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FORECAST OF HOUSING MARKET IN MIAMI

Bachelor thesis

Supervisor

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I declare I have written the bachelor thesis independently.
All works and major viewpoints of other authors, data from other sources of literature and elsewhere used for writing this paper have been referenced.

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Abstract

The purpose of this research is to find relationship between macroeconomic factors affecting to the apartment price changes and variables connected to the future price expectations in Miami-Dade County, Florida. Factors affecting apartment prices were research at the international level, but also locally in the area. These factors are described based on annual average observations. The aim is to achieve short-term apartment price forecast for the next 3 years, by using multiple regression model. The research data has been collected from 1980 to 2016 to obtain reliable results. Most of the research material for forecasting was collected from the United States Census Bureau and the United States Department of Labor. The original variables used in the analysis are population, interest rates, median household income, number of households, employment, income per capita, gross domestic product (GDP) and consumer price index (CPI). These variables created initial model, which assumed that apartment prices will increase 8.9% by 2017, 6.2% by 2018 and 27% by 2019. In the developed regression model, the insignificant variables were removed to obtain more reliable results. The significant variables in this research were population, income per capita and GDP. This final model assumed prices to increase 2.4%, 2.3% and 40.7% respectively during the becoming three years.

Keywords:

Macroeconomic factor, apartment market, apartment price, forecast, multiple regression model

INTRODUCTION

Apartments are generally considered as investment objects whose value is growing over time. As a result, buying an apartment could be thought to be a risk-free investment even with a mortgage loan. A single person's debt may be encouraged by the event of any repayment problems, an apartment can always be sold and thus debited the debt back to the lender. The assumption of ever-rising housing prices may be especially misleading, if we consider the housing prices in the United States.

In the past decade, there have been big movements in housing price level and housing cycles but somehow the history keeps repeating itself by the time to time. Movements in apartment prices can be related to changes of social and macroeconomic factors. It is important to briefly go through the economic factors and especially macroeconomic factors affected to price level changes in the past. The impact comes from the sum of industry level, wealth via finance sector to the whole production, consumption of households and employment rate. (Oikarinen 2007, 102-105).

Housing market researchers emphasize that the housing market research should be done locally. (Goodman 1998, Oikarinen 2007, and Reichert, 1990). According to these researchers, the housing market is local, because each apartment market has strong individual characteristics and the housing research carried out at national apartment trend data lead to wrong conclusions about the local housing market.

To find out answers for the research questions, we need to look at which factors are affecting apartment price trend changes in Miami-Dade County, Florida, and which macroeconomic factors have influence on these apartment trends. It requires a look to the previous researches and theory section of housing market. The thesis will show the price trend forecast of apartments for the next 3 years in Miami-Dade County and give theoretical framework of the factors influencing in residential apartment price level. Modifying the use of multiple regression model based on several drivers moving the price trend and by modifying correlations between the variables. As a result, real estate market is the driver in social and macroeconomic changes in the during period. The results help to predict the future residential apartment price changes for future investors. The major challenge in forecasting these parameters and predicting future value, is the quality of data.

1.1 Purpose of the thesis

The aim is to focus on residential apartments and find what are the macroeconomic factors affecting the price trend. The housing stock is generally greatest asset what society has. Built infrastructure is not the source of the recession or the other which can be carried away, it is immovable. The apartment scientists have researched and highlighted the importance of the housing stock in the economy. The housing stock is an important resource in our economy and its importance is significant for the well-being of our nation.

The study is quantitative and is based on time series analysis. The study is focused on residential apartments. Price changes are theoretically explained by the following macro variables: demand and supply, interest rates, number of households, household income, employment, population and migration, consumer price index (CPI) and gross domestic product (GDP). The material contains average of annual observations for the years 1980-2016. The interaction between the variables to be studied is determined by the model and hypothesis testing, coefficient of determination and the multiple regression model.

The housing market scientists have highlighted the importance of international housing market study. Nevertheless, the housing market of high-quality studies on the housing market can be found just a few. The households' debt is growing rapidly and most of the housing stocks are purchased with mortgages. It is important to research Miami-Dade County housing market to understand the structure and by this build the future expectations.

1.2 Assumptions and obstructions of the thesis

Assumption is that macroeconomic factors have a great impact on housing prices. As the macroeconomic factors are relatively increasing in Miami-Dade County, the apartment prices are also assumed to increase.

Given the wide scale of time which this research covers, there are limitations in comparability of data due to reporting methodologies and changes in data collection. In most of the data required for the economic variables was limited in the time scale of 1980-1990. Instead, the analysis and comparison are desired at the research level. The data needed to be gathered from numerous government reports

2. METHODS

The study material consists of annual averages including period 1980: Q1-2016: Q4 and also forecasted estimates for 2017-2019 were used for the analysis. The study examines time series of the following variables:

1) Apartment prices (Miami-Dade County)
2) Interest rates
3) Households
4) Median household income
5) Employment
6) Population
7) Income per capita
8) Consumer Price Index
9) Gross Domestic Product

In interest rate and consumer price index, there are 432 observations counted as an annual average per variable. Interest rates and apartment price data are collected from the Federal Reserve Bank of St. Louis (2017).

Data for CPI, employment and income per capita is gathered from the United States Bureau of Labor.

Population and number of households are collected from statistics and reports of U.S. Census Bureau. Part of the median household income is gathered U.S. Census Bureau reports and Florida Health Charts. Data from 1980-1990 is gathered from the Federal Reserve Bank of St. Louis 2017.

Gross domestic product is collected from the Bureau of Economic Analysis, U.S. Department of Commerce.

In following chapter, the variables used in the study are described in detailed. The purpose is to clarify how the variables are built. In addition, the development of variables has been attempted to illustrate with different figures.

3. MACROECONOMIC FACTORS AFFECTING THE PRICE TREND

Demand and supply drivers are controlling commodity prices and, by this also housing prices. The general economic situation has a central impact on the demand for housing and their prices. Accessibility to finance, employment rate and interest rates are essential factors affecting the activity of the housing market. Migration, the functionality of land use planning and the efficiency of the rent market are also important for housing prices. Households' ability and willingness to purchase an apartment and their favorite residential area are important issues affecting demand. The condominiums are also investments, whereby the viability of alternative investments influences house prices. Also, expectations for the development in the housing market indicates to the current prices. Economic cycles quickly respond to housing construction, which is why production slowdown reduces supply fast. Especially in the longer term, the rigidity of the supply of dwellings depends on whether housing prices are generated by household income growth or the overall inflation level.

Housing is a significant share of the whole macro economy but in the other hand, housing also constitutes a large part of household expenditures affecting to the total wealth.

The purpose of this chapter is to go through the most common theories that explain the price formation of dwellings. When looking at the theories, one must remember that dwellings are very heterogeneous commodities. Due to heterogeneity, the housing prices are not simple. The theory of hedonic prices assumes that the price function is indirectly derived from the market as the sum of the prices of different attributes. From the point of economic research, the housing market has been studied in terms of the housing demand and supply, thus focus has been on finding the balance.

3.1 Apartment prices

The apartment price information has collected from the U.S. Census Bureau, as the study also seeks to find out the impact of inflation on house prices. The data is collected on the price information of the studios and all the flats. In addition to macroeconomic variables, housing

prices are affected by the location, type of house and the number of rooms. That is why the U.S. Census Bureau uses the hedonistic method when building a housing price index. The composition of the annual sales of apartments varies, which is why the statistics are quality-corrected and this research is using annual averages. The number of condominiums sold this year is expected to be lower than last year. The composition of condominiums sold is considered when describing changes in housing prices. The statistics do not include cases where transaction price or square feet information is missing or is otherwise exceptional (e.g. trade between family members). The following figure shows the apartment price development in Miami Dade County from 1980 to 2016.

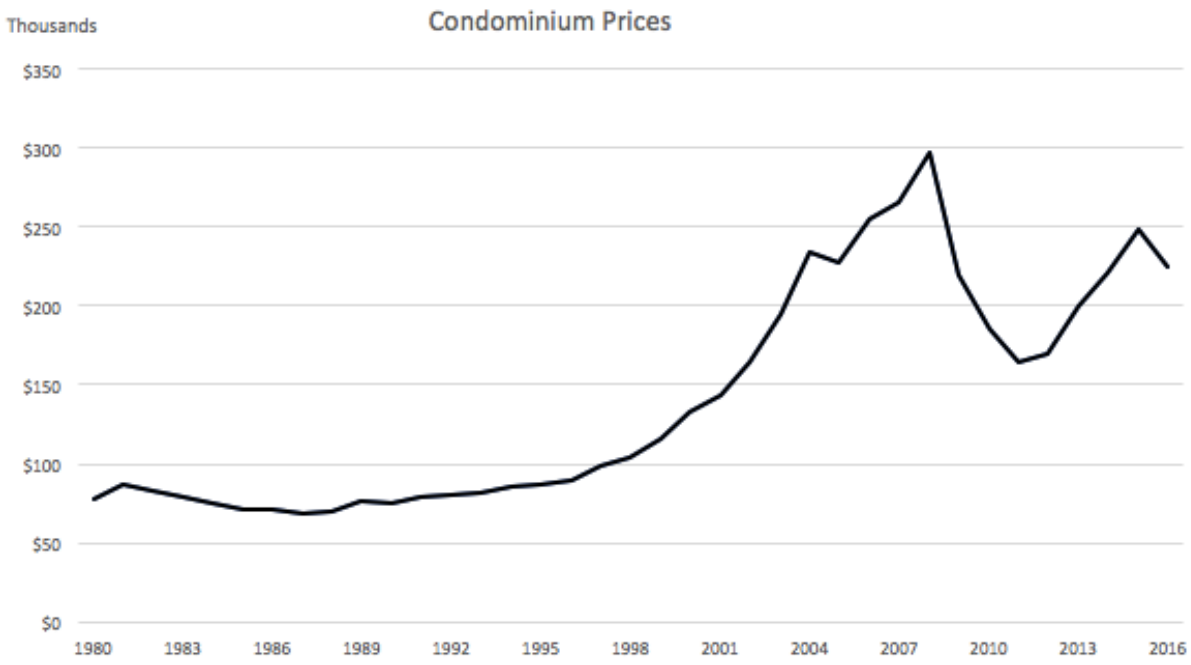

Figure 1. Condominium price development

In the early 2000s, United States experienced a subprime housing bubble. The fast growth in housing prices in the early 2000s did not slow down the willingness to take a mortgage, especially when the interest rate was relatively low. Credit institutions began issuing a high number of mortgages on light credit standards, resulting in a more active participation of the more poor population in the housing market. As a security for mortgages for housing loans were often the apartments which were being traded at that moment. The values were not expected to decrease. After 2005, when the debtor's mortgage debts and

foreclosures began to grow rapidly, housing prices began to decline. Because the securities for mortgages were most likely the apartments which lost its value, not only debtors were in problems but also many creditors.

3.2 Demand factors

Mainly, international housing market research is made from a national perspective. The biggest challenge of local housing study is the limitations of the data, which Goodman (1998, 43-53) also mentioned.

Goodman (Ibid.) has studied a lot of different areas of demographic impact on the price trends of housing and has noticed that in demand side the income and price elasticity may vary considerably because of the differences in the local market. Ethnic composition and population structure vary by location and so does the price elasticity of demand.

Another researcher of the American housing market, Alan Reichert (1990) has also studied the population structure effects on house prices in the United States. He also found regional differences in the considerable, which affected the demand for housing. These included uneven migration of traffic within the country, as well as minorities and the uneven distribution of the elderly.

3.3 Supply factors and price stability

Abraham and Hendershott (1996, 191-200) argues that the supply of new housing does not affect price stability at all. This applies to countries where the building site is scarce and the technical features and housing are carefully regulated. They also highlight how small the impact on the construction of new housing or the supply of dwellings is at the current housing stock price, because the price development of housing is largely due to national factors such as household income, mortgage interest rates or delays in house prices.

Another philosophy claims that the housing supply has a strong impact on housing prices. As prices rise, there is a temporary shortage of apartments in the market. In this situation, construction companies are responding by building new dwellings in the lots which

they have stored at low market when the prices are low. This is how the price equilibrium in the housing market has been created. (Colin, Jones, Michael, White and Neil, Dunse 2012, 187).

3.3.1 Bubbles

A bubble refers to the deviation of the price of a particular asset from its level of financial fundamentals. Factors affecting housing prices are called the fundamentals of housing prices. (Angela, Black et al. 2005, 3). Price deviations are due to the fact that the expectations of economic operators about the actual level of prices are incorrect. Bubbles are caused by the fact that demand pressures for a particular asset are due to cause or other excessive. Price bubbles may, for example, consist of prices for stocks, commodities or housing. (Marko, Melolinna and Katja, Taipalus 2006, 41-42).

When there is a bubble in the apartments prices, it is a housing bubble. The bursts of housing bubbles usually has devastating effects on economic activity. The impacts appear significantly in decreasing employment and reduction in production. The stability of the financial markets is in jeopardy if, in the situation of a housing bubble, lending decisions are made in accordance with delicate risk assessments. (Christian, Dreger and Konstantin A. Kholodil 2013, 16; M., Taipalus and K., Melolinna 2006, 43). The 'air' in the prices of dwellings may increase demand for goods in the short- and medium-run, causing inflationary pressures and inefficient allocation of resources. In other words, households and companies are living over their resources and incorrect allocation results in long-term unsustainable consumption and excess production. When the economic cycle changes to decline, imbalances are observed, usually with the most serious economic consequences (Dreger and Kholodilin 2013, 16).

When the bubble burst, prices change significantly. Since it is problematic to define the "right" price level in accordance with the economic fundamentals, it is ultimately paradoxical: the existence of a price bubble is basically impossible to prove to exist. (Melolinna and Taipalus 2006, 41-42).

When looking at the theories, there must be remembered that dwellings are very heterogeneous commodities. Due to heterogeneity, the price of housing is not simple.

Apartment price level can be estimated by considering, for example, the rental price ratio, or the ratio of housing operating cost and rents. They can be used to detect possible housing price under- or overvaluation of a housing bubble occurrence possibility in the market.

Bubbles have straight connection to housing supply, areas with larger share of elastic housing supply will have less bubbles and duration of bubbles are proved to be shorter and price increase will be low compared to the areas with lower elastic housing supply (Edward, Glaeser et al. 2008, 2). Bubbles can be more common in the areas with of high elasticity because those cities are responding to bubbles by overbuilding compared to the actual need (Ibid., 2, 14). Bubbles can have a significant the impact of changes on macroeconomic stability.

3.4 Mortgages

As mortgage rates increases, housing costs will increase. This will lead to lower housing demand, leading to decrease in apartment prices. As the cost of capital increases, it can be assumed that especially people with a lower level of wealth will move from the buyers' market to the rental market. According to the theory of finance, the increase in interest rates immediately affects by reducing the discounted present value of the dividends gained by the shares, which is why the price of the shares also falls. Thus, at the same time the P / E ratio (Price to Earnings) decreases, making the share purchases more attractive. The apartment owner is in the same situation with the shareholder of the stock exchange because it is a share of stock in a housing co. The present value of a housing stock decreases because of the rising interest costs, unless the rent paid by the tenant can be increased.

Housing is well-regulated as a commodity. Lawrence Smith, Kenneth Rosen & George Fallis (1988, 40) state that the public-sector controls housing in many ways, and thus the state affects the demand and supply of the asset. The State's means of regulating housing are:

1) taxation,
2) land policy,
3) location decisions of public institutions, and
4) regulation related to the granting of a loan.

Accessibility for finance affects housing prices through different liquidity impacts. Increased lending from banks improves household liquidity, which in turn activates economic activity. As economic activity increases, housing demand also increases and giving a significant effect on housing prices. As housing prices rise, the collateral values also rise, leading to a growth in loan size. The growth in mortgage lending and rising housing prices are mutually reinforcing spins, which often occur for example, as requiring additional collaterals from mortgage borrowers. A good historical example would be the subprime crisis breaking the balance in the US economy, where banks' irresponsible lending resulted in a housing bubble leading to the global financial crisis. Among others, Goodhart and Hofmann (2007) found an interaction in which changes in house prices have an impact on debt.

In Miami-Dade County, most households' assets are caught in housing units and mortgages. More than 58% of total households pay more than 30% of income for rent or mortgage costs. (Florida Housing Data 2016). Housing costs for households living in rental are also large even though the form of living does not tie the capital to housing. The wealth of the people there is therefore weakly decentralized and the fall in house prices can have massive consequences. Households may not understand that it is safer to invest in more than one type of asset to reduce the financial risk. One housing feature is the asset including its liquidity variability. The location, size, and economic cycle of an apartment can be important at the time of sale.

In Miami-Dade County, most of the houses are purchased at least partly by debt, the link between the mortgage accessibility and housing demand growth is clear by empirical studies. In the figure below, interest rates of mortgages are illustrated from 1980 to 2016.

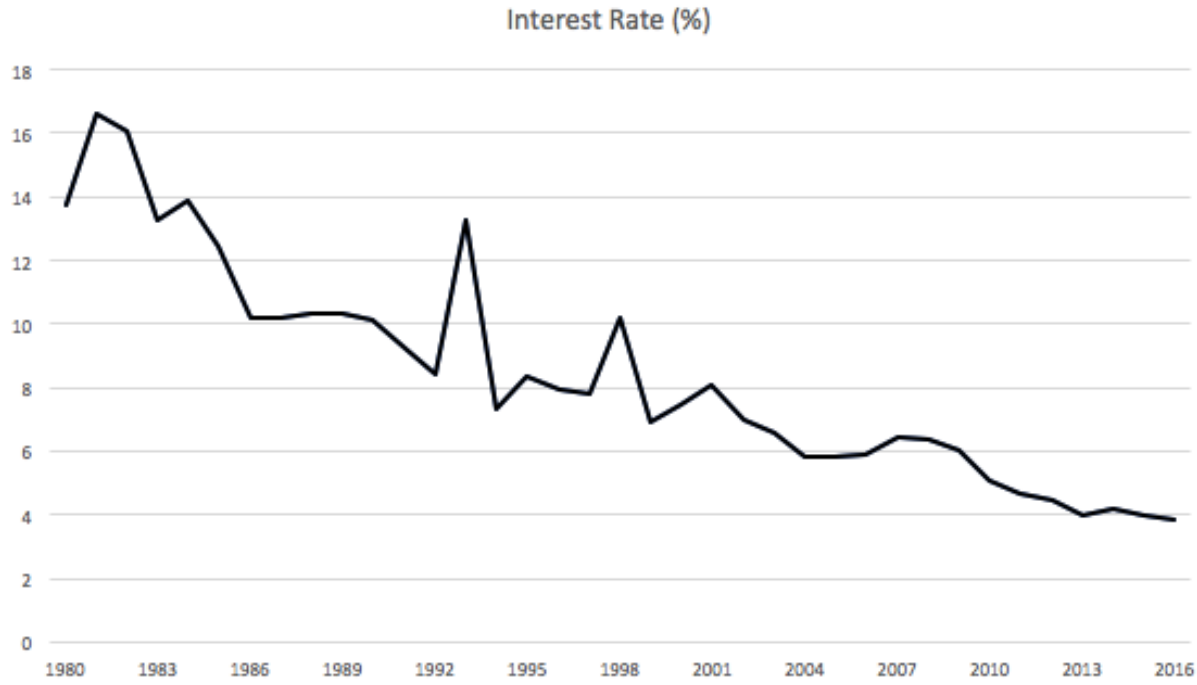

Figure 2. Interest rates

Researchers have studied housing demand when households' opportunities are limited for getting mortgage. When there is higher possibility for receiving a loan, for example through a reduction in the cash requirement, it had great impact on housing demand growth. The ability of young households to buy their own house also had a strong impact on housing demand. This result led to Ortalo-Magnè and Rady (2006, 459-485) who were suggesting that improving the availability of the loan increases the housing demand for young households. According to the researchers, the required cash deposit is so big that the potential buyers of the potential homeowners do not have enough savings or liquid money. As a result, the purchase of the dwellings move and the consumption decision will be postponed.

As interest rates increases, housing costs will increase. This will lead to lower housing demand, leading to decrease in house prices. As the cost of capital increases, it can be assumed that especially people with a lower level of wealth will move from the buyers' house market to the rental market.

According to the theory of finance, the increase in interest rates immediately affects by reducing the discounted present value of the dividends gained by the shares, which is why

the price of the shares also falls. Thus, at the same time the P / E ratio (Price to Earnings) decreases, making the share purchases more attractive. The apartment owner is in the same situation with the shareholder of the stock exchange because it is a share of stock in a housing co. The present value of a housing stock decreases because of the rising interest costs, unless the rent paid by the tenant can be increased.

If the mortgage is tied to a short-term interest rate, a negative consequence occurs for the owner immediately. Short-term interest rates are damaging the housing market balance than the long-term ones, because of its possible changes and higher volatility. In contrast, the long-term fixed-rate mortgage will not change the situation with the current owner, instead a long-term loan period offers the flexibility to live in balance during an economic cycle when the interest rates are rising.

Reichert (1990, 373-391) examined the effect of interest rates, income and employment on housing prices. He came to the conclusion that house prices reacted systematically to certain variables of the economy, such as the mortgage interest rate. On the other hand, regional factors, including migration, employment and income levels, showed a detailed impact on housing prices, depending on the residential area.

3.5 Household income

Income of households puts limit on their spending and investing in housing. Rationally behaved households choose their size, quality and location from their preferences, to maximize their benefits within their budget (Seppo, Laakso 1992, 143). Households differ and may have very different preferences regarding housing. Housing has many special features which are good to know before analyzing the housing market more closely.

Median household income is illustrated in the figure below. The numbers are in thousands of dollars ($).

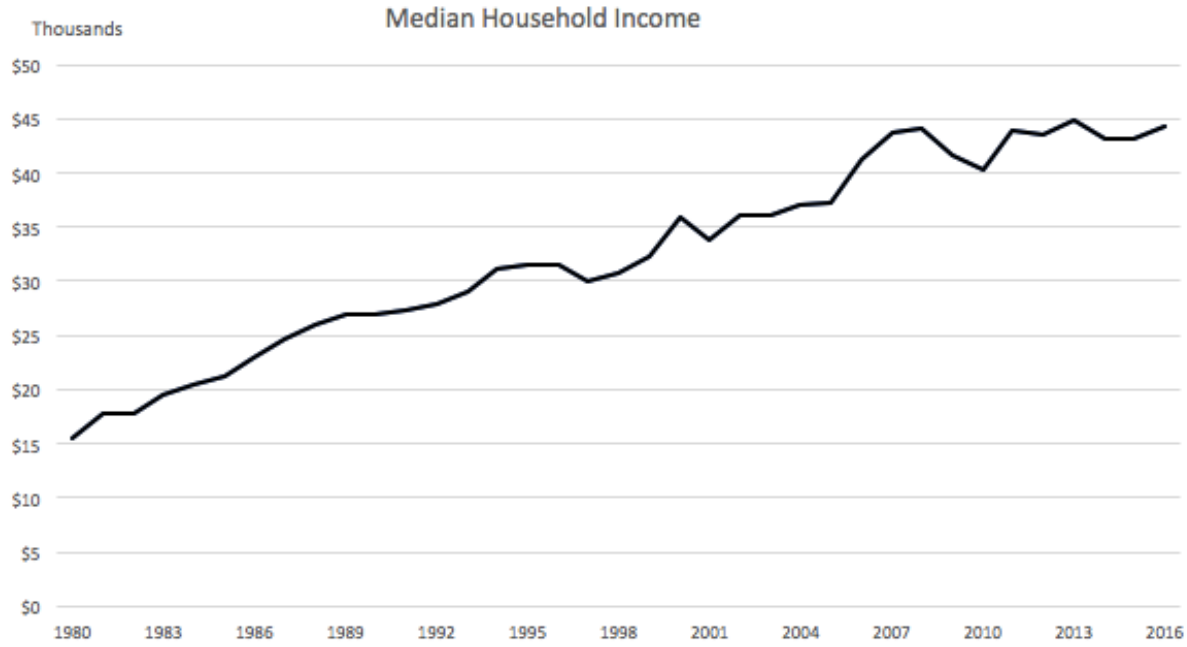

Figure 3. Median household income

The development of households' median income does not justify rising housing prices. The median household income has grown steadily. (United States Census Bureau.)

The disposable income of households is a financial fundament, which is usually being checked when evaluating apartment or any other housing prices. Housing prices should be relatively stable over the long term in relation to the disposable income of households. The deviations from long-term averages between earnings trend and price level gives signal that the housing price level is probably distorted. (Black et al. 2005, 3)

3.6 Employment

Straight linkage to the household income is employment. When individuals are working, they earn income and they can consume more, as their purchasing power increases. In the following figure, the individuals included to employment are the individuals who can work, are working or looking for job. They are considered as participated. Criteria range for the minimum age is 16 years.

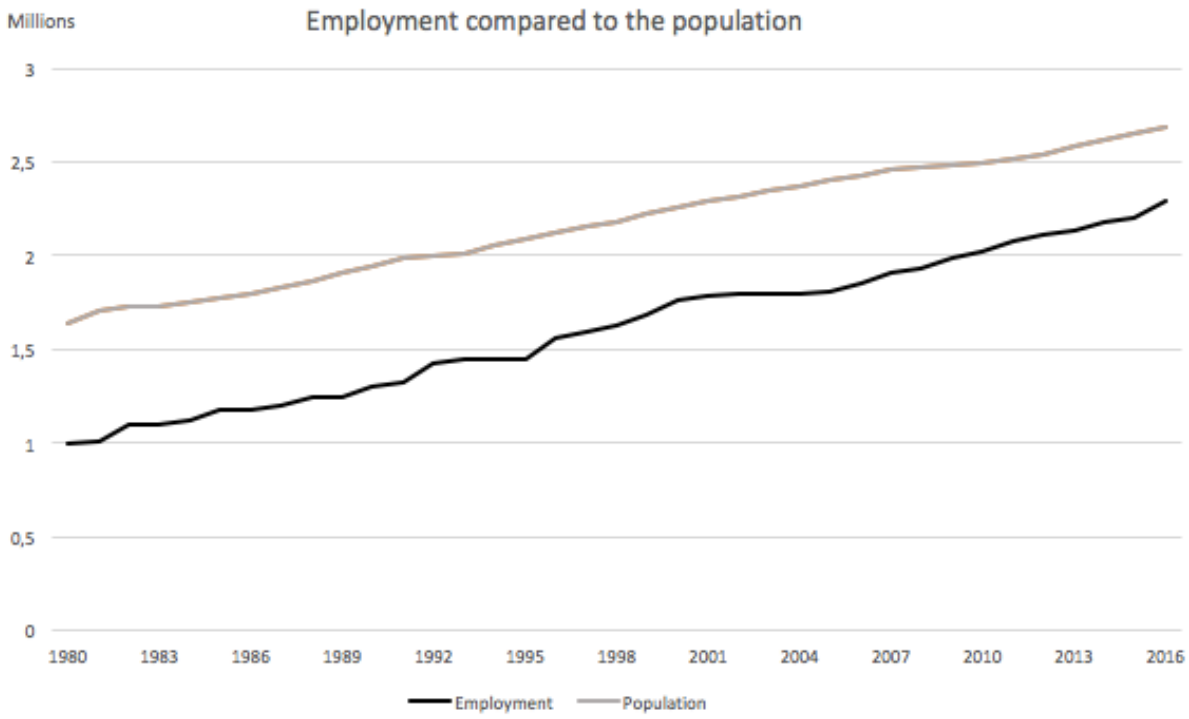

Figure 4. Employment compared to the population

3.7 Population and migration

As the population grows, the number of public services needed increases, and the need for households for housing increases. On the other hand, the decline in the number of populations often poses major challenges to local business. It is important to notice that housing market developments and housing prices vary a lot by the area. Therefore, it is necessary to find which factors are causing these differences and to understand the main reasons for regional differences, especially regarding this study, which examines the housing prices in Miami-Dade County.

Housing services are consumed by households, whereby the growth of households and the population is significant to the housing demand - nationally and regionally. Population growth is described in the following figure:

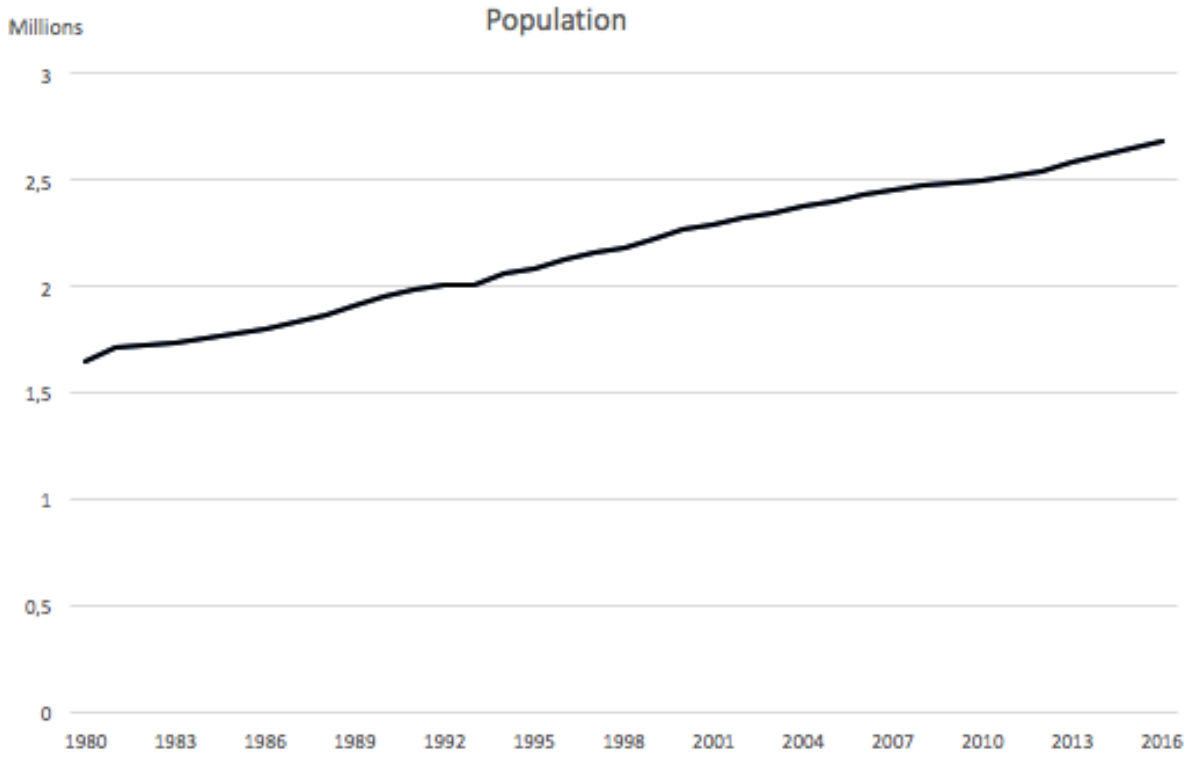

Figure 5. Population growth

In addition to natural population growth, migration has a significant impact on the regional housing market either by increasing or decreasing the number of housing demanders. Depending on the region, migrants are often young adults, people coming to the age of family planning, or the ones who change their place for study or work to the new areas. The migrants increase the pressure on housing production, especially when there is already demand for the area. An important element of demographic development is migration, which is especially affected regionally. Regional differences in demographic trends are big and the main reason for this is the inter-regional migration. This is one of the main reason why the population of Miami-Dade County is strongly increasing figure.

3.8 Consumer Price Index

A consumer price index has been developed for measuring the price level. It indicates the average price of services and goods purchased by consumers. The benefits and services

are emphasized by their consumption share. The method defines a basket of goods describing the consumption of the period under investigation. The commodities used in the basket can change because household consumption patterns are also constantly changing. The aim is to keep track of commodity prices that represent as much of the prevailing consumption as possible. The price index point was chosen as a change in value, as it illustrates the most cost-effective changes in time. The point is marked as one hundred.

The base year of the Consumer Price Index used in the study is 1982-84. In other words, the consumer price index for the 1982-84 is 100. The Consumer Price Index is one of the most widely used indicators of inflation observation. The consumer price index (CPI-U) is from Miami-Dade County. The figure below shows the trend in the consumer price index for the years 1980-2016.

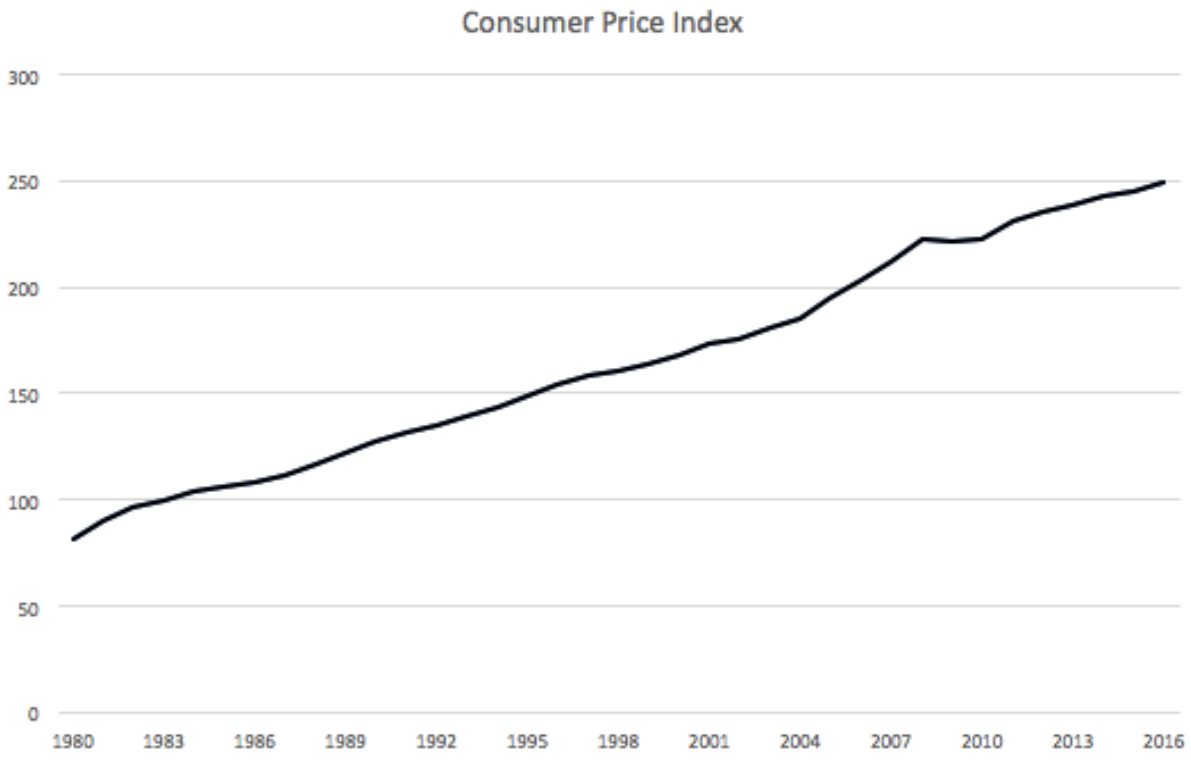

Figure 6. Consumer price index

The trend in the consumer price index has been fairly increasing over the whole period, which means that the individuals are increasingly consuming more goods and services in the researched county.

3.9 Gross domestic product

Housing markets are assumed to be in a positive relationship with the development of the Gross Domestic Product (GDP). GDP is an essential variable in this study due to its overall role as a measure of the national economy as a means of expressing whether the economy has grown or decreased. Housing-related economic activity represents a large part of GDP. Housing activities include, for example, construction, renovation, maintenance and housing services, such as mortgages, -insurance, -brokers, legal and migration services. GDP is reliable as a performance meter, and is generally accepted and internationally used. GDP growth means, among other things, that consumption and investment have increased. This in turn, raises demand, which in turn leads to a generally higher price level.
In the study, GDP correlated statistically significantly and relatively strongly (in initial 0.92, in developed 0.92) with the changes in apartment prices. Data of gross domestic product of Florida is collected from the Bureau of Economic Analysis, U.S. Department of Commerce. Data was gathered from the years 1980-2016.

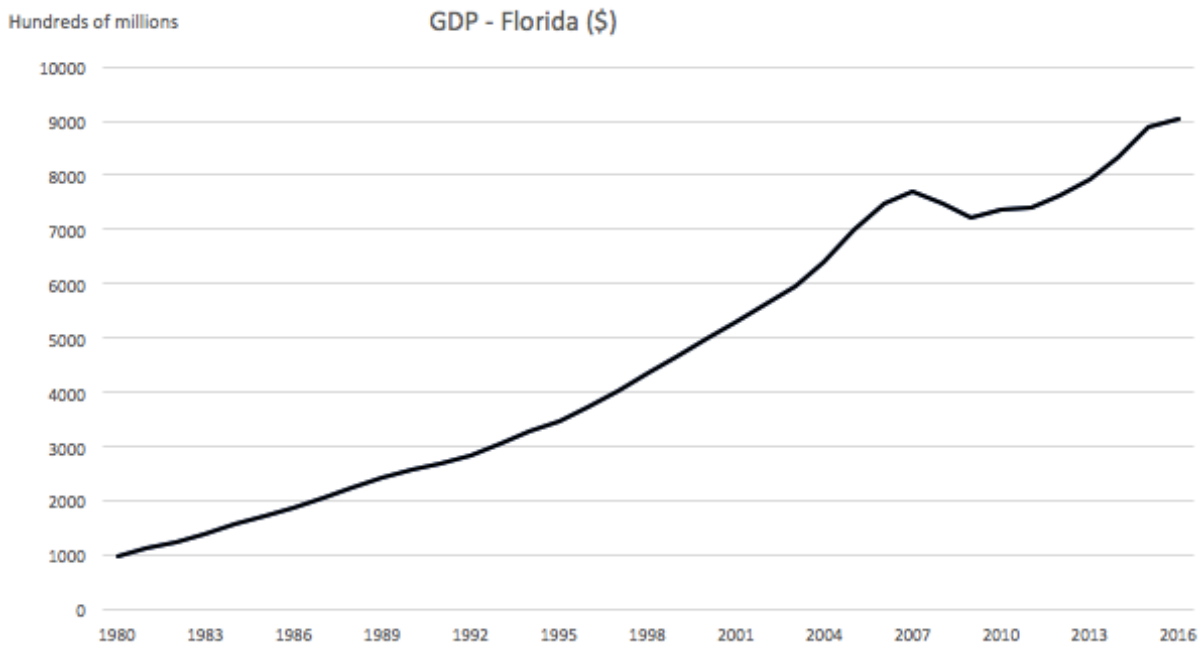

Figure 7. Gross domestic product

4. VALUATION OF THE APARTMENT PRICES

4.1 Linear regression

Linear regression has assumptions which needs to be noticed;

1) Linearity; when expressing the relationship between the response variable and the predictor variables, they are assumed to be linear on each other
2) Constant variance; variance of residuals which is constant
3) Multicollinearity missing; predictor variables are independent on each other
4) Normality; the residuals are assumed to be normally distributed in the model

Regression analysis is perhaps the most important statistical multivariate method. The central role of regression analysis is that other multivariate techniques are in one way or another derived from regression analysis. (Erkki, Liski and Simo, Puntanen 1976, 1.) Regression analysis is the most commonly used tool in economic research. Regression analysis describes and evaluates the relationship between two variables (explanatory variable and variable to be explained).

4.1.1 Simple regression

The regression model is based on the Arbitrage Pricing Theory which will be discussed in becoming section 4.2.2. The variable which we are predicting is the criterion variable (Y) and the variable which we are basing the predictions is predictor (X). The formula is defined as the following:

$$y = \alpha + \beta x + \varepsilon \qquad i = 1,...,n \tag{1}$$

or

$$y = X\beta + \varepsilon,$$

Where $\varepsilon \sim N(0,\sigma 2)$

4.1.2 Multiple regression

Multivariate statistics and multiple regression analysis allows to infer that there is a relationship between two or more variables.

Multivariate forecasting models use references to the movements in the current values or past values of other variables to explain changes in a variable.

The model of multiple linear regression equation is as follows:

$$y = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \ldots + \beta_m x_{mi} + \varepsilon_i, \quad (2)$$

If the variable is not linear, we can smoothen it with a natural logarithm (ln) to make it more linear.

$$\ln(y) = \ln \beta_0 + \beta_1 \ln(x_{1i}) + \beta_2 \ln(x_{2i}) + \ldots + \beta_m \ln(x_{mi}) + \varepsilon_i, \quad (3)$$

The models selected to the multiple regression analysis were as follows:

Table 1. Initial model's regression equation

Initial variable
$Y = \beta_0 + \beta_1$ Empl. $+ \beta_2$ Popul. $+ \beta_3$ Househ.$_{t-3} + \beta_4$ Med.HH.Income$_{t-3} + \beta_5$ Int.rate$_{t-2} + \beta_6$ Inc.per.cap $+ \beta_7$ CPI t-2 $+ \beta_8$ GDP $+ \beta_9$ Civ.lab.unempl.t-2 $+ \beta_{10}$ Unempl. $+ \varepsilon$

Table 2. Selected regression equations for further testing

	Selected variables
Model 4	$Y = \beta_0 + \beta_1$ Empl. $+ \beta_2$ Househ.$_{t-3} + \beta_3$ Med.HH.Income $_{t-3} + \beta_4$ GDP $+ \beta_5$ Unempl.$_{t-3} + \varepsilon$
Model 6	$Y = \beta_0 + \beta_1$ Empl. $+ \beta_2$ Househ.$_{t-3} + \beta_3$ Med.HH.Income $_{t-3} + \beta_4$ GDP $+ \varepsilon$
Model 14	$\ln Y = \ln \beta_0 + \beta_1$ Empl. $+ \beta_2$ Househ.$_{t-3} + \beta_3$ Med.HH.Income $_{t-3} + \beta_4$ GDP $+ \varepsilon$
Model 19	$\ln Y = \ln \beta_0 + \beta_1$ Empl.$_{t-1} + \beta_2 \ln$ (Househ.$_{t-3}$) $+ \beta_3 \ln$ (Med.HH.Income $_{t-3}$) $+ \beta_4$ GDP $+ \beta_5$ Unempl.$_{t-3} + \varepsilon$
Model 23	$\ln Y = \ln \beta_0 + \beta_1$ Househ.$_{t-3} + \beta_2$ Med.HH.Income $_{t-3} + \beta_3$ GDP $+ \beta_4 \ln$ (Unempl.$_{t-3}$) $+ \varepsilon$
Model 25	$Y = \beta_0 + \beta_1$ Med.HH.Income $_{t-3} + \beta_2$ GDP $+ \varepsilon$
Model 26	$Y = \beta_0 + \beta_1$ Popul. $+ \beta_2$ Incom.p.capita $+ \beta_3$ GDP $+ \varepsilon$

Initial model needed to be modified into 26 different models by its significance and the explanation level. These terms will be explained and described in the becoming section of coefficient of determination (4.1.3).

4.1.3 Coefficient of determination

The ratio of the regression sum to the total squared sum is denoted by R^2:

$$R^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST} \tag{4}$$

R^2 is the explanatory factor or explanatory force of the model. Usually in regression analysis we are interested in this chapter. The explanatory coefficient is used as a measure of model goodness, but it should be kept in mind that R^2 is just one meter (Liski 1976, 32-33). The most important objective of regression analysis is to explain the behavior of the variable to be explained.

In the regression analysis, the level of the explanation level is R^2, which explains how much the explanatory factors indicate the variation of the factor to be explained. The highest value is one which is needed. If there is no connection between the explanatory and explanatory variables, then R^2 is close to zero. In my thesis, the explanation is attempted to get the model as high as possible.

In this study, reverse regression is used in developed multiple regression analysis (5.2). The name of the method is due to the fact that the model of all the descriptors is first

formed and then dropped off one by one based on the given margin of significance. Bad explanatory factors are being removed which were unemployment rate and labor force of which unemployment.The data required for the developed multiple regression model needed to be modified in order achieve high R-Square (0.9-0.98).

P-value is required for coefficient of determination in this study, as R^2. P-value is for significance level, which describes what is the probability of the values being significant. Also, P-value is taking similar probability values than a Chi-square. P-value matches to x^2 – SS of n standard normal distribution. The highest degree of P-value researched in this study is in initial model, which is only created for comparison, as it does not give reliable results. All models with a P-value below the 0.5 value have been considered to be significant. As mentioned, the initial model needed to be modified into 26 different models, in order to achieve desired R-Square and P-values. Main outputs of these results are as follows:

Table 3. R-Square and P-Values by models

Model	R Square	Highest P-Value
Initial	0,97	0,973
Model 4	0,97	0,010
Model 6	0,97	0,003
Model 14	0,97	0,000
Model 19	0,97	0,044
Model 23	0,98	0,007
Model 25	0,92	0,000
Model 26	0,93	0,018

In this table we can observe, that the initial model has too high P-value at least in one variable. The initial model had even eight variables out of ten which had significantly high P-values in the initial model, this is a significant reason why initial model could not have been used for the reliable forecast. Other models' P-values are by the range so we will proceed forward to the hypothesis tests.

4.1.4 Hypothesis testing

To test the regression model statistics with a hypothesis, we need to complete the following steps:

1) Formulate a null hypothesis and alternative hypothesis
2) Build statistics to the hypothesis tests
3) Define criteria range, which will accept or reject model

We will go through three different hypothesis tests in the following sections.

4.1.4.1 Bayesian information criterion

Bayesian information criterion (BIC) is a measure of goodness of fit and the number of parameters which needs to be estimated to achieve the desired degree of fit. Algebraically, BIC is defined as follows (Chris, Brooks, Sotiris, Tsolacos 2010, 243);

$$\text{BIC} = \ln(\hat{\sigma}^2) + \frac{k}{T} \ln T \quad (5)$$

where,

$\hat{\sigma}^2$ is the residual variance or the residual sum of squares divided by the number of observations (T).

In this research, the used BIC formula is as follows:

$$\text{BIC} = k \ln (n) + n [\ln (\text{RSS}/n-k-1)] \quad (6)$$

where,

n = number of observations

k = number of free parameters

RSS = the residual sum of squares

In the following table, all the 7 models will be analyzed and tested with a BIC.

Table 4. Bayesian information criterion results

Model	**BIC**
Initial	768,8
Model 4	803,1
Model 6	822,5
Model 14	7,9
Model 19	-1,5
Model 23	1,9
Model 25	882,7
Model 26	859,5

The criteria in the BIC is that the lower the value is, the better is the result. Negative values are in the criteria range. By the table above, we can observe that models 14, 19 and 23 are accepted by the Bayesian information criteria.

BIC is argued to be inefficient because of the limited outcome of the data (Ibid.). Therefore, we could not rely only on this specific test. Instead, we see the overall picture from all the researched hypothesis tests. The reason why actually this specific hypothesis test is not suitable, is that it does not relatively evaluate right, the models in which natural logarithms are used. As a conclusion, we do not reject any model at this step, because the results can be observed to be incomparable.

4.1.4.2 Durbin-Watson test

Durbin-Watson (DW) test was developed for time series data. The model tests regression residuals for autocorrelation. DW tests only for a relationship between an error and its previous value. One way to motivate the test and to interpret the test statistic would be in the context of a regression of the time t error on its previous value (Chris, Brooks, Sotiris, Tsolacos 2010, 149),

$$u_t = pu_{t\text{-}1} + \nu_t \tag{7}$$

where,

$\nu_t \sim N(0, \sigma^2_v)$.

The Durbin-Watson test statistics has null and alternative hypotheses:

$$H_0: p = 0 \quad \text{and} \quad H_1: p \neq 0 \tag{8}$$

Under the hypothesis, errors at residuals (t-1) and residuals (t) are not dependent on each other. After defining residuals (t-1) and (t), it is necessary to run regression model. From the summary, we select coefficient of residuals (t-1). It will be deducted from number one and then multiplied by two.

In a simplified version,

$$DW \approx 2(1-\hat{p}). \tag{9}$$

Where, $\hat{p}$ is the assumed correlation coefficient from 2. To reject the null hypothesis, DW should be from the scale of 1.5 to 2.5. In the table below, the results of DW-tests are compared.

Table 5. Durbin-Watson test

Durbin-Watson test

Model	# of observations	**DW Coefficient**
Initial	33	1,39
Model 4	33	1,42
Model 6	33	**1,66**
Model 14	33	1,11
Model 19	33	**1,66**
Model 23	33	**1,67**
Model 25	33	**1,71**
Model 26	33	**1,67**

By this table, we can observe that the models 6, 19, 23, 25 and 26 were accepted as they had autocorrelation by the criteria. Other three models were outside the criteria range 1.5-2.5 and needed to be rejected. The rejected models will not be selected for the final multiple regression in a use of forecasting, except initial for the comparison. Tests will be still run through all the hypothesis tests.

4.1.4.3 White test

White test is for testing regression residuals for the heteroscedasticity. All computed test statistics had 34 observations and critical value (X^2, 5%) was 23.685 in models 4, 6, 19, 23 and in initial. Model 14, had a critical value of 22.362, model 25 had 11.07 and model 26 had 16.919.

Table 6. White test

White test

Model	Chi sq	Test statististics	Result
Initial	N/A	N/A	Rejected
Model 4	N/A	N/A	Rejected
Model 6	23,685	-4,2	Rejected
Model 14	22,362	**5,47**	Accepted
Model 19	N/A	N/A	Rejected
Model 23	23,685	-5,36	Rejected
Model 25	11,07	**1,90**	Accepted
Model 26	16,919	**2,51**	Accepted

In white test, the rule of thumb is that test a Chi-Square needs to be higher than the value of R-Square multiplied by number of observations (nR-Square T-Stat.). Based on this table of white test, we can give an assumption that there is no heteroscedasticity in models 14, 25 and 26. Models 6 and 23 had T-Stat value lower than the Chi-Square and needed to be rejected. The reason why initial model, model 4 and 19 did not provide any result, was the quality of the program used to run the white test. Excel has a limit of 20 variables to be used in the regression analysis, and these models had X-variables above the limit.

As a summary of these hypothesis tests, we provide a table in which, all the test results are compared.

Table 7. Hypothesis test comparison

Summary of hypothesis tests

Model	BIC	DW	White test statistics	Result
Initial	769	1,39	N/A	Rejected
Model 4	803	1,42	N/A	Rejected
Model 6	823	**1,66**	-4,2	Rejected
Model 14	**8**	1,11	5,473192972	Rejected
Model 19	-2	**1,66**	N/A	Rejected
Model 23	**2**	**1,67**	-5,36	Rejected
Model 25	883	**1,71**	**1,90**	**Accepted**
Model 26	860	**1,67**	**2,51**	**Accepted**
Criteria	**Low**	**1,5-2,5**	**High**	

From this table, we can observe that only model 25 and model 26 got accepted through the DW and white test. Though, these selected models did not give positive results in BIC, its' DW results are in the criteria range and white tests give positive results. The reason for their high BIC value is assumed to be the quality of the BIC as an estimator, as it did not consider natural logarithms in the data respectively. Model 25 will be selected for the further multiple regression analysis among with the model 26.

As an example, for BIC use, in model 23, the natural logarithm was used in condominium prices and in unemployment rate, the result is in the table 5 and 7. After removing natural logarithms from both variables, BIC increased to 831.843, DW increased to 1.68 and white test difference increased to 20.05. We can assume that BIC is not reliable test in the research, as the result values are relatively too high.

Though, model 25 and 26 are assumed to be the most significant models of the selection, we will go through the error analysis to double-check that error estimates are reasonable.

4.1.5 Forecasting errors

To obtain reliable results, it is necessary to estimate the percentage of the error. By running through the error analysis, the results will provide the uncertainty or quantity of error.

Table 8. Error analysis

Error analysis

Model	Min	Max	Median	MEAN	Strd dev
Initial	-21,68 %	30,06 %	1,52 %	0,48 %	0,0952128
Model 25	-22,12 %	20,20 %	-2,08 %	1,09 %	0,1162522
Model 26	-23,55 %	24,60 %	-0,52 %	0,76 %	0,1109611

The criteria for error estimate is, the smaller the MEAN percentage is, the better it is because it indicates of the smaller share of error.

MEAN values are absolute numbers, by this observation we can see that MEAN values are low, which is good.

This table gives an assumption, that model 26 is the most significant model for the final selection to forecast the future prices and it will be selected for the final. Also, because the model provided better results in white test. Initial model will be left as a baseline scenario, which reflects of the incorrect price estimate.

Standard deviation shows us the average spread of results, which is about the mean value.

4.2 Other valuation methods

4.2.1 Capital asset pricing model

The apartment owner is in the same situation with the shareholder of the stock exchange because it is a share of stock in a housing co. In Harry Markowitz's (1952, 77-79) theory, one can speak of a normative point of view where a "representative" or "typical" investor makes his investment decisions under certain assumptions. On the basis of this theory, the various researchers developed within a short time the equilibrium model for the

securities market (Mervi and Jyrki Niskanen 2000, 216). Main focus is on the studies of William Sharpe (1964), T.E. Copeland and J.F. Weston (1988) based on the Capital Asset Pricing Model (CAPM) and supplementary models complementing it for financial pricing.

Sharpe (1964) describes two main assumptions in capital asset market:
Firstly, all economic agents can, on the same terms, get loans or invest in a risk-free interest rate. Another assumption is that investors' expectations of different investment targets, their expected returns and standard deviations and correlation coefficients are homogeneous (William, Sharpe 1964, 433-434). In the CAPM literature, other major assumptions include, maximizing investor benefits and risk-taking, investing according to Markowitz's expectancy variance rule, and completeness of capital markets. Sharpe (Ibid.) highlighted the two main assumptions, but also the lack of transaction costs and taxes, the availability of infrequent small parts of the portfolio of investments, the availability of costly and timely information to all investors, as well as the one-period investment horizon. (Jyrki and Mervi, Niskanen 2000, 217).

To enable CAPM to calculate a yield requirement for a share, the risk-free interest rate r_f, which usually operates as a government issuance short-term debt instrument, and the so-called market risk premium as a difference between market portfolios and risk-free interest rates, should be alongside the beta coefficient. The CAPM formula can be represented in the following form (Fama and MacBeth 1973, 610):

$$E(R_i) = E(R_0) + \beta_i[E(R_m) - E(R_0)] \quad (10)$$

In the equation above, $E(R_i)$ is the required return on the share (i). $E(R_0)$ describes the risk-free interest rate, $E(R_m)$ stands for the return on market portfolio and β_i is the beta factor for the share (i).

It can be seen from the equation 2 that the required return on any share is equal to the risk-free interest rate plus the risk premium for the share. According to CAPM, all shares are in the securities market because it only gives the highest return on a given risk level or the lowest risk at a given yield level. If the stock is not in the securities market, it is temporarily over- or underpriced, and in the effective market assumptions, its expected return rises or falls immediately to the level required by CAPM (Ibid.).

CAPM has got extensive criticism since it was developed. The main weaknesses in the model are the assumptions it requires, which poorly reflects the reality. Among other reasons, CAPM has not been used in this study.

4.2.2 Arbitrage pricing theory

The Arbitrage Pricing Theory (APT) which was developed by Ross (1976). APT is often used as an alternative asset model pricing model for the CAPM. The CAP model is conditional on the expected return on the market, but the APT also utilize more macroeconomic variables. Alike to CAPM, the APT model generates a securities market rate by which the expected value-to-risk ratio of return on investment can be illustrated. The assumptions of APT are that the return on investment is predictable by a factor model (linear regression), sufficient investment is available (decentralized risk can be eliminated) and the capital markets operate and do not allow for permanent arbitrage, for example risk-free profit. (Ross 1976, Bodie et al. 2005, 348-350 and Gaston 2009, 18-19.) The APT model has gained popularity in business life.

5. RESULTS

This chapter is going through the results of the initial model, where all possible variables were included. The purpose of this, is to be able to compare the incorrect (initial model) results to the results of corrected model, where all the insignificant variables are being removed.

5.1 Initial multiple regression model

Purpose of results of initial multiple regression model is to build a baseline scenario of which we will compare to the developed multiple regression model which is modified by excluding factors with low correlation and insignificant variables from the model. In this regression analysis, all possible variables were selected in the model. There are total of ten variables, which are employment, population, number of households, median household income, interest rate, income per capita, CPI, GDP, civilian labor force which unemployed and unemployment rate.

$$Y = \beta_0 + \beta_1 \text{Empl.} + \beta_2 \text{Popul.} + \beta_3 \text{Househ.}_{t-3} + \beta_4 \text{Med.HH.Income}_{t-3} + \beta_5 \text{Int.rate}_{t-2} + \beta_6 \text{Inc.per.cap} + \beta_7 \text{CPI}_{t-2} + \beta_8 \text{GDP} + \beta_9 \text{Civ.lab.unempl.}_{t-2} + \beta_{10} \text{Unempl.} + \varepsilon \quad (11)$$

This model achieved high R Square value of 96.8%, which indicates the percentage of how well we can explain the variation. Instead, the P-values in variables were above the criteria range, except median household income (0.0385) and GDP (0.0004). As mentioned, P-value is required for coefficient of determination, which describes what is the probability of the values being significant. By P-values, we can assume that values were not significant, which causes errors in the forecasted values.

5.2 Developed multiple regression model

An attempt has been made to explain the price behavior of dwellings over the examined period using a multiple regression model. There needed to be eliminated the coefficients one at a time, which were not statistically significant. Meaning that the variables were not correlating, the P-values were relatively high compared to the standard (0.05) or the models did not pass hypothesis test. Removing theses explanatory variables was continued until the model contained only statistically significant variables to explain the change in the prices of apartments. The initial model was modified to 26 different models. Selection after this point was based on R Square value and acceptable P-value. Several hypothesis tests and error analysis proved model 26 to be the most significant for this research. Equation used for the selected model is as follows:

$$Y = \beta_0 + \beta_1 \text{ Popul.} + \beta_2 \text{ Inc.per.cap.} + \beta_3 \text{ GDP} + \varepsilon \quad (12)$$

Data used for the equation is since 1980 until 2016. Also, forecasted values for population, income per capita and GDP were used in this model for the years 2017, 2018 and 2019. We obtained 34 observations and the statistics is to be explained with the $R^2 = 93{,}2\%$, which is the percentage of how strongly the explanatory factors indicate the variation of the factor to be explained. P-values were below 0.05 and were considered strongly significant. Model 26 got accepted through Durbin-Watson and white test, and will provide the final forecast of the apartment prices in Miami-Dade County.

5.2.1 Predictive Ability

Forecasted values, which were used for data for years 2017, 2018 and 2019 are based on estimates and the actual future values may differ from these expectations. These estimators were gathered from the County reports. The possible difference of actual and expected variable data is assumed to increase the error and by this, cause changes in the results.

5.2.2 Annual forecast in a short term

The assumption of this thesis was that the condominium prices will increase during the next three years in Miami-Dade County. The assumption was based on increasing macroeconomic factors in the researched area, which positively affects apartment prices changes.

Based on the results of multiple regression analysis, we can assume that prices will increase. In the table below, the final results of forecasted prices for the next three years are presented.

Table 9. Forecasted apartment prices

Forecasted prices

Model	2016	2017	2018	2019
Initial	$ 225 450	$ 245 608	$ 260 772	$ 331 296
Growth	-	8,94 %	6,17 %	27,04 %
Model 26	$ 225 450	$ 230 876	$ 236 280	$ 332 394
Growth	-	2,41 %	2,34 %	40,68 %

Initial model assumes apartment prices to increase 8.94% in 2017, 6.17% in 2018 and 27.04% in 2019. Initial model's purpose was to create comparison to the final model (26), how price forecast will differ when using unsuitable model compared to the modified model.

We can assume that model 26 provides reliable results, as it provided the most significant results in hypothesis tests and in error estimate. Apartment prices are assumed to increase 2.41% in 2017, 2.34% in 2018 and 40.68% in 2019. The largest increase in forecasted prices will be in 2019, which may indicate of becoming housing bubble. After a large increase, the prices usually drop when the bubble bursts.

Comparing the results of initial and final model, we can observe from this forecast assumption that the forecasted values in 2019 are very close to each other. Considering that initial model did not provide significant results in coefficient of determination or in hypothesis tests.

6. DISCUSSION

A large part of the housing of researchers think that especially in the macro-economic factors are mostly explaining the changes in housing prices, which depends on the size of the scale. In macro-economic level, the apartment price changes are affected by the macroeconomic factors. Macroeconomic factors are factors affecting demand; changed in interest rates, income and expected housing price changes. Expected housing price changes has a big impact on how housing price trends will grow. Also, monetary policy, fiscal policy decisions and interest rates on loans affected to the price growth in all the regions of the United States. Interest rate was used as a variable in the forecasted equation of initial model, but could not be included in the final model because it did not provide significant results.

As the table 9 assumes that prices will grow slowly during the next two years, I assume that prices will grow slowly or even decrease a little in this year 2017. The new president of the United States was elected in January 2017, change from a democrat to a republican politician has caused a lot of discussion around the world. Fear from the war with the North-Korea has recently awaked discussion in the media. Even the fear can have significant negative consequences to the economy and by this effect the housing market negatively. As Miami is known from its foreign investors, the investors and the consumers can be assumed to become more sceptic where the market will go next.

It can be already seen in the housing market of Miami, that the sellers' and the buyers' apartment markets are slow, rents and apartment sales prices are slightly decreasing, but instead, construction for new apartments is increasing.

7. CONCLUSION

Factors affecting housing prices are the fundamentals. In addition to rental income and household income, housing prices are also influenced by other factors. These include interest rate, employment, population growth or reduction, available income, consumer price index and gross domestic product. These factors were research internationally, as they cause changes to the market situation without depending on any specific country.

By evaluating the households' indebtedness, there can be predicted the development of housing prices. Household indebtedness is generally a consequence of facilitating access to credit, which increases the demand for housing. Based on the laws of demand and supply, increasing demand will raise housing prices. The low rate of interest naturally attracts homeowners, as high interest rates can make the total costs of loan much higher. In the case of low interest rates, households' indebtedness may increase rapidly. This instead causes a phenomenon, that rising house prices are tempting household to buy dwellings, which is originally a result of debt. As concluded, borrowing will raise housing prices and rise in prices will increase the borrowing again, which can be a problematic cycle.

Population growth naturally affects housing demand positively and by this increases the housing prices. Population growth was used in the final model as one variable out of three. Assumption is that population growth has a significant importance on apartment prices, as the results were the most significant compared to the other 25 models which were tested.

Gross domestic product is considered to be the strongest variable in this research, as it correlates to apartment prices 0.92. The achieved correlation result is the same in initial model and in the selected final model number 26.

Initial model which was used as a baseline scenario for the comparison, was tested into 26 different models to obtain reliable results for running the final forecast with the multiple regression model. One of the criteria was to get acceptable values in coefficient of determination, which were P-value and R-Square. P-value is for significance level, which describes what is the probability of the values being significant. Acceptable P-values were under 0.05. R-Square indicates the percentage of how much we can explain the variables. The aim was to achieve high R-Square and hypothesis test results by the criteria.

Hypothesis tests used were Durbin-Watson test and white test. This selection removed three models including the initial model, because the Durbin-Watson results were not in the

criteria range of 1.5-2.5. White test was tested to total of five models out of eight, because of the limitations of data tool. Only two models got acceptable results from both, Durbin-Watson and white test.

The final model selection occurred via error analysis, where the number of uncertainty was estimated. Model 26 got selected to the multiple regression model to analyze the forecasted apartment prices for 2017, 2018 and 2019.

The assumption of this thesis was that the condominium prices will increase during the next three years in Miami-Dade County. The assumption was based on increasing macroeconomic factors in the researched area, which positively affects apartment prices changes. Based on the achieve results from the multiple regression analysis, we can assume that prices will increase 2.41% in 2017, 2.34% in 2018 and 40.68% in 2019. Median apartment price is assumed to be approximately $225,450 in 2016. The forecasted price is assumed to be $230,876 in 2017 and $236,280 in 2018. Instead, forecasted price for 2019 is expected to increase up to $332,394.

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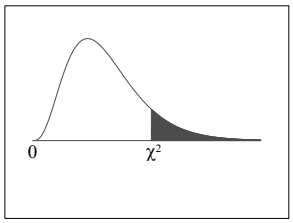
Appendix

Appendix 1.

Chi-Square Distribution Table



The shaded area is equal to α for $\chi^2 = \chi^2_\alpha$.

df	$\chi^2_{.995}$	$\chi^2_{.990}$	$\chi^2_{.975}$	$\chi^2_{.950}$	$\chi^2_{.900}$	$\chi^2_{.100}$	$\chi^2_{.050}$	$\chi^2_{.025}$	$\chi^2_{.010}$	$\chi^2_{.005}$
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169