TALLINN UNIVERSITY OF TECHNOLOGY
School of Economics and Business Administration
Department of Finance and Economics
Chair of Finance and Banking

Mark Varlashev

# ASSET SELECTION IN PRIVATE WEALTH MANAGEMENT

Bachelor thesis

Supervisor: associate professor Ilzija Ahmet

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I do solemnly declare that I have written the presented research thesis by myself without undue help from a second person. Where I have used thoughts from external sources, directly or indirectly, published or unpublished, this is always clearly attributed.

Mark Varlashev
28.05.2014

Matriculation No.: 112531
E-Mail address: mark.varlashev@gmail.com

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# ABSTRACT

The purpose of this study is to create a solution which can be used to make an informed decisions regarding the goals-based asset allocation for private individuals. The solution is designed to address the prevalent issues of private wealth management. Author reviews different frameworks of goals-based investing and wealth management and combines them with behavioural approach and Multi-Criteria Analysis. Overview of assets' characteristics and their relative performance show that depending on period and clients' risk tolerance different asset classes offer different Multi-Criteria Analysis scores, therefore the findings may be useful as a base for further studies of assets' characteristics or long-term investment strategy development.

Keywords: investing, wealth management, decision making, asset selection

# INTRODUCTION

Industry of investing has changed drastically since the Global Financial Crisis in 2008, so did society's opinion regarding investments. While interest rates are at their historical lows, investors search returns everywhere. Moreover, there are risks to active managers - to show true skill, as they potentially obliged to present results, which should be better, than market averages. All these tendencies require proactive and creative approach in the wealth management from banks and advisors, especially in terms of customer-oriented risk management and asset allocation.

If we are looking at investing from non-professionals perspective e.g. individual investors such as private individuals or households (hereinafter private clients) it is reasonable might assume that this class of investors would be interested in simple way to achieve their goals by investing in a single asset or fund for example. This is true especially if clients' financial possibilities are limited, as the cost of active portfolio management, complex diversification and trading expenses might undermine the benefits and lower the returns of their investments.

Common problem in such services as consulting or finance planning is hidden conflict of interests, which due to the specific bonus systems or performance-based reward schemes cannot be eliminated or regulated. Service providers are usually interested in selling particular plan or solution and therefore clients' needs may be ignored. Example of retirement planning is used because it is quite problematic for clients to choose the most appropriate solution to meet their goals as they are not sure about their real goals and needs, while professional do not help them with any clarification. It might be explained by the lack of practical decision making tool, which can be offered for client to help him make decision from simpler, more comprehensive point of view.

In this research author's attention is focused on private clients for two reasons, which are in fact connected to each other: low popularity of investing between middle-class private

clients, narrowness of effective and comprehensive tools and solutions for this particular group of clientele.

Private clients can be attractive group for banks and investment professionals as one of the biggest wealth holders by absolute number of wage receivers, therefore client advisors or investments professionals can find good possibilities for value creation as for clients as for themselves.

The tool or solution can be built around the problem of asset selection in different time horizons, therefore the key hypothesis is that different asset classes offer different risk premiums over the different periods, and thus have variable suitability in terms of client goals, risk tolerance and expectations. For example in short term it might be more appropriate to invest in bonds to mitigate the volatility of investment during market turmoil, while for long-term investing it would be more reasonable to prefer equity to fixed income in order increase expected return.

Asset classes, which are available to every investor one way or another should be examined: developed and emerging markets equities, bonds, gold, real estate and commodities. Each of these classes can be represented by corresponding index so data can be studied with the help of quantitative analysis methods along with simulations for forecasting the return in different scenarios, also quantitative methods offer good possibilities for comparison.

Hereby the aim of this research is to create a tool with can be used to make an informed decisions regarding asset allocations in terms of goals-based investment using the strategy risk and expected return. It can be achieved by completing the following set of tasks:

1. Examine the characteristics of different asset classes.
2. Compare them on the basis of performance depending on selected period.

Author considers that this model would be useful as for clients (in terms of better understanding of risks and perspectives of their investments) as for advisors, wealth management and financial planning professionals by the way of an effective tool for effective comparison of different alternatives and preparing thoughtful solutions for clientele.

This research is divided into three sections. The first part contains the theoretical base used, a special place is paid to the key theories, Shefrin and Statman's Behavioural Portfolio Theory, H. Markowitz's Modern Portfolio Theory and review of quantitative methods as

Stochastic Modelling and Weighted Product Model. The second section contains information about selected indices, important limitations and how selected methods are being used in this research (formulas and calculation analysis). The third sections contains results, graphs and tables along with discussion. The summary together with proposals for further researches presented in the last section. References and appendix can be found in the end of research.

# 1 THEORETICAL BASE

Study of different approaches is needed to define the most appropriate and efficient framework. In this section author reviews key components that may find implementation in this work from the perspective of previous researches as it will help to separate important aspects from those, which may be inappropriate or less efficient, and also to check the adequacy of selected components.

## 1.1 Particular Qualities of Investing for Private Individuals

There is significant difference between investing possibilities of private client and those of wealthy ones or corporate clients for example. Because of this difference the one should search and apply those investment techniques, which are affordable, simple and yet effective, to be able to present competitive offer for clientele. One of the main specialties of private clients are their goals, e.g. if we will take an average middle-age person we can assume that from investment perspective, this person's goal may be described as need or intent is to accumulate some amount of wealth by the certain time in future, it may be personal retirement plan, house purchase, child's education etc. The key element of these goals is that they are fixed in time, i.e. it is unlikely that person may decide to reject the retirement plan completely nor the investment scheme will change significantly.

That is where it is incomparable with goals of any enterprise, as it may be interested in hedging interest rate risks with Interest Rate Swap during the period when it receiving interest and repays the loan at the same time, and on the next year the main priority will be capital preservation outside the home markets to exclude the impact of economic turmoil of home market on asset's value. Same with wealthy clients, as unlike the smaller ones, they can afford the services of professional wealth, fund managers or private bankers, who can allocate their assets, and develop more complex strategies to achieve better results in constantly changing

world of investment opportunities. Still this client segment may be quite perspective in terms of business, as they represent majority of customers (see addendum 5).

For an average private client there is not enough funds to cover the expensive services of professional manager, but on the other hand there the need of excessive flexibility in client's investment strategy is disputable, as it can be defined once for a longer period of time due to the fact, that it is unlikely that client's goal will drastically change in future as author mentioned before. Plenty of alternatives are available to such investors, from ETF-s to Mutual Funds which cover single asset classes, portfolios and even more complex structures. In this research author focuses attention on the most widespread asset classes, as their characteristics may offer robust framework for data interpretation and check accordingly to the common market knowledge and also for future researches.

## 1.2 Wealth Management

Importance of client interest plays significant role in wealth management. Nevertheless, in case of small private clients (whose servicing regarding wealth management problems often does not fall into this category) this interest tend to be ignored because of many reasons. The most widely spread of them are clients' absence of financial knowledge and conflict of advisors' and clients' interests.

Recent discussions of fiduciary duty rightly insist that professional advisors such as financial planners for example must put the interest of the client before their own interest. In considering what that ethical duty amounts to, one always comes back to something like the Golden Rule (Duska, 2012, p. 17), "Do unto others as you would have them do unto you" or a pledge such as, "In all my professional relationships…I shall…make every conscientious effort to ascertain and understand, render that service, in the same circumstances, I would apply to myself."[1]

Lest take the case of retirement planning, for example, advisors generally speculate on such fund characteristics as growth or risk depending on fund (LHV Bank usually promote their

[1] The Commitment of a Chartered Financial Consultant®, the American College.

performance while Danske Bank speculate on risk, Swedbank has wider range of risk-based funds with different weights between equities and fixed income instruments, but still they have their own scheme of promoting them and there is no any particular way to adapt solution to clients' goals).

Unlike the big, professional or institutional investor, average middle class representatives cannot afford themselves an active asset management, diversificated portfolio, moreover, due to the high transaction costs, their access to the different asset classes is limited to such alternative as collective investment schemes as Mutual Funds or ETF-s, of course the one can invest through certain selection of equities, but in most of the cases this choice is far from the most rational because of limited possibilities to properly diversify it, high risks and such important characteristics as taxation, liquidity or trading volume can vary significantly from share to share.

The mandatory retirement plan scheme is a good example of collective investment scheme implementation. Retirement investment is being accumulated as fixed percentage of salary, and it is a flexible method for private individual because of next advantages (Berle & Means, 1932, pp. 317-325):

1) managed by investment professionals, who can offer higher returns and more adequate risk management;
2) benefit from economy of scale (lower transaction costs);
3) better asset diversification and therefore total risk is reduced.

Therefore only decisions regarding size of investment and asset/fund selection (not security allocation) are left to the client. But before making decisions regarding the fund or any other investment scheme, the proper understanding of different asset classes' characteristics needed.

## 1.3 Asset Allocation versus Security Selection

The evidence in prior researches about relative importance of asset and securities allocation is mixed. Brinsond, Hood and Beebower (1991) using historical data, argue that the asset allocation is superior to the stock selection while Kritzman and Page (2002) used

simulation approach show, that asset allocation have greater impact on investment result that security selections.

R. Raue in his 2013 asset allocation study used slightly different approach and investigated whether the asset allocation and security selection is more important over a considerably longer time horizon than researches mentioned before. His research data spanned a large spectrum of non-traditional asset classes (stocks, bonds, real estate, commodities and cash) results showed, that asset allocation strategies yield a superior dispersion in returns than security selection strategies. It is also true especially for crises. The most important periods for asset allocations from 1991 to 2011 were February 1991, during the Gulf War; August 1998, when Russia defaulted; March 2000; the beginning of the bursting of dotcom bubble; and September 2008, when Lehman Brothers collapsed near the start of the subprime mortgage crisis (Rau, 2013, p. 10).

From the practical point of view, it is reasonable to assume, that investment decisions start from asset selection and only after that investor seeking further possibilities of risk mitigation through security diversification. Cash, bonds and equities are the most common elements of portfolios and they are also elements of the portfolio puzzle discussed by Canner, Mankiw and Weil (1997). They also note, that investment advisors recommend that aggressiveness of their portfolios can be increased by proper allocation between bonds and stocks.

In this research author takes slightly different approach and assumes that for selected class of clientele decisions regarding the class and risk are tightly connected and therefore should be made at the same time, i.e. before opening the position as further changes in allocation mean additional costs.

## 1.4 Behavioural Approach and Markowitz's Framework

Harry Markowitz emphasized concept of diversification in his pioneering work in 1952. His concept still remains the cornerstone of any sound wealth management strategy. According to one Markowitz statements, investor should diversify his funds among all those securities which give maximum expected return. But in this research author limits concept of Markowitz's diversification to choice between different asset classes over different periods, yet the main

elements of Markowitz's fundamental work find application in current research in terms of different relative performance of asset classes to each other and relatively to the market.

Particular implementation of Markowitz's concepts can be found in studying the performance of selected assets, using their correlations. This basic tool offers very sound information on different possibilities to investor in terms of adapting to the market situation and also by showing the importance of asset choice and its impact on results.

To analyse behaviour of investors it is necessary to select the most important criteria in their decision making. Shefrin and Statman (2001) in their Behavioural Portfolio Theory suggest that investors generally have varied aims and that they construct their portfolios to meet specific goals. Optimal Behavioural Portfolio Theory portfolios are also different from optimal Capital Asset Pricing Model portfolios, as they are typically off the mean-variance efficient frontier. While Capital Asset Pricing Model uses portfolio beta and return as main factors, for clients the main characteristics of their investments are return and risk, which can be represented by standard deviation.

During the early 1990s the state-of-the-art in asset allocation was pre-tax mean-variance optimization, based on Markowitz's ground-breaking work in the 1950s. The best advisers for wealthy individuals at the time tended to be mostly in two camps – those that focused on stock picking with a sufficient allocation to municipal bonds for income and those that emphasized asset allocation and diversification by applying mean-variance optimization. In a typical implementation of Markowitz' framework, the application sought to find the highest returning mix of assets for a given level of risk (or vice versa) (Chhabra, Koneru, & Zaharoff, 2008).

Wang, Suri, Laster and Almadi (2011), propose an incremental step forward with their goals-based wealth management approach. From the authors perspective, their work provide robust base for further research of goals-based asset allocation. They developed approach that finds a specific subportfolio to address each of an investor's goals and then derive the least-cost solution. By combining mentioned approach with risk component we can get the simple, and what is more important transparent tool for clients, the question is how to get sufficient number of samples.

Therefore, what began as a journey to implement the Markowitz framework for individual investors by adding different features evolved into a larger mission to develop a sound, customized, and implementable strategy based on individual goals and risk constraints

(Chhabra, Koneru, & Zaharoff, 2008, p. 55), through the years they have built on these concepts to create an integrated framework, whose objectives are:

1) protection maximization (minimization of uncertainty i.e., of not meeting essential goals);
2) maximization of the probability of maintaining one's standard of living (higher possible return of investment or lower cost of goal achievement);
3) ability to define the possibility to reach for „Aspirational" goals.

Author uses the framework mentioned above as the baseline but in slightly simplified version. Using the assumption that key factors in asset selection are chance of achieving positive return and its rate, we have stable base which can be used to analyse asset effectiveness in seeking different investment goals depending of period, moreover this approach it would guarantee that clients will find solution conceptions useful and understandable. These factors seem to be comprehensive especially in terms of long-term investing, with the assumptions that client may want to stick to the selected asset until the end of period.

## 1.5 Stochastic Modelling

For the framework described above necessary amount of samples can be received with the use of Stochastic Modelling, which according to the authors opinion is an effective way to calculate expected return (and therefore the cost of achieving the goal) and the chance of positive return of particular strategy. At the time, Michael Kraten, writing about using Monte Carlo Simulation in constantly changing environment, proposed defining clients' alternative scenarios in a traditional financial planning format by classifying these individuals in terms of their behavioural willingness and the ability to absorb risk.

Even though the investment environment has become far more volatile, there is no need to change fundamental modelling approach, financial planners may find helpful to modify certain business practices to address the evolving needs of clientele (Kraten, 2009, pp. 53-54). In this particular research the Stochastic Modelling is used to forecast the variation of returns thus we are dealing with stochastic modelling.

Stochastic projection has become the 'best practice' methodology for modelling risk in institutional insurance and pensions business and is now commonly used too in the retail wealth

management process (Mowbay, 2010, p. 2). Stochastic models allow to assign reasonable probabilities to each of possible scenarios of the future behaviour of economic variables along with the ability make predictions regarding expected returns of investment.

Mowbay maintains that in the most real-life financial planning cases objective is some future cash flow or value. Also the financial planning outcome for the client is the extent to which some chosen investment strategy or product is able to meet the objective. In most cases, this objective will depend not only on the total (cumulative) investment return over the investment term, but also, critically, on the sequence of those returns.

Because of increasing reliance on Stochastic Modelling, a number of market participant have raised number of questions about its reliability. In general, the primary concern expressed by wealth managers is whether information or recommendations they may have provided to clients based on outputs from Stochastic Modelling could have been 'flawed' or 'unreliable' in some respect, as a result of relying on a model which failed to capture the events of 2008 (Mowbay, 2010, p. 1). In order to assess model 'performance' in the context of the financial planning process, he considers three common characteristics (Mowbay, 2010; p. 3):

1. **Decision supportive.**
   Ability to use the outputs from the stochastic projection to identify a financial planning strategy which best meets the clients' goals and risk profile.
2. **Risk illustrative.**
   Enables the client to make sure whether he or she feels comfortable with the risk level of selected financial planning strategy.
3. **Controllability of advice compliance.**
   Where the stochastic output has been used as an integral part of the planning and decision process (as above), these outputs can provide a record which enables the advisor to evidence an objective basis for the recommendation.

Stochastic Modelling applications correspond very well to concepts of Behavioural Portfolio Theory, therefore author assumes that mentioned elements can find good practical application and therefore uses mentioned concepts as a core in methodology construction.

## 1.6 Multi-Criteria Analysis

Comparing results current research result would require the use of dimensionless analysis because of different units of measure (cost and chance of positive return). Weighted product model suits as in this method each decision alternative is being compared with the others by multiplying their weighted criteria scores. Some of the first references to this method can be found in Bridgman and Miller and Starr works.

There is also recent researches that contain substantial analysis of Multi-Criteria Decision making methods. A thorough analysis of the design of multi-criteria decision making mechanisms been provided by Yeon-Koo Che. He develops a two-attribute procurement auction model. In the model, bidders bid on both price and quality, and a scoring function converts each bid into a single score number, and the bidder achieving the highest score wins the auction.

In this research the mention concept can be applied by converting price of achieving the goal and chance of positive return into scores of each alternative and thus. This model is also useful because of its flexibility in changing the weight of each criteria, more complex concept of this method described in methodology section.

# 2 METHODOLOGY

To be able to effectively implement Stochastic Modelling, the one should start from robust historical data to get appropriate measures of statistical dispersion, as they are going to be used to predict chance of positive return of chosen asset and its rate.

## 2.1 Asset Selection

Besides the most widely used asset classes (equities and fixed-income instruments) it is necessary to include alternatives such as gold, real estate and commodities. These asset classes are available to all investors in some way of another, at least through funds (mutual funds, funds of funds or ETF-s) or structured products. Selected period of historical data is 21 years (from beginning of 1992 till end of 2013), source Bloomberg Terminal by Bloomberg Finance LP (see figure 1).

Equities (also called stocks) represent shares of ownership in publicity held companies, according to the common knowledge they are usually more volatile in short term than other instruments. In this research due to the lack of appropriate index data for aggregated global equity return is divided into two categories: Developed Markets (DM) and Emerging Markets (EM). This also seems reasonable because of difference in return and risk perspectives connected with this two different markets. Both markets are represented by MSCI indexes: MSCI World Index (MXWO) and MSCI Emerging Markets (MXEM).

The MXWO captures large and mid-cap representation across 23 DM countries[2]. With 1611 constituents, the index covers approximately 85% of the free float-adjusted market capitalization in each country (MSCI Inc., 2014). The MSCI World Index has been calculated

[2] DM countries include: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Ireland, Israel, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, the UK and the US.

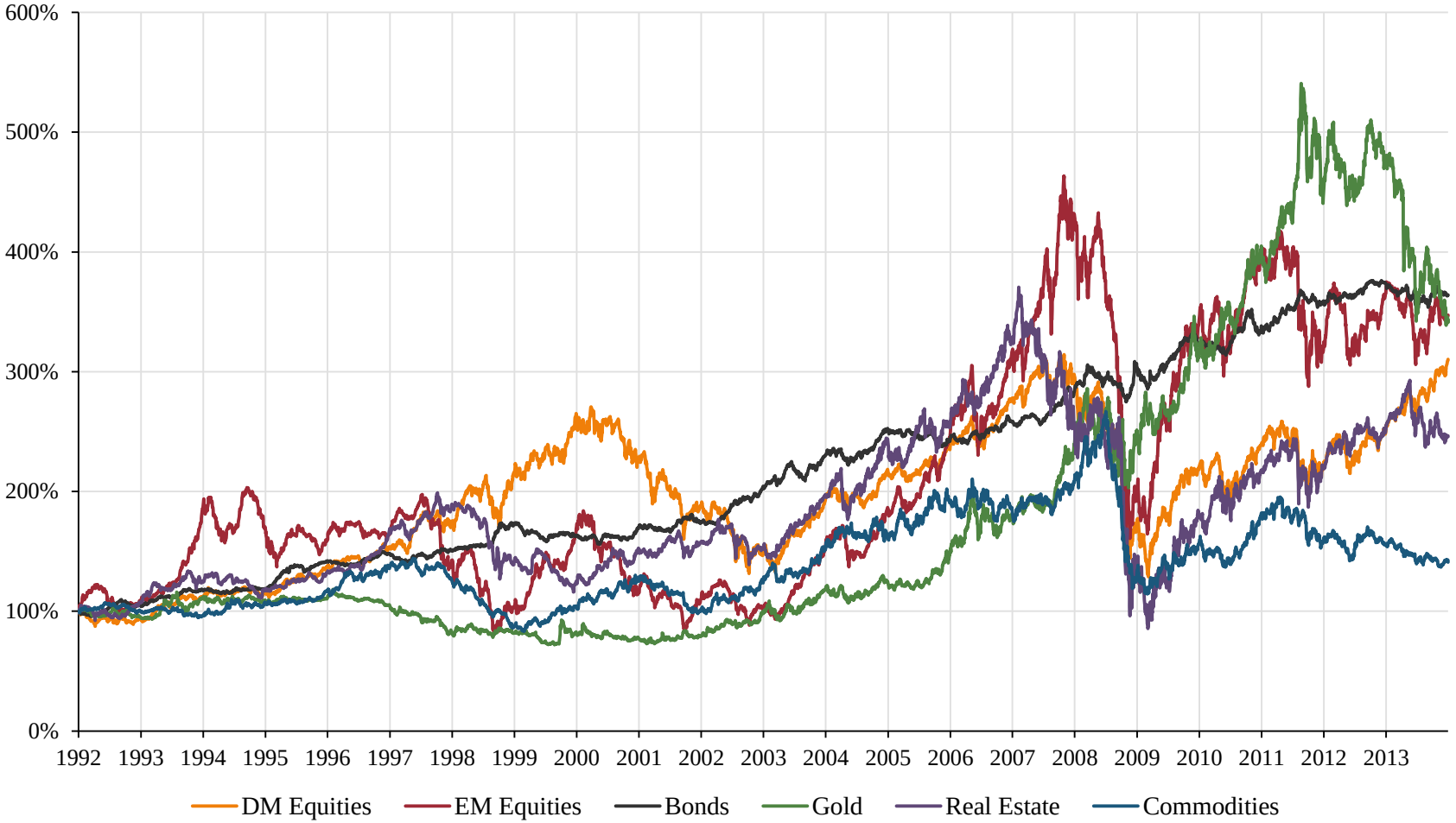


Figure 1. Selected asset classes indexed price dynamics 1992-2013.

Source: Bloomberg Terminal, 2014.

since 1969, in various forms: without dividends (Price Index), with net or with gross dividends reinvested (Net and Gross Index), in US dollars, Euro and local currencies. In this research, author uses US price indices for each asset representation. The MXEM captures large and mid-cap representation across 21 EM countries[3]. With 822 constituents, the index covers approximately 85% of the free float-adjusted market capitalization in each country (MSCI Inc., 2014), for top constituent see addendum 1-4.

Fixed Income or simply bonds, generally pay a set rate of interest over a given period (usually once or twice a year), principal is returned to investor on the maturity date. They commonly perceived as more stable in terms of return (and less volatile) but their value fluctuates due to interest and inflation rates. Author uses J.P. Morgan Global Aggregate bond Index (GABI) as it is a global investment grade (IG) benchmark that represents nine distinct fixed income classes (Table 1).

Table 1. GABI Index Criteria.

| Debt Class | Maturity | Coupon |
|---|---|---|
| **DM Treasuries** | >13 months | Fixed |
| **EM Local Sovereign** | >13 months | Fixed |
| **EM External Sovereign** | >2.5 years to enter<br>>1 year to remain | Fixed/Float |
| **US Credit** | >13 months | Fixed/Step |
| **Euro Credit** | >19 months to enter<br>>13 months to stay | Fixed/Step |
| **EM Credit** | >5 years to enter<br>>1 year to stay | Fixed/Step |
| **US Mortgage Backed Securities** | >1 year average | Fixed |
| **US Agencies** | >19 months to enter | Fixed |
| **Pfandbriefe** | >13 months | Fixed |

Source: J.P. Morgan Fixed Income Index Product Guide, January 2013.

[3] EM countries include: Brazil, Chile, China, Colombia, Czech Republic, Egypt, Greece, Hungary, India, Indonesia, Korea, Malaysia, Mexico, Peru, Philippines, Poland, Russia, South Africa, Taiwan, Thailand and Turkey.

This index is constructed from over 5000 instruments issued from over 60 countries denominated in over 25 currencies, collectively representing US$35 trillion in market value. Eligible constituents are IG quality (BBB-/Baa3/BBB- or higher) as rated by S&P, Moody's and Fitch respectively. EM external debt instrument eligibility is determined by the higher rating from only two rating agencies (S&P and Moody's). US MBS and US Agencies are excluded from this criterion (Ram, 2013, p. 1).

Real estate is usually home or investment property along with shares of funds that invest in commercial real estate and in some cases real estate backed securities. Author have chosen Dow Jones U.S. Real Estate Index (DJUSRE) to represent Real Estate Investment Trusts (REIT) and other companies that invest directly or indirectly in real estate through development, management or ownership, including property agencies. The index is a subset of the Dow Jones U.S. Index, which covers 95% of U.S. securities based on float-adjusted market capitalization. Base price first calculated on 31/12/1991, quoted in USD (for top components see appendix).

The difference with commodities is that they are highly leveraged and trade in contract sizes instead of shares. In this research they are represented by The Dow Jones-UBS Commodity Index (DJUBSCI), as it provides broadly diversified structure of commodity markets as an asset class. Its key features as mentioned by S&P Dow Jones Indices LLC:

- The index is made up of exchange-traded futures on physical commodities.
- The index currently represents 20 commodities, which are weighted to account for economic significance and market liquidity.
- Weighting restrictions on individual commodities and commodity groups promote diversification (see appendix for detail weightings).

Table 2. Dow Jones-UBS Index sector sub index weightings.

| **Sub index** | **Weight (%)** |
|---|---|
| **Agriculture** | 33,55 |
| **Energy** | 30,74 |
| **Precious Metals** | 15,05 |
| **Industrial Metals** | 15,00 |
| **Livestock** | 5,66 |
| ***Total*** | *100,00* |

Source: S&P Dow Jones Indices LLC, March 2014.

Gold is separated from other commodities as independent asset class because historically it plays a special role apart from other commodities, its price is influenced by the following factors stated by Liverpool Derivatives Group Co:

- Gold has developed widespread commercial use as a coating on electrical connectors. It can be found on various devices, from audio and video cables to computer and component cables and connectors.
- Worldwide gold production continues to underperform against worldwide demand. At the current level of production, an assumption is that in less than 45 years, our gold supply will not be able to meet the demand.
- The World Gold Council estimates that the total gold mined annually is approximately 2,500 metric tons. Currently, 3,500 metric tons of gold is used in the jewellery, investment and commercial industry.
- The International Monetary Fund (IMF) and the Washington Agreement on Gold (WAG) have very strict requirements in gold sales: less than 400 tons per year, and members cannot use gold to back or replace their currency.

The advantage of using these indices is that they find widespread implementation in market practice by many financial professionals as market condition indicators, and also as benchmarks in fund management.

## 2.2 Stochastic Modelling

In this research author uses stochastic investment model to forecast possible variation of each asset class along with their returns. As the main object is comparison between different periods of investments, model should include time component which would define the amount of variation (standard deviation, formula 1) added to final result at the end of the period.

$$\boldsymbol{\sigma_d} = \sqrt{\frac{\sum_{i=1}^{n}(\boldsymbol{r_i} - \bar{\boldsymbol{r}})^2}{\boldsymbol{n-1}}} \quad (1)$$

Where:

$\boldsymbol{r}$ $-$ *daily return of an asset*;

$\boldsymbol{n}$ $-$ *number of data points*.

Standard deviation is usually used as measure of risk associated with price-fluctuation of asset, in this research author uses it as a part component in random variable. In the original concept it is said that as variance increases, the expected return on an asset should increase as well, so investor should expect a higher return on investment with higher risk (risk premium, formula 2), the question is, which asset offer the highest premium among others. Same principles are used in this research as we seek the maximal return with minimal chance of failure (negative return in the end of investment period).

A starting point for model is concept of Brownian Motion as Markov process: *the future given the present state is independent of the past.* Karl Sigman (Geometric Brownian Motion, 2006; p. 2) brings the following interpretation (3):

> $\boldsymbol{S(t+h)}$ *(The future, h time units after time t) is independent of* $\{\boldsymbol{S(u : 0 \leq u < t}\}$ *(the past before time t) given* $\boldsymbol{S(t)}$ *(the present state now at time t). To see that this is so we note that*

$$\begin{aligned} \boldsymbol{A(t+\Delta t)} &= \boldsymbol{A_{0e}}^{\boldsymbol{X(t+\Delta t)}} \\ &= \boldsymbol{A_{0e}}^{\boldsymbol{X(t)+X(t+\Delta t)-X(t)}} \\ &= \boldsymbol{A_{0e}}^{\boldsymbol{X(t)}_{\boldsymbol{e}}\boldsymbol{X(t+\Delta t)-X(t)}} \\ &= \boldsymbol{A(t)_{0e}}^{\boldsymbol{X(t+\Delta t)-X(t)}} \end{aligned} \tag{3}$$

Where:

$\boldsymbol{\Delta t} - time\ units;$

$\boldsymbol{A}\ - asset\ price;$

$\boldsymbol{t}\ - certain\ point\ in\ time.$

*Thus given* $\boldsymbol{S(t)}$*, the future* $\boldsymbol{S(t+h)}$ *only depends on the future increment of the BM* $\boldsymbol{X(t\ +\ h) - X(t)}$*. But Geometric Brownian Motion has independent increments, so this future is independent of the past; we get the Markov property.*

In this particular interpenetration, logarithmic component might be confusing and also complex for calculation. Nevertheless it is an important part of model because price levels are log-normally distributed (figure 2 and 3). However it is possible to explain it from the compounding effect perspective: while the price decreases on the downside, the base is also getting smaller, so lose certain amount in price and there is less left to lose next time.

Therefore Geometric Brownian Motion can be implemented by combining previous interpretation with asset average return, variance and rearranging it to calculation of returns instead of price, so the logarithmic component is excluded (4, as proposed by D. Harper).

$$r_a = \frac{\left(\overline{r_d}\Delta t + \sigma_d \xi \sqrt{\Delta t}\right)}{y}, for -3 \leq \xi \leq 3 \quad (4)$$

Where:

$\overline{r_d} - average\ daily\ return\ of\ an\ asset;$

$\Delta t - number\ of\ days;$

$\xi\ - random\ variable;$

$y\ - number\ of\ years.$

The first part $\{\overline{r_d}\Delta t\}$ is a "drift" and the second part $\left\{\sigma_d \xi \sqrt{\Delta t}\right\}$ is a "shock" (figure 4). For each time period, this model assumes the price will "drift" up by the expected return. But the drift will be shocked (added or subtracted) by a random shock. The random shock will be the

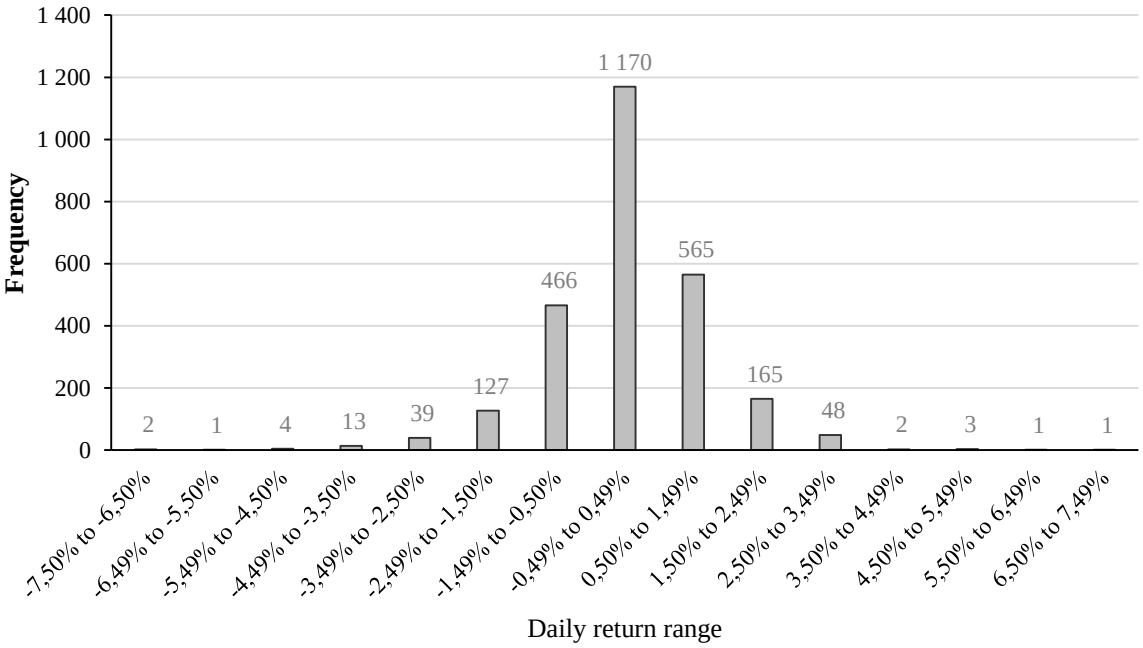


Figure 2. Gold Spot return normal distribution during 2000-2010.
Source: Bloomberg Terminal, 2014.

standard deviation $\sigma_d$ multiplied by a random number $\xi$. This is simply a way of scaling the standard deviation. Selected range for shock components seems sufficient because even for the most volatile instrument (DJUSRE, figure 5), the square of filed between -3 and 3 standard deviation is more than 98% of the whole bell (5456 out of 5540, table 3). Similar formula is proposed by Humphreys (Energy Risk, 2008, p. 83).

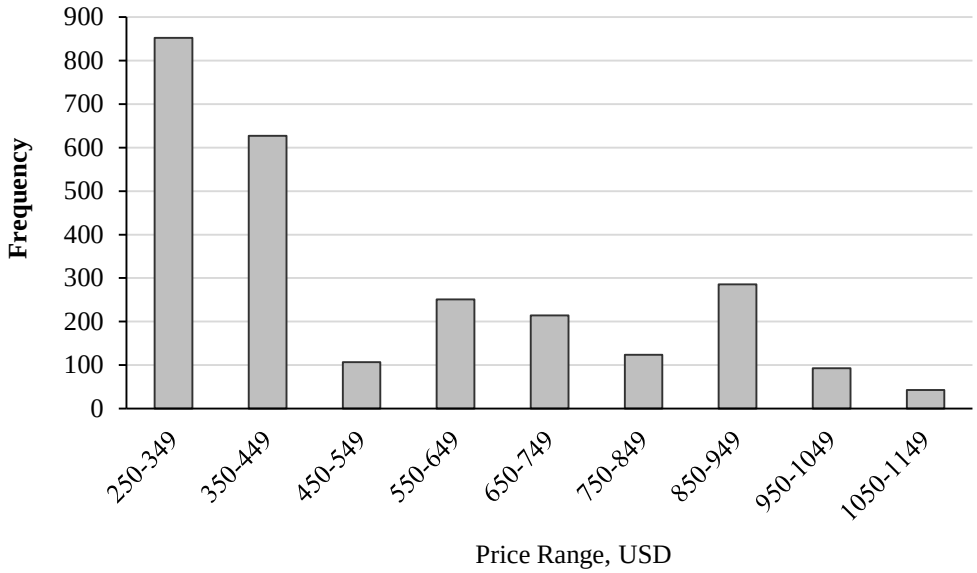


Figure 3. Gold Spot price log-normal distribution, during 2000-2010 (normal returns produce log-normal prices).
Source: Bloomberg Terminal, 2014.

To simplify the examination process, return component can be aggregate within different periods. By using compound interest formula we can calculate expected return of asset (formula 5) which can demonstrate the relative performance against other classes.

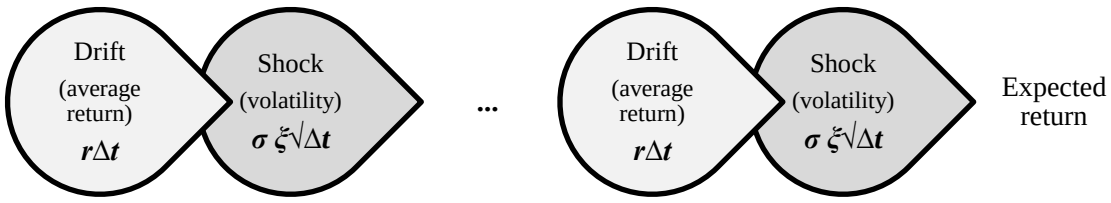


Figure 4. Essence of Geometric Brownian Motion component.
Source: David Harper, 2009.

By using this method author generates 1000 values of expected return for each asset class for 40 periods from 1 to 40 years. Rate of confidence correspond with the total weight of values in the range of ±3 standard deviations. For example in same case with gold price there is 2580 values out of 2608, so the rate of confidence is 98.9%.

$$\boldsymbol{R} = (1 + \bar{r_a})^n - 1 \quad (5)$$

Where:

$\boldsymbol{n} - lenght\ of\ the\ period\ in\ years.$

Also by calculating the weight of negative values of expected return in the whole sample we get the risk associated with specific asset in specific period or chance of positive return (formula 6) in getting positive return on investment, this value will be used as second component in Multi-Criteria Decision Making.

Table 3. Dow Jones U.S. Real Estate Index daily return distribution

| **Rounded return, standard deviations** | **Frequency** |
|---|---|
| -9 | 1 |
| -8 | 2 |
| -7 | 3 |
| -6 | 5 |
| -5 | 16 |
| -4 | 16 |
| -3 | 37 |
| -2 | 119 |
| -1 | 729 |
| 0 | 3 506 |
| 1 | 927 |
| 2 | 105 |
| 3 | 33 |
| 4 | 15 |
| 5 | 13 |
| 6 | 4 |
| 7 | 4 |
| 8 | 2 |
| 9 | 3 |
| *Sample size* | *5 540* |

Source: Bloomberg Terminal, 2014.

$$\boldsymbol{C} = 100\% - \frac{j}{n} \tag{6}$$

Where:

$\boldsymbol{j} - number\ of\ negative\ returns$;

$\boldsymbol{n} - sample\ size$ (1000).

## 2.3 Multi-Criteria Decision Making

As it was mentioned before, in this research author uses several factors as movers of investor's behaviour in terms of decision making regarding the selection of asset class, therefore the multiple-criteria analysis may suit well. As we have two different measures (chance of positive return – percentage, cost of achieving goal – currency units) it is possible to apply weighted product model (formula 6), which is widespread method in multi-criteria decision analysis and mainly it is similar to the weighted sum model, the key difference is that instead of summing the score, this method uses multiplication.

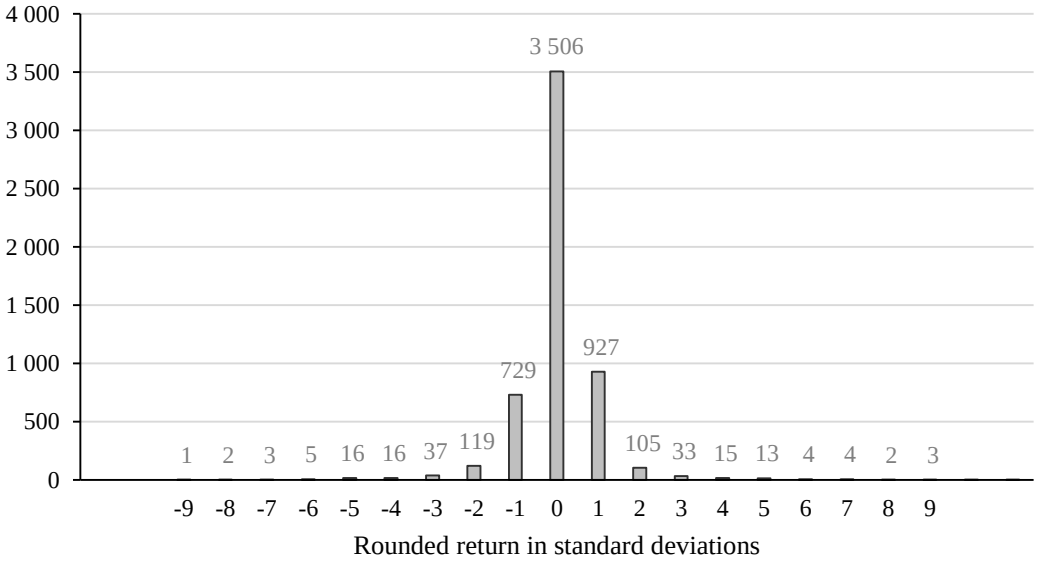


Figure 5. Gold Spot daily returns distribution histogram.
Source: author calculations based on Bloomberg data.

Given set of two factors can be used to calculate separate scores for each alternative (equities, bonds, gold, real estate, etc.) in different time periods from 1 to 40 years but by no

means should these scores be used for decision making separately from each other as result may suffer from a common flaw, in that more-aggressive asset allocation (e.g. EM equity) show higher score without a clear mechanism to emphasize the possible impact of the additional risk incurred.

For the chance of positive return the top alternative will have the value equal to 1 as it's chance of positive return will be used as denominator in calculation score of other alternatives in given period. For the cost of achieving the goal the top alternative will still be equal to 1 but with the exception, that base value of alternative will be used as numerator. Then these scores are weighted by subjective criteria importance in decision making and therefore the best alternative among others is the one with the highest score (formula 6, L. Laidroo; 2014).

$$\boldsymbol{P} = \left(\frac{a_{1,i}}{A_1}\right)^{\omega_1} * \left(\frac{A_2}{a_{2,i}}\right)^{\omega_2} \tag{6}$$

Where:

$\boldsymbol{i} - number\ of\ alternatives$

$\boldsymbol{A} - score\ of\ the\ top\ alternative\ in\ given\ period$

$\boldsymbol{\omega} - relative\ weight\ of\ importance\ of\ the\ criterion.$

This research decision problem is defined with 6 alternatives and 2 decision criteria. Furthermore one is benefit criterion while the other is not. Relative weight of importance of criteria can be adjusted. If the $\boldsymbol{P}$ score value is close or equal to 1 it indicates that the corresponding alternative is more desirable than others. To apply this model for determining the best choice, the one should choose the alternative with the highest value (the one which is closer to 1).

## 2.4 Assumptions and Limitations

As the main purpose of this research is to develop effective and transparent decision making tool, criteria implementation plays significant role in the methodology, therefore different aspects of adjusted performance measure may not find reasonable application in cases described here, as many of them use the same base characteristics and therefore may multiply their impact on final result. By using the Stochastic Modelling as sample generator, author assumptions on the assets are following:

- Variance of assets is constant (remains the same as it was during the selected historical period), therefore the return of assets remains in historical interval and its deviation does not exceed historical mean.
- 1000 simulations are sufficient for reasonable convergence.

There is also some market conditions omitted for simplification of model, some of assumptions are purely formal but still they have to be mentioned:

- There is no way to make riskless profit (no arbitrage opportunity).
- Transactions do not incur any costs or fees (frictionless market).
- There is no corporate events[4] associated with selected assets (e.g. dividend payments, additional cash flows, bankruptcies, stock splits, etc.).

Also, as author focuses attention on individual investors, important limitations of approach should be summarized:

- Impact of taxation on decision making is omitted as insignificant.
- There is no cash flows associated with an investment.
- Investor's goals and risk tolerance remain constant over the investment period.

[4] While there might be no important events associated with indices whose characteristics are used as object of stochastic modelling (as even in case of bankruptcy of index agency, the maintenance may be transferred to another company as it was in case of Lehman Brothers for example), author explains that he assumes that there will be no important events associated with those securities and assets which constitute used indices.

# 3 RESULTS AND DISCUSSION

It is reasonable to start discussion from analysing the key factors of chosen assets, as it will help to interpret the results of MCDM. In this work author uses the chance of positive of achieving the goal (certain future value in the period) and cost of achieving.

Table 4. Assets' common characteristics.

| Period, years | 1 | 5 | 10 | 20 | 30 | 40 | Average |
|---|---|---|---|---|---|---|---|
| **Developed Markets Equity** | | | | | | | |
| Average Annual Return | 6.8% | 6.4% | 6.4% | 6.5% | 6.9% | 6.5% | 6.6% |
| Risk | 42.2% | 36.1% | 27.8% | 17.3% | 9.0% | 5.3% | 23.0% |
| Range | 93.5% | 41.8% | 29.5% | 20.9% | 17.1% | 14.8% | 36.3% |
| **Emerging Markets Equity** | | | | | | | |
| Average Annual Return | 8.3% | 7.8% | 7.6% | 7.6% | 8.2% | 7.8% | 7.9% |
| Risk | 43.8% | 35.5% | 29.5% | 19.9% | 10.0% | 7.5% | 24.4% |
| Range | 115.7% | 51.7% | 36.6% | 25.8% | 21.1% | 18.3% | 44.9% |
| **Bonds** | | | | | | | |
| Average Annual Return | 6.5% | 6.4% | 6.1% | 6.2% | 6.3% | 6.3% | 6.3% |
| Risk | 30.8% | 6.3% | 0.0% | 0.0% | 0.0% | 0.0% | 6.2% |
| Range | 33.7% | 15.0% | 10.6% | 7.5% | 6.1% | 5.3% | 13.1% |
| **Gold** | | | | | | | |
| Average Annual Return | 7.2% | 7.0% | 7.6% | 7.0% | 6.9% | 7.3% | 7.2% |
| Risk | 41.8% | 34.9% | 25.3% | 17.7% | 11.1% | 4.9% | 22.6% |
| Range | 100.1% | 44.6% | 31.6% | 22.4% | 18.3% | 15.8% | 38.8% |
| **U.S. Real Estate** | | | | | | | |
| Average Annual Return | 8.5% | 8.7% | 7.9% | 7.9% | 8.4% | 7.9% | 8.2% |
| Risk | 44.6% | 39.5% | 35.1% | 27.6% | 23.3% | 19.6% | 31.6% |
| Range | 160.9% | 72.0% | 50.9% | 36.0% | 29.4% | 25.4% | 62.4% |
| **Commodities** | | | | | | | |
| Average Annual Return | 3.2% | 2.6% | 2.8% | 3.1% | 3.2% | 2.9% | 3.0% |
| Risk | 45.8% | 43.9% | 39.5% | 33.4% | 30.6% | 30.4% | 37.3% |
| Range | 92.8% | 41.5% | 29.3% | 20.7% | 16.9% | 14.7% | 36.0% |

Source: author calculations based on Bloomberg data.

Table 3 demonstrates averaged expected return of Stochastic Modelling samples, risk level which is presented by the weight of negative returns received in Stochastic Modelling and scatter or range of Stochastic Modelling returns. United States Real Estate has the highest average return but also the highest spreading of all at the same time (which means that over the period of 1 year, the difference between the highest generated return and the lowest one will be approximately 160%), which matches the common investor dilemma of risk and return.

Commodities offer the lowest return and risk premium which may be explained by the characteristics of this asset in terms of associated trading activities (mainly hedging) and bigger exposure to larger amount of moving factors including seasonality, natural disasters, etc.

Table 5. Assets' correlations.

| | | **DM Equities** | **EM Equities** | **Bonds** | **Gold** | **Real Estate** | **Commodities** |
|---|---|---|---|---|---|---|---|
| **DM Equities** | Pearson Correlation | 1 | | | | | |
| | Sig. | | | | | | |
| **EM Equities** | Pearson Correlation | .662** | 1 | | | | |
| | Sig. | .000 | | | | | |
| **Bonds** | Pearson Correlation | .073** | .090** | 1 | | | |
| | Sig. | .000 | .000 | | | | |
| **Gold** | Pearson Correlation | .132** | .175** | .295** | 1 | | |
| | Sig. | .000 | .000 | .000 | | | |
| **Real Estate** | Pearson Correlation | .579** | .297** | -.052** | .014 | 1 | |
| | Sig. | 0.000 | .000 | .000 | .309 | | |
| **Commodities** | Pearson Correlation | .329** | .377** | .148** | .416** | .110** | 1 |
| | Sig. | .000 | .000 | .000 | .000 | .000 | |

Source: author calculations based on Bloomberg data.

* Correlation is significant at the 0.01 level.

Correlations between asset shows that almost every single class will usually move together, especially equities with real estate, the only negative but on the other hand small ratio is between real estate and bonds, which is very interesting and according to the author's opinion, may be explained by impact of Global Financial Crisis in 2008 where investors were forced to switch to safe-haven investments, particularly to low risk munis and government bonds. TED spread, which represent the difference between the interest rates on interbank loans and on short-terms U.S. government debt was extremely high during 2008 (see figure 6).

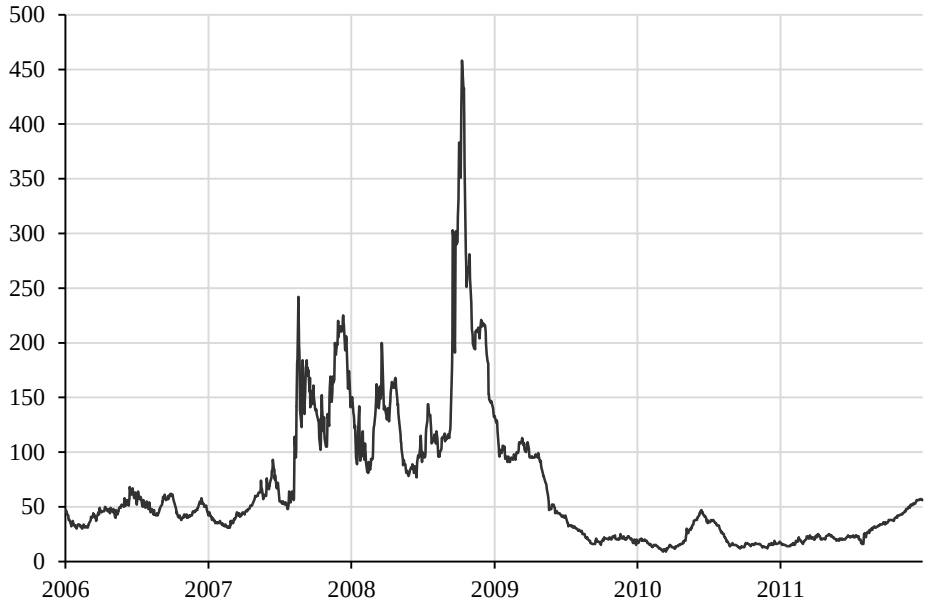


Figure 6. History of the TED spread.
Source: Economic Research of Federal Reserve Bank of St. Louis.

## 3.1 Expected Return

Expected return is obviously the leading factor in decision making especially if we are talking about investing in growth. Before moving to Multi-Criteria Decision Making it is essential to review return rates for every single asset chosen. Statistically there is more chance to get positive return assuming that other conditions stay the same and market situation will not change drastically. Therefore the process of allocating assets in short-term investments is purely tactical, as in short-term returns tend to be more derivative from historical averages as these deviations are typically based on current economic and market fluctuations, despite the fact that

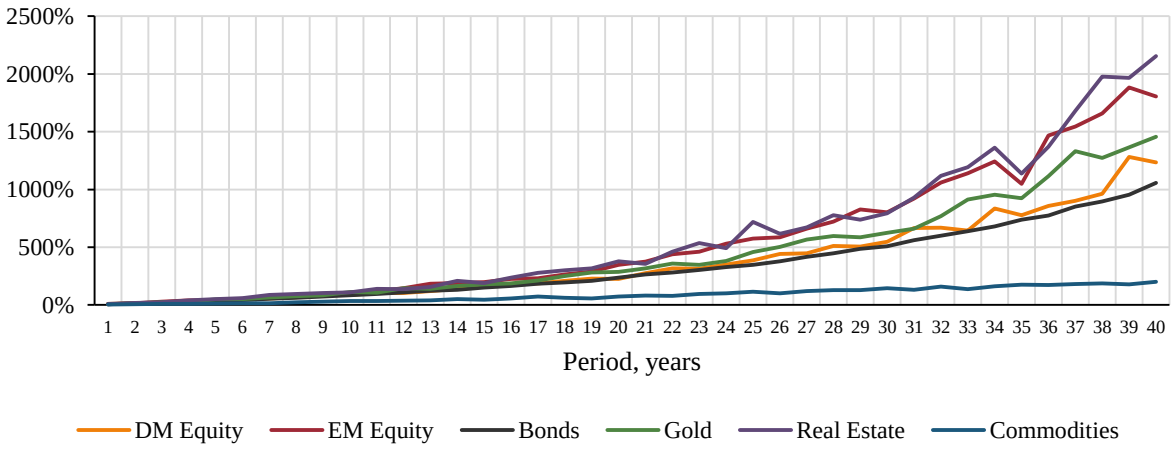


Figure 7. Asset chance of positive return according to Stochastic Modelling results.
Source: author calculations.

uncertainty is higher in longer periods, investors tent to expect higher returns from more volatile asset classes over the longer periods as statistically in many cases average historical return outperform variance component. But there is also a problem as there is no specific definition of "long-term".

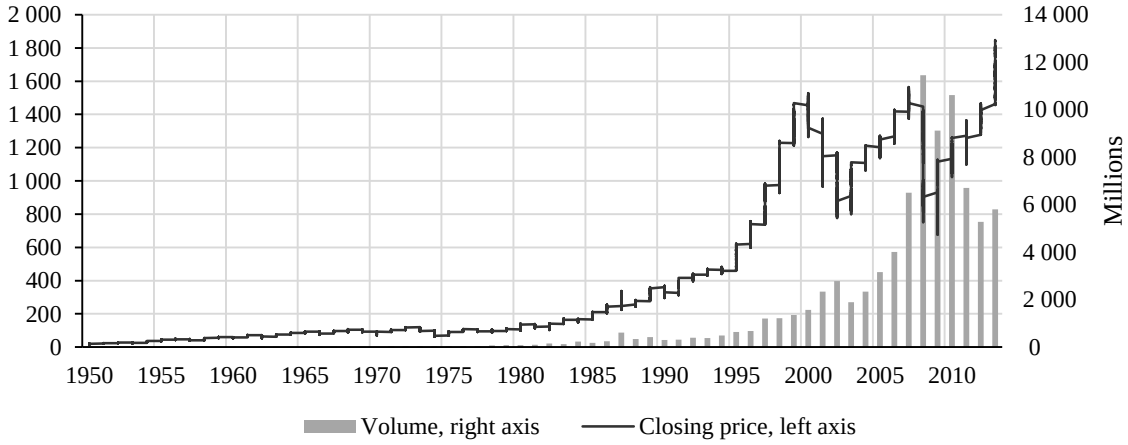


Figure 8. S&P 500 Index historical price and volume dynamic.
Source: Yahoo! Finance.

According to the widely accepted rule of thumb, long-term period is more than 10 years, in corporate banking practice, longer-term investments are those which are longer than 5 years so investment solutions are developed exactly according to this 5 year division.

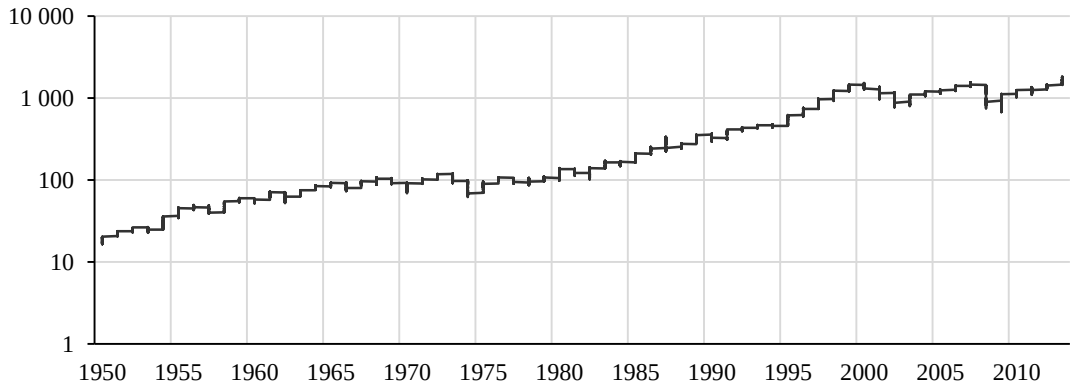


Figure 9. S&P 500 Index historical price logarithmic scale.
Source: Yahoo! Finance.

In AQR working paper of 2001 Clifford Asness claims that investor cannot lose in the stock market while investing in the long-term horizon. According to his result there are many sources for this belief, but the most influential is Jeremy Siegel's book[6], Stocks for the Long Run. This book documents that over long periods the equity market's Premium over inflation and over alternative assets like nominally risk-free cash has been consistent and relatively high (figure 8). Let us take both arithmetic and logarithmic scales (figure 9) of S&P 500 Index for example. There is an obvious change in cycle dynamics from 90s and seem like after dotcom crisis market moved sideways similar to 70s dynamics. That might be one of the reasons why equities did not show significantly better performance relatively to real estate and gold.

## 3.2 Probability of Positive Return

For private individual interested in investing this is the one of key decision motivators as in fact it shows what is the chance that a part or all investment will not be lost. In methodology section author described the calculation of the chance of positive return (the weight of negative returns generated by Stochastic Modelling). Because of chosen calculation method it is obvious that this chance will be increasing with selected period. Interesting to

[6] Siegel does not say that stocks will always win over any 20-year period. He only points out how consistently well stocks have done, but gives no silly assurances going forward (Asness, 2001).

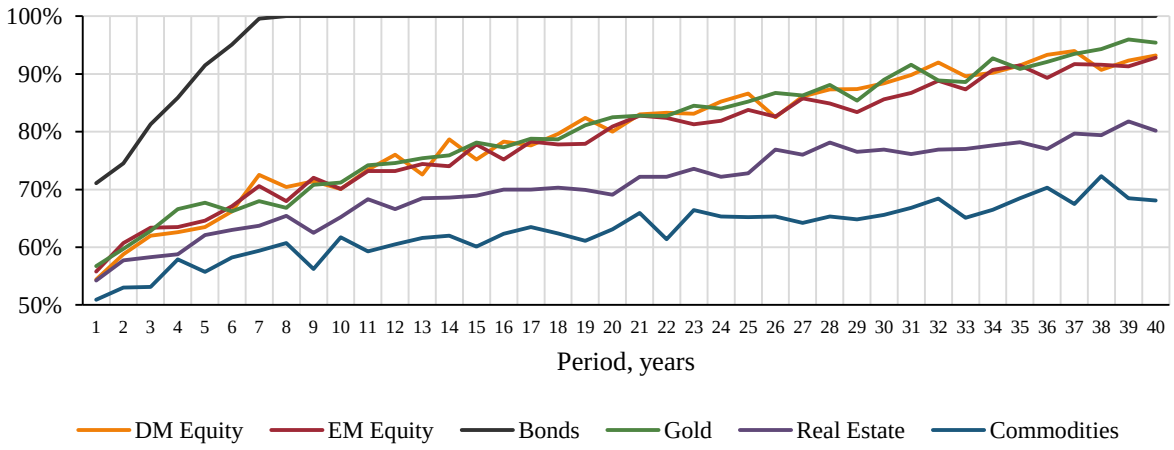


Figure 10. Probability of asset to generate positive return in Stochastic Modelling
Source: author calculations.

mention that the only asset class becoming statistically risk-free are bonds, and what more remarkable – in periods longer than 8 years. There is no other asset to be 100% successful in offering positive return during periods of 1 to 40 years.

## 3.3 Multi-Criteria Decision Making Process

Basic knowledge examined above allows to proceed to Multi-Criteria Decision Making. For better understanding of using the solution in customer oriented environment author examine three different cases based on client's risk tolerance (different time horizons included in model

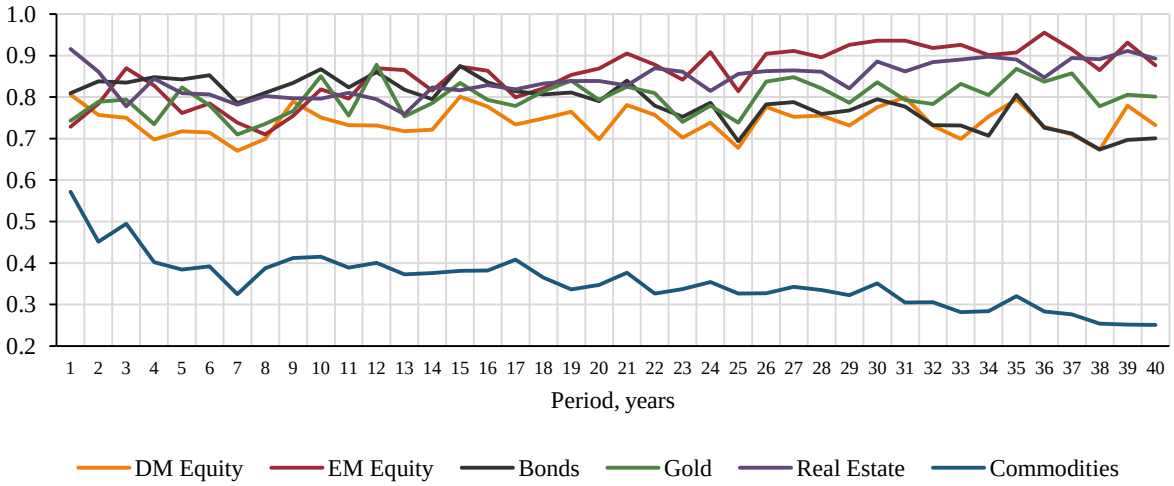


Figure 11. Assets' score chart in case of moderate risk tolerance (risk/return equal importance).
Source: author calculations.

by default). The first case is when risk and return play equal roles in customer's decision making process, therefore criteria weights are both equal to 0.5, in other words this may correspond to the investment strategy commonly known as moderate. Besides moderate author will examine aggressive strategy (0.25 risk, 0.75 return) and conservative (0.75 risk, 0.25 return).

By examination of moderate strategy score chart (figure 11) it is seen that asset score is in fact depend on investment period which is an evidence of key hypothesis confirmation. Bonds tend to be the best choice to meet selected criteria, while EM Equity and Real Estate may be more appropriate choice in long periods as they may offer sufficient expected return to cover the risks of high variance. Remarkable is that gold and real estate offer relatively consistent score, which makes them good alternative in case when period is unclear to investor (especially real estate, due to the higher score) or in cases when flexibility is required. Investing in commodities seem to be the most unattractive alternative which is mainly due to the lower return per risk (mentioned in current section preamble). So basically by applying more weight to the risk component, the score of assets with lower risk premium drops. Also, for a conservative investor it might be acceptable that all his investments are bound in low risk government bonds. The good thing about being conservative is that the probability of losing the money is very low, however, the return may be low.

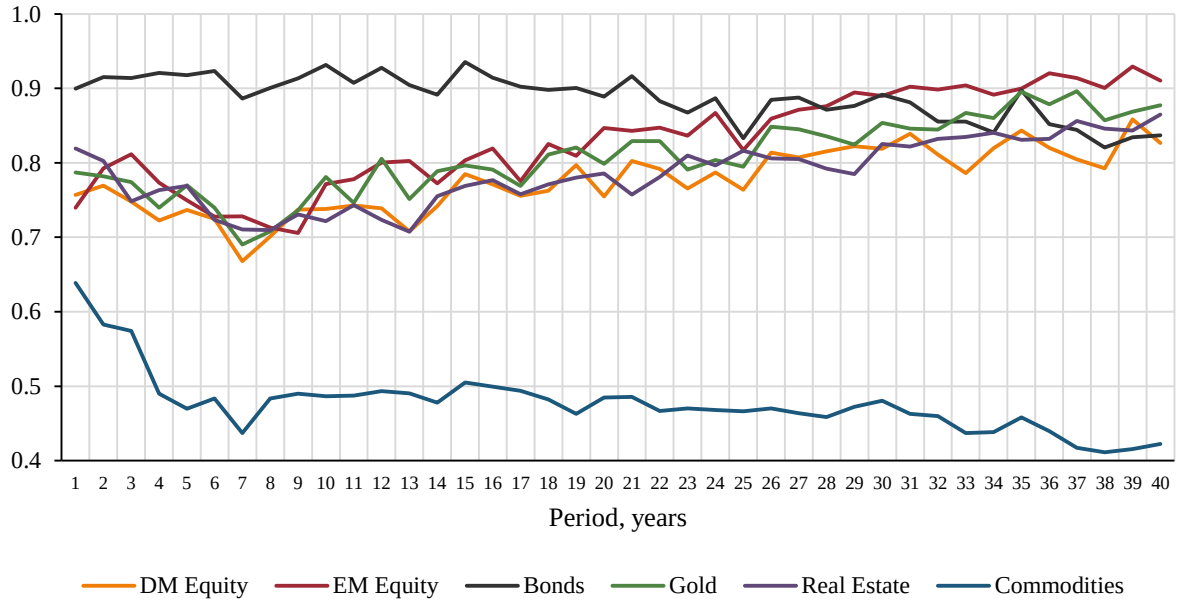


Figure 12. Assets' score chart in case of lower risk tolerance (risk/return importance ratio 2:1). Source: author calculations.

There is also another problem with conservative investments: return may not be sufficient to cover the inflation. And if investor will take taxation[7] component into consideration, the overall return may be negative i.e. investor's savings will worth less and less over the period. So the switching to the conservative approach may be reasonable in later periods of retirement planning as it will lower the exposure of investment to market turmoil, on the other hand it will lower the return, but if we examine the years left until retirement as the new separate period with the same risk appetite we are getting back to the bond as the safest assets.

The problem with bonds in this particular case is that there were two stock market crises in selected period which may make bond more attractive than equities and also key rate dynamics which has obvious impact on the price of fixed income instruments. Further researches may study the significance of this impact with implementation of duration concepts for example. Therefore by constantly reviewing the score charts investor may switch to other assets class if some asset offer higher score (based on his risk tolerance) for the residual period.

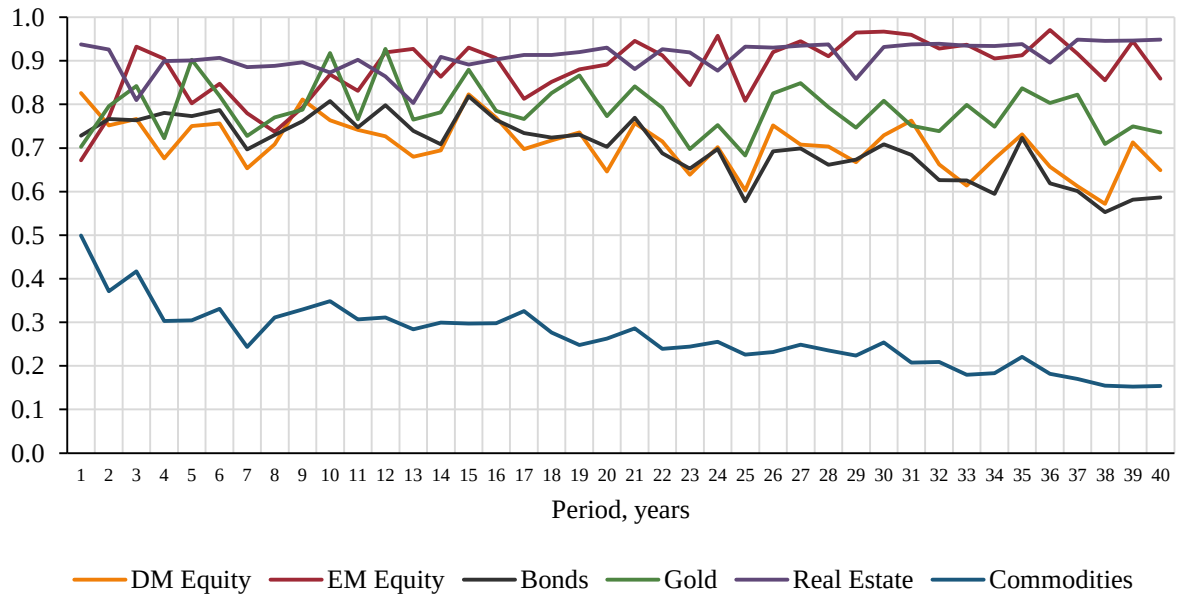


Figure 13. Assets' score chart in case of high risk tolerance (risk/return importance ratio 1:2). Source: author calculations.

[7] Taxation is very complex topic as in different law systems taxes on investment return may vary very dramatically from scheme to scheme. This is one of the reason why this element is omitted in this research, as it would require laborious legal research, analysis and preparation.

The last case, describes the risk appetite of an aggressive investor (figure 13) and it is seen that commodities are the special asset in all cases and from decision making perspective investor has to choose between real estate or EM equity, their position relatively other assets is constant and thus aggressive investor may not benefit from switching strategies According to the score charts commodities are the worst alternative for the long-term investments as in every case the longer the period, the lower the score. The main reason is very low historical return compared to variation.

To summarize, the results mentioned before show that the key hypothesis of research is fully confirmed. Using the Multi-Criteria Decision Making approach we see that depending on risk tolerance and period of investment, score can change dramatically. Also worth to mention that result correspond with the common knowledge regarding selected asset specifications, which on one hand speaks in favour of practical possibilities of tool implementation and its reliability and on the other hand may be directly connected to the features of selected indices as indicators of market situation.

From the practical point of view, advisors and investment professionals may do some research on the basis of their client risk tolerance and clarification of expectation (strategy letters may be a good way of collecting the needed information). After the goals and features mentioned before are available, scorecards of suitable asset classes can be demonstrated as an illustrative object lesson so the decision can be worked out cooperatively.

# CONCLUSION

In this particular study author address the prevalent issues of private wealth management such as unseen conflict of interest. Being able to offer clients simple, effective and transparent solutions is essential in post 2008 crisis era, this qualities mean competitively and ability for sustainable growth of business especially if we are talking about wealth management in customer-oriented environment.

As the diversification remains the cornerstone of any sound wealth management, advisor and financial professionals should be aware that standard Modern Portfolio Theory methods may not be applicable to small-size private individuals, thus the other approach should be implements, which first of all should be effective in value creation for customer.

This research proposes a framework for asset selection in private wealth management. From a review of previous works author defines the baseline and implements concepts of goals-based investing, Wealth Management Behavioural Portfolio Theory and Multi-Criteria Analysis. Preliminary results (assets' elemental characteristics and correlations) confirm the key hypothesis "different asset classes offer different risk premiums over the different periods, and thus have variable suitability in terms of client goals, risk tolerance and expectations".

As author establish connection between assets' relative performance with the length on investing period further study extent to the more practical case of decision making problems in customer-oriented environment. Solution is derived by aggregating two main factors in private client decision making perspective, the rate chance of positive return on the investment. Author examines three basic cases of clients' risk tolerances and finally show how to apply the results of this study to the general case of private wealth management of small size clients.

Although this framework facilitates problems of decision making process for small private clients, it can also be used to enrich service quality of other client segments as well, but there is work to be done before.

First this approach assumes that transactions do not incur any costs or fees and that means that essentially markets is frictionless, which does not correspond with reality. Further

researches may be significantly enrich by including this component, especially if used along with real securities instead of indices. Second, corporate events may play a significant role in the process of clients' decision making. Numerous researches as well as market practice show evidence that corporate event may cause very drastic market moves. Third, the solution of this research is single-goal oriented, thus does not consider in detail the uncertainties of potential life events. As components mentioned above are not critical in selected client segment niche, author leaves these extensions for further studies.

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# APPENDIX

## Addendum 1. MSCI World Index Top 10 Constituents

| | Country | Market Cap (USD Billions) | Index Weight (%) | Sector | Sector Weight (%) |
|---|---|---|---|---|---|
| **Apple** | US | 482,93 | 1,50 | IT | 12,3 |
| **Exxon Mobil Corp** | US | 426,72 | 1,33 | Energy | 13,9 |
| **Microsoft Corp** | US | 325,07 | 1,01 | IT | 8,3 |
| **Google A** | US | 308,41 | 0,96 | IT | 7,8 |
| **Johnson & Johnson** | US | 277,15 | 0,86 | Health Care | 7,4 |
| **General Electric Co** | US | 261,94 | 0,81 | Industrials | 7,2 |
| **Wells Fargo & Co** | US | 248,91 | 0,77 | Financials | 3,7 |
| **Nestle** | CH | 242,95 | 0,76 | Consumer Staples | 7,7 |
| **Chevron Corp** | US | 228,68 | 0,71 | Energy | 7,5 |
| **JPMorgan chase & Co** | US | 228,22 | 0,71 | Financials | 3,4 |
| ***Total*** | | *3 030,98* | *9,43* | | |

Source: MSCI Indices Factsheet, March 2014.

## Addendum 2. MSCI Emerging Markets Index Top 10 Constituents

| | Country | Market Cap (USD Billions) | Index Weight (%) | Sector | Sector Weight (%) |
|---|---|---|---|---|---|
| **Samsung Electronics Co** | KR | 139,38 | 3,69 | IT | 22.1 |
| **Taiwan Semiconductor MFG** | TW | 95,85 | 2,54 | IT | 15,2 |
| **Tencent Holdings Lim (CN)** | CN | 71,13 | 1,88 | IT | 11,3 |
| **China Mobile** | CN | 55,20 | 1,46 | Telecom Services | 21,1 |
| **China Construction BK H** | CN | 50,49 | 1,34 | Financials | 5,0 |
| **Gazprom (RUB)** | RU | 45,61 | 1,21 | Energy | 11,2 |
| **ICBC H** | CN | 45,37 | 1,20 | Financials | 4,5 |
| **Naspers N** | ZA | 43,63 | 1,15 | Consumer Discretionary | 12,5 |
| **Itau Unibanco PN** | BR | 37,71 | 1,00 | Financials | 3,7 |
| **Hyundai Motor Co** | KR | 36,36 | 0,96 | Consumer Discretionary | 10,4 |
| ***Total*** | | *620,73* | *16,43* | | |

Source: MSCI Indices Factsheet, March 2014.

## Addendum 3. Dow Jones U.S. Real Estate Index Top 10 Constituents

| Company | Float Factor[8] | Adjusted Weight (%) |
|---|---|---|
| **Simon Property Group** | 1,00 | 7,77 |
| **American tower Corp A** | 1,00 | 4,88 |
| **Crown Castle Intl Corp** | 1,00 | 3,72 |
| **Public Storage** | 0,84 | 3,67 |
| **ProLogis Inc** | 1,00 | 3,08 |
| **Equity Residential** | 0,93 | 2,94 |
| **Ventas Inc** | 1,00 | 2,69 |
| **HCP Inc** | 1,00 | 2,67 |
| **Boston Properties Inc** | 1,00 | 2,64 |
| **Health Care REIT Inc** | 1,00 | 2,61 |
| ***Total*** | *0,98* | *36,67* |

Source: S&P Dow Jones Indices LLC, March 2014.

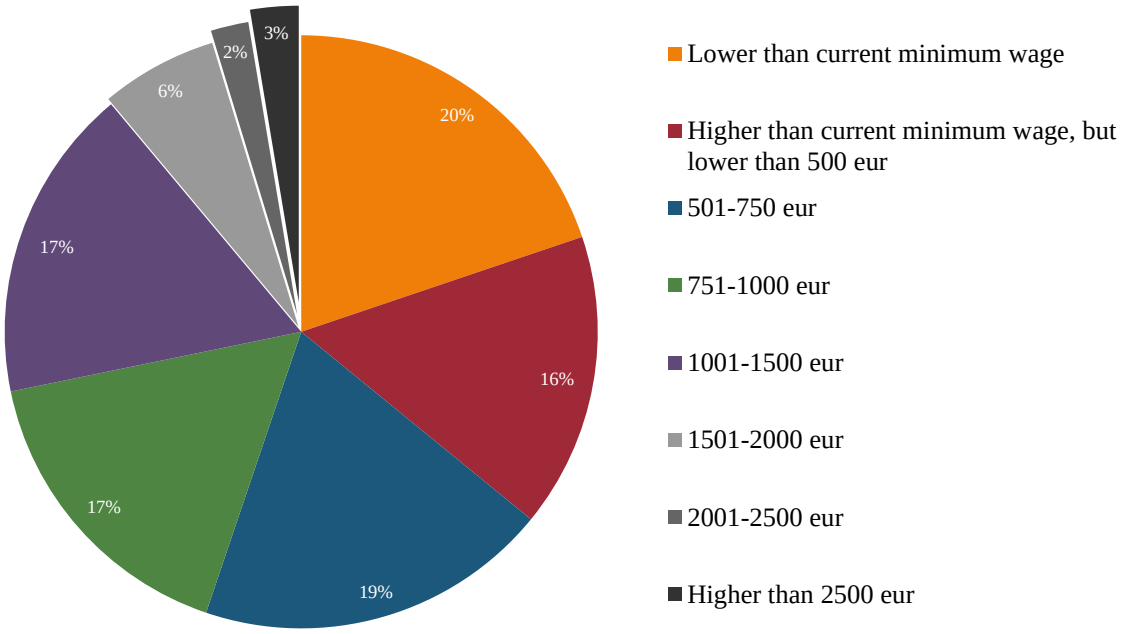
[8] Float factor is assigned to each stock to account for the proportion of outstanding shares that are held by the general public, as opposed to "closely held" shares owned by the government, royalty, or company insiders.

## Addendum 4. Dow Jones-UBS Index Commodity breakdown

| Commodity | Weight (%) |
|---|---|
| **Gold** | 11,21 |
| **Natural Gas** | 9,00 |
| **WTI Crude Oil** | 8,55 |
| **Corn** | 7,89 |
| **Copper (COMEX)** | 6,29 |
| **Brent Crude** | 6,08 |
| **Soybeans** | 6,07 |
| **Aluminium** | 4,38 |
| **Sugar** | 4,07 |
| **Silver** | 3,85 |
| **Unleaded Gasoline** | 3,65 |
| **Wheat** | 3,60 |
| **Heating Oil** | 3,45 |
| **Coffee** | 3,28 |
| **Live Cattle** | 3,07 |
| **Soybean Meal** | 2,88 |
| **Soybean Oil** | 2,81 |
| **Lean Hogs** | 2,59 |
| **Nickel** | 2,25 |
| **Zinc** | 2,08 |
| **Cotton** | 1,62 |
| **Kansas Wheat** | 1,34 |
| ***Total*** | *100,00* |

Source: S&P Dow Jones Indices LLC, March 2014.

## Addendum 5. The number of persons receiving salary by wage groups




Source: Estonian Tax and Customs Board, *http://tiny.cc/sivcgx*.