THESIS ON CIVIL ENGINEERING F16

Long Wave Dynamics in the Coastal Zone

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Declaration:
Hereby I declare that this doctoral thesis, my original investigation and achievement, submitted for the doctoral degree at Tallinn University of Technology has not been submitted for any academic degree.

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The ocean is a wilderness reaching round the globe, wilder than a Bengal jungle, and fuller of monsters, washing the very wharves of our cities and the gardens of our sea-side residences.

Henry David Thoreau
"The Writings of Henry David Thoreau", vol. 4, 1906

Contents

Keywords: nonlinear wave dynamics, coastal zone, long wave runup, wave transformation, waves in inhomogeneous media, marine natural hazards, tsunami, freak-waves, storm surges.

List of Tables

List of Figures

Introduction

State-of-the-art of the runup problem

Giant surface waves approaching the coast frequently cause extensive coastal flooding, destruction of coastal constructions and loss of lives. Such waves can be generated by various phenomena: strong storms and cyclones, underwater earthquakes, high-speed ferries, aerial and submarine landslides. The most resent examples of such events are the catastrophic tsunami in the Indian Ocean, which occurred on 26 December 2004 (Lay et al., 2005) and hurricane Katrina (28 August 2005) in the Atlantic Ocean (Kim et al., 2008). The huge storm in the Baltic Sea on 9 January 2005, which produced unexpectedly long waves in many areas of the Baltic Sea (Soomere et al., 2008) and the influence of unusually high surge created by long waves from high-speed ferries (Soomere, 2005; Parnell et al., 2007), should also be mentioned as examples of regional marine natural hazards connected with extensive runup of certain types of waves.

The prediction of possible flooding and properties of the water flow on the coast is an important practical task for a coastal and port engineering. That explains the multitude of empirical formulas describing runup characteristics, available in the engineering literature (see, for instance, Le Mehaute et al., 1968; Stockdon et al., 2006). For the most part these formulas are specific for different geographic areas due to particularities of local wave regimes (wind direction, coastal effects of wave refraction and diffraction).

The wave transformation and shoaling of water waves in the basin of variable depth is a well developed task of fluid dynamics and has numerous applications in physical oceanography (Le Blond & Mysak, 1978; Massel, 1989; Mei, 1989; Dingemans, 1996). Asymptotic methods are widely applied to describe the wave field for slow variations of water depth (Shen, 1975; Mei, 1989; Dingemans, 1995; Berry, 2005; Dobrokhotov et al., 2006, 2007). In the simplified case of the 1D linear shallow-water wave propagation, asymptotic methods lead to well-known Green's law $A \sim h^{-1/4}$ for the changes in the wave amplitude A (h is water depth), derived from the energy flux conservation. Not all amplitude changes follow this law; for example, the height of a solitary wave (soliton) may vary as $A \sim h^{-1}$ in the framework of the weakly nonlinear and dispersive theory (Grimshaw, 1970; Ostrovsky & Pelinovsky, 1970). A more complicated formula can be obtained for a solitary wave of arbitrary height (Pelinovsky, 1996). The particular law of dependence of the wave amplitude on the combination of the properties of the attacking wave and of the medium, and the related problem of wave runup is one of the central questions in tsunami modelling and the modelling of flooding.

If the water depth in the coastal zone varies rapidly, the exact analytical solutions for the wave transformation can be found within a linear shallow-water theory for different bottom profiles. Such solutions are usually expressed in terms of special functions (Le Blond & Mysak, 1978; Massel, 1989; Mei, 1989).

Analytical rigorous solutions of the nonlinear shallow-water system are only known to exist for the beach of constant slope in the vicinity of the shoreline (Carrier & Greenspan, 1958). The solution of the nonlinear problem strongly depends on the initial wave shape. Various shapes of the periodic incident wave trains have been analysed: the sine wave (Kaistrenko et al., 1991; Madsen & Fuhrman, 2007) and cnoidal wave (Synolakis 1991).

Relevant analysis has also been performed for a variety of solitary waves and single pulses, such as soliton (Pedersen & Gjevik, 1983; Synolakis, 1987; Kanoglu, 2004), sine pulse (Mazova et al., 1991), Lorentz pulse (Pelinovsky & Mazova 1992), Gaussian pulse (Carrier et al., 2003; Kanoglu & Synolakis, 2006), *N*-waves (Tadepalli & Synolakis, 1994) and "characterized tsunami waves" (Tinti & Tonini, 2005). It is important to mention that many analytical formulas of wave runup have been confirmed in laboratory tanks (Lin et al, 1999; Li & Raichlen, 2002) and are now actively used in predictions of marine natural hazards (see, for instance Curtis & Pelinovsky, 1999; Pelinovsky & Kharif, 2008).

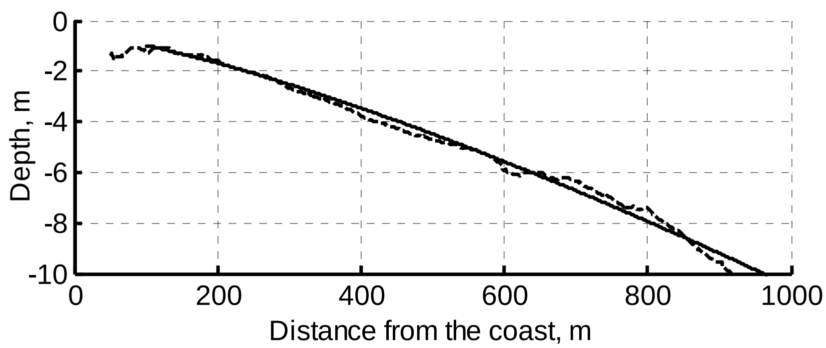

Fig. 1. The bottom profile measured at Pirita Beach, Estonia (dashed line) and its approximation with a power law with $d = 4/3$ (solid line). Modified from Soomere et al. (2007)

Most of the studies into wave transformation and runup have been performed for linearly varying depth. This approximation is not always particularly realistic. Various bottom profiles in the vicinity of the shoreline following power laws $h(x) \sim x^d$ have been discussed in literature. The most popular profile is the famous Dean's Equilibrium Profile with $d = 2/3$ (see, for example, Dean & Dalrymple, 2002). This approximation with $d = 0.78$ fits for Dutch dune profiles better (Steetzel, 1993). Kit & Pelinovsky (1998) found the range of $d = 0.73 - 1.1$ for Israeli beaches. The power law approximation for beach profiles is used also in theoretical models (Kabayashi, 1987; Kit & Pelinovsky, 1998). However, in many cases, bottom profiles have a complex structure and their shape in the immediate vicinity of the shoreline differs from that of the profiles at larger depths. Figure 1 demonstrates the bottom profile measured at Pirita Beach, Estonia (Soomere et al., 2007).

It is clearly seen that the bottom profile for the depths of 2–10 m can be reasonably described by a power law with $d = 4/3$. Similar types of profiles following power laws with $d > 1$ can be found for the continental Pacific shelf of Northern Chile for the coastal line up to 5 km (Fig. 2).

So, the wave transformation and runup should be analysed for various bottom profiles following more general power laws (not only the popular case $d = 1$). The analysis of wave properties along a specific type of convex bottom profile with $d = 4/3$ is one of the main goals of this study.

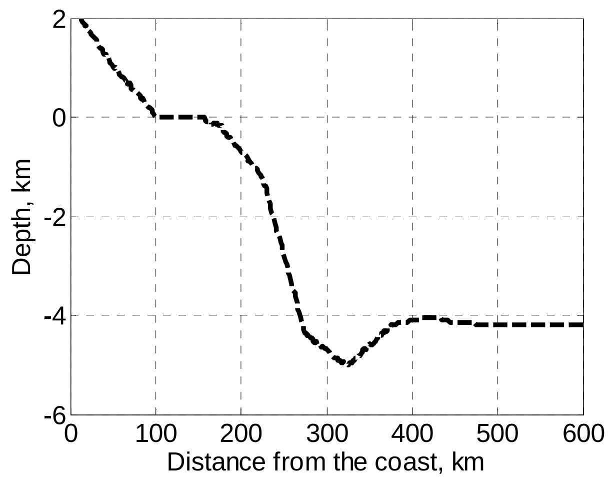

Fig. 2. The bottom profile extracted from Historical Tsunami Data Base (Gusiakov, 2002) for the Pacific coast of Northern Chile (coordinates of the coastal point are 7.70ºS, 78.51ºW)

Outline of the thesis

The thesis constitutes a study of the long wave dynamics in the coastal zone for various approximations of the bottom profile. The case $d = 4/3$ is analysed in Chapter 1 within a linear approximation. The relevant results are presented in Paper V. This case is of special interest, because the solution of linearized shallow-water equations can be obtained in closed form in terms of elementary functions for this profile (Cherkesov, 1975; Pelinovsky, 1996; Tinti et al., 2001). In the cited papers this solution was considered mainly to simplify the final expressions describing wave dynamics, but a comprehensive analysis of wave properties and transformation along this type of coastal slope is missing.

Main attention is paid to the unsteady dynamics of water displacement and depth-averaged flow induced by the wave field. The structure of the travelling wave is described in detail in Section 1.2. It is shown that the surface elevation is always sign-variable. The full solution of the relevant Cauchy problem is obtained in Section 1.3. It describes the generation of waves by arbitrary initial disturbances, their transformation and reflection from the beach. It is found that a sort of "relict" weak non-uniform current is formed in the region of the initial disturbance after the waves have left this zone.

The wave reflection from the shore is studied in Section 1.4. It is shown that amplification of waves due to the shoaling effect along a convex bottom slope is much larger than for other bottom profiles. The reflection of waves occurs only in the vicinity of the shoreline. Some interesting features are found for a special case of wave reflection from the zone of increased depth (Section 1.5). It is shown that the reflected wave always has a sign-variable shape, even when the incident wave is a pure elevation wave.

The results of analytical studies of long wave runup on a plane beach in a more complex nonlinear framework are presented in Chapter 2. Basic equations and the method of solving the nonlinear shallow-water system suggested by Carrier & Greenspan (1958) are briefly described in Section 2.2. A two-step approach for determining the runup characteristics is developed in Section 2.3. The fundamental advantage of this method is that extreme characteristics of the runup process (runup and rundown amplitudes, extreme values of on- and offshore velocities, the wave breaking condition) of nonlinear waves can be found within a linear approximation. The "real" nonlinear dynamics of the moving shoreline requires the nonlinear theory which is also described in Section 2.3. As the developed theory is correct only for non-breaking waves, it is important to specify the conditions (increase in the amplitude) at which wave breaking occurs exactly on the shoreline. In this case the velocity of the shoreline has the shape of a shock wave and the function describing the water displacement has a jump of the first derivative in the wave trough.

The runup of solitary waves of various shapes is analysed in Section 2.4. It is shown that with the use of a convenient definition of the "significant" wavelength, the dependence of the extreme runup of waves with symmetric profiles on the incident wave shape is very weak. This feature allows derivation of universal formulas for rapid estimation of runup characteristics provided the length and height of the approaching waves are known. Such formulas can be used for engineering applications and for mitigation of the tsunami hazard. The runup of asymmetric waves with a steep front is discussed in Section 2.5. It is shown that such waves penetrate much deeper inland and that inland-moving water flow is faster than in the case of symmetric waves of the same height and length.

Particularly high and steep (freak or rogue) waves are one of the most dangerous events that a traveller at sea may encounter. Often such waves (frequently called sneaker waves) occur in the coastal zone. Onshore freak wave events are analysed in Chapter 3. The onshore freak waves that occurred in 2005 (Paper I) are described in Section 3.2. These accidents are related to unexpected wave impact upon the coast and engineering structures or sudden intensive flooding of the coast. Runup of irregular waves, including freak waves, modelled as superposition of Fourier harmonics with random phases, is studied in the framework of nonlinear shallow-water theory (Section 3.3). It is shown that an average runup height for waves with a wide spectrum is higher than for waves with a narrow-band spectrum. The possibility of the appearance of freak waves on a beach is analysed in Section 3.4. The distribution functions of runup characteristics are computed under an assumption that the incident wave represents an irregular sea state with a Gaussian spectrum. The asymptotic behaviour of probability distribution functions of the occurrence of large amplitude waves for estimation of freak wave formation at the shore is studied.

Approbation of the results

The basic results described in this thesis have been presented in the following international conferences, symposiums, and workshops:

Conferences:

Solutions to Coastal Disasters Conference 2008, Oahu, Hawaii (2008);
Joint workshops "Implications of climate change for marine and coastal safety" and "Applied Wave Mathematics" of Marie Curie networks SEAMOCS and CENS-CMA, and Eco-NET network "Wave Current Interaction in Coastal Environment", Palmse, Estonia (2007);
General Assembly of the International Union of Geodesy and Geophysics (IUGG) (2007);
European Geosciences Union (EGU), Vienna, Austria (2006, 2007).

Seminars:

Seminar paper "Shoaling and runup of long waves generated by high-speed ferries" at Department of Civil & Environmental Engineering, Cornell University (2008);
Seminar paper "New Trends in the Nonlinear Theory of Long Wave Runup on a Beach" at the Department of Civil & Environmental Engineering, Massachusetts Institute of Technology (2008);
Seminar paper "Long waves in a coastal zone" at Lund University (2007);
Seminar paper "Mathematical modelling of long waves (tsunami waves)" at the Institute of Cybernetics, Tallinn University of Technology (2007)
Seminar paper "Runup of nonlinear asymmetric waves on a plane beach" at the University of Oslo (2006);
Seminar paper "Runup of nonlinear deformed waves" at Det Norske Veritas, DNV Research (2006).

The thesis is based on the following publications:

Papers indexed by the ISI Web of Science:

Didenkulova, I., Slunyaev, A., Pelinovsky, E., Kharif, Ch. Freak waves in 2005. – Natural Hazards and Earth System Sciences 6; 2006, p. 1007–1015.

Didenkulova, I., Kurkin, A., Pelinovsky, E. Run-up of solitary waves on slopes with different profiles. – Izvestiya, Atmospheric and Oceanic Physics 43 /3; 2007, p. 384–390.

Didenkulova, I., Pelinovsky, E. Runup of long waves on a beach: influence of the initial wave shape. – Oceanology 48 /1; 2008, p. 1–6.

Peer-reviewed papers in other international research journals:

Didenkulova, I., Pelinovsky, E., Zahibo, N. Long wave reflection from "non-reflecting" bottom profile. – Fluid Dynamics 43 /4; 2008, p. 101–107.

Chapters in books published by leading international publishing houses:

Didenkulova, I., Pelinovsky, E., Soomere, T., Zahibo, N. Runup of nonlinear asymmetric waves on a plane beach. – In: Tsunami & Nonlinear Waves (Ed: Anjan Kundu). Springer, 2007, p. 175–190.

Didenkulova, I., Pelinovsky, E., Sergeeva, A. Runup of long irregular waves on a plane beach. – In: Extreme Ocean Waves (Ed: Efim Pelinovsky and Christian Kharif). Springer, 2008, p. 83–94.

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1. Long wave dynamics along a convex bottom profile

1.1. Introduction

In this chapter, results of the study of the linear dynamics of shallow-water waves are presented for the convex depth profile $h(x) \sim x^{4/3}$ for a wide class of initial conditions. The main goal is to establish potential threats to the coastal zone through enhanced amplitude amplification of approaching waves and potentially larger runup height of long waves along beaches containing convex sections of the coastal slope. The relevant results have been presented in Paper V.

The properties of travelling waves along convex bottom are described in Section 1.2. The uniqueness of such travelling wave solutions is proved in Section 1.3 by means of introducing a 1:1 transformation of the governing wave equation with varying coefficients to the constant-coefficient wave equation. This transformation makes it possible to obtain the solution of the Cauchy problem and to study wave evolution for various initial conditions in a straightforward manner. Wave runup on a beach with the profile $h(x) \sim x^{4/3}$ is analysed in Section 1.4, with an important implication that the wave amplification for such a beach can be much more significant than for a plane beach. The wave propagation along the beach containing a shallow coastal area of constant depth and a section of convex beach is studied in Section 1.5. The main results are summarized in the conclusion.

1.2. Travelling waves above an uneven bottom

The basic model for the linear 2D shallow-water waves in the basin of variable depth is a linear wave equation for the vertical displacement of the water surface $\eta(x,t)$:

$$\frac{\partial^2 \eta}{\partial t^2} - \frac{\partial}{\partial x}\left[c^2(x)\frac{\partial \eta}{\partial x}\right] = 0, \qquad c(x) = \sqrt{gh(x)}, \tag{1.2.1}$$

where $c(x)$ is the wave speed, $h(x)$ is the water depth and g is the gravity acceleration. The domain, boundary and initial conditions for Eq. (1.2.1) will be discussed later.

Travelling wave solutions for the wave equation with slowly varying coefficients, equivalently, for the waves above slowly varying bottom relief, are usually studied with the use of asymptotic methods. These methods lead to an exact solution of the wave equation for a sine wave above a convex beach with $d = 4/3$ (Cherkesov, 1975; Pelinovsky, 1996). These results are shortly recalled here from the viewpoint of the structure of travelling water waves.

Travelling (progressive) waves are sought in the form

$$\eta(x,t) = A(x)\exp\{i[\omega t \text{-} \Psi(x)]\}, \tag{1.2.2}$$

where $A(x)$ and $\Psi(x)$ are real functions (the local amplitude and phase, respectively) which should be determined, and ω is the wave frequency. After substitution of Eq. (1.2.2) to Eq. (1.2.1) the real and imaginary parts of the resulting equation are two ordinary differential equations:

$$\left[\frac{\omega^2}{gh(x)} - k^2(x)\right]A + \left[\frac{d^2A}{dx^2} + \frac{1}{h}\frac{dh}{dx}\frac{dA}{dx}\right] = 0, \tag{1.2.3}$$

$$2k\frac{dA}{dx} + A\frac{dk}{dx} + \frac{1}{h}\frac{dh}{dx}kA = 0. \tag{1.2.4}$$

Here $k(x) = d\Psi / dx$ is the local wave number. Equation (1.2.3) can be interpreted as the generalized dispersion relation for waves in an inhomogeneous medium, whereas Eq. (1.2.4) has the meaning of the energy flux conservation law. While Eq. (1.2.4) can be easily integrated:

$$A^2(x)k(x)h(x) = \text{const}, \tag{1.2.5}$$

Eq. (1.2.3) is a second-order differential equation with variable coefficients and generally has no analytic solutions in closed form. This equation is not simpler than the initial wave equation (1.2.1).

Further progress in analytical solving of Eq. (1.2.3) can be made when a wave propagates above slowly a varying bottom. In this case variations of both the water depth and the wave amplitude are slow. The terms in the second bracket of Eq. (1.2.3) are small compared to other additives and can be ignored in the first approximation. In this case the solution of Eq. (1.2.3) is simple:

$$k(x) = \frac{\omega}{\sqrt{gh(x)}}. \tag{1.2.6}$$

Equation (1.2.6) is a generalization of the well-known dispersion relation for water waves in the basin of slowly varying depth. Solution (1.2.6), together with Eq. (1.2.5), determines the wave amplitude (which in the case of question evidently follows Green's law) and phase. The relevant asymptotic procedure and all higher-order corrections of the wave amplitude and phase are described in detail in (Maslov, 1987, 1994; Babich & Buldyrev, 1991; Berry, 2005).

Basically, Eq. (1.2.3) can be solved numerically for an arbitrary function $h(x)$. Analytical solutions exist for specific bottom profiles. After solving Eq. (1.2.3), solution (1.2.2) can be determined completely. Sometimes, solutions of this type are called travelling waves in an arbitrarily inhomogeneous medium (without any specific applications for water waves). Strictly speaking, however, such solutions can be interpreted as complicated physical processes of wave transformation and reflection in the basin of variable depth (Ginzburg, 1970; Brekhovskikh, 1980).

One of the central problems of this thesis is the analysis of the potential existence of exact travelling wave solutions to Eq. (1.2.1) and their propagation and reflection properties. There exists no comprehensive description of the procedure to select the travelling wave solution from the entire set of solutions of Eq. (1.2.3) in scientific literature. Historically, a subset of such solutions has been found by requesting that the equations

$$\frac{\omega^2}{gh(x)} - k^2(x) = 0 \tag{1.2.7}$$

and

$$\left[\frac{d^2 A}{dx^2} + \frac{1}{h}\frac{dh}{dx}\frac{dA}{dx}\right] = 0 \tag{1.2.8}$$

are satisfied simultaneously. Obviously, any set of solutions $\{A, k, h\}$ to Eqs. (1.2.7) and (1.2.8) also solves Eq. (1.2.3) [although generally solutions to Eq. (1.2.3) do not solve Eqs. (1.2.7) and (1.2.8) simultaneously]. The solution of Eq. (1.2.7) is straightforward and given by Eq. (1.2.6); thus the function $k(x)$ is uniquely defined. The system of Eqs. (1.2.5) and (1.2.8) is overdetermined for the wave amplitude. Its consistent solution can be achieved if and only if

$$h(x) = p(x+b)^{4/3}, \tag{1.2.9}$$

where p and b are arbitrary constants. The desired solution therefore only exists for beaches having a specific convex bottom profile. As constant b can be eliminated by a shift $\tilde{x} = x - b$ of the *x*-axis, we can assume $b = 0$ without the loss of generality. Doing so simply means that the origin $x = 0$ is located at the coastline. For the bottom profile presented by Eq. (1.2.9) the components of the travelling wave in ansatz (1.2.2) are then completely and uniquely defined:

$$k(x) = \frac{\omega}{\sqrt{gp}} x^{-2/3}, \qquad \Psi(x) = \frac{3\omega}{\sqrt{gp}} x^{1/3} + \text{const}, \qquad A(x) = \frac{\text{const}}{x^{1/3}}. \tag{1.2.10}$$

The corresponding full solution to Eq. (1.2.1) can be re-written as a travelling wave:

$$\eta(x,t) = A(x)\exp\{i\omega[t - \tau(x)]\}, \quad A(x) = A_0\left[\frac{h_0}{h(x)}\right]^{1/4}, \quad \tau(x) = \int_{x_0}^{x} \frac{dy}{c(y)}, \tag{1.2.11}$$

where A_0 and h_0 are the amplitude and the water depth at the point $x = x_0$, respectively. The location of the point $x = x_0$ can be chosen arbitrarily. This feature makes it possible to analyse the evolution of both waves approaching from offshore and waves generated in the vicinity of the coast. The solutions given by Eq. (1.2.11) correspond to right-going (propagating offshore in this geometry) monochromatic wave trains and are equivalent to those found in Cherkesov (1975) and Pelinovsky (1996). The resulting expressions coincide with the asymptotic wave solution for a slowly varying bottom profile, but are correct for any bottom slope.

A similar solution can be evidently obtained for a wave propagating to the left (onshore direction) by simply picking up another sign of $\tau(x)$ in Eq. (1.2.11). In the linear framework these waves do not interact with each other: the resulting surface displacement in the areas where they excite water displacement or local current, the resulting wave profile or current speed is just the sum of displacements or currents caused by the counterparts.

Previous studies into the problem in question have been limited to the analysis of properties of monochromatic or sine waves. An obvious generalization of the existing results consists in the use of Fourier analysis to obtain the superposition of such sine waves with different frequencies, the technique obviously being applicable in this linear framework. With the use of the Fourier integral of spectral components (1.2.11), the travelling wave of an arbitrary shape can be presented in a general form (Paper V)

$$\eta(x,t) = A(x) f[t - \tau(x)], \qquad (1.2.12)$$

where $f(t)$ describes the wave shape (interpreted here as the variation with time of the surface elevation at a fixed point). An important feature is that representation (1.2.12) allows considering wave pulses of finite duration – generalized solutions of the wave equation.

Another important property of the shallow-water wave field is the wave-induced, depth-averaged flow velocity. This velocity can be calculated from the water displacement using one of the equations of the linear shallow-water system:

$$\frac{\partial u}{\partial t} + g \frac{\partial \eta}{\partial x} = 0 . \qquad (1.2.13)$$

In particular, the velocities induced by the monochromatic wave (1.2.11) and by a pulse (1.2.12) are

$$u(x,t) = U(x)\left[1 + \frac{\sqrt{gh}}{4hi\omega}\frac{dh}{dx}\right]\exp\{i\omega[t - \tau(x)]\}, \qquad (1.2.14)$$

$$U(x) = A(x)\sqrt{\frac{g}{h(x)}} = A_0\sqrt{\frac{g}{h}}\left[\frac{h_0}{h(x)}\right]^{1/4},$$

$$u(x,t) = U(x)\left\{f(\xi) + \frac{\sqrt{gh}}{4h}\frac{dh}{dx}\Phi(\xi)\right\}, \tag{1.2.15}$$

where $\Phi(\xi) = \int f(\xi)d\xi$ and $\xi = t - \tau(x)$. The details of derivation of Eqs. (1.2.14) and (1.2.15) can be found in Paper V. Notice that the first terms in Eqs. (1.2.14) and (1.2.15) correspond to the asymptotic solution of Eq. (1.2.1) above a slowly varying bottom, for which the shapes of the water displacement and the wave-induced water flow coincide. The second term becomes important in the vicinity of the shoreline.

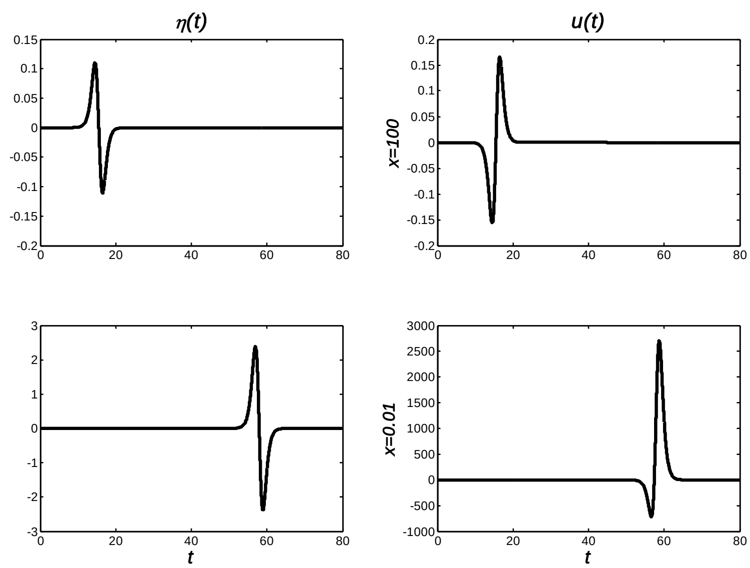

Fig. 1.2.1. The shape of a travelling wave (left) and the water flow (right) at various distances (m) from the shoreline. Time is given in seconds

The natural restriction for realistic pulses is that the disturbance should have a limited energy (equivalently, finite effective wave duration). This assumption leads to the condition

$$\int_{-\infty}^{+\infty} f(t)dt = 0, \tag{1.2.16}$$

from which it follows that the shape of the water displacement should be sign-variable. This condition is valid for a travelling wave only (Paper V). As it is not obvious from the viewpoint of the classical d'Alembert solution of the generic wave equation (which may consist of two sign-constant impulses propagating in different directions), we will discuss this feature in more detail in Section 1.3.

Figure 1.2.1 shows the evolution of the shape of a travelling wave propagating onshore along a coast with the bottom profile (1.2.9) with the coefficient $p = 0.01$ m$^{-1/3}$. This value will be used in all computations below. The figure illustrates the conservation of the shape of the water displacement and strong deformation of the water flow. The shapes of the vertical displacement and the water flow are almost identical offshore (at great depths), but different near the shoreline (at small depths). While the wave shape remains symmetric, the wave-induced water flow is asymmetric: at small depths it is directed offshore rather than onshore (for a given shape of wave elevation). Considerable amplification of wave amplitudes occurs when such a wave approaches the shoreline. From Eqs. (1.2.12) and (1.2.15) it follows that the amplitude of the "velocity wave" varies more strongly than the amplitude of surface displacement.

1.3. Generation of waves by initial disturbances

From Eq. (1.2.11) it follows that the function $f[\tau(x) \pm t]$ should satisfy a wave equation with constant coefficients. The key component of the analysis of the existence and uniqueness of solutions to Eq. (1.2.1) corresponding to travelling waves in a basin of variable depth is establishing a 1:1 transformation of Eq. (1.2.1) to a similar equation with $c(x) = const$.

Let us seek the solution of Eq. (1.2.1) in the form

$$\eta(x,t) = B(x) H[\tau(x), t], \tag{1.3.1}$$

where $B(x)$ and $\tau(x)$ should be determined, and the function H satisfies the constant-coefficient wave equation with $c = 1$:

$$\frac{\partial^2 H}{\partial t^2} - \frac{\partial^2 H}{\partial \tau^2} = 0. \tag{1.3.2}$$

Substitution of Eq. (1.3.1) into Eq. (1.2.1) results in Eq. (1.3.2) if and only if the unknown functions $B(x)$ and $\tau(x)$ satisfy the following three equations:

$$\frac{d}{dx}\left[h(x)\frac{dB}{dx}\right] = 0, \tag{1.3.3}$$

$$h(x)\frac{dB}{dx}\frac{d\tau}{dx} + \frac{d}{dx}\left[h(x)B(x)\frac{d\tau}{dx}\right] = 0, \tag{1.3.4}$$

$$gh(x)\left(\frac{d\tau}{dx}\right)^2 = 1 . \tag{1.3.5}$$

These equations are generalizations of Eqs. (1.2.5), (1.2.7) and (1.2.8). They are also overdetermined in the sense that they have a solution if and only if $h(x)$ is given by Eq. (1.2.9). In other words, the desired transformation exists if and only if the bottom profile is $h(x) \sim x^{4/3}$. This solution is unique for a reasonable choice of initial or boundary conditions and coincides with that of Eqs. (1.2.10), (1.2.11) if $B(x) = A(x)$. Moreover, if $B(x)$ and $\tau(x)$, together with $h(x) \sim x^{4/3}$, solve Eqs. (1.3.3)–(1.3.5), then the transformation given by Eq. (1.3.2) reduces Eq. (1.2.1) to Eq. (1.3.2) for the unknown function H.

The existence of transformation (1.3.1) if and only if the bottom profile is $h(x) \sim x^{4/3}$ proves that exact travelling wave solutions for the above considered type of varying bottom relief are unique to this shape of the bottom profile[1].

There is another important consequence from the existence of transformation (1.3.1). Namely, wave equation (1.3.2) has been extensively studied in mathematical physics, and many theorems and approaches can be directly applied to the particular solutions in question. In what follows this connection is used for constructing the general solution of Eq. (1.2.1).

First of all, wave equation (1.2.1) has a clear meaning in the given geometry and should be solved on a semi-axis ($0 < \tau < \infty$) only, whereas the origin $x = 0$ is a singularity point of the solution. An important simplification of the problem is that the point $\tau = 0$ corresponding to the shoreline ($x = 0$) is not singular in Eq. (1.3.2). The natural boundary condition for Eq. (1.3.2) at this point is

$$H(\tau = 0, t) = 0 . \tag{1.3.6}$$

This condition implies that the water displacement $\eta(x = 0, t)$ always remains bounded at the shoreline. In this case the domain for Eq. (1.3.2) can be formally extended to the whole axis ($-\infty < \tau < +\infty$). The extension is physically meaningful if the initial conditions are continued for $\tau < 0$ as $H(-\tau,0) = -H(\tau,0)$. Nevertheless, the wave field has a clear physical interpretation in the domain $\tau \geq 0$ only.

The general solution of the Cauchy problem for Eq. (1.2.1), describing free evolution of waves generated from the generic initial disturbance of water surface and given velocity field

$$\eta(x,0) = \eta_0(x) , \qquad u(x,0) = u_0(x) , \tag{1.3.7}$$

---

[1] This result does not exclude the existence of analogous solutions obtainable with the use of transformation of Eq. (1.2.1) to some other type of exactly solvable equation.

can be expressed as

$$\eta(x,t) = \frac{1}{x^{1/3}}\{f_+[\tau(x)-t] + f_-[\tau(x)+t] - f_-[-\tau(x)+t]\}, \tag{1.3.8}$$

$$u(x,t) = \sqrt{\frac{g}{p}}\frac{1}{x}\left[f_+(\tau-t) - f_-(\tau+t) - f_-(-\tau+t)\right]$$
$$-\frac{g}{3x^{4/3}}\left[\Phi_+(\tau-t) - \Phi_-(\tau+t) - \Phi_-(-\tau+t)\right], \tag{1.3.9}$$

where functions f_+ and f_- (representing the waves propagating offshore and onshore, respectively) can be found from initial conditions (1.3.7) and $\Phi_\pm(\xi) = \int f_\pm(\xi)d\xi$. Condition (1.3.6) is satisfied automatically.

In the theory of tsunami wave generation above an inclined bottom only the vertical displacement of the source is usually used (Pelinovsky, 1996; Carrier et al., 2003; Tinti & Tonini, 2005; Dutykh et al., 2006). In this case

$$f_+ = f_- = f_0[\tau(x)] = 0.5x^{1/3}\eta_0(x). \tag{1.3.10}$$

Generally, function f_0 can have an arbitrary shape determined by the initial displacement.

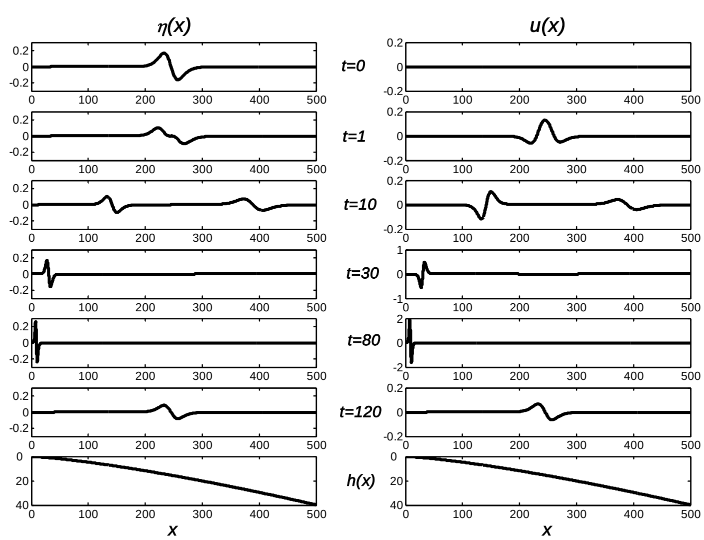

Fig. 1.3.1. Water displacement (left) and velocity (right) for initial disturbance (1.3.11). The bottom profile is shown at the bottom

Figure 1.3.1 displays the water displacement and velocity for the case where the initial displacement in the source (located approximately at a depth of 20 m) is a sign-variable function (*N*-wave) that satisfies Eq. (1.2.16):

$$f_0(\tau) = -\frac{4s}{3}\frac{\tanh[2(\tau-60)/3]}{\cosh^2[2(\tau-60)/3]}, \tag{1.3.11}$$

where s is a numerical coefficient with dimension $m^{4/3}$. In all following calculations it is assumed that $s = 1$.

The initial disturbance is split into two waves after some time. The right-going wave moves quickly offshore. Its amplitude decreases rapidly and it propagates out of the domain ($x \leq 500$ m) after 20 s. The amplitude of the left-going wave increases as it approaches the shore. The maximum amplitude occurs at the coastline. The solution experiences perfect reflection from the shore and propagates to the right with the amplitude decreasing afterwards.

Another instructive example (Fig. 1.3.2) is the propagation of an initial disturbance located entirely above the calm water level. Let us consider evolution of the wave system generated from a disturbance in the form of a solitary wave:

$$f_0(\tau) = s \cdot \mathrm{sech}^2[2(\tau-60)/3]. \tag{1.3.12}$$

An interesting feature here is the formation of a weak current between left-going and right-going pulses. The existence of a non-zero current follows from the behaviour of functions $\Phi_\pm(\xi)$. The magnitude of this current is very small, only a few per cent from the maximum flow velocities near the wave crests (Fig. 1.3.3).

Initially, only positive disturbances of the water surface are present in the system. As in the previous example, the right-going wave propagates soon out of the computational domain without qualitative changes in its shape. The sign of the water elevation caused by the left-going wave, however, is inverted in the process of reflection from the coastline. After this reflection, two right-going waves exist in the system, forming together a sign-variable disturbance as expected from Eq. (1.2.11).

It is straightforward to extend the above analysis to the case of waves propagating along an ambient current. The latter can be expressed via a non-zero initial velocity field. The procedure of finding the solution is then as follows. One of the functions, for instance f_+, can be expressed through the initial displacement (1.3.8)

$$f_+(\tau) = x^{1/3}\eta_0(x) - f_-(\tau). \tag{1.3.13}$$

For the other function, the following differential equation for f_- (or Φ_-) can be derived from (1.3.9):

$$-f_-(\tau)+\frac{1}{\tau}\Phi_-(\tau)=\Gamma(\tau) \quad \Gamma=\frac{h^{3/4}}{2\sqrt{g}}u_0(x)-\frac{h^{1/4}}{2}\eta_0(x)+\frac{1}{2\tau}\int h^{1/4}\eta_0 dx\,. \tag{1.3.14}$$

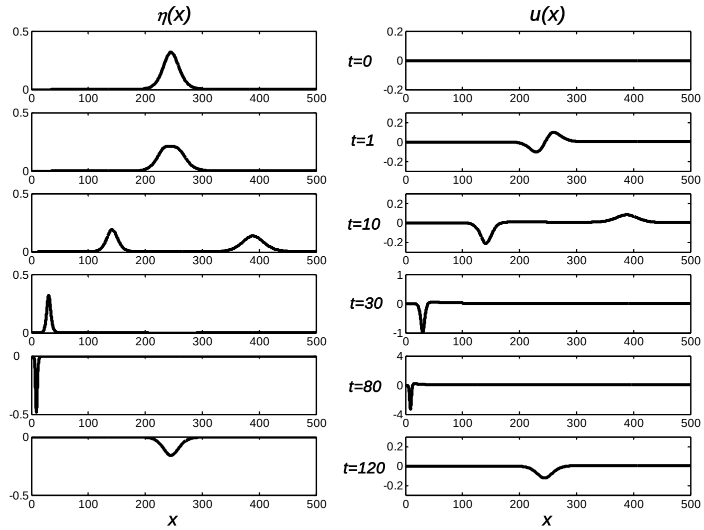

Fig. 1.3.2. Water displacement (left) and velocity (right) for the initial disturbance presented by Eq. (1.3.12)

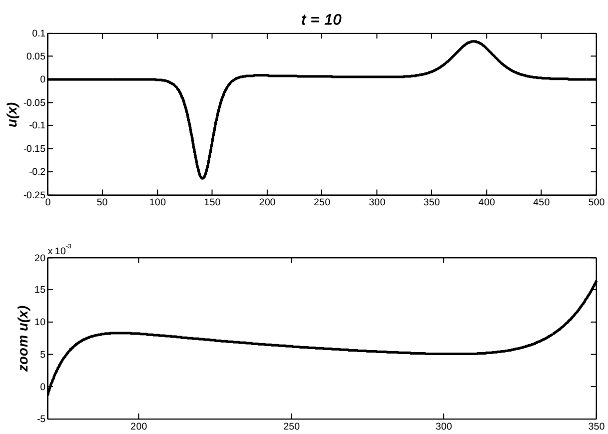

Fig. 1.3.3. Formation of space-variable current between two pulses in Fig. 1.3.2

This equation can be easily integrated to give

$$\Phi_{-}(\tau) = -\tau \int \frac{\Gamma(\zeta) d\zeta}{\zeta} . \qquad (1.3.15)$$

The effect of the initial velocity is manifested in an additional difference between the left-going (onshore) and right-going (offshore) waves.

1.4. Wave reflection from the shore

From the practical point of view the behaviour of the wave field on the shoreline ($x = 0$) is the most interesting. Details of process of wave reflection, accompanying amplitude amplification and potential runup have important applications in tsunami modelling, forecast and mitigation studies. Formally, the linear theory is not valid in the vicinity of the shoreline where the wave amplitude becomes comparable with the water depth. In the case of a plane beach of constant slope it has been demonstrated that the extreme runup characteristics can be calculated rigorously from the linear shallow-water theory even for the nonlinear problem (see Chapter 2 and Paper II). The approach used in these studies is applied to the case of a convex beach bottom profile (1.2.9).

If the wave approaches the beach from the infinity, the wave solution of Eq. (1.2.1), satisfying also the boundary condition at the shoreline (1.3.6), has the following form (see Eqs. (1.3.8) and (1.3.9)):

$$\eta(x,t) = \frac{1}{x^{1/3}} \left\{ f[t + \tau(x)] - f[t - \tau(x)] \right\} , \qquad (1.4.1)$$

$$u(x,t) = -\sqrt{\frac{g}{p}} \frac{1}{x} \left[f(t+\tau) + f(t-\tau) \right] + \frac{g}{3x^{4/3}} \left[\Phi(t+\tau) - \Phi(t-\tau) \right], \qquad (1.4.2)$$

where $f(t+\tau)$ is the shape of an incident wave approaching the shoreline $x = 0$ ($\tau = 0$). The vertical displacement of the water surface at $x = 0$ can be found from Eq. (1.4.1) exactly by using Taylor's series in the vicinity of $\tau = 0$:

$$R(t) = \eta(0,t) = \frac{6}{\sqrt{gp}} \frac{df(t - \tau_0)}{dt} , \qquad (1.4.3)$$

where τ_0 is the travel time from a fixed point $x = L$ (chosen far offshore) to the shore. Taking into account that the shape of the incident wave at the point $x = L$ is

$$\eta_{in}(t) = \frac{f(t)}{L^{1/3}} , \qquad (1.4.4)$$

Eq. (1.4.3) can be re-written as

$$R(t) = 2\tau_0 \frac{d\eta_{in}(t-\tau_0)}{dt}. \quad (1.4.5)$$

Thus, the amplitude of water level oscillations at the shoreline is proportional to the vertical velocity of water particles in the incident wave. If the incident wave has the form of a solitary crest, the water level on the shoreline experiences first runup, followed by rundown. The runup height is determined by the ratio of the travel time τ_0 to the wave period T. Therefore it is greater if the incident wave approaches from deeper waters. This feature suggests that beaches that have extensive convex slopes offshore may experience considerable amplification of waves compared to beaches with linearly increasing depth.

The maximum velocity of water particles in the vicinity of the shore $x = 0$ is unbounded and proportional to

$$u(x \to 0, t) \approx 2\sqrt{\frac{g}{p}}\frac{f(t)}{x}. \quad (1.4.6)$$

This feature may be interpreted as an implicit manifestation of wave breaking. However, wave breaking is not accounted for in the framework of Eq. (1.2.1). Although water velocity becomes infinitely large at the shoreline, the water discharge is bounded, because

$$h(x)u(x,t) \to 2\sqrt{gp}x^{1/3} f(t) \to 0. \quad (1.4.7)$$

The shore therefore plays a role of a vertical wall perfectly reflecting the wave energy.

The singularity of the water velocity in the vicinity of the shoreline can be excluded by a small variation of the bottom profile, more precisely, by variations of the face slope which is zero in a given geometry. However, the water level is not sensitive to bottom variations in the vicinity of the shoreline. That is why we do not study in detail characteristics of the velocity field at the shoreline.

To illustrate the processes in the vicinity of the coastline, time records of the water displacement during the runup of a sign-variable wave (*N*-wave) [Eq. (1.3.11)], computed numerically with the use of Eq. (1.4.1), are presented in Fig. 1.4.1 for selected points of the coastal slope. Far from the shoreline, the time series contain both incident and reflected waves (the latter having an inverted shape as discussed above). Wave amplification when the wave approaches the shore and the transformation of the wave shape at the shoreline are clearly seen in this figure. For this particular wave shape, the rundown amplitude significantly (approximately three times) exceeds the runup amplitude. According to Eq. (1.4.5), the maximum runup height is 5.7 m and rundown depth 17 m for the initial amplitude of 11 cm.

The wave amplitude on such a beach can be amplified by an order of magnitude and even more.

As an example, let us calculate the runup height analytically for the case where the incident wave is the soliton solution of the Korteweg–de Vries (KdV) equation:

$$\eta(t) = A\operatorname{sech}^2\left[\sqrt{\frac{3Ag}{4h^2}}t\right]. \tag{1.4.8}$$

The runup height induced by the approaching solitary wave is

$$R_{\max} = 4L\left(\frac{A}{h}\right)^{3/2}. \tag{1.4.9}$$

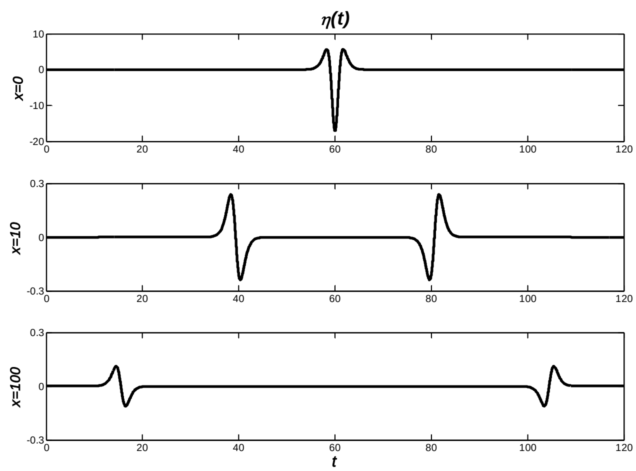

Fig. 1.4.1. Time series of the water surface of the wave system generated from the initial disturbance given by Eq. (1.3.12) at the shoreline and at two offshore points

If we introduce the mean slope of a beach $\alpha = h/L$, expression (1.4.9) can be rewritten as

$$R_{\max} = 4\frac{A}{\alpha}\sqrt{\frac{A}{h}} \sim A^{3/2}. \tag{1.4.10}$$

Comparison of this result with the asymptotic formula for the runup of a solitary wave on a plane beach (Synolakis, 1987)

$$R_{\max} = 2.8312 \frac{A}{\sqrt{\alpha}} \left(\frac{A}{h} \right)^{1/4} \sim A^{5/4} \tag{1.4.11}$$

suggests that the runup of solitary waves of moderate amplitudes on convex beaches may lead to considerably wider inundation than a similar process next to beaches of constant slope.

The runup of waves of arbitrary shape can be studied in a similar way. Recently it has been shown in the framework of the Carrier – Greenspan transformation for a plane beach that the runup height of asymmetric incident waves (the face slope of which exceeds the back slope) is higher than the runup of symmetric waves (Paper II). This feature may occur for beaches of various profiles. It is inherently evident from Eq. (1.4.5) for a convex beach.

1.5. Wave reflection from a zone of increasing depth

For engineering purposes it is important to establish what happens with the wave that once approached the coast from offshore, was then reflected from the shore and propagated back from offshore over a shallow area. It is well known that in the case of a step-like bottom profile the offshore-going wave may be re-reflected from the step and the wave energy may be trapped in the shallow area (Dean & Dalrymple, 1991). Analogous effects may also occur for the wave reflection from the border of the zone of increasing depth.

To analyse such effects, consider a situation when a wave moves in a channel of small but finite depth, which serves as a prolongation of a convex coastal slope. This situation mimics processes occurring in entrance channels of several ports or in a small river with a weak current. This can be done by considering the geometry of the following bottom relief in which the origin separates the shallow area of constant depth from the convex slope (Fig. 1.5.1):

$$h(x) = \begin{cases} h_0 & x < 0 \\ h_0 (1 + x/L)^{4/3} & x > 0 \end{cases}. \tag{1.5.1}$$

In this case the velocity field is bounded everywhere. The coastal slope is discontinuous at the origin. The presence of this inflection point gives rise to a specific problem of transmission of wave energy between different areas and reflection from this point.

Let us first consider the case where an incident sine wave approaches a convex coast from the zone of constant depth ($x < 0$). Following the classical theory of long wave reflection, the wave field in this zone is presented by the superposition of the incident and reflected waves:

$$\eta(x,t) = A_i \exp[i\omega(t - x/c_0)] + A_r \exp[i\omega(t + x/c_0)]. \tag{1.5.2}$$

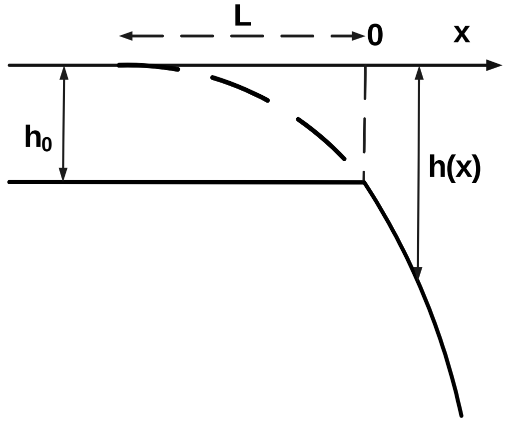

Fig. 1.5.1. Sketch of geometry

Here $c_0 = \sqrt{gh_0}$ is the long wave speed along the even bottom and A_i and A_r are the amplitudes of the incident and reflected waves, respectively. The monochromatic wave along the convex slope is described by Eq. (1.2.11) and has an amplitude A_0 at the point $x = 0$. At the inflection point the solutions expressed by Eqs. (1.5.2) and (1.2.11) must match each other in terms of the continuity of water level and total discharge. These boundary conditions allow calculating the relative amplitudes of the reflected and transmitted waves from the following expressions for the coefficients of reflection and transmission (Paper V):

$$\frac{A_r}{A_i} = -\frac{1}{1 + i\omega\tau}, \qquad \frac{A_0}{A_i} = \frac{i\omega\tau}{1 + i\omega\tau}, \tag{1.5.3}$$

where $\tau = 6L / c_0$. These amplitudes depend on the ratio of the wave period and the travel time to the zone of variable depth. As expected, for steep bottom slopes ($\omega\tau << 1$) the wave is almost completely reflected and experiences a phase shift of 180º. For gentle slopes ($\omega\tau >> 1$) the incident wave passes to the zone of variable depth almost without reflection.

Another important particular case is the reflection of a solitary wave propagating offshore. In this case Eq. (1.5.3) presents the operator form of the ordinary differential equation (that can be obtained from this equation by replacing $i\omega$ by d / dt):

$$\eta_r(t) + \tau \frac{d\eta_r}{dt} = -\eta_i(t). \tag{1.5.4}$$

This equation allows determination of the reflected wave in the vicinity of the inflection point if the incident wave at the same point is known. The details of dispersion-related transformation to differential equations in a general case are described in Whitham (1974). The reflected wave can be calculated as an integral:

$$\eta_r(t) = -\frac{\exp(-t/\tau)}{\tau}\int_0^t \eta_i(z)\exp(z/\tau)dz\,, \qquad (1.5.5)$$

whereas it is assumed that the reflected wave is absent before the incident wave approaches the inflection point. If the incident wave is a pulse of finite duration T ($0 < t < T$), then from Eq. (1.5.5) it follows that the reflected wave amplitude at the inflection point decreases exponentially after passing the incident wave $t > T$:

$$\eta_r(t > T) = -\frac{\exp(-t/\tau)}{\tau}\int_0^T \eta_i(z)\exp(z/\tau)dz\,. \qquad (1.5.6)$$

From Eq. (1.5.6) it follows that the solitary wave in the channel may entirely cross the convex slope and the inflection point without any loss of its energy. This happens for specific shapes of the incident wave and specific values of beach parameters, for which integral (1.5.6) is equal to zero. The analysis of these specific cases is out of the scope of the present thesis.

From Eq. (1.5.4) it follows that

$$\int_{-\infty}^{+\infty}\eta_r(t)dt = -\int_{-\infty}^{+\infty}\eta_i(t)dt\,. \qquad (1.5.7)$$

Thus, if the incident wave is a wave of elevation (pure crest), a wave of depression (pure trough) dominates in the reflected wave. This feature can be interpreted as a generalization of the above-discussed property of inversion of the shape in the process of reflection from the coastline.

As an example of the transformation of a wave pulse of limited duration we consider an incident sine pulse (Fig. 1.5.2)

$$\eta_i(t) = A\begin{cases}\sin(\Omega t) & 0 < \Omega t < \pi \\ 0 & \text{out of the interval}\end{cases}. \qquad (1.5.8)$$

An instructive feature of such a pulse is that it originally contains discontinuities of the surface slope that are gradually smoothed in the process of propagation. The profile of surface elevation in the reflected wave, computed from Eq. (1.5.5), is

$$\eta_r(t) = -A\frac{q}{1+q^2}\begin{cases} 0 & \Omega t < 0 \\ \exp(-q\Omega t) + q\sin(\Omega t) - \cos(\Omega t) & 0 < \Omega t < \pi \\ (1+\exp(q\pi))\exp(-q\Omega t) & \pi < \Omega t \end{cases}, \quad (1.5.9)$$

where

$$q = \frac{1}{\Omega\tau} = \frac{\tan\theta}{8\Omega}\sqrt{\frac{g}{h}}. \quad (1.5.10)$$

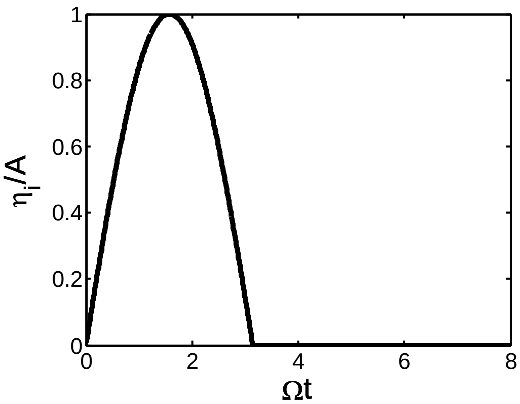

Fig. 1.5.2. Relative water surface elevation in the incident wave described by Eq. (1.5.8)

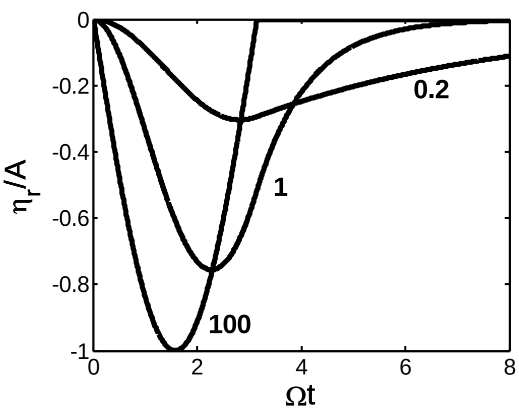

Fig. 1.5.3. The shape of reflected waves for various values of the parameter q

In accordance with the above analysis, the reflected wave is inverted for all values of the parameter q (Figs. 1.5.2 and 1.5.3). Its amplitude decreases and its tail gets gradually longer. The growth of the tail is more pronounced for gentle beaches. In the case of steep beaches the shape of the reflected wave is almost the same as for the incident wave but has opposite polarity.

Expressions (1.5.9) and (1.5.10) describe the shape of the reflected wave near the inflection point. It is straightforward to show, using Fourier superposition of the spectral components [Eq. (1.5.2)], that the reflected wave preserves its shape at all distances from the inflection point.

In the immediate region of the inflection point the transmitted wave can be found from the boundary condition of continuity of water displacement:

$$\eta_t(t) = \eta_i(t) + \eta_r(t). \tag{1.5.11}$$

Due to Eq. (1.5.7), condition (1.2.16) is satisfied automatically. This feature was expected for the travelling wave solution (see Chapter 2) and is confirmed here by Eqs. (1.5.11) and (1.5.7).

The oscillations of the water level in the immediate vicinity of the inflection point are of specific importance, because they can be the starting point of further description of the wave attack and runup with the use of more detailed models of the coastal zone. The time series of water surface at this point are presented in Fig. 1.5.4 for the incident sine pulse. The figure shows that a sign-variable wave is excited and propagates onshore after the inflection point. As expected, the amplitude of this wave is quite small in the case of steep convex beaches, yet almost full transmission may occur if the convex section of the beach has a moderate slope.

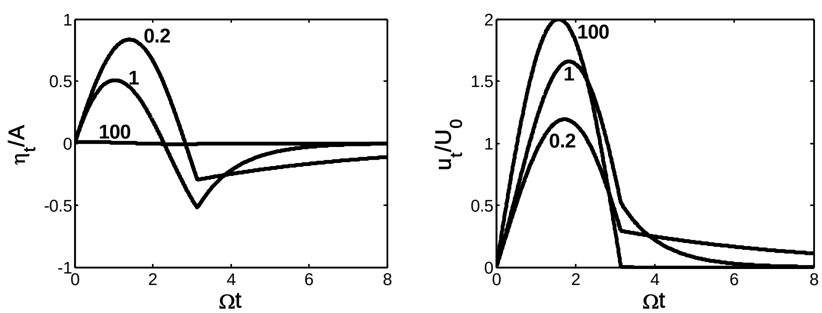

Fig. 1.5.4. The shape of the transmitted wave (left) and transmitted velocity (right) after the inflection point for various values of the parameter q

According to Eq. (1.2.12), the transmitted wave does not change its shape, but its amplitude and phase do change with the distance from the inflection point. The shape of the velocity field in the transmitted wave changes with the distance as well [see Eq. (1.2.15)]. In the immediate vicinity of the inflection point the velocity of wave particles can be found from the boundary condition of the continuity of discharge:

$$u_t(t) = \sqrt{\frac{g}{h_0}}\left[\eta_i(t) - \eta_r(t)\right]. \tag{1.5.12}$$

The time series of velocity are presented in Fig. 1.5.4 for the case of an incident sine pulse for several values of the parameter q. The velocity is always positive (as for the incident wave). The velocity pulse is, however, somewhat modified and contains an elongated tail, the effective duration of which is larger for gentle beaches. The shape of the velocity variations in a transmitted wave varies with distance according to Eq. (1.2.15) and not necessarily follows the shape of the water surface displacements. Nevertheless, far from the inflection point the first term in Eq. (1.2.15) dominates and the shape of the velocity variations matches the shape of surface displacements. These processes are illustrated in Fig. 1.5.4.

1.6. Concluding remarks

The above analysis of linear long wave dynamics in a basin of variable depth first confirms the intuitively clear opinion that exact travelling wave solutions of the variable-coefficient wave equation (1.2.1) exist for a very limited number of situations. In fact, such solutions only exist for a convex bottom, the water depth along which increases as $h(x) \sim x^{4/3}$. For this particular case a 1:1 transformation converts the general 1D wave equation into an analogous equation with constant coefficients. In other words, the analysed situation is the sole case in which the complex dynamics and reflections of long waves propagating over an uneven bottom can be fully described in terms of simple solutions for basins of constant depth. Notice that the other generic form of the wave equation (valid for water velocity) allows such solutions for another profile ($h(x) \sim x^4$) of the coastal slope (Didenkulova et al., 2008).

The obvious gain from the existence of such solutions it that quite complex wave phenomena can be analysed with the use of the large pool of results obtained for a much simpler framework. This similarity allows getting an important insight into how long travelling waves behave when approaching convex sections of the ocean coasts. While the majority of properties of wave propagation along a convex bottom mirrors those occurring in the basin of linearly varying depth, some interesting distinguishing features become evident; for example the shapes of water displacement and velocity in the travelling wave do not coincide. As expected, the general solution of the Cauchy problem to the wave equation in the case of the convex bottom profile in question is expressed through two travelling waves propagating in opposite directions. A zone of space-variable current generally exists between these two waves.

A deeply interesting feature is that shoaling and runup of certain wave classes on a beach with this sort of bottom relief may be considerably higher than for a beach with a linear profile and an equal mean slope. This property has been shown to hold for shallow-water KdV solitary waves (solitons) that are frequently used as a convenient model of tsunami waves. It is also shown that the shape of water oscillations at the shoreline is determined by the first derivative of the incident wave shape. As a result, if the incident wave has a steep front, the runup height will be higher.

Although the exact results of the above studies are valid for a limited class of bottom profiles, they are eventually approximately correct for a much wider class of basins with a convex bottom slope, or containing extensive sections of such slopes. An important aspect to be mentioned once more is that the performed analysis does not require slow variation of the basin depth and remains valid for quite large slopes. This property allows extensive use of the obtained solutions and results for developing practically usable models of, e.g., tsunami and freak waves and also opens new perspectives in developing the weakly nonlinear theory of water waves in a basin of variable depth.

2. Long wave runup on a plane beach

2.1. Introduction

The above studies allowed establishing certain important properties of long wave dynamics in the linear framework, however realistic description of processes occurring in the vicinity of the coastline is only possible with the use of more complicated, nonlinear models. Still, the results obtained in the linear approximation have a key role in such studies, because in many important applications the nonlinear problem can be (partially) reduced to a linear one. An example of extensive use of such an approach is the theory of nonlinear long wave runup.

The first rigorous solutions describing the nonlinear long wave runup on a plane beach were obtained by Carrier & Greenspan (1958). They reduced a nonlinear shallow-water system to a linear wave equation. Their approach was actively applied in tsunami studies (Pedersen & Gjevik, 1983; Synolakis, 1987; Mazova & Pelinovsky, 1992; Tinti & Tonini, 2005) and is used as the basis of the relevant studies described in this thesis. The key and novel moments of the developments of the analytical theory of long wave runup on a plane beach presented in this chapter are: (i) parameterization of basic formulas for extreme runup characteristics and (ii) runup analysis of asymmetric waves with a steep front. The basis of the former development is the weak dependence of several runup features on the initial wave shape (which is usually unknown in real sea conditions), provided the shape is symmetric with respect to the wave crest. The central message from the latter development is that waves with a steeper face slope penetrate inland over larger distances and with greater velocities than symmetric waves. The chapter reflects the results published in Papers II, III and IV.

An analytical model of wave runup, which is based on nonlinear shallow-water equations and is valid only for non-breaking waves, is described in Section 2.2. A method of reducing the nonlinear shallow-water system to the linear wave equation, suggested in the original paper of Carrier & Greenspan (1958), is briefly reproduced. The basic advantage of their approach, as shown below (Paper II), is the proof that extreme characteristics of the nonlinear runup process (runup and rundown amplitudes, extreme values of on- and offshore velocities, the wave breaking condition) can be found within a linear approximation. The nonlinear dynamics of the moving shoreline is described in Section 2.3 for the case of non-breaking waves. With the increase in the amplitude first the wave breaks on the shoreline. In this case the velocity of the shoreline has the shape of a shock wave, whereas the water displacement has a jump of the first derivative in the trough. A simplified criterion of wave breaking is obtained. The runup height of solitary waves of various shapes is analysed in Section 2.4. It is shown that with the use of a convenient definition of the "significant" wavelength, formulas for extreme runup characteristics become universal in the sense that their dependence on the incident wave shape is very weak. Such formulas can be used for engineering applications. The runup of asymmetric waves with a steep front is discussed in

Section 2.5. It is shown that such waves penetrate inland over larger distances and with greater velocities than symmetric waves.

2.2. Analytical model of wave runup

The main equations describing the processes analysed in this chapter are nonlinear shallow-water equations

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial H}{\partial x} = -g\alpha \,, \tag{2.2.1}$$

$$\frac{\partial H}{\partial t} + \frac{\partial}{\partial x}(Hu) = 0 \,, \tag{2.2.2}$$

where

$$H(x,t) = -\alpha x + \eta(x,t) \tag{2.2.3}$$

is the total water depth. Equations (2.2.1) and (2.2.2) represent a hyperbolic system of partial differential equations with constant coefficients. It allows using the hodograph transformation to reduce nonlinear equations to a linear system with variable coefficients. This transformation was first obtained by Carrier & Greenspan (1958) for water waves. The details of this transformation are given in Paper II.

According to this method, Eqs. (2.2.1) and (2.2.2) can be reduced to the linear wave equation

$$\frac{\partial^2 \Phi}{\partial \lambda^2} - \frac{\partial^2 \Phi}{\partial \sigma^2} - \frac{1}{\sigma}\frac{\partial \Phi}{\partial \sigma} = 0 \,, \tag{2.2.4}$$

and all variables are expressed through the wave function $\Phi(\lambda,\sigma)$:

$$t = \frac{1}{\alpha g}(\lambda - u), \quad x = \frac{1}{2\alpha g}\left(\frac{\partial \Phi}{\partial \lambda} - u^2 - \frac{\sigma^2}{2}\right), \tag{2.2.5}$$

$$\eta = \frac{1}{2g}\left(\frac{\partial \Phi}{\partial \lambda} - u^2\right), \qquad u = \frac{1}{\sigma}\frac{\partial \Phi}{\partial \sigma}. \tag{2.2.6}$$

Wave equation (2.2.4) is solved for the fixed semi-axis $\sigma \geq 0$ (the point $\sigma = 0$ corresponds to the moving shoreline). Notice that the initial shallow-water equations are generally solved in the domain with an unknown, moving boundary. The natural condition at the point $\sigma = 0$ is the boundedness of physical variables η and u. This follows from Eq. (2.2.6):

$$\frac{\partial \Phi}{\partial \sigma}(\lambda, \sigma = 0) = 0 . \qquad (2.2.7)$$

The general initial conditions for η and u

$$\eta(x,0) = \eta_0(x), \qquad u(x,0) = u_0(x), \qquad (2.2.8)$$

should be transformed into initial conditions for $\Phi(\lambda, \sigma)$. Thus, for example for $u_0(x) = 0$ (a popular presentation of the tsunami source in the framework of the piston model), the initial conditions are formulated for $\lambda = 0$ instead of $t = 0$:

$$\Phi(\sigma,0) = 0, \qquad \frac{\partial \Phi}{\partial \lambda}(\sigma,0) = \Phi_1(\sigma), \qquad (2.2.9)$$

where Φ_1 is parametrically defined by Eqs. (2.2.6).

Linear wave equation (2.2.4) defined on a semi-axis is well studied in mathematical physics and the corresponding Green's function can be written in the integral form. Several authors have used the Green's function approach in the runup problem (Carrier et al., 2003; Kanoglu & Synolakis, 2006). The transformation of the initial wave field $\eta_0(x)$ and $u_0(x)$ to the wave function $\Phi(\lambda, \sigma)$ and back to the water wave field $\eta(x,t)$ and $u(x,t)$ is described by implicit formulas (2.2.5) and (2.2.6). There are only a few examples when the solution of Eqs. (2.2.1) and (2.2.2) can be found explicitly (Carrier & Greenspan, 1958; Spielfogel, 1976; Pedersen & Gjevik, 1983). In practice, this transformation can be done numerically.

Waves usually approach the shore from the open sea where the depth is great and wave amplitudes are small compared to the depth. Therefore it is correct to assume that the incident wave is mostly (almost) linear (except for laboratory experiments, where the mechanically generated wave frequently has an amplitude comparable with the depth). Under this assumption the formulas of the Carrier – Greenspan transformation (2.2.6) can be simplified far from the coast:

$$t = \frac{1}{\alpha g}\lambda, \qquad x = -\frac{\sigma^2}{4\alpha g}, \qquad \eta = \frac{1}{2g}\frac{\partial \Phi}{\partial \lambda}. \qquad (2.2.10)$$

As a result, all formulas become explicit, and the transformation of initial conditions (2.2.8) from the physical space (x,t) to the space (λ, σ) is trivial:

$$\frac{\partial \Phi}{\partial \lambda}(\sigma,0) = 2g\eta_0[x(\sigma)], \qquad \Phi(\sigma,0) = \int \sigma u_0[x(\sigma)] d\sigma . \qquad (2.2.11)$$

Since initial conditions for wave equation (2.2.4) are fully determined for a general case of non-zero velocities, the general solution of the Cauchy problem can be obtained by applying the described procedure. It is the main advantage of the use of the linear approximation for solving a nonlinear problem.

Moreover, if we consider the linear shallow-water system

$$\frac{\partial u}{\partial t} + g\frac{\partial \eta}{\partial x} = 0, \qquad \frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}(-\alpha x u) = 0, \tag{2.2.12}$$

and apply the linearized Carrier-Greenspan transformation (2.2.5), (2.2.10), we reduce Eqs. (2.2.12) to wave equation (2.2.4). Equation (2.2.4) should be solved for the same initial conditions (2.2.11) and boundary condition (2.2.7) on a semi-axis. The most important conclusion from the presented line of reasoning is that the formal solutions $\Phi(\lambda,\sigma)$ and $\Phi_l(\lambda_l,\sigma_l)$ of the basic nonlinear and linear systems, respectively, are identical.

The physical meaning of variables and certain components of these solutions is of course different. First, the interpretation of the boundary point $\sigma = 0$ is different in nonlinear and linear cases. In the nonlinear problem the point $\sigma = 0$ corresponds to the moving shoreline: the total water depth (2.2.3) is equal to zero at $\sigma = 0$, while in the linear problem the same point corresponds to the unperturbed shoreline (still sea level): the unperturbed water depth $h(x)$ is equal to zero.

The dynamics of the moving shoreline in the nonlinear problem is described by the function $\Phi(\lambda,\sigma = 0)$. In particular, the maximum runup height is achieved when $u = 0$:

$$R = \max(\eta) = \frac{1}{2g}\max\frac{\partial \Phi}{\partial \lambda}(\lambda,\sigma = 0) \tag{2.2.13}$$

[see (2.2.6)]. The wave field at the shoreline ($x = 0$) in the linear problem is described by the function $\Phi_l(\lambda_l,\sigma_l)$. The maximum wave height at this point is

$$R_l = \max(\eta_l) = \frac{1}{2g}\max\frac{\partial \Phi_l}{\partial \lambda_l}(\lambda_l,\sigma_l = 0)\,. \tag{2.2.14}$$

At the same time, as we pointed above, if initial conditions are given far from the shoreline where the wave is linear, functions $\Phi(\lambda,\sigma)$ and $\Phi_l(\lambda_l,\sigma_l)$ are identical. It means that the maximum value of the runup height in the nonlinear theory is equal to the wave height on the unperturbed shoreline in the linear theory. Therefore, the maximum runup height can be found in the framework of the linear theory, a feature which is extremely important for engineering applications. This non-trivial conclusion is rigorously proved, for instance, in Pelinovsky & Mazova (1992); see also Paper II.

The same results can also be applied in order to find the maximum value of rundown depth and the onshore and offshore velocities of the moving shoreline. Thus, all important extreme characteristics of the runup process can be found in the framework of the linear shallow-water theory. It is the main advantage of the use of the approach originally developed by Carrier & Greenspan (1958).

As an example we consider the runup of a sine wave with frequency ω on a plane beach. The well-known bounded solution of the linear wave equation (2.2.4) is expressed through Bessel functions:

$$\eta(x,t) = R_0 J_0\left(\sqrt{\frac{4\omega^2\,|x|}{g\alpha}}\right)\cos(\omega t)\,, \tag{2.2.15}$$

where $J_0(z)$ is the Bessel function of zeroth order. Far from the shoreline the wave field can be presented asymptotically as the superposition of two sine waves of equal amplitude propagating in opposite directions:

$$\eta(x,t) = A(x)\left\{\sin\left[\omega(t-\tau)+\frac{\pi}{4}\right]+\sin\left[\omega(t+\tau)-\frac{\pi}{4}\right]\right\}, \tag{2.2.16}$$

$$A(x) = R_0\left(\frac{\alpha g}{16\pi^2\omega^2\,|x|}\right)^{1/4}, \qquad \tau(x) = \int\frac{dx}{\sqrt{gh(x)}}\,, \tag{2.2.17}$$

where $A(x)$ is the instantaneous wave amplitude and $\tau(x)$ is the propagation time of this wave over the distance x in a fluid of variable depth.

The ratio of the maximum amplitude R_0 of the approaching wave (with the wavelength λ_0 determined from the shallow-water dispersion relation $\omega = 2\pi\sqrt{gh(L)}/\lambda_0$) to the initial amplitude A_0 at the fixed point $|x| = L$ can be found from Eq. (2.2.17):

$$\frac{R_0}{A_0} = \left(\frac{16\pi^2\omega^2 L}{g\alpha}\right)^{1/4} = 2\pi\sqrt{\frac{2L}{\lambda_0}}\,. \tag{2.2.18}$$

The amplification factor R_0/A_0 in Eq. (2.2.18) is a generalization of the shoaling coefficient in the linear surface wave theory. As pointed above, it is the same in the nonlinear theory. This feature allows determining the extreme runup characteristics in both linear and nonlinear cases as soon as the initial wave amplitude and wavelength at a fixed offshore point $|x| = L$ are known.

Formally, solutions of wave equation (2.2.4) can be obtained for different depth profiles either numerically or analytically (as it was done in Chapter 1 for a special beach profile). An important limitation of the approach used in this chapter is that the equivalence between nonlinear and linear theories in calculating the extreme characteristics has been established only for a plane beach. This limitation is only substantial in the region where the wave is essentially nonlinear. Therefore the approach can still be used far offshore (where the wave is linear) where the bottom relief can have any shape, but in the vicinity of the shoreline the rigorous nonlinear results are only applicable if the shallow-water coastal region has an almost plane slope. A more realistic situation can thus be modelled by a combination of a flat bottom (roughly representing the wave propagation in the area where the wave is almost linear) and a plane beach (on which nonlinear effects may become substantial), presented in Fig. 2.2.1.

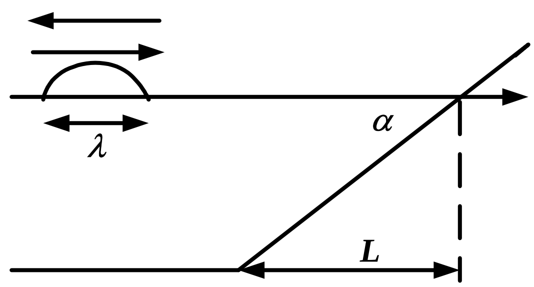

Fig. 2.2.1. Sketch of geometry

The wave field on such a beach is described by Eq. (2.2.15). On a flat bottom the solution of wave equation (2.2.4) is

$$\eta(x,t) = A_0 \exp[i\omega(t - x/c)] + A_r \exp[i\omega(t + x/c)], \qquad (2.2.19)$$

where A_0 and A_r are the amplitudes of the incident and reflected waves, and c is a long wave speed on a flat bottom. Matching solutions (2.2.15) and (2.2.19) requires the continuity of water level and velocity in the joint point. These conditions allow finding A_r (see Pelinovsky, 1982; Madsen & Fuhrman, 2008) and the runup height R:

$$\frac{R}{A_0} = \frac{2}{\sqrt{J_0^2(2kL) + J_1^2(2kL)}} . \qquad (2.2.20)$$

Here $k = \omega / c$ is the wave number of the incident wave. For large values of kL Eqs. (2.2.18) and (2.2.20) coincide (Fig. 2.2.2). If the beach width L tends to zero, the runup amplitude twice exceeds the incident wave amplitude.

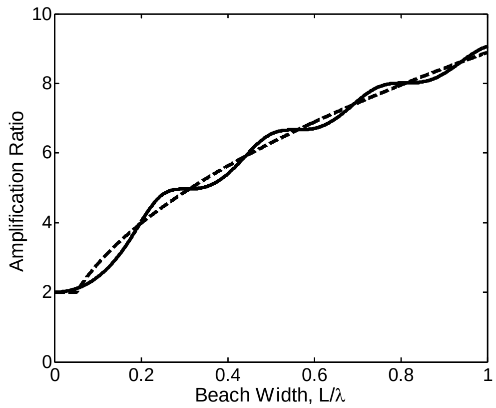

Fig. 2.2.2. Runup height versus beach width

2.3. Dynamics of the moving shoreline

As discussed above, the linear theory predicts extreme characteristics of the long wave runup. This theory can also be used for calculating "real" (nonlinear) dynamics of the moving shoreline. For instance, the velocity of the moving shoreline in the nonlinear theory can be derived from Eq. (2.2.6) for $\sigma = 0$:

$$u(\lambda) = \lambda - \alpha g t \, . \tag{2.3.1}$$

This equation implicitly determines the time dependence of the velocity of the moving shoreline and can be written as

$$u(t) = U\left(t + \frac{u}{\alpha g} \right). \tag{2.3.2}$$

The physical meaning of the function $U(t)$ is a "linear" velocity on the shoreline. Implicit formula (2.3.2) demonstrates that the "nonlinear" velocity of the moving shoreline can be obtained from the "linear" solution by a specific alteration of the time axis (so-called Riemann transformation of time). It is clear from Eq. (2.3.2) that extreme values of functions $u(t)$ and $U(t)$ coincide, which confirms the conclusion made in Section 2.2. Thus, the presence of nonlinearity only modifies velocity time series but does not influence the maximum velocity.

It is straightforward to find horizontal and vertical coordinates of the moving shoreline by integrating the "nonlinear" velocity of the moving shoreline:

$$x(t) = \int u(t)dt\,, \qquad z(t) = \frac{x(t)}{\alpha}\,. \tag{2.3.3}$$

The vertical displacement of the water level at the shoreline in the linear theory $Z(t) = \eta_l(t, x = 0)$ can be calculated from the solution of linear wave equation (2.2.4) with the use of traditional methods of mathematical physics or numerical modelling. It is related to the "linear" velocity as follows:

$$U(t) = \frac{1}{\alpha}\frac{dZ(t)}{dt}\,. \tag{2.3.4}$$

The "nonlinear" vertical displacement of the moving shoreline $r(t)$ can be obtained from Eqs. (2.2.6) and (2.3.2):

$$r(t) = \eta(t, \sigma = 0) = Z\left(t + \frac{u}{\alpha g}\right) - \frac{u^2}{2g}\,. \tag{2.3.5}$$

An important conclusion from Eq. (2.3.5) is that extremes of the vertical displacement (the runup and rundown heights) in the linear and nonlinear theories coincide, as expected from the discussion in Section 2.2. Therefore, the linear theory adequately describes the runup height, which is an extremely important characteristic of the action of long waves (tsunami, storm surges) on the shore.

The central outcome of the presented analysis is that the solution of the linear problem together with the Riemann transformation of time (two-step analysis) allows calculating the runup characteristics. This is much easier than the use of the complicated Carrier–Greenspan transformation. Such an approach was obviously first suggested in Pelinovsky & Mazova (1992) and then applied in several cases in Paper II, III and IV.

Another important outcome of the proposed approach is a simple definition of the first breaking condition of long waves on a beach. Very long waves with small amplitudes do not break at all. They just result in a slow rise in the water level resembling a surge-like flooding. With the increase in the wave amplitude, breaking appears seawards from the runup maximum. Depending on the wave amplitude and the bottom slope, breaking may occur relatively far offshore in the form of, e.g., plunging breakers.

The two-step approach described above allows exact determination of certain properties of waves with the limitation that the first wave breaking occurs precisely on the shoreline. The temporal derivative of the "nonlinear" velocity of the moving shoreline, calculated from Eq. (2.3.2),

$$\frac{du}{dt} = \frac{dU/_{dt}}{1 - \frac{dU/dt}{\alpha g}}, \tag{2.3.6}$$

tends to infinity when the denominator of the right-hand side of Eq. (2.3.6) approaches zero. As follows from the theory of hyperbolic equations, it leads to the so-called gradient catastrophe. This instant is usually identified as the start of the (plunging) breaking of long water waves, which implies the condition of the first wave breaking

$$Br = \frac{\max(dU/dt)}{\alpha g} = \frac{\max(d^2Z/dt^2)}{\alpha^2 g} = 1, \tag{2.3.7}$$

where the parameter Br has the meaning of the breaking criterion. Condition (2.3.7) has a simple physical interpretation: the wave breaks if the maximum acceleration of the shoreline $Z''\alpha^{-1}$ along the sloping beach exceeds the along-beach gravity component (αg). This interpretation, although convenient for illustration of the process, is figurative, because formally Z'' only presents the vertical acceleration of the shoreline in the linear theory, but the "nonlinear" acceleration du/dt actually tends to infinity at the breaking moment. Criterion (2.3.7) can be re-written with the use of the relation between dU/dt and $\partial\eta/\partial x$ from linear equation (2.2.12) on the shore $x = 0$:

$$Br = \frac{\max(\partial\eta/\partial x)}{\alpha} = 1. \tag{2.3.8}$$

The physical meaning of this notation is that the wave steepness should be equal to the bottom slope. In this case curves of the water level and the bottom profile do not cross. This form of the criterion is popular in oceanography where it was obtained heuristically; see for instance Massel (1989).

Thus, the nonlinear dynamics of the moving shoreline can be fully determined from Eqs. (2.3.2) and (2.3.5) by finding first the solution of the linear problem and calculating water displacement on the unperturbed shoreline. The breaking condition can be found in the same way. In the particular case of a sine wave runup on a plane beach the breaking parameter is

$$Br = \frac{\omega^2 R}{\alpha^2 g}, \tag{2.3.9}$$

where ω is the frequency of a sine wave. The maximum runup height of a non-breaking wave can be found from the breaking condition $Br = 1$:

$$R_{\max} = \frac{g\alpha^2 T^2}{4\pi^2}, \qquad (2.3.10)$$

where T is a wave period. This height depends on the bottom slope and the wave period (Fig. 2.3.1).

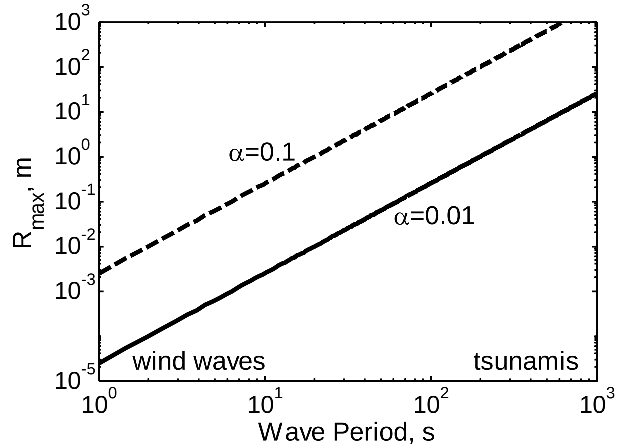

Fig. 2.3.1. The breaking criterion for long waves on a plane beach

Wind waves are relatively short and have a characteristic period of 10 s. In many cases they start to break if their amplitudes exceed 10 cm. That is why the process of the wind wave breaking on a beach is observed almost always. Tsunami waves have a characteristic period of 10 min and break if their heights exceed 10 m. As a result, we come to a paradoxical conclusion that huge tsunami waves may climb the beach without breaking, while small-amplitude wind waves almost break offshore. According to the observations by Pelinovsky (1982), approximately 75% of tsunamis approach the shore without breaking.

Detailed analysis of the nonlinear dynamics of the moving shoreline for the case where a monochromatic wave approaches the beach can be performed based on the Riemann transformation (2.3.2). It is convenient to use non-dimensional variables

$$u' = \alpha u / \omega R, \qquad U' = \alpha U / \omega R, \qquad t' = \omega t. \qquad (2.3.11)$$

In this case Eq. (2.3.2) becomes

$$u(t) = U(t + Br\, u), \qquad (2.3.12)$$

where Br is determined by Eq. (2.3.9) and primes are omitted for simplicity. The "linear" vertical displacement of water on the shoreline can be found from Eq. (2.3.4) by introducing non-dimensional variables:

$$Z(t) = \int U(t)dt \; . \qquad (2.3.13)$$

The “real”, nonlinear vertical displacement of the moving shoreline in non-dimensional variables is defined by Eq. (2.2.6):

$$r(t) = Z(t + Br\,u) - \frac{Br}{2}u^2(t) \; . \qquad (2.3.14)$$

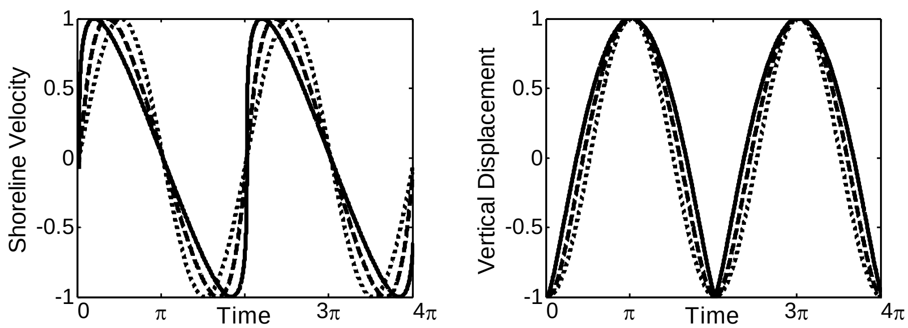

Fig. 2.3.2. Velocity and vertical displacement of the moving shoreline (dimensionless variables) when the sine wave approaches the coast for $Br = 0.2$ (dotted line), $Br = 0.5$ (dashed line) and $Br = 1$ (solid line)

Equations (2.3.12) and (2.3.14) give a parametric presentation of the moving shoreline for any shape of the incident wave. In particular, for a sine wave the parametric formulas are

$$t = \lambda - Br\cos\lambda \; , \qquad r = \sin\lambda - \frac{Br}{2}\cos^2\lambda \; . \qquad (2.3.15)$$

The dynamics of the moving shoreline under the influence of a sine wave (Fig. 2.3.2) can be computed from Eq. (2.3.15). If the wave amplitude is small enough ($Br << 1$), the shoreline changes in time as a sine function. If the wave amplitude increases and the parameter Br tends to 1, the water moves onshore faster and recedes to the sea more slowly. The first wave breaking with the increase in the amplitude occurs at the stage of the maximum rundown.

2.4. Runup of solitary waves

The above-described two-step procedure for calculating runup characteristics can be applied to all types of incident waves approaching the shore from the open sea, provided waves can be assumed as linear in a certain sea area. This assumption allows using the principle of linear superposition for offshore wave description. The first step of the above procedure – the (general) solution of the linear wave equation for runup characteristics – can be presented as a Fourier superposition of elementary solutions (2.2.15) with various frequencies and spectral amplitudes. The Fourier superposition is valid for both an incident wave field (its elementary solution for the fixed point $x = -L$ is given by the left term in Eq. (2.2.16)) and for vertical oscillations of the water level at the shoreline $x = 0$ (its elementary solution is given by Eq. (2.2.15)). An incident wave field given far from the shoreline (at $x = -L$) is

$$\eta(t) = \int_{-\infty}^{\infty} A(\omega)\exp(i\omega t)d\omega , \tag{2.4.1}$$

where the complex amplitude $A(-\omega) = A^*(\omega)$ provides a real part of the function $\eta(t)$. A similar formula can be written for the vertical oscillations of the water level on the shoreline:

$$R(t) = \int_{-\infty}^{\infty} \sqrt{|\omega|}A(\omega)\exp\left[i\left(\omega t + \frac{\pi}{4}\operatorname{sign}(\omega)\right)\right]d\omega . \tag{2.4.2}$$

If a solitary wave approaches the coast, the incident wave is characterized by two parameters: wave amplitude (height) H_0 and wave duration T_0. The third parameter determining the wave phase can be eliminated by an appropriate time shift. It is convenient to present the incident wave, its runup displacement and velocity in a non-dimensional form:

$$\eta(t) = H_0 f(t/T_0) = H_0 \int_{-\infty}^{\infty} B(\Omega)\exp(i\Omega\zeta)d\Omega , \tag{2.4.3}$$

$$R(t) = \sqrt{\frac{4\pi L}{\lambda_0}} H_0 \int_{-\infty}^{\infty} \sqrt{|\Omega|}B(\Omega)\exp\left[i\left(\Omega\zeta + \frac{\pi}{4}\operatorname{sign}(\Omega)\right)\right]d\Omega , \tag{2.4.4}$$

$$U(t) = \sqrt{\frac{4\pi L}{\lambda_0}} \frac{H_0}{\alpha T_0} \int_{-\infty}^{\infty} \sqrt{|\Omega|^3}B(\Omega)\exp\left[i\left(\Omega\zeta + \frac{\pi}{4}\operatorname{sign}(\Omega)\right)\right]d\Omega , \tag{2.4.5}$$

where

$$\zeta = t/T_0 , \quad \Omega = \omega T_0 , \quad \lambda_0 = \sqrt{gh_0}T_0 , \quad B(\Omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(\zeta)\exp(-i\Omega\zeta)d\zeta . \tag{2.4.6}$$

As the breaking parameter is determined by the linear solution, it can also be presented by a Fourier superposition:

$$Br = \frac{H_0 L}{\alpha \lambda_0^2} \sqrt{\frac{L}{\lambda_0}} \max\left\{ \int_{-\infty}^{\infty} |\Omega|^{5/2} B(\Omega) \exp\left[i\left(\Omega\zeta + \frac{\pi}{4}\mathrm{sign}(\Omega) \right) \right] d\Omega \right\}. \quad (2.4.7)$$

The convergence of integrals (2.4.3)–(2.4.7) gives a limitation on the smoothness of the incident wave shape. From Eq. (2.4.7) it follows that a non-breaking runup may only occur if the spectrum $B(\omega)$ decays faster than $\omega^{-7/2}$. Notice that a sufficient criterion is that the wave amplitude is bounded. Therefore, disturbances of finite duration (usually used in numerical simulations) cannot contain points of inflection and generally should be quite smooth if one aims at the simulation of non-breaking runup processes. For example, the runup of a single sine-shaped wave crest cannot be considered as non-breaking, because its spectrum decay is proportional to ω^{-2} and such a wave has to break for all values of its amplitude. In fact, only the third- and higher-order derivatives of the pulse shape may have discontinuities for a non-breaking runup.

Formulas (2.4.3)–(2.4.7) can be re-written in a more convenient form:

$$R_{\max} = R_0 p_R, \qquad U_{\max} = \frac{R_0}{\alpha T_0} p_U, \qquad Br = \frac{H_0 L}{\alpha \lambda_0^2} \sqrt{\frac{L}{\lambda_0}} p_b, \quad (2.4.8)$$

where

$$R_0 = \sqrt{\frac{4\pi L}{\lambda_0}} H_0, \qquad \lambda_0 = \sqrt{g h_0} T_0. \quad (2.4.9)$$

Here λ_0 is the wavelength, h_0 is the water depth at the point $|x| = L$ and numerical coefficients p_R, p_U and p_b characterize the incident wave shape.

In many cases it is not clear how to determine the wavelength λ_0 and the duration T_0 of a solitary pulse. In particular, most of the wave shapes (represented by analytical functions that are continuous by all derivatives) are non-zero everywhere at $-\infty < t < \infty$. There is obvious ambiguity in the definition of their wavelength (or duration) that can be interpreted as their width at any level of elevation, or by the value of an appropriate integral (see for details Papers III and IV). Even if the incident wave has finite duration, its definition through the length of its carrier not necessarily characterizes the wave shape properly.

A convenient definition of the wavelength is the extension (spatial or temporal) of the wave profile elevation exceeding the 2/3 level of the maximum wave height. This choice is inspired by the definition of the significant wave height and length in physical oceanography and ocean engineering. Detailed justification of this choice is presented in Papers III and IV. For symmetric solitary waves, the significant wave duration and significant wavelength are

$$T_s = 2T_0 f^{-1}\left(\frac{2}{3}\right), \qquad \lambda_s = \sqrt{gh_0}\,T_s . \tag{2.4.10}$$

Here f^{-1} is the inverse function of f. Thus formulas (2.4.8) for the maximum displacement, velocity of the moving shoreline and the breaking parameter can be expressed as

$$R_{\max} = \mu_R^+ H_0 \sqrt{\frac{L}{\lambda_s}}, \quad U_{\max} = \mu_U^+ \frac{H_0 L}{\lambda_s}\sqrt{\frac{g}{\alpha\lambda_s}}, \quad Br = \mu_{Br}\frac{H_0 L}{\alpha\lambda_s^{\,2}}\sqrt{\frac{L}{\lambda_s}}, \tag{2.4.11}$$

where coefficients μ_R^+, μ_U^+ and μ_{Br} (called form factors) depend on the wave shape:

$$\mu_R^+ = 2\sqrt{2\pi f^{-1}\left(\frac{2}{3}\right)}\,p_R, \quad \mu_U^+ = 4\sqrt{2\pi\left[f^{-1}\left(\frac{2}{3}\right)\right]^3}\,p_U,$$
$$\mu_{Br} = 8\sqrt{2\pi\left[f^{-1}\left(\frac{2}{3}\right)\right]^5}\,p_{Br}. \tag{2.4.12}$$

Analogous formulas can be derived for the rundown depth and velocity (see Paper IV).

A remarkable property of this choice of significant wave duration is that the difference in solitary wave shapes has a fairly small effect on runup characteristics. This feature will be discussed in more detail below. The analytical expressions for the maximum runup characteristics (runup height, rundown depth, runup and rundown velocities, and breaking parameter) become universal and depend on the height and duration of the incoming onshore wave only.

This statement is proved by considering a variety of solitary pulses of different shape (Papers III and IV). Let us first consider incident wave crests having the shape of various "powers" of a sine pulse:

$$f(\zeta) = \cos^n(\pi\zeta), \qquad n = 3,4,5\ldots, \tag{2.4.13}$$

which are defined on the segment (– 1/2, 1/2). Their shapes have certain similarity, but their wave characteristics (mean water displacement, energy and wave duration on various levels) differ considerably (Fig. 2.4.1). The functions representing such impulses have different smoothness: their nth-order derivatives are discontinuous at their ends.

Form factors for runup and rundown heights (μ_R^+ and μ_R^-), runup and rundown velocities (μ_U^+ and μ_U^-) and the breaking parameter μ_{Br}, calculated for sine power pulses (2.4.13) with the use of the definition of the characteristic wavelength λ_s at the 2/3 level of the maximum height, are presented in Figs. 2.4.2 and 2.4.3. Their values for the maximum wave runup $\mu_R^+ = 3.61 \cdot (1 \pm 0.02)$ and rundown $\mu_R^- = 1.78 \cdot (1 \pm 0.28)$ have a fairly limited variation in terms of means and root-mean-square deviations (Table 2.4.1).

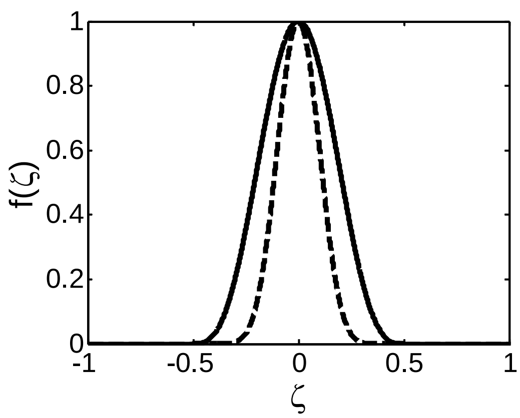

Fig. 2.4.1. Family of sine power pulses (2.4.13): solid line $n = 3$ and dashed line $n = 10$

First of all, it is significant that the runup height is higher than the rundown depth. This feature is observed for all sets of positive impulses. The form factor for the maximum wave runup is almost independent of the power n, showing that the influence of the initial wave shape on the extreme runup characteristics can be made fairly small by an appropriate choice of the characteristic wavelength. The above choice of the (significant) wavelength reduces the variation of the form factor for the sine power pulses to a notably small value, about 2%.

The deepest rundown is more affected by the wave shape: the relevant form factor varies by up to 28%. This feature can be explained by the presence of a complex field of motions in the rundown phase. A positive wave first executes runup and only later rundown. Therefore runup is predominantly governed by the incident wave dynamics, while rundown occurs under the influence of a set of distributed wave reflections and re-reflections from the slope and consequently is more sensitive to wave shape variations.

Similar analysis is applied to maximum runup and rundown velocities of the moving shoreline in Paper IV. The calculated form factors μ_U^+ and μ_U^- for the maximum runup and rundown velocities are presented in Fig. 2.4.2. The maximum values for the rundown velocity are always greater than for the runup velocity for initial unidirectional impulses. The form factor for the rundown velocity $\mu_U^- = 6.98 \cdot (1 \pm 0.01)$ is almost constant for all values of n (root-mean-square deviation is 1%), whereas that of the runup velocity $\mu_U^+ = 4.65 \cdot (1 \pm 0.30)$ changes in a wider range (±30%); see Table 2.4.1.

Variations of the form factor for the breaking parameter are also weak (see Fig. 2.4.3, triangles). The relevant form factors $\mu_{Br} = 13.37 \cdot (1 \pm 0.10)$ can be considered a constant with reasonable accuracy (Table 2.4.1).

Thus, form factors for the most important parameters such as runup height, rundown velocity, and to some extent for the breaking parameter, are universal and weakly depend on the particular shape of the sine power impulse. Variations of form factors for rundown height and runup velocity are more significant (about 30%), but they can also be neglected for express engineering estimates.

Similar analysis is performed for the family of solitary waves, described by the expression

$$f(\zeta) = \operatorname{sech}^n(4\zeta), \quad n = 1,2,3\ldots . \tag{2.4.14}$$

These impulses are unlimited in space, whereas they exponentially decay for large values of $|\zeta|$. The case $n = 2$ corresponds to the well-known soliton solution of the Korteweg–de Vries (KdV) equation, which is frequently used as a generic example of shallow-water solitary waves. The runup of the KdV solitons on a beach of constant slope was studied previously by Synolakis (1987). In our notation, Synolakis obtained that

$$\frac{R_{max}}{H_0} = 2.8312\sqrt{\frac{L}{h_0}}\left(\frac{H_0}{h_0}\right)^{1/4} . \tag{2.4.15}$$

The significant length of the soliton is easily calculated from the well-known analytical expression for a soliton in a constant-depth basin

$$\eta(x) = H_0 \operatorname{sech}^2\left(\sqrt{\frac{3H_0}{4h_0}}\frac{x}{h_0}\right) \tag{2.4.16}$$

and has the explicit form:

$$\lambda_s = 4\operatorname{sech}^{-1}\left(\sqrt{\frac{2}{3}}\right) h_0 \sqrt{\frac{h_0}{3H_0}} , \tag{2.4.17}$$

where $\mathrm{sech}^{-1}(z)$ is the inverse function of $\mathrm{sech}(z)$. Substituting this expression into the right-hand side of Eq. (2.4.15), we obtain:

$$\frac{R_{\max}}{H_0} = 3.4913\sqrt{\frac{L}{\lambda_s}} . \qquad (2.4.18)$$

Numerical calculations lead to the same value of the form factor, $\mu_R^+ = 3.4913$ at $n = 2$. This example indicates that the theory of soliton runup on a beach, which leads to a nonlinear relation between the runup height and the soliton amplitude, is consistent with a general theory of the runup of solitary waves on a beach and represents a special case.

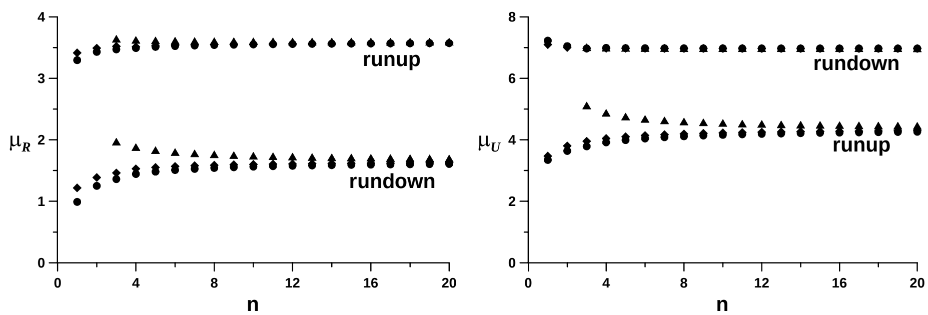

Fig. 2.4.2. Calculated form factors for the maximum runup (μ_R^+) and rundown (μ_R^-) heights and runup (μ_U^+) and rundown (μ_U^-) velocities for sine power pulses (triangles), soliton-like (diamonds) and Lorentz-like (circles) impulses

Similar results are obtained for solitary ridges of a Lorentz-like shape with algebraic decay

$$f(\zeta) = \frac{1}{\left[1 + (4\zeta)^2\right]^n}, \qquad n = 1,2,3\ldots . \qquad (2.4.19)$$

The maximum runup and rundown characteristics of a solitary wave on a beach (Table 2.4.1, Figs. 2.4.2 and 2.4.3) virtually do not depend on the form of the incident wave if the wave duration is appropriately defined. It is especially evident for the runup height and rundown velocity; their variations for all given classes of wave shapes do not exceed 8%.

Based on the analysis in Paper IV, the following semi-empirical formulas for express estimations of the runup and rundown characteristics of long waves on a beach are recommended:

$$R_+ = 3.5 H_0 \sqrt{\frac{L}{\lambda_s}}, \qquad R_- = 1.5 H_0 \sqrt{\frac{L}{\lambda_s}}, \qquad (2.4.20)$$

$$U_+ = 4.5 \frac{H_0 L}{\lambda_s} \sqrt{\frac{g}{\alpha \lambda_s}}, \quad U_- = 7 \frac{H_0 L}{\lambda_s} \sqrt{\frac{g}{\alpha \lambda_s}}, \quad Br = 13 \frac{H_0 L}{\alpha \lambda_s^{\ 2}} \sqrt{\frac{L}{\lambda_s}}, \quad (2.4.21)$$

where R_+ and R_- are the maximum values of runup and rundown heights, and U_+ and U_- are the maximum values of runup and rundown velocities.

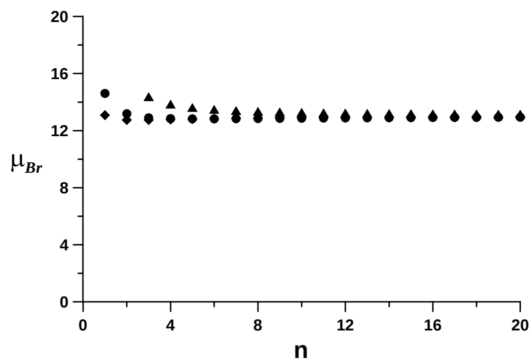

Fig. 2.4.3. Calculated form factors for the breaking parameter μ_{Br} for sine power pulses (triangles), soliton-like (diamonds) and Lorentz-like (circles) impulses

Table 2.4.1. Calculated form factors for different wave shapes

μ	Sine power		Soliton power		Lorentz pulse power	
	Mean	Normalized standard deviation	Mean	Normalized standard deviation	Mean	Normalized standard deviation
μ_R^+	3.61	0.02	3.55	0.05	3.53	0.08
μ_R^-	1.78	0.28	1.56	0.28	1.51	0.44
μ_U^+	4.65	0.3	4.15	0.22	4.07	0.26
μ_U^-	6.98	0.01	6.98	0.02	6.99	0.04
μ_{Br}	13.37	0.1	12.90	0.03	12.99	0.13

As indicated above, extreme runup characteristics can be found within the linear theory. For calculating the real dynamics of the moving shoreline, the nonlinear theory based on transformations (2.3.2) and (2.3.5) should be used. The computed shoreline velocity and vertical displacement for the incident KdV soliton are shown in Fig. 2.4.4. As for a sine wave, the first breaking occurs on the stage of maximum rundown, which takes place after the flooding induced by a long wave runup on a beach.

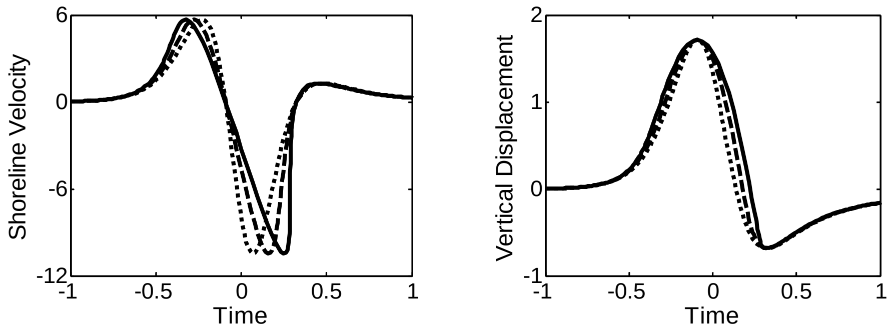

Fig. 2.4.4. Velocity and vertical displacement of the moving shoreline (dimensionless variables) for the incoming KdV soliton; the breaking parameter $Br = 0$ (dotted line), $Br = 0.5$ (dashed line) and $Br = 1$ (solid line)

2.5. Runup of asymmetric periodic waves

Asymmetric waves, the front slope steepness of which exceeds the back slope steepness, constitute another class of incident wave shapes of specific interest. Such waves are rather often observed in a coastal zone (Fig. 2.5.1).

Such a wave shape is intrinsically formed in the process of realistic, nonlinear propagation of initially symmetric (incl. sine) waves in the ocean, even if the water depth is constant (Paper II). The wave shape gradually deforms due to the difference in the speed of the crest and trough, which is always present in a nonlinear system. The wave steepness therefore increases when it propagates.

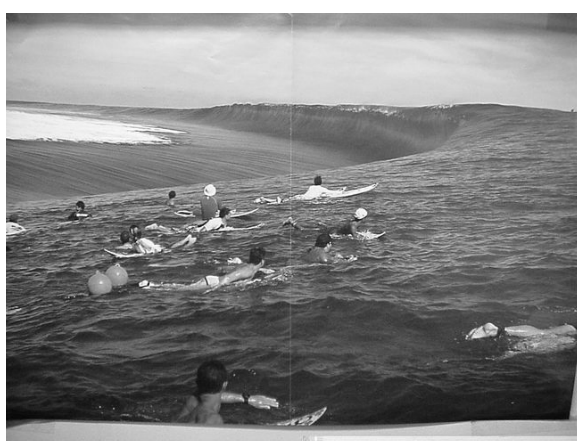

Fig. 2.5.1. Asymmetric wave

The theoretical model for the runup of periodic, asymmetric waves on a beach is similar to a solitary wave runup, and the basic formulas are the Fourier series instead of Fourier integrals in Eqs. (2.4.3)–(2.4.7). In particular, the incident wave is presented as

$$\eta(t) = H_0 f(\omega t) = H_0 \operatorname{Re}\sum_{n=1} B_n \exp(in\zeta)\,, \tag{2.5.1}$$

where

$$\zeta = \omega t\,, \qquad B_n = \frac{1}{2\pi}\int_0^{2\pi} f(\zeta)\exp(-in\zeta)d\zeta\,, \tag{2.5.2}$$

and Re means a real part of the complex sum in Eq. (2.5.1).

Similarly to Section 2.4, the linear "runup" wave field can be written in non-dimensional form:

$$R(t) = \sqrt{\frac{4\pi L}{\lambda_0}} H_0 \operatorname{Re}\sum_{n=1} \sqrt{n} B_n \exp\left[i\left(n\zeta + \frac{\pi}{4}\right)\right], \tag{2.5.3}$$

$$U(t) = \sqrt{\frac{4\pi L}{\lambda_0}} \frac{H_0}{\alpha T_0} \operatorname{Re}\sum_{n=1} n^{3/2} B_n \exp\left[i\left(in\zeta + \frac{\pi}{4}\right)\right]. \tag{2.5.4}$$

Here

$$\lambda_0 = \sqrt{gh_0}T_0 = 2\pi\sqrt{gh_0}/\omega \tag{2.5.5}$$

is the wavelength of the incident wave at the point $|x| = L$ with a water depth h_0. The breaking parameter can also be presented through the Fourier series:

$$Br = \frac{H_0 L}{\alpha {\lambda_0}^2}\sqrt{\frac{L}{\lambda_0}} \max \operatorname{Re}\sum_{n=1} n^{5/2} B_n \exp\left[i\left(n\zeta + \frac{\pi}{4}\right)\right]. \tag{2.5.6}$$

It is natural to request that the series in Eq. (2.5.6) converges. This is only true if spectral amplitudes B_n of the incident wave decrease with number n increasing faster than $n^{-7/2}$. In other words, the shape of the incident wave should be smooth enough.

Several periodic incident wave shapes have been described in literature. Synolakis (1988) considered the runup height of a cnoidal wave – the steady-state solution of the Korteweg–de Vries equation

$$\eta(x,t) = H\left\{cn^2\left[2K\left(\frac{x}{\lambda} + \frac{t}{T}\right)\middle| m\right] - < cn^2 >\right\}. \tag{2.5.7}$$

Here cn is the elliptic Jacobi function, λ and T are the wavelength and the period of a periodic wave, K is the complete elliptic integral of the first kind, m is an elliptic parameter and $< cn^2 >$ is a mean value for the whole period. The cnoidal wave (2.5.7) is a monochromatic (cosine) wave for $m \to 0$. In the other limit $m \to 1$ it is transformed to a KdV soliton.

All extreme runup characteristics of a cnoidal wave can be computed by calculating its Fourier spectrum (Synolakis, 1988). The runup height of the cnoidal wave is greater than for a sine wave, and therefore, the contribution of high harmonics (obertones) is important.

The runup of a periodic nonlinear asymmetric (deformed) wave is studied in Paper II. The considered geometry is a flat bottom joined with a plane beach (Fig. 2.2.1). The spectral amplitudes can be expressed through the face-slope steepness

$$B_n(s) = \frac{2B}{n(1 - s_0 / s)} J_n\left(n\left[1 - \frac{s_0}{s}\right]\right), \tag{2.5.8}$$

where s_0 is the initial steepness (in the case in question it is the steepness of a sine wave) and B is the non-dimensional amplitude of the incident (sine) wave in the open sea. The analysis in Paper II demonstrates that the distribution of spectral amplitudes becomes universal for waves with steep fronts (Fig. 2.5.2).

Runup characteristics of such asymmetric (deformed) waves on a plane beach are studied within the above-described approach in Paper II. The runup properties are appreciably different as compared to symmetric waves. Non-dimensional runup and rundown amplitudes (normalized by R_0) turn out to be functions of the wave steepness (Fig. 2.5.3). The rundown depth weakly depends on the wave steepness: it differs from the rundown caused by a sine wave by no more than 30%. Therefore Eq. (2.3.13) can still be used for the express evaluation of its magnitude. However, the runup height significantly depends on the wave steepness. It tends to infinity for a shock wave within a given model of non-breaking waves. The realistic runup is limited by the wave breaking.

Thus the wave asymmetry is the most important parameter of the runup process. An asymmetric wave with a steep front penetrates inland over a much larger distance than a symmetric wave of the same height and length. This result partially explains why tsunami waves with a steep front (for example, the 2004 tsunami in the Indian Ocean) penetrate extremely deep inland.

A similar analysis for the extreme characteristics of the shoreline velocity shows that the runup velocities of asymmetric waves may considerably exceed the rundown velocities (Fig. 2.5.3). The “nonlinear” time history of the water level and velocity of the moving shoreline for different values of the breaking parameter Br shows that time records of asymmetric waves are substantially asymmetric even when the wave amplitude is small (Fig. 2.5.4).

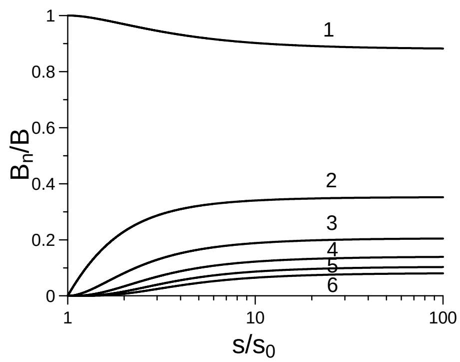

Fig. 2.5.2. Amplitudes of high harmonics versus face-slope steepness. Adapted from Paper II

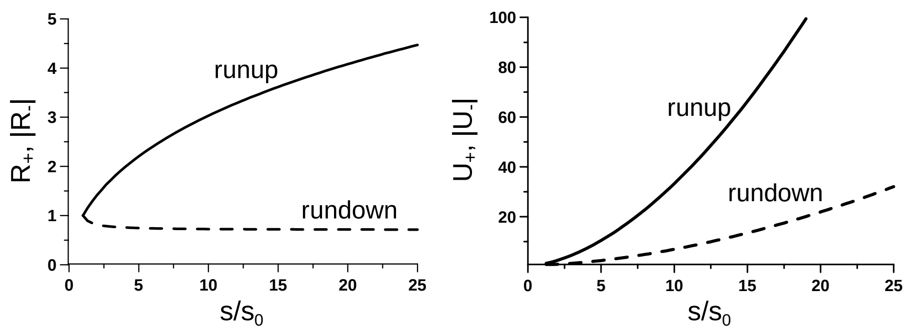

Fig. 2.5.3. Dependence of extreme runup (R_+) and rundown (R_-) heights and runup (U_+) and rundown (U_-) velocities (dimensionless variables) on the wave steepness

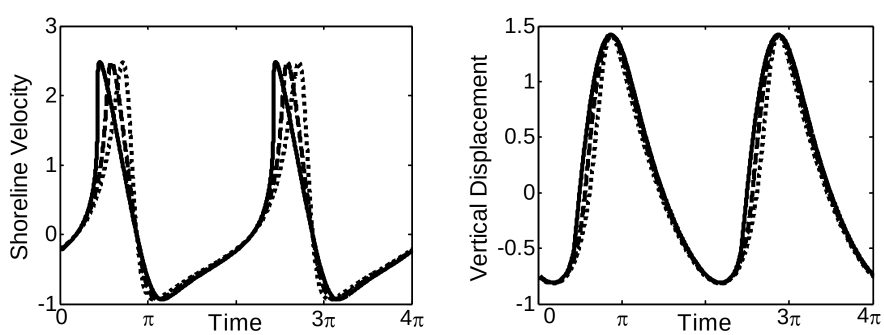

Fig. 2.5.4. Velocity and vertical displacement of the moving shoreline for a nonlinear deformed wave ($s / s_0 = 2$) in non-dimensional variables; the breaking parameter $Br = 0$ (dotted line), $Br = 0.5$ (dashed line) and $Br = 1$ (solid line)

The runup height and velocity are higher than rundown depth and velocity even for very low-amplitude but asymmetric waves. The runup process of a very asymmetric wave ($s = 10s_0$) causes extremely strong flow moving inland during a short time (Fig. 2.5.5). The runup height is considerably higher than the rundown depth. Such intense flows can be identified on many photos of the 2004 tsunami in the Indian Ocean. Very steep waves, however, break relatively fast.

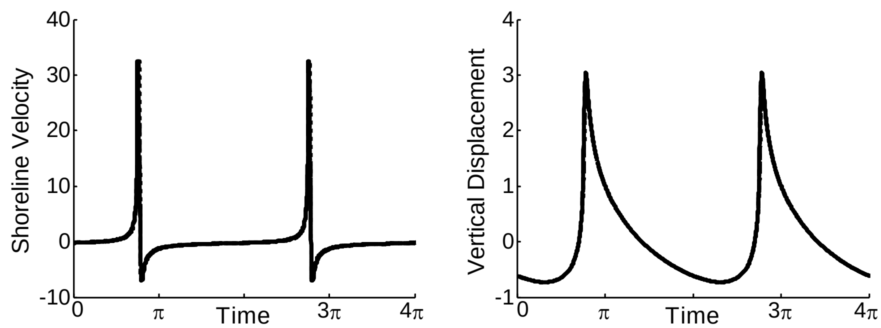

Fig. 2.5.5. Velocity and vertical displacement of the moving shoreline for a nonlinear deformed wave ($s/s_0 = 10$) in non-dimensional variables; the breaking parameter $Br = 0$ (dotted line), $Br = 0.5$ (dashed line) and $Br = 1$ (solid line)

2.6. Concluding remarks

The long wave runup on a plane beach is described analytically in the framework of rigorous solutions of the nonlinear shallow-water theory. Such solutions can be obtained with the use of the hodograph transformation of the nonlinear shallow-water system to the linear wave equation (Carrier & Greenspan, 1958). A two-step algorithm for computing the nonlinear dynamics of the moving shoreline is developed. This approach allows calculation of extreme characteristics of the runup process (runup and rundown amplitudes, extreme values of on- and offshore velocities and the wave breaking condition) within the linear approximation, although the realistic dynamics of the moving shoreline is described by the nonlinear theory.

This approach also allows considering properties of breaking waves if the first breaking of the wave occurs on the shoreline. In this case the velocity of the shoreline has the shape of a shock wave and the water displacement has a jump of the first derivative in the trough. The criterion for such wave breaking is obtained in a form that is convenient for the use in practice.

The runup of solitary waves of different shapes (various powers of a sine crest, soliton and Lorentz pulses) is analysed within the same approach. It is shown that the use of the significant wavelength (defined as the extent of the elevation exceeding 2/3 of the maximum elevation) leads to universal formulas of appreciable accuracy for extreme runup characteristics, the dependence of which on the particular incident wave shape becomes very weak. Such formulas can be used for engineering estimates.

Analysis of the runup process of asymmetric waves has revealed that such waves penetrate inland over larger distances and with greater velocities than symmetric waves of the same height and length.

3. Freak waves in coastal areas

3.1. Introduction

Descriptions of unusually high waves appearing on the sea surface for a short time (freak, rogue or killer waves) have long been considered as part of marine folklore. Recently, however a number of instrumental registrations have appeared, which have made the community pay attention to this problem and reconsider the known observations of freak waves (some of them are presented in Mallory, 1976; Torum & Gudmestad, 1990; Olagnon & Athanassoulis, 2001; Kharif & Pelinovsky, 2003; MaxWave, 2003). Nowadays it has become clear that such waves may have played a crucial role in many accidents that have led to ship damages and losses of lives (Lavrenov & Porubov, 2006; Toffoli et al., 2006).

Rogue waves in the open sea may be detected by altimeters installed on offshore platforms or deployed buoys, or via synthetic aperture radar (SAR) image processing. These data are trustworthy and may be used for accurate analysis. Similar events are also observed nearshore. These are seldom registered by e.g. tide gauges and have mostly been reported by eyewitnesses. Observations of such events become more frequent, and they broaden the area of possible freak wave occurrence.

Usually freak events occurring onshore result in a short-time sudden flooding of the coast, or strong impact upon the steep bank or coastal structures. Some descriptions of these accidents are given in the above-mentioned reviews and Rabinovich & Monserrat (1998) and Dean & Dalrymple (2002). Some accidents are explained as “meteorological tsunamis”, but similar phenomena may be caused by a much larger class of water motions (see, for instance, theoretical study of edge waves by Kurkin & Pelinovsky (2002).

In this chapter nearshore and onshore freak wave events are analysed on the basis of the results published in Papers I and VI. Descriptions of freak waves that occurred onshore in 2005 are presented in Section 3.2. These accidents are related to unexpected wave impact upon the coast and coastal engineering structures or with sudden intensive flooding of the coast. Runup of irregular waves, including freak waves, modelled as superposition of Fourier harmonics with random phases, is studied in the framework of the nonlinear shallow-water theory (Section 3.3). The possibility of the appearance of freak waves on a beach is analysed in Section 3.4. The distribution functions of runup characteristics are computed for a case where an incident wave represents an irregular sea state with a Gaussian spectrum. The asymptotic behaviour of probability functions in the range of large amplitudes for estimation of freak wave formation in the nearshore is studied. It is shown that the average runup height of waves with a wide spectrum is higher than that of waves with a narrow spectrum.

3.2. Observations of freak wave events

Numerous descriptions of dangerous events that occurred in the nearshore or in the coastal area include six accidents nearshore that took place in 2005. These can be interpreted as true freak events. The reasons for limiting the analysis to this set are described in detail in Paper I. The basic distinguishing feature is that, according to the information about significant wave height in the nearby offshore area (NOAA data of satellite observations) the impact of such waves was much larger than expected. Distinction is to some extent involuntary, because no quantitative measurement of wave impacts has been used in literature to select nearshore freak wave events.

Fig. 3.2.1. A wave over 9 m high washed two people off the breakwater in Kalk Bay on 26 August. Photo by Mr Philip Massie

26 August, Kalk Bay. A wave washed two people off the breakwater in Kalk Bay (South Africa). Both were rescued, although one received serious head injuries (Hunter, 2005). The wave height was over 9 m (Fig. 3.2.1). The offshore significant wave height near South Africa coast was up to 4.5 m (Live Access Server, 2006). A similar case was registered at the same place on 21 April 1996, when three people were washed off; only one survived that time (Hunter, 2005). On 22 July 2006 a 60-year-old man was swept off Kalk Bay harbour (Ndenze, 2006).

16 October, Maracas Beach. Panic arose at Maracas Beach (Trinidad Island, the Antilles), when a series of towering waves, many over 7 m high according to eyewitnesses, sent sunbathers, vendors and lifeguards running for their lives (Fig. 3.2.2), taking everyone by surprise around 14:15 local time. The waves raced past the shoreline onto vending stalls, crossed the roadway and flooded the car park and bake and shark vendors' stalls on the northern side of the main thoroughfare (Paper I). Refrigerators, stoves and gas tanks stood in knee-high water inside vendor stalls. The swells, which began pounding the North Coast shoreline around 11:00, continued late into the evening. There were reports that pirogues at Las Cuevas, Blanchisseuse and La Fillette were destroyed by the swells (Stapleton, 2005). The offshore significant wave height for Maracas Bay was about 1.5 m (Live Access Server, 2006).

Fig. 3.2.2. The waves at Maracas Beach on 16 October (Stapleton, 2005)

11 November, Port Orford. A wave swept three people into the Pacific Ocean, killing two and injuring one in Port Orford (Southern Oregon, USA) (The Seattle Times, 2005). No reliable estimates of the wave are available, but one of the three persons was reported to be elderly (72 years old). This most probably implies that the rogue wave was exceptionally high. The offshore significant wave height on that day was up to 3.8 m.

The observation data show wide distribution of freak wave events in the coastal zone. During the time of writing, one such event occurred at the western coast of Korea near Kunsan at Boryeong, 200 km southwest of Seoul on 4 May 2008. An unusually high wave of 4–5 m swept many people (tourists and fishermen) from a bulwark and nine people died. According to the first reports, the event occurred without any prior notice of increasing wave heights in the region.

More or less regular occurrence of such waves at different coasts suggests that a sub-population of (possibly nonlinear) waves may exist, similar to the set of freak waves in deep ocean or in shallow areas, which exert extremely large runup. Thus, developing adequate theoretical and prognostic models is necessary.

3.3. Runup of irregular waves

The analytical model described in Chapter 2 can be applied to describe the runup of irregular long waves as well. As mentioned above, it is rather difficult to calculate all the wave characteristics, because the Carrier–Greenspan transformation is implicit. The key benefit of the method used in Chapter 2 is that calculations of the extreme runup characteristics can be made with the use of the linear approach. In this case we need to find extremes of the Fourier series

$$\eta(t, x=0) = \int \left(\frac{16\pi^2 h}{g\alpha^2} \right)^{1/4} \omega^{1/2} A(\omega) \exp\left[i\left(\omega(t-\tau) + \phi(\omega) + \frac{\pi}{4} \right) \right] d\omega , \quad (3.3.1)$$

$$u(t, x=0) = \frac{1}{\alpha} \int \left(\frac{16\pi^2 h}{g\alpha^2} \right)^{1/4} \omega^{3/2} A(\omega) \exp\left[i\left(\omega(t-\tau) + \phi(\omega) + \frac{3\pi}{4} \right) \right] d\omega, \quad (3.3.2)$$

where A and ϕ are spectral amplitudes and phases, ω is the basic frequency of the incident wave

$$\eta(t, x=L) = \int A(\omega) \exp[i(\omega t + \phi(\omega))] d\omega, \quad (3.3.3.)$$

and τ is the travel time to the coast. Notice that series (3.3.1) and (3.3.2) can be used to calculate positive and negative runup amplitudes but they cannot be applied for many other purposes; for example, to calculate moments and distribution functions of the water displacement at the shoreline.

The ensemble of realizations of incident wave fields with random phases ϕ is taken for a numerical simulation of irregular waves. For this purpose we discretize Fourier series (3.3.1)–(3.3.3) and use real functions. As a result, equations for the shape of the incoming wave, water surface displacement and velocity of the shoreline in non-dimensional variables can be rewritten as

$$\bar{\eta}(t, x=L) = \sum_{n=1}^{N} A_n \cos(\omega_n t + \phi_n), \quad (3.3.4)$$

$$\bar{\eta}(t, x=0) = \sum_{n=1}^{N} \sqrt{\omega_n} A_n \cos\left(\omega_n t + \phi_n + \frac{\pi}{4} \right), \quad (3.3.5)$$

$$\bar{u}(t, x=0) = \sum_{n=1}^{N} \omega_n^{3/2} A_n \cos\left(\omega_n t + \phi_n + \frac{3\pi}{4} \right), \quad (3.3.6)$$

where $A_n = \sqrt{2S(\omega_n)\Delta\omega}$ are calculated with the use of the frequency spectrum of the incoming wave field $S(\omega)$, $\Delta\omega = 2\pi / T$ is the sampling rate, T is the size of time domain and $\omega_n = n\Delta\omega$. It is assumed that random spectral phases ϕ_n are distributed uniformly within the interval $(0, 2\pi)$.

First, let us consider a random wave field with Gaussian statistics, where the frequency spectrum $S(\omega)$ of the incoming field is

$$S(\omega) = Q \exp\left[-\frac{(\omega - \omega_0)^2}{2l^2} \right], \quad (3.3.7)$$

with the central frequency ω_0 and the spectrum width l. The constant Q in Eq. (3.3.7) can be found from the condition

$$\sigma^2 = 2\int_0^\infty S(\omega)d\omega\,; \tag{3.3.8}$$

then

$$Q = \frac{\sigma^2}{\sqrt{2\pi}l\,\mathrm{erfc}(-\omega_0/\sqrt{2}l)}\,, \tag{3.3.9}$$

where

$$\mathrm{erfc}(z) = \frac{2}{\sqrt{\pi}}\int_z^\infty \exp(-t^2)dt \tag{3.3.10}$$

is a complementary error function. In this case the frequency spectra for the shoreline displacement $S_r(\omega)$ and the shoreline velocity $S_u(\omega)$ are

$$S_r(\omega) = \frac{4\pi L\omega}{c}Q\exp\left[-\frac{(\omega-\omega_0)^2}{2l^2}\right], \tag{3.3.11}$$

$$S_u(\omega) = \frac{4\pi L\omega^3}{c\alpha^2}Q\exp\left[-\frac{(\omega-\omega_0)^2}{2l^2}\right]. \tag{3.3.12}$$

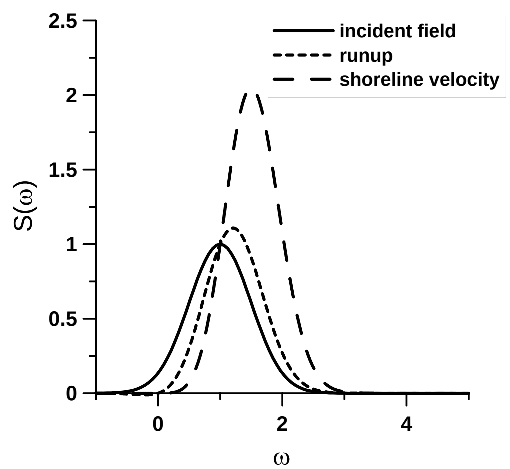

Fig. 3.3.1. Incident field, runup and shoreline velocity spectra for $l = 0.5$

All these spectra in non-dimensional variables for $l = 0.5$ are shown in Fig. 3.3.1. It is obvious that spectra for the shoreline displacement $S_r(\omega)$ and the shoreline velocity $S_u(\omega)$ are asymmetric and shifted to the high-frequency area. Distribution functions for maximum amplitudes (positive and negative) of the wave field, defined as a maximum (minimum) between two zero points, are important for applications.

3.4. Distribution functions of runup characteristics

Detailed calculations of the distribution functions of runup amplitudes are given in Paper VI. The Fourier series of $N = 512$ harmonics with the sampling rate of $\Delta\omega = 0.01$ are used. The spectrum width l varies from 0.1 to 0.7. The statistical characteristics are obtained with the use of ensemble averaging over 500 realizations.

The occurrence probability of the wave with an amplitude A for a Gaussian narrow-band process can be described by the Rayleigh distribution (Massel, 1996):

$$P(A) = \exp\left(-2A^2\right), \tag{3.4.1}$$

where A is the wave amplitude normalized on the significant amplitude A_s. The latter is defined as $A_s \approx 2\sigma$. For the numerical estimation of positive (negative) amplitude distribution the statistical "frequency" F (the ratio of the number of waves *m* with a fixed amplitude *a* to the total number of waves),

$$F = \frac{m}{N}, \tag{3.4.2}$$

and the statistical distribution function of amplitudes (equivalently, the frequency of occurrence of waves with the amplitude *A* larger than *a*),

$$P(a) = F(A > a), \tag{3.4.3}$$

are calculated. As expected due to the applicability of expressions for extreme characteristics based on the linear approach, for the narrow-band incident wave field ($l = 0.1$) the distribution functions of the runup characteristics are described by the Rayleigh distribution. If the spectrum of incident waves is wider ($l = 0.7$), the asymmetry of the displacement and velocity spectra increases, but nevertheless the distribution functions of the maximum shoreline displacement (Fig. 3.4.1) and the maximum shoreline velocity (Fig. 3.4.2) differ insignificantly from the Rayleigh distribution.

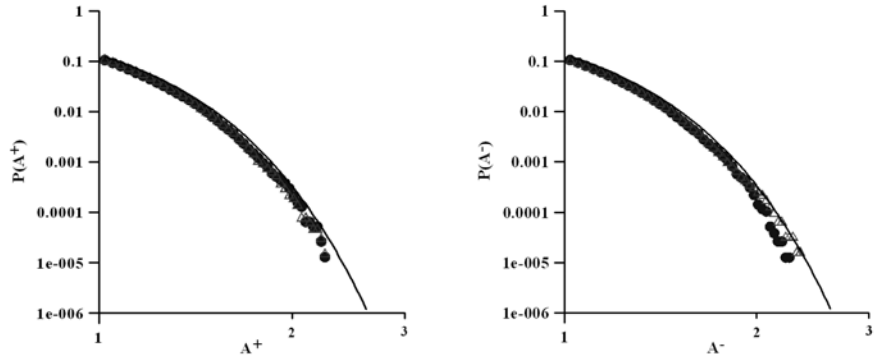

Fig. 3.4.1. Distribution functions of maximum positive (left) and negative (right) amplitudes for incident wave (triangles) and shoreline displacement (circles) with the incident wave spectrum width $l = 0.7$. The solid line corresponds to the Rayleigh distribution

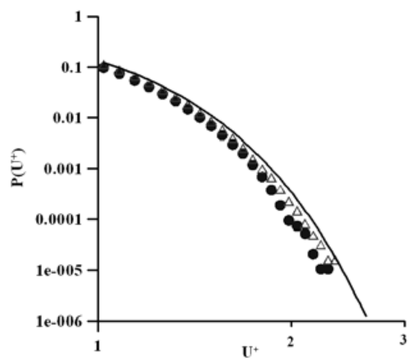

Fig. 3.4.2. Distribution functions of maximum velocities for incident wave (triangles) and shoreline displacement (circles) with the incident wave spectrum width $l = 0.7$. The solid line corresponds to the Rayleigh distribution

The spectral and probability distributions of the wave field serve as a basis for calculation of the runup characteristics on a beach. The (significant) runup height of the wave on a beach is

$$R_s = \sqrt{\frac{4\pi\omega_0 L}{c}} A_s F\left(\frac{\omega_0}{l}\right) = 2\pi\sqrt{\frac{2L}{\lambda}} A_s F\left(\frac{\omega_0}{l}\right), \tag{3.4.4}$$

where the function

$$F(z)=\sqrt{1+\frac{\exp(-z^2/2)}{\sqrt{\pi/2}z\,\mathrm{erfc}(-z/\sqrt{2})}} \quad (3.4.5)$$

describes the influence of the width of the incident wave spectrum (Fig. 3.4.3). This function tends to one ($F=1$) for the narrow-band process ($l \ll \omega_0$) and the significant runup height of the resulting wave field (which is equivalent to a monochromatic wave) can be described by the formula for the runup of a sine wave. The significant runup height increases with the increase in the spectrum width, especially when $l > \omega_0$. Thus, the Gaussian approximation in the problem of wave runup on a beach works not only for the case of $l \ll \omega_0$, but also for $l < \omega_0$, when the distribution function differs from the Gaussian.

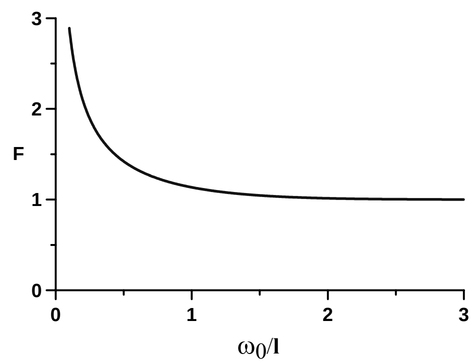

Fig. 3.4.3. Function of the influence of the incident wave spectrum width on the runup height of the wave

The performed analysis used the wave field presented as the superposition of independent spectral components. Such an approach is widely employed to describe random water waves (Massel, 1996). However, the wave field in shallow-water contains many coherent wave components, and the idea of presenting it as an ensemble of solitary waves with randomly distributed properties is very popular (see, for instance, Brocchini & Gentile, 2001). The runup of a solitary wave on a plane beach is well studied (Synolakis, 1987) and the runup amplitude can be expressed through the amplitude of the incoming soliton [Eq. (1.4.11)]. In fact, Eq. (1.4.11) can be derived from Eq. (3.3.1) by taking into account the relation between the soliton amplitude and duration.

If the wave field consists of random solitons that are well separated from each other, the runup of each individual soliton represents an independent process. The (joint) distribution function of runup amplitudes can be found analytically if the distribution functions of individual solitons are known. Assuming for simplicity the Rayleigh distribution for soliton amplitudes and using Eq. (1.4.11), the exceedance frequency of the runup amplitude is

$$P(R) = \exp\left[-0.378\alpha^{4/5} \frac{(R/h)^{8/5}}{(A/h)^2}\right]. \tag{3.4.6}$$

Expression (3.4.6) suggests that the probability of the appearance of large waves on the coast is high, provided the wave field is dominated by well-separated solitons. More detailed estimates of statistical runup characteristics of realistic ensembles of solitons are found in Brocchini & Gentile (2001).

So, the modification of the incident wave field during the wave runup process on a plane beach leads to an increase in the probability of large-amplitude waves. This result suggests that freak (sneaker) wave phenomena may be more common in the immediate vicinity of the seaward border of the surf zone than in the open sea.

3.5. Concluding remarks

The observation data of freak wave events that occurred in 2005 on the shore suggest that in certain cases freak events become evident as unusual flooding on a beach or as huge slash on breakwaters. These data have revealed a wide range of magnitude and extension of rogue waves in the nearshore and on the shore, and the necessity of developing adequate theories to describe and forecast rogue waves in the context of coastal engineering.

Freak wave events connected with specific features of long wave propagation are considered in the framework of the nonlinear shallow-water theory, with emphasize on the distribution functions of the extreme wave characteristics (displacement and velocity), caused by a wave coming from the open sea. A modelled (Gaussian) spectrum of the incident wave field is used for numerical simulations of the runup characteristics. It is shown that variations of the distribution functions for the maximum shoreline displacement and shoreline velocity are small for narrow-band processes. For this case the significant runup height of the wave can be described by the formula for the runup of a sine wave. For wide-band processes the significant runup height is notably larger. This effect may be one of the reasons why freak (sneaker) wave phenomena are relatively frequently observed.

Conclusions

Summary of the results

Long wave dynamics in the coastal zone is studied for two realistic bottom profiles. In the case of a convex profile described by $h(x) \sim x^{4/3}$, the general solution of the Cauchy problem is obtained within the linear shallow-water theory. The wave system consists of two travelling waves propagating in opposite directions, whereas generally a zone of a weak current is formed between these two waves. The long wave runup on such beaches is analysed. It is shown that the runup height on this kind of bottom profile is significantly larger than the runup height of waves of the same height and length along a plane beach. The study of the reflection and transmission of waves from a zone of increasing depth, described by the same profile $h(x) \sim x^{4/3}$, shows that a transmitted wave always has a sign-variable shape.

In the case of a beach of a constant slope the problem of the long wave runup is discussed in the framework of the nonlinear shallow-water theory. A key finding of this study is that the definition of the wavelength for symmetric solitary incident waves at a 2/3 level of the maximum height is optimal for express prediction of their runup properties. The maximum runup and drawdown characteristics of a solitary wave on a beach virtually do not depend on the shape of the incident wave if the wave duration is appropriately defined. It is especially evident for the runup height and drawdown velocity; their variations for all given classes of wave shapes do not exceed 8%.

The wave steepness is shown to have very strong influence on the runup characteristics of long waves. Although established in the framework of the analytical theory of nonlinear shallow-water waves, this finding is supported by numerous evidence of unexpectedly large extent of coastal flooding caused by several tsunamis. Among waves of a fixed amplitude and period (length), the steepest wave penetrates inland over the largest distance.

The data on the freak wave events that occurred in 2005 on the shore show that some events are manifested as unusual flooding on a beach. A model to explain these data is developed in terms of the theory of runup of irregular wave fields. Distribution functions of the extreme wave characteristics are computed in the framework of the nonlinear shallow-water theory. Variations of the distribution functions for extreme runup heights and shoreline velocity were proved to be moderate for narrow-band processes, for which the significant runup height can be appropriately described by the formula for the runup of a sine wave. For wide-band processes the significant runup height is considerably larger. Thus, the presence of waves with different periods may lead to an increase in the probability of the appearance of large-amplitude (freak) waves.

Main conclusions proposed to defend

1. The existence and uniqueness of linear travelling wave solutions above a convex bottom profile $h(x) \sim x^{4/3}$ is proved. The technique of the reduction of the relevant equation to the wave equation with constant coefficients allows solving the Cauchy problem and studying long wave dynamics above a variable bottom.
2. The wave system consists of two travelling waves propagating in opposite directions, whereas generally a zone of weak current is formed between these two waves.
3. Wave reflection and runup on the convex beach is studied within a linear approximation. It is shown that the runup height along the convex profile is considerably larger than for beaches with a linear slope.
4. Wave propagation and transformation between a shallow area of small but finite depth and a region with a convex coastal profile is studied within a linear model. Analysis of the wave reflection from and transmission to the zone of the increasing depth described by the convex profile shows that the transmitted wave always has a sign-variable shape.
5. Long wave runup on a plane beach is investigated analytically in the framework of rigorous solutions of the nonlinear shallow-water theory. A two-step algorithm for computing the nonlinear dynamics of the moving shoreline is developed.
6. The use of the "significant" wavelength allows a simple but efficient and reasonably accurate parameterization of design formulas for extreme runup characteristics. Detailed analysis is performed for sine-power, soliton-like, Gauss-like and Lorentz-like pulses. The resulting formulas are suitable for engineering estimates.
7. The runup characteristics of asymmetric waves are calculated. Such waves penetrate inland over larger distances and with greater velocities than symmetric waves of the same height and length. This can explain the observed large inland penetration of tsunami waves.
8. Distribution functions of the extreme runup characteristics caused by irregular wave fields are studied in the framework of the nonlinear shallow-water theory. The variations of distribution functions for the maximum shoreline displacement and shoreline velocity are small for narrow-band processes. For very narrow-band wave fields the significant runup height coincides with the runup height of a sine wave.
9. For a wide-band random process the significant runup height is considerably larger; therefore the presence of waves with different periods may lead to an increase in the probability of the appearance of large-amplitude (freak) waves in shallow-water.
10. Data on freak and sneaker wave events that occurred on the shore in 2005 are collected and analysed. These events are manifested as unusual flooding on the beach or a huge slash on breakwaters.

Recommendations for further work

Wave transformation above a convex bottom profile is studied here in the linear approximation. When the wave approaches the shore, its amplitude becomes larger and nonlinear effects should be taken into account. A comprehensive description of phenomena occurring during such processes is the major challenge for further work in this direction. First of all, a weakly nonlinear theory can be developed with the use of various asymptotic methods. It allows studying the nonlinear energy exchange between spectral harmonics and in many cases gives an insight into more complex phenomena occurring in the processes of nonlinear transformation and interaction of waves. In parallel, numerical methods should be applied to the analysis of the runup problem for getting quantitative information of the runup characteristics and breaking criterion.

The description of the runup of irregular waves is not completely investigated even in case of the problem of long wave runup on a plane beach. The distribution functions of extreme runup heights have been computed, but statistical moments of the runup field have not been analysed yet. These results can be obtained numerically.

Freak waves occurring in the immediate vicinity of the shoreline also require further study. The results obtained in this thesis are only the first step towards classification and quantification of such events. Relevant observations should be systematically collected, catalogued, analysed and modelled for developing methods for forecasting rogue waves.

It is also important for future analysis to study wave dynamics above realistic beaches that usually contain several sections with different types of bottom profile. Quite frequently realistic beaches are composed of planar, convex and concave profiles. The progress here could be achieved through application of numerical and asymptotic methods, and using *in situ* instrumental or experimental data. Such an experiment is already planned in Tallinn Bay for 2008–2010 in the framework of the European economic area (EEA) project "Shoaling and runup of long waves generated by high-speed ferries". The project addresses the problem of preventing and mitigation of natural coastal hazards associated with long wave dynamics by using fast ferries' wakes as a dynamically similar input, allowing modelling and measurements of the shoaling and runup properties of extreme large-scale oceanic waves in well-controlled, safe conditions. Tallinn Bay, a semi-enclosed body of water that hosts extremely intense fast ferry traffic and provides calm conditions during part of the high season, is a suitable natural laboratory for such studies.

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Papers constituting the thesis

I. **Didenkulova, I., Slunyaev, A., Pelinovsky, E., Kharif, Ch.** Freak waves in 2005. – Natural Hazards and Earth System Sciences 6; 2006, p. 1007–1015.

II. **Didenkulova, I., Pelinovsky, E., Soomere, T., Zahibo, N.** Runup of nonlinear asymmetric waves on a plane beach. – In book: Tsunami & Nonlinear Waves (Ed: Anjan Kundu). Springer, 2007, p. 175–190.

III. **Didenkulova, I., Kurkin, A., Pelinovsky, E.** Run-up of solitary waves on slopes with different profiles. – Izvestiya, Atmospheric and Oceanic Physics 43 /3; 2007, p. 384–390.

IV. **Didenkulova, I., Pelinovsky, E.** Runup of long waves on a beach: influence of the initial wave shape. – Oceanology 48 /1; 2008, p. 1–6.

V. **Didenkulova, I., Pelinovsky, E., Zahibo, N.** Long wave reflection from "non-reflecting" bottom profile. – Fluid Dynamics 43 /4; 2008, p. 101–107.

VI. **Didenkulova, I., Pelinovsky, E., Sergeeva, A.** Runup of long irregular waves on a plane beach. – In book: Extreme Ocean Waves (Ed: Efim Pelinovsky and Christian Kharif). Springer, 2008, p. 83–94.

Abstract

Long wave dynamics in the coastal zone is studied for two realistic bottom profiles. In the case of a convex profile the general solution of the Cauchy problem is obtained within the linear shallow-water theory. The wave system consists of two travelling waves propagating in opposite directions, whereas generally a zone of weak current is formed between these two waves. The long wave runup on such a beach is analysed. It is shown that the runup height on a convex bottom profile is significantly larger than that along a plane beach. The reflection and transmission of waves from a zone of increasing depth, described by the convex profile, is also studied. It is shown that a transmitted wave always has a sign-variable shape.

In the case of a beach of constant slope the problem of the long wave runup is discussed in the framework of the nonlinear shallow-water theory. A key finding of this study is that the definition of the wavelength for symmetric solitary incident waves at a 2/3 level of the maximum height is optimal for express prediction of their runup properties. The maximum runup and drawdown characteristics of a solitary wave on a beach virtually do not depend on the shape of the incident wave if the wave duration is appropriately defined. It is especially evident for the runup height and drawdown velocity; their variations for all given classes of wave shapes do not exceed 8%. It is shown that the wave steepness has very strong influence on the runup characteristics of long waves. Among waves of a fixed amplitude and period (length), the steepest wave penetrates inland over the largest distance.

The data on the freak wave events that occurred on the shore in 2005 show that some events are manifested as unusual flooding on a beach. A model to explain these data is developed in terms of the theory of runup of irregular wave fields. Variations of the distribution functions for extreme runup heights and shoreline velocity are moderate for narrow-band processes, for which the significant runup height can be appropriately described by the formula for the runup of a sine wave. For wide-band processes the significant runup height is considerably larger.

Resümee

Väitekirjas analüüsitakse pikkade lainete dünaamikat rannikuvööndis kahe realistliku rannanõlva profiili (konstantse kaldega ning kumera profiiliga rannad) jaoks. Esmakordselt vaadeldakse süstemaatiliselt kumera profiiliga rannanõlval levivate lainete dünaamikat analüütiliste meetoditega. On leitud lineaarse muutuvate kordajatega ühemõõtmelise madala vee lainevõrrandi Cauchy ülesande üldlahend. Mistahes alghäiritusest tekkiv lainesüsteem koosneb üldiselt kahest vastassuunas levivast lainest, mille vahel paiknevas alas esineb nõrk hoovus. Analüüsitakse kirjeldatud süsteemis esinevate lainete uhtekõrgust ja rannaäärsete alade üleujutuste kulgemist. On tõestatud, et kumera profiili korral on sama kõrgete ja pikkade lainete uhtekõrgus märksa suurem võrreldes lineaarse profiiliga randadega. Vaadeldakse lainete dünaamikat juhul, kui rannanõlv koosneb kahest osast: madalaveeline horisontaalse põhjaga alaga piirneb kumera profiiliga rannanõlva sügavam osa. Selliste randade puhul kujuneb madalamas vees levivast lainest alati muutuva märgiga veepinna häiritus ranna sügavamas osas.

Pikkade lainete uhtekõrguse problemaatikat konstantse kaldega rannanõlval vaadeldakse keerukamas raamistikus – mittelineaarsete madala vee võrrandite baasil. On näidatud et sümmeetriliste lainete poolt põhjustatud rannaäärsete alade üleujutuste ekstreemsed parameetrid (maksimaalne uhtekõrgus, maksimaalne veepinna alanemine jms.) praktiliselt ei sõltu lainete kujust ning on määratud lainete pikkusega. On demonstreeritud, et kõige olulisemad parameetrid – maksimaalne uhtekõrgus ja tagasivoolava veemassi maksimaalne kiirus – sõltuvad konkreetse laine kujust vähem kui 8% võrra. Optimaalseks uhtekõrgust iseloomustavaks suuruseks on lainete pikkus 2/3 kõrgusel laine maksimaalsest kõrgusest. On tuletatud uhtekõrguse operatiivprognoosiks sobivad empiirilised valemid nõnda defineeritud olulise lainepikkuse kaudu.

On tõestatud, et lainete profiili võimalik asümmeetria võib oluliselt modifitseerida uhtekõrguse suurust ja teisi rannaäärsete alade üleujutuse parameetreid. Suhteliselt järsema frondiga lainete uhtekõrgus võib olla kordades suurem sama kõrgete ja pikkade, kuid sümmeetriliste lainete uhtekõrgusest. On näidatud, et mida järsem on randa saabuva laine esinõlv, seda kaugemale sisemaale jõuab vesi.

Süstemaatiliselt analüüsitakse 2005. a. registreeritud hiidlainetega seonduvaid nähtusi. On näidatud, et mitmetel juhtudel on hiidlained põhjustanud ootamatuid üleujutusi rannaäärsetel aladel. On koostatud mudel, mis võimaldab kirjeldada hiidlainete anomaalselt suurt uhtekõrgust keerukate laineväljade uhtekõrguse teooria raames. On demonstreeritud, et maksimaalne uhtekõrgus ja laine frondi (veepiiri) liikumise kiirus varieerub suhteliselt vähe juhul, kui on tegemist kitsa spektriga lainesüsteemiga. Sellisel juhul saab nimetatud parameetreid adekvaatselt kirjeldada siinuslainete uhtekõrgust iseloomustavate seostega. Seevastu laia spektriga lainesüsteemide puhul on oodatav maksimaalne uhtekõrgus märksa suurem kui samade keskmiste parameetritega, kuid kitsa spektriga lainetuse puhul.

Appendix A: Curriculum Vitae

1. **Personal data**

Name	Ira Didenkulova
Date and place of birth	23.05.1980, Gorky
Citizenship	Russia

2. **Contact information**

Address	Akadeemia tee 21, 12618, Tallinn
Phone	(+372) 620 4260
E-mail	ira@cs.ioc.ee

3. **Education**

Educational Institution	Graduation year	Education (field of study/degree)
Nizhny Novgorod State Technical University	2006	Mechanics of fluids, gases and plasma / Cand. of Sc.
Nizhny Novgorod State University	2003	Radiophysics (acoustics) / MSc
Nizhny Novgorod State University	2001	Radiophysics / BA

4. **Language skills** (fluent; average, or basic skills)

Language	Level
Russian	native language
English	fluent

5. **Further training**

Period	Educational or other institution
Mar. 2008	Winter school “Nonlinear waves 2008”, Nizhny Novgorod, Russia
Jan. 2008 – Feb. 2008 Jan. 2007 – Aug. 2007 Nov. 2005 – Feb. 2006	Universite des Antilles et de la Guyane, Guadeloupe, French West Indies
Aug. 2007 – Sept. 2007	Summer School “Waves and Coastal Processes”, Tallinn, Estonia
Aug. 2007	Participation in the field survey after Hurricane Dean on Guadeloupe, French West Indies

Aug. 2006	Summer School on Air-Sea Interaction, Helsinki, Finland
Aug. 2005 – Sept. 2005	Participation in paleo tsunami field survey on Kunashir and Shikotan, Kuril Islands, Russia
May 2005 – Jul. 2005	Institut de Recherche sur les Phenomenes Hors Equilibre (IRPHE), Marseille, France

6. Professional employment

Period	Organisation	Position
2007 – to date	Institute of Cybernetics, Tallinn University of Technology	Experienced researcher
2006 – to date (on leave)	Institute of Applied Physics, Russian Academy of Sciences	Researcher
2003 – to date (on leave)	Nizhny Novgorod State Technical University	Assistant
Apr. 2006 – Sept. 2006	Institute of Cybernetics, Tallinn University of Technology	Researcher
2003 – 2006	Institute of Applied Physics, Russian Academy of Sciences	Junior Researcher
2000 – 2003	Institute of Applied Physics, Russian Academy of Sciences	Research Assistant

7. Scientific work

Conference presentations

US-EU-Baltic 2008 International Symposium, Tallinn, Estonia: “Analysis of tide-gauge records and their spectra of tsunami waves and background oscillations” (2008);

Solutions to Coastal Disasters Conference 2008, Oahu, Hawaii: “Influence of the initial wave shape on tsunami wave runup characteristics” (2008);

Joint workshops “Implications of climate change for marine and coastal safety” and “Applied Wave Mathematics” of Marie Curie networks SEAMOCS and CENS-CMA, and Eco-NET network “Wave Current Interaction in Coastal Environment”, Palmse, Estonia: “Long wave runup on the plane beach” (2007);’

General Assembly of the International Union of Geodesy and Geophysics (IUGG): “Long wave runup on the plane beach”, “Pointwise and distributed reflection of long waves from a beach”, “A comparison of tsunamis in Caribbean and Mediterranean; history, possibility, reality” (2007);

European Geosciences Union (EGU), Vienna, Austria: "Runup of nonlinear deformed waves on a beach", "Spectrum and steepness of nonlinear deformed shallow waves", "Freak runup of irregular waves", "Tsunamis in Russian lakes and rivers", "Freak waves in 2005", "Runup of solitary waves of different shapes on a beach", "Runup of irregular waves with various statistics", "Freak waves in 2006", "Characteristics of the nonlinear shallow water wave: shape, steepness and spectrum", "Spectrum of the tide-gauge records in Pointe-a-Pitre bay, Guadeloupe" (2006, 2007);

The Fifth International Symposium on Waves, Madrid, Spain: "Modelling of two global tsunamis in the Indian ocean (1883 Krakatau eruption and 2004 Sumatra earthquake)" (2005);

International Symposium "Topical Problems of Nonlinear Wave Physics", Nizhny Novgorod, Russia: "The Nizhny Novgorod tsunami on the Volga river" (2003);

International Workshop "Local Tsunami Warning and Mitigation", Petropavlovsk-Kamchatsky, Russia: "The 1597 Tsunami in the River Volga" (2002);

Nizhny Novgorod acoustical scientific session, Nizhny Novgorod, Russia: "Формирование волн большой амплитуды в рамках обобщенного уравнения Кортевега-де Вриза" (2002);

The IV International young scientist's scientific workshop "The future of technical science", Nizhny Novgorod, Russia: "Сравнение двух цунами: индонезийского 2004 года и Кракатау 1883 года" (2005);

The IX Nizhny Novgorod young scientist's session, Sarov, Russia: "Numerical simulation of tsunami Krakatau" (2004);

The Workshop "Ecological and Industrial Safety", Sarov, Russia: "Солитоны и кинки огибающей в решетках солитонов", "Цунами на Волге", "Численное моделирование цунами в реке" (2001, 2003, 2004);

The Scientific Radiophysics Workshop, Nizhny Novgorod, Russia: "Солитоны и кинки огибающей в решетках солитонов модели Гарднера", "Реконструкция волнового источника на примере цунами Кракатау" (2001, 2003).

Seminars:

10 Apr. 2008	Seminar paper "Shoaling and runup of long waves generated by high-speed ferries" at the Department of Civil & Environmental Engineering, Cornell University (Ithaca, USA)
4 Apr. 2008	Seminar paper "New trends in the nonlinear theory of long wave runup on a beach" at the Department of Civil & Environmental Engineering, Massachusetts Institute of Technology (Boston, USA),
20 Dec. 2007	Seminar paper "Long waves in a coastal zone" at Lund University (Lund, Sweden)
9 Oct. 2007	Seminar paper "Mathematical modelling of long waves (tsunami waves)" at the Institute of Cybernetics, Tallinn University of

	Technology (Tallinn, Estonia)
25 Aug. 2006	Seminar paper “Runup of nonlinear deformed waves” at Det Norske Veritas, DNV Research (Høvik, Norway)
22 Aug. 2006	Seminar paper “Runup of nonlinear asymmetric waves on a plane beach” at the University of Oslo (Oslo, Norway)

Peer-reviewed publications

1.1. Articles indexed by ISI Web of Science

I. Didenkulova, E. Pelinovsky, and T. Soomere. Run-up characteristics of tsunami waves of “unknown” shapes. *Pure and Applied Geophysics* (2008) (accepted).

B. H. Choi, E. Pelinovsky, D. C. Kim, I. Didenkulova. Two- and three-dimensional computation of solitary wave runup on non-plane beach. *Nonlinear Processes in Geophysics* (2008) (accepted).

I. Didenkulova, E. Pelinovsky. Run-up of long waves on a beach: the influence of the incident wave form. *Oceanology*, **48**, No 1, 1–6 (2008).

N. Zahibo, I. Didenkulova, A. Kurkin, E. Pelinovsky. Steepness and spectrum of nonlinear deformed shallow water wave. *Ocean Engineering*, **35**, No 1, 47–52 (2008).

I. Didenkulova, A. Kurkin, E. Pelinovsky. Run-up of solitary waves on slopes with different profiles. *Izvestiya, Atmospheric and Oceanic Physics*, **43**, No 3, 384–390 (2007).

I. Didenkulova, N. Zahibo, A. Kurkin, E. Pelinovsky. Steepness and spectrum of a nonlinearly deformed wave on shallow waters. *Izvestiya, Atmospheric and Oceanic Physics*, **42**, No 6, 773–776 (2006).

I. Didenkulova, N. Zahibo, A. Kurkin, B. Levin, E. Pelinovsky, T. Soomere. Runup of nonlinear deformed waves on a beach. *Doklady Earth Sciences*, **411**, No 8, 1241–1243 (2006).

I. Didenkulova, A. Slunyaev, E. Pelinovsky, Ch. Kharif. Freak waves in 2005. *Natural Hazards and Earth System Sciences,* **6**, 1007–1015 (2006).

1.2. Peer-reviewed articles in other international research journals

I. Didenkulova, A. Zaytsev, E. Pelinovsky. The 1806 tsunami in Kozmodemyansk on Volga. *Marine Hydrophysical Journal*, Sevastopol, **1**, 73–76 (2007).

I. Didenkulova, E. Pelinovsky, N. Zahibo. Long wave reflection from “non-reflecting” bottom profile. *Fluid Dynamics*, **43**, No 4, 101–107 (2008).

N. Zahibo, I. Didenkulova, E. Pelinovsky. Spectra of nonlinear shallow water waves. *Journal of Korean Society of Coastal and Ocean Engineers*, **19**, No 4, 355–360 (2007).

I. Didenkulova, E. Pelinovsky. Phenomena similar to tsunami in Russian internal basins. *Russian Journal of Earth Sciences,* **8**, No 6, ES6002, doi:10.2205/2006ES000211 (2006).

I. Didenkulova, E. Pelinovsky, A. Kurkin. Nonlinear shallow wave characteristics: shape, spectrum and steepness. *Izvestiya, Russian Academy of Engineering Science*, **18**, 18–32 (2006).

I. Didenkulova, E. Pelinovsky. Comparison of two global tsunami data in the Indian Ocean. *Izvestiya, Russian Academy of Engineering Science*, **18**, 58–64 (2006).

A. Sergeeva, I. Didenkulova. Runup of irregular long waves on a sloping beach. *Izvestiya, Russian Academy of Engineering Science* **14**, 98–105 (2005).

I. Didenkulova, C. Kharif. Runup of biharmonic long waves on a beach. *Izvestiya, Russian Academy of Engineering Science* **14**, 9–97 (2005).

I. Didenkulova. Tsunamis in Russian lakes and rivers. *Izvestiya, Russian Academy of Engineering Science* **14**, 82–90 (2005).

I. Didenkulova, A. Zaytsev, A. Krasilshikov, A. Kurkin, E. Pelinovsky and A. Yalchiner. Nizhny Novgorod tsunami on the Volga river. *Izvestiya, Russian Academy of Engineering Science* **4**, 170–180 (2003).

3.1. Articles and chapters in books published by internationally renowned the publishers (including collections indexed by the ISI Web of Proceedings)

I. Didenkulova, E. Pelinovsky, T. Soomere. Influence of the initial wave shape on tsunami wave runup characteristics. *In: Proceedings the Conference Solutions to Coastal Disasters 2008. Tsunamis*. American Society of Civil Engineers, 94–105 (2008).

I. Didenkulova, E. Pelinovsky, A. Sergeeva. Runup of long irregular waves on a plane beach. *In: Extreme Ocean Waves (Ed: Efim Pelinovsky and Christian Kharif)*. Springer, 83–94 (2008).

N. Zahibo, I. Nikolkina, I. Didenkulova. Extreme waves generated by cyclones in Guadeloupe. *In: Extreme Ocean Waves (Ed: Efim Pelinovsky and Christian Kharif)*. Springer, 159–177 (2008).

I. Didenkulova, E. Pelinovsky, T. Soomere, N. Zahibo. Runup of nonlinear asymmetric waves on a plane beach. *In: Tsunami & Nonlinear Waves (Ed: Anjan Kundu)*, Springer, 175–190 (2007).

E. Pelinovsky, B. Choi, A. Stromkov, I. Didenkulova, H. Kim. Analysis of tide-gauge records of the 1883 Krakatau tsunami. *In: Tsunamis: case studies and recent developments (Ed: Kenji Satake)*, Springer, 57–78 (2005).

3.4. Articles and presentations published in other conference proceedings

I. Didenkulova, E. Pelinovsky. Tsunami like events in Russian inland waters. *Preprint of IAP RAS* No754 (2008).

I. Didenkulova, A. Zaytsev, A. Krasilshikov, A. Kurkin, E. Pelinovsky and A. Yalchiner. The 1597 Nizhny Novgorod tsunami on the Volga river. *Preprint of IAP RAS* No632 (2003).

I. Didenkulova. Runup of waves on a beach. *In: Proceedings of the Fifth scientific workshop "Young people in science",* Sarov, 83–89 (2007).

I. Didenkulova, E. Pelinovsky, N. Zahibo. Analytical expressions for runup characteristics of nonlinear long waves on a plane beach. *In: Proceedings of the International Symposium Tsunami Disaster Mitigation for East Korean Coast*, Korea, 1–4 (2007).

I. Didenkulova, A. Kurkin, E. Pelinovsky, O. Polukhina, A. Sergeeva, A. Slunyaev. Onshore freak waves: observation and modelling. *In: Proceedings of the VIII International Symposium "Modern methods of natural and anthropogenic hazards mathematical modelling"*, Kemerovo, 147–157 (2005).

E. Pelinovsky, B. Choi, A. Zaitsev, and I. Didenkulova. Modelling of two global tsunamis in the Indian ocean (1883 Krakatau eruption and 2004 Sumatra earthquake). *In: Proceedings of the Fifth International Symposium Waves*, Madrid, Paper No 213 (2005).

I. Didenkulova, A. Zaytsev, A. Krasilshikov, A. Kurkin, Numerical simulation of tsunami in river, *Proc*. of the III Workshop "Ecological and Industrial Safety", VNIIEF, Sarov, 227–234 (2004).

I. Didenkulova, A. Zaytsev, A. Krasilshikov, A. Kurkin, E. Pelinovsky and A. Yalchiner. The Nizhny Novgorod tsunami on the Volga river. *In: Proceedings of the International Symposium "Topical Problems of Nonlinear Wave Physics"*, Nizhny Novgorod, 299–300 (2003).

I. Didenkulova, E. Pelinovsky. Tsunami in the River Volga. *In: Proceedings of the II Workshop "Ecological and Industrial Safety"*, VNIIEF, Sarov, 311–315 (2003).

I. Didenkulova, E. Pelinovsky, A. Stromkov. Reconstruction of the wave source on example of tsunami Krakatau. *In: Proceedings of the VII Scientific Radiophysics Workshop*, Nizhny Novgorod, 225–226 (2003).

I. Didenkulova, E. Pelinovsky. The 1597 Tsunami in the River Volga. *In: Proceedings of the International Workshop "Local Tsunami Warning and Mitigation"*, Moscow, 17–22 (2002).

I. Didenkulova, A. Slunyaev. Generation of large amplitude waves in the framework of extended Korteweg–de Vries equation. *In: Proceedings of the Nizhny Novgorod acoustical scientific session,* Nizhny Novgorod, 241–244 (2002).

K. Gorshkov, I. Didenkulova. Envelope solitons and kinks in soliton lattices of Gardner model. *In: Proceedings of the V Scientific Radiophysics Workshop*, Nizhny Novgorod, 284–286 (2001).

8. Defended theses

Runup of long waves on the sloping beach and analyses of real events. Candidate of Science's degree.
Reconstruction of wave source. Master's degree.
Envelope solitons and kinks in the framework of Gardner model. Bachelor's degree.

9. Main areas of scientific work/Current research topics

Wave motion in the sea, wave runup on the beach, wave transformation, tsunami and freak waves, nonlinear theories, numerical simulation.

10. Other research projects

Leader of grant projects

EEA grant "Shoaling and runup of long waves generated by high-speed ferries", 2008–2010;
ProVention Consortium Research and Action Grants "Tsunamis in Russian Lakes and Rivers" No 3019, 2007–2008.

11. Honours and awards

Marie Curie Fellow, 2006–2009;
INTAS Young Scientist Postdoctoral Fellowship "Study of the tsunami and freak wave runup on a beach" No 06-1000014-6046, 2007;
The Medal of the Ministry of Education and Science of Russian Federation as "The best scientific student's work" (MSc thesis "Reconstruction of the wave source"), 2005;
Scholarship of the French Embassy for a three-month period, for stay at the Institut de Recherche sur les Phenomenes Hors Equilibre (IRPHE) (Marseille, France), 2005;
Scholarship for young scientists of Academician Razuvaev, 2004.

Appendix B: Elulookirjeldus

1. Isikuandmed

Ees- ja perekonnanimi	Ira Didenkulova
Sünniaeg ja -koht	23.05.1980, Gorki (Nižni Novgorod)
Kodakondsus	Venemaa

2. Kontaktandmed

Aadress	Akadeemia tee 21, 12618, Tallinn
Telefon	(+372) 620 4260
E-posti aadress	ira@cs.ioc.ee

3. Hariduskäik

Õppeasutus	Lõpetamise aeg	Haridus (eriala, kraad)
Nižni Novgorodi Riiklik Tehnikaülikool, aspirantuur	2006	Vedelike, gaaside ja plasma mehaanika, füüsika-matemaatikakandidaat
Nižni Novgorodi Riiklik Tehnikaülikool	2003	Raadiofüüsika (akustika), magistrikraad
Nižni Novgorodi Riiklik Tehnikaülikool	2001	Raadiofüüsika, bakalaureuse kraad

4. **Keelteoskus** (alg-, kesk- või kõrgtase)

Keel	Tase
Vene	emakeel
Inglise	kõrgtase

5. Täiendõpe

Õppimise aeg	Täiendõppe koht või üritus
märts 2008	Talvekool "Mittelineaarsed lained 2008", Nižni Novgorod, Venemaa
jaanuar – veebruar 2008 jaanuar – august 2007 november 2005 – veebruar 2006	Universite des Antilles et de la Guyane, Guadeloupe, Prantsuse Ida-India
august – september 2007	Rahvusvaheline suvekool "Lained ja rannikuprotsessid", Tallinn
august 2007	Välitööd orkaani Dean järel, Guadeloupe,

	Prantsuse Ida-India
august 2006	Rahvusvaheline suvekool “Mere ja atmosfääri interaktsioon”, Helsingi
august – september 2005	Osalemine paleotsunamide uuringute välitöödel, Kunashir ja Shikotan (Kuriilid), Venemaa
mai – juuli 2005	Institut de Recherche sur les Phenomenes Hors Equilibre (IRPHE), Marseille, France

6. Teenistuskäik

Töötamise aeg	Organisasioon	Ametikoht
2007 – tänaseni	Tallinna Tehnikaülikooli Küberneetika Instituut	Erakorraline vanemteadur
2006 – tänaseni (tööleping peatatud)	Venemaa Teaduste Akadeemia Rakendusfüüsika Instituut	Teadur
2003 – tänaseni (tööleping peatatud)	Nižni Novgorodi Riiklik Tehnikaülikool	Assistent
aprill – september 2006	Tallinna Tehnikaülikooli Küberneetika Instituut	Erakorraline teadur
2003 – 2006	Venemaa Teaduste Akadeemia Rakendusfüüsika Instituut	Nooremteadur
2000 – 2003	Venemaa Teaduste Akadeemia Rakendusfüüsika Instituut	Tehnik

7. Teadustegevus

Ettekanded rahvusvahelistel teaduskonverentsidel:

USA-Euroopa Liidu ja Baltimaade rahvusvaheline sümpoosium (Tallinn, 2008): “Analysis of tide-gauge records and their spectra of tsunami waves and background oscillations”;

Rahvusvaheline konverents “Solutions to Coastal Disasters 2008” (Oahu, Havai, 2008): “Influence of the initial wave shape on tsunami wave runup characteristics”; Marie Curie võrgustike SEAMOCS ja CENS-CMA ning Eco-NET’i võrgustiku “Wave Current Interaction in Coastal Environment” ühised konverentsid “Implications of climate change for marine and coastal safety” ja “Applied Wave Mathematics” (Palmse, 2007): “Long wave runup on the plane beach”;

Rahvusvahelise Geodeesia ja Geofüüsika Liidu (IUGG) Peaassamblee (Perugia, 2007): “Long wave runup on the plane beach”, “Pointwise and distributed reflection of long waves from a beach”, “A comparison of tsunamis in Caribbean and Mediterranean; history, possibility, reality”;

Euroopa Geoteaduste Liidu (EGU) Peaassambleed 2006 ja 2007 (Viin, 2006, 2007): “Runup of nonlinear deformed waves on a beach”, “Spectrum and steepness of nonlinear deformed shallow waves”, “Freak runup of irregular waves”, “Tsunamis in Russian lakes and rivers”, “Freak waves in 2005”, “Runup of solitary waves of different shapes on a beach”, “Runup of irregular waves with various statistics”, “Freak waves in 2006”, “Characteristics of the nonlinear shallow water wave: shape, steepness and spectrum”, “Spectrum of the tide-gauge records in Pointe-a-Pitre bay, Guadeloupe”;

Rahvusvaheline konverents “The Fifth International Symposium on Waves” (Madriid, 2005): “Modelling of two global tsunamis in the Indian ocean (1883 Krakatau eruption and 2004 Sumatra earthquake)”;

Rahvusvaheline sümpoosium “Topical Problems of Nonlinear Wave Physics” (Nižni Novgorod, 2003): “The Nizhny Novgorod tsunami in the Volga River”;

Rahvusvaheline konverents “Local Tsunami Warning and Mitigation” (Petropavlovsk-Kamtšatski, 2002): “The 1597 Tsunami in the River Volga”;

Rahvusvaheline akustikakonverents (Nižni Novgorod, 2002): “Формирование волн большой амплитуды в рамках обобщенного уравнения Кортевега-де Вриза”;

IV rahvusvaheline noorte teadlaste konverents “The future of technical science” Nižni Novgorod, 2005): “Сравнение двух цунами: индонезийского 2004 года и Кракатау 1883 года”;

IX Nižni Novgorodi noorte teadlaste konverents (Sarov, 2004): “Numerical simulation of tsunami Krakatau”;

Konverentsid “Ecological and Industrial Safety” (Sarov, 2001, 2003, 2004): “Солитоны и кинки огибающей в решетках солитонов”, “Цунами на Волге”, “Численное моделирование цунами в реке”;

Rahvusvahelised raadiofüüsika konverentsid (Nižni Novgorod, 2001, 2003): “Солитоны и кинки огибающей в решетках солитонов модели Гарднера”, “Реконструкция волнового источника на примере цунами Кракатау”.

Teadusseminarid:

10. aprill 2008	Ettekanne “Shoaling and runup of long waves generated by high-speed ferries” (Department of Civil & Environmental Engineering, Cornelli Ülikool, Ithaca, USA)
4. aprill 2008	Ettekanne “New trends in the nonlinear theory of long wave runup on a beach” (Department of Civil & Environmental Engineering, Massachusettsi Tehnoloogiainstituut, Boston, USA),
20. detsember 2007	Ettekanne “Long waves in a coastal zone” (Lundi Ülikool, Rootsi)
9. oktoober 2007	Ettekanne “Mathematical modelling of long waves (tsunami waves)” (ülelinnaline mehaanikaseminar, TTÜ Küberneetika

	Instituut)
25. august 2006	Ettekanne "Runup of nonlinear deformed waves" (Det Norske Veritas, DNV Research, Høvik, Norra)
22. august 2006	Ettekanne "Runup of nonlinear asymmetric waves on a plane beach" (Oslo Ülikool, Norra)

Eelretsenseeritud publikatsioonid:

1.1. Artiklid, mis on kajastatud ISI Web of Science andmebaasis

I. Didenkulova, E. Pelinovsky, and T. Soomere. Run-up characteristics of tsunami waves of "unknown" shapes. *Pure and Applied Geophysics* (2008) (accepted).

B. H. Choi, E. Pelinovsky, D. C. Kim, I. Didenkulova. Two- and three-dimensional computation of solitary wave runup on non-plane beach. *Nonlinear Processes in Geophysics* (2008) (accepted).

I. Didenkulova, E. Pelinovsky. Run-up of long waves on a beach: the influence of the incident wave form. *Oceanology*, **48**, No 1, 1–6 (2008).

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1.2. Artiklid teistes rahvusvahelistes eelretsenseeritud teadusajakirjades

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3.1. Artiklid ja peatükid rahvusvaheliselt tunnustatud teaduskirjastuste poolt välja antud raamatutes ja kogumikes (sh. artiklid, mis on kajastatud ISI Web of Proceedings andmebaasis)

I. Didenkulova, E. Pelinovsky, T. Soomere. Influence of the initial wave shape on tsunami wave runup characteristics. *In: Proceedings the Conference Solutions to Coastal Disasters 2008. Tsunamis*. American Society of Civil Engineers, 94–105 (2008).

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E. Pelinovsky, B. Choi, A. Stromkov, I. Didenkulova, H. Kim. Analysis of tide-gauge records of the 1883 Krakatau tsunami. *In: Tsunamis: case studies and recent developments (Ed: Kenji Satake)*, Springer, 57–78 (2005).

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I. Didenkulova, A. Kurkin, E. Pelinovsky, O. Polukhina, A. Sergeeva, A. Slunyaev. Onshore freak waves: observation and modelling. *In: Proceedings of the VIII International Symposium "Modern methods of natural and anthropogenic hazards mathematical modelling"*, Kemerovo, 147–157 (2005).

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I. Didenkulova, A. Zaytsev, A. Krasilshikov, A. Kurkin, E. Pelinovsky and A. Yalchiner. The Nizhny Novgorod tsunami on the Volga river. *In: Proceedings of the International Symposium "Topical Problems of Nonlinear Wave Physics"*, Nizhny Novgorod, 299–300 (2003).

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8. Kaitstud lõputööd ja väitekirjad

Runup of long waves on the sloping beach and analyses of real events. Nižni Novgorodi Riiklik Tehnikaülikool. Füüsika-matemaatikakandidaat.
Reconstruction of wave source. Nižni Novgorodi Riiklik Tehnikaülikool. Magistrikraad.
Envelope solitons and kinks in the framework of Gardner model. Nižni Novgorodi Riiklik Tehnikaülikool. Bakalaureusekraad.

9. Teadustöö põhisuunad

Pinnalained meres, lainete uhtekõrguse problemaatika, lainete ümberkujunemine madalas vees, tsunami ja hiidlained, mittelineaarne laineteooria, lainete evolutsiooni numbriline modelleerimine.

10. Teised uurimisprojektid

Grantid hoidjana:

Pikkade lainete uhtekõrguse analüüs kiirlaevalainete baasil (EEA, hoidja, 2008–2010).
Tsunamid Venemaa järvedes ja jõgedes (ProVention Consortium Research and Action Grant No 3019, hoidja, 2007–2008).

11. Tunnustused

Marie Curie stipendiaat (SEAMOCS, TTÜ Küberneetika Instituut, 2006–2009).
INTAS noorteadlaste järeldoktori stipendium "Study of the tsunami and freak wave runup on a beach" No 06-1000014-6046, 2007.
Vene Föderatsiooni Haridus- ja Teadusministeeriumi medal parima üliõpilaste teadustöö eest (magistritööd, väitekiri "Reconstruction of the wave source" 2005).
Prantsusmaa saatkonna stipendium tööks Marseille's, Institut de Recherche sur les Phenomenes Hors Equilibre (IRPHE), 2005.
Akadeemik Razuvajevi nimeline noorteadlaste stipendium, 2004