Scalar BEKK	Equation 2.13 $\mathbf{A} = a\mathbf{I}; \mathbf{B} = b\mathbf{I}$	SBEKK
Constant Conditional Correlation	Equation 2.18	CCC
Dynamic Conditional Correlation	Equation 2.21 Equation 2.22 Equation 2.23	DCC
Asymmetric Dynamic Conditional Correlation	Equation 2.21 Equation 2.22 Equation 2.26	ADCC

Source: author's compilation

## 4.2. Data

Two portfolios, one consisting of equity market instruments and the other consisting of currency market instruments are provided in Table 4.2.

Table 4.2. Portfolios used in correlation model performance evaluation

	Equity portfolio (hedged <sup>28</sup> )	Currency portfolio (euro)
Asset 1	WIG Index (Poland)	CHF (Switzerland)
Asset 2	PX Index (Czech Republic)	PLN (Poland)

Source: Authors compilation

<sup>26</sup> Dynamics in GARCH(1,1) specification was defined again as  $h_{i,t}^2 = \omega + \alpha r_{i,t-1}^2 + \beta h_{i,t-1}^2$ .

<sup>27</sup>  $vech(\mathbf{H}_t) = \mathbf{c}(1 - a - b) + a(\mathbf{r}_{t-1}\mathbf{r}'_{t-1}) + b\mathbf{H}_{t-1}$ .

<sup>28</sup> Portfolio currency effect is hedged, i.e. portfolio return is only influenced by local currency returns.

Equity portfolio constituent indices were chosen from the sample of Central Eastern Europe country equity indices based on two criteria. First, the markets needed to be with sufficient liquidity and second, the market closing times needed to be approximately the same. Latter two criteria are important to avoid asynchronous data in correlation modelling. As Polish and Czech equity markets are the biggest in Central Eastern Europe (by market capitalization) and the markets close at 18:00 (CET, Central European Time) and 18:15 (CET) respectively, then the portfolio was considered as sufficiently representative for correlation analysis.

For sample time series data, daily local currency index closing values were collected from Bloomberg database. The sample period included calendar days<sup>29</sup> between (and including) January 3rd, 2000 to July 31st, 2009. Furthermore, time series data was filtered to include only the days when both markets were open for trading. Latter filtering decreased the end-sample from 2 500 observations (not including weekends) to 2 283 observations (not including weekends and non-trading weekdays).

Unconditional volatility<sup>30</sup> of log returns for Polish WIG index and Czech PX index time series for sample period was calculated to be 24.6% and 26.2% respectively. Unconditional correlation between WIG and PX index log returns were calculated to be 0.58.

Figure 4.1. provides further insights into the volatility and correlation dynamics during the sample period. In-sample conditional volatility in Figure 4.1. was modelled using univariate GARCH(1,1)<sup>31</sup> model and conditional correlations with Dynamic Conditional Correlation (DCC) and Constant Conditional Correlation (CCC) models, both with univariate GARCH(1,1) dynamics in their volatilities (see Chapter 2 for further details).

---

<sup>29</sup> Calendar days means all days in a selected period including all weekends and holidays.

<sup>30</sup> Unconditional volatility is a simple measure of sample standard deviation calculated as  $\sigma_x = \sqrt{\frac{1}{T-1} \sum_{t=1}^T (x_t - \bar{x})^2}$

<sup>31</sup> Conditional variance dynamics in GARCH(1,1) specification was defined as  $h_{i,t}^2 = \omega + \alpha r_{i,t-1}^2 + \beta h_{i,t-1}^2$

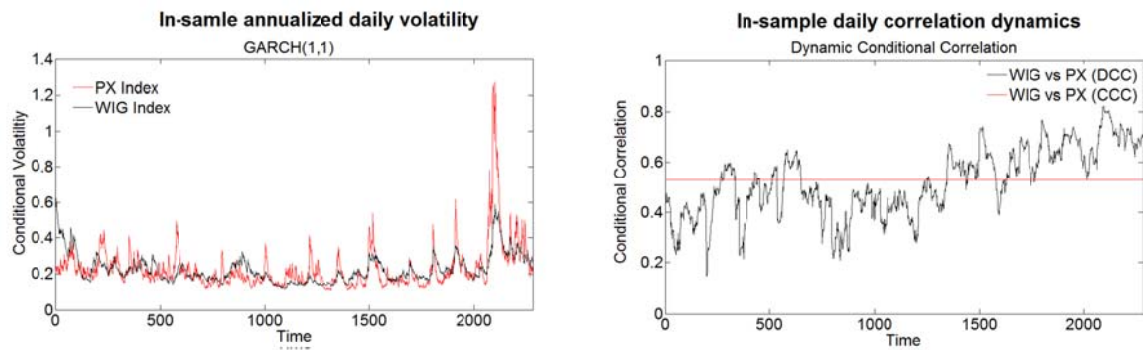


Figure 4.1. WIG and PX index local currency log return volatility and correlation dynamics during the period of 03.01.2000 – 31.07.2009 (observation number 1 through 2,283)

Currency portfolio constituents were selected with a goal to have an average negative correlation between chosen currencies. In order to achieve that, a portfolio made up of two currencies were chosen: one belonging to the class of „risk-on“ currencies and the other belonging to the class of „risk-off“ currencies<sup>32</sup>. Selected currencies were Polish Złoty (PLN) as „risk-on“ currency and Swiss franc (CHF) as „risk-off“ currency. Portfolio base currency was selected to be Euro (EUR).

Daily end-of-day values were then collected from Bloomberg database for EUR/CHF and EUR/PLN currency pairs. Taking into consideration that currency markets are trading 24/7<sup>33</sup>, then no filtering was done for public holidays. Nevertheless, due to lower liquidity, weekends were still removed from final dataset. Latter resulted in 2 500 daily observations for both currency pairs in the end-sample.

Based on log returns, in-sample unconditional volatilities for EUR/CHF and EUR/PLN were 4.7% and 11.1%, respectively. Unconditional correlation between EUR/CHF and EUR/PLN log returns during the same period was -0.19.

Figure 4.2. provides further insights into the volatility and correlation dynamics during the sample period. As with equity indices, in-sample conditional volatility in Figure 4.2. is modelled with the univariate GARCH(1,1) model and conditional correlations with Dynamic Conditional

<sup>32</sup> In general „risk-on“ assets outperform „risk-off“ assets during less volatile periods (usually in good economic conditions) and „risk-off“ assets outperform „risk-on“ assets during more volatile (usually in bad economic conditions).

<sup>33</sup> 24/7 means 24 hours a day and 7 days a week.



Correlation (DCC) and Constant Conditional Correlation (CCC) models, both again with univariate GARCH(1,1) dynamics.

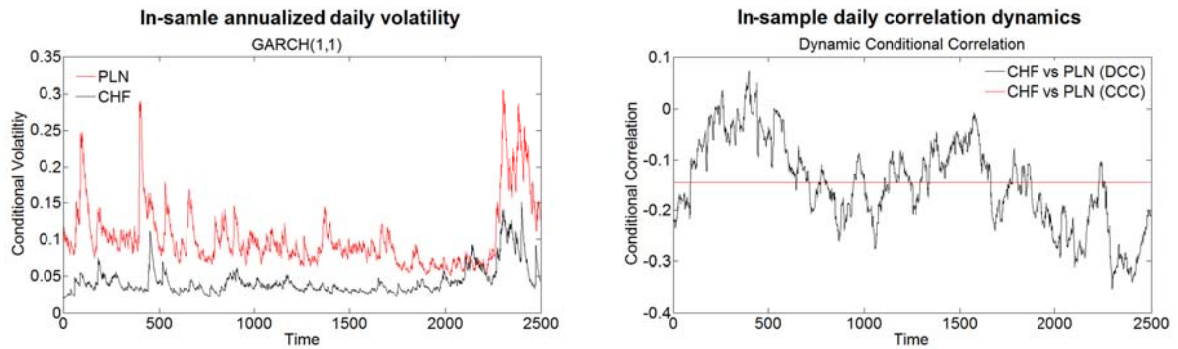


Figure 4.2. EURCHF and EURPLN log return volatility and correlation dynamics during the period of 03.01.2000 – 31.07.2009 (observation number 1 through 2,500)

In order to evaluate correlation model out-of-sample performance, Clements et al. (2009, 2012), Laurent et al. (2010), Caporin and McAleer (2012) and Boudt et al. (2013) used low and high volatility sub-samples to assess the consistency in model performances across different market conditions. Both Figure 4.1. and Figure 4.2. indicate that there are periods when markets are calm and there are periods when volatility increases substantially. Consider for example charts on the left in Figure 4.1. and Figure 4.2. One can see that starting from around observation number 2 000 in case of equity indices and 2 250 in case of currencies, the volatility jumps up and stays high for the following 250-300 days. Such a substantial increase in volatility levels was due to the financial crisis of 2007-2008 which culminated with Lehman Brothers<sup>34</sup> bankruptcy in September 2008. Following the bankruptcy announcement, global equity markets went into freefall causing drastic increases in the return volatilities of risky assets.

Alternatively, the period up to the observation number 2 000 for equity indices and between 1 000 and 2 000 for currencies, showed very little volatility. This was the period preceding the financial crisis when equity markets were rising and volatility levels were low.

As the market dynamics and hence correlation model forecasting performance during such periods can be substantially dissimilar from each other, and following the testing principles

<sup>34</sup> Lehman Brothers was United States domiciled investment bank.

of the aforementioned empirical surveys, equity and currency samples were divided into three parts: initial in-sample training set, calm period sub-sample and turbulent period subsample. The propose of the initial in-sample training set was to calibrate correlation models for the first out-of-sample one day ahead forecast. Subsample observations together with the corresponding statistics are provided in Table 4.3. and Table 4.4.

Table 4.3. Equity portfolio division into initial training set, calm period subsample and turbulent period subsample

	Dates	Observations	Annualized unconditional volatility		Unconditional correlation
			WIG Index	PX Index	WIG - PX
Initial training set	03.01.2000 - 30.09.2005	1 – 1 343	21.5%	20.8%	0.40
Calm period	30.09.2005 - 28.12.2007	1 343 – 1 893	20.9%	19.0%	0.65
Turbulent period	28.12.2007 - 31.07.2009	1 893 – 2 283	32.4%	44.8%	0.77

Source: author's calculations

Table 4.4. Currency portfolio division into initial training set, calm period subsample and turbulent period subsample

	Dates	Observations	Annualized unconditional volatility		Unconditional correlation
			EUR/CHF	EUR/PLN	EUR/CHF - EUR/PLN
Initial training set	03.01.2000 - 30.09.2005	1 – 1 500	3.8%	10.7%	-0.03
Calm period	30.09.2005 - 28.12.2007	1 500 – 2 085	3.3%	6.9%	-0.20
Turbulent period	28.12.2007 - 31.07.2009	2 085 – 2 500	8.2%	16.1%	-0.36

Source: author's calculations

### 4.3. Forecast Evaluation

Out-of-sample performances of nine correlation models in Table 4.1. were assessed using Model Confidence Set (MCS) with 25% significance level ( $\alpha=25\%$ ) and with QLIKE loss function as defined in equation 3.5.

Initial model parameter estimation was done based on the initial training set defined in Table 4.3. and Table 4.4.

Based on the parameter values, one day ahead out-of-sample forecast was made which in turn was fed into QLIKE loss function for the calculation of loss for a given day. Next, the model in-sample was adjusted to include the first out-of-sample observation day and model parameters were re-evaluated. Again, based on the new parameter values, new out-of-sample forecast was made for each model, QLIKE loss calculated and in-sample once again lengthened by one day. The procedure was continued until the last sample observation (in case of equities until observation number 2 283 and in case of currencies until observation number 2 500). Once all losses were calculated, model losses from calm and turbulent periods were fed into MCS to find out which were the winning models.

Furthermore, as in Clements et al. (2012) calculation cost was calculated for each model. For a given model, calculation cost was defined as time that was needed to estimate and forecast all parameters and results for all iterations.

All calculations were made in MATLAB, with Kevin Sheppards' MFE Toolbox to estimate parameters in correlation models other than SMA. SMA model, one step ahead forecast and loss calculations were performed using code written by the author. MCS procedure was again carried out using MFE Toolkit<sup>35</sup>. One step ahead forecasts for all models were reconciled against secondary independent forecasts made using MS Excel to ensure validity of authors code. The same validation was also carried out for the loss function results.

---

<sup>35</sup> All MCS calculations were made with block length of 1 and 10 000 bootstrap replications.

#### 4.4. Equity Sample Results

Equity sample results from the initial training set are provided in Figure 4.3 and Appendix 2<sup>36</sup>. The figure is divided into two parts. First part – left chart – gives the in-sample results for all nine models. For further clarity, the second part – right chart – gives the in-sample results for six models. Three models omitted from right chart are SVECH, DBEKK and FBEKK. The first two models, SVECH and DBEKK are omitted due to their similar results compared to EWMA and SBEKK models. FBEKK is omitted due to its higher volatility and range that influences the visibility of other model dynamics. As dynamics of the omitted models are very similar across asset classes and in-sample versus out-of-sample plots, the aforementioned structure is followed throughout the rest of the chapter.

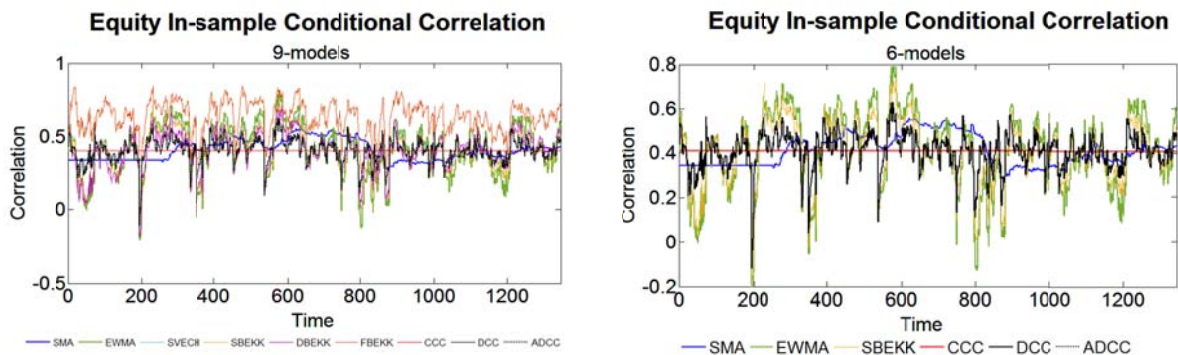


Figure 4.3. WIG and PX in-sample correlation dynamics (initial training set)

In-sample result plots are mostly for illustrative purposes, but still convey some useful information. Namely, one can see that more flexible models also experience higher volatility. Also, there is evidence of similar dynamics in case of EWMA, SVECH, SBEKK and DBEKK and in case of DCC and ADCC. Contrary to the first group, where SVECH and DBEKK were omitted for similar dynamics from right charts, ADCC was not omitted for reasons to be seen in the following section discussing the results for currency sample.

Equity portfolio one day ahead out-of-sample forecast model performance results are provided in Table 4.5. Table results can be interpreted as follows. The number in each result cell

<sup>36</sup> For better visibility, larger versions of Figure 4.3 through Figure 4.6 are provided in the corresponding appendices

represents corresponding models' p-value for a given sample period. The grey cells indicate which models were included in the final model confidence set at the 25% significance level (i.e.  $p \geq 0.25$ ).

Table 4.5. MCS results for equity sample. Winning models in grey

	Calm Period	Turbulent Period	Full Period
SMA	0	0	0
EWMA	0	0	0
SVECH	0	0	0
SBEKK	0	0	0
DBEKK	0	0	0
FBEKK	1	0	0.0026
CCC	0.1731	1	1
DCC	0	0	0
ADCC	0	0	0

One day ahead out-of-sample forecast model performance using QLIKE loss function and MCS  $\alpha=25\%$ . p-values provided in result cells. Source: author's calculations

During calm periods FBEKK model tends to outperform all other models. This might be due to its high flexibility that is not compromised by volatility surprises. Latter means that in case the market volatility and correlation structure does not change dramatically, FBEKK is the best model out of our sample models to forecast one day ahead correlations for Polish and Czech equity market indices.

MCS results for the turbulent period gave strong preference to CCC model (see Table 4.5). This is a rather surprising result considering that most of the previous empirical studies have found DCC and ADCC models outperforming CCC model in more volatile periods. As the QLIKE loss is calculated for the entire covariance matrix, i.e. including volatility loss, one could argue that volatility loss might play a role in the total loss calculation. However, as the three conditional correlation models, CCC, DCC and ADCC, were all modelled with GARCH(1,1) univariate dynamics, then volatility loss effect in the model relative performance is neutralized and the loss difference is explained by correlation forecast loss only.

Hence, the most probable explanation for CCC model outperformance is the daily instability of realized correlation between WIG and PX indices. Latter causes dynamic models to systematically under-/overshoot the realized correlation generating losses greater than for the con-

stant estimate. Nevertheless, when omitting CCC model from MCS and re-running the procedure, ADCC model followed by DCC model outperform the other models in turbulent period. Latter agrees with the results obtained in Chapter 3.

According to MCS results, by reversing the losses (from bads to goods)<sup>37</sup>, SMA model is the worst performing model in all sample periods (calm, turbulent and full period). This most probably is due to its naïve specification across the entire covariance matrix. By equally weighting the observations from the last 252 trading days, the model is incapable of adjusting to the most recent changes in the market volatility and correlation structure.

Figure 4.4. (Appendix 3) provides some further illustrative evidence for one day ahead out-of-sample correlation dynamics.

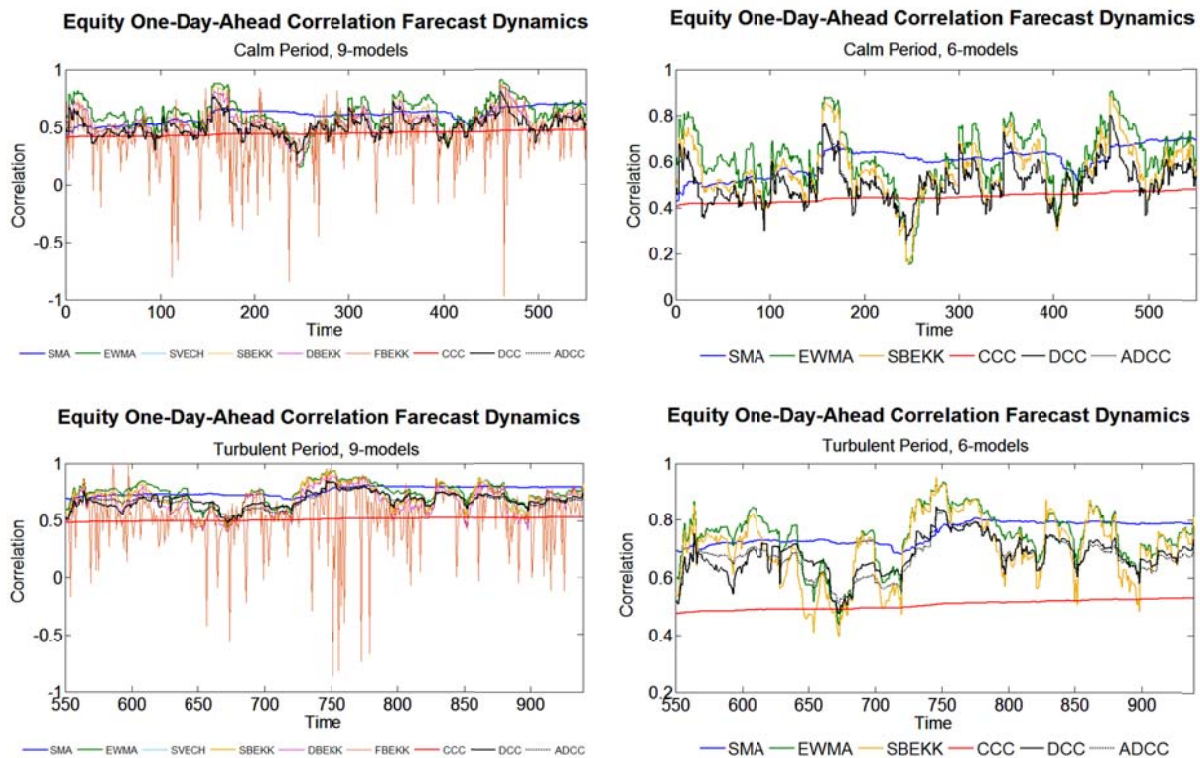


Figure 4.4. WIG and PX one day ahead correlation forecast dynamics

<sup>37</sup>  $\mathbf{L}^{Good} = -1 \times \mathbf{L}^{Bad}$ , where  $\mathbf{L}^{Bad}$  is a  $T \times M$  matrix of losses for  $M$  models and  $T$  periods.

A couple noteworthy observations can be made from Figure 4.4. First, FBEKK model is highly unstable varying between -1 and +1. However, as explained, this flexibility pays off in the low volatility period where correlation structure remains intact. During the more volatile period, FBEKK model is in fact one of the worst performing models, outperformed by CCC, ADCC, DCC, EWMA and DBEKK. Second, the results for DCC and ADCC models are almost identical in calm periods and differ only slightly in more turbulent periods. Nevertheless latter difference is sufficient for ADCC to outperform DCC in the turbulent period.

One additional interesting observation can be made from the above results. As can be seen from Figure 4.4, correlation dynamics is relatively static in SMA model and close to completely static in CCC model. Considering now that one model is the best performer in almost all states (CCC) and the other is the worst performer in all states (SMA) and also taking into account how the volatilities are modelled in both cases, then it can be argued that volatility dynamics is far more important to the covariance matrix than correlation dynamics. In fact, as CCC model outperforms DCC and ADCC models (all having the same underlying volatility dynamics), it can be further argued that in case of the sample data used, dynamic correlation modelling can be even hurtful to the overall covariance (and correlation) out-of-sample forecasting performance.

Turning now to the issue of computational time cost, Table 4.6. provides evidence about the time it took to estimate and forecast each model.

Table 4.6. Simulation computational time cost for equity sample (ordered by time)

Model	Time
SMA	00:00:05
EWMA	00:00:13
CCC	00:23:10
SVECH	01:30:15
DCC	04:24:03
SBEKK	05:19:22
ADCC	08:06:58
DBEKK	08:30:14
FBEKK	20:19:38

Time format: HH:MM:SS. Processor: Intel(R) Core i5-2520M CPU @ 2.50GHz, 2501 Mhz, 2 Cores, 4 Logical Processors. Source: author's calculations

FBEKK model results took by far the most time to be simulated. This is due to the reason that the model has the highest number of parameters to be estimated at each step. Not surprisingly, naïve models were computationally two of the least costly models to be simulated, followed by CCC model. All in all, it took more than two days to estimate and forecast all nine model one day ahead out-of-sample results. Considering that we are dealing with only two asset portfolio, it is a clear evidence of the costliness of correlation modelling.

#### 4.5. Currency Sample Results

Currency sample results from the initial training set is provided in Figure 4.5 (Appendix 4).

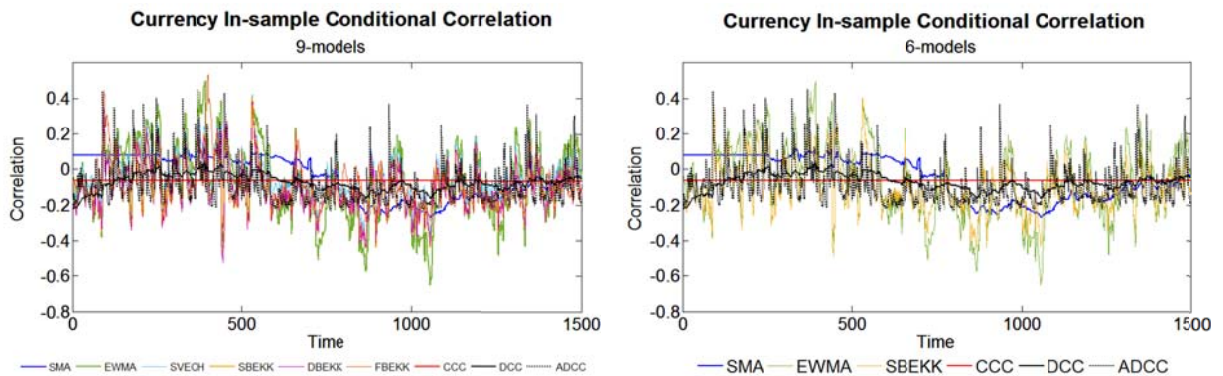


Figure 4.5. EURCHF and EURPLN in-sample correlation dynamics (initial training set)

One interesting observation can be made from Figure 4.5. Namely DCC and ADCC model results deviate substantially. This is due to the reason that when the correlation between two assets is negative, then the asymmetry component is almost always present in the correlation estimate. In the currency markets, it is also interesting to interpret the impact of negative return to volatility, as negative return in one currency is always positive return in the other currency. For instance, when Swiss franc experiences negative return while being a risk off currency, would that mean that volatility rises? Usually, volatility asymmetry is linked to market stress. Now, when Swiss franc declines against euro, that should mean that we are in a risk-on, or alternatively



bull market, and volatility should be low (in relative terms). All in all, it is interesting to see whether ADCC can still outperform DCC in case of our risk-on/risk-off currency portfolio.

Currency portfolio one day ahead out-of-sample forecast model performance results are provided in Table 4.7.

Table 4.7. MCS results for currency sample. Winning models in grey

	Calm Period	Turbulent Period	Full Period
SMA	0	0	0
EWMA	0	0	0
SVECH	0	0	0.0493
SBEKK	0	0	0.0493
DBEKK	0	0	0
FBEKK	1	0	1
CCC	0	1	0.0493
DCC	0	0	0
ADCC	0	0.0014	0

One day ahead out-of-sample forecast model performance using QLIKE loss function and MCS  $\alpha=25\%$ . p-values provided in result cells. Source: author's calculations

As with equity sample, FBEKK model was found to outperform all other models during the calm period and CCC model in turbulent period. Possible reasons for this outperformance are the same as with equity sample. In the calm period, the flexibility of FBEKK model once again appears to fit the market correlation dynamics the best and in the turbulent period, constant conditional correlation seems to outperform any dynamic estimate. Furthermore, the worst performing models for calm, turbulent and full periods were tested and the results were as follows. SMA type model was the worst in turbulent and full period, whereas EWMA was the underperformer during calm period sub-sample.

As mentioned in the beginning of current section, it would be interesting to assess ADCC relative performance against DCC. Eliminating CCC from model set and re-running MCS procedure for turbulent period ends up having three models in the final model confidence set ( $\alpha=25\%$ ): ADCC, DCC and EWMA, with ADCC having the highest p-value. So even when considering the nature of our currency portfolio, an asymmetric version of DCC model tends to outperform the general DCC model specification. Illustrative dynamics of one day ahead out-of-sample forecasts is provided in Figure 4.6 (Appendix 5).

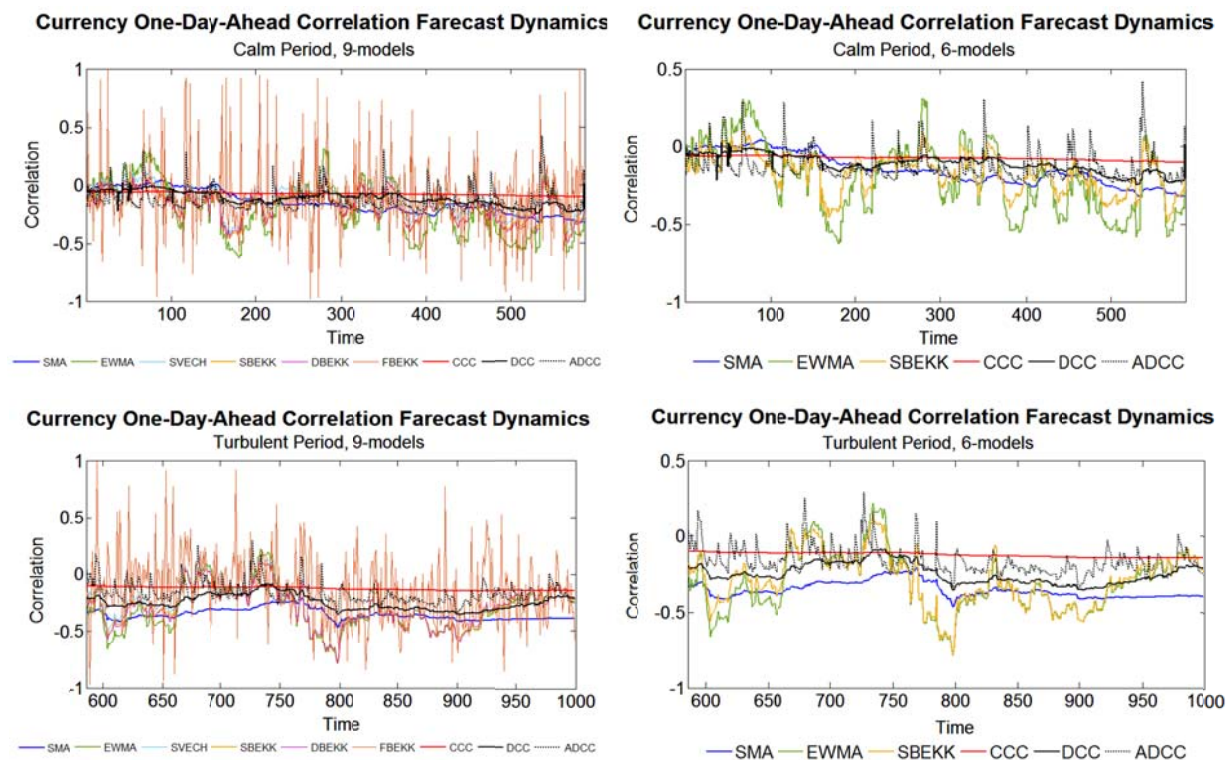


Figure 4.6. EURCHF and EURPLN one day ahead out-of-sample correlation dynamics

Simulation computational time cost for currency sample is provided in Table 4.8.

Table 4.8. Simulation computational time cost for currency sample (ordered by time)

Model	Time
SMA	00:00:00
EWMA	00:00:13
CCC	00:26:31
SVECH	01:50:27
DCC	04:44:23
SBECK	06:18:10
DBECK	09:31:07
ADCC	09:45:57
FBECK	24:45:50

Time format: HH:MM:SS. Processor: Intel(R) Core i5-2520M CPU @ 2.50GHz, 2501 Mhz, 2 Cores, 4 Logical Processors. Source: author's calculations

Similarly to the results for equity sample, FBEKK model results took the most time to be simulated, while naïve models again providing quick results.

## 4.6. Conclusions

In this chapter, using data from emerging markets, one day ahead forecasting performance of nine correlation models was evaluated. Based on the data from the beginning of year 2000 until end of July 2009, two two-asset portfolios were constructed, one with Polish and Czech equity indices and the other with Swiss and Polish currencies. Model out of sample performance was then evaluated separately for low volatility (calm) period and high volatility (turbulent) period.

In general, the results were quite similar for equity and currency portfolio samples. In both cases the Full BEKK model outperformed the rest of the models during the calm period before the global financial crisis and Constant Conditional Correlation (CCC) during turbulent period. The results for turbulent periods are in large part at odds with the results from previous empirical studies on the same subject. Most of these studies have found evidence of Dynamic Conditional Correlation (both asymmetric and symmetric) model outperforming CCC model under more volatile market conditions. The possible reason for CCC outperforming its dynamic counterparts might be the structural differences between developed and emerging market volatility and correlation structure. Furthermore, based on the very poor results of Simple Moving Average model and very good results of CCC model, it was argued that when forecasting the full covariance matrix the best performance in case of emerging market assets is achieved with the combination of dynamic volatility and nearly constant correlation.

Further evidence in favour of using CCC model in out-of-sample forecasting was provided with the computational time cost evaluation results. Namely, in relative terms, CCC model estimation and forecasting time was roughly 50 times less than for Full BEKK, 20 times less than for Dynamic Conditional Correlation (DCC) model and 10 times less than for Asymmetric DCC model.

Based on the previous empirical research as well as the results from this study, it can be argued that when considering different market environments, asset types as well as modelling

time costs there is no single best model that would work equally well under any given condition. However, it would be interesting to see whether there would be any changes to the results when some statistical loss function is used with outlier adjusted covariance proxy. Using alternative loss function in combination with emerging market data might therefore be of interest in future studies on similar subject. Also, another area of potential future research could be the incorporation of theoretical correlations into some naïve or Constant Conditional Correlation model<sup>38</sup>. This might further improve correlation model forecasting ability without compromising its efficiency.

---

<sup>38</sup> It was explained in Chapter 1 that both empirical as well as theoretical correlations break during periods of high volatility.

## SUMMARY

Since the introduction of modern portfolio theory in 1952 (Markowitz 1952), expected correlations have played a significant role in the field of asset allocation. Expected correlations together with expected volatilities and expected returns have been the cornerstones of any portfolio allocation decision, being it risk minimization or return maximization. In this thesis correlation model out-of-sample forecasting performance was evaluated with a goal to find a correlation model that would consistently outperform others in its forecasting ability, i.e. would consistently provide the best estimate of expected correlation. Such model could then be used in risk systems (or equivalently in any other allocation driven system) to model portfolio expected variance. Having measured expected portfolio variance correctly, institutions are able to make both better risk budgeting decisions as well as avoid regulation-driven forced liquidation of risky assets in stressed market environments.

The thesis presented various correlation modelling related theoretical concepts and empirical results as follows. In Chapter 1 the properties of time-varying correlations were investigated. It was argued that a correlation model should be able to handle six so called stylized facts about correlation dynamics. These properties, or stylized facts were said to be of importance when assessing the theoretical soundness of various correlation models. The six facts introduced in Chapter 1 included the tendency for correlations to strengthen over time and the fact that they are mean-reverting. Also, it was demonstrated that correlations can change significantly during market stress and that they exhibit asymmetric tendency by changing significantly only during downmarkets. The last two properties discussed in Chapter 1 were correlation persistence and possible outliers. It was then shown that correlation does indeed tend to be autocorrelated with its lagged observations and that outliers do exist and can potentially have substantial effect on correlation estimate. After the aforementioned stylized facts were established, eight correlation models were formally defined in Chapter 2 and benchmarked against the aforementioned stylized facts. Presented models were as follows: (i) Simple Moving Average correlation (SMA) and RiskMetrics version of Exponentially Weighted Moving Average correlation (RiskMetrics EWMA) from the

class of naïve correlation models; (ii) VEC, BEKK, and Orthogonal GARCH (O-GARCH) models from the class of models with conditional covariance matrix; and (iii) Constant Conditional Correlation (CCC), Dynamic Conditional Correlation (DCC) and Asymmetric Dynamic Conditional Correlation (ADCC) from the class of models with conditional variances and covariances. It was argued that out of the eight models presented, ADCC model by handling mean-reversion, correlation breaks, asymmetry and persistence is by construction the most flexible model while CCC model by not handling any of the stylized facts presented in Chapter 1 is the least flexible model. It was further argued that some models, such as more flexible versions of VEC model, cannot guarantee the positive semidefiniteness of the resulting covariance matrix (the result could be negative portfolio expected volatility). Nevertheless, all models with conditional variances and covariances (CCC, DCC, ADCC) as well as naïve models (SMA, RiskMetrics EWMA) were guaranteed to provide covariance matrix estimates that are positive semidefinite.

An extensive review of the literature on correlation model out-of-sample forecasting performance was provided in Chapter 3. It was argued that additional complications arise when evaluating correlation model forecasting performance because the true correlation is unobservable (i.e. is a latent variable). For this reason various robust statistical and economic loss functions were introduced that would overcome latter problem and make it possible to assess how far off we are with our forecast compared to the „realized outcome“<sup>39</sup>. Finally, Model Confidence Set was used as a procedure for selecting winning models with superior forecasting ability<sup>40</sup>. With regard to data samples, the results from the existing literature were based on developed market data during calm and turbulent market environments. Further sampling was done to differentiate between various asset types (equity, currency and mixed) as well as portfolio size. The results were as follows.

In general, ADCC model outperformed others in the developed market samples. The preference for the ADCC model was stronger during turbulent (i.e. high volatility) period than during the calm (i.e. low volatility) period. In fact, neither the assumption of constant correlation nor the assumption of symmetry could be rejected during calm periods. Furthermore, no significant differences in results was found between equity, currency and mixed asset samples. Relating to

---

<sup>39</sup> „realized outcome“ is in quotation marks because we still do not exactly know what is „realized“ (as true correlation is unobservable).

<sup>40</sup> Model Confidence Set was described to be a statistical procedure that selects a set of equally superior models from the initial set of competing models.

large portfolio sample, it was found that the most flexible models (such as DCC) become computationally too costly to use in large portfolios.

In Chapter 4, nine correlation model specifications were tested for Polish and Czech equity market index sample and for Czech krona and Swiss franc currency sample. All calculations were performed by author using same procedures and sampling principles as were used in Chapter 3. The results were as follows.

The best performing model in the calm period was Full BEKK model and in turbulent period CCC model. The finding is interesting as it is at odds with the results from developed market samples in Chapter 3. The fact that CCC model outperformed its dynamic peers during turbulent markets indicates that at least within the selected samples dynamic models are incapable of dynamically forecasting one day ahead correlations well enough to beat constant estimate. As in Chapter 3, additional analysis was carried out in Chapter 4 to clarify the costliness of using various correlation models in practice. Even though author only dealt with two-asset portfolios, Full BEKK model took around 24 hours to be simulated (approximately 1 000 one day ahead forecasts with the same number of model parameter re-estimations). ADCC model took around 8-9 hours, CCC model around half an hour and RiskMetrics EWMA took only 13 minutes to be simulated.

Overall conclusion based on both literature review and own analysis was that with regard to forecasting performance no single model can be considered unconditionally superior to others. Furthermore, taking into account the computational time cost that is needed for the estimation of dynamic versions of conditional correlation models as well as for models with conditional covariance matrix, it might also be suboptimal to implement such models into real life portfolio risk measurement systems.

Some areas were mentioned that could be of interest in future investigation. First, it would be interesting to see the results for model performance based on alternative emerging market data sample together with same other robust loss function. Second, further research could be done in the field of theoretical correlation modelling with an aim to incorporate theoretical correlation dynamics into some naïve or constant conditional correlation model. This might make it possible to incorporate correlation dynamics into correlation models without compromising its estimation efficiency.

## **RESÜMEE**

### **KORRELATSIOONI MODELLEERIMINE NING PROGNOOSIMINE FINANTSADMETEL**

Matis Tomiste

Alates 1952. aastast mil Markowitz avaldas oma mõjuvõimsa portfelliteooria aluspõhimõtteid käsitleva artikli (Markowitz 1952), on varade vahelisi oodatavaid korrelatsioone peetud üheks portfelli risijuhtimise nurgakiviks. Sellest ajast alates on ülemaalmine regulatiivne keskkond tervitanud kõikvõimalike portfelliteooriale tuginevate riskisüsteemide teket, milles kõigis on olulist rolli etendanud nii alusvarade oodatavad dispersioonid kui ka nende vahelised oodatavad korrelatsioonid. Vastupidiselt dispersiooni modelleerimisele, mis on olnud akadeemilite uurimustööde tähelepanu keskpunktis alates Engle'i publikatsioonist teemal autoregressiivne tingimuslik heteroskedastiivsus (Engle 1982), on korrelatsiooni modelleerimine selle aja jooksul mõnevõrra vähem tähelepanu saanud. Võimalikke põhjusi selleks on kaks. Esiteks on korrelatsioonide olulisus riskimudelites mõnevõrra väiksem dispersioonide omast. Teiseks kannatab korrelatsiooni modelleerimine niinimetatud "dimensionaalsuse probleemi" käes. Viimane tähendab seda, et instrumentide arvu poolest suuremate portfelledele korrelatsioonidünaamika modelleerimine võib muutuda paljude korrelatsioonimudelite jaoks juba ületamatuks probleemiks.

Käesoleva magistr töö eesmärgiks oli korrelatsioonimudelite võrdlev analüüs ning parima prognoosivõimega korrelatsioonimudeli väljaselgitamine. Selleks uuris autor esmalt korrelatsiooni dünaamikat finantsturgudel, tuues välja kuus stiliseeritud fakti millega korrelatsioonimudel peaks toime tulema. Viimasteks olid korrelatsiooni tugevnemine ajas ning keskvaartuse juurde tagasipöördumine, korrelatsiooni murdumine ja viimase asümmeetria, autokorrelatsioon korre-



latsiooni dünaamikas ning võimalikud erandid. Järgnevalt esitas autor formaalsed definitsioonid kaheksale korrelatsioonimudelile, uurides samas ka nende võimet tulla toime eelnimetatud stiliseeritud faktidega. Defineeritud mudelite hulka kuulusid: (i) Tavalise libiseva keskmise mudel (SMA, Simple Moving Average) ning RiskMetrics'i versioon Eksponentsiaalselt kaalutud libiseva keskmise mudelist (RiskMetrics EWMA, RiskMetrics Exponentially Weighted Moving Average), mis kuuluvad niinimetatud naiivsete korrelatsioonimudelite klassi; (ii) VEC, BEKK ja Ortogonaalne GARCH (O-GARCH, Orthogonal Generalized Autoregressive Conditional Heteroskedasticity) tingimuslike kovariatsioonimudelite klassist; ning (iii) Konstantne tingimuslik korrelatsioon (CCC, Constant Conditional Correlation), Dünaamiline tingimuslik korrelatsioon (DCC, Dynamic Conditional Correlation) ja Asümmeetriline dünaamiline tingimuslik korrelatsioon (ADCC, Asymmetric Dynamic Conditional Correlation) tingimuslike dispersiooni- ja kovariatsioonimudelite klassist. Lähtudes vaid teoreerilisest mudeli ülesehitusest, vastas ADCC mudel stiliseeritud faktidele kõige paremini ning CCC mudel kõige halvemini.

Magistritöö kolmandas ja neljandas peatükis uuris autor eelnimetatud mudelite prognoosimisvõimet arenenud ning arenevatel turgudel. Kui arenenud turgude puhul tugines autor oma järeldustes juba olemasolevatele uurimustöödele, siis arenevate turgude andmetel tehtud järeldused tuginesid juba autori enda arvutustel. Nii eelneva kirjanduse kasutamises kui ka enda arvutustes kasutas autor nn. robustseid kaotusfunktsioone ning mudeli usaldus komplekti (MCS, Model Confidence Set) protseduuri. Lisaks arenenud ja arenevate turgude eristamisele, eristati tulemusi ka turu volatiilsuse (madala ja kõrge volatiilsuse valimid) ja vara tüübi (aktsiad, valuutad ning sega) lõikes. Kuigi tulemused polnud üheselt määratavad, kippus ADCC mudel edestama teisi mudeleid arenenud turgudel ning CCC mudel arenevatel turgudel<sup>41</sup>. Põhjusteks miks ADCC ei suutnud ületada CCC mudelit arenevatel turgudel võis autori hinnangul olla ADCC mudelile ebasobiv ebakorrapärane dünaamika autori poolt valitud valimites. Lisaks korrelatsioonimudelite sooritusvõime uurimisele, uuris autor ka korrelatsiooni modelleerimise suurest dimensionaalsusest tingitud arvutustest tulenevat ajakulu. Nii autori enda kui eelneva uurimustöö tulemustele tuginedes leidis autor, et seoses suure ajakuluga võib nii tingimuslike kovariatsioonimudelite kui

---

<sup>41</sup> Autori aktsiaportfelli valimisse kuulusid Poola ja Tšehhi börsiindeksid ning valuutaportfelli Poola ja Šveits valuutad.

ka dünaamiliste tingimuslike dispersiooni- ja kovariatsioonimudelite (ehk välja arvatud CCC) rakendamine praktilises riskimõõtmises osutada ületamatuks probleemiks.

Lõpliku järeldusena leidis autor, et ei eksisteeri üheselt parimat korrelatsioonimudelit, mille sooritusvõime prognoosimisel oleks ühtmoodi hea erinevates turu-, vara tüübi ja geograafilistes tingimustes ning mille rakendamine suuremahulistes valimites oleks praktiliselt teostatav.

Edasisteks võimalikeks uurimisteedeks pakkus autor välja uutel arenevate turgude andmetel põhinevate valimite peal alternatiivsete kaotusfunktsioonide kasutamise. Nimelt kasutas autor käesolevas tööd QLIKE kaotusfunktsiooni. Kasutades aga näiteks mõnda kaotusfunktsiooni, mille kaotus mõõdetakse otseselt mõne realiseeritud kovariatsioonimaatriksi vastu, võivad tulemused potentsiaalselt olla erinevad. Teiseks, stiliseeritud faktide juures mainis autor korrelatsiooni dünaamika eksisteerimist teoreetilistes korrelatsioonides. Seetõttu oleks autori hinnangul huvitav näha kas teoreetilise korrelatsioonidünaamika sisseviimine naiivsesse või CCC tüüpi mudelisse annaks veelgi paremaid tulemusi.

## BIBLIOGRAPHY

- Alexander, C. O. (2001). A primer on the orthogonal GARCH model. [Online]  
Available at: [www.icmacentre.ac.uk/pdf/orthogonal.pdf](http://www.icmacentre.ac.uk/pdf/orthogonal.pdf)
- Alexander, C. O., Chibumba, A. M. (1997). Multivariate orthogonal factor GARCH. -*University of Sussex Discussion Papers in Mathematics*.
- Ali, G. (2013). EGARCH, GJR-GARCH, TGARCH, AVGARCH, NGARCH, IGARCH and APARCH Models for Pathogens at Marine Recreational Sites. - *Journal of Statistical and Econometric Methods*, 2(3), pp. 57-73.
- Andersen, T. G., Bollerslev, T., Christoffersen, P. F., Diebold, F. X. (2007). Practical volatility and correlation modeling for financial market risk management. *NBER*.
- Andersen, T. G., Bollerslev, T., Diebold, F. X., Labys, P. (2003). Modeling and Forecasting Realized Volatility. - *Econometrica*, 71(2), pp. 579-625.
- Anon (2005). Theoretical and empirical properties of dynamic conditional correlation multivariate GARCH. *Working Paper, Oxford University*.
- Barndorff-Nielsen, O. E., Shephard, N. (2004). Measuring the impact of jumps in multivariate price processes using bipower covariation. *Discussion paper. Nuffield College. Oxford University*.
- Bauwens, L., Laurent, S., Rombouts, J. V. K. (2006). Multivariate GARCH models: A survey. - *Journal of Applied Econometrics*, Volume 21, p. 79–109.
- Bollerslev, T. (1990). Modeling the coherence in short-run nominal exchange rates: A multivariate generalized ARCH model. - *Review of Economics and Statistics*, Issue 72, p. 498–505.
- Bollerslev, T., Engle, R. F., Wooldridge, J. M. (1988). A capital asset pricing model with time-varying covariances. - *Journal of Political Economy*, 96(1), pp. 116-131.
- Boudt, K., Croux, C. (2010). Robust M-estimation of multivariate GARCH models. - *Computational Statistics and Data Analysis*, Volume 54, p. 2459–2469.
- Boudt, K., Croux, C., Laurent, S. (2011). Outlyingness weighted covariation. - *Journal of Financial Econometrics*, p. 657–684.
- Boudt, K., Danielsson, J., Laurent, S. (2013). Robust forecasting of dynamic conditional correlation GARCH models. - *International Journal of Forecasting*, pp. 244-257.

- Boyer, B. H., Gibson, M. S., Loretan, M. (1999). Pitfalls in tests for changes in correlations. *Federal Reserve Board, IFS Discussion Paper, Volume No. 597R.*
- Caporin, M., McAleer, M. (2012). Robust ranking of multivariate GARCH models by problem dimension. *Unpublished manuscript, University of Canterbury, Department of Economics and Finance, Working Paper No. 06/2012.*
- Cappiello, L., Engle, R. F., Sheppard, K. (2006). Asymmetric dynamics in the correlations of global equity and bond returns. - *Journal of Financial Econometrics*, pp. 537-572.
- Chua, D. B., Kritzman, M., Page, S. (2009). The Myth of Diversification. - *The Journal of Portfolio Management*, 36(1), pp. 26-35.
- Clements, A., Doolan, M., Hurn, S., Becker, R. (2009). Evaluating multivariate volatility forecasts. *NBER Working Paper #41.*
- Clements, A., Scott, A., Silvennoinen, A. (2012). Forecasting multivariate volatility in larger dimensions: some practical issues. *NBER Working Paper #80.*
- Engle, R., Colacito, R. (2006). Testing and valuing dynamic correlations for asset allocation. - *Journal of Business and Economic Statistics*, pp. 238-253.
- Engle, R. F. (1982). Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation. - *Econometrica*, 50(4), pp. 987-1007.
- Engle, R. F. (2002). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. - *Journal of Business & Economic Statistics*, Issue 20, p. 339–350.
- Engle, R. F., Kelly, B. (2012). Dynamic equicorrelation. - *Journal of Business & Economic Statistics*.
- Engle, R. F., Kroner, K. F. (1995). Multivariate simultaneous generalized ARCH. - *Economic Theory*, Volume 11, pp. 122-150.
- Engle, R. F., Sheppard, K. (2008). Evaluating the specification of covariance models for large portfolios. *available at www.kevinsheppard.net.*
- Forbes, K., Rigobon, R. (2002). No contagion, only interdependence: Measuring stock market comovements.. - *Journal of Finance*, Volume 57, p. 2223–2261.
- Hon, M. T., Strauss, J., Yong, S.-K. (2004). Contagion in financial markets after September 11: myth or reality?. - *Journal of Financial Research*, 27(1), p. 95–114.
- Jorion, P. (2006). *Value at Risk: The New Benchmark for Managing Financial Risk*. 3rd ed. New York: McGraw-Hill.
- Kotkatvuori-Örnberg, J., Nikkinen, J., Äijö, J. (2013). Stock market correlations during the financial crisis of 2008–2009: Evidence from 50 equity markets. - *International Review of Financial Analysis*, Volume 28, pp. 70-78.

- Laurent, S., Rombouts, J. V., Violante, F. (2012). On the forecasting accuracy of multivariate GARCH models. - *Journal of Applied Econometrics*, 27(6), pp. 934-955.
- Laurent, S., Rombouts, J. V., Violante, F. (2012). On the forecasting accuracy of multivariate GARCH models. - *Journal of Applied Econometrics*, 27(6), pp. 934-955.
- Laurent, S., Rombouts, J. V., Violante, F. (2013). On loss functions and ranking forecasting performances of multivariate volatility models. - *Journal of Econometrics*, pp. 1-10.
- Ledoit, O., Santa-Clara, P., Wolf, M. (2003). Flexible multivariate GARCH modeling with an application to international stock markets. - *The Review of Economics and Statistics*, 85(3), pp. 735-747.
- Longin, F., Solnik, B. (1995). Is the correlation in international equity returns constant: 1960-1990?. - *Journal of International Money and Finance*, 14(1), pp. 3-26.
- Longin, F., Solnik, B. (2001). Extreme correlation of international equity markets. - *The Journal of Finance*, Volume 2, pp. 649-676.
- Loretan, M., English, W. B. (2000). Evaluating “correlation breakdowns” during periods of market volatility. *Board of Governors of the Federal Reserve System*, Volume 658.
- Markowitz, H. (1952). Portfolio Selection. - *The Journal of Finance*, 7(1), pp. 77-91.
- Patton, A. J., Sheppard, K. (2009). Evaluating volatility and correlation forecasts. In: *Handbook of Financial Time Series*. Springer.
- Riskmetrics (1996). Riskmetrics Technical Document. 4th ed. New York: J.P.Morgan.
- Sandoval Junior, L., De Paula Franca, I. (2012). Correlation of financial markets in times of crisis. - *Physica A: Statistical Mechanics and its Applications*, 391(1-2), p. 187–208.
- Sheppard, K. (2013). Oxford MFE Toolbox. [Online]  
Available at: [http://www.kevinsheppard.com/MFE\\_Toolbox](http://www.kevinsheppard.com/MFE_Toolbox)
- Silvennoinen, A., Teräsvirta, T. (2008). Multivariate GARCH models. *Working Paper, SSRN*.
- Tse, Y. K., Tsui, A. K. C. (2002). A multivariate generalized autoregressive conditional heteroskedasticity model with time-varying correlations. - *Journal of Business & Economic Statistics*, Volume 20, pp. 351-362.
- Tuckman, B., Serrat, A. (2011). Empirical approaches to risk metrics and hedging. In: *Fixed Income Securities: Tools for Today's Markets*. s.l.:Wiley.
- van der Weide, R. (2002). GO-GARCH: a multivariate generalized orthogonal GARCH model. - *Journal of Applied Econometrics*, Volume 17, pp. 549-564.

## APPENDICES

### Appendix 1. Selected features of empirical surveys

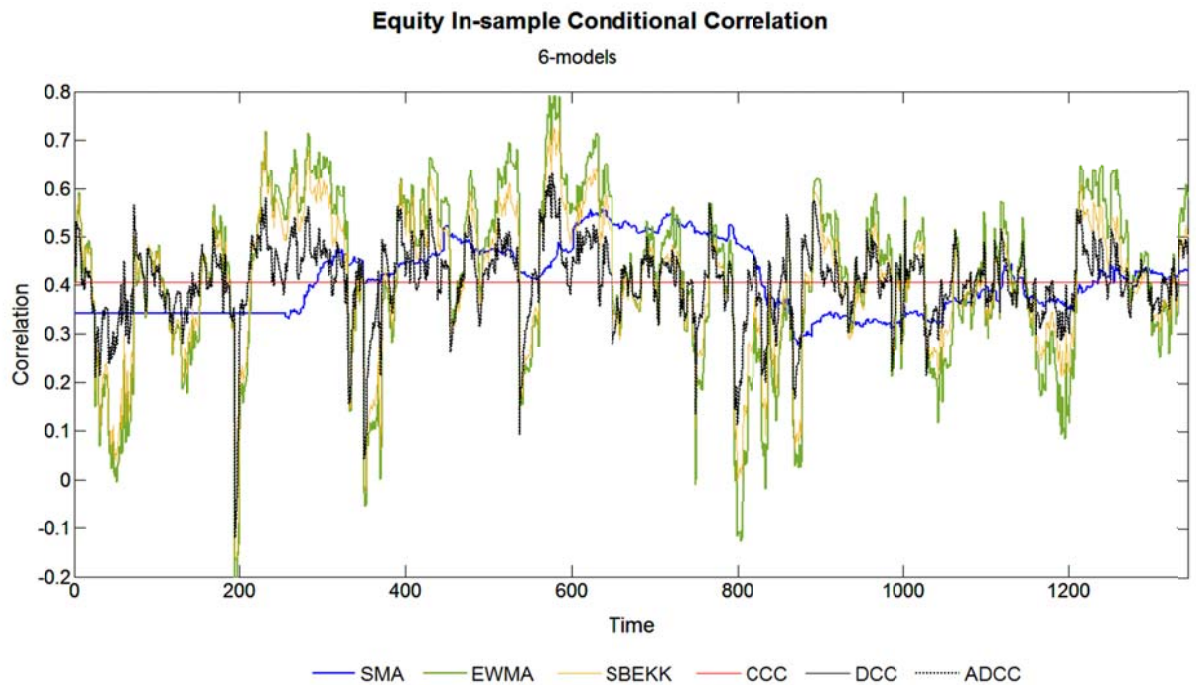
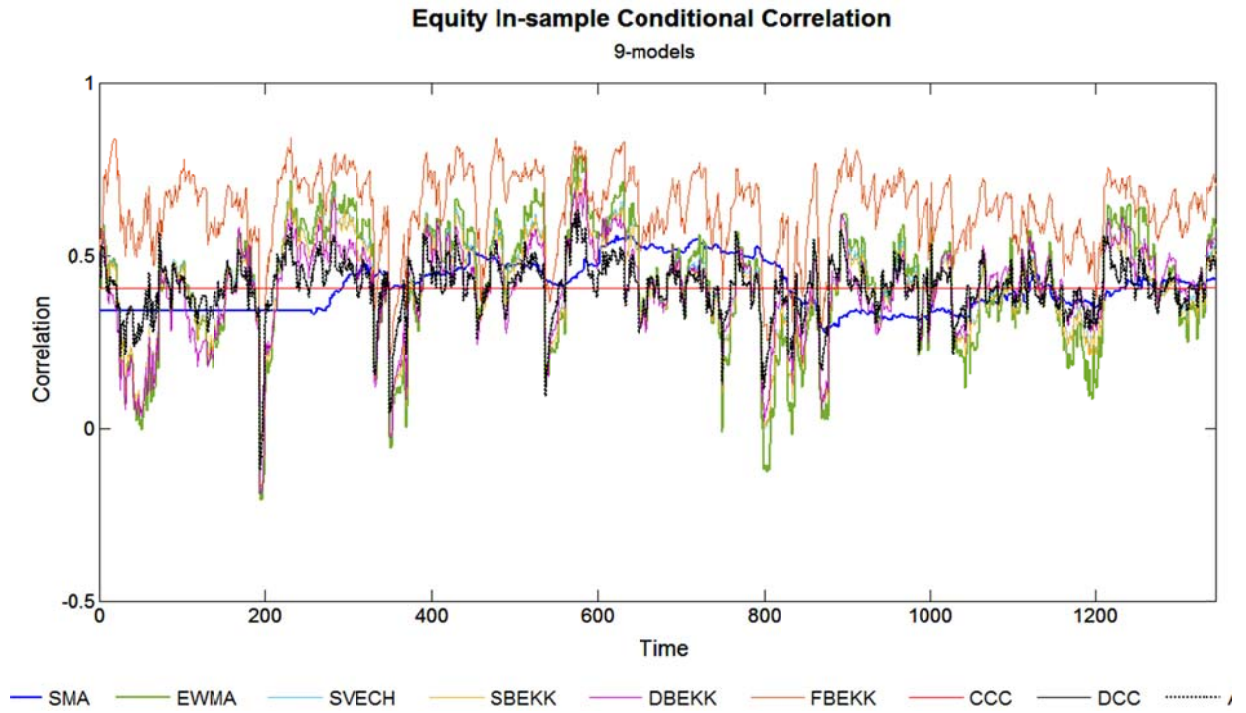
	Laurent et al. 2010	Caporin and McAleer 2012	Clements et al. 2012	Boudt et al. 2013	Laurent et al. 2013	Clements et al. 2009
Asset Class	Equity	Equity	Equity	Currency	Currency	Multi Asset
Sample	10 stocks NASDAQ NYSE	89 stocks S&P 100	200 stocks S&P 1500	EUR/USD Yen/USD	EUR/USD Yen/USD GBP/USD	5 futures on: S&P 500 NASDAQ US Tr. Bonds Gold Crude Oil
Covariance models	EWMA <sup>RM</sup> SBEKK DBEKK CCC <sub>vol</sub> DCC <sub>vol</sub> ADCC <sub>vol</sub> DECO <sub>vol</sub> OGARCH <sub>vol</sub> * (VC <sub>vol</sub> )	EWMA <sup>RM</sup> BEKK CCC <sub>vol</sub> DCC <sub>vol</sub> ADCC <sub>vol</sub> cDCC <sub>vol</sub> OGARCH <sub>vol</sub> *	EWMA <sup>FKO</sup> DCC <sub>vol</sub> DECO <sub>vol</sub> (SMA) (MIDAS)	EWMA <sup>RM</sup> SBEKK DBEKK CCC <sub>vol</sub> DCC <sub>vol</sub> cDCC <sub>vol</sub> BIP-DCC <sub>vol</sub> BIP-cDCC <sub>vol</sub>	EWMA <sup>RM</sup> DBEKK CCC <sub>vol</sub> DCC <sub>vol</sub> OGARCH <sub>vol</sub> (GOGARCH <sub>vol</sub> ) <sup>42</sup>	EWMA <sup>RM</sup> EWMA <sup>FKO</sup> CCC DCC ADCC (SMA) (STAT)
Volatility models	Arch* Aparch* Egarch* Garch* Gjr* Hgarch Igarch Figarch Rm	Garch* Gjr	Gjr	Bip-garch	Garch Aparch Egarch Gjr Igarch Rm	Garch
Loss Function	Euclidean Frobenius	MSE QLIKE	GMVP	$\sum_{1 \leq i, j \leq N}  \sigma_{i,j,t} - h_{i,j,t} $	Frobenius	MSE QLIKE GMVP MVP

<sup>42</sup> GO-GARCH or Generalized Orthogonal GARCH model of van der Weide (2002) is an extension of O-GARCH where  $m = N$  (see O-GARCH subsection in Chapter 2 for definitions of  $m$  and  $N$ ).

Proxy	RCov <sub>5min</sub>	RBPCov <sub>5min</sub>	NA	RCov <sub>30min</sub> RBPCov <sub>30min</sub> ROWCov <sub>30min</sub>	RCov <sub>5min</sub>	RCov <sub>30min</sub>
MCS significance level (authors choice)	$\alpha=25\%$	$\alpha=25\%$	$\alpha=25\%$	$\alpha=25\%$	$\alpha=10\%$	$\alpha=25\%$

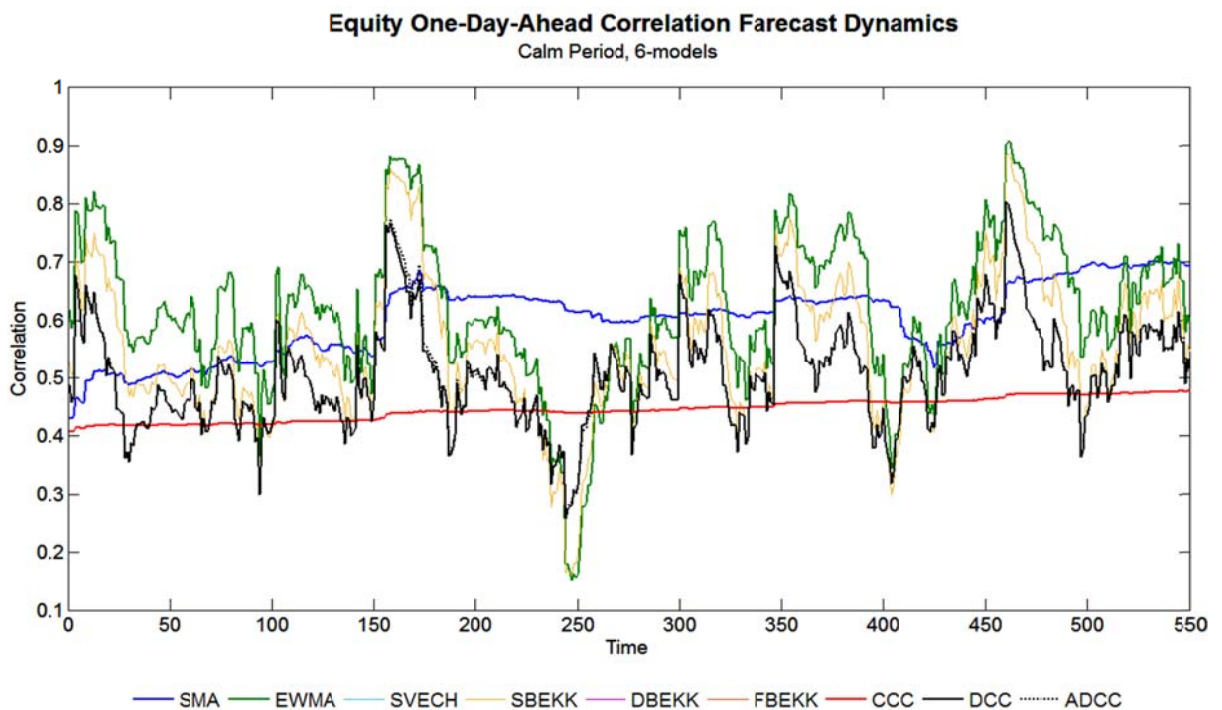
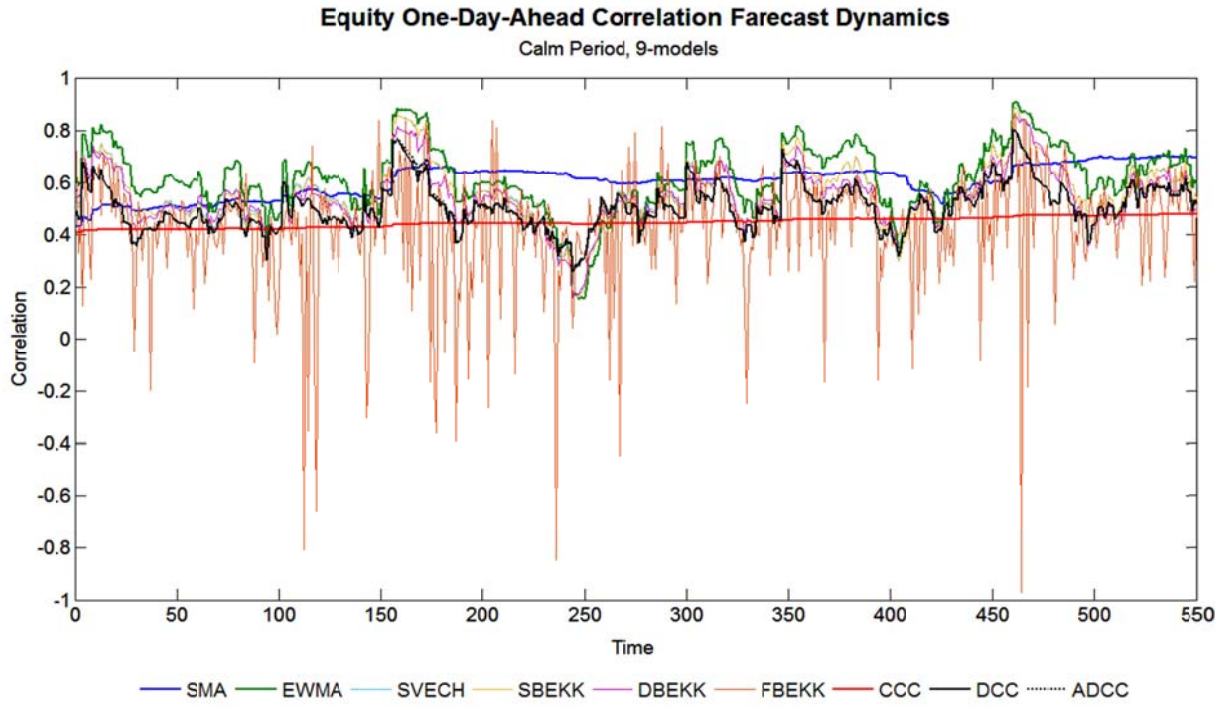
Subscript *vol* indicates that univariate volatility has been modeled according to specification under volatility models section (*vol* includes all volatility models, *vol\** includes only those models that are denoted with \*). Covariance models in parentheses were included in source literature, but were omitted from current survey. Source: Authors compilation

## Appendix 2. WIG and PX in-sample correlation dynamics (initial training set)

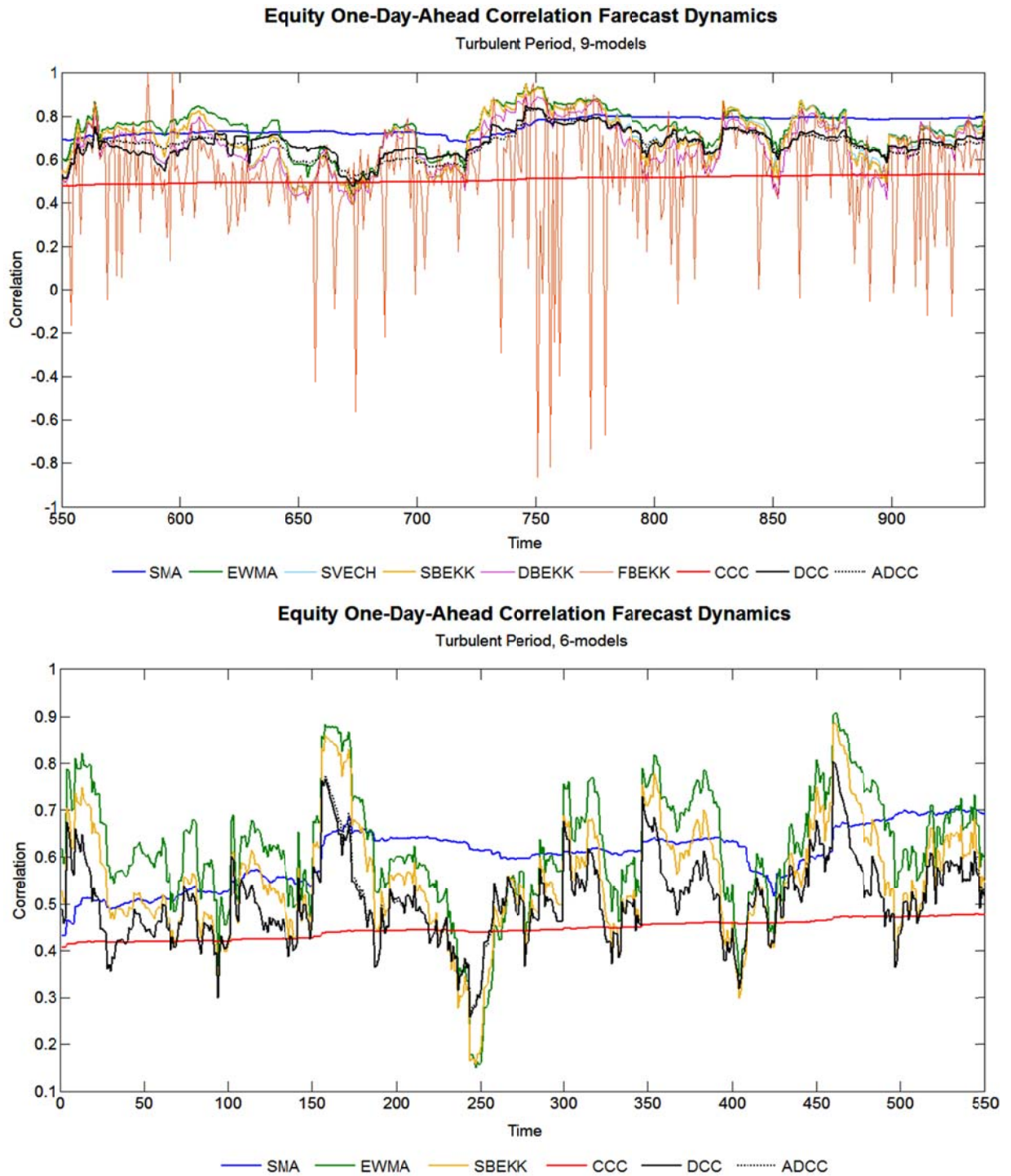




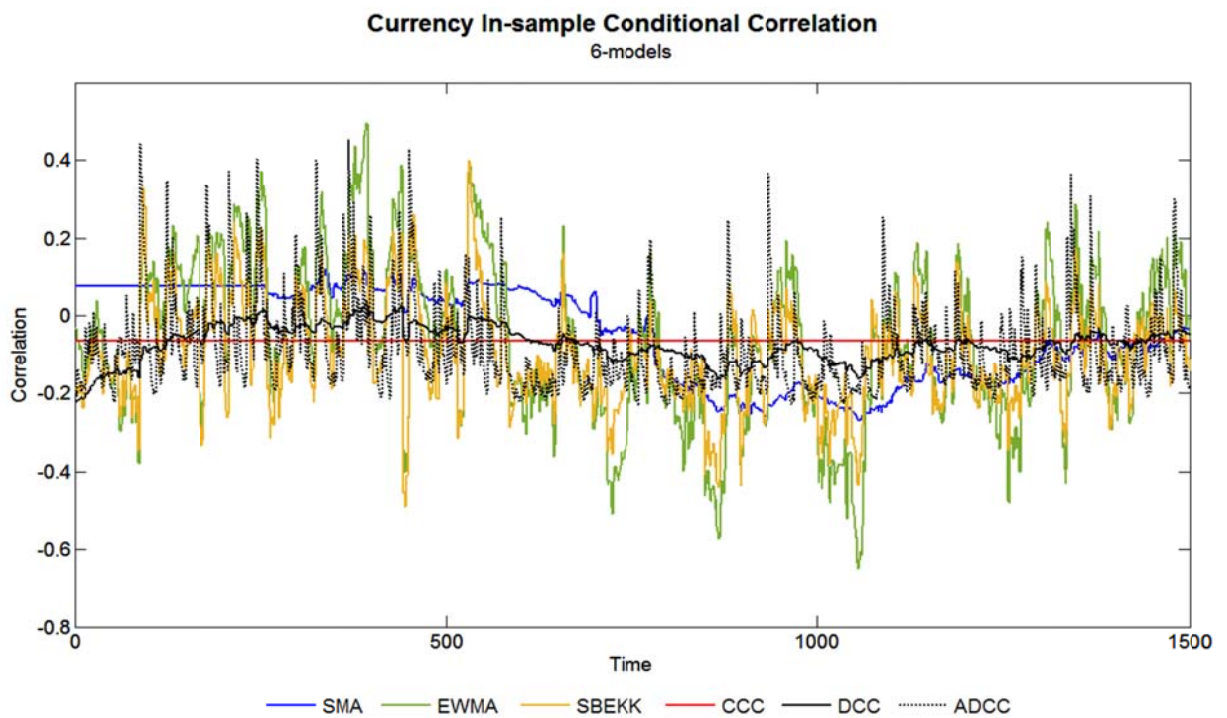
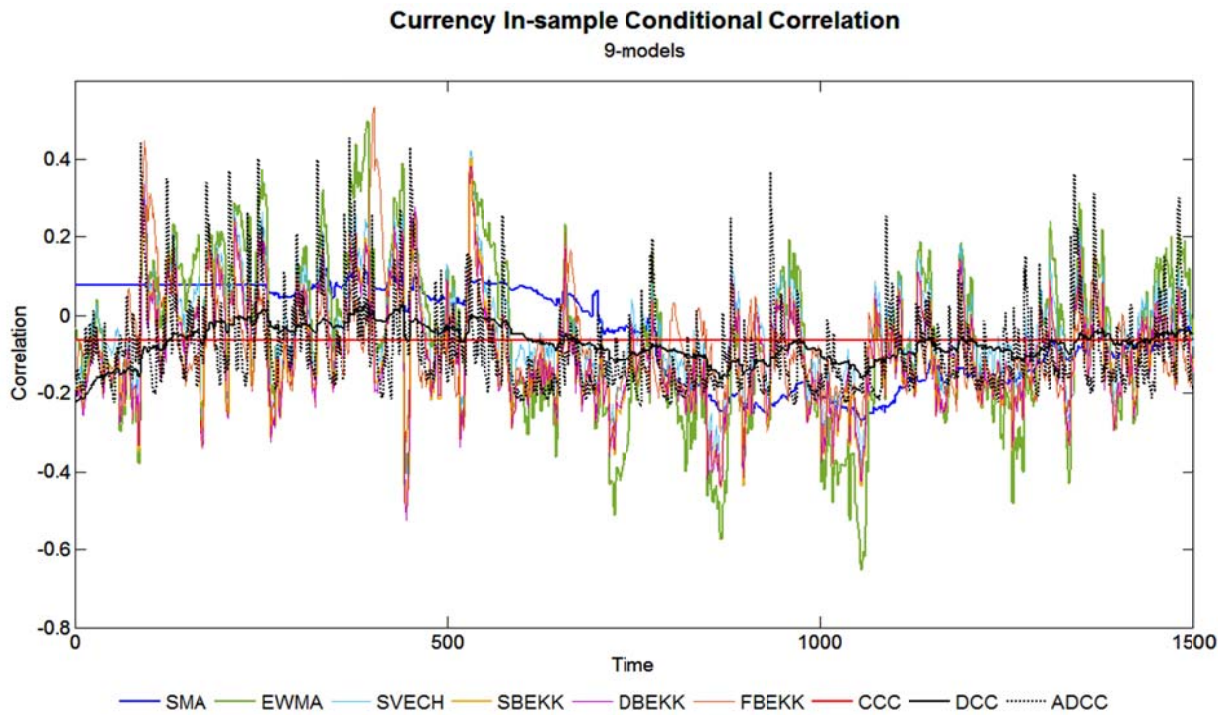
### Appendix 3. WIG and PX one day ahead correlation forecast dynamics



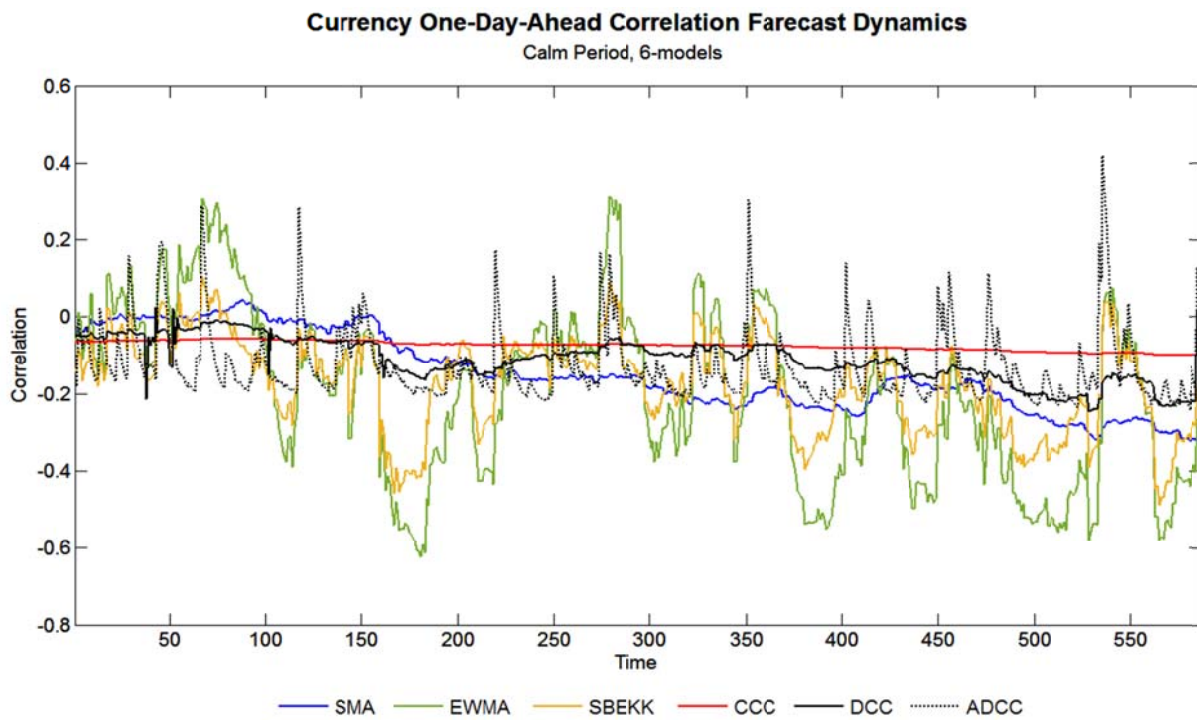
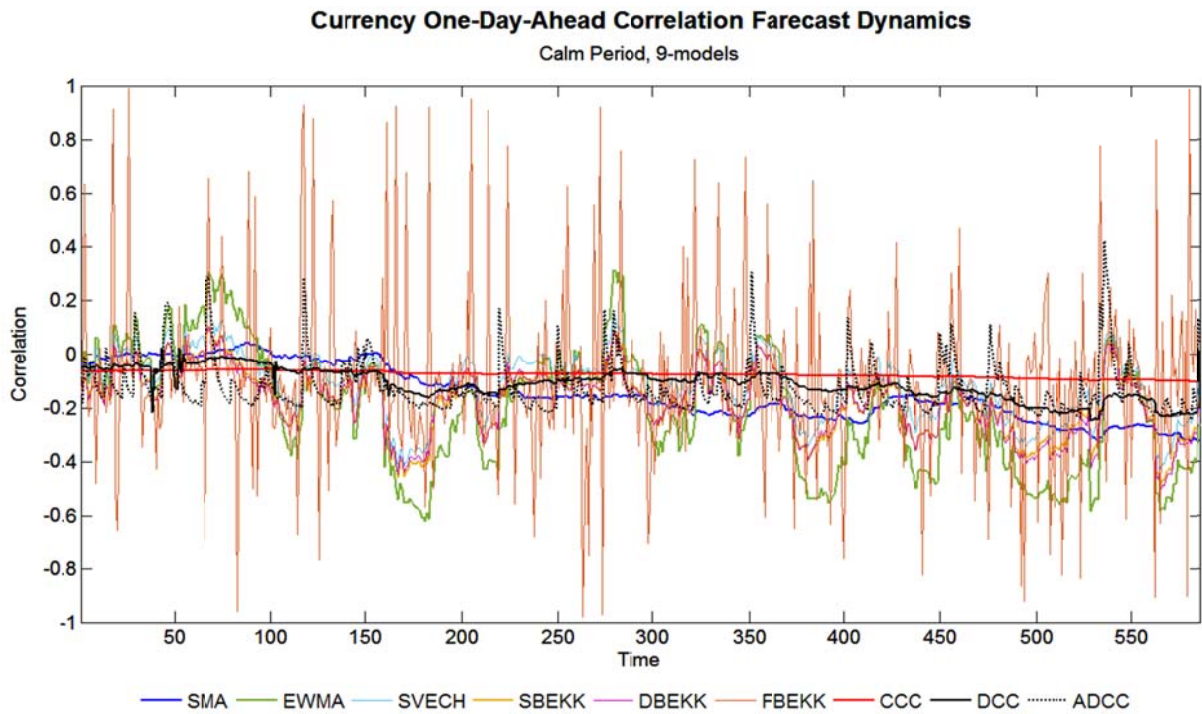
## Appendix 3 continued



## Appendix 4. EURCHF and EURPLN in-sample correlation dynamics (initial training set)

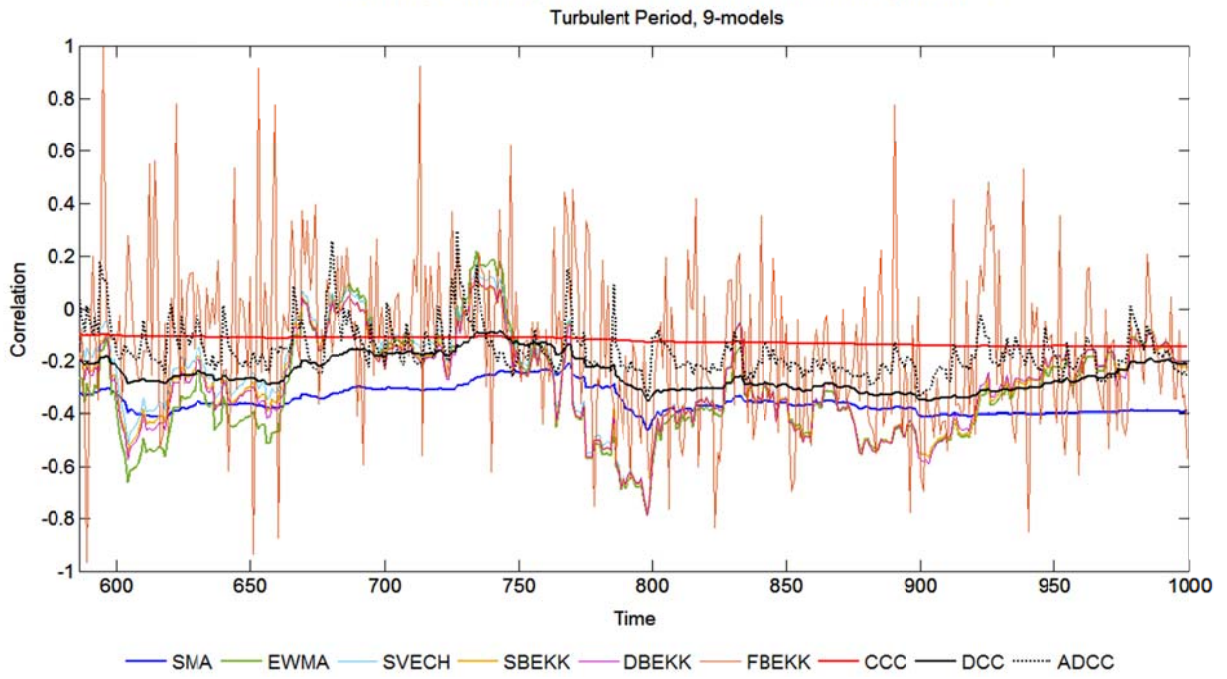


## Appendix 5. EURCHF and EURPLN one day ahead out-of-sample correlation dynamics



## Appendix 5 continued

### Currency One-Day-Ahead Correlation Forecast Dynamics



### Currency One-Day-Ahead Correlation Forecast Dynamics

