TALLINN UNIVERSITY OF TECHNOLOGY
SCHOOL OF ENGINEERING

Department of Electrical Power Engineering and Mechatronics

DESIGN OF A ROBUST CONTROLLER USING SLIDING MODE TECHNIQUE FOR A LINEAR BELT-DRIVEN SYSTEM

MASTER THESIS

MECHATRONICS PROGRAM

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Tallinn, 2017

AUTHOR’S DECLARATION

Hereby I declare, that I have written this thesis independently.

No academic degree has been applied for based on this material.

All the works, major viewpoints and data of the other authors used in this thesis have been referenced.

The thesis was completed under the supervision of **Prof. Trieu Minh Vu**.

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The thesis complies with the requirements for graduation thesis.

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TALLINNA TEHNIKAÜLIKOOL
INSENERITEADUSKOND

Elektroenergeetika ja mehhatroonika instituut

ROBUST KONTROLLERI DISAIN LINEAARSELE RIHUMÜLEKANDEGA SÜSTEEMILE KASUTADES SLIDING MODE TEHNIKAT

MAGISTRITÖÖ

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Tallinn, 2017

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THESIS TASK

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THESIS TOPIC:

(in English) Design of a Robust Controller using Sliding Mode Technique for a Linear Belt-Driven System

(in Estonian) Robust Kontrolleri Disain Lineaarsele Rihumülekandega Süsteemile Kasutades Sliding Mode Tehnikat

Assignments to be completed and the schedule for their completion

No.	**Description of the assignment**	**Completion date**
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2.	Implementing the control strategy	18.02.2017
3.	Mathematical Modeling of the system, Debugging the complications	25.02.2017
4.	Simulating the system in MATLAB-SIMULINK	11.03.2017
5.	Attempts to see the changes in behavior with the parameters change	25.03.2017
6.	Attempt to improve the assumed values and the results obtained so far	01.04.2017
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Table of Contents

PREFACE

This thesis deals with designing a robust controller for Linear Belt Driven Systems with the help of Sliding Mode Control Approach. The thesis task is carried out at Tallinn University of Technology, Tallinn, Estonia for the Masters Level Education (MSc. in Mechatronics). The idea of the topic is originated from a real project in Kurresaare city of Estonia being overtaken by the university.

The linear belt-driven system is already being used in Kurresaare and is used to conduct experiments over a ship model which is connected to a carriage in a towing tank which is 60m long. DC Servos are used to drive the system. The system is controlled by a Proportional Integrator plus Derivative (PID) Controller. PID controller is a generic controller and is not precise enough to control the position of the carriage. The elasticity of the belt, the presence of frictions and un-modeled dynamics lead to error in position control of the carriage. For dealing with this kind of system, a robust controller is needed. The chattering-free Sliding Mode Control (SMC) approach is used in the thesis to control the system. The simulations are made on Simulink – MATLAB and the simulation experiments showed that the system is free from “chattering”, is robust to eliminate the noises/disturbances added to the system in a very short duration. Additionally, reference tracking is done with the errors which are minimal to an extent that can be considered negligible.

I would like to thank Prof. Trieu Minh Vu for giving me an opportunity to work on this topic and guiding me throughout. A special thanks also due to Prof. Mart Tamre for his consistent support and words of encouragement.

PREFACE IN ESTONIAN LANGUAGE (EESSÕNA)

Käesolevas lõputöös on välja töödatud kontroller lineaarsele rihmajamile. Lõputöö ülesanded on lahendatud Tallinna Tehnikaülikoolis, mehhatroonika magistriõppe lõputööna. Teema idee pärineb reaalsest projektist Kuressaares, mis on Tallinna Tehnikaülikooli poolt üle võetud.

Lineaarseid rihmajamid on juba Kuressaares juba kasutusel, nende abil tehakse katseid laevamudelitega 60 m pikkuses basseinis. Süsteemi käitamiseks kasutatakse alalisvoolu servomootoreid ja süsteemi juhtimiseks PID (Proportional Integrator plus Derivative) kontrollereid, kuid nende täpsus ei ole piisav. Rihma elastsus, hõõrdejõud ja modelleerimata dünaamika tekitavad vea veose asukoha määramisel. Sellise süsteemi puhul on tarvis kasutada robustsemat kontrollerit. Antud töös on süsteemi juhtimiseks kasutatud vibratsioonivaba libisevat juhtimisrežiimi (Sliding Mode Control - SMC). Simulatsioonide tegemiseks on kasutatud tarkvara Matlab Simulink keskkonda. Testid näitasid, et süsteemis ei esine kõrvalekaldeid ning süsteem suudab kõrvaldada lühiajalise müra ja häired.

Omalt poolt tahaksin tänada Prof. Trieu Minh Vu-d juhendamise ja võimaluse eest töötada antud teemal. Erilised tänusõnad lähevad Prof. Mart Tamrele tema järjepideva toetuse ja julgustavate sõnade eest.

CHAPTER1: INTRODUCTION

1.1 OVERVIEW

Belt-drives usage gathered significant attention and is the most common method of power transmission from past almost two centuries. The most common technological devices like laptops, computers, cars, media devices, ventilation systems etc. are predominantly dependent on belt-drives. Another significant sphere where the belt-drives are exhaustively used includes industrial transportation systems often called as conveyor belts. The first conveyor belt assembly line was introduced by Henry Ford in 1913 in the Ford Motor Company Factory.

The efficiency of belt-drives can be inferred from the fact that in certain applications its level has reached to a whopping 98%. Additionally, many scholars have cited varied advantages of using belt-drives as a better substitute for power transmission over the chain and gears. Few of the advantages of belt-drives are: simple installation, low maintenance, smooth and noise-free operations, cleanliness, no lubrication needed, can absorb sudden shocks, wide speed-ratio range operations, easy to visualize the failure etc. other than the major advantage of being a low-price transmission mean. [1].

Now-a-days, Timing-belts (also called as Toothed-belts) are a leading edge in industrial sectors. In industrial applications, the precedence is given to motion control performance and in furtherance to that on position tracking and rapidly changing dynamics. These timing-belts are also not without their merits and demerits. Some noteworthy advantages include high efficiency, long travel length, suitability to higher speed applications and of course the low cost make them ideal choice to be used [2] and the disadvantages comprise high transmission errors and uncertain dynamics [3]. Also, belt-drive dynamics do have more resonance frequencies which causes de-stabilizing in feedback control [4].

To simulate the original system which is being installed in Kurresaare, a city of Estonia, the need is to get the Motion Equations of the system. The mathematical model of such a linear belt-driven system is discussed in [5]. The same system has been simulated in this research endeavor.

Incongruities are bound to creep-in when comparisons are made between a real plant and a mathematical model for the same. Mathematical Model is usually designed to conduct simulations to see the behavior of the system for any desired purpose. Generally, for the ease of the calculations, it is a common practice to make few assumptions and reduce the complexities. These assumptions become one of the key reasons that account for those inconsistencies. Un-modeled dynamics of the system and the variations occurring in the plant variables are other major factors contributing to irregularities and discrepancies.

To control the systems being considered, the controllers like PD or PI or even PID are conventionally used by re-designing them to cope-up higher-order-nodes [6]. However, they can't control the systems that introduce more resonant frequencies [7]. Hence, the controllers just mentioned are not sufficient enough to deal with the errors and for the system considered in this thesis. The method of control signal filtering could be used to improve the performance [8] but it also not sufficient as mechanical vibrations, plant variations, load-torque disturbances and uncertain dynamic behavior are few of the key issues which are needed to be pondered upon. The controller must be capable of stabilizing the load oscillations in the case of toothed-belt systems.

The above discussion creates an inevitable need of designing a robust controller for the complex systems which is a nice area of research. Many researchers have contributed and are contributing to a continual improved outcome in the area. As a consequence, time has witnessed new controllers being designed by the enthusiastic researchers that not only reach the required performance levels and deal with system mismatches but they also aid in improving the efficiency of control.

For the Linear Belt-Driven System discussed in current thesis, the crux of the task revolves around making a robust controller for the system. Out of many other approaches like Proportional-Integral plus Derivative (PID) Control, optimal Linear Quadratic Regulator (LQR) Control, Composite Non-Linear Feedback (CNF) Control, Adaptive Control etc., Sliding Mode Control (SMC) [9] [10] [11] [12] [13] [14] [15] [16] approach has been recommended by few authors to improve the robustness of the elastic system [17] [18]. Hence, SMC has been employed in this task too.

Every technology replete with pros and cons. SMC has a major drawback of controller's discontinuous switching action so as to keep the system stable and to work in accordance to the desired expectations. This in theoretical terms means that the control must switch with infinite frequency to provide the total rejection to the uncertainties present in the system. This discontinuous control signal leads to a phenomenon called "chattering" [12] [14] [19] [20] which is dangerous for mechanical systems and may lead to wear and tear of the parts involved and in extreme cases may even the failure of the whole system.

In this research, a chattering-free SMC is used to control the system's behavior. Belt-Stretch and its control are important to achieve the vibration-free performance [21] [22]. Thus, paramount importance is given to the belt-stretch in the present research endeavor. The control law is derived by the combination of the Lyapunov Theory and the SMC Theory.

The simulated system has proven its effectiveness in reducing and finally subsiding all the vibrations and noises which are introduced in the system intentionally at a quite quick pace. For simulating the system, Simulink – MATLAB is used. The plant is modeled and the position tracking error is determined.

1.2 OBJECTIVES OF THE RESEARCH

The objective of the research is to simulate the Linear Belt-Driven system in Kurresaare, Estonia and to design a robust controller so as to control the position of the carriage on which the ship model is attached. The Motor Inertia is the main motive force of the system. The inaccuracies currently occurring in the system are found to be primarily because of the fact that the simplest controller i.e. PID is currently installed on the system. For such a dynamic system, a robust controller based on Sliding Mode Control (SMC) Theory is to be built which can deal with the un-modeled dynamics, plant variations, frictions involved and improve the system behavior. The motive is to make the system robust enough to deal with all the unknown parameters and disturbances whilst keeping the position error as low as possible.

1.3 SCOPE OF THESIS

The thesis deals with understanding the system and then formulating the mathematical model of the system by making the suitable assumptions. The system (called "The Plant") is to be controlled with the help of a robust controller and the error in the position tracking is mainly determined with the help of the simulations.

The controller should be developed on the basis of Variable Structure System (VSS) Theory [15]. VSS are the systems which have discontinuities in the differential equations describing the dynamical behavior of the system. The systems have a state-space that has Hyperplane Segments. In the each segment, the system can be modeled by a continuously-differentiable function. Such characterization of system changes as the state-trajectory crosses a discontinuous hyperplane.

A special form of VSS approach referred to as Sliding Mode Control abbreviated as SMC is used in this research by which a better position control can be achieved. Chattering-free SMC approach is used in the research; the control scheme is simulated in Simulink – MATLAB and the results of simulations are obtained.

1.4 ORGANIZATION OF THESIS

The Chapter 2 deals with the basics of Sliding Mode Control. In Chapter 3, Conventional Sliding Mode Control is discussed and an example is taken to explain the concept in detail. The System's Mathematical Model Formulation is discussed in Chapter 4 which is followed by SMC Controller Design in Chapter 5. The tests conducted and the results of the simulations are shown in Chapter 6. The summary which includes conclusion, limitations of the research, suggestion for further improvement and future scope is discussed in Chapter 7 follwed by the summary in Estonian Language.

CHAPTER 2: BASICS OF SLIDING MODE CONTROL

2.1 SLIDING MODE CONTROL

Sliding Mode Control (SMC) evolved in 1960's from pioneering work in former Soviet Union [9] [15] [23]. It has been applied to many types of systems like linear and non-linear systems, continuous and discrete-time systems, small scale-large scale and infinite-dimensional system. It is a robust technique for controlling the systems which includes the control achieved on electric drives, mobile drives, robotic arms, underwater vehicles, drones and space crafts and many more.

An important characteristic of Variable Structure Systems (VSS) includes the capability of possessing new properties which are actually not inherent in the continuous systems. As an example, an asymptotic-stability can be realized by adding the two structures among which none is asymptotically stable. An "astatic" control method known as Variable Structure Control (VSC) was proposed by a researcher named Emelyanov [24]. In VSC, a discontinuous hyperplane (also called as Switching Hyperplane/Switching Surface/Sliding Surface) is introduced artificially in the system with the help of a control input. When the system-states are made to direct towards the hyperplane and are further made to slide along it or it its bounded vicinity, consequently a special form of VSC is obtained. This special form is known as Sliding Mode Control.

In SMC, the control law is dynamic in nature which means that the control law is changing continuously as per some pre-defined rules. These rules in SMC's technical terminology are called as a "Switching Functions". Ideally, the controller pushes the system so as to make the state-trajectories reach the pre-defined sliding surface in a finite time and then keeps the system on it or its close vicinity. In this course, the desired dynamics which are well-specified by the switching hyperplane are assured.

The moment the state trajectory is "above" the pre-defined sliding surface i.e. in sliding manifold, a feedback controller uses one gain to bring it towards the sliding surface. On the contrary, when the state trajectory is "below" the sliding surface then another gain acts to bring it towards the sliding surface. And, once the system reaches the sliding surface, the contol input is maintained and the controller makes sure that the trajectory will be locked on sliding surface. The choice of the switching function is directly proportional to desired behavior of the system.

With closed loop response, the mechanical system which is controlled by SMC methodology becomes unaffected to matched uncertainties [16] .

SMC has three major advantages which are:

a) Order reduction
b) Parametric invariance i.e. the robustness
c) Desired performance definition

Designing a controller based on SMC has been a significant area of research. Many other techniques associated with it have also been proposed time to time. The basis for all of those however is the same wherein, the first step is to design the sliding surface in the state-space so as to get the reduced order sliding motion. The second step involves determining the control law for the system so that system is made to move towards the sliding surface defined in first step and then the system stays on the surface or in its close vicinity then-after.

Stability for any system is a critical issue. For the time-invariant systems, two main stability criterias are available which are: Nyquist Stability Criterion and Routh Stability Criterion. In few of the cases it is easy to check the stability via Bode-Plots. But, for the non-linear systems (and even for the linear systems) which are varying with the time, Lyapunov's method is used.

The Lyapunov's first method demands the explicit solutions of the system's differential equations. But, in Lyapunov's second method (also called as the Direct Method of Lyapunov), we can determine the stability of any equilibrium point without solving the rigorous state-equations [25]. To be precise, while dealing with any SMC based task, Lyapunov's Direct Theorem is used which has been used in designing the Sliding Mode Controller in this research work.

In Lyapunov's approach, if we assume $V(x)$ to be a Continuously Differentiable Function (Scalar) in Ω domain containing the origin, then the function is called as a Negative-Definite Function if $V(x) = 0$ for $x = 0$ and also $V(x) < 0$ for $|x| > 0$. And, on the contrary the function is called as a Positive-Definite Function if $V(x) = 0$ for $x = 0$ and also $V(x) > 0$ for $|x| > 0$.

This Lyapunov Approach assures that derivative of any fully defined positive-definite function is negative definite. The derivative of any function holds a negative value when the function has a positive value and vice-versa. Therefore, system's stability is assured about origin of state-space. Lyapunov's Candidate Function which describes the motion of the state of the system to the sliding surface is always defined. For every switched control structure chosen, the gains are so opted that the derivative of the chosen Lyapunov's candidate is always negative-definite so that the sliding surface tends to zero [26].

Once this stage of defining and designing of sliding surface is completed, a controller is then designed such that the state-trajectories of the system are aimed towards the sliding surface. As the system reaches the sliding surface, then, the controller makes sure that the system stays over it or in a bounded domain in its vicinity.

There are two phases involved in SMC which are:

a) Reaching Phase
b) Sliding Phase

Figure 2.1 is the diagrammatic representation of the two stages.

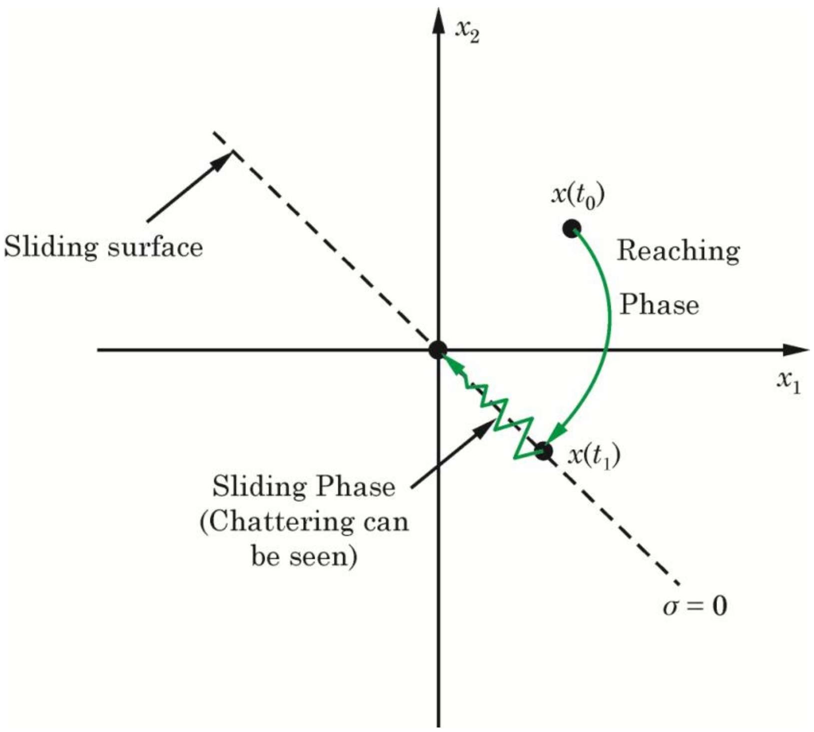

Figure 2.1: Reaching phase and Sliding phase

In reaching phase, the dynamics of the system are directed towards the surface in a finite time. The condition that guarantees the system's reachability is known as reachability condition.

Mathematically, reaching condition may be expressed as:

$$\begin{cases} \dot{\sigma} < 0 \;\; when \;\; \sigma > 0 \\ \dot{\sigma} > 0 \;\; when \;\; \sigma < 0 \end{cases} \tag{2.1}$$

Or, in simple terms, reaching condition can also be written as:

$$\sigma\dot{\sigma} < 0 \tag{2.2}$$

The reaching condition ensures the reaching and the stability but no information is provided about how the sliding surface is reached. The information about the system reaching the sliding surface is important as this manner of reaching determines the final characteristics of the transient behavior of the system.

The three most common approaches of reaching the sliding surface are:

1. Constant Approaching Law

This law is given by:

$$\dot{\sigma} = -\rho \operatorname{sgn}(\sigma)\,; \rho > 0 \tag{2.3}$$

As per this law, the switching function will approach zero with a constant velocity.

2. Proportional Approaching Law

This law is given by:

$$\dot{\sigma} = -\rho \operatorname{sgn}(\sigma) - k\sigma\,; \rho > 0, k > 0 \tag{2.4}$$

As per this law, the switching function will approach towards zero with velocity equal to the sum of a constant and an amount which is proportional to distance of the state-trajectory from the sliding surface.

3. Exponential Approaching Law

This law is given by:

$$\dot{\sigma} = -k|\sigma|^m \operatorname{sgn}(\sigma); k > 0, \ 1 > m > 0 \tag{2.5}$$

As per this law, the switching function will approach to zero with velocity of an exponential function of the distance of the state-trajectory from the sliding surface.

When the sliding surface is reached, the reaching phase terminates and the system enters the next phase of sliding mode which is called as sliding phase.

The main advantage of using SMC approach is that the system's response is insensitive to the modeling uncertainties and the unknown disturbances while being in the sliding phase. But, on the contrary, the same is not true while the system is in the reaching phase. The system is sensitive to both the disturbances and the uncertainties (bounded as well as unbounded) while it is in the reaching phase. So, to increase the robustness of the system, the attempts should be made to reduce the reaching phase.

Once the system reaches the sliding surface i.e. σ = 0, then, due to effects of sampling, switching and delays in the system used to implement the control, there occurs "chattering". Thus, the discontinuous control action ensures the desired performance but it leads to chattering which is highly undesirable for mechanical applications of SMC and especially in the type of system covered in this thesis.

Figure 2.2 shows chattering effect in more comprehensive way and is followed by the paragraph illustrating and elaborating the same.

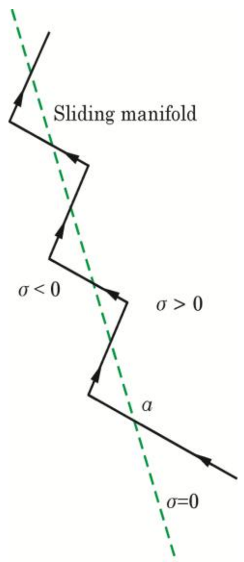

Figure 2.2: Chattering Phenomenon Detailed

Referring to Figure 2.2, let us say we are initially in the region $\sigma > 0$ and the controller is bringing the trajectory on the sliding surface i.e. at $\sigma = 0$. Assuming that for the first time it hits the sliding surface at a point "*a*". Ideally, after reaching the sliding surface, the trajectory should start sliding over the manifold but, in real life, the controller works as per some sampling time. There is no control for the time in-between the consequent sampling times and hence the trajectory moves to the other side where $\sigma < 0$. When the next sampling time activates the control again, the controller again tries to bring the trajectory over the sliding surface. The same phenomenon repeats itself leading to chattering.

Chattering doesn't only result in inaccurate controls but also leads to wear and tear of the mechanical parts which may reduce the life of the actuators [20], heat losses in power circuits and sometimes may excite the un-modeled high frequency dynamics which ultimately will lead to system's instability. So, chattering-free SMC with continuous control action is required [27]. A combination of chattering-free SMC and a disturbance observer have shown the improved performance [8] [28]. Chattering-free SMC is implemented in this thesis work.

CHAPTER 3: CONVENTIONAL SMC

3.1 CONVENTIONAL SMC APPROACH

Consider a simple unit mass mechanical system with no damper and also no spring attached to it as shown in Figure 3.1.

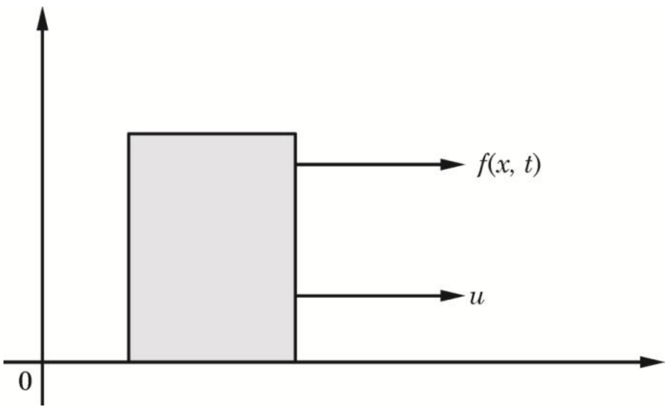

Figure 3.1: A Unit Mass System

The mathematical model for the system mentioned in Figure 3.1 above is:

$$M\ddot{x} = u + f(x_1, x_2, t) \tag{3.1}$$

where M – mass of the object,

$\ddot{x}$ – Acceleration

u – control force; $u \in \mathbb{R}$

$f(x_1, x_2, t)$ – the term which includes the disturbances and uncertainties in which the terms are assumed to be bounded; $\left|f(x_1, x_2, t)\right| > 0$

But, as we have assumed a unit mass system, so,

$$\ddot{x} = u + f(x_1, x_2, t) \tag{3.2}$$

The state-variable description for such a system can easily be obtained by introducing the position and velocity variables. Let the position be defined as:

$$x = x_1 \tag{3.3}$$

Now, differentiating the position yields velocity i.e.

$$\dot{x} = \dot{x}_1 = x_2 \tag{3.4}$$

Differentiating Equation 3.4 and substituting Equation 3.2 to the differentiation result gives:

$$\ddot{x} = \dot{x}_2 = u + f(x_1, x_2, t) \tag{3.5}$$

In simplest terms, if $f(x_1, x_2, t)$ is neglected for a moment from Equation 3.5 then, we get:

$$\ddot{x} = u \tag{3.6}$$

The control law for this system is given by:

$$u = -\operatorname{sgn}(x) \tag{3.7}$$

The function "sgn ( )" in the formula above refers to Signum function which is explained graphically in the Figure 3.2 below.

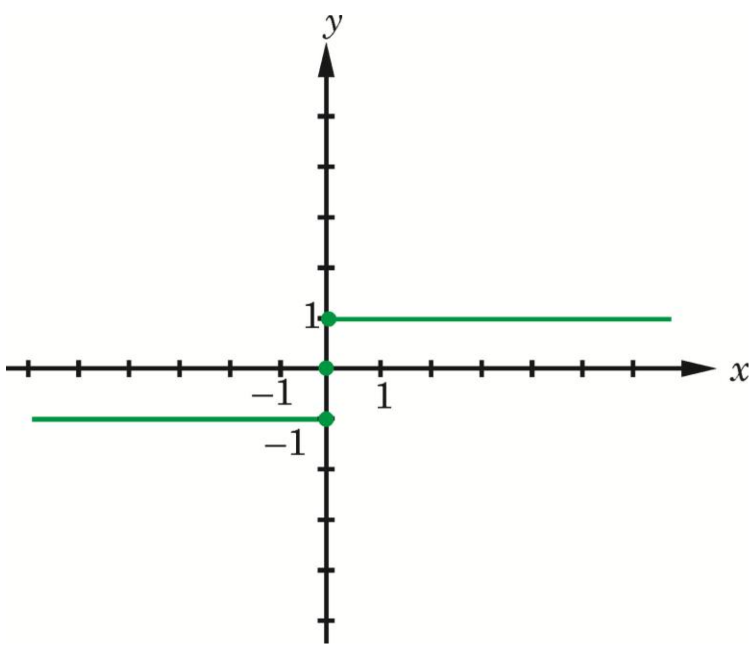

Figure 3.2: Signum Function Explanation

In very simple words, anything which goes under brackets of the signum function, i.e. in "( )" will ultimately go to zero.

The signum function denotes the discontinuous part of the control law and the continuous part is called as Equivalent Control. If needed in some applications, then, the discontinuous control function may be made continuous by using some continuous function like “Sigmoid” or “Tangent Hyperbolic” function as being used by authors in [29].

Mathematically, the control law in Equation 3.7 seems good but practically it is not possible in such mechanical systems as the forces are continuous function and may have high frequency dynamical terms. Also, to add more to explanation, in real life, each system has its own pace and inertia so, this control law is not applicable in reality. Hence, a fictious variable, " σ " should be involved to relate it to the real-life. This variable σ is the Sliding Variable.

When the sliding mode dynamics are defined by selection of the sliding surface appropriate for that system then it is called as Solving the Existence Problem. Then, the control law to assure that the designed sliding surface is reached and finally the system stays over or in the bounded domain in vicinity then this is called as Solving the Reachability Problem.

Having the knowledge of the advantages associated to SMC (as discussed in Section 2.1 of the thesis) we define the sliding surface for this Second-Order System as:

$$\sigma = \dot{x} + cx \tag{3.8}$$

Or, we can also write it in terms of state-variables as:

$$\sigma = x_2 + cx_1 \tag{3.9}$$

This Equation 3.9 is called as Sliding Surface Equation. Here, it is to be noted that the sliding variable σ is reduced-ordered (i.e. of first-order) where the coefficient “ c ” is our design variable. Hence, the performance of the system depends on our choice of the variable “ c ”.

Now, the new control law for the system would be:

$$u = -\operatorname{sgn}(\sigma) \tag{3.10}$$

Following the control law in Equation 3.10, the controller will make sure that σ will go to zero.

When σ goes to zero, i.e. putting $\sigma = 0$ in Equation 3.9 we get:

$$x_2 + cx_1 = 0 \tag{3.11}$$

Or, re-arranging the terms gives:

$$x_2 = -cx_1 \tag{3.12}$$

Here, it is to be noted that $c > 0$ for converging towards origin

For example, let us consider $c = 10$ in Equation 3.10.

So,

$$x_2 = -10x_1 \tag{3.13}$$

Or, in terms of state variables,

$$\dot{x} = -10x \tag{3.14}$$

The general solution of this First-Order Linear Differential Equation (FOLDE) is given by:

$$x(t) = x(0)\,e^{-10t} \tag{3.15}$$

Graphically, the system will appear as shon in Figure 3.3 below.

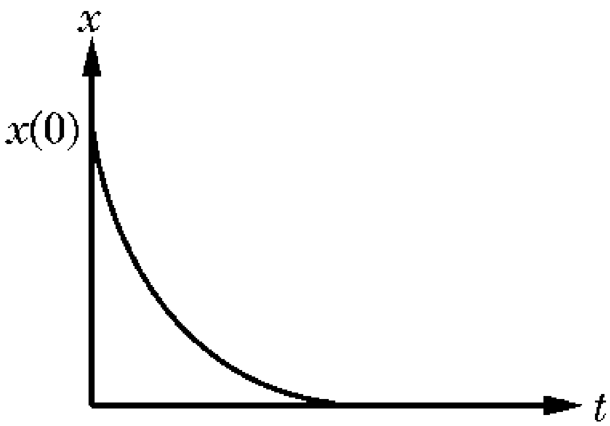

Figure 3.3: Exponential Convergence of the System

Substituting Equation 3.6 in Equation 3.10 we get,

$$\ddot{x} = -\text{sgn}\,(\sigma) \tag{3.16}$$

Now, substituting Equation 3.9 in Equation 3.16 gives:

$$\ddot{x} = -\text{sgn}\left[x_2 + cx_1\right] \tag{3.17}$$

But, actually in the system actually considered we have another term i.e. $f(x_1, x_2, t)$ which involves all the bounded unknown terms and can't be neglected in real life. Hence, now we will include this term for finding the proper solution. So, it means that we have to bring the system to zero in the presence of bounded disturbances $f(x_1, x_2, t)$.

This can be done by following the following two steps:

(1) Designing the Sliding Surface or Switching Hyperplane
(2) Proving the Reachability Condition

As discussed earlier, proving the reachability condition means proving $\sigma\dot{\sigma} < 0$

So, we take the sliding surface to be:

$$\sigma = x_2 + cx_1 \tag{3.18}$$

Or, expressing in terms of state variables we get:

$$\sigma = \dot{x} + cx \tag{3.19}$$

Differentiating Equation 3.19 gives:

$$\dot{\sigma} = \ddot{x} + c\dot{x} \tag{3.20}$$

Substituting the value of $\ddot{x}$ from Equation 3.5 in Equation 3.20 we get:

$$\dot{\sigma} = u + f(x_1, x_2, t) + c\dot{x} \tag{3.21}$$

But, from Equation 3.4 we know that $\dot{x} = x_2$

So,

$$\dot{\sigma} = cx_2 + u + f(x_1, x_2, t) \tag{3.22}$$

Multiplying both sides of Equation 3.22 by σ we get:

$$\sigma\dot{\sigma} = \sigma(cx_2 + f(x_1, x_2, t) + u) \tag{3.23}$$

The new control law will now be defined by Equation 3.24 which was also stated in [30].

$$u = -cx_2 - \rho \operatorname{sgn}(\sigma) \tag{3.24}$$

where ρ – a constant

c – another constant

The controller which obeys the control law stated in Equation 3.24 is called as a Sliding Mode Controller. Relay component (i.e. the discontinuous component) "– ρ sgn (σ)" is present to keep the effects of bounded disturbances and uncertain perturbations hidden and the component "– cx_2" is there to provide energy dissipation of system's oscillations.

Putting Equation 3.24 in the Equation 3.23 we get:

$$\sigma\dot{\sigma} = cx_2 + f(x_1, x_2, t) - cx_2 - \rho \operatorname{sgn}(\sigma) \tag{3.25}$$

Simple mathematical operation leads to:

$$\dot{\sigma} = -\rho \operatorname{sgn}(\sigma) + f(x_1, x_2, t) \tag{3.26}$$

Here, the constant ρ is assumed a proper value by considering the condition stated in Equation 3.27.

$$|\rho| > f(x_1, x_2, t) \tag{3.27}$$

So, as $|\rho| > f(x_1, x_2, t)$ hence σ and $\dot{\sigma}$ will always have opposite signs. Hence the reachability condition i.e. $\sigma\dot{\sigma} < 0$ is proved too.

To have a better understanding, the example discussed in [31] is used. For the system considered in Figure 3.1 and using Equation 3.5 where the disturbance term $f(x_1, x_2, t)$ is not included, the system is shown in Figure 3.4.

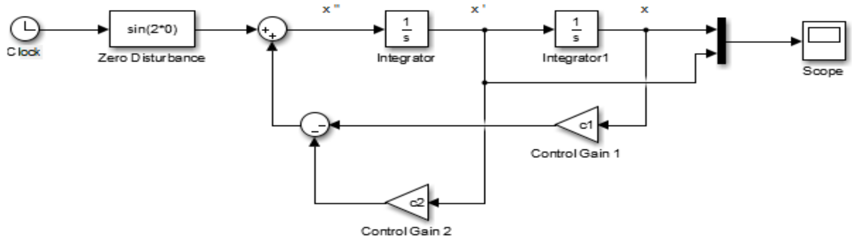

Figure 3.4: Simulink Model of the system when the disturbance is not considered

The control law should be designed so that it can drive the state-variable to zero. In mathematical terms, finite time convergence is defined as $x_1(t) = x_2(t) = 0; \forall t > t_r$ where $t_r =$ reaching time which actually depends on initial conditions. Considering the initial conditions to be $x_1(0) = 1$, $x_2(0) = -2$ and assuming the control law in Equation 3.28, the scope results are shown in Figure 3.5 below the control law.

$$u = -c_1 x_1 - c_2 x_2; c_1, c_2 > 0 \tag{3.28}$$

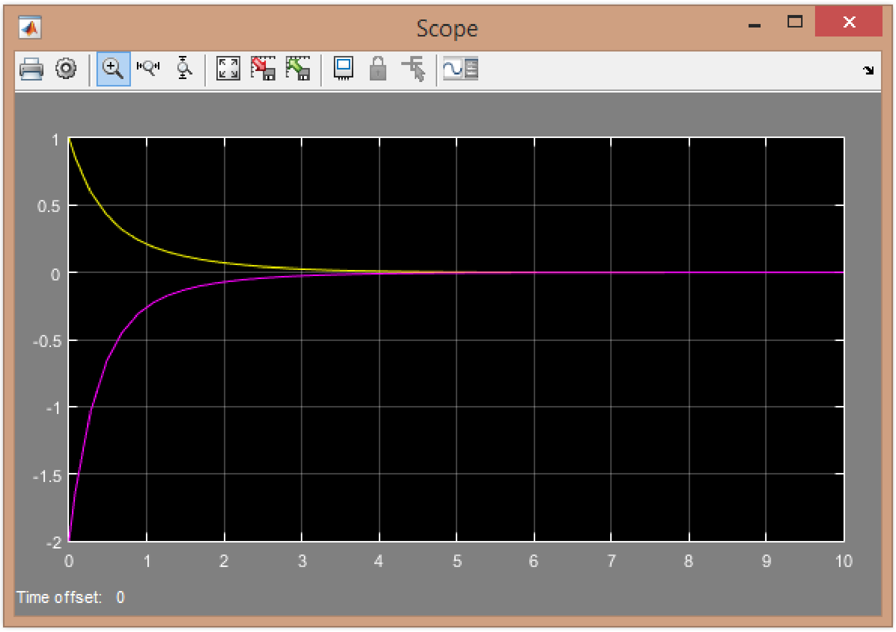

Figure 3.5: Asymptotic Convergence when the disturbance is not added

It can be seen from Figure 3.5 that the asymptotic stability of the system is achieved when there is no disturbance added to the system.

Now, as we have disturbance term included in the original system, the Simulink Model for the original system is shown in Figure 3.6.

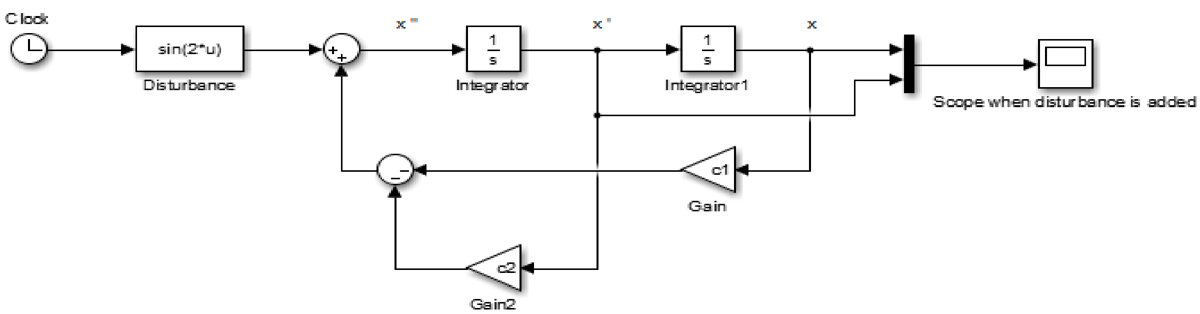

Figure 3.6: Simulink Model of the system when the disturbance is present

The results of the simulation using the same initial conditions and the control law stated in Equation 3.28, the scope result is shown in Figure 3.7 below.

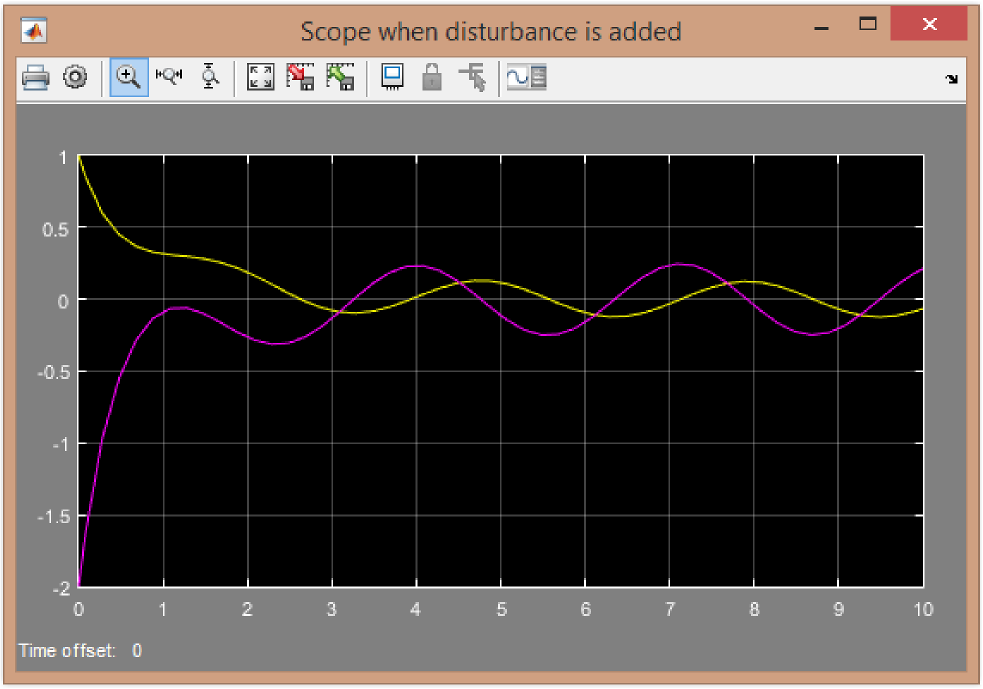

Figure 3.7: Convergence to a bounded domain when disturbance is added

Figure 3.7 shows that the system converged to a bounded domain when the bounded disturbance with the magnitude of *sin (2t)* is added. Hence, the control law is not sufficient enough to bring the asymptotic stability to the system.

Now, using Equation 3.24 as the control law for the system where the same initial conditions are assumed with the additional value of ρ = 2 and c = 1.5. Also, the same disturbance magnitude of *sin (2t)* is considered. The Simulink model is shown in Figure 3.8.

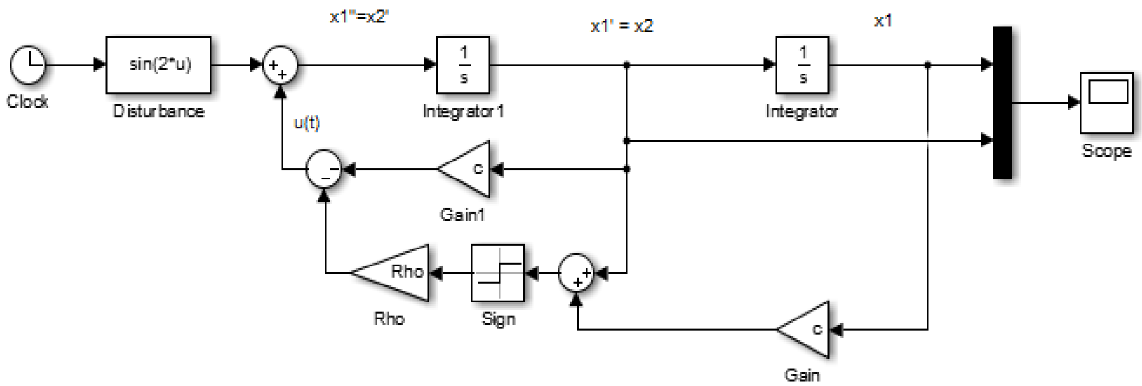

Figure 3.8: Simulink Model of the System with new control law

The scope results using the new control law are shown in Figure 3.9 below.

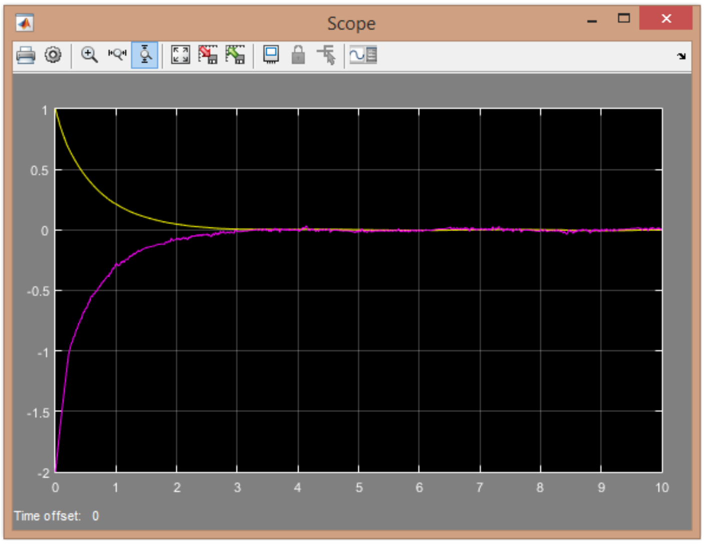

Figure 3.9: Scope Results with new control law

It can be seen from Figure 3.9 that the asymptotic convergence of the state variables is achieved in the presence of the bounded disturbance assuring the robustness of the system. The chattering phenomenon can also be seen which is inferred to the fact that the discontinuous control applied.

3.2 LITERATURE SURVEY

Specifically in terms of belt-driven systems, many researches were made. The early stages of the belt-drives were focusing on belt-drive mechanisms regarding their design and manufacturing. Then in 1980s, the dynamic response of the belt-drives was a nice area of research as the belt-drives usage was getting popular among the industries those days. In the current scenario, both the studies i.e. the mechanics and the dynamics are merged together so as to generate more accurate model and to achieve better control and efficiency of the belt drives.

In late 20th century, the Proportional-Integrator plus Differentiator (PID) control algorithms were quite famous among the researchers. In 1994, a PID controller with the provision of the adaptive-compensation of inertia force for a high-speed positioning table driven by belt-drive was presented [32]. Then, a PID control algorithm with offline trajectory-planning for accurate position control with compensation of delay dynamics and the vibrations while under maximum velocity and maximum acceleration constraints was presented [33]. Later, a PID algorithm with the feature of auto-tuning of the belt-driven system for tracking control was implemented [34]. The blend of PID with Sliding Mode Control (SMC) was also used to fetch the good characteristics of both the approaches.

SMC theory originated in 1930s within Soviet Union when the relay-type regulators (also known as “ON-OFF” type regulators) captured a huge market in feedback control system design. The relay-type regulators were not only easy to install but they were also highly efficient. It was the same time when few concepts of modern control theory like switching line, phase plane and sliding mode started finding places in the research publications. From 1950s to 1970s SMC focused on the systems in the phase-canonical forms. Thereafter, in 1970s to 1980s, the generalized SMC theory for multi-input linear systems was established completely but nothing captured the enthusiasm of the researchers, as the robustness property was not recognized by that time fully.

Owing to its simplicity and robustness, SMC approach garnered significant attention of the researchers once again in 1980s. The achievements included almost all kinds of the systems like: the SMC for the non-linear systems [14] [35] [36], SMC for discrete systems [37] [38] [39], SMC for time-delay systems [40] [41], SMC for large systems [42] and SMC for infinite-dimensional systems [43]. State observation was also dealt-with later [44]. Additionally, invariance to the parameter variations and disturbances were explored [45].

Studies related to both linear and non-linear systems being controlled by SMC were made and the research was basically focused on two areas which are:

a) Static SMC i.e. Reduced-Order SMC [9] [46]
b) Dynamic SMC i.e. Full-Order SMC [47]

The robust controllers for linear systems are well established since many decades now. A few non-linear systems are also known to have a robustness property. Generalized approach for determining the control law for non-linear systems is discussed in [48].

Mostly, the full-state feedback type control laws are used but, in real life applications, it is not always possible to know the exact system-states either because of complications of the system or because of the need of high-end expensive technology to fetch them. In such cases, observer-based SMC are often used [12]. Hence, an observer can estimate the unknown states and then provide SMC law.

It has been found that the state-trajectories are ideally insensitive to the matched-disturbances and also to the parametric variations while being in the sliding-mode phase [9] but, in the reaching-phase they are sensitive to matched as well as unmatched disturbances [49]. To deal with this drawback, few methods for example Integral SMC (ISMC) [50] have been proposed which aim at either reducing the system's sensitivity or reduce the reaching phase to as little as possible, or eliminating it altogether if possible. Generally, the approach is to incorporate a linear-cum-constant sliding surface using classical SMC method. The design problem can then be reduced to properly selecting the parameters of sliding surface which correspondingly will determine the performance of the considered system [51].

Plentiful research publications were made in the area but Utkin's work overrode all others. Initially his research was mainly made on designing phase canonical form for SMC and designing the principles for SMC systems which were having multi-inputs. Later, Utkin in [51] presented a time-varying sliding surface solution scheme for multi-input case as a solution of time-varying surfaces in state-space. He did it by deriving the control law for linear sliding surface nests so as to assure the sliding mode during the tracking control. The idea of moving sliding surfaces was generalized later for higher order system [52] [53] after a linear translating and rotating scheme for second-order systems was proved in [54]. Lately, for same linear translating and rotating schemes, a new approach of "Fuzzy-tuning" was also proposed [55]. A SMC control algorithm based on pre-defined structure of Lyapunov candidate function's time derivative was suggested for linear timing-belt drives in [22]. Further many issues related to it were investigated in [5] [21].

Authors in [56] surveyed about incorporating "Intelligence" in the SMC using famous computational intelligent methods like Fuzzy Logic, Artificial Neural Networks and also the Probabilistic Reasoning. Later, the current progress and the future possibilities of integrating SMC with other computational intelligence methods were examined in [57].

To address the problems of chattering and unknown bound of the uncertainties present, various methods like the method of boundary layer [14], the method of higher order SMC [31] [58], the adaptive SMC methodology [59] [60] [61], the method of Disturbance Estimation [62] [63] and then Adaptive Fuzzy SMC method which combines the advantages of both SMC and Fuzzy-Logic method discussed in [64] [65] were proposed but each of these has its pros and cons.

Chattering-Free SMC is discussed in [9] [20] [27] [63] [66]. For the present system, the one considered in this thesis, chattering-free SMC by addition of a new state variable (as discussed in [27]) is used.

CHAPTER 4: SYSTEM'S MATHEMATICAL MODEL FORMULATION

4.1 THE PLANT

The considered system i.e. The Plant consists of a DC motor which provides the main motive force to the system, two pulleys of same dimensions that stretch the belt where the one which is connected to motor is the driving pulley and the other is a driven pulley; a carriage (which is considered to be the load-side of the system) whose position error is to be minimized ultimately; and the belt for force transmission from the driving pulley to the load carriage. The arrangement represents a complex-cum-non linear distributed parameter system.

It is to be noted here that the Gear-Reduction Ratio (also known as Gear Ratio) which is an important unit in the real industrial applications is assumed to have a value of unity in this research.

The system is shown diagrammatically in Figure 4.1 below.

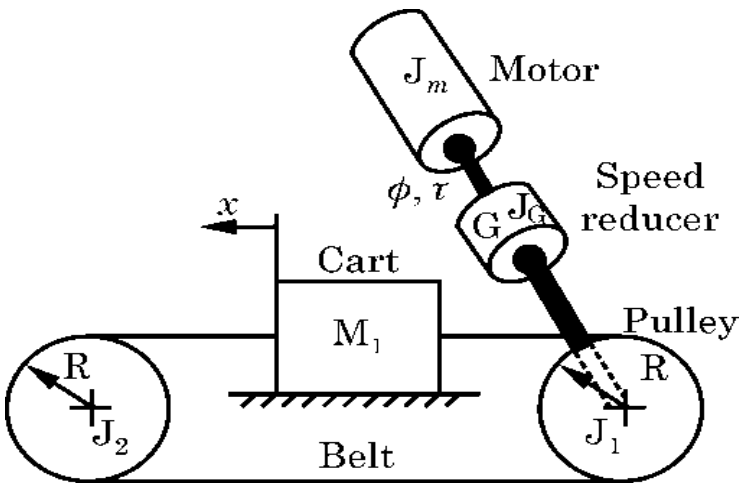

Figure 4.1: The Considered System

4.2 ASSUMPTIONS MADE

The following assumptions were made for the system:

- Gear reduction ratio has its value equal to unity in the simulated system
- The motor has negligible delay in providing high-dynamic torque response
- The system is free from backlash of belt drives due to pre-tensioning
- The link between motor shaft and belt drive's driving pulley is rigid

- The belt elasticity is equivalent to a mass-less spring
- The unknown disturbances/noises consist of friction present between: (a) DC motor and the driving pulley; (b) carriage guiding rail transmission guiding channels

4.3 EQUIVALENT SPRING-MASS SYSTEM

Figure 4.2 below is the diagrammatic representation of Equivalent Spring-Mass System in lieu of the underlying assumptions.

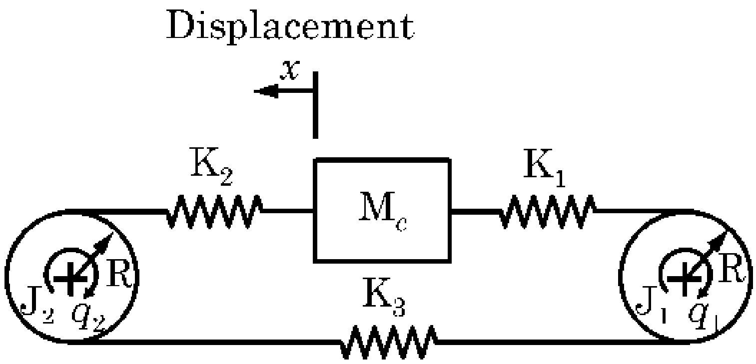

Figure 4.2: The Equivalent Spring-Mass System

4.4 MATHEMATICAL MODEL

4.4.1The 6th Order Model

Using modal analysis, the Sixth-Order Lagrenze's Equations for the system are depicted in Equations 4.1a, 4.1b and 4.1c. [5]. For generating these equations the detailed analysis is made in [22].

$$(J_1 + G^2 (J_G + J_m)).\ \ddot{q}_1 + \tau_{f1} = G\tau - R.\ [K_1(x).\ (\mathrm{R}q_1 - x) - K_3.\ (Rq_2 - Rq_1)] \tag{4.1a}$$

$$\mathrm{J}_2 \ddot{q}_2 + \tau_{f2} = R.\ [K_2\ (x).\ (x - Rq_2) - K_3.\ (Rq_2 - Rq_1)] \tag{4.1b}$$

$$M_c\ddot{x} + f_f = K_1(x).\ (Rq_1 - x) - K_2(x).\ (x - Rq_2) \tag{4.1c}$$

where J_G, J_m – Moment of Inertia of the speed reducer and the motor, respectively,

J_1, J_2 – Moment of Inertia of driving and the driven pulley, respectively,

M_c – Carriage mass,

G – Speed reducer gear ratio,

R – Radius of the pulleys,

K_1, K_2, K_3 – Position dependent belt elasticity coefficients,

q_1, q_2, ϕ – Angular position of the driving pulley, driven pulley, and the motor, respectively,

x – Position of carriage,

τ – Torque developed by the motor,

τ_{f1}, τ_{f2} – Friction torque which affects the pulleys,

f_f – Friction force on the carriage

4.4.2 The 4th Order Model

The 6th Order Model discussed above is a Three-Mass Model and it is a highly-coupled and non-linear system with external disturbances which enter both in the driving-side i.e. motor-side and also on the load-side of the system. An important point to be considered is that the inertias at the load-side and the motor-side are high as compared to that which is available at the driving and driven pulleys [22]. So, the 6th Order Model (Three-Mass model) is reduced to 4th Order Model (Two-Mass Model) as described below in Equations 4.2a, 4.2b and 4.2c [5]. The Two-Mass Model includes only the first resonance.

$$J\ddot{\varphi} + \tau_f = \tau - L.Kw \tag{4.2a}$$

$$M\ddot{x} + f_f = Kw \tag{4.2b}$$

$$w = L\varphi - x \tag{4.2c}$$

where J – Motor inertia;

M – Mass on the load side (approx. to the Cart Mass);

τ_f – Motor Side friction torque that perturb the system ;

f_f – Force of friction in the system;

K – Coefficient of Elasticity of the belt;

w – Belt-stretch;

L – Transmission constant of the linear belt-drive = *R/G;*

4.4.3 The 4th Order Model with Belt-Stretch Consideration

Belt-stretch is an important consideration as stated earlier in the thesis. The model discussed in Section 4.4.2 can hence be modified according to the vibration analysis [7]. So, we can express the system dynamics in terms of belt-stretch. It is to be noted that to simplify the calculations, the parameter "*L*" i.e. transmission coefficient is assumed to be unity while finding the new set of equations for the modified model. The modified model can be written as:

$$J\ddot{w} + K_w w = \tau - \tau_{wf} \tag{4.3a}$$

$$M\ddot{x} + f_f = Kw \tag{4.3b}$$

where $\tau_{wf} = \tau_f - \kappa f_f$,

$K_w = K(1+\kappa)$,

$\kappa = J/M$ (inertia ratio)

The resonance frequency of the elastic belt-drive system is given by Equation 4.4 below:

$$\omega_0 = \sqrt{\frac{K}{J}(1+\kappa)} \tag{4.4}$$

The control model for the modified system where belt-stretch is considered is shown in Figure 4.3. The system has a control input signal τ. It has two parts where first one is for the Belt-Stretch Dynamics and the second one is for the Load Side Dynamics. Both these parts are described by their independent nominal and linear 2^{nd}-order dynamics. The disturbance torque perturbs the belt-stretch side and the friction force perturbs the load side as shown in Figure 4.3.

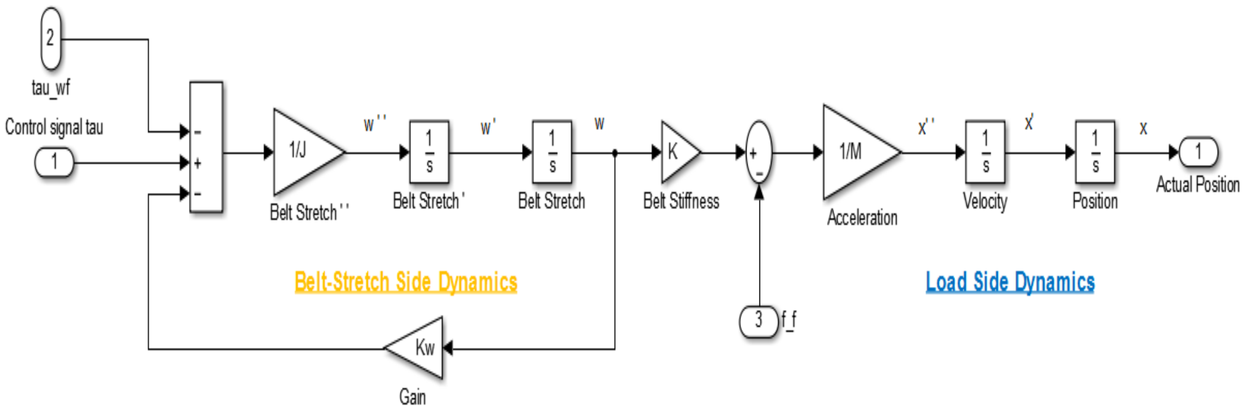

Figure 4.3: Block Scheme of Linear Belt-Driven System

In real life, the main problems in such a system arise from many factors some of which are enlisted below.

a) Elasticity of the belt
b) Load-position dependent friction
c) Non-linearity of induced belt forces
d) Large stiction effects

The prime interest is to generate the control scheme for the system mentioned in Figure 4.3 so that the position tracking can be simulated and the position error of the cart position can be minimized. Also from the simulation, results of various other parameters can also be fetched.

CHAPTER 5: SMC CONTROLLER DESIGN

5.1 CONTROL DESIGN

As in the current system, the uncertainties are not known, so, a robust control law is needed. VSS control approach assures stability for such a system where close loop behavior is obtained by incorporating the proper technique to select a sliding manifold. The idea is to find the control input which restricts the system-states to sliding manifold after which sliding mode occurs. SMC is one such approach in which the sliding motion is insensitive to the parametric variations and the unknown disturbances [9].

SMC approach for systems with bounded control input can be used if the model structure assures that the uncertainties are bounded [9]. The complete disturbance rejection is possible after fulfilling the matching conditions [49].

The design of SMC is two-stepped process. These are:

1. Defining the switching hyperplane or switching surface or sliding surface
2. Designing the control Law

First, the switching hyperplane often designated by "σ" is defined which is basically dependent on the desired dynamical behavior of the system. This σ actually provides a measure of the distance of the state-trajectory from the sliding surface located at $\sigma = 0$. Secondly, a control law is designed to drive the states onto the pre-defined sliding surface. Also, it makes sure that they remain there and slide along the sliding surface or in the bounded vicinity of that sliding surface. The control is continuous in all the segmented regions but it alters the structure when the dynamics of the state-trajectory crosses the boundary defined by the sliding surface i.e. sliding manifold.

5.1.1 Derivation of Control Law

The ultimate goal is to find the control input which takes the system towards the sliding surface manifold and then restricts it there despite of the perturbations present in the system which basically include all the uncertainties of the system.

Let a Single Input-Single Output (SISO) non-linear considered mechanical system in the state-space be defined as:

$$\dot{z}_i = z_{i+1} \tag{5.1}$$

$$\dot{z}_n = f(z) + b(z)\, u + d(t) \tag{5.2}$$

where $i = 1, \ldots, n–1$,

$z^T = [z_1 \ldots, z_n]$ is a system state vector,

u - scalar input,

$f(z)$ - the bounded non-linear driving term of system-state vector,

$b(z)$ - the bounded non-linear control gain of system-state vector,

$d(t)$ - the bounded scalar disturbances.

Now, the prime goal is to determine control signal "u" that restricts the motion of the system-states z to the pre-defined sliding surface $\sigma(z,t) = 0$ even in the presence of $f(z)$, $b(z)$ and $d(t)$. The convergence (of system states) to the sliding surface is called reaching phase. The motion within the sliding manifold is called as sliding phase.

The control with discontinuities on sliding manifold is working on the following defined principles (termed as switching function) as also stated in [15]:

$$u = \begin{cases} u^+, & \sigma(z,t) > 0 \\ u^-, & \sigma(z,t) < 0 \end{cases} \tag{5.3}$$

Here, u^+ and u^- are so selected that the Lyapunov function candidate has its derivative to be negative-definite as discussed in Section 2.1 of the thesis. But, the discontinuous control in many applications leads to failure of VSS which further leads to "chattering" [27] [62] which is an important issue while dealing with the system under consideration. Hence, the chattering-free SMC is to be used in this task. For this purpose, another additional state [5] of the system is needed as introduced in [21] and also discussed in [17] to eliminate the discontinuities on the control signal. So, it yields,

$$\dot{z}_i = z_{i+1} \tag{5.4}$$

$$z_{n+1} = g(z,u) + b(z)\dot{u} + \dot{d}(t) \tag{5.5}$$

where $g(z, u) = \dot{f}(z) + \dot{b}(z)u$,

$i = 1, \ldots, n$

The condition which is to be fulfilled so that the system states start moving towards; and finally reach the sliding surface is called as reaching condition (expressed in Equation 2.1 and Equation 2.2 already). The solution given by $\sigma = 0$ will be having at least Asymptotically Stability if the control satisfies the condition that the derivate of any assumed Lyapunov's candidate function will be semi-negative definite. To meet this criterion, considering the Lyapunov Candidate of the form:

$$V(\sigma) = \sigma^2/2 \quad (5.6)$$

[5] [67]

The squared distance to the surface (as measured with term σ^2) decreases along the system trajectory. Hence, it will constrain the trajectories to actually point towards the pre-defined sliding surface. Once the system reaches the surface, the system trajectories will then be on the surface.

The reachability condition can be proved when the condition $\sigma.\dot{\sigma} < 0$ is fulfilled. As suggested in [27], the robust law is chosen with proportional rate given by $\dot{\sigma} = -D.\sigma, D > 0$ where D is the damping coefficient arbitrarily chosen to achieve a required rate of robustness of the closed-loop system. The value of D determines the disturbance rejection and ideally the value of gain D should be as high as possible. But, it is not possible in the practical applications. The value of D is limited by various factors few of which are:

- Un-modeled dynamics of electro-magnetic torque
- Noise levels
- Discrete control algorithm implementation

The derivative of Lyapunov's Candidate Function V as stated by Equation 5.6 will thus have the form [22] :

$$\dot{V} = -D\sigma^2, D>0 \quad (5.7)$$

Now, from the condition $\dot{V} = \sigma\dot{\sigma} = -D\sigma^2$ and by application of the knowledge regarding the Equivalent Control Method [9] in which the equivalent control signal u_{eq} is not the control action applied to the nominal plant. It can be imagined as representing (at an average) the same effect as shown by the applied discontinuous control and can hold the system on the sliding surface, we can derive:

$$\dot{u} = \dot{u}_{eq} + D.\sigma \tag{5.8}$$

The obtained equivalent control u_{eq} is actually the solution of $\dot{\sigma}\big|_{\sigma=0} = 0$

The control law u can be obtained as:

$$u = \int_0^t \dot{u}dt \tag{5.9}$$

This control assures invariant motion of the system in the sliding mode if the value of the disturbance *d(t)* complies to matching conditions [49]. The control will be achieved when the derivative of Lyapunov Candidate is negative i.e.

$$\dot{V} = \sigma.\dot{\sigma} = -D.\sigma^2 < 0 \tag{5.10}$$

5.1.2 Designing the Sliding Surface

The task of designing the sliding surface or sliding manifold is an important step which is constructed in a system state-space. Thus, σ combines system's state-variables in such a way that the system's motion in the sliding manifold is asymptotically stable. The definition of the system states is in accordance to the adopted system model. In case of higher-order sliding mode approach, the original system states can be extended so as to achieve the smooth control.

For the system considered, the conventional SMC design approach involves switching function of first order which could be taken as:

$$\sigma = \Delta\dot{x} - c\Delta x \tag{5.11}$$

where Δx – The Error Signal = $r\,(t) - x$ i.e. Reference Position – Actual Position

c – Our design variable

The SMC which is free from chattering for the linear belt-driven system must have a switching function of second-order in the motion control [5].

Hence, taking the switching function to be:

$$\sigma = \Delta\ddot{x} + K_v\Delta\dot{x} + K_p\Delta x \tag{5.12}$$

where Δx - Corrected position

$\Delta\dot{x}$ - Corrected velocity

$\Delta\ddot{x}$ - Corrected acceleration

K_v and K_p - positive control gain values to shape the second-order dynamic behavior of the error in desired position.

But, as proposed by the authors in [68], for elastic systems, the switching function is given by:

$$\sigma = \Delta\ddot{x} + K_v\Delta\dot{x} + K_p\Delta x + \gamma(\ddot{w} + \alpha\dot{w}) \tag{5.13}$$

where α and γ - arbitrarily chosen positive control gains in order to reduce the vibrations due to belt compliance and elasticity so as to shape asymptotically stable motion dynamics on the sliding manifold.

The portion of belt-stretch dynamics is added to switching function definition ($\ddot{w} + \alpha\dot{w}$). It basically aims at coping with resonant frequency ω_0 and also to achieve asymptotically stable dynamics of motion.

A little consideration to the Equation 5.13 says that if the belt is stiff then both $\ddot{w}$ and $\dot{w} = 0$ and we will reach the Equation 5.12.

For the controller implementation, the only signals we require are position and the velocity of the system. The other required model parameters are the Equivalent Mass i.e. $J + M$, the Moment of Inertia of the motor i.e. J and lastly the resonance frequency i.e. ω_0 of the system. And then the other control parameters i.e. K_v, K_p, α, γ are designed.

The Simulink description of the belt-stretch consideration being implemented and the calculation of the control law for the system is shown in Figure 5.1.

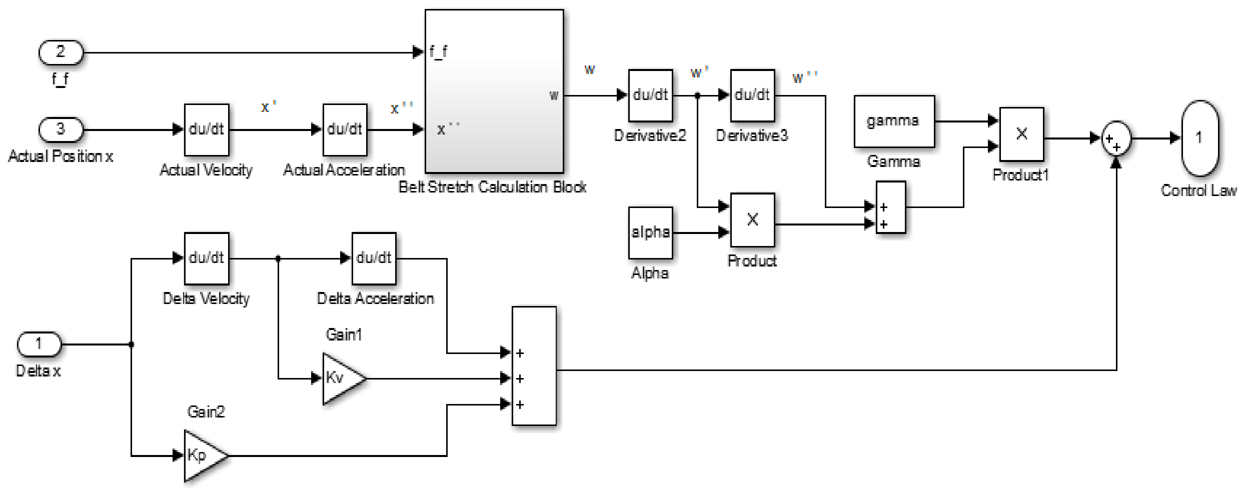

Figure 5.1: Control Law Deriving Procedure including the Belt-Stretch

The constructed sliding manifold as described in Equation 5.13 allows the steady state position error to reach the zero value assuring the operation to be vibration-free. In order to give a little more explanation about choice of the switching function, σ has been opted so such that it complies with the order of the system. Also, it involves position tracking error and its order up to the 2nd-order along-with the dynamics of vibration suppression i.e. $\dot{w}$ and $\ddot{w}$.

It is to be noted here that higher order derivatives of position error are simply not available in the practice. All the variables involved in σ are then available in the implemented control law which actually requires the integral of σ.

Considering control engineer’s perspective, σ is a measure for a distance to sliding manifold. The sliding manifold is constructed so as to bring the driving position error to zero while operating at vibration-free mode. In practice, the ultimate convergence is hard to guarantee, especially, in the case of simplified version of the control law. However, by the high-gain approach, it can asymptotically converge to a vicinity of the sliding manifold at a fair robustness.

In practical scenarios, it is difficult to observe belt-stretch’s first and second order derivatives. So, Equation 4.3b from Section 4.4.3 of the thesis is used. Hence, belt-stretch is computed as:

$$w = \frac{M}{K}\ddot{x} + \frac{1}{K} f_f, \tag{5.14}$$

The first and second order derivatives are hence calculated by differentiating "*w*". In MATLAB Simulation, it can be done by using the "Derivative" Blocks from Simulink Library (also described in Figure 5.1).

Now, as discussed earlier that the control law needs the derivative of switching function σ. So, differentiating Equation 5.13 we get,

$$\dot{\sigma} = \Delta\dddot{x} + K_v\Delta\ddot{x} + K_p\Delta\dot{x} - \gamma(\ddot{w} + \alpha\dddot{w}) \tag{5.15}$$

Here, $\ddot{w}$ is obtained by rearranging the terms involved in Equation 4.3a as:

$$\ddot{w} = -\omega_0^2 w + \frac{\tau - \tau_{wf}}{J} \tag{5.16}$$

$\dddot{w}$ is calculated by differentiating the Equation 5.16 i.e. calculating $d\ddot{w}/dt$

So, we get,

$$\dot{\sigma} = \Delta\dddot{x} + K_v\Delta\ddot{x} + K_p\Delta\dot{x} + \gamma\left(-\omega_0^2\dot{w} + \frac{\dot{\tau} - \tau_{wf}}{J} + \alpha\ddot{w}\right) \tag{5.17}$$

Now, from condition $\dot{\sigma} = 0$ where $\dot{\tau}$ becomes $\dot{\tau}_{eq}$ we get:

$$0 = \Delta\dddot{x} + K_v\Delta\ddot{x} + K_p\Delta\dot{x} + \gamma\left(-\omega_0^2\dot{w} + \frac{\dot{\tau}_{eq} - \tau_{wf}}{J} + \alpha\ddot{w}\right) \tag{5.18}$$

Re-arranging the terms leads to:

$$\gamma\dot{\tau}_{eq} = -J\left[\Delta\dddot{x} + K_v\Delta\ddot{x} + K_p\Delta\dot{x}\right] + J\gamma\omega_0^2\dot{w} - J\gamma\alpha\ddot{w} + \gamma\dot{\tau}_{wf} \tag{5.19}$$

Further rearrangement of terms yields:

$$\dot{\tau}_{eq} = -\frac{J}{\gamma}\left[\Delta\dddot{x} + K_v\Delta\ddot{x} + K_p\Delta\dot{x}\right] + J\left[\omega_0^2\dot{w} - \alpha\ddot{w}\right] + \dot{\tau}_{wf} \tag{5.20}$$

Using the knowledge of resonance frequency (from Equation 4.4) we can deduce that,

$$\omega_0^2 = \frac{K}{J}\frac{(M + J)}{M} \tag{5.21}$$

Rearranging the terms in Equation 5.21 above gives the following:

$$\frac{M}{K} = \frac{(M + J)}{J} \cdot \frac{1}{\omega_0^2} \qquad (5.22)$$

Multiplying and dividing the equation of $\dot{\tau}_{eq}$ by $\frac{K}{M}$ we get:

$$\dot{\tau}_{eq} = -\frac{M}{K} J\beta \left[\Delta\dddot{x} + K_v \Delta\ddot{x} + K_p \Delta\dot{x} \right] + J \left[\omega_0^2 \dot{w} - \alpha \ddot{w} \right] + \dot{\tau}_{wf} \qquad (5.23)$$

where β = K/Mγ = A gain value

High value of β can although extend the robust bandwidth of the operation but it is always limited in the real life. For example, in belt-drives, it is normal to have force-transmission delay which may lead to unstable belt response. Hence, β value is actually a result of design compromise. Also, $\beta = \gamma^{-1} K/M > 0$ is the design parameter which shape the dynamics of system motion when $\sigma = 0$.

Substituting $\frac{M}{K}$ from Equation 5.22 to Equation 5.23 we get:

$$\dot{\tau}_{eq} = -\frac{\beta}{\omega_0^2} (M + J) \left[\Delta\dddot{x} + K_v \Delta\ddot{x} + K_p \Delta\dot{x} \right] + J \left[\omega_0^2 \dot{w} - \alpha \ddot{w} \right] + \dot{\tau}_{wf} \qquad (5.24)$$

Now, equivalent control torque signal can be obtained by integrating the $\dot{\tau}_{eq}$ over the time limit *0* to *t* i.e.

$$\tau_{eq} = \int_0^t \dot{\tau}_{eq} dt \qquad (5.25)$$

So, integrating Equation 5.24 we get,

$$\tau_{eq} = -\frac{\beta}{\omega_0^2} (M + J) \left[\Delta\ddot{x} + K_v \Delta\dot{x} + K_p \Delta x \right] + J \left[\omega_0^2 w - \alpha \dot{w} \right] + \tau_{wf} \qquad (5.26)$$

Here, in Equation 5.26, we can see that we need acceleration signal which is generally not available. We have only position and velocity sensors installed with-in the system.

So, to find $\ddot{x}$ we have to use Equation 4.3b where rearranging the terms provides:

$$\Delta\ddot{x} = \frac{Kw - f_f}{M} \tag{5.27}$$

Hence,

$$\tau_{eq} = -\frac{\beta}{\omega_0^2}(M+J)\left[K_v\Delta\dot{x} + K_p\Delta x\right] - \frac{\beta}{\omega_0^2}(M+J)\left(\frac{Kw - f_f}{M}\right) + J\left[\omega_0^2 w - \alpha\dot{w}\right] + \tau_{wf} \tag{5.28}$$

$$\tau_{eq} = -\frac{\beta}{\omega_0^2}(M+J)\left[K_v\Delta\dot{x} + K_p\Delta x\right] - \frac{\beta}{\omega_0^2}(Kw - f_f) + \frac{\beta}{\omega_0^2}\frac{J}{M}(Kw - f_f) + J\left[\omega_0^2 w - \alpha\dot{w}\right] + \tau_{wf} \tag{5.29}$$

$$\begin{aligned}\tau_{eq} = &-\frac{\beta}{\omega_0^2}(M+J)\left[K_v\Delta\dot{x} + K_p\Delta x\right] - \frac{\beta}{\omega_0^2}Kw + \frac{\beta}{\omega_0^2}f_f - \frac{\beta}{\omega_0^2}\frac{J}{M}Kw + \frac{\beta}{\omega_0^2}\frac{J}{M}f_f \\ &+ J\left[\omega_0^2 w - \alpha\dot{w}\right] + \tau_{wf}\end{aligned} \tag{5.30}$$

Now, substituting the value of τ_{wf} (as defined in Section 4.4.3) in the Equation 5.30 we get:

$$\begin{aligned}\tau_{eq} = &-\frac{\beta}{\omega_0^2}(M+J)\left[K_v\Delta\dot{x} + K_p\Delta x\right] - \frac{\beta}{\omega_0^2}Kw + \frac{\beta}{\omega_0^2}f_f - \frac{\beta}{\omega_0^2}\frac{J}{M}Kw + \frac{\beta}{\omega_0^2}\frac{J}{M}f_f \\ &+ J\left[\omega_0^2 w - \alpha\dot{w}\right] + \tau_f - \kappa f_f\end{aligned} \tag{5.31}$$

After re-arranging the terms we get:

$$\begin{aligned}\tau_{eq} = &-\frac{\beta}{\omega_0^2}(M+J)\left[K_v\Delta\dot{x} + K_p\Delta x\right] - \frac{\beta}{\omega_0^2}Kw - \frac{\beta}{\omega_0^2}\frac{J}{M}Kw + J\omega_0^2 w - J\alpha\dot{w} + \tau_f \\ &+ \left[\frac{\beta}{\omega_0^2} + \frac{\beta}{\omega_0^2}\kappa - \kappa\right] f_f\end{aligned} \tag{5.32}$$

$$\tau_{eq} = -\frac{\beta}{\omega_0^2}(M+J)\left[K_v\Delta\dot{x} + K_p\Delta x\right] - J\left[\alpha\dot{w} + \left(\frac{\beta}{\omega_0^2}\frac{K}{J} + \frac{\beta}{\omega_0^2}\frac{K}{M} - \omega_0^2\right)w\right] + \tau_f + \xi f_f \tag{5.33}$$

where $\xi = \kappa\left(\frac{\beta}{\omega_0^2} - 1\right) + \frac{\beta}{\omega_0^2}$

Solving the Equation 5.33 and substituting $\omega_0^2 = \frac{K}{J}\left[\frac{M+J}{M}\right]$ we get:

$$\tau_{eq} = -\frac{\beta}{\omega_0^2}(M+J)\left[K_v\Delta\dot{x} + K_p\Delta x\right] - J\left[\alpha\dot{w} + (\beta - \omega_0^2)w\right] + \tau^{dist} \tag{5.34}$$

where $\tau^{dist} = \tau_f + \xi f_f$ = the system disturbance signal

It is to be noted that ξ is related to both the plant-model parameters and enforced belt-stretch dynamics parameters.

In order to obtain a continuous control of the signal τ, the control law applies the condition $\dot{\sigma} = -D\sigma$ in order to obtain control signal of the form $u = u_{eq} + D\sigma$ as explained earlier.

But we don't have the perfect knowledge about the whole system and in reality, τ^{dist} is not a measurable entity so, τ_{eq} is replaced with the estimated value $\hat{\tau}_{eq}$. The estimate $\hat{\tau}_{eq}$ doesn't assure the convergence to the sliding surface hence the discontinuous term is also added to it as described in Equation 5.35 below.

$$\tau = \hat{\tau}_{eq} + \frac{\beta}{\omega_0^2}(J+M).D\sigma \tag{5.35}$$

Performance of robust controller as obtained from Equation 5.35 plays an important role in desensitizing the system from the disturbances. This also allows for implementation of the rapid vibration-free belt response which can further aid in rejecting the load side equation.

Equivalent Control Estimation is determined by:

$$\hat{\tau}_{eq} = \frac{\beta}{\omega_0^2}(J+M)a^c - J(\alpha\dot{w} + (\beta - \omega_0^2)w) \tag{5.36}$$

where $a^c = K_v\Delta\dot{x} + K_p\Delta x$ (5.37)

Now, the control torque signal is given by Equation 5.38 as:

$$\tau = \int_0^t \dot{\tau}dt = \hat{\tau}_{eq} + \int_0^t \dot{\tau}_{SMC}dt \tag{5.38}$$

The Equation 5.38 has 2 components

a) Estimation of equivalent control
b) Estimation of disturbance and convergence to pre-defined sliding manifold

Using Equations 5.35 to 5.38, the system motion projection on σ-space is governed by:

$$\dot{\sigma} + D\sigma = \frac{\omega_0^2}{\beta}\frac{\dot{\tau}^{dist}}{J + M} \tag{5.39}$$

This Equation 5.39 in combination with the condition $\sigma\dot{\sigma} = -D\sigma^2;\ D > 0$, proves the system's asymptotically stable reaching phase. The convergence is dictated by right side of Equation 5.39 above.

Having a stable solution where $\sigma = 0$ can be guaranteed if $\dot{\tau}^{dist} = 0$ i.e. it should be constant. Also, then the derivative of considered Lyapunov's function candidate is negative definite i.e.

$\dot{V} = -D\sigma^2, D > 0$

In systems with fast sampling rate, the fast convergence rate can be achieved. If the rate of change of disturbance is low i.e. $\dot{\tau}^{dist} = 0$ then the control law will keep the system states in the vicinity of the pre-defined sliding manifold.

CHAPTER 6: SIMULATION AND RESULTS

6.1 SIMULATED SYSTEM

The system considered in the research being simulated in MATLAB-Simulink is depicted in Figure 6.1 below.

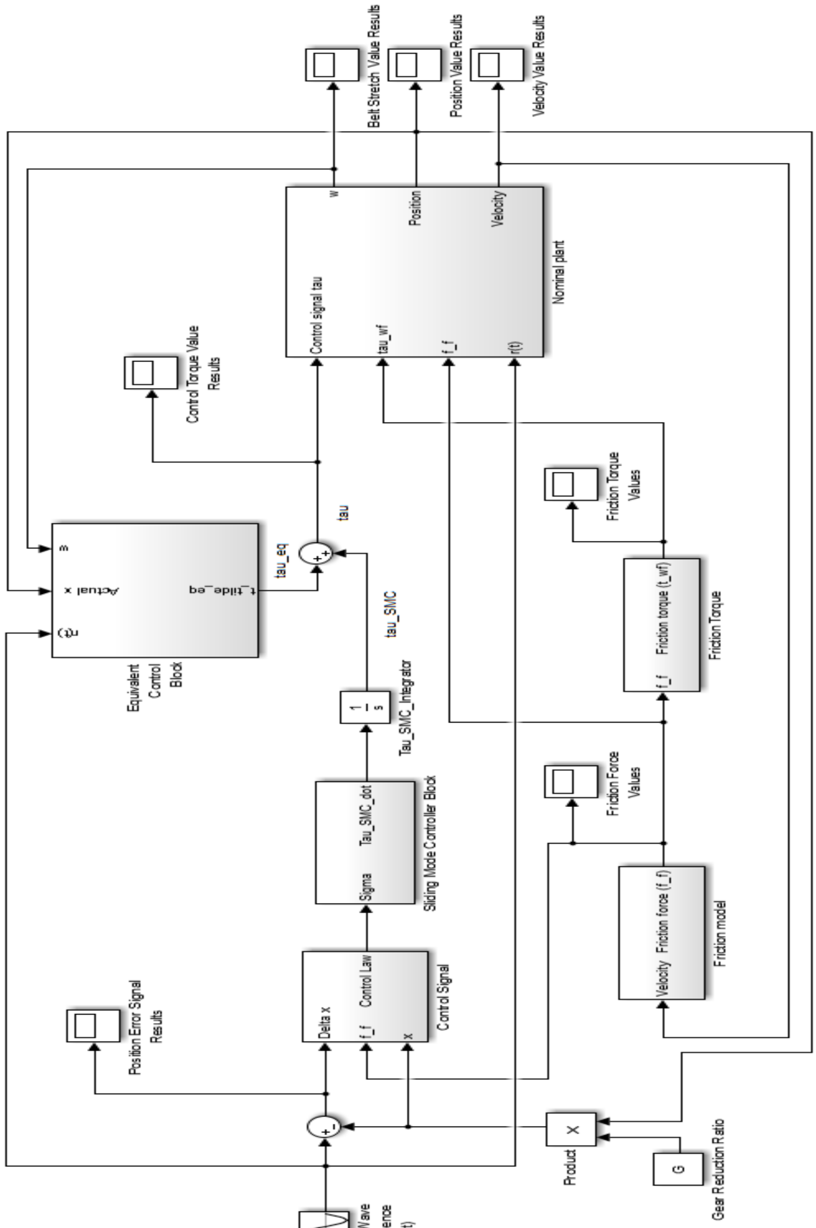

Figure 6.1: The Simulated System

6.2 THE PLANT WITH DISTURBANCES ADDED

The noise/disturbance with amplitude of 0,5 m was added to the system to check the robustness of the simulated system. The addition of the disturbances has been highlighted in Figure 6.2.

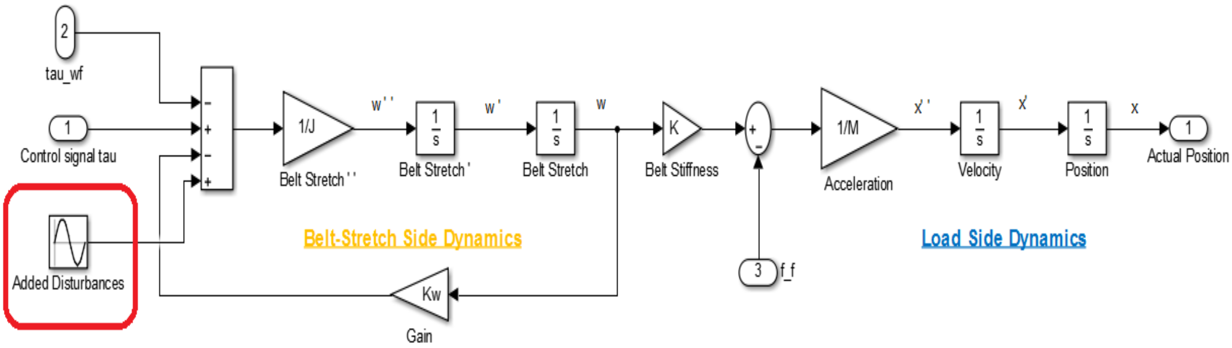

Figure 6.2: The Addition of Disturbances to the System

6.3 MATLAB SIMULATION PARAMETERS

The values of various parameters fed for the MATLAB simulation of this task are obtained from [5] [68]. The values below are copied from MATLAB Script file alongwith the commented portion mentioning the Friction Torque and Friction Coefficient used in this task.

```
D = 75;
Kp = 625;
Ks = 0;
Kv = 50;
G = 1;
J = 0.00202;
K = 2.7e5;
R = 0.06;
L = R/G;
M = 90;
alpha = 125.6;
beta = 7888;
gamma = K/(beta*M);
g = 20;
w0 = 62.8;
```

```
zeta = 2.000022;
kappa = J/M;
Kw = K*(1+kappa);
%Friction Torque
t_f = 0.13;
%Friction Coefficient
Fv=15;
```

6.4 SOLVER SETTINGS

To run the simulation, the default settings of MATLAB Software are to be modified as shown in Figure 6.3 below.

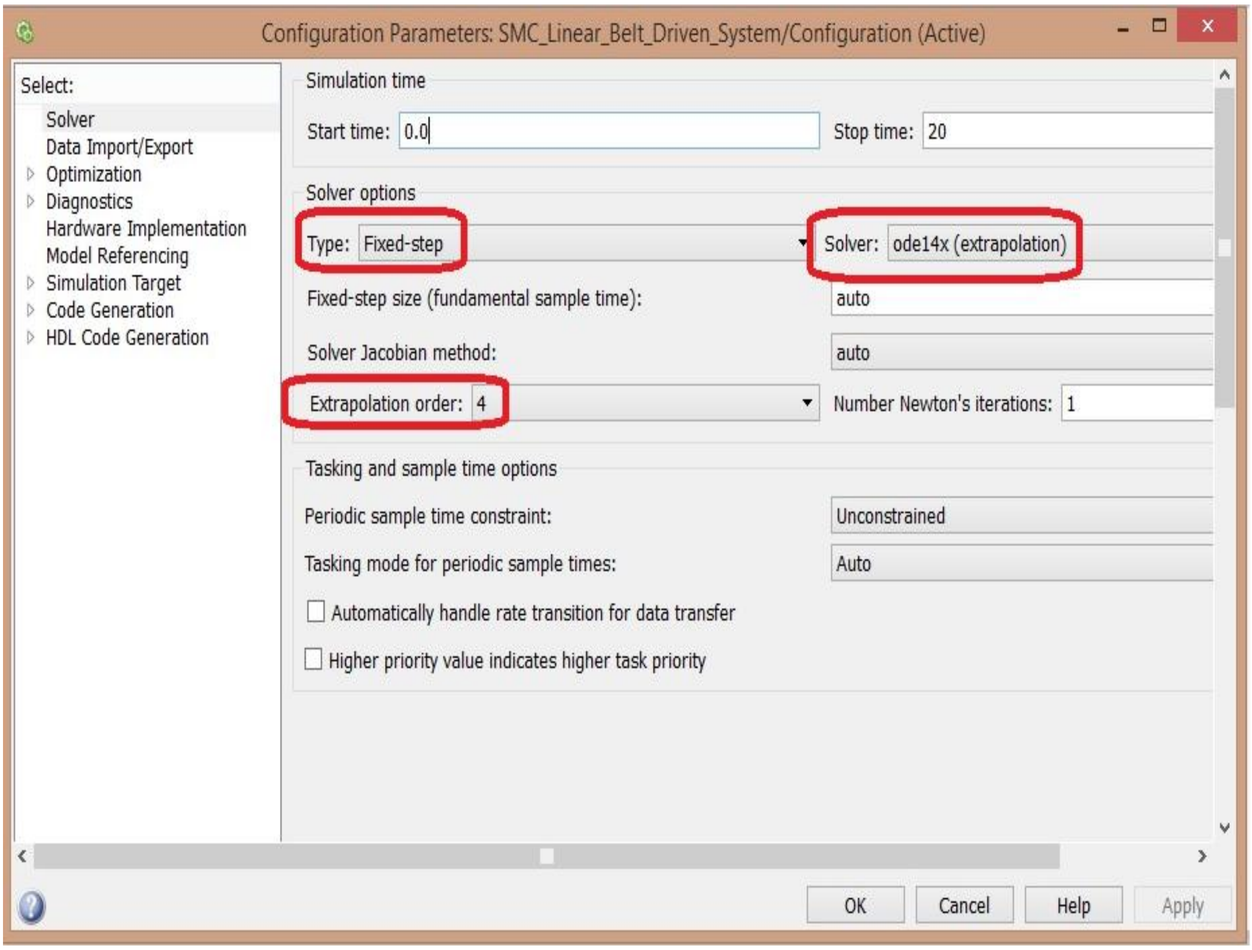

Figure 6.3: The solver settings needed to run the simulation

6.5 SIMULATION RESULTS

The following texts discuss the various results obtained from the simulated system where the robustness is checked initially without the phase-shift. Then the position error magnitude was obtained followed by the belt-stretch value determination. The control torque magnitudes and the friction force magnitudes are then fetched.

Then the robustness of system when the phase-shift of 90° was added to the system was also checked. Later, the belt-stretch which was stated to be an important factor is removed from the simulated system for the next simulation test and the value of the new magnitudes of the belt-stretch are then obtained.

To start with, the DC motor provides a sinusoidal wave i.e. the reference of amplitude 0,2m with a frequency of 0,5rad/sec and phase shift is defined to be 0rad as can be seen from Figure 6.4 below.

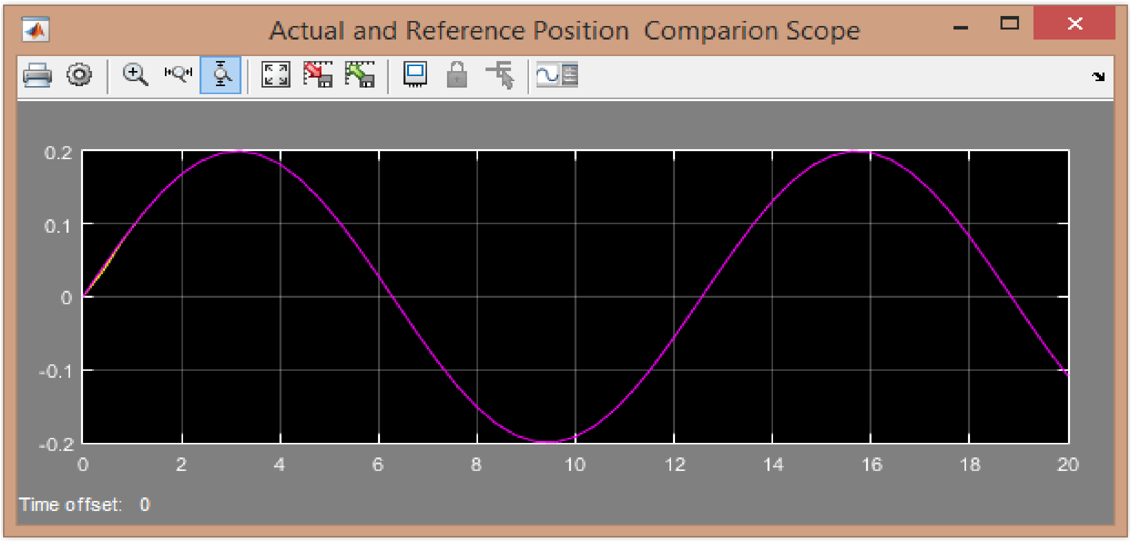

Figure 6.4: The Robustness of the Chattering-Free System

As seen from Figure 6.4, the system is robust enough to deal with the disturbances added very quickly and the output obtained is chattering-free which was one of the prime interests while dealing with the system of this type. The reference is tracked very quickly by the control law implemented in the task.

Now, error magnitude which is the difference between the reference position and the actual position is found shown in Figure 6.5.

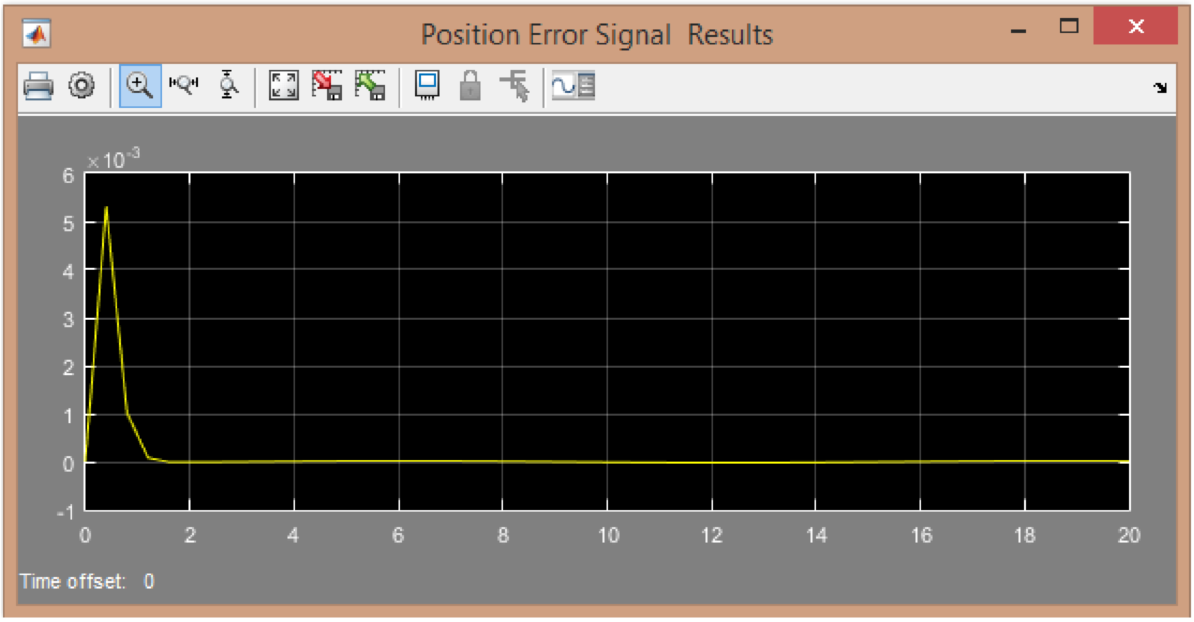

Figure 6.5: The Position Error Results of the System

It can be witnessed from Figure 6.5 that the position error is very less. Initially when the system started, due to jerk, the position error is around 5,4e-03m and then it comes very close to 0m. This shows the accuracy being achieved is quite-high.

The belt stretching results are now shown in Figure 6.6 below where initially the magnitude of the belt-stretch is of the order 1,9e-03m and then later it is oscillating around the magnitude of 0,8e-03m in correspondence to the varying reference input signal.

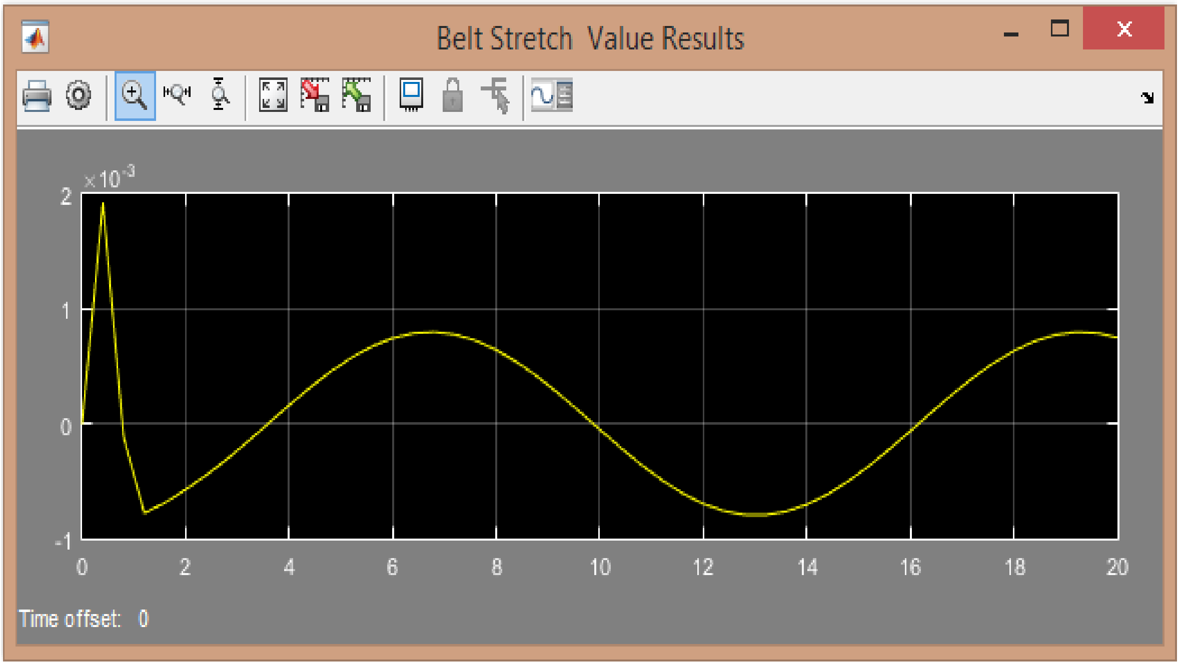

Figure 6.6: The Belt-Stretching Magnitudes

Now, the Control Torque results are fetched as shown in Figure 6.7 below.

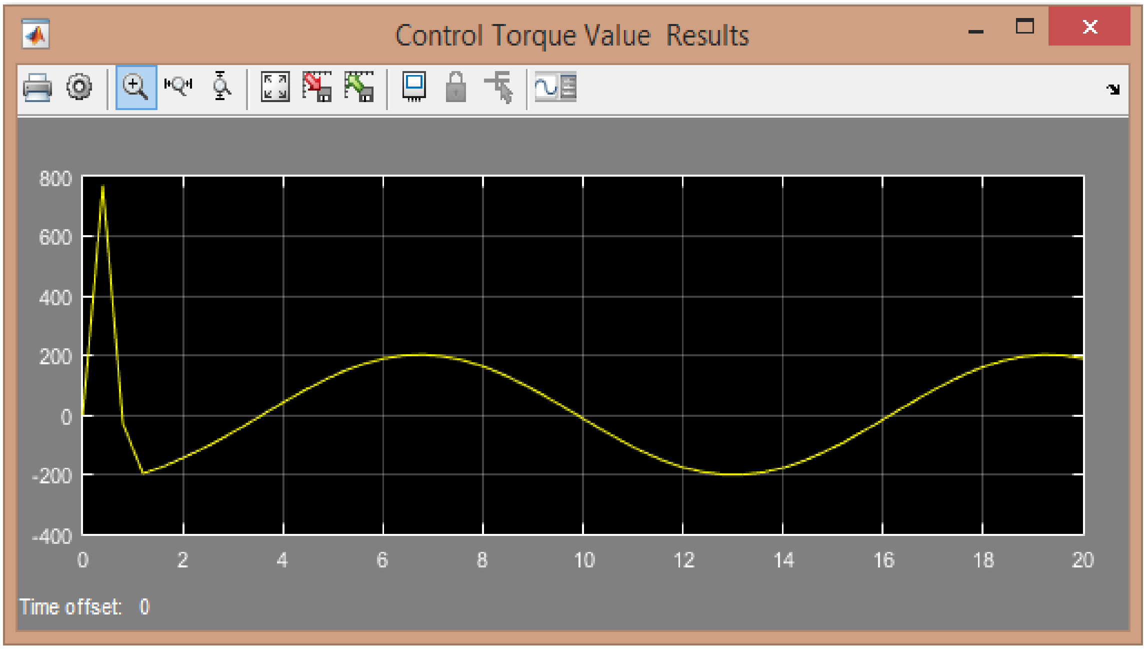

Figure 6.7: Control Torque Curve

It can be seen from the Figure 6.7 that initially, to control the system and minimizing the position error value, the control torque magnitude is quite high with the approximate magnitude of 775Nm and later stays with-in the range of +200 to -200Nm.

The friction force is then obtained from the simulation curves as shown in Figure 6.8.

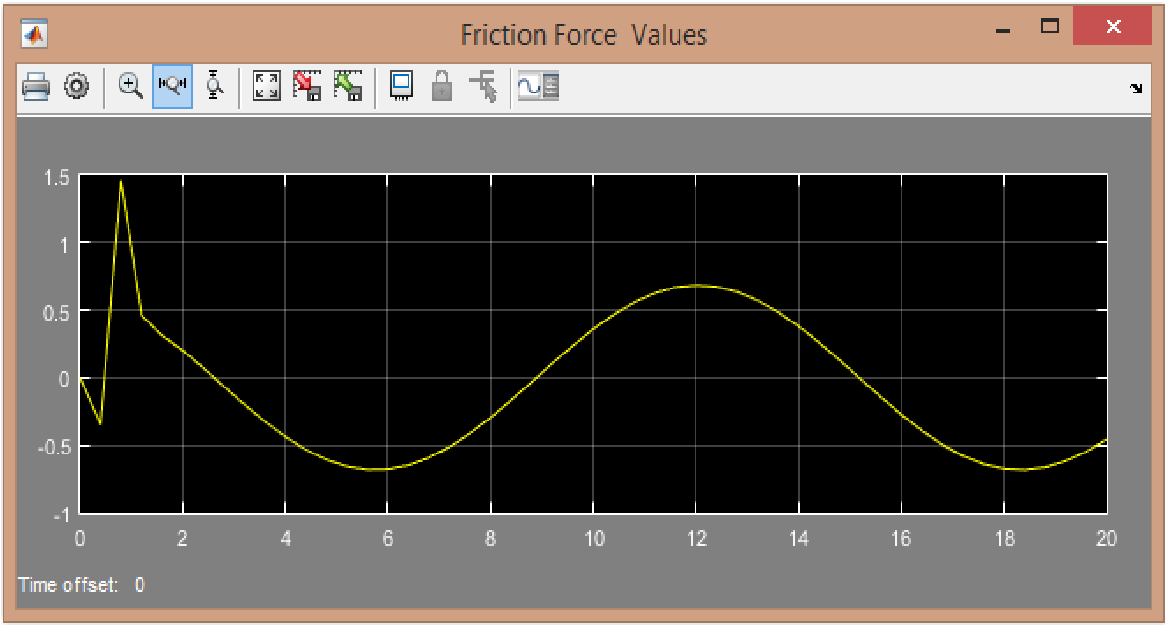

Figure 6.8: Friction Force Curve

Now, to check the robustness of the system with phase-shift, another test was conducted. Keeping the reference amplitude and the frequency to be the same i.e. at the value of 0,2m and 0,5rad/sec respectively, the phase shift is now defined to be pi/2 i.e. 90° and the scope results are shown in Figure 6.9 below.

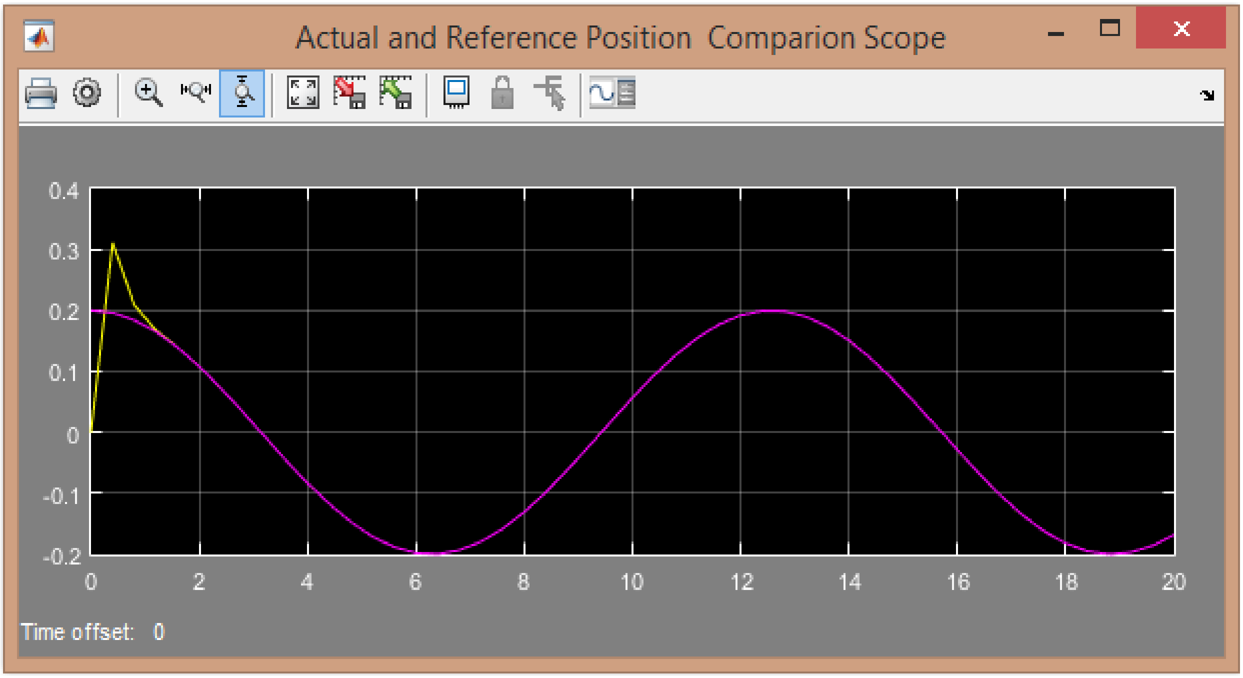

Figure 6.9: The robustness test curve with phase-shift of 90 degrees.

In Figure 6.9 we can see that the system is robust enough to deal with the new reference signal and it took 1,6 seconds to track it and the results are again free from chattering.

Now, to make another experiment regarding the importance of belt-stretch consideration while designing the control law, the changes are made in the control law calculation blocks. The belt-stretch is removed and the sliding surface is now defined according to Equation 5.12.

The changes made are highlighted in Figure 6.10 where the output of Belt Stretch Calculation Block is terminated and new Control Law is derived for the new sliding surface.

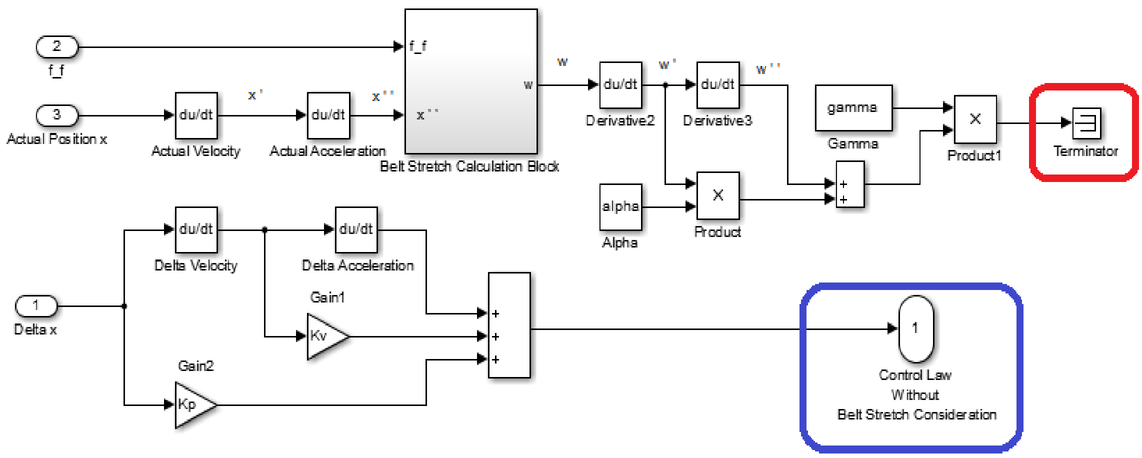

Figure 6.10: Removing the Belt Stretch Component from the Sliding Surface

The simulation scope results, when the belt-stretching derivatives i.e. when $\dot{w}$ and $\ddot{w}$ are not considered while designing the control law for the new sliding surface are shown in Figure 6.11.

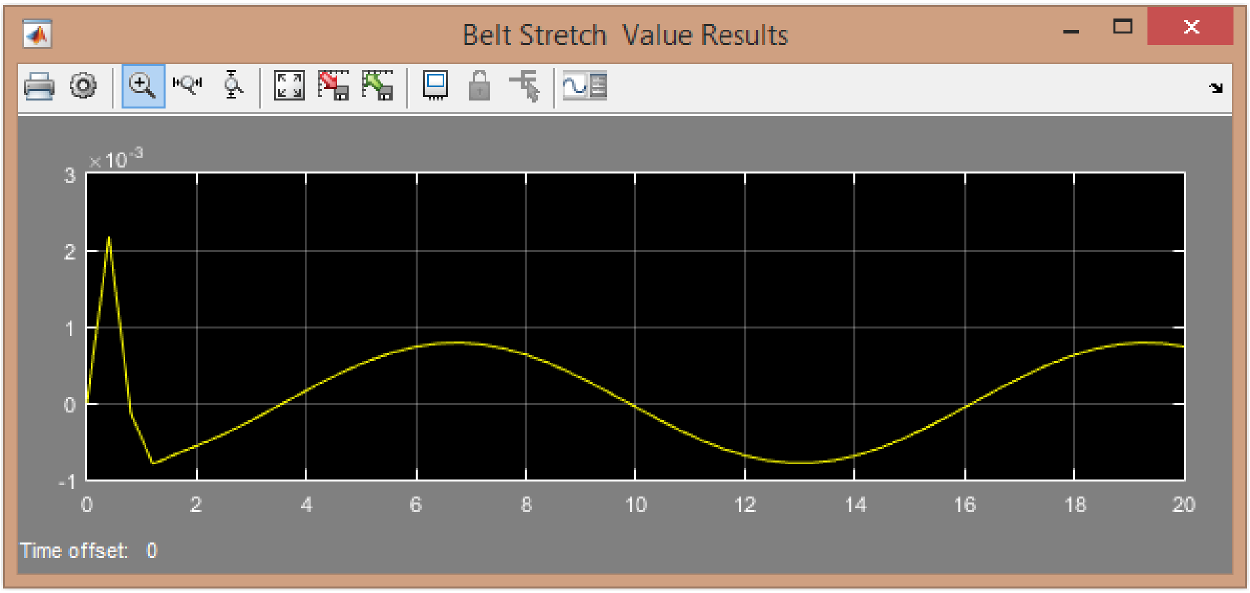

Figure 6.11: Belt Stretch Simulation Curve without Belt-Stretch Derivatives in Sliding Surface Design

When a comparison is made between Figure 6.6 and Figure 6.11, we can clearly see the rise in the belt-stretching magnitudes which ultimately is responsible for decreasing the accuracy of the system.

CHAPTER 7: SUMMARY

7.1 CONCLUSION

The system was mathematically formulated and the Simulink simulations were made to run on MATLAB Software. The results of simulations were quite impressive as the controller worked as it was expected. The major drawback of SMC, "chattering" was not found and hence the controller was found to be a safe choice to be used for the mechanical systems. The position error was found to be very less which proves the accuracy of the controller. The reference position tracking was achieved at quite a fast pace and the controller ably eliminated all the disturbances entering into the system. The magnitude of belt-stretching was considerably reduced after the belt-stretch control term was incorporated in the control law. This further reinforced the accuracy of the system.

The tests were also conducted to test the robustness of the system by changing the phase of the input terms. The system swiftly detected and consequently responded to those changes. Tests were also conducted to see the belt stretching when the belt-stretch control was not considered while designing the control law. It showed that the belt-stretching magnitudes increased i.e. the accuracy of the system dropped.

In nutshell, the proposed system was robust, accurate and vibration-free. The researcher thus advocates using the projected approach as a better alternate to the system installed in Kurresaare.

7.2 LIMITATIONS OF THE RESEARCH

The control law derived and proposed as a solution to the thesis task shows robustness against dynamic friction but it will not be able to deal-with and compensate the friction effects immediately and instantaneously.

The position errors can be further minimized by selecting the large values of damping coefficient D but as described in the texts above that practically we have an upper limit to the selection. If we take a very high value of term D then it may create oscillations and may even lead to unstable plant response.

7.3 FUTURE SCOPE

A combination of Chattering-Free SMC and a Disturbance Observer can be used to reduce or even get rid of the problems in dealing with the friction and also with the rapid changes in the disturbances. So, in such a case, the solution will assume that the system dynamics will be represented by the plant dynamics which is actually perturbed by disturbance signal of the system. Hence, combining both the approaches, we will have equivalent control signal i.e. $\hat{\tau}_{eq}$ and disturbance estimation signal i.e. $\hat{\tau}^{dist}$. The control law will then be given by Equation 7.1.

$$\tau = (\hat{\tau}^{dist} + \hat{\tau}_{eq}) + \int_0^t \dot{\tau}_{SMC} dt \tag{7.1}$$

The effect of keeping Gear-Ratio "*G*" into consideration may be studied and the behavior of the system may be noted. The results of the simulations can be compared to the results obtained in [5] where the real time experimental set-up is made with *G* was also taken into account.

The comparative study where the results obtained from SMC and other control methods can be an interesting research area. The comparison can be made in the Simulation Environment where MATLAB can be used.

A desire to achieve a faster pace in the reaching phase, (to keep the effects of bounded and unbounded uncertainties away) may increase the chances of chattering and correspondingly its magnitude. To solve this issue, Time Delay Control in the Sliding Mode Control (TDC-SMC) can be used to remove the interdependence between the chattering level and the reaching-time.

SUMMARY IN ESTONIAN LANGUAGE (KOKKUVÕTE)

JÄRELDUS

Süsteemi koostamiseks ja simulatsioonide tegemiseks kasutati MATLAB tarkvara Simulink keskkonda. Tulemused olid muljetavaldavad ja kontroller töötas ootuspäraselt. SMC suurimat puudust "vibreerimist" õnnestus vältida ja seetõttu on kontroller mehaanilistes süsteemides kastuamiseks igati ohutu valik. Asukoha määramisel tekkiv viga oli minimaalne. Kontroller suutis lähteasendi kindlaks teha väga kiiresti ja suutis eemaldada süsteemi sisenevad haired. Lisaks suutis kontroller vähendada rihma venimist, mis suurendas täpsust veelgi.

Läbi viidi katsed süsteemi töökindluse testimiseks. Faasimuutused suutis kontroller kiiresti tuvastada ja nendele reageerida. Lisaks tehti katseid rihma venimisesest tekkinud ebatäpsuste kohta. Võib järeldada, et kui rihma venimist kontrolleris arvesse ei võta, tekib süsteemis suuri ebatäpsusi.

Lühidalt, väljapakutud lahendus on robustne, täpne ja vibratsioonivaba. Seetõttu oleks mõistlik antud lahendus parema alternatiivina hetkel Kuressaares eksisteeriva süsteemi vastu välja vahetada.

UURIMISTÖÖ PIIRANGUD

Antud töö lahendusena väljapakutud juhtsüsteem suudab hakkama saada dünaamilise hõõrdumisega, kuid ei suuda neid silmapilkselt kompenseerida.

Asukoha määramisel tekkivaid vigu saab veel enam vähendada valides sumbuvustegurile D suuremaid väärtusi, kuid nagu eelpool mainitud on peaaegu suurimad väärtused juba kasutusel. Väga suure D väärtuse kasutamine võib kaasa tuua võnkeid ja muuta süsteemi ebastabiilseks.

TULEVIK

Vibratsioonivaba SMC-d ja häirevaatleja (Disturbance Observer) kombineerimisel on võimalik vähendada või isegi elimineerida hõõrdumisega tekkivaid probleeme, lisaks saab neid kasutada ka häirete kiirete muutuste vähendamiseks. Sellisel juhul eeldab lahendus, et süsteemi dünaamikat esitatakse jaama dünaamikana, mida tegelikult häirivad süsteemi signaalihäired. Kombineerides mõlemat lahendust, on meil kontrollsignaal $\hat{\tau}_{eq}$ ja prognoositav häiresignaal $\hat{\tau}^{dist}$. Juhtseadus on antud võrrandis S.1.

$$\tau = (\hat{\tau}^{dist} + \hat{\tau}_{eq}) + \int_0^t \dot{\tau}_{SMC} dt \tag{S.1}$$

Ülekandesuhtest tekkivaid võimalikke muutusi tuleks samuti arvesse võtta ja süsteemi käitumist jälgida.

Võrdlusuuring, kus võrreldakse SMC-lt saadud tulemusi mõne teise juhtimismeetodiga võiks olla huvitav uurimisala. Võrdlust saab teha simulatsioonikeskkonnas, kus MATLABi kasutamine on võimalik.

Soov saavutada suuremat kiirust võib tõsta vibreerimise tõenäosust ja selle suurust. Selle probleemi lahendamiseks võib kasutada ajalist viivitust (Time Delay Control in the Sliding Mode Control - TDC-SMC), mille abil oleks võimalik eemaldada vibratsiooni taseme sõltuvust kohale jõudmise ajast.

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