

THESIS ON NATURAL AND EXACT SCIENCES B76

**Technology of integrated photoelasticity  
for residual stress measurement in  
glass articles of axisymmetric shape**

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Declaration: Hereby I declare that this doctoral thesis, my original investigation and achievement, submitted for the doctoral degree at Tallinn University of Technology has not been submitted for any academic degree.

Johan Anton

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LOODUS- JA TÄPPISTEADUSED B76

**Integraalse fotoelastsuse tehnoloogia  
jäakpingete määramiseks  
telgsümmeetrilistes klaasobjektides**

JOHAN ANTON

TALLINN 2008



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## Introduction

Glass is the oldest man-made material, which due to its transparency and resistance is nowadays widely used in architecture, tableware, automotive industry, telecommunication etc.

Glass has two characteristic peculiarities. The first is the fragility of glass – it breaks easily due to tensile stresses. The second is the fact that in every glass item there exist residual stresses due to the complicated process during which glass changes from the state of a viscous liquid at high temperature into a solid state upon cooling. As during the cooling process the temperature field in the glass is inhomogeneous, residual stresses appear which can considerably reduce or increase the strength of the article. Measurement and control of residual stresses are inseparable parts of modern glass technology. In this thesis the residual stress measurements are carried out by using photoelastic effect.

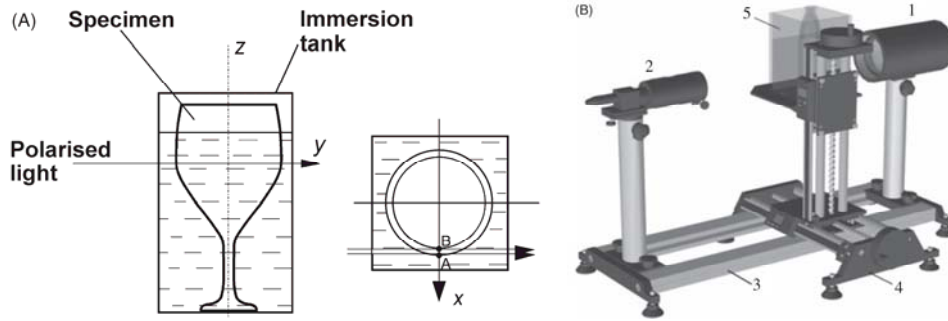
The photoelastic effect in glass was discovered in the beginning of the 19<sup>th</sup> century independently by the Estonian-German physicist Seebeck and the British physicist Brewster. Since then, photoelasticity has been widely used for the estimation of residual stress in glass. However, until the middle of the previous century this estimation has been mostly qualitative, based on the observation of the interference colors in simple polariscopes. Later on many quantitative photoelastic techniques for glass stress analysis have been elaborated. More popular of them include frozen stress method and scattered light method. An exhaustive review can be found in [1].

By the end of last century the theory of integrated photoelasticity was well developed, but there were no technologies or algorithms for quick and accurate automated measurements. Popular methods of evaluating residual stress in glass still include spring punch test, pendulum test, weight dropping or just measuring the optical retardation field.

The aim of this thesis is to develop nondestructive measurement technology in terms of software algorithms and automatic polariscopes for a quick and simple automated residual stress measurement in glass articles of axisymmetric shape. The result should be a practical quality control tool for the manufacturers of axisymmetric glass objects like drinking glasses, bottles, CRT tubes, lamps, fibre preforms, etc. Due to different nature of the objects and the different stress distributions in them several algorithms has to be developed. The technique used in this thesis for stress measurement is integrated photoelasticity [2].

In integrated photoelasticity the test object is placed in an immersion tank to avoid refraction of light and a beam of polarised light is passed through it (Figure 1A). The light rays which pass the axisymmetric hollow glass article parallel to the midsurface, between points A and B, are the most informative. This is termed ‘illumination by tangential incidence’. Measurements are carried out with a computer controlled polariscope (Figure 1B). Transformation of the polarisation of light in the specimen is measured on many light rays. The light source 1 contains a

set of polaroids and quarter-wave plates that permit precise photoelastic measurements by using the fringe or phase-stepping method [3, 4].



**Figure 1:** Experimental set-up in integrated photoelasticity (A) and automatic polariscope AP-06C (B): 1 – box of the light source, 2 – CCD camera, 3 – optical rail, 4 – coordinate device, 5 – immersion tank with the specimen

As the values and directions of the principal stresses vary on the light rays, optical phenomena in integrated photoelasticity are complicated and the relationships between the measurement data and parameters of the stress distribution are non-linear. However, it has been shown [1, 5] that if birefringence is weak or rotation of the principal stress directions on the light rays is weak, a 3D specimen can be investigated in a conventional transmission polariscope similar to 2D specimens.

On every light ray it is possible to determine the parameter of the isoclinic  $\varphi$  and the optical retardation  $\Delta$ . The latter are related to the components of the stress tensor on the ray by the simple integral relationships

$$\Delta \cos 2\varphi = C \int (\sigma_z - \sigma_x) dy, \quad (1)$$

$$\Delta \sin 2\varphi = 2C \int \tau_{zx} dy, \quad (2)$$

where  $C$  is the photoelastic constant and  $\sigma_x$ ,  $\sigma_z$ , and  $\tau_{zx}$  are components of the stress tensor in the plane perpendicular to the light ray  $y$ . Formulas (1) and (2) express the integral Wertheim law.

It is valid if either optical retardation is less than one-third of the wavelength or rotation of the principal stress along the light ray is less than  $\pi/6$ . If no rotation of the principal stress axes is present, Equations (1) and (2) are always valid. The integral Wertheim law is a generalization of the ‘classical Wertheim law’, used in 2D photoelasticity, for the case of 3D photoelasticity when stresses on the light beam are not constant.

Experimental measurements have shown that in annealed hollow glass articles and in glass fibre preforms the birefringence is almost always weak. In tempered drinking glasses birefringence is high but mostly rotation of the principal stress axes on the light rays can be ignored. Thus, with certain exceptions (for example,

knuckles of highly tempered thick drinking glasses), measurement of stresses in annealed as well as in tempered axisymmetric glass articles can be based on equations (1) and (2). It has been shown that if the parameter of the isoclinic  $\varphi$  and optical retardation  $\Delta$  have been measured on many light rays in two parallel sections, perpendicular to the specimen axis  $z$ , then radial distribution of the axial stress  $\sigma_z$  and shear stress  $\tau_{zx}$  can be determined [1]. Using the equilibrium equation and the sum rule, all the stress components can be determined [6, 7]. Influence of the measurement errors on the calculation of stress components is considered in [8].

This thesis considers residual stress measurement only in axisymmetric glass articles. The reason for that is that only in the axisymmetric case formulas (1) and (2) permit complete measurement of the residual stress field. Photoelastic tomography [9, 10] opens the possibility for stress measurement also in glass articles of arbitrary shape, but such a measurement technology is considerably more complicated and out of the scope of this thesis.

The present thesis is based on four academic papers, which are referred to in the text by their Roman numerals I-V.

- I. **J. Anton**, A. Errapart, H. Aben. Measurement of tempering stresses in axisymmetric glass articles. *Internat. J. Forming Processes*, 2004, **7**, No. 4, DOI 10.3166/ijfp.7.543-554, 543-554. (12p)
- II. H.Aben, L.Ainola, **J.Anton**. Half-fringe phase-stepping with separation of the principal stress directions. *Proc. Estonian Acad. Sci. Eng.*, 1999, **5**, No. 3, 198-211. (13p)
- III. H.Aben, **J.Anton**, J.Josepson. Nonaxisymmetric residual stress distribution in axisymmetric glass articles. *Glastech. Ber. Glass Sci. Technol.*, 1996, **69**, Nr. 3, 75-81. (7p)
- IV. **J. Anton**, A. Errapart, H. Aben, L. Ainola. A discrete algorithm of integrated photoelasticity for axisymmetric problem. *Exp. Mech.*, 2008, DOI 10.1007/s11340-008-9121-9. (8p)



### **Approbation**

1. 8<sup>th</sup> International Congress on Glass, San Francisco, USA, 1998.
2. International Conference on Advanced Technology in Experimental Mechanics ATEM'99, Ube, Japan, 1999.
3. Danubia-Adria Symposium 2003, Győr, Hungary, 2003.
4. International Conference on Advanced Technology in Experimental Mechanics ATEM'03, Nagoya, Japan, 2003.
5. Danubia-Adria Symposium 2007, Sibiu, Romania, 2007.

### **Author's contribution**

The author of this thesis developed the fringe method (Paper I) and the method for non-axisymmetric stress distribution (Paper III). The author took part of the development and validation of the phase-stepping method (Paper II) and the onion-peeling method (Paper IV). All methods were realized by the author in the software of automatic polariscope. Automatic polariscopes AP were constructed and built by the author to carry out practical measurements.

### **Acknowledgement**

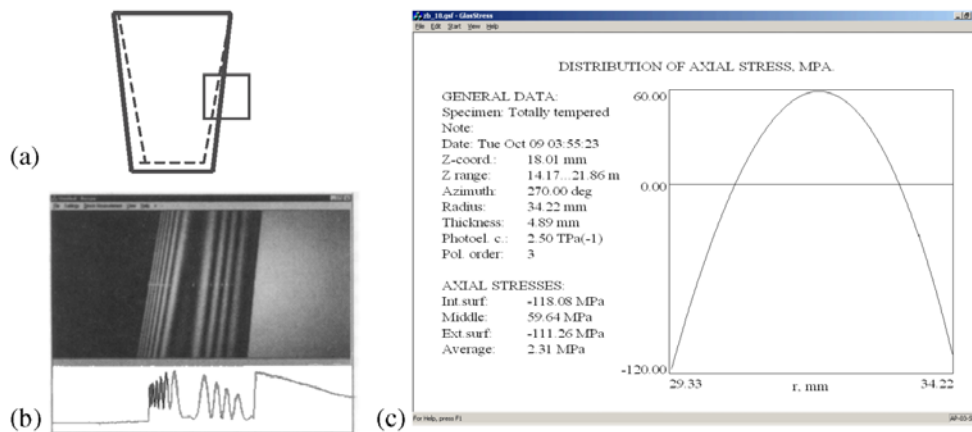
This research was carried out at the Laboratory of Photoelasticity at Institute of Cybernetics at Tallinn University of Technology.

I am deeply grateful to my supervisor acad. Hillar Aben for introducing me into the field of photoelasticity and for encouraging me to bring my doctoral studies to the procedure of defence. I wish to thank all my colleagues at the Laboratory of Photoelasticity for providing the pleasant working atmosphere and for all the help during the research.

## 1. Determining stresses in tempered glass objects

Thermal tempering is the most popular way to increase the mechanical resistance and safety of glass products. The aim of tempering is to introduce strong compression stresses onto the surfaces of the glass and tension inside the glass. The magnitude and distribution of residual stresses caused by tempering is one of the most important quality characteristics of the final product.

In strongly tempered tableware the birefringence is high and in a circular polariscope usually many interference fringes can be observed (Figure 2). In this case the fringe pattern can be digitized and using this information the stresses can be calculated [11].



**Figure 2.** Geometry of a tempered tumbler (a), physical and digitized fringe patterns (b), and axial stress distribution (c)

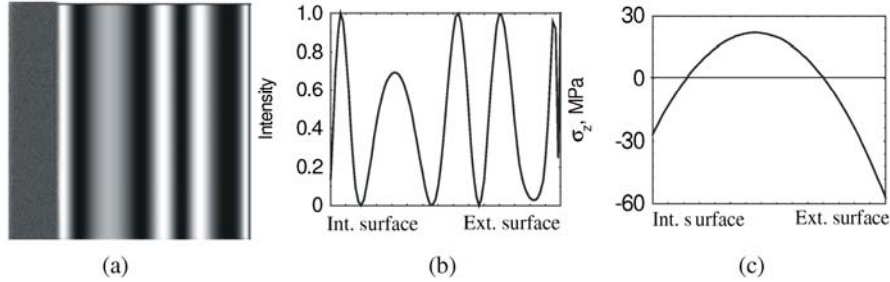
### 1.1. Method of fringes

In case of multiple fringes one gets enough information to determine stresses by recording the locations of the fringes and their order.

Using modern image processing software, the fringe locations are easily found. Determination of fringe orders, however, is not trivial. Different approaches have been used in literature to get some initial value for the unwrapped retardation. They include using phase unwrapping with additional measurements at different wavelengths, using different compensation methods or phase-stepping methods [3], applying additional loads [12], etc. In **Paper I**, all information about stress distribution is extracted from only one image, taken by a CCD camera.

In case of tempered axisymmetric glass objects the best way to start identifying fringes is locating the fringe with maximum retardation. For that usually the light intensity gradient is used. However, due to using non-collimated light and aberrations in optical system the intensity amplitude of fringes diminishes when

fringe pattern gets denser and the intensity gradient gives distorted information about the location of the maximum order fringe. The problem is amplified if maximum-order fringe is not full or half order (Figure 3).

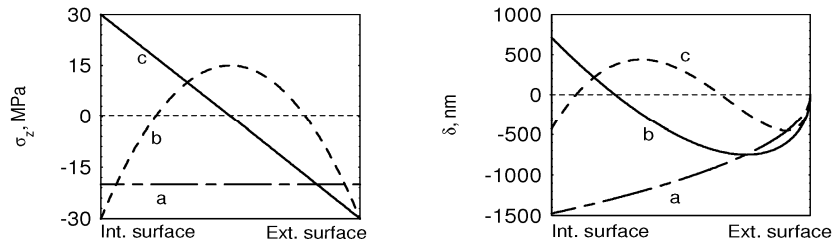


**Figure 3.** Fringe pattern in a bottle (a), digitized fringe pattern (b), and stress distribution (c)

To find correct fringe orders in **Paper I** a new criteria is used [13]. It is the product of the intensity gradient  $\nabla I(x, y)$  and its Laplacian  $\nabla^2 I(x, y)$ :

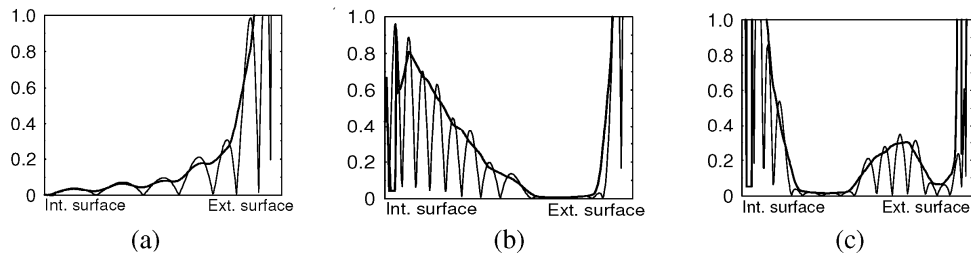
$$K(x, y) = \nabla I(x, y) \cdot \nabla^2 I(x, y) \quad (3)$$

Total stress in hollow glass is considered as a sum of membrane, bending, and tempering stresses (Figure 4, left). The corresponding optical retardation curves are shown on the right in Figure 4.



**Figure 4.** Membrane (a), bending (b), and tempering (c) stress distributions through the wall of hollow glassware and corresponding retardation distributions

After applying the operator  $K(x, y)$  to the fringe pattern, it is easy to find the fringe with the highest order by locating the minimums in Figure 5. The rest of the fringes are sequentially numbered from the maximum-order fringe.



**Figure 5.** Images of the modified fringe patterns representing membrane (a), bending (b), and tempering stress (c) after applying the operator K.

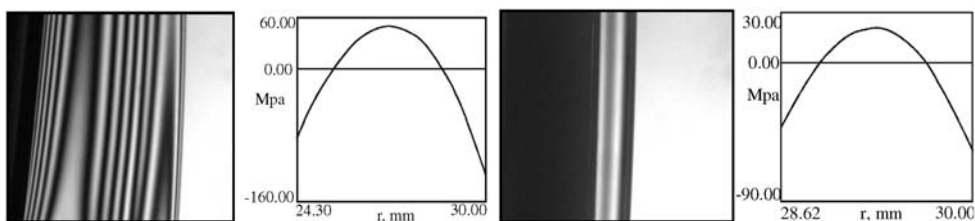
This procedure gives only the correct shape of the retardation curve. The absolute values of fringe orders are determined by shifting the retardation curve up and down by the amount of one wavelength and each time calculating stresses from it. The minimum average stress over the thickness is a reliable criterion for selecting the final stress distribution.

### 1.2. Optimizing the number of terms in stress polynomial

By using polynomial approximation of the axial stress, one has to decide how many terms to keep in the polynomial that is used to approximate stresses. A reasonable approach is to interpret the measurement data with different numbers of terms and to compare the results. Practical measurements have shown that minimum number of terms is 3. If the number of terms is too big, the stress values at the boundaries start to jump hectically. Usually most stable and close to each other are stress distributions described by 3, 4, or 5 terms. The higher the number of fringes, the more terms in the approximating polynomial are justified. If fringes have been recorded correctly, stress distributions with 4, 5, and 6 terms should be close to each other indicating that the results are reliable.

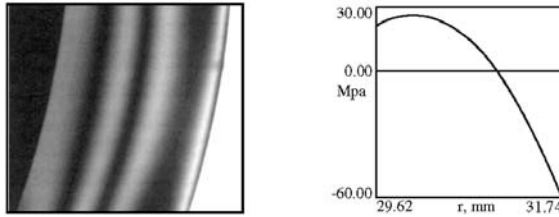
### 1.3. Examples

Figure 6 shows two examples of stress distributions measured with fringe counting method where tempering stresses prevail. Small bending component is also visible due to inferior tempering conditions inside the glass.



**Figure 6.** Fringe patterns and residual stress distributions in two tempered drinking glasses with different thicknesses and numbers of fringes

Only one extreme in curve in Figure 5b indicates that bending stresses prevail. In that case one additional procedure may be needed – the retardation curve is flipped around zero axis if the first term in the cubic stress polynomial is not negative. This permits measurement of the specimens with high bending stresses (Figure 7).



**Figure 7.** Fringe pattern and residual stress distribution near the neck of a tempered bottle with high bending stresses

## 2. Determining stresses in case of weak birefringence

Since the stress-optic coefficient of glass is low then in glass objects with low stresses or just small objects the total optical retardation due to birefringence usually stays below half the wavelength and no fringes appear.

In **Paper II** it is shown that if optical retardation is less than half the wavelength, the true values of optical retardation and of the parameter of the isoclinic, including the direction of the first principal stress, can be uniquely determined using phase-stepping. The result is especially important for integrated photoelasticity where a priori information about the stress field is limited. The influence of the measurement errors on the results is investigated and application of the method described.

### 2.1. Classic phase-stepping methods

Voloshin and Burger [14] showed that optical retardation can be comparatively easily determined automatically if it is less than half the wavelength. However, they did not consider determination of the isoclinics.

Nowadays phase-stepping is most widely used in automatic photoelastic measurements. The idea of the method was introduced by Hecker and Morche [15] and later considerably developed by Patterson and Wang [3, 16, 17], Kihara [18], Asundi [19], Umezaki et al. [20, 21], Quiroga and González-Cano [22, 23], Ramesh and Mangal [24] and many others. A review of different phase-stepping methods is given in [25].

A number of problems are associated with the phase-stepping technique. Both the optical retardation  $\Delta$  and the isoclinic angle  $\varphi$  are found using an arctangent operator. The data obtained lie in the ranges

$$-\frac{\pi}{4} \leq \varphi \leq \frac{\pi}{4}, \quad (4)$$

$$-\frac{\pi}{4} \leq \Delta \leq \frac{\pi}{4}. \quad (5)$$

The following questions arise. First, which of the principal stress axes,  $\sigma_1$  or  $\sigma_2$  is determined by  $\varphi$ ? Second, how to find the true value of  $\Delta$ ?

Optical retardation is determined by the formula

$$\Delta = C \frac{2\pi}{\lambda} (\sigma_1 - \sigma_2) \cdot t, \quad (6)$$

where  $C$  is the photoelastic constant,  $\lambda$  is the wavelength, and  $t$  is the thickness of the model. Thus,  $\Delta$  is essentially a positive parameter, although phase-stepping algorithms may give it also a negative value.

In interpreting the measurement data, the operator has to identify optical retardation at least at one point [26]. The sign and the first derivative of optical retardation are required to generate its continuous distribution [17]. For the separation of the  $\sigma_1$  and  $\sigma_2$  directions, no algorithm has been described in the literature on phase-stepping. The method for determining the fast axis of a wave plate, described in [27], needs interferometric measurements and cannot be applied with standard phase-stepping polariscopes.

Thus, in applying phase-stepping in two-dimensional photoelasticity, some a priori information about the stress field is needed for interpretation of the measurement data. Measurement data are interpreted for the whole field, using conditions of continuity, boundary values, etc.

Integrated photoelasticity [2] is nowadays mostly used for residual stress measurement in axisymmetric glass articles [1, 28]. Since stress distribution is determined after solving an inverse problem for a system of Fredholm integral equations, a priori information about the stress field is very poor.

Generally, in integrated photoelasticity the characteristic parameters are to be measured [1, 2]. A phase-stepping method for this purpose has been elaborated by Tomlinson and Patterson [29]. However, if birefringence is weak, optical measurements are similar to those applied in two-dimensional photoelasticity [1, 5], only interpretation of the measurement data is different. In measuring stresses in axisymmetric glass articles which are not tempered (e.g., bottles, neck tubes of CRT bulbs, electric lamps, optical fibre preforms, etc.), optical retardation is usually small (less than half the wavelength). In this case the algorithms of integrated photoelasticity demand measurement of the isoclinic angle and of optical retardation and ordinary phase-stepping can be used.

In two dimensional photoelasticity measurement data on the boundaries can be interpreted directly, but in integrated photoelasticity the light ray, which is tangent to the boundary does not give any information. Stresses can be determined only

after measurements in the entire section have been carried out. Besides, algorithms of integrated photoelasticity demand that the direction of  $\sigma_1$  should be unambiguously known.

Therefore, using phase-stepping in integrated photoelasticity, it is of paramount importance to reveal from the measured values of  $\varphi$  and  $\Delta$  their true values by identification of the direction of the first principal stress without any a priori information about the stress (and birefringence) field.

## 2.2. Calculating true values of $\varphi$ and $\Delta$

In **Paper II** an algorithm for single-valued interpretation of the phase-stepping measurement data for the case of optical retardation being less than half the wavelength is elaborated. The algorithm gives also the direction of the first principal stress.

The algorithm described in **Paper II** is easy to realize in software and does not need any interference of the operator. Since the angle  $\varphi$  gives the azimuth of the slow axis of the model, the directions of  $\sigma_1$  and  $\sigma_2$  are determined unambiguously. The same algorithm is valid when four light intensities are recorded [30].

## 3. Determining non-axisymmetric stress distribution

Residual stress measurements in many bottles, tumblers and in other axisymmetric glass articles by using integrated photoelasticity have shown that the residual stress distribution often deviates from the axially symmetric one. Deviations from axial symmetry lead to less favorable distribution of the residual stresses; e.g. they may lead to tensile residual stresses on the surfaces.

Approximate values of the stresses can be calculated using the algorithm which is based on the assumption that the stress distribution is axisymmetric. There is no controversy in this statement, e.g., if stresses vary symmetrically relative to the meridional section measured, the measurement data corresponds exactly to the stresses in the meridional section. However, with a more complicated variation of stresses around the perimeter the simplified algorithm may not be sufficiently precise.

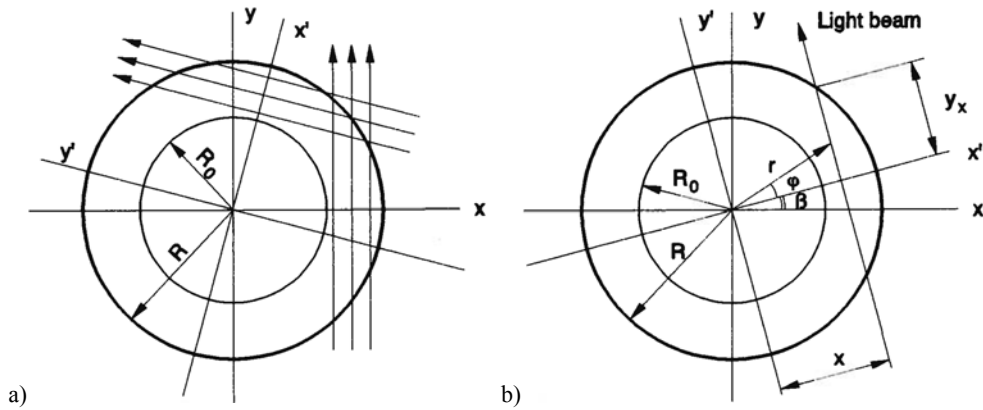
Therefore, in **Paper III** an algorithm, which is free from the assumption about axial symmetry has been developed for calculation of the stresses. The case when the stress gradient in the direction of the axis (axis of the article) is absent or weak is considered. In this case the only stress component which influences the polarization of light is the axial stress  $\sigma_z$ .

On investigating an axisymmetric glass article, the light can be passed through the latter perpendicularly to different meridional sections (Figure 8a). Assuming that stresses vary both in radial and circumferential directions,  $\sigma_z$  can be presented as

$$\sigma_z(\varrho, \beta) = f_0(\varrho) + \sum_{m=1}^M [f_m^c(\varrho) \cos m\beta + f_m^s(\varrho) \sin m\beta] \quad (7)$$

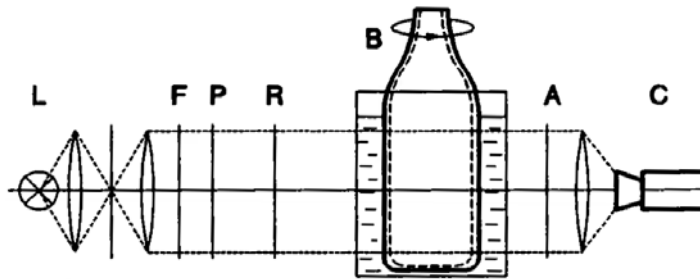
where the angle  $\beta$  is shown in Figure 8b, and  $\rho = r/R$ .

The functions  $f_0(\rho)$ ,  $f_m^c(\rho)$  and  $f_m^s(\rho)$  are expressed as power series, whose coefficients are to be determined from the measurement data by solving an overdetermined system of equations with least squares method.



**Figure 8.** The light is passed through the article perpendicularly to different meridional sections (a) and geometrical notations for tangential incidence (b)

The measurements are carried out with an automatic polariscope (Figure 9) that enables quick measurement of the axial stress distribution through the wall of the specimen. A software package has been developed which calculates stresses in the section where measurements were made. After the photoelastic measurements in a position of the article are made, it is turned by a stepper engine for a certain angle  $\Delta\beta$ , and again the photoelastic measurements and stress calculation are carried out. Stress distribution through the wall at any value of  $\beta$  as well as distribution of the surface stresses over the perimeter can be shown on the monitor.



**Figure 9.** Optical system of polariscope. L: light source. F: filter, P: polarizer. R: retarder. B: bottle in immersion bath, A: analyzer. C: CCD camera

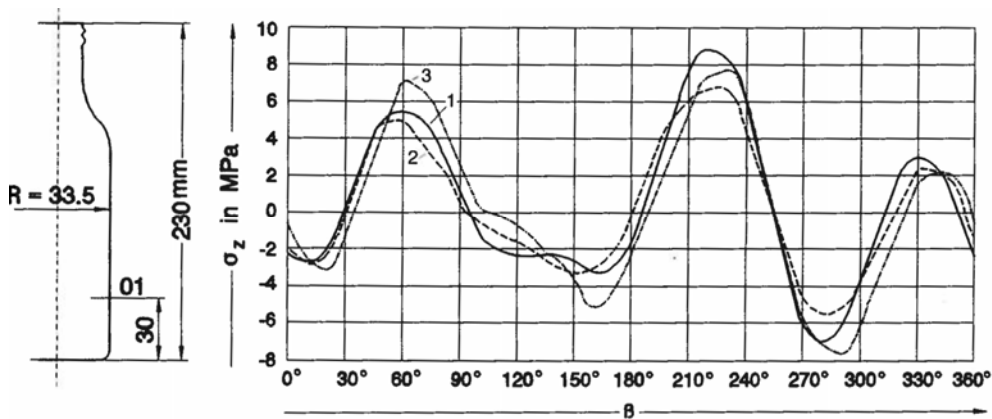


If the bottle is under internal pressure, the value of the latter is also calculated. The algorithm for that is based on the fact that since residual stresses must be in equilibrium over the section, the resulting axial force in the bottle is caused by internal pressure.

### 3.1. Comparison of the stress calculation algorithms

One important indicator of the residual stresses in a glass article is the distribution of the average through-wall (membrane) axial stress over the perimeter. This distribution may be measured by different methods. Firstly, by using tangential incidence, one may interpret the measurement data for a meridional section assuming that stress distribution is axisymmetric. Secondly, membrane stresses can be determined by using passage of light through the bottle wall in the direction of the normal to the latter (using a reflection polariscope). Thirdly, most precise results are obtained by using the algorithm described **Paper III**.

Figure 10 shows the geometry of a 0.5 l bottle and the distribution of the axial membrane stress in section 01, obtained by using three methods. It shows that also simplified interpretation of the measurement data gives rather satisfactory results. However, at the sections with extreme values of the stresses simplified methods give for the stresses somewhat diminished absolute values.



**Figure 10.** Geometry of the 0.5 l bottle and distribution of the axial membrane stress in section 01 over the perimeter. Curve 1: exact algorithm, curve 2: local interpretation of the measurement data obtained by tangential incidence, curve 3: using normal incidence

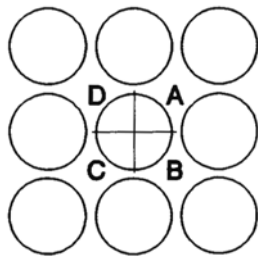
### 3.2. Origin of non-axisymmetric stresses

The reason for the deviation of the residual stress distribution from the axisymmetric one is probably the nonaxisymmetric temperature of the article during different stages of the production process as well as nonaxisymmetric cooling conditions. Significance of glass flow symmetry in the feeder bowl and

possibilities to improve it have already been considered in the literature [31]. Thus, it is important to have an axisymmetric temperature distribution in the gob.

The next reason for nonaxisymmetric temperature distribution in the bottle may be the noneven temperature of the mold.

Thirdly, in the lehr cooling conditions for the article are not the same over the perimeter. In Figure 11, the points A, B, C and D of the article in the middle cool down more quickly than other parts. This has been considered in [32] as the main reason for nonaxisymmetric residual stresses.

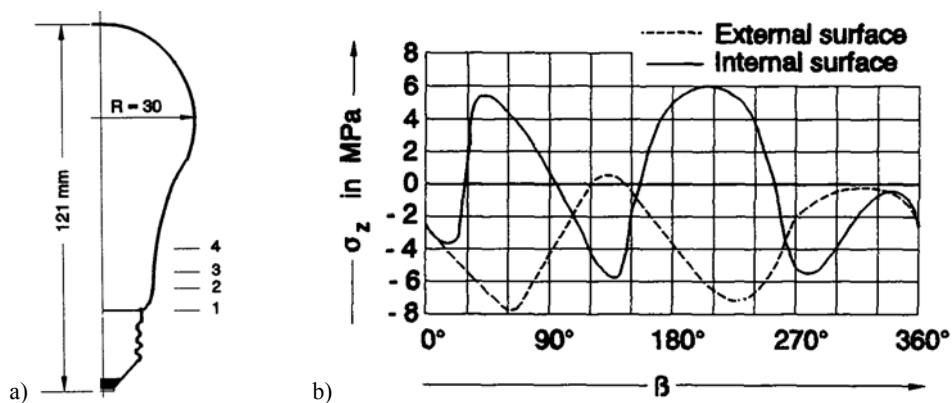


**Figure 11.** Position of axisymmetric glass articles in the lehr

There may be other reasons for the nonaxisymmetric distribution of the residual stresses. e.g. variation of the residual stresses over the perimeter is related to the variation of the wall thickness over the perimeter.

### 3.3. Example

The geometry of a lamp is shown in Figure 12a with the distribution of the meridional surface stresses over the perimeter of the lamp in section 3. The deviation of stress field from axisymmetric one is extreme.



**Figure 12.** Geometry of the electric lamp (a) and distribution of the meridional surface stresses around the perimeter in section 3 (b)

The measurement of residual stresses in many axisymmetric glass articles by integrated photoelasticity has shown that deviation of the residual stress distribution from the axisymmetric one may be considerable (like current example). This deviation causes less favorable residual stresses in the articles and decreases the resistance of the latter.

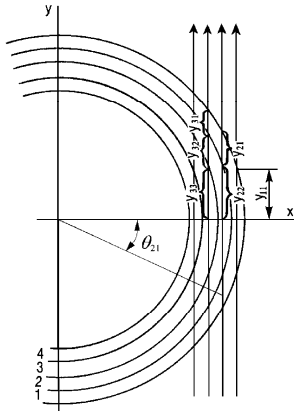
## **4. Discrete algorithm**

In integrated photoelasticity of axisymmetric stress fields until now mostly polynomial approximation of the stress components has been used [17, 18, 22, 23]. That presumes comparatively smooth stress distributions. In **Paper IV** the classical onion-peeling method is generalized for the case when stress gradient in axial direction of an axisymmetric specimen is present. An algorithm for the determination of all the stress components is presented. The method is based on the linear approximation of integrated photoelasticity when equations (1) and (2) are valid.

### **4.1. Classic onion-peeling method**

Classic onion-peeling method deals with simplest case – stress measurement in cylinders with no stress gradient in axial direction. In this case one of the principal directions is parallel to the cylinder axis and optical retardation is due only to axial stress [2, 33]. Thus determination of the axial stress distribution leads to the Abel integral equation and Abel inversion, which was first used by Sutton [34] and later on by many authors for measuring stresses in optical fibers and fiber preforms [35-37]. For the calculation of the stress components  $\sigma_r$  and  $\sigma_\theta$ , the equilibrium equation and the classical sum rule [6] can be used.

The direct use of Abel inversion for inverting the data requires precise knowledge of measurement data, but in practice, there are inevitable errors in measured optical retardations. In the literature [38] it has been stated that fitting of polynomials to the data and integrating Abel inversion directly may lead to erroneous values of  $\sigma_z$ . This problem is considered in detail by Minerbo and Levy [39].



**Fig. 13** Discrete representation of a section of the specimen

At the same time, Drucker and Woodward [40] proposed a simple method for numerical calculation of optical retardation distribution. Let us consider the cylinder as consisting of a number of cylindrical rings (Figure 13), in each of which the axial stress  $\sigma_z$  may be considered constant.

Figure 13 shows that  $\sigma_z$  in ring 1 is determined through phase retardation on the ray, which is tangent to the middle surface of the first ring. The  $\sigma_z$  in any inner ring is determined through phase retardation on that ray minus the cumulative retardation contributed by all outer rings.

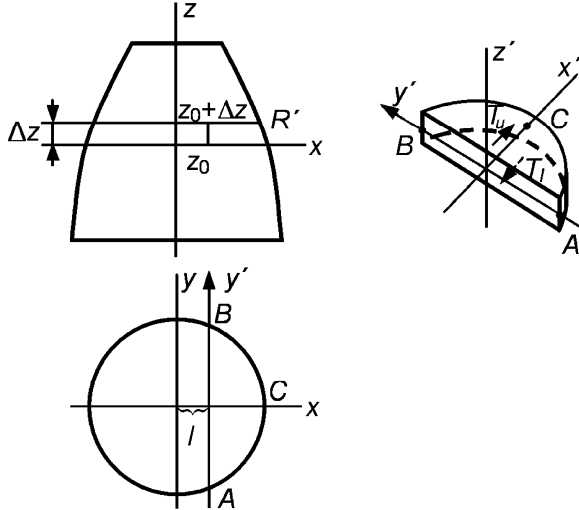
This so-called onion-peeling method has been used for stress measurement in cylindrical epoxy models [41] and in optical fibers [42, 43].

## 4.2. Generalized onion-peeling method

The aim of **Paper IV** is to generalize the onion-peeling method for the general axisymmetric state of stress when stress gradient in the axial direction  $z$  is present. In comparison with the cylinder, the problem is much more complicated since also shear stress  $\tau_{rz}$  is present. We limit ourselves with the case of weak birefringence while in this case simple integral relationships between the stress components and measurement data are valid [5]. Besides, integrated photoelasticity is nowadays mostly used for residual stress measurement in glass and in annealed glass birefringence is usually weak.

Several authors [44-46] have considered generalization of the onion-peeling method using general expressions of integrated photoelasticity. In this case every layer of the specimen is characterized by a Jones matrix. For stepwise determination of Jones matrices in all the layers, in [44] the equilibrium equation and the compatibility equation are used. In [45, 46] the authors use also the method of oblique incidence. In both cases a great number of multiplications of the Jones matrices may lead to the accumulation of errors. Application of the oblique incidence is undesirable since that excludes independent measurement of stresses in a single section of the specimen. In **Paper IV**, by generalizing the onion-peeling

method, simple expressions (1) and (2) of linearized integrated photoelasticity are used.



**Figure 14.** Investigation of an axisymmetric specimen with stress gradient in axial direction

The equations (1) and (2) were first applied by Davin [47]. Brosman used them by elaborating integrated photoelasticity for cubic single crystals [48]. Doyle and Danyluk developed, from their basis, a method of solving the axisymmetric problem using Abel inversion with polynomial approximation [49, 50]. In a different way equations (1) and (2) were derived in [5], where for the first time the domain of their applicability was investigated.

The distributions of the axial stress and shear stress are determined directly from the measurement data, using linear approximation of integrated photoelasticity. Let us consider stress measurement in an axisymmetric specimen (Figure 14). Sections  $z=z_0$  (main section) and  $z=z_0+\Delta z$  (auxiliary section) are scanned with light rays parallel to the  $y$  axis and on each light ray the parameter of the isoclinic  $\varphi$  and optical retardation  $\Delta$  are measured in both sections.

In **Paper IV**  $\tau_{rz}$  and  $\sigma_r$  are calculated directly from the measurement data using equilibrium of a three-dimensional segment ABC in Figure 14.

### 4.3. Determining all stress components

Determination of the stress components  $\sigma_r$  and  $\sigma_\theta$  has been considered by Ahmetzyanov and Solov'ev [44] in the case when stresses are due to external loads. Since integrated photoelasticity is mostly used by measuring residual stress in glass [1, 51], we consider complete stress analysis in this case.

In case of residual stresses one can not use the compatibility equation since the residual stresses are caused by incompatible initial deformations [52]. Since residual stresses in glass have thermal origin [53, 54], additional relationship

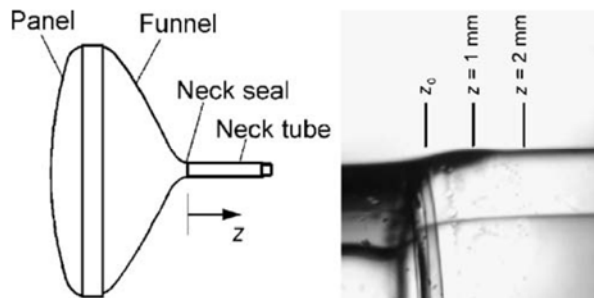
between the stress components can be derived from the equations of thermoelasticity.

For complete determination of stresses in cylinders without stress gradient in axial direction, the classical sum rule has been used [35, 36]. If stress gradient in the axial direction is present, one should use the generalized sum rule in second approximation [55], from which the  $\sigma_\theta$  can be recursively calculated.

In **Paper IV** stress components  $\sigma_r$  and  $\sigma_\theta$  are calculated from equilibrium equation and generalized sum rule. Thus the residual stress distribution in the specimen can be completely determined.

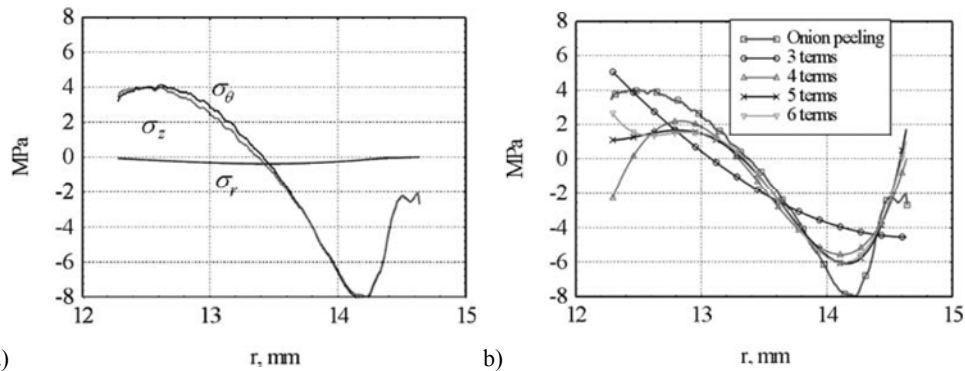
#### 4.4. Example

Photoelastic measurements were carried out near the neck seal of a CRT glass bulb (Figure 15) with a computer-controlled polariscope using the phase-stepping method. The measurement data was obtained in a section, located at 1 mm from the neck seal and in the auxiliary section, located at 2 mm from the neck seal.



**Figure 15.** Geometry of the CRT glass bulb and photoelastic pattern of the neck tube near the neck seal

Figure 16a shows distribution of the axial stress  $\sigma_z$ , determined with the onion-peeling method and with polynomial approximation, using different number of terms. It is seen that determination of the optimal number of terms is difficult – 3 and 4 terms give a totally non-satisfactory description of the distribution of  $\sigma_z$ . With 5 and 6 terms the distribution of  $\sigma_z$  is more or less satisfactorily described except in the vicinity of the internal surface of the neck tube, where these two cases give drastically different values of the axial stress. Thus advantages of the onion-peeling method are evident. Let us mention that strong dependence of the stress profile on the number of terms in the polynomial indicates that measurement data is noisy. The reason for this is optical inhomogeneity of the glass near the neck seal.



**Figure 16.** All the stress components in the main section (a) and axial stress distributions at  $z=1$  mm, determined with the onion-peeling method and with polynomial approximation, using different number of terms in the expression (b)

Figure 16b shows distribution of all the stress components through the tube wall at  $z=1$  mm, determined with the onion-peeling method described above. Since at the surfaces of the tube  $\sigma_r=0$ ,  $\sigma_r$  has low value also in the middle part of the tube wall. The plots of the stresses  $\sigma_\theta$  and  $\sigma_z$  are close to each other.

## Conclusions

It has been shown that integrated photoelasticity can be efficiently used for the measurement of residual stresses in axisymmetric glass articles. Four developed algorithms cover the stress range from zero up to breakage and enable to measure most axisymmetric glasses in areas, where linear approximation of integrated photoelasticity can be applied. The technology in this thesis has been successfully realized in automatic polariscopes and applied in numerous glass companies over the world, proving its usefulness to glass technologists.

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## **Abstract**

### **Technology of integrated photoelasticity for residual stress measurement in glass articles of axisymmetric shape**

Four algorithms for residual stress measurement in glasses of axisymmetric shape has been developed during this thesis.

The fringe counting method for tempered glasses is very quick and has practically no upper stress limit if there is no stress gradient in the direction of symmetry axis. New criteria for determining the location of characteristic maximum order fringe makes the fringe counting more robust and the algorithm works well with just a few fringes and with dozens of fringes.

The second algorithm works for glasses with optical retardation less than half the wavelength and uses phase-stepping to uniquely determine the true values of optical retardation and of the parameter of the isoclinic, including the direction of the first principal stress. The result is especially important for integrated photoelasticity where a priori information about the stress field is limited.

Third algorithm enables to measure glasses, where stresses may vary strongly around perimeter. The variation may often be large enough to pose a threat to the safety of the product by introducing tensile stresses on surfaces. Although measurement procedure is longer due to several measurements, the knowledge of such stresses is important for adjusting the production technology.

Finally, the onion-peeling method was generalized for the case of axisymmetric stress distributions when stress gradient in the axial direction is present. Measurement of the integrated isoclinic and optical retardation is carried out in two parallel sections, which are perpendicular to the specimen axis. The distributions of the axial stress and shear stress are determined directly from the measurement data, using linear approximation of integrated photoelasticity. Other stress components are determined using the equilibrium equation and the compatibility equation (if stresses are due to external loads) or using the generalized sum rule (if residual stresses in glass are measured). The method is less sensitive to measurement errors than the Abel inversion. In comparison with the polynomial approximation of the stress distributions, the onion-peeling method gives more adequate results if stress distribution is not smooth.

All presented research results are published in close to 40 scientific articles and realized in the software of automatic polariscopes AP, which are successfully used for residual stress measurement in numerous glass companies over the world.



## **Kokkuvõte**

### **Integraalse fotoelastsuse tehnoloogia jääkpingete mõõtmiseks telgsümmeetrilistes klaasobjektides**

Doktoritöös on välja töötatud neli algoritmi jääkpingete mõõtmiseks telgsümmeetrilistes klaasides.

Ribade meetod on karastatud klaasidele, kus kõrgete pingete tõttu on näha mitu interferentsriba. Meetod on väga kiire, kuna kasutab pingeanalüüsiks vaid ühte kaamera pilti ning praktiliselt puudub maksimaalse mõõdetava pinge piirang. Uus kriteerium suurima käiguvahega riba leidmiseks lisab oluliselt meetodi töökindlust ning võimaldab edukalt mõõta väga erinevate ribade arvuga klaase.

Teine algoritm on klaasidele, kus käiguvahe suurim väärtus jääb alla poole lainepikkuse. Näidatakse, et sellisel juhul saab faasisammude meetodiga üheselt määrata optilise käiguvahe ja isokliini parameetri väärtused, sealjuures ka esimese peapinge suuna. Tulemus on väga oluline integraalses fotoelastsuses, kus a priori informatsioon pingevälja kohta on tihti piiratud.

Kolmas algoritm võimaldab mõõta klaase, kus pinged võivad üle perimeetri tugevalt muutuda. Klaastoode võib muutuda ohtlikuks, kui pinnale tekivad tõmbepinged. Info sellise pingejaotuse kohta on oluline õnnetuste ennetamiseks ja tootmistehnoloogia parandamiseks.

Viimaseks on sibulakooremise meetod üldistatud juhule, kus teljesuunaline pingegradient on olemas. Integraalne optiline käiguvahe ja isokliini parameeter mõõdetakse kahes paralleelses lõikes, mis on objekti teljega risti. Kasutades integraalse fotoelastsuse lineaarset lähendust, saab mõõteandmetest otse arvutada telgpinge ja nihkepinge jaotused. Teised pingekomponendid arvutatakse tasakaaluvõrrandi ja summareegli abil (juhul, kui välist koormust pole). Meetod on mõõtevigadele vähem tundlik kui Abeli inversioon ning kirjeldab ebaühtlaseid või hüppelisi pingejaotusi (nt. fiibri toorik) paremini kui polünoomidega lähendamine.

Kõik tulemused on avaldatud ligi 40 teaduslikus artiklis ning on realiseeritud automaatpolariskoobi AP tarkvaras. Väljatöötatud tehnoloogia efektiivsust kinnitab polariskoobi AP ja selle tarkvara kasutamine paljude tuntud klaasifirmade poolt.