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EXPERIMENTAL ANALYSIS OF THE EFFECTS OF A QUOTA ON MANIPULATIONS IN WEIGHTED VOTING GAMES

Master Thesis

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EKSPERIMENTAALNE ANALÜÜS - KVOODI MÕJU MANIPULATSIOONI EFEKTIIVSUSELE KAALUTUD HÄÄLETUSTEGA MÄNGUDES.

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Author’s declaration of originality

I hereby certify that I am the sole author of this thesis. All the used materials, references to the literature and the work of others have been referred to. This thesis has not been presented for examination anywhere else.

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Abstract

Weighted voting system is adopted by many organizations, both in public and private sectors, as the main decision making procedure. Its main advantage is an opportunity to recognize a difference between voters, by assigning the appropriate weights to them. Because of this nature, weighted voting becomes a subject to specific *dishonorable behaviour* that can be conducted by voters. Even though this system has been in use since the ancient Rome a question of the effects that different factors have on these manipulations haven't been fully addressed.

The main focus of this work lies within study of the effects that a quota have on manipulations' payoff in weighted voting games of different sizes, using experimental analysis for Shapley-Shubik index. The authors also show a connection between *payoff bounds* of *merging* and *splitting manipulations*, that makes it possible to find some of the maximal and minimal bounds of *manipulation by merging that* weren't proven in previous studies. The work is concluded with a manipulation analysis of voting system in Council of the European Union under the Treaty of Lisbon and Treaty of Nice.

The results of the work have shown that it is possible to control manipulation vulnerability of the system to some extend by choosing the right quota, but also that there are many different factors that affect it.

This thesis is written in English and is 94 pages long, including 8 chapters, 35 figures and 2 tables.

Annotatsioon

Eksperimentaalne analüüs - kvoodi mõju manipulatsiooni efektiivsusele kaalutud hääletustega mängudes.

Kaalutud hääletussüsteemi kasutatakse paljudes era kui avalikus sektori organisatsioonides peamise otsustustamise protseduurina. Sellise süsteemi peamine eelis on võimalus tuvastada erinevusi hääletajates, määrates neile kindlaid osakaale. Osakaalud aga võimaldavad teatud juhtudel ebaausat käitumist. Sõltumata sellest, et sellist hääletussüsteemi on kasutatud vana rooma aegadest alates, ei ole erinevate tegurite mõju manipulatiivsele tegevusele hääletustes põhjalikult uuritud.

Selle töö peamine eesmärk on uurida kvootide mõju manipulatsiooni tulemuslikkusele erineva suurusega kaalutud hääletustega mängudes. Uuringus kasutatakse ekperimentaalanalüüsi Shapley-Shubik indeksi põhjal. Sealhulgas autorid näitavad seoseid, kus mõjutades resultaadi piire manipulatsioonides läbi ühinemise ja jagunemise on võimalik leida maksimaalseid ja minimaalseid piire manipulatsiooniks läbi ühinemise - mis ei leidnud kinnitust eelnevates uuringutes. Töö epiiriline osa keskendub Euroopa Liidu Lissabony ja Nice Ministrite Nõukogu manipulatsiooni analüüsile.

Selle töö tulemused võtavad kokku, et hääletussüsteem on võimalik mõjutatada läbi manipulatsiooni valides õigeid kvoote, kuid esineb ka muid tegureid mis võivad tulemust mõjutada.

Lõputöö on kirjutatud inglise keeles ning sisaldab teksti 94 leheküljel, 8 peatükki, 35 joonist, 2 tabelit.

List of abbreviations and terms

TTU	Tallinn University of Technology
WVG	Weighted Voting Game
S-S	Shapley-Shubik power index
BZ	Banzhaf power index
CEU	Council of the European Union
I-Power	Power as an Influence
P-Power	Power as a Price
IMF	International Monetary Fund
UN	United Nations

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1 Introduction

1.1 Overview

A weighted voting is one of the ways to make a group decision that found an application in politics, international relations, business, finance, computer systems and other areas. Shareholders meetings in corporations, Council of the European Union (Council of the European Union, 2015), U.S. Electoral College (U.S. Electoral College, 2016), etc. adopted weighted voting as a decision making model. Main difference from a simple majority voting is that one voter doesn't equal to one vote, instead each voter is assigned with a number of votes (weight). For a bill or decision to be adopted the sum of weights of voters who voted in favor of it must be higher than an established number, known as quota, whereas quota can represent a particular number of votes or percentage of the total weight all voters possess. Weighted voting is used when differences between voters have to be recognized, for example, when a voter represents interests of a group of people, the size of the group can be used as a weight. Furthermore, parties in parliament can be seen as single voters with weight equal to a number of seats party holds.

Considering such a wide use of the system, it is important that the weighted voting system is well studied from all perspectives. One of the side of the problem is mathematical justification of the system, as behind any voting algorithm lies mathematics. Well design mathematical model for voting can improve decision making, ensuring systems' consistency, predictability and stability. In this work weighed voting would be studied from the mathematical perspective.

In mathematics weighted voting systems are abstracted into the Weighted Voting Games (here and after WVG), which is a part of a Game Theory. WVGs received close attention and was being developed primarily between the middle and the end of the 20th century (Felsenthal & Machover, 2004) with the introduction of indexes that measure power of voters, as it was shown that weight doesn't exactly represent power (influence over the result of voting) of voter. Several power indexes were introduced and the most significant among them - Banzhaf and Shapley-Shubik *priori* power indexes (Felsenthal & Machover, 2004). Term *priori* means that indexes don't account for any voter's preferences and other voter-specific or system-specific properties. Today these two indexes are widely used for analysis of real-life voting systems and WVGs in general. The work of Banzhaf (1964), for example, helped to find flaws in voting system of Nassau County board, showing that three of the voters have actually no power and couldn't affect the outcome of an election.

Power indexes opened many possibilities for analysis of WVGs from different perspectives, but the contribution is not limited to analysis alone. For example, they can be used as an instrument of precise design of WVGs with desired powers of voters in mind (Aziz, Paterson, & Leech, 2007). Power indexes also proved to be useful for analysis of dishonest behavior of involved parties – voters, election authorities or third parties. Voting algorithms are very sensitive, even a slight change of rules or voters' behavior may lead to a shift of powers and ultimately to a completely different outcome, which is an important issue, considering a wide use of weighted voting in politics and business.

Dishonest behaviors in WVGs can be divided into 3 categories (Zuckerman, Faliszewski, Bachrach, & Elkind, 2008): *manipulation* (dishonest behavior by voters), *control* (dishonest behavior by the election authorities), and *bribery* (dishonest behavior by a third party). In this work we are focusing on manipulations.

There are three types of manipulations (Aziz & Paterson, 2009): manipulation by *splitting*, *merging* and *annexation*. Study of manipulations is in a very active stage, as there are a big number of recent works, which are addressing different sides of this issue. For examples works by Lasisi and Allan (2014), Aziz and Paterson (2009), R. O. Lasisi and A. A. Lasisi (2016), Bachrach and Elkind (2008) and others. This indicates that there are still major uncovered areas in analysis of manipulations in WVGs.

Power indexes give a possibility to numerically evaluate results of dishonest behavior. In regards to manipulations, the ratio between player power after and before manipulation is called a *manipulation payoff* and is used to examine the success of manipulation. Most of manipulation studies are centered around this concept, as there are many factors that influence payoff behavior.

In this work the effects that changes of a quota have on properties of manipulation by splitting and merging would be shown through an analysis of a payoff behavior. The work also addresses gaps in the literature and research of the subject by proving bounds for minimal and maximal values of payoff for merging manipulation (Section 5.4). Furthermore, in this work authors discuss the effects that number of voters in WVG has on manipulations and apply manipulation analysis in a simulation of a voting in Council of the European Union under the recently abolished Treaty of Nice and its successor Treaty of Lisbon, to find whether the transition to a new voting system has improved its manipulation resistance measured with Shapley-Shubik index.

1.2 Used Methods

In this paper three hypotheses on effects of payoff to quota and reverse properties of manipulation by splitting and merging are tested. We used experimental analysis, with application of game theory, mathematical analysis and probability theory. Sufficient simulation environments were developed using MatLab for creation of random samples of WVGs, random manipulations in them and calculation of manipulation payoff (simulation environment described in chapter 4.5.1). Results achieved by the simulation were analyzed using mathematical analysis and probability theory to find proof for proposed hypotheses. We also have achieved some theoretical results for connection between manipulations by splitting and merging in scope of the game theory.

1.3 Structure of the Thesis

In this work the effects that changes of a quota have on manipulation behavior measured with Shapley-Shubik index are analyzed.

In Chapter 2 the necessary background for understanding of weighted voting games, *priori* power indexes and manipulations is provided.

Chapter 3 is aimed to familiarize a reader with the discussion around relevancy of a *priori* power indexes, as they are the main measure used in this work. Also the state of manipulation study is presented to clearly identify gap in existing knowledge and formulate thesis hypotheses.

In Chapter 4 the analysis of manipulation by splitting is presented, Chapter 5 has the same structure and is dedicated to merging manipulation. In Chapter 5.4 the new bounds for maximal and minimal payoff for merging manipulation are proved.

Chapter 6 gives an overview of manipulation by annexation and explains reasoning for avoiding its analysis. This chapter also provides a conclusion on effects of quota merging and splitting manipulation.

In Chapter 7, we make a suggestion on the effectiveness of manipulation for real life weighted voting system and then apply experiential analysis to compare results. Also this chapter touches upon finding the influence that multi-majority systems have on behavior of manipulation indexes.

2 Preliminaries

2.1 Definitions and Notations

Weighted voting system is an electoral system in which voters are not necessarily having the same influence on the outcomes of elections. There could be different rules and weight assignment procedures. In general terms, voting systems where one voter equals to one vote, also can be considered as a weighted voting systems with equal weights of votes.

Notations and definitions that would be used in this work are described below.

Weighted voting is a well-known concept in Game Theory (Felsenthal & Machover, 1998). In mathematical terms it is referred to as a **Weighted Voting Game**.

Definition 2.1 *Weighted Voting Game and Coalition:*

Let $P = \{p_1, \ldots, p_n\}$, $n \in \mathbb{N}$ be a set of all **players** (voters) in WVG G and $\{w_1, \ldots, w_n\}$ their corresponding weights. Then WVG G with Quota $q \in \mathbb{R}$ can be denoted as:

$$[q; w_1, \ldots, w_n]$$

A non-empty sub-set $C \subseteq P$ is called **Coalition.** $w(C)$ is a total weight of Coalition C equal to the sum of all players weights in it $w(C) = \sum_{j \in C} w_j$. Coalition C is **winning** if $w(C) \geq q$ and **losing** otherwise. While a single voting is held, there could be only one winning coalitions, which is achieved by choice of $q > w(P)*0.5$. Coalition has a **veto power** if $w(P) - w(C) < q$, which means that any combination of players left out of C is a losing coalition. Any winning coalition has a veto power, but losing coalition can also have it.

Coalition C is a **Minimal Winning Coalition** if every subset of it is a losing coalition $w(C) \geq q$ and $\forall S \subset C; w(S) < q$.

Coalitions that consist of all players $C = P$ is called a **grand coalition**.

Definition 2.2 *Critical, Dummy and Dictator player:*

Player p_i is **critical** in a coalition C if $w(C) \geq q$ and $w(C \setminus p_i) < q$.

Dummy player is a player which is not critical in any coalition.

Player p_i is a **dictator** if $w_i \geq q$. Dictator can pass or fail voting by casting his vote no matter what are the votes of other players.

Definition 2.3 *Unanimity and Non-Unanimity WVGs*:

A game G is **unanimity** WVG if the only winning coalition is a grand coalition. Otherwise game G is **non-unanimity** WVG.

2.2 Power Indexes

The concept of power was introduced in 1787 by Luther Martin for the first time, but it didn't receive much attention until the middle of 20 century (Felsenthal and Machover, 2004), when the works of Penrose (1946), Shapley and Shubik (1954) and Banzhaf (1965) were published. This work is based on the concept of player's power and extensively uses Banzhaf (Normalized Penrose index) and Shapley-Shubik power indexes.

Definition 2.4 *Power vector*:

Power vector of WVG G is a n-dimensional vector with powers of players: $(\phi_1(G),\ldots,\phi_n(G)) \in [0,1]^n$, $\phi_i(G) \in \mathbb{R}$.

It may seem obvious that player power should be equal to percentage of his weight of total weights of players, but consider the following example of WVG G with 3 players with weights equal to 50, 45, 5 and quota equals to 51.

Example 2.1:

$G = [51; 50, 45, 5]$, $W = 100$, $Q = 51$.

Player p_1 has weight 10 times greater than p_3 and p_2 has weight 9 times greater. Presumably power of both p_1 and p_2 has to be much greater too. But if we write down all possible winning coalitions, we see that there is an equal amount of winning coalitions for each player, so in fact, player p_3 has the same amount of influence over the results of voting as players p_1 and p_2. All winning coalitions:

$[p_1, p_2] = 95$, $[p_1, p_3] = 55$, $[p_2, p_3] = 54$, $[p_1, p_2, p_3] = 100$;

If we will count number of times each player is critical in a coalition it will also be equal for each player. This idea the main idea behind the Banzhaf power index (here and after BZ) (Banzhaf, 1965). He proposed to evaluate voting power of a player by dividing the number of times he is critical in a coalition by a total number of times any player is critical.

Definition 2.5 *Banzhaf power index*:

The Banzhaf index for player p_i in WVG G is noted as $\beta_i(G)$ and is given by:

$$\beta_i(G) = \frac{\eta_i(G)}{\sum_{j \in I} \eta_j(G)} \tag{2.1}$$

where $\eta_i(G)$ is a number of times player p_i is critical.

Let's mark critical players in each coalition:

$[\underline{p_1}, \underline{p_2}]$, $[\underline{p_1}, \underline{p_3}]$, $[\underline{p_2}, \underline{p_3}]$, $[p_1, p_2, p_3]$;

For the given example Banzhaf powers of players will be equal:

$$\beta_1(G) = \beta_2(G) = \beta_3(G) = \frac{2}{6} \approx 0.333$$

The power vector of the game G is [0.333, 0.333, 0.333], which shows that even though the difference in weights is very significant; the powers of players are actually equal according to Banzhaf index.

Banzhaf index alongside **Shapley-Shubik** index (here and after S-S) are the most studied and used in researches of power of players in WVGs. In a contrast to Banzhaf index, Shapley-Shubik index (Shapley & Shubik 1954) takes into account the order in which players are joining coalitions.

Coalition in which order of players is important is called **sequential** coalition.

Player p is **pivotal**, if in sequential coalition C, players before p don't form a winning coalition, but with an addition of player p a winning coalition is formed. Unlike critical player, there can be only one pivotal player per coalition.

For player p Shapley-Shubik index is equal to a ratio between number of coalitions in which the player is pivotal to number of all possible grand coalitions.

Definition 2.6 *Shapley-Shubik power index*:

Denote $\prod_n$ as set of all possible permutations of players, then $\pi \in \prod_n$ is a permutation of players, so $\pi : \{1,\ldots,n\} \to \{1,\cdots,n\}$. Denote by $S_\pi(i)$ the set of predecessors of player p_i in π, i.e., $S_\pi(i) = \{j \mid \pi(j) < \pi(i)\}$. Then Shapley-Shubik index $\varphi_i(G)$ for each player p_i in a WVG G is given by:

$$\varphi_i(G) = \frac{1}{n!} \sum_{\pi \in \Pi} [v(S_\pi(i) \cup \{i\}) - v(S_\pi(i))], \tag{2.2}$$

where $v(C) = \begin{Bmatrix} 1, w(C) \geq q \\ 0, w(C) < q \end{Bmatrix}$;

Number of all permutations of all players in G equals to n!

For the example 2.1 given above all permutations of players are:

3! = 6.

$[p_1, p_2, p_3]$,

$[p_1, p_3, p_2]$,

$[p_2, p_1, p_3]$,

$[p_2, p_3, p_1]$,

$[p_3, p_1, p_2]$,

$[p_3, p_2, p_1]$;

Every second player in order in each coalition is pivotal, which gives us equal Shapley-Shubik indexes for each player:

$$\varphi_1(G) = \varphi_2(G) = \varphi_3(G) = \frac{2}{6} \approx 0.333$$

As mentioned above, Shapley-Shubik and Banzhaf indexes are the most studied and used, one of the reasons for that is, that they have several important properties: *symmetry property, normalization property and dummy player property*.

Symmetry property means that indexes of players, who make same contributions to all coalitions, have equal values of indexes. As the sum of indexes of all players equal to 1 for both indexes, they have the normalization property. Another property is that dummy players always have a power of 0. These properties make Banzhaf and Shapley-Shubik indexes very convenient measures of power (Felsenthal and Machover, 1998).

Interestingly power indexes are not limited to WVGs, as they can be applied for other voting systems, where players are unequal. In voting systems without weights there are other rules, which create inequality. For example, UN Security Council consists of 5 permanent and 10 non-permanent members. To pass a motion all 5 permanent and 4 non-permanent members have to make a “yes” vote.

In other words, all permanent members have veto power; any of the winning coalitions have to contain all 5 of them. For this system we can easily calculate S-S and BZ power indexes.

Banzhaf index calculation:

$C_n^k = \frac{n!}{k!(n-k)!}$ - number of combinations of *k* players from a set of *n* players.

First we should find number of winning coalitions, coalition is winning only if it consists of 5 permanent and at least 4 non-permanent members => there are $2^{10} - C_{10}^3 - C_{10}^2 - C_{10}^1 - C_{10}^0 = 848$ of them (all possible combinations of 10 non-permanent members minus all combinations with less than 4 members, as coalition is winning only if there are >= 4 of them), of which $C_{10}^4 = 210$ consists of 9 players (minimal winning coalition).

In each coalition all permanent players are critical, and a non-permanent member is critical only in minimal winning coalitions so the total number of critical players in all coalitions equals to = 848 * 5 + 4 * 210 = 5080. For permanent members Banzhaf power index = $\frac{848}{5080}$ = 16,69%, and for non-permanent members = $\frac{210*4}{10*5080}$ = 1,65%. Roughly, power that permanent member possess is 10 times greater than a non-permanent.

Shapley-Shubik index calculation:

There are 15! = 1307674368000 possible grand coalitions.

Non-permanent member p_i can be pivotal only when he follows all 5 permanent members and 3 other non-permanent, so we should multiply all possible permutations between 8 players before and 6 players after the p_i member. Also we should account for that all non-permanent members are equal therefore the 3 of them that have to be in winning coalition other than tested member are interchangeable.

Then there are 8! * 6! * C_9^3 = 2438553600 of such coalitions. Shapley-Shubik index for non-permanent would be $\frac{2438553600}{1307674368000}$ = 0.187%. All non-permanent players combined hold power of 0.187 * 10 = 1.87%, which leaves 98.13% for permanent members, the power of single one of them = $\frac{98.13}{5}$ = 19,63%, which is more than 100 times more than non-permanent member power.

So such voting system is unequal even without weights assignation. This example is important as it helps to understand the nature of Shaplay-Shubik and Banzhaf indexes, which would be used in chapter 4.3 and 5.3 for identifying hypotheses.

2.3 A Formal Problem

This work is mostly focusing on manipulation by splitting and merging, but also includes a discussion around annexation manipulation - formal definitions for all types of manipulations are presented below.

Players who are involved in manipulation are called **manipulators**. We assume that the single goal of manipulators is to improve their power by engaging in manipulation.

Definition 2.7 *Manipulation payoff:*

To calculate the success of manipulation we use a ratio *r* between a power of player before and after manipulation - it is called **payoff**. For each type of manipulation this value is calculated with different formula. In general payoff for player is p_i :

$$r_i = \frac{\phi_i'(G)}{\phi_i(G)} \tag{2.3}$$

Definition 2.8 *Advantageous and disadvantageous manipulation*:

If r_i>1 then manipulation is **advantageous**, otherwise **disadvantageous** or **non-beneficial** (r = 1).

2.3.1 Manipulation by Splitting

Manipulation by splitting is a form of manipulation where p_i - manipulator in WVG *G* splits into *k* false-players $P_{i_k} = \{p_{i_1},..., p_{i_k}\}$ with weights $\{w_{i_1},..., w_{i_k}\}, w_{i_j} > 0, j = 1,..,k$ and $\sum_{j=1}^{k} w_{i_j} = w_i$. He alters game *G* with WVG *G'*, which has same quota *q*, but larger number of players $P' = (P \setminus p_i) \cup P_{i_k}$ with length $n' = n + k - 1$. Then payoff for splitting manipulation:

$$r_i = \frac{\sum_{j=1}^{k} \phi_{i_j}(G')}{\phi_i(G)} \qquad (2.4)$$

2.3.2 Manipulation by Merging

Manipulation by splitting is a form of manipulation where $k > 2$ players, which form a coalition C in WVG G, agreed to create a bloc (merge) that we denote as a new player p_C in WVG *G'* (game, altered by manipulation) with players $P' = (P \setminus C) \cup p_C$ and length $n' = n - k + 1$. Weight of a bloc $w_C = \sum_{i \in C} w_i$. In this work we will limit coalition C to a losing coalition, so p_C wouldn't be a dictator in *G'*. Then payoff for merging manipulation is:

$$r_C = \frac{\phi_C(G')}{\sum_{i \in C} \phi_i(G)} \qquad (2.5)$$

2.3.3 Manipulation by Annexation

Manipulation by annexation is a form of manipulation where a manipulator player p_i in WVG G annexes coalition C $(p_i \notin C)$ of length *k>0* to create a bloc $(C \cup p_i)$. An altered by annexation game *G'* has players $P' = (P \setminus (C \cup p_i)) \cup p_{(C \cup p_i)}$ and length $n' = n - k$. Weight of a bloc $w_{(C \cup p_i)} = \sum_{j \in C} w_j + w_i$. In this work we will limit coalition $(C \cup p_i)$ to be losing, so $p_{(C \cup p_i)}$ would be a dictator in *G'*. Then a payoff for annexation manipulation is:

$$r_{(C \cup p_i)} = \frac{\phi_{(C \cup p_i)}(G')}{\phi_i(G)} \qquad (2.6)$$

3 State of the Art

3.1 Relevancy of Power Indexes

Power indexes since their invention were regularly criticized, as many researchers believe that they are oversimplified and don't account for vital properties of real-life voting systems. As this work is based around the concept of *priori* power indexes, we provide an overview of works on relevancy of power indexes to identify their place in weighted voting analysis.

Banzhaf and Shapley-Shubik are the two most prominent *priori* power indexes, but there are several others, which Felsenthal and Machover (1998) proposed to divide into two categories – **I-Power** and **P–Power** indexes. I-Power stands for Power as Influence, and Banzhaf index is in this category, P-Power stands for Power as a Prize and Shapley-Shubik index falls into that category. The difference is in what index measures, I-Power indexes measure ability of a player to influence results of a voting, and P-Power measures expected prize a player gets from participating in voting. In a case of WVG a prize is either 1 or 0. The theory of I- and P-powers found support even between critics of power indexes; in particular Geoffrey Garrett and George Tsebelis (2001) agreed that difference between indexes have to be recognized, as some of the criticism isn't applicable for both types of power indexes. And while in real life they are used interchangeably because in many cases the numerical difference is insignificant, still there are reasons (Felsenthal, Leech, List, & Machover, 2003) to differentiate them and use for appropriate measurement. In general Felsenthal and Machover (1998) propose to use Shapley-Shubik or Banzhaf indexes as they don't suffer from number of paradoxes which affect other indexes behaviors and are the most studied.

As was said earlier, power analysis of real case voting systems is under a strong criticism, Garrett and Tsebelis (1999) on an example of Council of the European Union, arguing that assumptions made for indexes calculation doesn't account policy positions of EU countries governments and their strategic voting behavior which contradicts the assumption of equal probability for coalitions to form. In their work they showed the power analysis may produce inaccurate and inconsistent results when applied to a real world WVGs. They even made a statement that attempt of improvement of power indexes in response to criticism is similar to "*epicycles generated by Ptolemaic astronomers in response to anomalies in their charts*."(Garrett & Tsebelis, 1999, p306). Garrett and Tsebelis see the main problem of power indexes in their simplicity as they represent dynamic voting environment and voters behavior as a simple probability distribution. Similar opinions were presented by Albert (2003) and Gelman. Katz and Bafumi (2004) who also highlighted similar issues of power indexes. The more philosophic approach to the critic was presented by Albert (2003) in work 'The Voting Power Approach: Measurement without Theory'.

The counter-arguments were put forward by Felsenthal, Leech, List and Machover (2003) as they pointed out that term *"priori"* means that these indexes are abstracted from political or other reasons that may affect the voting behavior and should be used to study voting rules and voting system itself. They showed several examples of power indexes application on real-life voting systems that lead to some important results;in particular - a powerless Luxemburg under the Treaty of Rome, directors elections in IMF constituencies in 2002 where Estonia belongs to Nordic/Baltic constituency and had a zero power in it. We also can remember Banzhaf (1964) work Nassau County board. Power analysis has brought up a political discussion about the weighted voting systems not once and as mentioned above even lead to changes in them. Also worth mentioning that with each enlargement of EU, the question of how it will affect decision making is raising and power analysis is a tool that can, at least partially, answer that question. Antonakakis, Badinger and Reuter (2014) tracked the whole history of voting in Council of the European Union (here and after CEU) showing the changes in power of participants with each enlargement.

While the negotiations in EU on new voting system, Poland proposed to use Penrose square root rule ("Germany gives ear to Poland in Reform of Treaty talks", Goldirova, 2007) as an alternative way of weight distribution. Other example of political debate started from power theory is the question of underrepresentation of countries with a large population such as Germany in EU Council (Felsenthal and Moshé Machover, 1998, p156) (Le Breton, Montero, & Zaporozhets, 2012). Also interesting results were achieved by Kauppi and Widgren (2007), who showed that 90% of EU budgeting division is corresponded to power distribution among countries measured with Shapley-Shubik index.

The criticism of priori power index, lead to introduction of posteriori power indexes, in particular several probabilistic interpretations of Shapley-Shubik and Banzhaf were created by Staffin (1978), Paterson (2005) and Garrett and George (1996). Probabilistic indexes can give more accurate measurements, but they still can't account for every detail, as any mathematical model is a simplification. It also needs data to calculate probabilities or some kind of estimation and are very sensitive for its change.

Another way for application of power indexes is the inverse problem of finding the weights of players from the given powers. The most research in this subject was done for weights approximation from Banzhaf index values. Several algorithms were proposed de Keijzer, Klos and Zhang (2010), Aziz, Paterson and Leech (2007), Kurz (2012). This reverse approach gives a possibility to precisely design WVGs and achieve desired powers of players, overall stability and predictability of a voting system. In particular this approach can be applied to help solve a problem of expansion of a voting system, which EU and other intergovernmental bodies are facing regularly.

Power analysis proved itself to be a useful tool, even though it does in some cases give empirically wrong results but when appropriately analyzed they lead to important findings. They proved to be particularly useful for finding edge case, like Felsenthal, Leech, List and Machover (2003) showed on several examples; power indexes can serve well for finding powerless (dummy) players in complex systems, where it isn't obvious from their weights. Kauppi and Widgren (2007) have gone further with their research and presented testable results on EU budgeting, which shows actually how powerful this tool can be if applied correctly. Indexes are numerical values that can help participants of a voting to better understand their possibilities and adjust voting behavior accordingly. The choice of power index between I-Power and P-Power as well as *posteriori* and *priori* is also important as it measures power from different perspectives. *Priori* power indexes are undeniable useful on the stage of designing of WVG model when there is no data of voters behavior as Roth (1998) stated, *priori* analysis give a possibility to study rules of the voting themselves, and not be deviated by dynamical political environment.

3.2 Manipulation Study

Manipulations in different domains received a wide attention especially in the end of 20 – beginning of 21st century. In 2004 manipulation by splitting were studied for combinatorial auctions by Yokoo, Sakurai and Matsubara, (2004). Later in 2005 the results were expanded for coalitional games (Yokoo, Conitzer, Sandholm, Ohta, & Iwasaki, 2005). Felsenthal and Machover (1998) in their work described several paradoxes of power indexes, which are related to manipulations, for example **bloc paradox** ($\phi_{(C \cup p_i)}(G') < \phi_i(G)$) and described situations when annexation and merging manipulations are advantageous a priori (Felsenthal & Machover, 2002). Specifics of manipulations in weighted voting games can be found in the works of Aziz, Paterson, (2007) and its more recent development in 2009 (Aziz & Paterson, 2009). These works describe behavior of manipulations for different classes of WVGs, and study a question of finding an advantageous manipulation. The bounds for maximal and minimal payoff of manipulation by splitting were extensively researched as well and were proven in several works by Lasisi and Alan (2014) and Bachrach and Elkind (2008), while the bounds for merging manipulation are still not fully justified, latest work on this topic by R. O. Lasisi and A. A. Lasisi (2016) is missing both upper and lower bounds for Shapley-Shubik and Banzhaf indexes for merge of $k > 2$ fake-players. Zuckerman, Faliszewski, Bachrach and Elkind (2008) studied how one can influence power of players by changing the quota.

Lasisi (2013) in his work experimentally studied behavior of 3 most prominent indexes - Shapley-Shubik, Banzhaf and Deegan-Packel, under manipulations. One of the results of the work showed how a voting system designer can affect a probability of an advantageous merge manipulation by choice of a quota. Then the results on the effects of a quota were extended with probabilities of advantageous manipulation and expected values of payoff in works of Lasisi and Alan (2011). In these works authors limited the number of players in WVGs to set from 5 to 20 players because of calculation complexity of Deegan-Packel index.

This work will extend these results with study of maximal, minimal, expected values and standard payoff for manipulation by splitting and merging, which gives a more comprehensive understanding of effects of a quota on manipulations in WVGs. In this work we prove minimal and maximal bounds for merging manipulation, which were missing before. We also will touch upon how the number of players in WVGs influence manipulation payoff and extend studies of effects of quota for length of WVGs to maximum of 50 players.

3.3 The Purpose of the Thesis

The main focus of this work is to study the effects that quota have on payoff of manipulation. The work is extending the results of previous works on this topic by Lasisi and Alan (2011), Lasisi (2013), Bachrach and Elkind (2008), R.O. Lasisi and A.A. Lasisi (2016) described in Chapter 3.2, where only the expected values were analyzed, with other correlations for payoff of manipulation.

In particular the authors analyze trends for maximal, minimal values and standard deviation of payoff and a probability of advantageous manipulation. The authors also derive how trends and values are changing for WVGs with different number of players (more about simulation environment in section 4.5.1).

Most of the results are achieved for Shapley-Shubik values, as they have a number of important properties also discussed in Chapter 2.2, and resistance to a bloc paradox (Felsenthal & Machover, 1998), which makes it a convenient tool for WVG analysis.

An important part of the work is a proof of connections between maximal and minimal bounds of manipulation by merging with minimal and maximal bounds of manipulation by splitting respectively. These analyses give an opportunity to fill the gap in research (R. O. Lasisi and A. A. Lasisi, 2016) of bounds for $k > 2$ for Banzhaf and Shapley-Shubik indexes for merging manipulation addressed in Chapter 3.2

In this work we propose three hypotheses:

Hypothesis 1: *Manipulation by splitting has higher probability of being advantageous and reaches maximum values of expected payoff for WVGs with high quotas, for Shapley-Shubik index.*

To prove hypothesis we answer following questions:

- What is the effect of a quota on the expected values of payoff?

- What is the effect of a quota on the standard deviation of payoff?

- What is the effect of a quota on probability of advantageous manipulation?

- How players number affect trends of expected values and probability of advantageous manipulation for changing quota?

Substantiation of the hypothesis can be found in Chapter 4.3. Related finding and conclusion are located in Chapter 4.5.

Hypothesis 2: *Bounds for maximal and minimal possible manipulation by merging are the reversed bounds for manipulation by splitting and vice versa for all power indexes.*

To prove hypothesis we answer following questions:

- What is the connection between manipulation by splitting and merging?

- How to find maximal/minimal bounds for merging manipulation from minimal/maximal bounds of splitting manipulation?

Substantiation of the hypothesis can be found in chapter 5.3. Related finding and conclusion are located in 5.4.

Hypothesis 3: *Manipulation by merging has higher probability of being advantageous and reaches its maximum values of expected payoff for WVGs with low quotas, for Shapley-Shubik index.*

To prove hypothesis we answer following questions:

- What is the effect of quota on the expected values of payoff?

- What is the effect of quota on the standard deviation of payoff?

- What is the effect of quota on probability of advantageous manipulation?

- How players number affect trends of expected values and probability of advantageous manipulation for changing quota?

Substantiation of the hypothesis can be found in chapter 5.3. Related finding and conclusion are located in 5.5.

4 Effects of a Quota on Manipulation by Splitting

4.1 An Overview and Examples

Manipulation by Splitting:

As was shown in works of Lasisi (2013), Bachrach and Elkind (2008) and others Banzhaf and Shapley-Shubik indexes are sensitive to the **manipulation by splitting**, suggesting that it could be beneficial for a player p_i to split into k false-players $P_{i_k} = \{p_{i_1},...,p_{i_k}\}$, dividing his weights between them $\{w_{i_1},...,w_{i_k}\}, w_{i_j} > 0, j = 1,..,k$.

Consider the example from work of Lasisi (2013):

G = [12; 6, 5, 4, 4, 3] is a WVG of 5 players, the power of last $w_5 = 3$;

Banzhaf index for p_5: $\beta_5(G)$ = 0.172

Shapley-Shubik index for p_5: $\varphi_5(G)$= 0.167

Suppose player p_5 split into 3 false-players with weights equal 1, then the altered by this action game *G* would be *G'*:

G'= [12; 6,5,4,4,1,1,1]

The powers of false-players would be:

Banzhaf index: $\beta_{5_1}(G) = \beta_{5_2}(G) = \beta_{5_3}(G) = 0.059$,

Combined Banzhaf index $\sum_{j=1}^{k} \beta_{5_j}(G') = 0.177 > \beta_5(G)$,

Shapley-Shubik index: $\varphi_{5_1}(G) = \varphi_{5_2}(G) = \varphi_{5_3}(G) = 0.057$, Their combined power $\sum_{j=1}^{k}\varphi_{5_j}(G')=0.171>\varphi_5(G)$.

Based on this result we can say that p_5 have benefited from the split manipulation. Therefore manipulation was **advantageous**.

4.2 Unanimity Weighted Voting Games

For unanimity games it is proven that split is always advantageous. We provide the proof as helps further understand the nature of used power indexes.

Theorem 4.1: In Unanimity WVGs with *n*>1 players manipulation by splitting is always advantageous for Banzhaf, Shapley-Shubik power indexes.

Proof:

1. For Unanimity WVG *G* Banzhaf index for each player is equal to $\beta_i(G)=\frac{1}{n}, i=1,...,n$ as there is only one winning coalition (grand coalition), each player of which is critical.

If player p_i in *G* splits into *k* > 1 false-players, the power of each player in altered WVG *G'* would be $\beta_i(G')=\frac{1}{n+k-1}, i=1,...,n+k-1$, and the combined power of false-players $\sum_{j=1}^{k}\beta_{i_j}(G')=\frac{k}{n+k-1}$.

Now subtract power of initial player p_i in *G* from combined power of false-players in *G'*:

$\sum_{j=1}^{k}\beta_{i_j}(G')-\beta_i(G)=\frac{k}{n+k-1}-\frac{1}{n}=\frac{nk-n-k+1}{n(n+k-1)}=\frac{n(k-1)-(k-1)}{n(n+k-1)}=\frac{(n-1)(k-1)}{n(n+k-1)}$, as *k* and *n* are greater than 1 => $\frac{(n-1)(k-1)}{n(n+k-1)}>0$ => $\sum_{j=1}^{k}\beta_{i_j}(G')>\beta_i(G)$ => **split was advantageous for Banzhaf index.**

2. For Shapley-Shubik index: In Unanimity WVG with n players, any player is pivotal only if he is the last one in a permutation, then Shapley-Shubik index for player p_i, $\varphi_i(G) = \frac{(n-1)!}{n!} = \frac{1}{n}$.

Then in G' power of each player $\varphi_i(G') = \frac{1}{n+k-1}, i = 1,...,n+k-1$, as it was shown above the value of $\frac{k}{n+k-1}$ is greater than $\frac{1}{n}$ => **split was advantageous for Shapley-Shubik index**, **theorem 4.1 proven**.

4.3 Non-unanimity Weighted Voting Games

It is obvious that any winning coalition with fake-players will be winning if they are replaced with initial player. In this sense, we can say that manipulation by splitting doesn't bring new winning coalitions to the game, and the increase in their number is done due to the increased number of combination with fake-players.

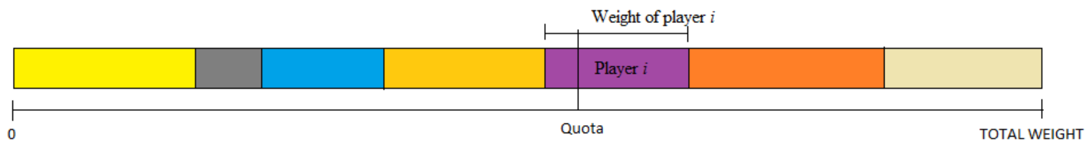

Figure 4.1. Sequential coalition where player p_i is pivotal.

If a player p_i is pivotal in a grand coalition (Figure 4.1), it doesn't matter in which order players to the left from him and to the right come, as long as they don't switch from left to right. So there is no need to check every possible coalition, just those with unique set of players to the left of the analyzed player and calculate all combinations of players from the left and right which is equal to:

$(m-1)!(n-m)!$, where m is a number of players in winning coalition.

This observation is lying behind the direct enumeration method of calculation of Shapley-Shubik index and can be applied to evaluate effects of splitting on it.

When we split p_i into *k* fake-players one winning coalition for p_i in *G* produces several winning coalitions in *G'* due to increased number of players. Following this line of thought, we can try to estimate number of produced winning coalitions.

$(m-1+l)!(n-m+k-l)!$, where *l* is a number of fake-players in the winning coalition, it is also possible that fake-players are interchangeable, for example if they have equal weights, then number of coalitions = $(m-1+l)!(n-m+k-l)!*h$, where *h* is a number of permutations between fake-players in which one of them has to be pivotal. This number is obviously maximal with maximal number of interchangeable fake-players in coalition, this is most likely when weights of fake-players are equal and have minimal possible weight. Let's investigate two other arguments of multiplication:

$$(m-1+l)!(n-m+k-l)! \to \max \tag{4.1}$$

as $(m-1+l)$ and $(n-m+k-l)$ are connected (with increase of one of them other one is decreasing) the maximum value of (4.1), because of the nature of factorial, (4.1) will be achieved $l = k$ and $(m-1) > (n-m)$ or when $l = 0$ and $(m-1) < (n-m)$.

$(m-1)$ is maximal when $m = n$ => $\max(m-1) = (n-1)$;

$(n-m)$ is maximal when $m = 2$ => $\max(n-m) = (n-2)$;

$(n-1) > (n-2)$ => maximum is achieved when m = n, which means that the player is pivotal in a grand coalition. These results support previously gained results in theorem 4.1 and also shows manipulation by splitting is not only always beneficial for a manipulator in unanimity but also reaches its greatest values for WVGs in which winning coalitions consist of most possible number of the players, which in most cases means that WVG is close to unanimity (*q* is close to $\sum W$).

Therefore, from these observations we can formulate ***Hypothesis* 1:**

Manipulation by splitting has higher probability of being advantageous and reaches maximum values of payoff for WVGs with high quotas, for Shapley-Shubik index.

4.4 Bounds

In the work by Bachrach, and Elkind (2008) were proven the theoretical bounds of manipulation by splitting for k = 2 fake-players for Shapley-Shubik and Banzhaf indexes, this result was expanded for $k > 2$ by Lasisi (2013).

Table 4.1. Bounds of splitting manipulation.

Bounds	Shapley-Shubik index	Banzhaf index
Upper $(k=2)$	$\varphi_{i_1}(G')+\varphi_{i_2}(G')\le\frac{2n}{n+1}\varphi_i(G)$	$\beta_{i_1}(G')+\beta_{i_2}(G')\le 2\beta_i(G)$
Lower $(k=2)$	$\varphi_{i_1}(G')+\varphi_{i_2}(G')\ge\frac{2}{n+1}\varphi_i(G)$	$\beta_{i_1}(G')+\beta_{i_2}(G')\ge\frac{1}{n}\beta_i(G)$
Upper $(k>2)$	$\sum_{j=1}^{k}\varphi_{i_j}(G')\le\frac{nk}{n+k-1}\varphi_i(G)$	$\sum_{j=1}^{k}\beta_{i_j}(G')\le k\beta_i(G)$
Lower $(k>2)$	$\sum_{j=1}^{k}\varphi_{i_j}(G')\ge\frac{k}{C(n+k-1,k-1)}\varphi_i(G)$	$\sum_{j=1}^{k}\beta_{i_j}(G')\ge\left[\frac{1}{1+\frac{(n-1)\cdot 2^{n+k-1}}{k\sum_{x\in P}\eta_x(G)}}\right]\cdot\beta_i(G)$

Where $\eta_x(G)$ - number of winning coalitions for player x.

As it is evident from the table 4.1, the upper bounds for Shapley-Shubik and Banzhaf indexes are getting higher with increase of n and k, which suggests that for higher number of players and weight of player p_i (manipulator) the maximal possible payoff is also higher.

4.5 Simulation

In this section we would make a computer simulation to provide experimental results to empirically support theoretical results achieved above (section 4.3) and test ***hypothesis 1*** for manipulation by splitting, as well as find other properties and correlations.

The main objectives of the simulation are:

1. Find an evidence to prove that maximal payoffs for manipulation by splitting are achieved for high quotas.

2. Find an evidence to prove that chance of finding a beneficial split is higher for a higher quota.

3. Find effects of quota on standard deviation of payoff.

4. Find effects of number of players in WVGs on payoff properties.

For these purposes the authors developed a set of MatLab programs that could be found in Appendix A. Similar simulations have been done previously by Lasisi (2013) and our results are comparable, however this work offers an extension to previous research as well as specific observations that were not addressed in the previous studies highlighted in chapter 3.3.

For a payoff measurement we have chosen Shapley-Shubik index, but as it is also evident from the results from Lasisi (2013) and Lasisi and Alan (2011) payoffs measured with Banzhaf and Shapley-Shubik indexes react similarly to changes in WVGs, therefore we can suggest that trends and ***hypotheses 1 and 2*** are also applicable for Banzhaf index. Testing of these hypotheses for Banzhaf index can be a field for future research.

4.5.1 Simulation Environment

We have developed several functions that create random non-unanimity weighted voting games, and measure ratio between S-S index of a player before manipulation and sum of S-S indexes of created by split fake-players. For each WVG a quota was described as a percentage from a total weight of players. Each WVG was tested for quotas from 0.5 to 0.95 (1 wasn't included, because as was proven above, split in unanimity games is always advantageous) with increments of 0.025. To create split simple uniformly distributed pseudorandom was used, while in work of Lasisi and Alan (2011) random distributions and specific algorithms for splits were used, the trends of measured properties proven to be very similar, so the choice towards uniform distribution was done to save computational time. Because maximal and minimal bounds of payoff (section 5.4) for splitting manipulation are dependent on number of fake-players manipulator splits into, for single test we used same number of fake-players (usually $k = 2$) for each random WVG for each quota, to avoid inconsistency in results and separate the effects of quota from other factors.

For WVGs creation two different algorithms were used, first uses independent uniformly distributed pseudorandom for generating a weight for each player; second algorithm assigns weights with a regression uniformly distributed pseudorandom to avoid big gaps between closest weights in small WVGs ($n < 10$). But results proved to be almost identical, so most of WVGs from the presented in this section results were created using first algorithm, as it works slightly faster and creates wider range of different WVGs.

Two algorithms were used to calculate Shapley-Shubik index two: simple algorithm of direct enumeration, which is slow but gives the most accurate results and it can be easily modified to take into account more complex voting systems with, for example, double majority system or other additional rules. It also works quickly enough for smaller WVGs ($n < 16$), but as its complexity $O(2^n)$ growing exponentially it is almost impossible to use it for WVGs with $n > 25$. Second algorithm that was used is based on generated functions proposed by Mann and Shapley (1962) specifically designed to calculate powers for large WVGs. We use interpretation given by Lambert presented in work of Leech (2002). This algorithm is much faster than the first one for WVGs with $n \geq 16$. Complexity of algorithm $O(qn^2)$ which makes it possible to calculate indexes for WVGs with even $n > 100$ players.

Nevertheless, even if using generated functions, big number of tests for large WVGs takes a very substantial time, that's why we have limited number of tested samples of WVGs to no more than 1000 and number of players n = from 5 to 50.

4.5.2 Simulation Results and Analysis

The results are presented in graphs with analysis trends rather than specific values, because of a random nature of tests, values are not representable as such. For the same reason and because of the limits of test samples the variance results, especially of extreme (max and min) values between different tests are quite noticeable, but general trends and average values are much closer between tests. Although we can't make conclusions on monotonicity (only suggestions) and exact values trends can be analyzed as they differ slightly from test to test, but are never contradictory.

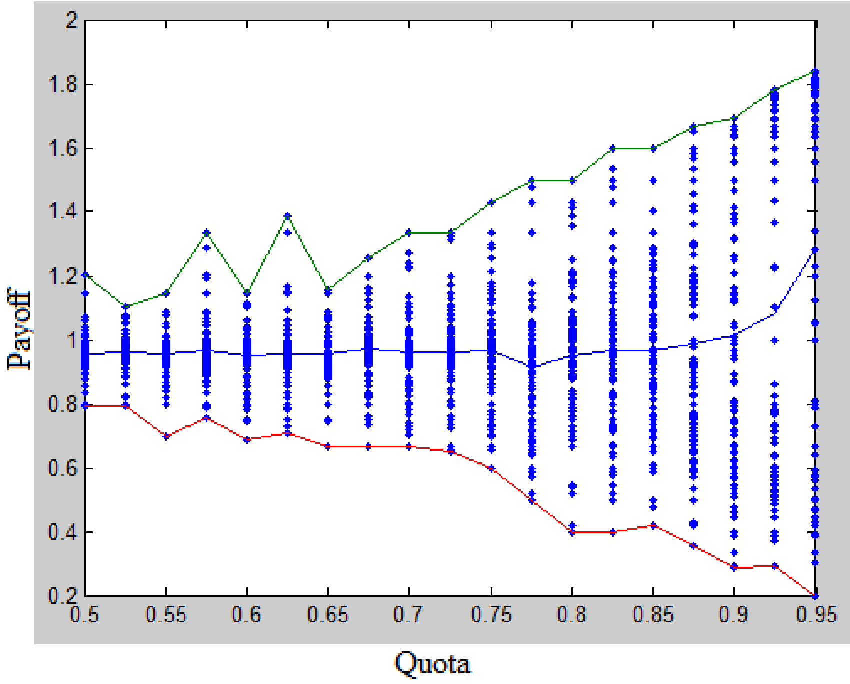

Figure 4.2. Minimal (red), Maximal (green) and Expected (blue) payoff for each quota for WVGs with 5 to 20 players.

Average values (**Expected Values**) of split are relatively steady (slightly lower than 1), with noticeable increase for close to 1 quotas, which means that expected payoff of a split for close to unanimity games is advantageous, which supports ***hypothesis 1***.

According to the results of the test, which is represented on the Figure 4.2, a manipulator can achieve the highest payoff when quota is high, while lowest values are reached for low quota. For any quota value the advantageous manipulation was found with at least ~1.4 payoff. The result differs for the games with high number of players, where even maximum values are lower than 1 until the very high quotas, which is shown on Figure 4.7.

Maximal disadvantage from split increasing with growth of quota (as can be seen on Figure 4.2), trend is opposite to maximal advantage, which shows instability of WVGs with high quotas, as splitting manipulation with high probability will have great effect on power distribution among the players.

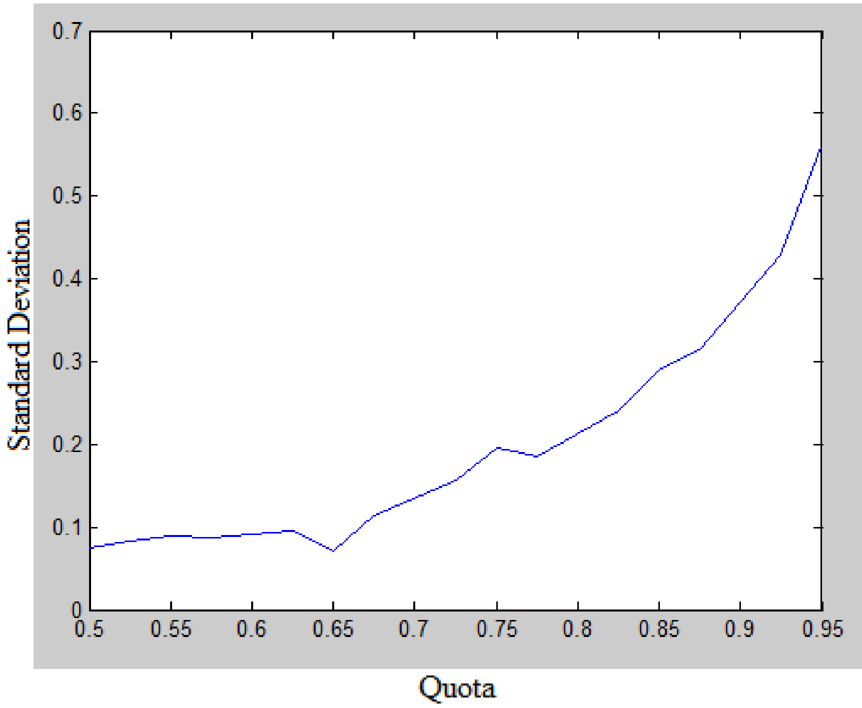

Figure 4.3. Standard deviation of expected payoff of a manipulation by splitting for WVGs with 5 to 20.

In Figure 4.2 minimal and maximal values represent maximal deviation from the expected value, Figure 4.3 shows standard deviation which is almost monotonically increasing. Payoffs for splits differ more from average value for high quotas; actually based on Figure 4.2 it visible that they tend to polarize to extreme values. Standard deviation is correlating with number of players and fake-players, which is evident from the upper and lower bound. Because of that for this test we limited number of fake-players to 2, to create equal environment for advantageous and disadvantageous splits (as in tests with random split greater instability of the results may happen). For large WVGs standard deviation tend to be equally small for low and medium quota and rapidly grow for high quotas.

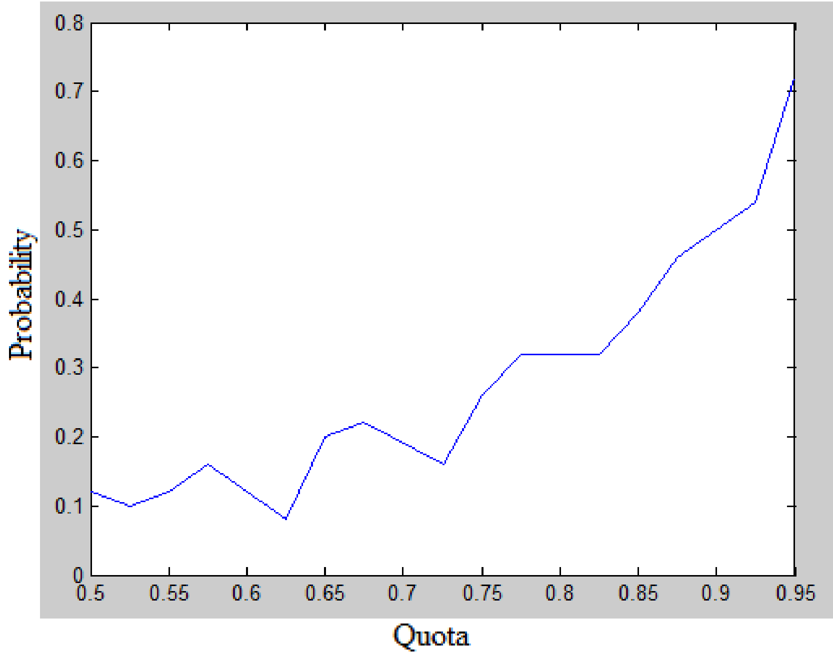

Figure 4.4. Probability of an advantageous split manipulation for WVGs with 5 to 20 players.

Figure 4.4 shows that probability for advantageous split has similar trend and is also growing with the quota. It has very low values (under 0.2) for small quotas, while for the highest quotas probability was higher than 0.5, so most of splits were advantageous, which supports ***hypothesis 1***. Thus high quotas are a great target for manipulation by splitting, as most of the splits were advantageous.

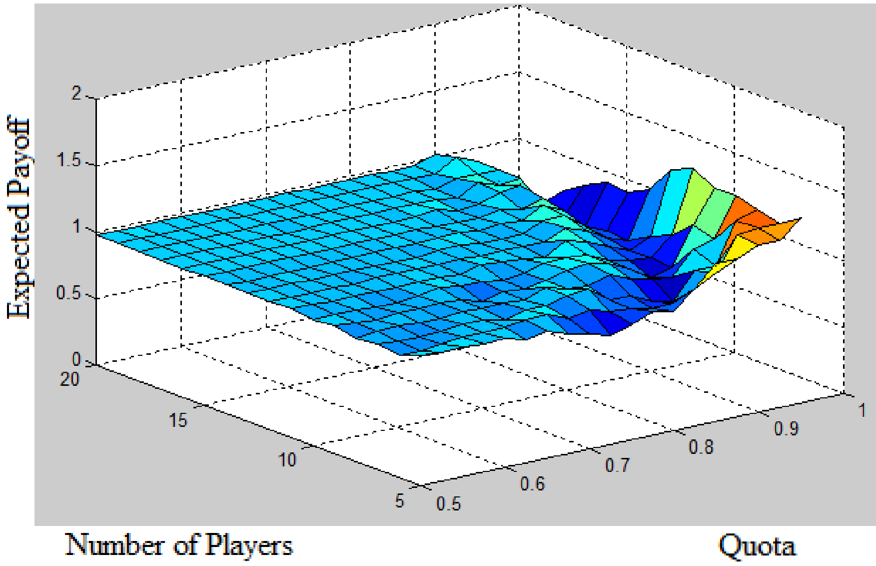

Figure 4.5. Expected value of payoff for splitting manipulation for quotas from 0.5 to 0.95 and number of players from 5 to 20. 50 tests for each number of players and quota value. Axis *y* (number of players) is reversed for presentation purposes.

Average payoff of manipulation also depends on a number of players in WVG, as shown on Figure 4.5. Games with higher number of players show the highest expected values for high quotas, but trend isn’t so clear, as for all tested number of players there is a dent, which gets closer to quota value of 1 with growth of number of players. On other hand, standard deviation and extreme values show the same trends (Appendix B.1) for any number of players, with the difference that maximal probability for larger game happens for values of quota around 0.9.

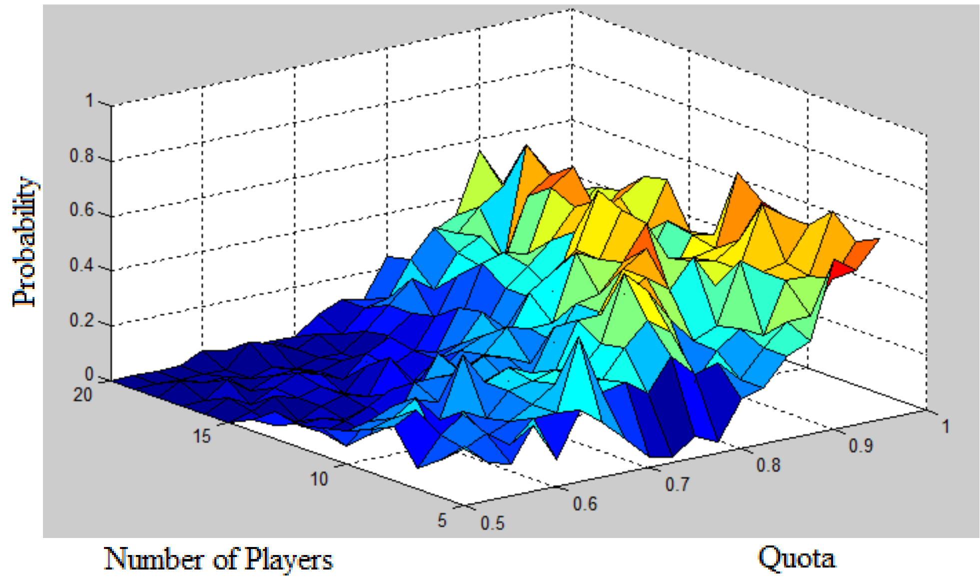

Figure 4.6. Probability of advantageous split for manipulation by merging for quotas for different number of players in WVGs. WVGs with 5 to 20 players. 50. Take a notice that axis *x* (quota) is reversed for presentation purposes.

Similarly to Figure 4.4, Figure 4.6 is very nonmonotonic with local extremes almost for each value of quota, but the trend is still visible. Overall, for any number of players (from 5 to 20) the trend of probability is growing, for WVGs with high number of players it is more smooth but is also more curved, as the values for small quotas are close to zero (on a contrary from small WVGs) and the maximal values are almost the same.

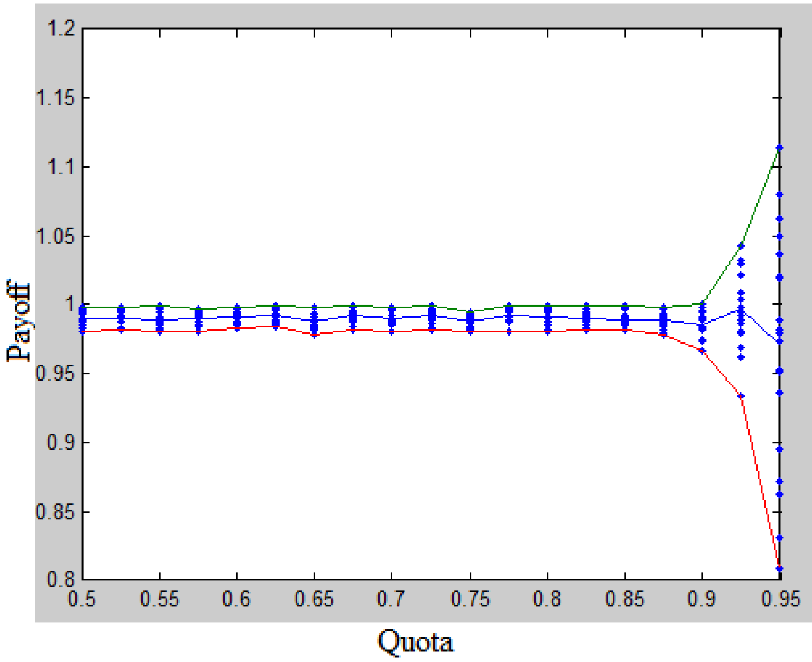

Figure 4.7. Minimal, maximal and expected values of payoff for each quota in WVGs with 50 players for split manipulation.

The difference for WVGs with 50 players (Figure 4.7) to, where WVGs with 5 to 20 players (Figure 4.2) is quite significant. We see that, even though trends are similar, the results for small, medium and even high quotas ($q < 0.9$) are very steady, with maximal, minimal and expected values all lower than 1. Still the highest values of payoff and probability of advantageous (Appendix B.2 and B.3) split are found for high quota, which supports ***hypothesis 1***. From this we can conclude that for large WVGs the splitting manipulation is very likely to be disadvantageous with expected payoff little lower than 1 for almost any quota except very high values > 0.9.

Power indexes are quite sensitive to number of players, but as most of the real life WVGs are in bounds from 5 to 50, (with few exceptions), in this work, for the most part, WVGs with n = [5,20] are evaluated, some results are separately achieved for WVGs with 50 players. The problem of larger WVGs can be seen as an area of future research, as there are only few works, which study their specifics even outside of manipulation context, for example work of Lindner (2004). This is especially true for experimental studies, because of high computational complexity and memory requirements of algorithms.

4.6 Conclusion

Results from the tests support theoretical results - best possible splitting manipulations happen with high quotas, as well as the probability of advantageous split. In other words, high quotas are good target for manipulation by splitting, while for smaller quotas advantageous splitting is less probable and less effective, also overall results of manipulations are more predictable for them. Manipulation by splitting in games with high quota will likely to change power distribution of players' significantly. And these results fully reflect ***hypothesis 1.*** We also found that WVGs from small to high number of players (5 – 50) behave slightly different, but don't contradict the ***hypothesis 1***. In general strong correlation of extreme values, standard deviation and probability of advantageous split to quota is evident; expected values are slightly less affected (Correlation coefficients can be found in Appendix C.1).

This result can be applicable in real life, even though it is unlikely that, for example, splitting may happen in Council of the European Union because of nature of players, but it may happen in national parliaments, so the relevance of manipulation analysis should be tied with realistic possibilities of such action. In some cases danger from splitting manipulation can be neglected.

5 Effects of a Quota on Manipulation by Merging

5.1 An Overview and Examples

Next possible manipulation is **manipulation by merging**, on a contrary to splitting manipulation it actually can happen quite easily, as it does require only an agreement between players involved in a manipulation, which in most cases isn't forbidden. If several players who can't create winning coalition agreed to act identically they form a bloc, they can be seen as a single player with a weight equal to the sum of weights of players in bloc. It may seem that the bloc power would be equal or greater than a sum of the players in it, but actually it can differ quite significantly. If bloc power is greater than cumulative power of its players before merge => $r > 1$ then the merge manipulation is advantageous (beneficial), if lesser - then disadvantageous.

Consider the slightly modified example of advantageous manipulation by merging in WVGs, from Lasisi (2013):

Example 5.1:

G = [18; 8, 7, 4, 4, 2, 1, 1] in which players from coalition C = [p_2, p_3, p_6], form a bloc with cumulative weight equal to $w(C) = 7 + 4 + 2 = 13$ in a G' = [18; 13, 8, 4, 1, 1]

Combined S-S index of p_2, p_3, p_6 in G: $\varphi_2(G) + \varphi_3(G) + \varphi_6(G) = 0.492$;

Combined BZ index of p_2, p_3, p_6 in G: $\beta_2(G) + \beta_3(G) + \beta_6(G) = 0.495$;

S-S index for p_C in G': $\varphi_C(G') = 0.617$;

BZ index for p_C in G': $\beta_C(G') = 0.524$.

Payoff of manipulation for S-S index = $\dfrac{0.617}{0.492} = 1.254$;

Payoff of manipulation for BZ index = $\frac{0.524}{0.495} = 1.05$.

=> For S-S and BZ indexes merging manipulation was advantageous.

5.2 Unanimity Weighted Voting Games

Similarly as for splitting manipulation, it is possible to evaluate merge manipulation theoretically for unanimity WVGs.

Theorem 5.1: In unanimity WVGs manipulation by merging of $2 \le k < n$ players is always disadvantageous for to BZ and S-S indexes.

Proof of this theorem is almost identical to proof of theorem 4.1

Proof:

As was shown in proof of theorem 4.1, for unanimity WVG *G* BZ index of a player equals to his S-S index $\beta_i(G) = \varphi_i(G) = \frac{1}{n}$, *i* = 1,…,*n* where *n* is a number of players before merging.

Players $P_C = [p_1,...,p_k]$ are involved in manipulation by merging. After merging number of players in altered from *G* WVG *G': n'* = $(n-k+1)$;

Cumulative power of manipulators before manipulation for both indexes $\sum_{i \in C} \beta_i(G) = \sum_{i \in C} \varphi_i(G) = \frac{k}{n}$, and after $\beta_C(G') = \varphi_C(G') = \frac{1}{n-k+1}$.

Payoff of this manipulation:
$$\sum_{i \in C} \beta_i(G) - \beta_C(G') = \sum_{i \in C} \varphi_i(G) - \varphi_C(G') = \frac{k}{n} - \frac{1}{n-k+1} = \frac{kn - k^2 + k - n}{n(n-k+1)} = \frac{(k-1)(n-k)}{n(n-k+1)},$$
because *1 < k < n,* (*k – 1*)*,* (*n – k*) *and* (*n – k* + 1) are positive => total power of manipulators was higher before manipulation than power of a bloc after it => **manipulation was disadvantageous, theorem 5.1 is proven**.

5.3 Non-unanimity Weighted Voting Games

Manipulation by merging can be looked at as a reversed procedure to manipulation by splitting, as it is obvious that if split in *G* into several fake-players in *G'* was disadvantageous, the reverse merge of fake-players in *G'* to original player in *G* will be advantageous. This observation will be further developed in the next section 5.4.

Based on this observation we propose **Hypothesis 2:**

Bounds for maximal and minimal possible manipulation by merging are the reversed bounds for manipulation by splitting and vice versa for all power indexes.

Because of that we can find properties of a payoff of merge using intermediate results and similar reasoning used for manipulation by splitting. Based on this line of thought, manipulation by merging is most likely to be advantageous (4.1) -> min, where k – number of players forming a bloc, which can be achieved, as it was shown above, when $(m-1+l)=(n-m+k-l)$.

$2m-n+1=k-2l$, *n* and *k* here are constants, then the greater *m* is, the lesser *l* should be; the more players are there in a winning coalition, the lesser number of manipulators have to be in it. This situation is more likely to happen to a lot of coalitions when quota is small. It should be pointed out, that for small non-unanimity WVGs this situation is also likely to happen for any quota values.

Therefore, we can formulate **Hypothesis 3:**

Manipulation by merging has higher probability of being advantageous and reaches its maximum values of expected payoff for WVGs with low quota, for Shapley-Shubik index.

5.4 Bounds

Boundaries for manipulation by splitting are well studied, when boundaries for merging manipulations are still a question. The problem of complexity of finding a beneficial merge received the most of an attention. Only in recent years R. O. Lasisi and A. A. Lasisi (2016) proved upper and lower boundaries for 2 players merge. Here we propose a method of finding bounds for merging using bounds for splitting manipulation, we proof that bounds of merging manipulation are, in fact, reverse bounds for splitting manipulation for all of power indexes.

Based on this we can formulate theorem:

Theorem 5.2 If r is maximal/minimal possible payoff of a split manipulation for *k* false-players in WVG *G* with n players that altered game *G* to *G'* with *n'* = (*n* + *k* – 1) players. Then payoff of a merge in WVG *G'* of false-players would be minimal/maximal possible for k players in WVG of (*n* + *k* -1) players and vice versa.

Proof:

Let player p_i be a manipulator in WVG $G^{(1)}$ with n players, who involved in the most advantageous split possible with k false-players to $P_k = \{p_{i_1}, ..., p_{i_k}\}$ alter $G^{(1)}$ with WVG $G^{(2)}$ with *n'* = (*n* + *k* – 1) players. $r^{(1)} = \frac{\sum_{j=1}^{k} \phi_{i_j}\left(G^{(2)}\right)}{\phi_i(G^{(1)})}$ is maximal possible (reached the upper bound). For simplicity we denote use $\phi^{(1)} = \phi_i(G^{(1)})$ and $\phi^{(2)} = \sum_{j=1}^{k} \phi_{i_j}\left(G^{(2)}\right)$ then $r^{(1)} = \frac{\phi^{(2)}}{\phi^{(1)}}$.

Assumption: There exist merge manipulation for WVG $G^{(3)}$ with *n'* players for *k* manipulators that form coalition C, with the minimal possible payoff $r^{(2)} = \frac{\phi^{(4)}}{\phi^{(3)}} \neq \frac{1}{r^{(1)}}$.

$\phi^{(3)}$ - cumulative power of manipulators before manipulation, $\phi^{(4)}$ is a power of a bloc P_C after manipulation.

Manipulation alters game $G^{(3)}$ is alternated with WVG $G^{(4)}$ with (n' – k + 1) = n players. => $r^{(2)} = \frac{1}{r^{(1)}}$.

Let's consider a split manipulation in game $G^{(4)}$ of a player P_C into fake-players with equal weights to weights of players of C in $G^{(3)}$, then payoff of this splitting manipulation is $r^{(3)} = \frac{\phi^{(3)}}{\phi^{(4)}} = \frac{1}{r^{(2)}}$.

Let's consider a split manipulation in game $G^{(4)}$ of a player P_C into fake-players with equal weights to weights of players of C in $G^{(3)}$, then payoff of this splitting manipulation is $r^{(3)} = \frac{\phi^{(3)}}{\phi^{(4)}} = \frac{1}{r^{(2)}}$.

We have two options:

1) If $r^{(2)} < \frac{1}{r^{(1)}}$:

=> $r^{(1)} < \frac{1}{r^{(2)}}$ => $r^{(1)} < r^{(3)}$ which contradicts to the conditions that $r^{(1)}$ is maximal possible payoff of splitting in WVG of *n* players into *k* false-players.

2) If $r^{(2)} > \frac{1}{r^{(1)}}$:

Let's consider a merge manipulation in $G^{(2)}$ of players P_k to create a WVG equal to $G^{(1)}$, as a $w(P_k) = w(p_i)$ and all other players are the same. Payoff of this manipulation would be equal to $r^{(4)} = \frac{\phi^{(1)}}{\phi^{(2)}} = \frac{1}{r^{(1)}}$ => $r^{(2)} > r^{(4)}$ => minimal possible payoff for merging manipulation of *k* manipulators in WVG of *n'* players, which contradicts the assumption.

From 1) and 2) => $r^{(2)} = \frac{1}{r^{(1)}}$, which means that the most advantageous split in WVG of *n* players to *k* false-players is reversed to a most disadvantageous merge of *k* players in WVG of (*n* + *k* – 1) players.

Proof on reversed properties of upper bounds of merge manipulation with lower bounds of splitting manipulation is analogous. => **Theorem 5.2 proven**.

Theorem 5.2 proves that ***hypothesis 2*** is justified.

Using Theorem 5.2 we can write full bounds of manipulation by merging reversing bounds for splitting manipulation:

Lower Shapley-Shubik index bound for merge from Upper Boundary for splitting manipulation:

$$\sum_{j=1}^{k} \varphi_{i_j}(G') \leq \frac{nk}{n+k-1}\varphi_i(G)$$

$$\varphi_i(G) \geq \frac{n+k-1}{nk}\sum_{j=1}^{k} \varphi_{i_j}(G') \tag{1}$$

k = number of fake-players in G', then for reverse merge manipulation k = to number of manipulators. n = number of players in G, $n' = n + k - 1$, then $n = n' - k + 1$, then (1):

$$\varphi_i(G) \geq \frac{(n'-k+1)+k-1}{(n'-k+1)k}\sum_{j=1}^{k} \varphi_{i_j}(G')$$

$$\varphi_i(G) \geq \frac{n+k-1}{nk}\sum_{j=1}^{k} \varphi_{i_j}(G') \tag{2}$$

After switching notations between G and G', n and n' we can rewrite (2) to achieve lower bound of merging manipulation for S-S index for $k > 2$ manipulators.

$$\textit{Lower bound} = \varphi_m(G') \geq \frac{n}{(n-k+2)k}\sum_{i=1}^{k} \varphi_{m_i}(G);$$

Using analogous transformation, we can achieve Upper and Lower bounds for both Shapley-Shubik and Banzhaf indexes from table 5.1.

Table 5.1. Bounds of merging manipulation.

Bounds	**Shapley-Shubik index**	**Banzhaf index**
Upper $(k=2)$	$\varphi_m(G') \leq \frac{n-k+2}{2}\left(\varphi_{m_1}(G)+\varphi_{m_2}(G)\right)$	$\beta_m(G') \leq (n-1)\left(\beta_{m_1}(G)+\beta_{m_2}(G)\right)$
Lower $(k=2)$	$\varphi_m(G') \geq \frac{n-k+2}{2(n-k+1)}\left(\varphi_{m_1}(G)+\varphi_{m_2}(G)\right)$	$\beta_m(G') \geq \frac{1}{2}\left(\beta_{m_1}(G)+\beta_{m_2}(G)\right)$

Upper $(k > 2)$	$\varphi_m(G') \geq \frac{C(n,k-1)}{k} \sum_{i=1}^{k} \varphi_{m_i}(G)$	$\beta_m(G') \leq 1 + \frac{(n-k)2^n}{k \sum_{x \in P'} \eta_x(G')} \sum_{i=1}^{k} \beta_{m_i}(G)$
Lower $(k > 2)$	$\varphi_m(G') \geq \frac{n}{(n-k+2)k} \sum_{i=1}^{k} \varphi_{m_i}(G)$	$\beta_m(G') \geq \frac{1}{k} \sum_{i=1}^{k} \beta_{m_i}(G)$

Where $\eta_x(G')$ - number of winning coalitions for player *x* in *G'*.

5.5 Simulation

This section provides experimental results to empirically support achieved theoretical result for manipulation by merging, as well as shows other properties and correlations. Main objectives were:

Main objectives of simulation:

1. Find an evidence to prove that chance of finding a beneficial merge is higher for lower quotas.

2. Find effects of quota on standard deviation.

3. Find effects of number of players in WVGs on payoff properties.

5.5.1 Simulation Environment

To test results for manipulation by merging the same simulation environment was used as for manipulation by splitting, as well as the same set of programs, which were modified to create a random bloc (of random length) that is not a dictator. Lasisi (2013) used both random and specific algorithm for creation of a bloc, and the results are similar to our findings with a random split based on uniformly distributed pseudorandom. Because maximal and minimal bounds of payoff (section 5.4) for merging manipulation are dependent on number of manipulators conducting a merge, for single test we used same number of manipulators or small range ([2,4]) in each game, to avoid inconsistency in results and separate the effects of a quota from other factors.

5.5.2 Simulation Results and Analysis

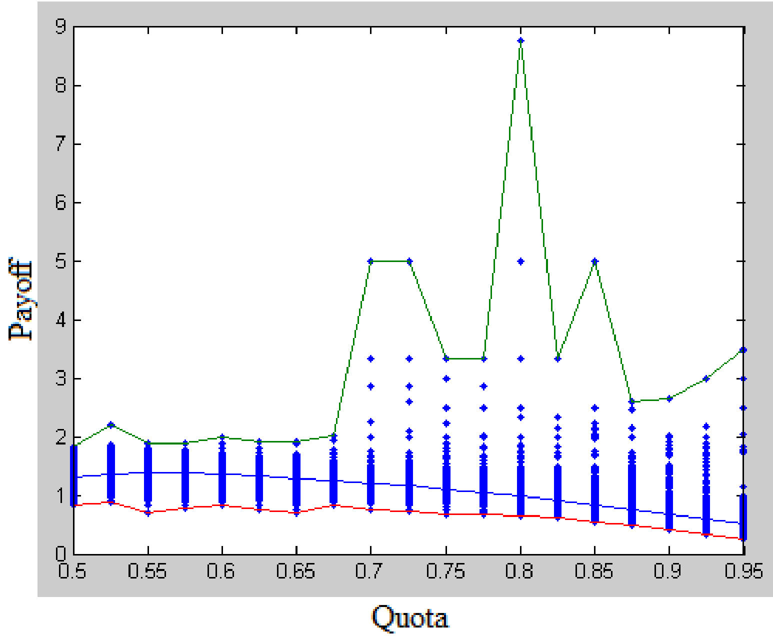

Figure 5.1. Maximal (green), Minimal (red), Expected (blue) payoff for Merge manipulation for WVGs with 5 to 20 players.

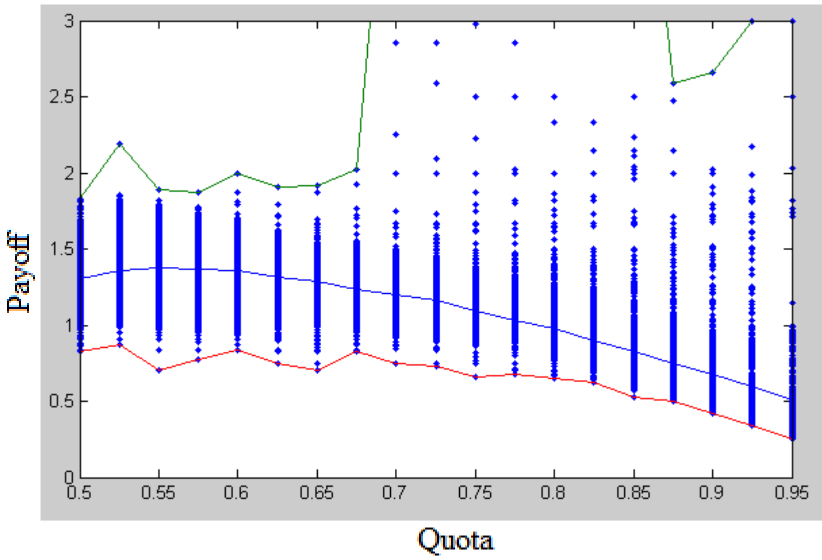

Figure 5.2. Maximal (green), Minimal (red), Expected (blue) payoff for Merge manipulation for WVGs with 5 to 20 players, clamp Figure 5.1 to clearly see trend for expected values.

Results of the tests can be seen on a Figure 5.1 and 5.2 - one can notice several interesting trends. Expected payoff is decreasing with growth of a quota, which supports the theoretical findings, as the probability of right conditions for advantageous split are decreasing. High payoffs are also achieved for high quotas which proves the reverse to manipulation by splitting properties, as the most disadvantageous split would be a most advantageous merge. And the highest value achieved through all handled tests = 8.74, it was achieved in a WVG G = [136; 48, 45, 39, 13, 12, 7, 5] (quota ratio = 0.8) where p_4, p_5, p_6, p_7 form bloc to create new WVG G' = [136; 48, 45, 39, 37].

The cumulative Shapley-Shubik index before merge = 0.0286;

After Merge = 0.25;

Accordingly to formulas for payoff bounds of merge, which were derived in the previous section, for WVGs with 8 players with 4 manipulators, upper bound is 8.75, so in this case maximum was almost reached.

It is easy to notice that G' is, in fact, a unanimity game, as only a grand coalition is winning, which can be considered as a border case. This is an example of gaining of extreme payoff, which shows that small weighted players can greatly improve their power if they form a bloc, which would transform WVG to unanimity.

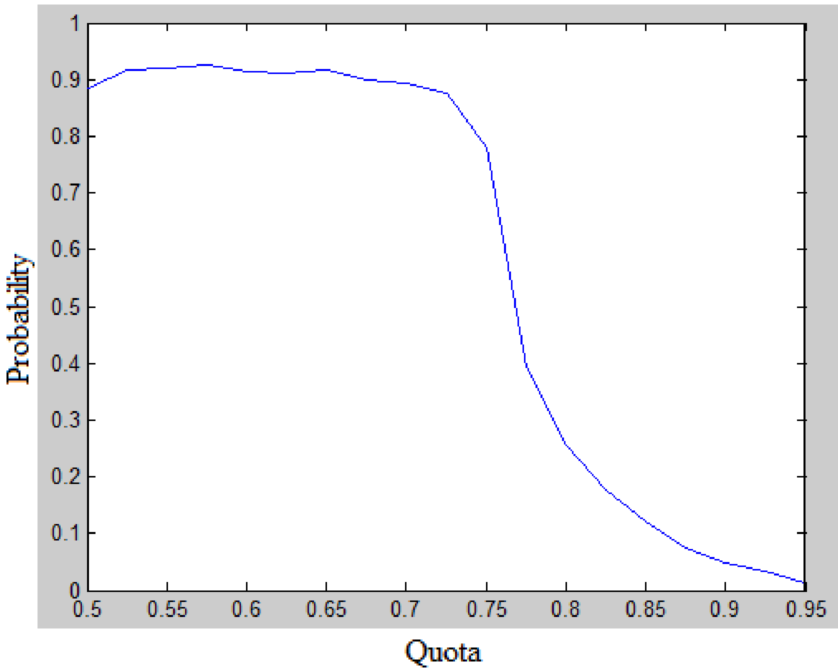

Figure 5.3. Probability of advantageous merge manipulation for WVGs with 5 to 20 players.

The influence on probability of advantageous merge, which can be seen on Figure 5.3, was expected from theoretical findings, and is decreasing with a growth of a quota after a certain point (around 0.725) and with a very high rate, to the values close to 0, as a chance of coalitions that consists of half of all players is very low. For quotas from 0.5 to 0.725 the probability is close to 1, which means that almost every merge is advantageous, which makes this values of quota a good target for manipulation by merging. This result supports ***hypothesis 3***.

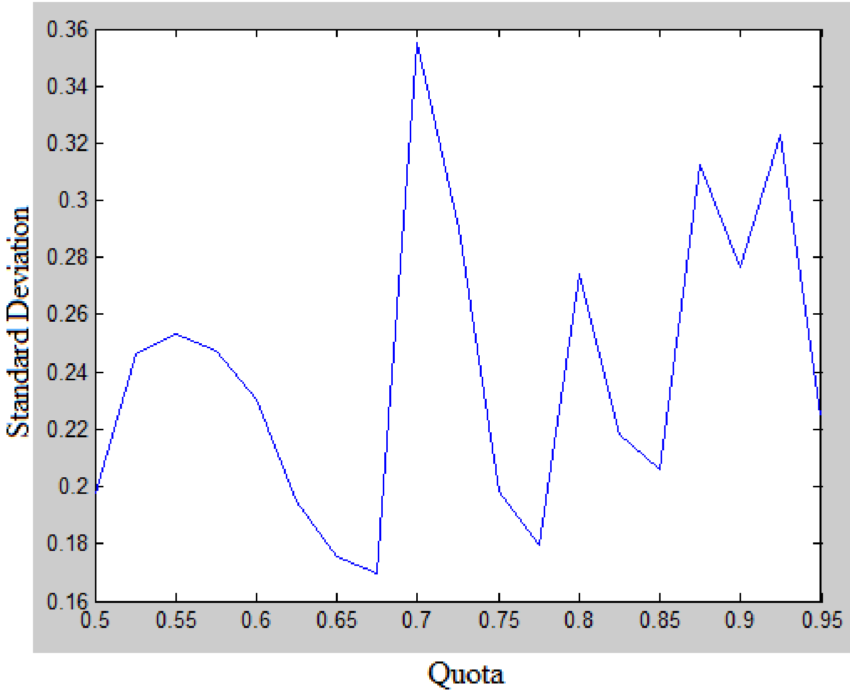

Figure 5.4. Standard deviation of expected payoff for manipulation by merging for WVGs with 5 to 20 players.

In contrast to manipulation by splitting, standard deviation for merging doesn't have predictable pattern for standard deviation (Figure 5.4) and results for different tests vary significantly, so from empirical results there isn't strong correlation between quota and standard deviation. But it is immediately visible that maximal value of standard deviation for merge is less than maximal value for split, which means that possible payoff has a better predictability.

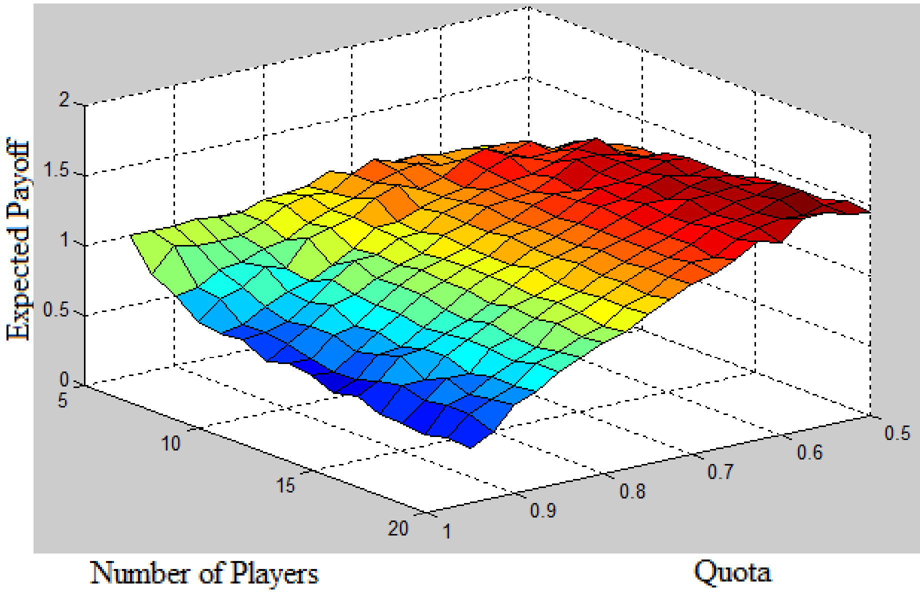

Figure 5.5. Expected value of payoff for merging manipulation for quotas from 0.5 to 0.95 and number of players from 5 to 20. Take a notice that axis *x* (quota) and axis *y* (number of players) are reversed for presentation purpose.

In the Figure 5.5 one can see that for small number of players expected payoff is quite steady, and is very slightly influenced by quota with a trend to slow and non-monotony decrease with increase of a quota. While for WVG with high number of players - expected value drops significantly with increase of a quota.

For small quotas decreasing number of players will result in decreasing expected payoff, and for high quotas it will lead to increase in expected payoff. Based on these observations, we can conclude that quota and number of players in WVG affects expected payoff of manipulation by merging oppositely.

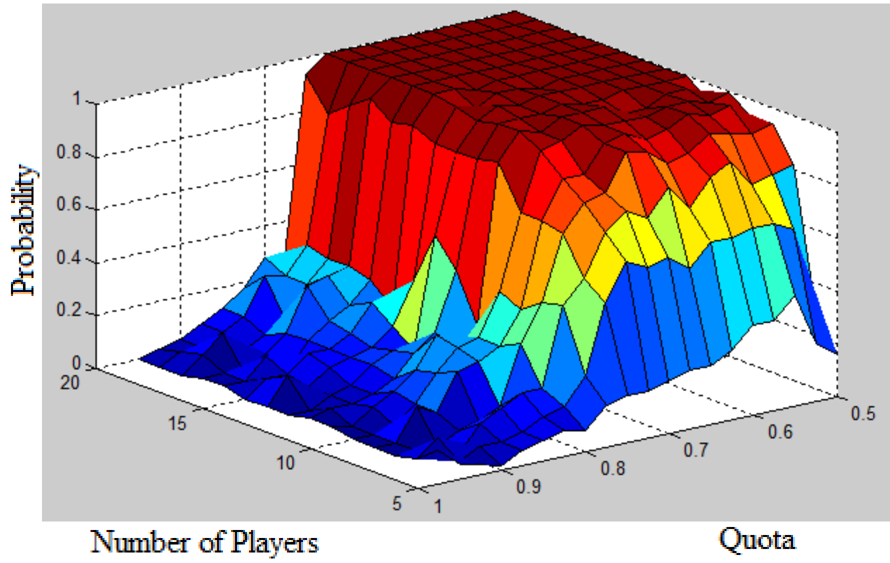

Figure 5.6. Probability of advantageous merge manipulation for quotas for different number of players in WVG. WVG from 5 to 20. Take a notice that axis *x* (quota) is reversed for presentation purposes.

The same as expected value, the probability of advantageous merge is getting more sensitive to a change of quota with increase in number of players and very rapidly (Figure 5.6). After number of players $n > 10$ probabilities react on changes of quota almost identically, very similar to results of simulation in Figure 5.3. The overall trend for any tested number of players is similar, and it is decreasing with increase of quota, which supports ***hypothesis 3***.

Also it is important to notice that for WVGs with $n > 10$ the chance for advantageous merge for low quotas is 1, which means that all randomly generated merges were advantageous. Based on this observation we can assume that in large WVGs for small quotas it is always advantageous to merge. Finding a theoretical proof for this assumption may be seen as a field of future work.

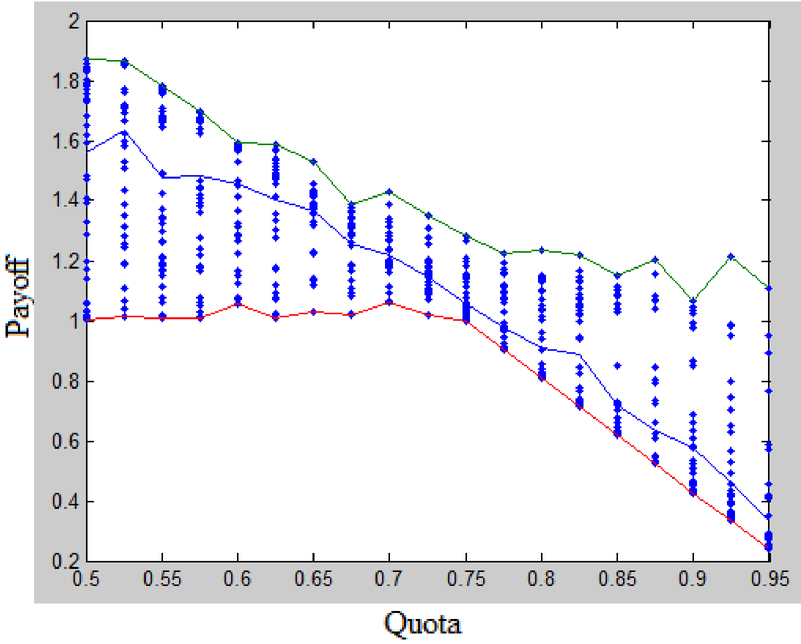

Figure 5.7. Minimal, maximal and expected values of payoff for each quota in WVGs with 50 players for merge manipulation.

Results presented n Figure 5.7 support the trend found in Figure 5.5 - for WVGs with higher number of players the difference between maximal value of expected value and minimal is growing. The trend for expected values is much more rapid, but has same direction – decreasing with increase of quota. We also can notice that maximal values of payoff tend to behave much more predictable, on contrary to Figure 5.5, which can be explained by rarity of extreme advantageous merges in WVGs with high number of players. It is evident that probability of advantageous merge (probability Figure can be found in Appendix B.5) is highest for small quotas, which supports ***hypothesis 3***.

5.6 Conclusions

Results achieved for manipulation by merging are showing that in overall high quotas ensure smaller expected payoff and low chance of advantageous manipulation. It is especially true for large WVGs, while small have more stable expected payoff. Also for large WVGs the maximum values of payoff were achieved for the smallest quota, while for games with small number of players there is no such correlation (Appendix B.4). For small WVGs probability of advantageous split is also more stable and in overall has a lower value.

Based on these observations we can suggest to use high quotas for WVGs with medium and high number of players, while for small WVGs (*n* around 5) the choice of quota has much smaller effect. In general strong correlation of extreme values, standard deviation and probability of advantageous merge to quota is evident; expected values are slightly less affected (Correlation coefficients can be found in Appendix C.2).

Creation of blocs, in most of the real life voting systems are not forbidden, and considered a normal player behavior so in such cases analysis of voting system on possibility for manipulation by merging is serving for informational purposes rather than for measurement of vulnerability. The results shown above shows that in most cases it is if players, who can't create a winning coalition, form a bloc with high probability it will be advantageous for low and disadvantageous for high quotas.

The results achieved in Chapter 5 support ***hypotheses 2 and 3***.

6 Conclusion on Manipulations

6.1 Overview of Manipulation by Annexation

In a scientific literature manipulation by annexation very often comes together with merging and splitting manipulation, and although it is important to understand its nature, we have decided not to conduct analysis similar to other manipulations described , because of the below reasons.

The nature of annexation is similar to manipulation by merging, as in both cases several players are combined into one, but calculation of payoff is done differently. The power of a bloc created by annexation is divided by power of manipulator before manipulation. What may come as a surprise is that annexation can be disadvantageous in some cases for some of the power indexes; this situation is called **bloc paradox** and was defined by Felsenthal and Machover (1998). They also proved that for Shapley-Shubik index manipulation by annexation is always advantageous (satisfies **bloc postulate** $\phi_{(C \cup p_i)}(G') \geq \phi_i(G)$). But for other P-Power indexes and Banzhaf index in some cases annexation can be disadvantageous. This result furthermore developed and supported by Aziz, Bachrach, Elkind, & Paterson (2011) and Lasisi (2013) with simulation results.

To conclude on manipulation by annexation consider the example from Felsenthal and Machover (1998) which shows different behavior of S-S and BZ indexes.

WVG G = [11; 6, 5, 1, 1, 1, 1, 1];

$\beta_1(G)$ = 0.47826;

$\varphi_1(G)$ = 0.47826;

p_1 annexes p_3 to create new WVG G' = [11; 7, 5, 1, 1, 1, 1], in which weight of $p_1 = p_{1 \cup 3}$ = 7.

$\varphi_1(G')= 0.53$;

$\beta_1(G') = 0.47222$;

$\varphi_1(G') > \varphi_1(G)$ => manipulation by annexation is advantageous for Shapley-Shubik index.

$\beta_1(G') < \beta_1(G)$=> manipulation by annexation is disadvantageous for Banzhaf index.

For analysis of payoff we ue Shapley-Shubik index, for which annexation manipulation is alwas advantageous => whenever chosen quota, manipulation would be advantageous. Because of this, used analysis methods are not applicable for it.

6.2 Conclusions on Manipulation by Splitting and Merging

In Chapter 4 and 5 we were able to prove all three proposed ***hypotheses***. We found that splitting manipulation reaches its maximal values for high quotas, as well as probability of advantageous split, which supports ***hypothesis 1***. We proved that bounds for merging manipulation are reversed to bounds of splitting manipulation, which supports ***hypothesis 2***. And finally, showed that merging manipulation reaches greatest values of payoff and probability of being advantageous for small quotas, which supports ***hypothesis 3***.

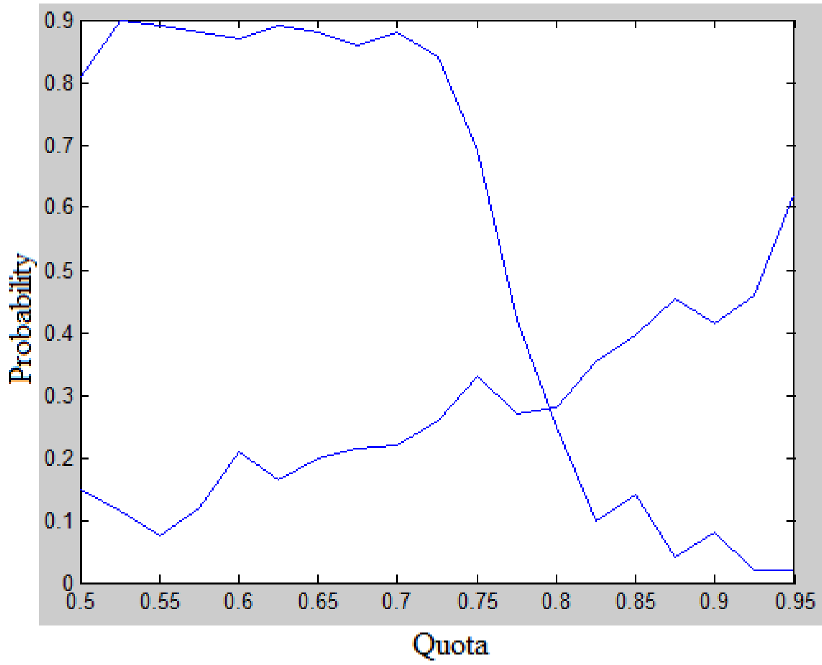

Figure 6.1. Probabilities of advantageous split and merge manipulations for WVGs with from 5 to 20.

To better summarize the results on Manipulation by merging and splitting we show results in a graphical form outlining both probabilities of advantageous manipulations on it (Figure 5.6). Our findings prove that he affects that quota has on probabilities of manipulation should be considered when designing a weighted voting system. In different voting systems certain actions by voters can be undesirable, but very hard to forbid by limiting possible behaviors. For example, for *manipulation by merging* voters doesn't have to officially create a bloc, they can reach an agreement to vote the same way. As can be seen from Figure 6.1 and Figure 5.3 the probability of such action is getting smaller with increase of a quota, but on the other hand probability of advantageous *manipulation by splitting* and maximal payoffs from both reviewed manipulations are increasing. The meeting point of these probabilities falls on the values around 0.75 to 0.8 but their average probability reaches its lowest values closer to quota that equals to 0.95. However, because standard deviation of splitting and maximal payoffs of both manipulations for such quotas are high (Figure 4.3), these values cannot be considered as a "safe choice". Our empirical results suggest that the most stable quotas are between 0.7 and 0.75. These results may differ for WVGs with different number of players, that's why it cannot be used as a measurement but rather they have value as guide for voting system designers and players. It may also be a signal to conduct a comprehensive analysis of a WVG. From the point of a manipulator, who is seeking to achieve highest possible payoff, the high quotas are good for both manipulations, while it is easier to find an advantageous merge for low quotas, maximum values are achieved for high quotas. The number of players also affects probabilities and expected values, as our experiments have shown, higher number of players will greatly decrease overall effectiveness of a splitting manipulation and improve predictability, while for merging manipulation lower number of players means lower probability of advantageous manipulation and more stable trend of expected values.

7 Evaluation

The analysis conducted in Chapter 3 could be applied to any existing weighted voting systems, to test its manipulation resistance. In this Chapter we will answer the question, how well a weighted voting system in Council of the European Union under the Treaty of Nice, which was in effect until March 2017, was designed, from a point of manipulation analysis. Sequentially the same analysis would be carried out for a weighted voting system of CEU under the Treaty of Lisbon, which substituted the Treaty of Nice. The results would be compared to find out if a change of the system brought a better stability and resistance to manipulations by splitting and merging. In addition, since both of these systems aren't simple WVGs, we can unveil how manipulations are affected by multi-majority systems. In 2003 the problem of computation of power indexes in multiple-majority systems was researched by Algaba, Bilbao, Garcıa and López, and we apply their findings in next setction to present CEU case-study.

7.1 Case Study

7.1.1 Weighted Voting System in Council of the European Union under the Treaty of Nice

CEU under Treaty of Nice was in use from 2003 to 2017. It is based on a triple majority system: majority of weights (>74%), majority of population (>62%), that also can be seen as a rule with weights, and majority of countries (>50% + 1) which is a simple majority rule. The WVG for this system can be noted as follows:

WVG = [256; 29, 29, 29, 29, 27, 27, 14, 13, 12, 12, 12, 12, 12, 10, 10, 10, 7, 7, 7, 7, 7, 4, 4, 4, 4, 4, 3];

$$v(C) = \begin{Bmatrix} 1, w(C) \geq q, length(C) \geq round(0.5 * n) + 1 \\ 0, w(C) < q \\ 0, length(C) < round(0.5 * n) + 1 \end{Bmatrix}$$

Based on the theoretical results and simulations in Chapter 3 we can suggest that because of a medium quota (0.74) and a large number of players, this WVG has a balanced resistance, even though it is a good target for manipulation by merging, it is resistant to splitting manipulation. It is also interesting to notice how a simple majority rule for a number of countries affects the payoffs of manipulations.

We have used the same simulation environment described in Chapter 3 with several modifications that allowed us to evaluate single simple WVG and WVG with addition of simple majority rule.

Even though, the nature of players in EU Council makes manipulation by splitting very unlikely, we are presenting the results below to show what affect multiple-majority rule has on manipulation by splitting. To clearly see the difference tests were done in 2 variance - first only for the first rule of Nice Treaty (weights > 74%, Figure 7.1) and second for all of the rules (Figure 7.2).

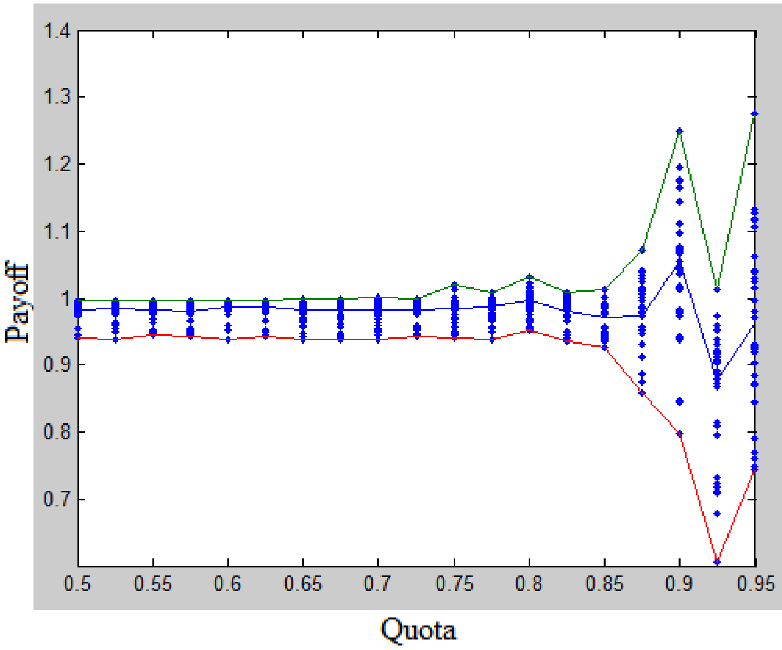

Figure 7.1. Manipulation by splitting in Council of the European Union considering only the first rule (weight > 0.74*W).

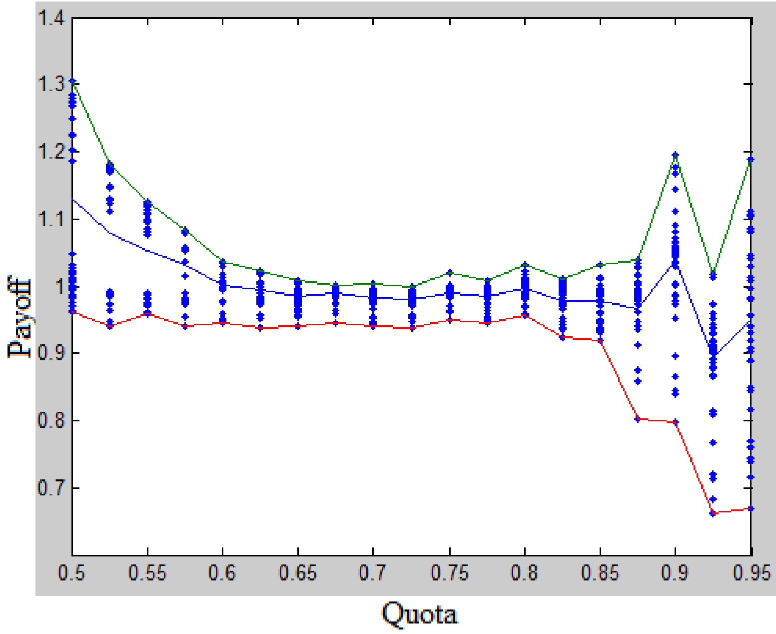

Figure 7.2. Manipulation by splitting in Council of the European Union.

As it follows from the Figure 7.2, trend of expected values is similar to the one in Figure 7.1 and Figure 4.5 with an exception for low quotas, as both the highest values for payoff and expected payoff are achieved. With growth of the quota, Figures 7.1 and 4.5 are becoming almost identical, so from manipulation analysis point, adding a simple majority rule to a weighted voting system will increase expected value of splitting manipulations for low quotas.

Under the Treaty of Nice the quota = 0.74 was used, according to a Figure 7.2 it is a quite stable value with expected value around 1 and small standard deviation (Figure 7.3).

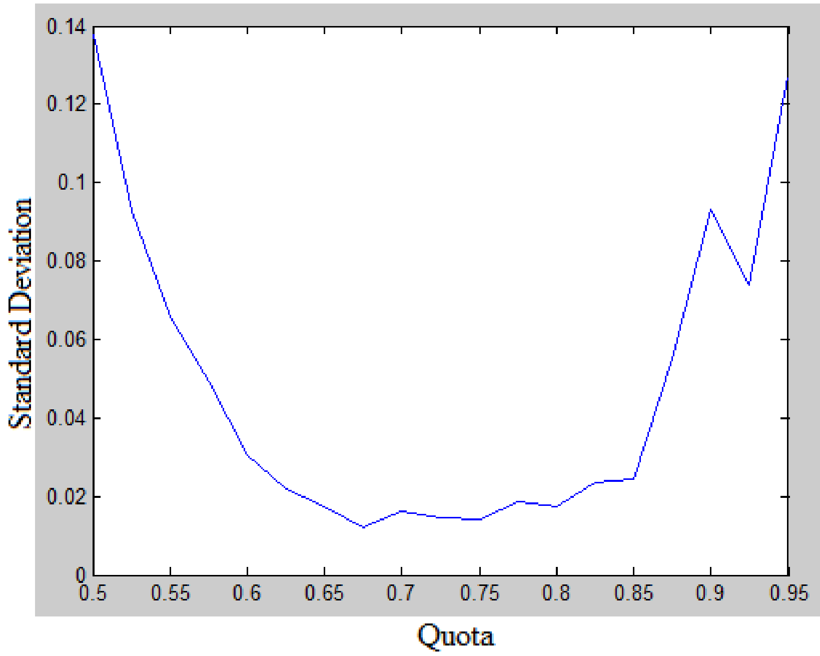

Figure 7.3. Standard deviation of manipulation by splitting in Council of the European Union.

Hereby we can conclude that quota for voting in Council of the European Union is chosen optimally for ensuring stability and resistance of a system to manipulation by splitting.

In contrast to splitting, merging manipulation is quite likely to happen in Council of the European Union, so the appropriate analysis isn't only interesting for the purposes of identifying how multiple-majority system affects manipulation, but also can be used to achieve some practical results, which we will do in the next section about Treaty of Lisbon, for the voting system that is currently in use in EU Council.

Results of simulation:

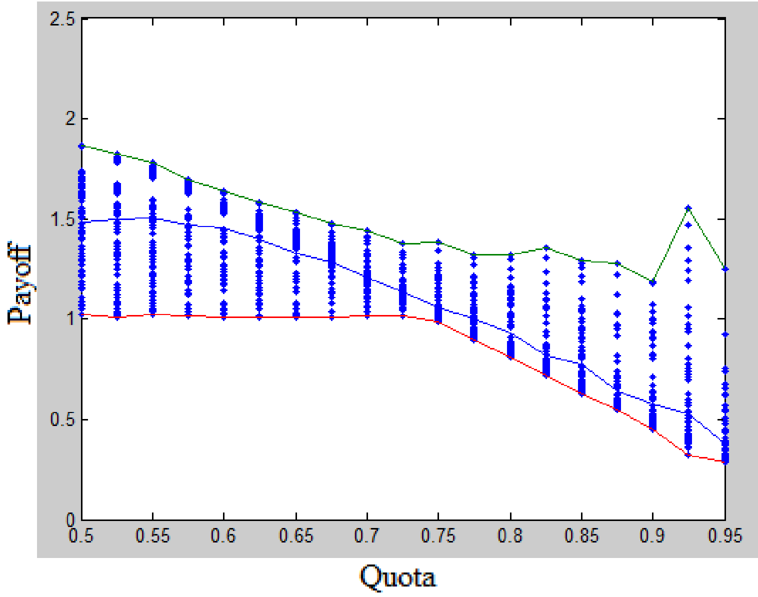

Figure 7.4. Manipulation by merging in Council of the European Union considering only the first rule (weight > 0.74*W).

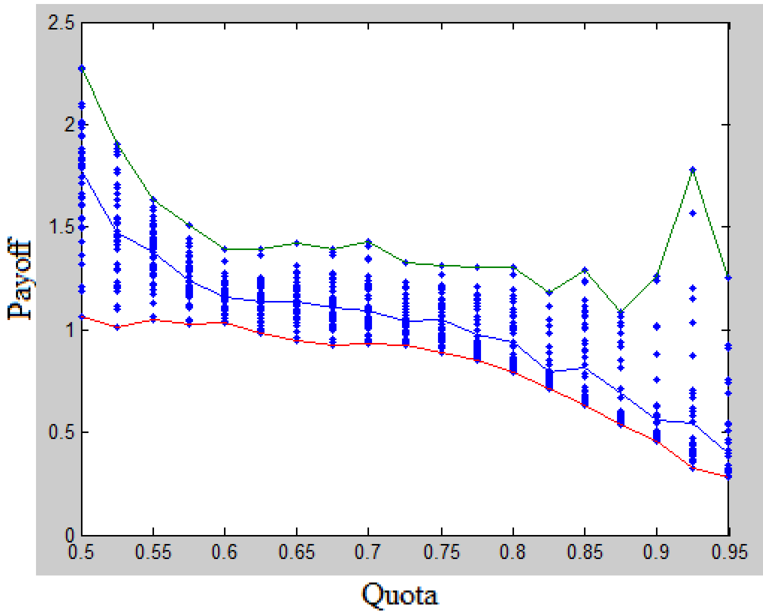

Figure 7.5. Manipulation by merging in Council of the European Union.

When adding the simple majority rule to WVG, same as for splitting manipulation, the main difference in tendency of expected value happens for low quotas, which can be seen from Figure 7.4 and Figure 7.5. Also maximum values for multiple-majority system for low quotas are noticeably higher. Based on this observations, we can conclude that simple majority rule (in this case >50% of players), when added to WVG affects the behavior of manipulation by merging significantly. The question on correlation between changes of simple majority rule and its effects on WVG manipulations could be a field of further research.

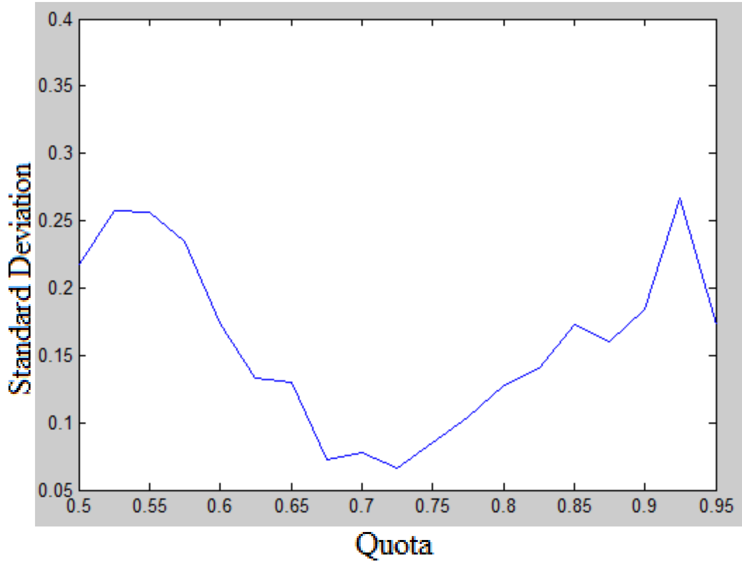

Figure 7.6. Standard deviation of manipulation by splitting in Council of the European Union considering only the first rule (weight > 0.74*W).

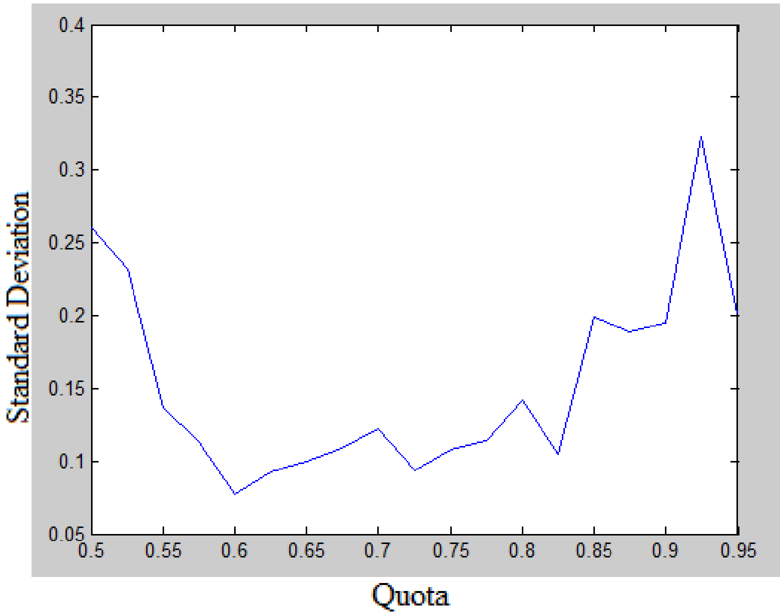

Figure 7.7. Standard deviation of manipulation by splitting in Council of the European Union.

The picture for standard deviation is quite similar, except that WVG with simple majority rule seems to be more stable for changes of quotas between average values (Figures 7.6 and 7.7).

In general, chosen value of quota = 0.74 seems to be a balanced choice between the values of expected payoff and its standard deviation. Even though, for this quota the expected payoff >1, the standard deviation is quite small, which shows a good predictability and stability of the system

Conclusion on weighted voting system of decision making in Council of the European Union under the treaty of Nice:

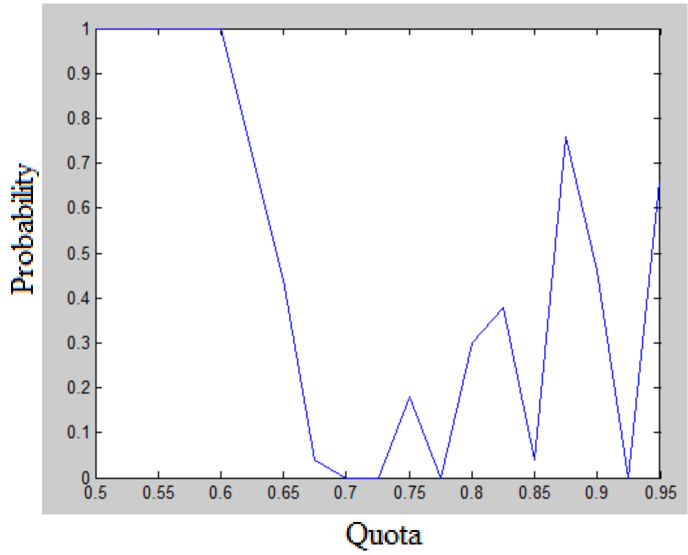

Figure 7.8. Probability of advantageous manipulation by splitting in Council of the European Union.

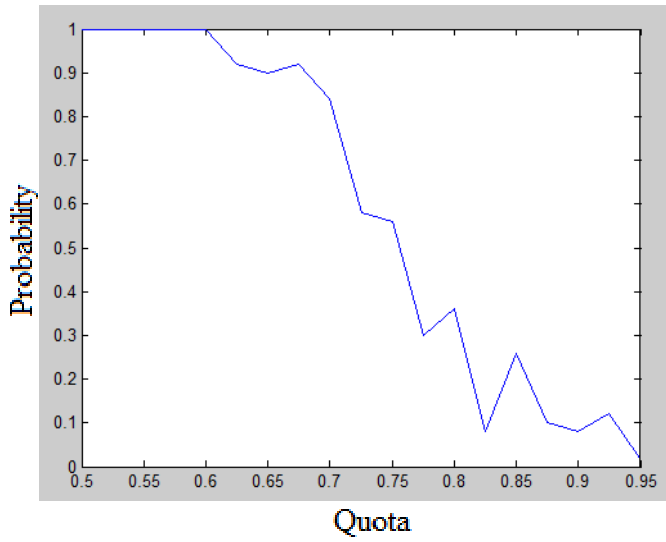

Figure 7.9. Probability of advantageous manipulation by merging in Council of the European Union.

As it follows from the analysis the chosen value of quota for number of voters in the EU Council provides the system with a good stability and predictability for manipulation by splitting and merging. Players can expect slight benefits from merging while splitting expected to be disadvantageous, both have moderate extreme values. Probability of advantageous merge for chosen quota is slightly higher than 0.5, which means that more than a half of randomly created blocs were advantageous. Split on the other hand almost doesn't have chances to be advantageous (Figure 7.8 and 7.9). Addition of a simple majority rule affects the payoff behavior for both manipulations quite significantly for lower quotas.

7.1.2 Weighted Voting System in Council of the European Union under the Treaty of Lisbon

Treaty of Lisbon was adopted in 2014, and took full effect in 2017. It uses a double majority system, majority of population (>65%) which can be seen as rule with weights and majority of countries (>55%) which is a simple majority rule. Because of a complexity of an algorithm and the fact that it is sensitive to the cumulative weight of players, it is not possible to calculate Shapley-Shubik indexes in a reasonable amount of time if exact population figures were used. To solve this problem we approximated states populations with weights, using a rounding formula round (population/100). Then WVG can be noted as follows:

WVG = [3317; 87, 113, 71, 42, 9, 106, 57, 13, 55, 667, 822, 108, 98, 47, 607, 20, 29, 6, 4, 170, 380, 103, 198, 54, 21, 464, 99, 653];

$$v(C) = \begin{cases} 1, w(C) \geq q, length(C) \geq round(0.55 * n) + 1 \\ 0, w(C) < q \\ 0, length(C) < round(0.55 * n) + 1 \end{cases}$$

Under the Treaty of Lisbon, the quota equals to 0.65, which is lower than under the Treaty of Nice, based on the results from chapter 3 we can suggest that it may lead to increase chance of advantageous merge manipulations and its expected payoff. For simple WVG with small quota splitting manipulation is less likely to be advantageous with an expected payoff little under 1, but because of a double-majority system with a simple majority rule, as we saw from the previous example, the results for small quotas is curved compared to simple WVG. Therefore we can predict that for weighted voting system in CEU under the Treaty of Lisbon both manipulation by splitting and merging would have an expected payoff > 1 and with high chance (>0.5) of advantageous split.

Results of simulation:

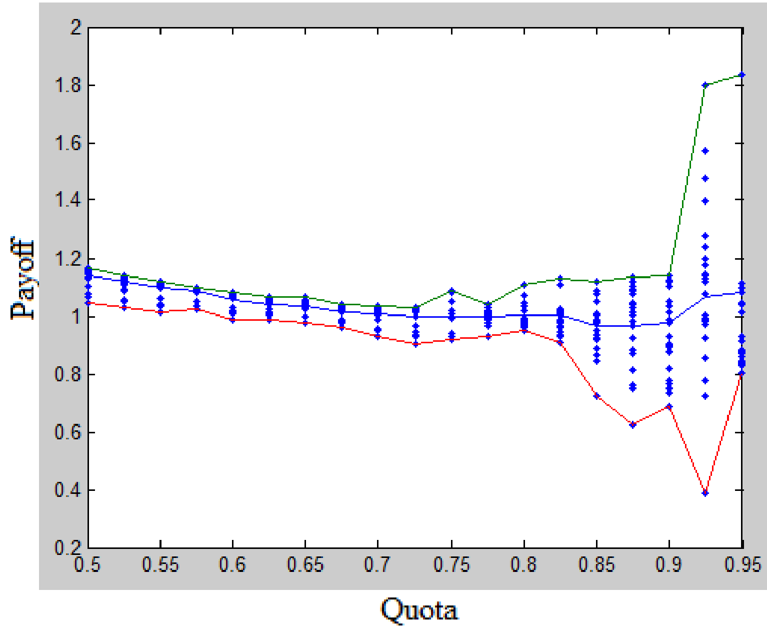

Figure 7.10. Manipulation by splitting in Council of the European Union under Treaty of Lisbon.

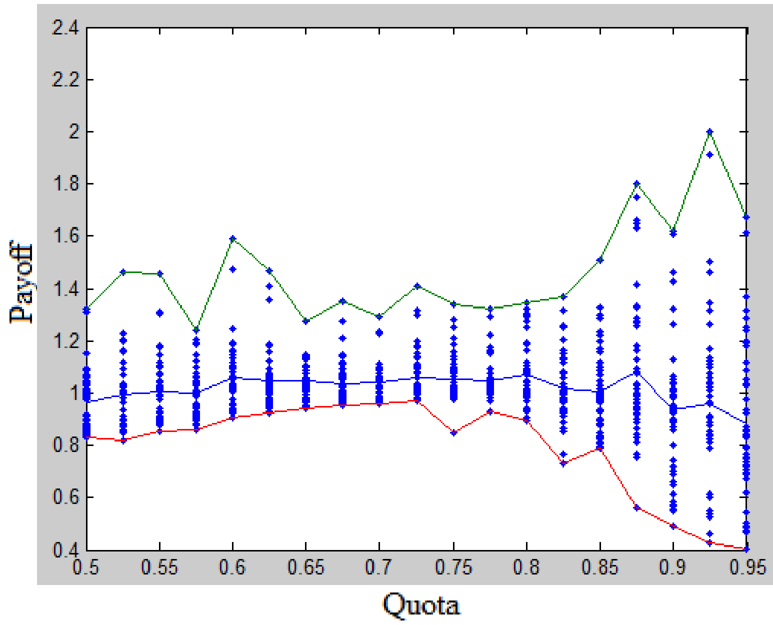

Figure 7.11. Manipulation by merging in Council of the European Union under Treaty of Lisbon.

For the splitting manipulation, the picture for expected payoff (Figure 7.10) is similar to the voting under the Treaty of Nice (Figure 7.2), but noticeably more smooth for high quotas and the deviation for low quotas as well as expected payoff are much lower. Quite opposite happened to a merging manipulation, as can be seen on Figure 7.11, it is more similar to the Figure 5.1 than to the Figure 7.5. Expected value at first is growing and after a value of quota ratio around 0.8 it starting to decrease to the lowest point, which is achieved for the highest quota. Chosen for CEU voting quota of 0.65 gives a value of expected payoff for splitting and merging manipulation slightly higher than 1, which shows that a manipulator can expect a gain in power for both analyzed manipulations.

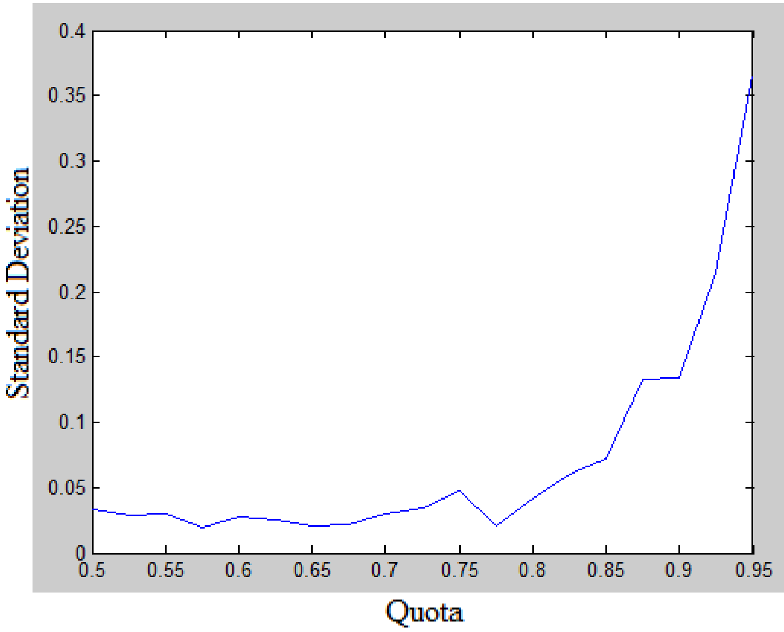

Figure 7.12. Standard deviation of expected value of manipulation by splitting in Council of the European Union under Treaty of Lisbon.

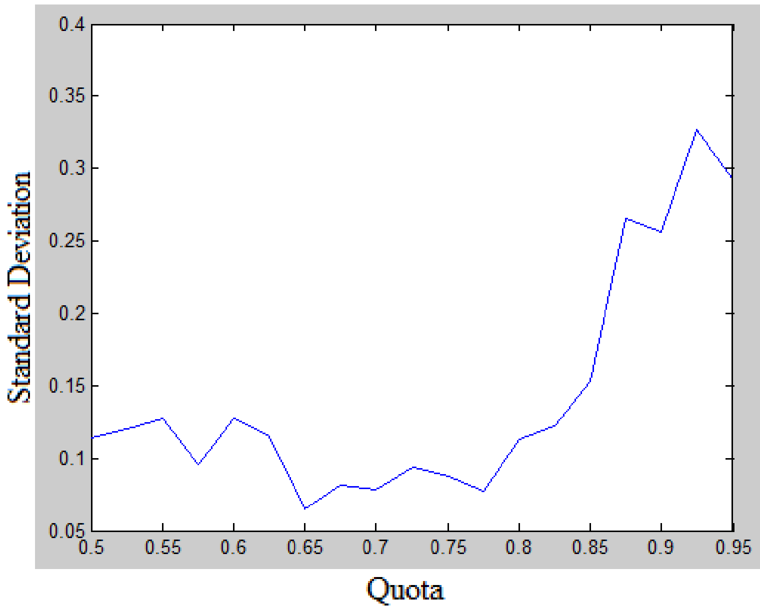

Figure 7.13. Standard deviation of expected value of manipulation by merging in Council of the European Union under Treaty of Lisbon.

Interestingly in contrast to voting under the Nice Treaty, simple majority rule hasn't affected standard deviation (Figure 7.12) for splitting manipulation, and it is very similar to the result from chapter 4 (which is a little bit flatter, because of the differences in number of players of tested samples). Standard deviation for merging has a noticeable difference for low quotas, where its values are much lower, but for medium and high quotas trend is identical. Chosen value of quota = 0.65 ensures low standard deviation, therefore high predictability of effects of manipulation by splitting and merging on the voting system.

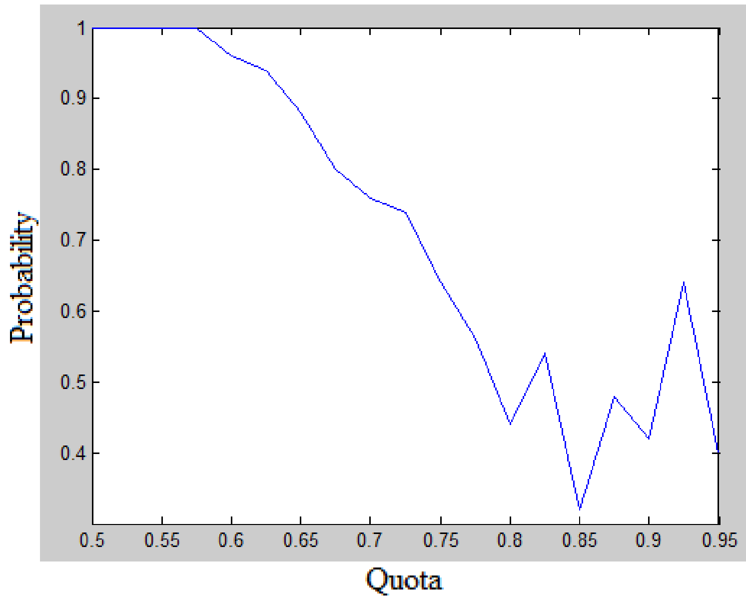

Figure 7.14. Probability of advantageous manipulation by splitting in Council of the European Union under Treaty of Lisbon.

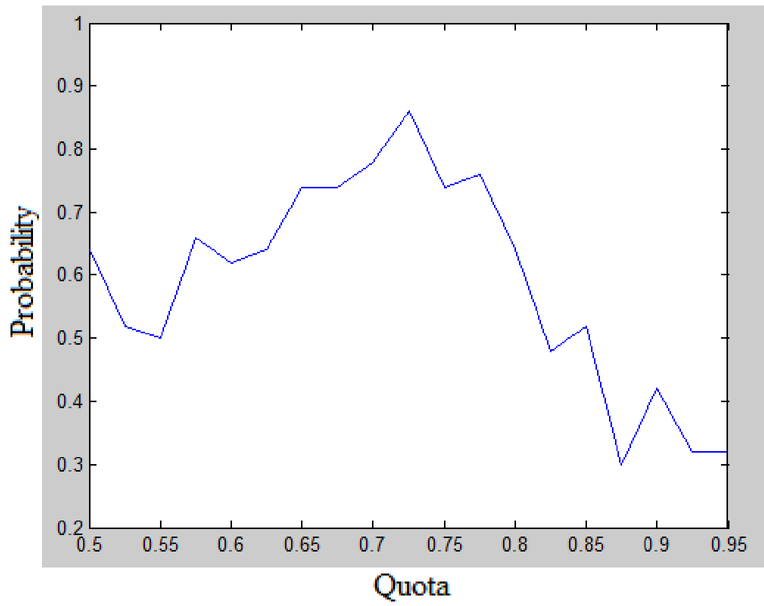

Figure 7.15. Probability of advantageous manipulation by merging in Council of the European Union under Treaty of Lisbon.

On a contrary to voting under the Treaty of Nice, where only merging manipulation is more likely to be advantageous, both manipulations in voting under the Treaty of

Lisbon have high probability of being advantageous >50%. This probability for splitting manipulation is almost 80% and for merging around 60%, these are very high values, which show that it is easy to find an advantageous manipulation in this voting system.

7.2 Conclusions on the Case Study

From the presented analysis it is evident that transition of voting rules of Treaty of Nice to Treaty of Lisbon has negatively affected the manipulation stability of a system. Splitting manipulation has become much more likely to be advantageous with higher expected payoff and standard deviation. But as was discussed before, splitting is unlikely to happen in Council of the European Union, therefore the results on splitting manipulation have rather theoretical value on influence of multi-majority system of payoff behavior. From the practical point - it is expedient to evaluate results of merging manipulation. The change in resistance of a system for merging manipulation isn't so clear, the expected payoff has slightly decreased, but the probability of advantageous merge has noticeably increased. Standard deviation for both voting systems is quite low. Using the study from chapters 4 and 5, we were able to roughly predict the results of manipulation analysis for both examined WVGs.

The results on the effects of multiple-majority systems on a payoff behavior for changes of quota, turned out to be heterogeneous, because of that it is hard to make precise conclusions, what is evident is that addition of simple majority rule to WVG influence correlation of payoff to quota quite significantly. This area can be seen as a field of future work as more advanced study techniques can be applied to test the systems, such as multiple-majority system in CEU.

7.3 Discussion of the Results

In this thesis an effect of a size of a quota on a payoff in the conditions of manipulation by merging and splitting was studied using experimental analysis. A specific simulation environment was created with the set of programs for creation of different random WVGs, simulation of manipulations and calculation of payoffs for Shapley-Shubik index. We found that expected values for manipulation by splitting are growing with increase of a quota as well as extreme values of payoff, standard deviation and probability of advantageous split (Chapter 4). For manipulation by merging, on the other hand, expected payoff and probability of advantageous merge is decreasing with a growth of a quota (Chapter 5), while there is no strong link between standard deviation of payoff and value of quota. These results supported ***hypotheses 1*** and (see section 3.3.). In section 5.3 we have proved a theorem 5.2 that satisfies conditions of ***hypothesis 2***, and we have used these results to find new bounds for merging manipulation.

In addition, the effects of a quota for different sizes of WVGs were studied, and we have found that depending on a number of players in the game the trends for expected and extreme values, standard deviation of payoff and probability of advantageous manipulation may significantly differ. Nevertheless, achieved results don't conflict with any of the ***hypotheses***.

Finally, we applied methods used in the simulation to a real world WVGs on the example of voting system in Council of the European Union under the Treaty of Nice (active 2003 – 2014/2017) and the Treaty of Lisbon (took full effect in March of 2017). We found that transition to the new system decreased overall manipulation resistance of the voting system.

The results of the work fill the gap in previous research of manipulation payoff properties, such as correlation with a value of a quota, number of players, minimal and maximal bounds for merging manipulation for $k > 2$.

7.4 Limitations of the Thesis

To achieve reliable results, we have limited the number of fake-players for splitting manipulation and number of manipulators for merging manipulation, but as it is evident from the maximal and minimal bounds of manipulations, these factors can significantly affect the payoff. In future studies the effects of these factors can be studied to extend presented analysis.

Number of test WVGs samples was limited for the sake of saving computational time, as indexes calculation for large WVGs takes significant amount of time. For WVGs with more than 30 players number of test samples was set to 100 for each quota value. The smallest number of test samples was 50 when payoffs for WVGs with number of players from 5 to 20 for each value of quota were calculated (Figures 4.5, 4.6, 5.5 and

5.6). However it lead to some limitations, in particular some level of inconsistency of results, especially for extreme values of payoff. In future works may be used faster algorithms to allow for higher number of tests.

In this work we focused on the Shapley-Shubik power index and have presented some additional results on Banzhaf index, but chosen indexes have limitations, for example they doesn't give an opportunity to study annexation manipulation for the reasons described in section 6.1.

For the efficiency of the simulations we used simple algorithms for creation of WVGs and manipulations, for that reason created WVGs are likely to have similar properties: average weigh values, average gap between weights and other properties - as a continuation of this research it is possible to replicate similar test using more advance algorithms. Also in this work only unanimity and non-unanimity WVGs were studied, while there are several more classes of WVGs, like excess unanimity ($w(P) > q$). This area can be a field of future studies.

8 Thesis Summary and Future Work

In this work were experimentally investigated the effects a quota has on manipulations by splitting and merging. To measure results we have used Shapley-Shubik value, which the most prominent P-Power index. This work aimed to expand results that were found by Lasisi (2013), Aziz and Paterson (2009) and Lasisi and Allan (2011). on effects of manipulations on power indexes. The main objectives were to find correlation between value of quota and effectiveness, predictability of weighted voting systems.

We presented theoretical observations based on a nature of a Shapley-Shubik index for both considered manipulations to formulate three ***hypotheses*** and found evidential proof for them through the experiments with large number of samples. The results showed that expected value, probability and standard deviation of payoff of manipulations is dependent on choice of a quota. While splitting manipulation is less likely to be advantageous and less effective for WVGs with low quota, manipulation by merging behaves oppositely, and has the lowest probability of being advantageous and lowest expected payoff for high quotas. These results show that it is possible for voting system designer to decrease likelihood and average payoff by choosing appropriate quota.

Based on observation of nature of splitting and merging manipulation we proposed ***hypothesis 2*** and were able to prove it and apply findings to discover full bounds for maximal and minimal possible payoff for manipulation by merging, which were not seized before.

Along with the main objectives of the work, we have found several other interesting results and correlations. Most importantly, the correlation between manipulations effectiveness and number of players in WVGs were found, showing how trends for expected payoff and probability of advantageous manipulation are changing for number of players from 5 to 20 for both examined manipulations.

We concluded the work with an application of manipulation analysis to a real world weighted voting system. The voting in Council of the European Union under the Treaty of Nice and Treaty of Lisbon were analyzed and we have find that the change of voting system from triple-majority to double-majority has negatively affected the manipulation resistance of a system. We also discovered some additional results on effects of simple majority rule to WVG on the manipulations based on these specific examples.

Several areas of future research were highlighted in this work. In particular, emerging from results for ***hypotheses 1 and 3,*** the question of manipulations in large WVGs can be investigated; as it is evident from the results in chapter 4 and 5 that number of players has a great effect of manipulation payoff values. The relevancy of ***hypotheses 1 and 3*** for Banzhaf index can also be researched, as well as extend the results on the manipulation by annexation. Furthermore, manipulations study in multiple-majority systems can show some unpredictable results, as there is a significant difference in effects of manipulation for different voting systems.

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Appendix A – Source Code of Used Programs

Source code for MatLab 7.9.0(R2009b).

Program to create a random WVG.

```
function [ result_output ] = RandomWVG( num_of_players, maximal_weight )
i = 0;
result(1)=1;
r(1)=0;
maximal_weight = maximal_weight;
j=0;
minR = 0;
j1=0;
minR1 = 0;
s=1;
result(1:num_of_players)=1;
while(0.95*sum(result)>sum(r)+minR || 0.95*sum(result)>sum(r)+minR1 || s ==
1)%unanimity check
    for i = 1:num_of_players
        result(i) = randi([1, maximal_weight]);
    end
    r=result;
    [minR,j] = min(r);
    r(j)=[];
    [minR1,j1] = min(r);
    r(j1)=[];
    s=0;
    for l=1:length(result)
        if(result(l)>=round(0.5*sum(result))+1)
            s=1;
        end
    end
end
result_output = result;
end
```

Program to create a random split of the player.

```
function [ v, j ] = SplitElem( w, i )
%v - result vector
%j - number of fake-players
v=w;
v(i)=[];
splittingWeight=w(i);
j=0;
while(splittingWeight>1)
    j=j+1;
    newWeight = randi([1,splittingWeight-1]);
    splittingWeight = splittingWeight-newWeight;
    v(length(v)+1)=newWeight;
end
r = randi([1,j+1]);
if(length(w)-1+r>length(v) || j<2)
   v(length(v)+1)=1;
   j=j+1;%
else
   v(length(w)-1+r)= v(length(w)-1+r)+1;
end


end
```

Program to create random merge of players.

```
function [ resultV, mergedElems ] = MergeElem( v )
numOfMerge = min(randi([2,4]),length(v)-2);
result = v;
j = 0;
tryCount=0;
mergedElems=19999;
while(j==0 || (sum(v(mergedElems))>0.5*sum(v)))
    h = 1:length(v);
    mergedElems(1:length(mergedElems))=[];
    for i = 1:numOfMerge
        j = randi([1,length(h)]);
        mergedElems(i) = h(j);
        h(j)=[];
    end
    tryCount=tryCount+1;
    if(tryCount>10)%if too many tries
        numOfMerge = max(numOfMerge-1,2);
        tryCount=0;
    end
end

resultV(mergedElems) = [];
resultV(length(result)+1)=sum(v(mergedElems));

end
```

Program to calculate S-S index of player using direct enumeration method.

```
function [ result ] = ShapleyShubik( w, q, i )
v = w;
v(i) = [];
result = 0;
wLength = length(w);
for j = 1:(wLength-1)
    comb = combnk(v, j);
    combsum = sum(comb, 2);

    for k = 1:length(combsum)
        elem = combsum(k);
          if(elem<q)
             if(elem+d>=q)
                 l = length(comb(k,:));
                 result = result + factorial(l)*factorial(wLength-1-l);
             end
          end
    end
end
result = result/factorial(wLength);
end
```

Program to calculate C matrix for Generating functions Method (Leech, 2002).

```
function [ c ] = SSGen( v )
vLength = length(v);
J=0:sum(v);
K = 1:(vLength+1);
R = 1:vLength;
d(1:length(J),1:(vLength+1),1:vLength+1) = 0;
d(1,1,1)=1;
p=0;
for j = 1:length(J)
    for r = 2:vLength+1
        for k = 1:(vLength+1)
            d2=0;
            if(r-1<1)
                d(j,k,r)=0;
            else
                if(j-v(r-1)<1 || k-1<1)
                    d2=0;
                else
                    d2=d(j-v(r-1),k-1,r-1);
                end
                d(j,k,r) = d(j,k,r-1)+d2;
            end
        end
    end
end
c(1:length(J),1:(vLength+1),1:length(v))=0;
for r=1:vLength
    for j=1:length(J)
        for k=1:(vLength+1)
            if(j-v(r)<1 || k-1<1)
                c2=0;
            else
                c2=c(j-v(r),k-1,r);
            end
                c(j,k,r) = max(d(j,k,vLength+1)-c2,0);
        end
    end
end
end
```

Program to calculate Indexes of players from matrix C (Leech, 2002).

```
function [ result ] = CalcGenIndex( v, c, q )
vLength=length(v);
majority = round(vLength*0.55)+1;
index(1:vLength)=0;
for r = 1:vLength
    for k = 1:vLength
        sumC=0;
        for j=(q-v(r)+1):(q)
            if(j<1)
                q
                v
                v(r)
            end
            %if(k>=majority) %for multiple-majority voting
                sumC = sumC + c(j,k,r);
            %end
        end
        index(r)=index(r)+sumC*factorial(k-1)*factorial(vLength -1-(k-
1))/factorial(vLength);
    end
end
result = index;
end
```

Program to evaluate effects of quota on manipulation by merging.

```
function [ result ] = AssertMerge( iterations )
q=[0.5, 0.525, 0.55, 0.575, 0.6, 0.625, 0.65, 0.675, 0.7, 0.725, 0.75,
0.775, 0.8, 0.825, 0.85, 0.875, 0.9, 0.925, 0.95];

%%%%%%%%%%%% TREATY OF LISBON %%%%%%%%%%%%
%v =
[29,29,29,29,27,27,14,13,12,12,12,12,12,10,10,10,7,7,7,7,7,4,4,4,4,4,3,];
%%%%%%%%%%%% TREATY OF NICE %%%%%%%%%%%%
%v =
[87,113,71,42,9,106,57,13,55,667,822,108,98,47,607,20,29,6,4,170,380,103,1
98,54,21,464,99,653];
%c = SSGen(v);

maxI = 0;
vmax = [0,0];
jmax = [0,0];
%%%%%%%%%%%% FOR 3D FIGURES %%%%%%%%%%%%
%s = 5:20;
%for k = 1:length(s)
 %   s(k)
for i = 1:iterations
    i
    v = RandomWVG(randi([5,20]), randi([50,150]));%s(k), randi([50,
100+100*(s(k)/20)]));%% FOR 3D FIGURES %%%%%%%%%%%%
    c = SSGen(v);
    for j = 1:length(q)
        mergedElemsValues = 0;
        before = 0;
        after=0;
        q(j)
        quota=min(round(q(j)*sum(v)) + 1, sum(v));
        [v2, mergedElems] = MergeElem(v);
        vInd = CalcGenIndex(v,c,quota);
        for p=1:length(mergedElems)
            before = before + vInd(mergedElems(p));
        end
        afterC = SSGen(v2);
        v2Index = CalcGenIndex(v2,afterC,quota);
        after = v2Index(length(v2));
        if(before==0)
            payoff(i,j) = 1;
        else
            payoff(i,j) = after/before;
        end
        if(payoff(i,j)>1)
            isBeneficial(i,j) = 1;
        else
            isBeneficial(i,j) = 0;
        end
```

```
    end

end

for j = 1:length(q);
    p(j)=max(payoff(:,j));%-
min(payoff(:,j));%sum(payoff(:,j))/length(payoff(:,j));%%;%;
    h(j) = sum(payoff(:,j))/length(payoff(:,j));
    z(j) = min(payoff(:,j));
    beneficialChance(j) =
sum(isBeneficial(:,j))/length(isBeneficial(:,j));
end
%%%%%%%%%%%% FOR 3D FIGURES %%%%%%%%%%%%
 %   r(k,:)=beneficialChance;
  %  r1(k,:)=h;
   % r2(k,:)=p;
    %r3(k,:) = std(payoff);
%end

result = payoff;
standartDeviation = std(payoff);
figure('Name','Chance of Advantageous Merge','NumberTitle','off');
plot(q, beneficialChance);
figure('Name','Standard Deviation Merge','NumberTitle','off');
plot(q,standartDeviation);
figure('Name','MinMaxExpected Merge','NumberTitle','off');
plot(q,payoff,'b.','MarkerSize',5); hold on;
plot(q,h,q,p,q,z);

%%%%%%%%%%%% FOR 3D FIGURES %%%%%%%%%%%%
%figure('Name','Chance','NumberTitle','off');
%surf(q,s,r);
%figure('Name','Expected Payoff','NumberTitle','off');
%surf(q,s,r1);
%figure('Name','Maximum','NumberTitle','off');
%surf(q,s,r2);
%figure('Name','Std','NumberTitle','off');
%surf(q,s,r3);

end
```

Program to evaluate effects of quota on manipulation by splitting.

```
function [ result ] = AssertSplit( iterations )
q=[0.5, 0.525, 0.55, 0.575, 0.6, 0.625, 0.65, 0.675, 0.7,
0.725, 0.75, 0.775, 0.8, 0.825, 0.85, 0.875, 0.9, 0.925, 0.95];

%%%%%%%%%%%% TREATY OF LISBON %%%%%%%%%%%%
%v = [29, 29, 29, 29, 27, 27, 14, 13, 12, 12, 12, 12, 12,10,10,10,
7,7,7,7,7,4,4,4,4,4,3,];
%%%%%%%%%%%% TREATY OF NICE %%%%%%%%%%%%
%v = [87,113,71,42,9,106,57,13,55,667,822,108,98,47,607,20,29,6,4,170,380,
103,198,54,21,464,99,653];
%c = SSGen(v);
%%%%%%%%%%%% FOR 3D FIGURES %%%%%%%%%%%%
%s = 5:20;
%for k = 1:length(s)
 %   s(k)
for i = 1:iterations
    i
    v = RandomWVG(randi([5,20]), randi([20,100]));
%s(k), randi([50, 100+100*(s(k)/20)]));%% FOR 3D FIGURES %%%%%%%%%%%%
    c = SSGen(v);
    for j = 1:length(q)
        before = 0;
        after=0;
        splittedElemsNum = 0;
        quota=min(round(q(j)*sum(v)) + 1, sum(v));
        q(j)
        splitIndex=randi([1,length(v)]);
        vInd = CalcGenIndex(v,c,quota);
        before = vInd(splitIndex);

        [v2, splittedElemsNum] = SplitElem(v, splitIndex);
        afterC = SSGen(v2);
        v2Ind = CalcGenIndex(v2,afterC,quota);
        for p=1:splittedElemsNum
            after = after + v2Ind(p+length(v)-1);
        end

        if(before==0)
            ratio(i,j) = 1;
        else
            ratio(i,j) = after/before;
        end
        if(ratio(i,j)>1)
            isBeneficial(i,j) = 1;
        else
            isBeneficial(i,j) = 0;
        end
    end
```

```
end

for j = 1:length(q);
    p(j)=max(ratio(:,j));
    h(j) = sum(ratio(:,j))/length(ratio(:,j));
    z(j) = min(ratio(:,j));
    beneficialChance(j) =
sum(isBeneficial(:,j))/length(isBeneficial(:,j));
end
%%%%%%%%%%%% FOR 3D FIGURES %%%%%%%%%%%%
%    r(k,:)=beneficialChance;
 %   r1(k,:)=h;
  %  r2(k,:)=p;
%end
result = ratio;
standartDeviation = std(ratio);
figure('Name','Chance of Advantageous Split','NumberTitle','off');
plot(q, beneficialChance);
figure('Name','Stanard Deviation Split','NumberTitle','off');
plot(q,standartDeviation);
figure('Name','MinMaxExpected Split','NumberTitle','off');
plot(q,ratio,'b.','MarkerSize',5); hold on;
plot(q,h,q,p,q,z);


%%%%%%%%%%%% FOR 3D FIGURES %%%%%%%%%%%%
%figure('Name','Chance','NumberTitle','off');
%surf(q,s,r);
%figure('Name','Expected Payoff','NumberTitle','off');
%surf(q,s,r1);
%figure('Name','Maximum','NumberTitle','off');
%surf(q,s,r2);

end
```

Appendix B – Additional Results

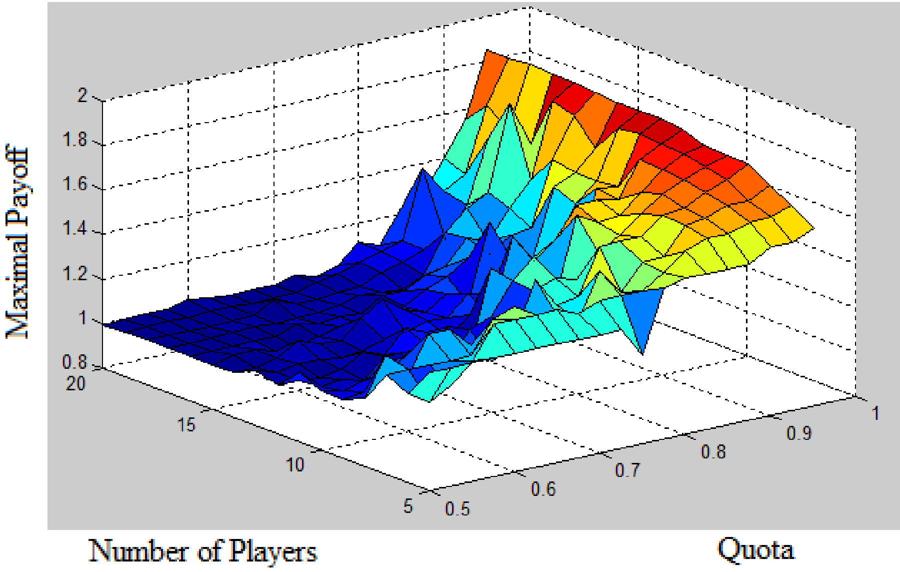

Figure B.1. Maximal values of payoff for manipulation by splitting for WVGs with 5 to 20 players.

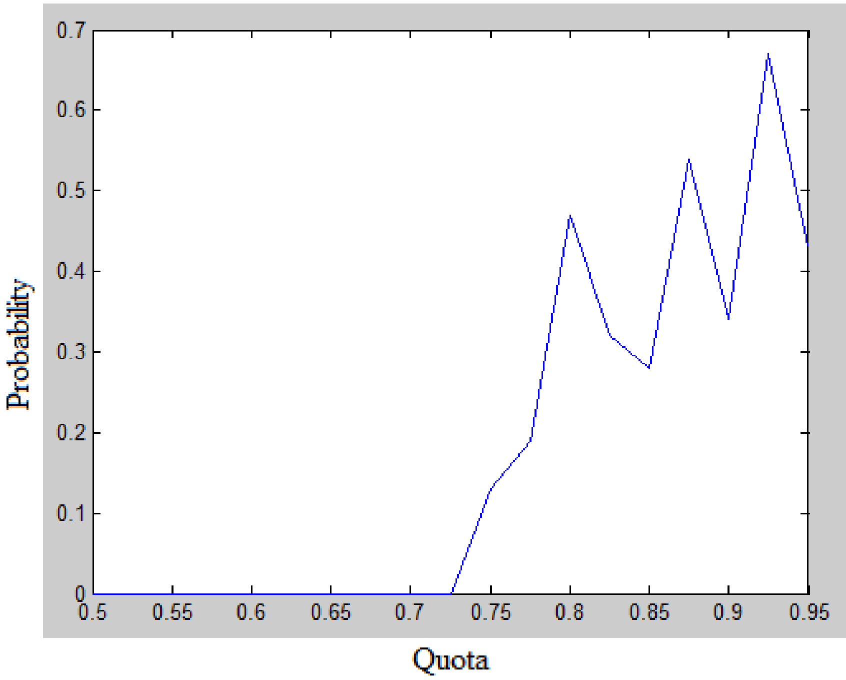

Figure B.2. Probability of advantageous split for WVGs with 50 players.

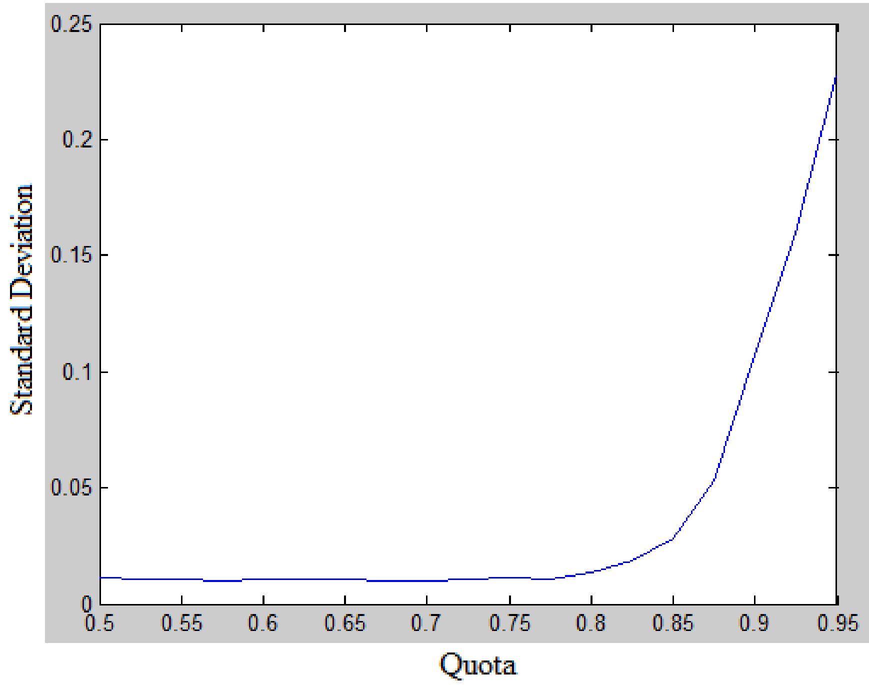

Figure B.3. Standard deviation of payoff for splitting manipulation for WVGs with 50 players.

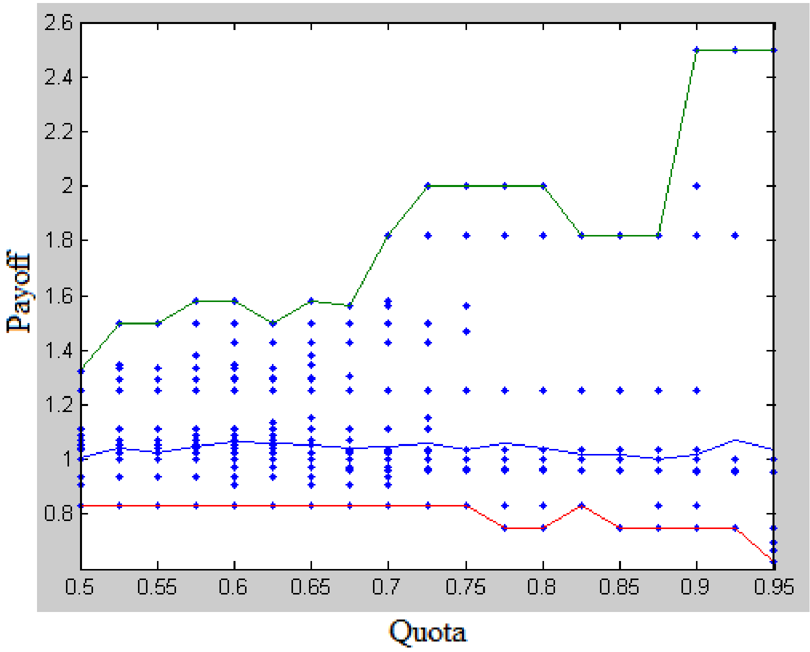

Figure B.4. Payoff values for merging manipulation for WVGs with 5 players.

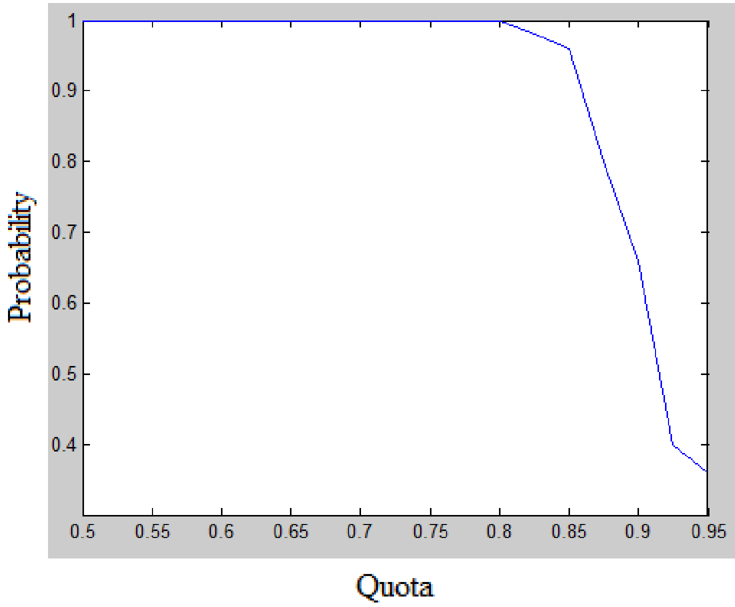

Figure B.5. Probability of advantegeous merge in WVGs with 50 players.

Appendix C – Correlation Analysis Results

C.1. Correlation Coefficient for split manipulation for WVGs with 5 to 20 players.

C.1.1. Correlation Coefficient for Maximal payoff values.

$p(MaxPayoffs, q) = 0.8414$;

C.1.2. Correlation Coefficient for Expected payoff values.

$p(ExpectedPayoffs, q) = 0.5216$;

C.1.3. Correlation Coefficient for Probability of advantageous split.

$p(\Pr obability, q) = 0.8165$;

C.1.4. Correlation Coefficient for Standard Deviation of payoff.

$p(\Pr obability, q) = 0.9121$;

C.2. Correlation Coefficient for merge manipulation for WVGs with 5 to 20 players..

C.2.1. Correlation Coefficient for Maximal payoff values.

$p(MaxPayoffs, q) = 0.8223$;

C.2.2. Correlation Coefficient for Expected payoff values.

$p(ExpectedPayoffs, q) = -0.5534$;

C.2.3. Correlation Coefficient for Probability of advantageous merge.

$p(\Pr obability, q) = -0.8672$;

C.2.4. Correlation Coefficient for Standard Deviation of payoff.

$p(\Pr obability, q) = 0.7982$.