E. Machinery and Fine Mechanics

TALLINNA TEHNIKAÜLIKOOLI VÄITEKIRJAD

THESES OF TALLINN TECHNICAL UNIVERSITY

ДИССЕРТАЦИИ ТАЛЛИННСКОГО ТЕХНИЧЕСКОГО УНИВЕРСИТЕТА

Jakub Kõo

Determination of Residual Stresses in Coatings and Coated Parts

TALLINN 1994

DECLARATION

I declare that this thesis is my original, unaided work. It is being submitted for the degree of Doctor of Engineering of Tallinn Technical University, Estonia. It has not been submitted before for any degree or examination in any other university.

Signature of candidate: *[signature]*

Date: *05. 11. 93*

ISBN 9789949483099 (pdf)

KOKKUVÕTE

Jäakpingete maäramine pinnetes ja pindega detailides

Väitekirjas kasitletakse eksperimentaalsete meetodite valjatöötamist ja täiustamist jääkpingete ja -deformatsioonide maaramiseks pinnetes ja pindega detailides.

Esitatakse uldine algoritm, mis võimaldab maarata jääkpinged kas pinde kasvamisel (pindamisel) või kahanemisel (eemaldamisel) aluse vabal pinnal või pinde liikuval pinnal mõõdetud deformatsiooniparameetrite jargi. On koostatud algoritmid jaakpingete maaramiseks paksu mitmekihilise pindega mitmekihilistes ristkülikvarrastes, plaatides, silindrites ja kerades.

Tapsustatakse algpingete arvutust sirge ribaaluse, umartraataluse, õhukeseseinalise sfaarilise aluse, õhukeseseinalise rõngasaluse ja kruvijoonelise ribaaluse deformatsiooniparameetrite jargi. Esitatakse õhukeseseinalise toruaEluse ja lahtise rõnga kujulise aluse deformatsiooniparameetrite mõõtmise meetodid algpingete maaramiseks vastavalt galvaanilistes ja tampoongalvaanilistes pinnetes.

Fotoelastsusmeetodiga on kontrollitud jaakpingete jaotust sirge varrasaluse ühepoolses pindes. Termobimetallanaloogiat kasutades on naidatud, et uhepoolse pinde kasvamisel või kahanemisel paindub riba- või plaatalus sfaariliselt. Eksperimentaalselt on kontrollitud uhepoolse pindega kruvijoonelise ribaaluse puhta painde teooriat. On uuritud jaakpingeid paksudes galvaanilistes teraspinnetes.

ACKNOWLEDGEMENTS

I would like to thank all who have, in one way or another, contributed to this thesis and in particular, the following persons:

– Prof. I. A. Birger (Moscow), Prof. L. I. Dekhtyar (Kishinev), Prof. S.I. Ivanov (Samara), Dr. M. L. Kozlov (Lvov), Dr. A. A. Martynenko (Rovno), Prof. A. Peiter (Saarbrücken) and Prof. M. Perakh (Woodland Hills) for the expressing an interest in my work.

– Dr. S. Idnurm (Tallinn), Dr. A. Kiviste (Tartu), Dr. H. Lille (Tartu), Dipl.- Eng. M. A. Meshcheryakov (Moscow), Prof. V. A. Smirnov (Moscow) and Dipl.-Eng. J. Valgur (Tartu) for their helpful collaboration.

CONTENTS

INTRODUCTION

Motivation and presentation of the thesis

In recent decades, strengthening and protecting of new machine parts and rebuilding of worn-out ones by the application of various coatings has been the accepted procedure. Almost all of the coating technologies cause residual stresses and deformations in the coated parts.

It is well known that residual stresses may have both detrimental and favourable consequences for the service characteristics of coated parts. The experiences range from the cracking of the coating up to a considerable increase in fatigue strength in a coated part as a result of the residual stress state.

As all the residual stress state influences the service characteristics of the coated parts, it is of great importance to know where the advantages of residual stresses actually occur and can successfully be used. The optimized utilization of residual stress states is a challenge to modern coatings engineering.

The following conclusions can be drawn:

a) one must take residual stresses into account, if reliable coated parts are to be produced;

b) reliable methods for the determination of residual stresses in coatings and coated parts are of great theoretical and practical importance;

c) great necessity for the information about the really existing residual stress states in the coated parts exists.

Taking the conclusions above into account this thesis is to advance the existing methods and to create new ones for the determination of the residual stresses in the coatings and coated parts. The residual stresses in some coatings are investigated as well.

The thesis is based on the investigations carried out during the period of 1962-1992 at the Estonian Agricultural University. The results of these investigations, published in the series of articles [1-25], are used as grounds for the thesis.

Out of the 25 main publications on the subject of the thesis 12 articles have been included in the editions of an international distribution and preliminarily reviewed. In collective papers [16, 18-25] the candidate has been the supervisor and an equal coauthor as well. In the thesis [88] written under the supervision of the candidate, results of investigations published in papers [16, 18, 19, 22, 24, 25] are partly used.

The main results of the thesis have been presented at the following international and former All-Union conferences and colloquiums:

1. Всесоюзная научно-техническая конференция "Остаточные напряжения и несущая способность деталей машин" (Харьков, 1969).
2. XIV научное совещание по тепловым напряжениям в элементах конструкций (Киев, 1977) [26].
3. The 6th International Conference "Experimental Stress Analysis" (Munich, 1978).
4. V и XIII Всесоюзные научно-технические конференции по конструкционной прочности двигателей (Куйбышев, 1978, Самара, 1991) [27,28,31].
5. II-V Kolloquien "Eigenspannungen und Oberflächenverfestigung "mit Iternationaler Beteiligung (Zwickau, 1979, 1982, 1985, 1989) [29].
6. The 9th International Conference on Experimental Mechanics (Copenhagen, 1990).
7. The European Conference "Residual Stresses" (Frankfurt a. M., 1992) [32,33].

The main part of the thesis consists of the introduction, list of symbols, five chapters, conclusions and bibliography. Three examples of the application are included in the appendix.

Background

The system of classification which is widely used arranges systematically the residual stresses in three different kinds [59]. From the engineering point of view the first kind or macro residual stresses are considered in the present thesis to be the most interesting.

Residual stresses cannot be determined directly. Distinct deformation parameter of a substrate or coating should always be measured by which the value of residual stresses can be calculated. At the same time the difference must be made between the non-destructive and destructive measuring methods. In the non-destructive methods the deformation parameter is measured by the application of coating to the substrate, in the destructive methods by removing it from the substrate.

In the course of time various methods to determine the residual stresses in the coatings have been developed.

E.J. Mills, the author of the article "On electrostriction" [49] is generally considered as the first researcher of residual stresses in coatings (see, e.g. [58]). By his method the galvanic coating is deposited on the outer surface of the silvered vessel of a mercury thermometer and residual stresses are conventionally valued on the scale of the thermometer.

G.Stoney [57] is the first author of the quantitative method of residual stress determination in galvanic coatings. By his method the initial stress (the stress in the superficial layer of the coating) is calculated by the curvature measured after the unilateral coating of a free straight strip substrate. At the same time the state of stress of the coating and substrate is considered uniaxial and the initial stress - constant. It is worth mentioning that Stoney observed the coating process as a consecutive application of parallel elementary layers, i.e. he used the model that is known as the model of continuous growth in layers.

In the article [56] the method of measuring the curvature of the straight strip substrate with unilateral coating was developed for the case of fixed ends. A. Brenner and S. Senderoff [39] have essentially improved this method. They have used the substrate with slipping ends and formed the theory which allows us to determine the initial as well as the residual stresses and consider the difference of elastic module of coating and substrate.

In the candidate s article [81] the attention is paid to the fact that the strip substrate, the width of which is usually considerably bigger than thickness, cannot be treated as a beam. The theory of the method of measuring the curvature of a unilaterally coated plate substrate for the various fixing conditions of substrate edges is presented.

In the article [91] an error has arisen in the developed theory of the method of measuring the curvature of a straight strip substrate with unilateral coating because of the identification of the radii of gyration of a bimetal and homogeneous bar. The author has given up the constancy hypothesis of initial stress.

The idea of the method of measuring the deformation of a unilaterally coated straight strip substrate belongs to the authors of the article [93], who have used it for the determination of initial stresses in thin galvanic coatings supposing the existence of uniaxial state of stress.

The method of measuring the angular deflection of a helical warped strip substrate with unilateral coating is dealt with for the first time in the article [38], where constant initial stress is calculated by the use of modified Stoney s formula, on the assumption that the helix angle of the substrate coil is negligible. In the monograph [92] the method is advanced considering biaxial state of stress and abandoning the hypothesis of the constant initial stress. The formula obtained includes the error mentioned above in connection with the article [91].

In the article [87] the theory of the method of measuring the deflection of an unclosed ring strip substrate with slipping edges and unilateral coating on the assumption of uniaxial state of stress is presented. The determination of initial stresses by measuring of longitudinal deformation of the round wire substrate is considered in the article [54] on assumption that the circumferential stress is constant and prestressing takes place by the force of gravity.

In the article [36] the determination of residual stresses by measuring the longitudinal deformation during the etching of the outer surface (coating) of the boron tungsten fibre is considered on the assumption of uniaxial state of stress.

To determine the residual stresses in the thick non-homogeneous coating of non-homogeneous cylinder the destructive method, presented in the article [47], may be used. It is essential to mention that the identity hypothesis of deformation distribution of non-homogeneous and homogeneous cylinders decreases the precision of the method. The disadvantage of the non-destructive method covered in the monograph [77] is a disregard of the radial displacement which corresponds to the longitudinal stresses.

In the case of equal elastic parameters of coating and substrate it is possible to determine the residual stresses in cylindrical and spherical coatings by the deformation parameters measured by X-ray method on the free surface of the coating during the removal process [50, 41]. The method presented in the articles [52, 53] allows us to determine the residual stresses in the coating of the hollow sphere of the same conditions by the use of circumferential deformation measured on the inner surface of the substrate during the removal of coating.

Thermal stresses and deformations in the straight or curvilinear substrate with unilateral coating on assumption of uniaxial state of stress are treated in the article [39]. Methods presented in the article [81] allow us to determine the thermal stresses and deformations in the multilayer coatings of the plate substrate.

The survey of the literature allow us to draw the following conclusions:

1. There are no algorithms available for the determination of residual stresses in the non-homogeneous parts with non-homogeneous coating.

2. The existing theory of methods for the determination of initial stresses requires improving.

3. There are no modern methods for the determination of initial stresses, primarily such ones meant for the application of strain gages.

4. Methods for the determination of thermal stresses and deformations in non-homogeneous coating of non-homogeneous substrate require improving.

5. Several problems connected with the determination of residual stresses in coatings need experimental studies (reliability of the model of continuous growth in layers, stress distribution in the edge region of coated parts, etc.).

Outline of the thesis

Having studied the literature an attempt was made to find solutions to the following problems:

1. To compose the algorithms for the determination of residual stresses in the thick multi-layered coating of multi-layered rectangular bars, plates, cylinders and spheres.

2. To advance the theory of methods for the determination of initial stresses:

a) to consider substrate and coating non-homogeneous and the state of stress biaxial in the deformation parameters measuring method of the unilaterally coated strip or plate substrate;

b) to consider the state of stress biaxial and possibility to prestress the substrate with elastic element in longitudinal deformation measuring method of bilaterally coated straight strip substrate;

c) to desist from the hypothesis of constant circumferential stress and consider the possibility to prestress the substrate with elastic element in the longitudinal deformation measuring method of the round wire substrate;

d) to deduce the formula for the calculation of initial stress for the method of measuring the displacement of the inner surface of thin-walled spherical substrate;

e) to consider the substrate as a short cylindrical shell in theory of the circumferential deformation measuring method of a thin-walled ring substrate;

f) to deduce the formula for the angular deflection measuring method of the helical warped strip substrate as a cylindrical shell with curvilinear edges.

3. To elaborate the method for the determination of initial stresses in a thick cylindrical galvanic coating and the method for the determination of initial stresses in tampon-galvanic coating designed for the use of strain gages.

4. To improve the method for the determination of thermal stresses and deformations in unilaterally coated strip and plate substrate considering non-homogeneous coating and substrate and biaxial character of state of the stress. To compose algorithms for the determination of thermal stresses and deformations in multi-layered cylinders and spheres with multi-layered coating.

5. To verify experimentally the residual stress distribution determined by the model of continuous growth in layers.

6. To determine the distribution of residual shear stresses in the edge region of unilaterally coated strip substrate and to check it up experimentally.

7. To verify experimentally the deformation of strip or plate substrate at unilateral coating growth or removal.

8. To verify experimentally the theory of pure bending of a helical warped strip substrate with unilateral coating.

9. To investigate residual stresses in thick galvanic steel coatings used to rebuild the machine parts. To find a suitable function for the description of initial stress distribution in galvanic coatings.

10. To compose the general algorithm for the determination of residual stresses in coated parts.

To solve the problems above the methods of linear theory of elasticity and those of technical theory of shells and plates were used. In the case of strip and plate substrate the coating and substrate are considered continuously non--homogeneous, in the case of cylindrical and spherical substrate - piecewise non-homogeneous.

In the experimental investigations strain gages, holographic interferometry and the photoelastic method were used.

LIST OF SYMBOLS

In the thesis following symbols are repeatedly used:

c – dimensionless parameter

E_m – modulus of elasticity of the substrate (m=1) and coating (m=2)

$E'_m = E_m/(1-\mu_m)$ – elastic parameter of the substrate (m=1) and coating (m=2)

e – distance of the reduction surface from the interface between the coating and bar, plate or shell substrate

$f = 1 + \nu(4\zeta+6\zeta^2+4\zeta^3+\nu\zeta^4)$ – quartic polynomial

$f_1 = 1 + 3\zeta+3\nu\zeta^2+\nu\zeta^3$ – polynomial of the third degree

$f_2 = 1 + 2\zeta+\nu\zeta^2$ – polynomial of the second degree

h – variable thickness of the coating

h_1 – thickness of the substrate

h_2 – thickness of the coating

$k = h_2/h_1$ – dimensionless thickness of the coating

$k_o = r_o/r_1$ – dimensionless inner radius of the cylindrical or spherical substrate

$k_2 = r_2/r_1$ – dimensionless outer radius of the cylindrical or spherical coating

R – radius of the moving surface (superficial layer) of the cylindrical or spherical coating

r – radial coordinate

r_o – inner radius of the cylindrical or spherical substrate

r_1 – outer radius of the cylindrical or spherical substrate

r_2 – outer radius of the cylindrical or spherical coating

α_m – coefficient of thermal expansion of the substrate (m=1) and coating (m=2)

ΔT – temperature change

ε – relative linear deformation on the free surface of the bar, plate or shell substrate

$\zeta = h/h_1$ – dimensionless variable thickness of the coating

η – z/h_1 – dimensionless coordinate

$\varkappa$ – curvature of the free surface of the bar or plate substrate

$\theta = E_2/E_1$ – ratio of moduli of elasticity of the coating and substrate

μ_m – Poisson's ratio of the substrate (m=1) and coating (m=2)

$\nu = E_2'/E_1'$ - ratio of elastic parameters of the coating and substrate

$\xi = R/r_1$ - dimensionless radius of the moving surface of the cylindrical or spherical coating

$\rho = r/r_1$ - dimensionless radial coordinate

σ_{ijm} - residual stress components of the substrate (m=1) and coating (m=2) in rectangular coordinates (i, j=x, y,z)

σ_{im} - residual and thermal stress components of the substrate (m=1) and coating (m=2) in cylindrical (i=r, θ, z) or spherical (i=r, θ) coordinates

σ_m - residual stress in the substrate (m=1) and in the coating (m=2) in the case of uniaxial or planar stress state with equal principal stresses

$\bar{\sigma}_2$ - initial stress

$\bar{\sigma}_2(0)$ - initial value of the initial stress

σ^*_{ij2} - components of the additional stress in rectangular coordinates (i, j=x, y, z)

σ^*_{i2} - components of additional stress in cylindrical (i = r, θ, z) and spherical (i = r, θ) coordinates

1. DETERMINATION OF RESIDUAL STRESSES IN COATED PARTS: GENERAL ALGORITHM

1.1. Model of continuous growth in layers. Initial stress

Formation of residual stresses in the coating is directly connected with the coating process. Since a precise mathematical interpretation of this process is complicated, it is of necessity to give a growth model of the coating.

The basis of the author's studies is the model of continuous growth in layers, according to which the coating process is regarded as a continuous growth of the equidistant to substrate surface elementary layers. The general algorithm for the determination of residual stresses based on this model is presented below in detail.

Let us consider the shell-like substrate (part) with the constant thickness h_1, one lateral surface of which is covered with the coating (Fig.1.1). We take the lines of principal curvature of the interface between the substrate and coating for coordinates x, y, i.e. in general x and y are curvilinear. Rectilinear coordinate z is taken along normal of the interface towards the coating.

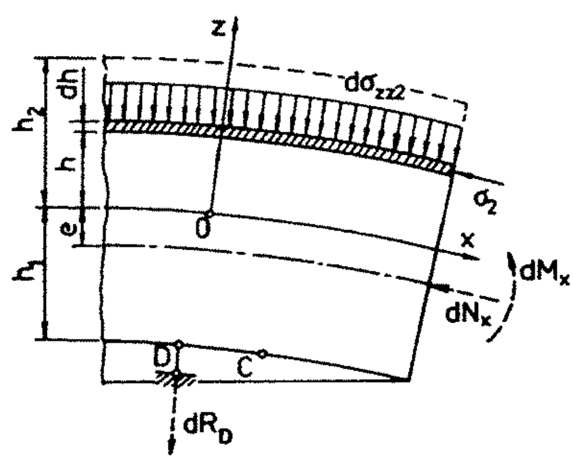

Fig.1.1 Coated substrate. The formation of an elementary layer dh of the coating creates the loads $d\sigma_{zz2}$, dN_x, dM_x and reaction dR_D of redundant constraint

We assume the outer surface of the substrate to be smooth with positive Gaussian curvature and the edges of the substrate coincident with the line of principal curvature of the interface. We assume the substrate to be sufficiently fixed, i.e. the degree of freedom of the substrate as a rigid body is zero or negative.

According to the model of continuous growth the coating is formed as a result of the successive addition of elementary layers equidistant from the substrate surface. Let us denote the variable thickness of the coating by $\mathbf{h}$. We assume that the surface $z=\mathbf{h}+d\mathbf{h}$ is free, i. e. there are no stresses on the surface and displacements are cannot be prevented.

Physico-chemical processes in the superficial layer cause a change in volume [72], which in the case of isotropic material is characterized by deformations

$$\bar{\varepsilon}^{o}_{ij2} = \bar{\varepsilon}^{o}_{2}\delta_{ij} \quad (i, j = x, y, z), \tag{1.1}$$

where $\bar{\varepsilon}^{o}_{2} = \bar{\varepsilon}^{o}_{2}(\mathbf{h})$ is the initial deformation and δ_{ij} is Kronecker s symbol.

Assuming the superficial layer being in ideal mechanical contact with the coating we came to the conclusion that of the deformations (1.1) only the deformation $\bar{\varepsilon}^{o}_{zz2}$ in the normal direction can freely be realized. The deformations $\bar{\varepsilon}^{o}_{xx2}$ and $\bar{\varepsilon}^{o}_{yy2}$ of a superficial layer element in tangential plane are prevented due to the processes which cause the change in volume also occur in the adjacent elements. Hence, the biaxial state of stress arises in the superficial layer.

Depending on the intensity of the initial deformation $\bar{\varepsilon}^{o}_{2}$, the material of the superficial layer may deform either elastically or plastically. Since the results obtained by the mechanical methods to determine the residual stresses do not depend on the mechanical state of the superficial layer, the treatment is limited to the elastic state. (More general elastoplastic treatment is given in the articles [1,84]).

According to the principle of superposition it is possible to calculate the total deformations in the superficial layer as the sum of elastic deformations $\bar{\varepsilon}^{*}_{ij2}$ and initial deformations (1.1):

$$\bar{\varepsilon}_{ij2} = \bar{\varepsilon}^{*}_{ij2} + \bar{\varepsilon}^{o}_{2}\delta_{ij}. \tag{1.2}$$

Expressing the elastic deformations in the formulas

(1.2) by stresses using Hooke's law, the following equations can be obtained:

$$\bar{\varepsilon}_{ij2} = (1/E_2)[(1+\mu_2)\bar{\sigma}_{ij2} - \mu_2\bar{\sigma}_{\Sigma 2}\delta_{ij}] + \bar{\varepsilon}_2^{o}\delta_{ij}, \qquad (1.3)$$

where $\bar{\sigma}_{ij2} = \bar{\sigma}_{ij2}(h)$ are stresses and $\bar{\sigma}_{\Sigma 2}$ is the sum of normal stresses in the superficial layer.

Bearing in mind that after formation, the superficial layer forms the whole with the coating, five of the six total deformations may be taken equal to zero:

$$\bar{\varepsilon}_{xx2} = \bar{\varepsilon}_{yy2} = \bar{\varepsilon}_{xy2} = \bar{\varepsilon}_{yx2} = \bar{\varepsilon}_{zx2} = 0. \qquad (1.4)$$

Taking condition $\bar{\sigma}_{zz2} = 0$ into account, we obtain from the expressions (1.3) a system of equations whence we find the stresses in the superficial layer:

$$\bar{\sigma}_{xx2} = \bar{\sigma}_{yy2} = \bar{\sigma}_2 = -E_2\bar{\varepsilon}_2^{o}/(1-\mu_2),$$
$$\bar{\sigma}_{xy2} = \bar{\sigma}_{yz2} = \bar{\sigma}_{zx2} = 0. \qquad (1.5)$$

With the help of the equations (1.2) and conditions (1.4) the elastic deformations in the superficial layer can be expressed in terms of the initial deformation. We obtain

$$\bar{\varepsilon}^{*}_{xx2} = \bar{\varepsilon}^{*}_{yy2} = \bar{\varepsilon}^{*}_2 = -\bar{\varepsilon}^{o}_2. \qquad (1.6)$$

Thus, a biaxial state of stress with equal principal stresses arises in the superficial layer. The stress $\bar{\sigma}_2 = \bar{\sigma}_2(h)$ in the superficial layer is called the initial stress. It is essential to note that this stress does not depend on the rigidity and fixing conditions of the coated substrate.

1.2. General algorithm for determination of residual stresses

Let us observe the formation of the stress state in the internal layer $z<h$ of the coating. The initial stress (1.5) in the layer dz arises immediately after its formation ($h=z$). The additional stresses $\sigma^{*}_{ij2}(X)$ will be added to the initial ones during the growth of the coating in a interval $z<h\leq h_2$, For brevity X denotes here and below the coordinates of a point (x, y, z).

Thus, residual stresses in the coating are expressed as the sum of initial and additional stresses:

$$\sigma_{ij2} = \bar{\sigma}_2(z)\delta_{ij} + \sigma^{*}_{ij2}(X) \qquad (\delta_{zz} = 0). \qquad (1.7)$$

The additional stresses are calculated by integrating the elementary additional stresses $d\sigma^*_{ij2}(X,h)$ arising in the layer dz by the formation of the superficial layer dh:

$$\sigma^*_{ij2} = \int_z^{h_2} d\sigma^*_{ij2}(X,h). \qquad (1.8)$$

The residual stresses, deformations and displacements in the substrate ($-h_1 \le z \le 0$) are expressed as follows:

$$\begin{bmatrix} \sigma_{ij1} \\ \varepsilon_{ij1} \\ u_{i1} \end{bmatrix} = \int_0^{h_2} \begin{bmatrix} d\sigma_{ij1}(X,h) \\ d\varepsilon_{ij1}(X,h) \\ du_{i1}(X,h) \end{bmatrix}, \qquad (1.9)$$

where $d\sigma_{ij1}$, $d\varepsilon_{ij1}$ and du_{i1} are elementary stresses, deformations and displacements, which arise in the substrate by the formation of the superficial layer dh.

We explicate the initial stress in the equations (1.7) and (1.8). For this it is possible to use the solution of the corresponding thermoelastic problem assuming, according to the formula (1.5) thermal deformation

$$\varepsilon_t = \bar{\varepsilon}^0_2 H(z-h) = -[(1-\mu_2)\bar{\sigma}_2/E_2]H(z-h) \qquad (1.10)$$

in the substrate and coating is present. In the latter formula H(z-h) is Heaviside s function.

The articles [82-84] are based on this method. For the same purpose a mechanical effect caused by the formation of the superficial layer dh could be replaced by the surface and edge loads acting on the coating.

To determine the surface load the equilibrium equation $\Sigma Z = 0$ for an element of the superficial layer dh is composed. Hence we can find

$$d\sigma_{zz2} = \mp \bar{\sigma}_2(R_x^{-1} + R_y^{-1})dh, \qquad (1.11)$$

where R_x and R_y are the radii of principal curvature of the superficial layer. The sign (-) is valid for the coating on the convex and (+) on the concave side of the substrate.

If the coating has a free edge (Fig.1.1), the homogeneous state of stress (1.5) at the edge of the superficial layer will be possible only on the condition that the edge will be loaded with the initial stress $\bar{\sigma}_2$. As in practice this load does not exist, an opposite-directed stress $-\bar{\sigma}_2$ which relieves the edge from the stress is applied to the edge of the coating. In the case of a shell-like or plate-like substrate (Fig.1.1) the proper way is to reduce the stress $-\bar{\sigma}_2$ to an edge load which consists of edge force and edge moment:

$$dN_x = -\bar{\sigma}_2 dh, \quad dM_x = \bar{\sigma}_2 (e+h) dh, \tag{1.12}$$

where $e = e(h)$ is the distance from the substrate and coating interface to the reduction surface.

In the case of isotropic substrate and coating with elastic parameters variable through the thickness, the equations of the theory of shells simplify considerably, if we determine the position of the reduction surface from the condition [65]:

$$\int_{-h_1}^{h} [E/(1-\mu)](e+z)dz = 0, \tag{1.13}$$

where $E = E(z)$ is the modulus of elasticity, and $\mu = \mu(z)$ is Poisson s ratio. If the elastic parameters of the substrate and coating are constant and equal, from the condition (1.13) we can find $e = (h_1-h)/2$, i.e. the reduction surface is the middle surface.

We note that depending on the fixing conditions of the substrate the edge load (1.12) may be either partly or fully balanced by reactions of constraints, e.g. in the case of a slipping edge the edge moment is balanced and in the case of fixed edge both the edge moment and force are balanced.

The use of the edge load as forces and moments reduced to the edge of the reduction surface means to satisfy the statical edge conditions in Saint-Venant s meaning. Therefore the solution obtained may be considered quite exact outside the edge region, the size of which usually does not exceed the total thickness of the substrate and coating.

Solving the thermoelastic problem for a certain substrate with coating by thermal deformation (1.10) or the problem of theory of elasticity by loads (1.11), (1.12), we can get the elementary stresses, deformations and displacements through the initial stress within the framework of linear elasticity [62] as follows:

$$\begin{bmatrix} d\sigma^{*}_{ij2} \\ d\sigma_{ij1} \\ d\varepsilon_{ij1} \\ du_{i1} \end{bmatrix} = \bar{\sigma}_2(h) \begin{bmatrix} f^{-1}_{ij2}(X,h) \\ f^{-1}_{ij1}(X,h) \\ g^{-1}_{ij1}(X,h) \\ p^{-1}_{i1}(X,h) \end{bmatrix} dh, \qquad (1.14)$$

where f_{ij2}, f_{ij1}, g_{ij1} and p_{i1} are the functions characterizing the rigidity of the coated substrate which depend on the shape, dimensions and fixing conditions of the substrate and the elastic parameters of the substrate and coating.

Substituting the elementary stresses, deformations and displacements from (1.14) in the expression (1.8) and (1.9) the following general formulas to calculate the stresses, deformations and displacements in the coating and substrate can be obtained:

$$\sigma^{*}_{ij2} = \int_{z}^{h_2} \bar{\sigma}_2(h) f^{-1}_{ij2}(X,h)dh, \qquad (1.15)$$

$$\begin{bmatrix} \sigma_{ij1} \\ \varepsilon_{ij1} \\ u_{i1} \end{bmatrix} = \int_{0}^{h_2} \bar{\sigma}_2(h) \begin{bmatrix} f^{-1}_{ij1}(X,h) \\ g^{-1}_{ij1}(X,h) \\ p^{-1}_{i1}(X,h) \end{bmatrix} dh. \qquad (1.16)$$

The formulas (1.7), (1.15), (1.16) allow us to calculate the residual stresses in the coating and substrate, and the residual deformations and displacements in the substrate, if the initial stress $\bar{\sigma}_2(h)$ is known. Since in the surface physics of materials there are no theoretical methods for the determination of initial stress the following experimental methods are used:

1.In the case of deformation methods the initial stress is determined by the deformation parameters measured either on the free surface of the substrate ($z=-h_1$) or on the moving surface of the coating ($z=h$). For example, measuring the deformation $\varepsilon_{xx1C}(h)$ with strain gage or linear displacement $u_{z1C}(h)$ with the displacement gage at some point C of the free surface of substrate (Fig.1.1), the initial stress can be calculated by the formulas

$$\bar{\sigma}_2 = \begin{cases} g_{xx1C}(h)\ d\varepsilon_{xx1C}(h)/dh \\ p_{z1C}(h)\ du_{z1C}(h)/dh \end{cases} \qquad (1.17)$$

resulting from the expressions (1.14).

On the moving surface of the coating it is possible to determine the elastic deformation (1.6) corresponding to the initial stress by chrystallographic parameters of the deformation measured by the X-ray method.

2. In the case of force methods the initial stress is determined by reactions of redundant constraints preventing the deformation of the substrate during the coating process. For example, by measuring the reaction $R_D(h)$ of the constraint D (Fig.1.1) the initial stress is calculated by formula

$$\bar{\sigma}_2 = q_D(h)dR_D(h)/dh, \qquad (1.18)$$

where $q_D(h)$ is the function which characterizes the rigidity of the coated substrate.

It is possible to determine the initial stress by a variant of deformation method when the deformation parameter caused by the removal of a redundant constraint is measured. For example, if the displacement $u_{z1CD}(h)$ of the substrate point C after the removal of the redundant constraint D is measured, then according to Hooke s law

$$u_{z1CD}(h) = r_{CD}^{-1}(h)R_D(h),$$

where $r_{CD}(h)$ is the function which characterizes the rigidity of the coated substrate without the redundant constraint.

Substituting the reaction $dR_D(h)$ found from the formula (1.18) by integrating into the latter expression, an integral equation for the determination of the initial stress will be obtained

$$u_{z1CD}(h) = r_{CD}^{-1}(h)\int_0^h \bar{\sigma}_2(h)q_D^{-1}(h)dh. \qquad (1.19)$$

Usually the redundant constraints for constraining the deformation of the substrate are applied to the substrate edges. The suitable force methods and corresponding deformation methods for a bar-like or plate-like substrate with slipping and fixed edges are covered in the paper [6].

The theory of deformation methods of the bar or plate-like substrate during the coating process with fixed edges is presented in the articles [8,15].

The formulas (1.7) and (1.15)-(1.19) form the general algorithm to determine the residual stresses, deformations and displacements in coated parts by the deformation parameters measured on the free surface of the substrate or on the moving surface of the coating. We demonstrate that this algorithm is valid also in case the deformation parameters are measured during the removal process of coating, i.e. if the destructive method is applied.

We assume that the removal process of coating (electrolytic or chemical etching, polishing, laser treatment, etc.) is quite precisely described by the model of continuous decrease of the coating in layers, according to which the removal process of coating is regarded as a continuous removal of the equidistant to substrate surface elementary layers. As shown above, the formation of the superficial layer dh of the coating is statically equivalent to the application of the surface and edge loads (1.11), (1.12). The removal of the same superficial layer is statically equivalent to the releasing from the loads mentioned above, i.e. with the application of the negative loads (1.11) and (1.12). If the technology of removal has no effect on the initial stress in the superficial layer, the removal of the superficial layer causes elementary stresses, deformations and displacements in the substrate and coating, different from the corresponding quantity at the forming of the superficial layer only in sign.

Thus, the removal and growth processes of the coating are invertible under certain conditions, i.e. the initial stress value does not depend on that whether the deformation parameters used for its determination were measured either on the removal or growth process. It follows that the algorithm (1.7), (1.15)-(1.19) is universal for non-destructive and destructive methods.

The present algorithm supposes that the deformation parameters for the determination of initial stress are measured during the coating process of the part in which the residual stresses are determined. In practice such measurements are usually technically sophisticated or are altogether impossible. The point is that the substrate deformation parameters are often too little for common gages or there is no free surface of the substrate to measure the deformations and displacements. To overcome these difficulties the author has advanced the method [1, 2, 83, 84] which in literature is called the computational-experimental method (see, e.g. [70,79]). The algorithm of this method formally coincides with the universal algorithm above. The essential difference is that for the determination of the initial stress the coating is applied not to the part, but to a thin-walled substrate of the same material and also, if necessary which is geometrically similar to the part. Since the rigidity of the substrate is considerably smaller than rigidity of the part, it is possible to measure quite exactly the deformation parameters with the help of common instruments.

In some cases the use of a geometrically similar thin-walled substrate may be the only way for the investigation into residual stresses in the coated massive part. For example, we may take a coated sphere when the substrate has no free surface at all and thus it is impossible to measure the substrate deformation by common techniques. Nevertheless, the residual stress can be determined by applying the coating to a thin-walled spherical substrate and measuring the radial displacement of inner surface by the principle of liquid thermometer [49 ,84].

It is of expediency to distinguish thick and thin coatings. The importance of additional stresses in residual stresses may be taken as the basis of the classification. If we have a thicker coating, additional stresses are of greater importance. If we ignore the additional stresses up to 5%, we may consider the coating of a momentless substrate thin on the condition $\nu k < 1/20$ and the coating of a substrate with the moment on the condition $\nu k \leq 1/80$ [1].

Most of the coatings used in practice are thin. Residual stresses in the part with a thin coating are ignorable, but in the coating they are equal to the initial stress.

The algorithm (1.7), (1.15)-(1.19) is based on the model of continuous growth of coating, which by the use of the concept of initial stress enables us to reduce the mechanical effect of the forming of a elementary layer to the application of elementary loads (1.11), (1.12). Such a treatment in differential form used in the most of the author's studies (see, e.g. [4, 17, 25]), may be called the differential approach. The integral approach analogously with the destructive methods [69], is possible (see, e.g. [3, 9, 10, 13]).

To eliminate the edge region by the integral approach the modified Birger's scheme (Fig.1.2) can be used. Unlike his destructive method, based on the principle of the application of opposite-direction residual stresses [69] in the case of coating growth, the residual stresses must be applied in the original direction.

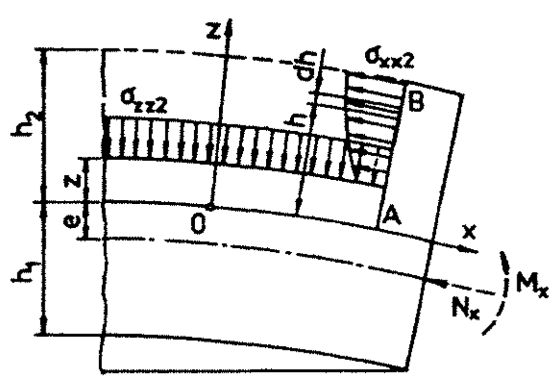

Fig.1.2. Modified Birger's scheme for the elimination of edge effect. The static effect of formation of the finite thickness layer h_2-z of the coating: loads σ_{zz2}, N_x, M_x

The differential and integral approaches mainly differ in that the model of continuous growth of coating and initial stress in the explicit form are not used in the latter case. If in the differential approach the treatment was based on the consideration of forming of an elementary superficial layer, the mechanical effects of which were elementary loads (1.11), (1.12), then in the integral approach the basis for the treatment is the formation of finite thickness layer h_2-z and the surface load $\sigma_{zz2}(X,z)$ and edge loads

$$N_x = \int_z^{h_2} \sigma_{xx2}(X,h)dh, \quad M_x = \int_z^{h_2} \sigma_{xx2}(X,h)[e(z) + h]dh \qquad (1.20)$$

statically equivalent to it. Here $\sigma_{zz2}(X,z)$ is the residual stress acting on the surface $h=z$ and $\sigma_{xx2}(X,h)$ is the residual stress acting on the surface AB in the Birger's scheme (Fig.1.2).

By the differential approach we assumed that the initial stress depended only on the variable thickness h of the coating and as a result we obtained the algorithm which helped us to determine the initial stress and residual stress by deformation parameters measured only at one point of the free surface of the substrate or moving surface of coating. In order to get the algorithm with the same possibilities by the integral approach we have to assume the residual stresses to be dependent only on the coordinate z. We know that such an assumption constrains the range of the problems solved by the integral approach because of the circumstance that unlike the initial stresses, the residual stresses depend on the geometric and mechanical parameters of the substrate.

For introducing the integral approach let us observe the determination of residual stresses by the growth of a freely deforming plate with arbitrary contour [3]. On the assumption that out of the edge zone $\sigma_{zz2}(z) = 0$, $\sigma_{xx2}(z) = \sigma_{yy2}(z) = \sigma_2(z)$ the problem is reduced to that of compression and bending of the plate with the edge force and edge moment

$$N = \int_z^{h_2} \sigma_2(h)dh, \quad M = \int_z^{h_2} \sigma_2(h)[(h_1-z)/2 + h]dh.$$

Solving this problem in the framework of the technical theory of plates and supposing that $E_1 = E_2 = E$ and $\mu_1 = \mu_2 = \mu$, an equation can be obtained:

$$2 \int_z^{h_2} \sigma_2(h)(h_1-2z + 3h)dh = E'(h_1+z)^2\Delta\varepsilon(z), \qquad (1.21)$$

where $\Delta\varepsilon(z) = \varepsilon(h_2)-\varepsilon(z)$ is the increment of deformation measured on the free surface of substrate during the coating growth at an interval of (z, h_2).

The equation (1.21) is Volterra's first kind integral equation from which, by means of the reduction to differential equation, the following one can be found:

$$\sigma_2=-E'\left[\frac{h_1+z}{2}\frac{d\Delta\varepsilon(z)}{dz}+2\Delta\varepsilon(z)-3(h_1+z)\int_z^{h_2}\frac{\Delta\varepsilon(h)}{(h_1+h)^2}\,dh\right]. \qquad (1.22)$$

It is easy to prove that the first member of the latter formula determines the initial stress in the superficial layer. Indeed, the passage to the limit $h_2 \to z$ gives

$$\bar{\sigma}_2 = -(E'/2)(h_1+z)d\Delta\varepsilon(z)/dz. \qquad (1.23)$$

Thus, in the case of integral approach the initial stress is included in the final result in an implicit form. If the function $\Delta\varepsilon(z)$ is to be found by integrating the differential equation (1.23) and then to express $\Delta\varepsilon(h)$ in the second and the third member of the equation (1.22) by initial stress, we get

$$\sigma_2 = \bar{\sigma}_2(z) + 2\int_z^{h_2}\frac{\bar{\sigma}_2(h)}{h_1+h}\,dh - 6(h_1+z)\int_z^{h_2}\frac{\bar{\sigma}_2(h)}{(h_1+h)^2}\,dh. \qquad (1.24)$$

The same result can be obtained by the differential approach while the solution process is easier, as there is no need to compose and solve the integral equation. Still we cannot conclude that the differential approach always permits us to avoid the integral equation. If the coating (e.g. on technological considerations) has been applied to the substrate with redundant constraint and the stresses are determined by the deformation parameters generated on releasing from redundant constraints, the determination of the initial stress is reduced to solving of equations which are similar to the integral equation (1.19). In some cases, e.g. by applying a coating to all the outer surface of the spherical substrate, the moving surface is closed and the edge load does not exist at all. In that case the formation of a coating layer of finite thickness $h_2 - z$ is statically equivalent to the surface load $\sigma_{zz2}(z)$ and the integral and differential approaches are practically of equal efficiency.

As it is shown above, the algorithm of the differential approach is valid on certain assumptions regardless of that whether the deformation parameters on the substrate or coating were measured at the process of a coating growth or removal. Naturally, that is valid for the algorithm of integral approach as well. In this connection we note, that in the integral approach algorithm the increment of deformation parameters of the substrate is used as an experimental information. For example, in the formulas (1.21)-(1.23) it is represented by the deformation increment $\Delta\varepsilon(z) = \varepsilon(h_2)-\varepsilon(z)$. If, for the determination of residual stresses the coating is removed and during the growing process the deformation has not been measured, then $\varepsilon(h_2)=0$, i.e. $\Delta\varepsilon(z)=-\varepsilon(z)$. The algorithm of differential approach does not need such a correction since the experimental information is used in the form of a derivative of the deformation parameter.

We add that the principles of the integral approach are formulated in the work [80] as general principles of non-destructive mechanical methods.

We have found that the residual stresses in the substrate (1.16) satisfy the usual compatibility conditions of linear elasticity.

If we have to create the compatibility conditions for the coating, taking the initial deformation (1.1) into consideration and paying no attention to the layer growth process of the coating, assuming that the physico-chemical processes generate the initial stress which appears after growth only, then the residual stresses calculated by the formulas (1.7) and (1.15) do not satisfy these conditions. For example, in the case of free plate growth, considered above the compatibility condition for the coating can be given as follows: $d^2\sigma_2^*(z)/dz^2=0$. Substituting here the additional stress (the second and the third member of the formula (1.24)), the result will be

$$d^2\sigma_2^*(z)/dz^2=[4/(h_1+z)]d\bar{\sigma}_2(z)/dz+2\bar{\sigma}_2(z)/(h_1+z)^2,$$

i.e. the compatibility condition is not satisfied.

The author has obtained the analogous results for the cylindrical and spherical coating [83, 84]. The fact that residual stresses in the coating do not satisfy the usual compatibility condition of a body with stationary configuration is explicable with the specific feature of the coating formation, which consists in discontinuity of deformation and stress distributions on the interface of coating and superficial layer dh.

1.3. Treatment of the experimental information

According to both algorithm variants above the residual stresses are calculated by the deformation parameters which are determined experimentally. Since the experimental data always contain errors, to gain the reliable results they have to be treated mathematically [48]. The measurement errors should carefully be considered as the presented algorithm requires the differentiation of experimental data, which, as it is known, is an ill-posed problem (small measurement errors of the initial data may cause big errors in their derivatives).

In the author s works the following methods for obtaining the reliable results by derivation of the experimental data have been used:

1. Preliminary smoothing of the experimental data with cubic splines [17, 18, 29] or polynomials, formed by the use of the least-squares method [13].

2. Calculating the derivative through solving the integral equation by the regularization method [24, 94].

3. Approximation of experimental data with semiempiric formulas, gained on the assumption that the dependence of initial stress on the coating thickness can be described by the linear fractional function

$$\bar{\sigma}_2 = \bar{\sigma}_2(0)(h_1 + \nu h)/(h_1 + c\nu h), \qquad (1.25)$$

where $\bar{\sigma}_2(0)$ is the initial value of initial stress and c is dimensionless parameter. If c = 1 then $\bar{\sigma}_2(h) = \bar{\sigma}_2(0)$ =const, if c < 1 the initial stress increases and if c > 1 decreases monotonically.

By using the formula (1.25) the following semiempiric formula for approximation of the deformation ε_{xx1C} can be obtained from the first expression (1.17):

$$\varepsilon_{xx1C} = \bar{\sigma}_2(0) \int_0^h g^{-1}_{xx1C}(h)[(h_1+ \nu h)/(h_1+c\nu h)]dh. \qquad (1.26)$$

To determine the parameters $\bar{\sigma}_2(0)$ and c in the author's works [2, 5, 85] the method of equal averages [76], in the article [3] - iterative method [76] and in papers [16, 22] - deforming polyhedron method [46] (with least-squares method where the integral was calculated by Simpson's rule), have been used.

Describing the initial stress distribution by the formula (1.25) has the following advantages:

a) the initial stress distribution in a coating is determined by two parameters, while parameter $\bar{\sigma}_2(0)$ determines usually the greatest value of the initial stress in the coating;

b) it is possible to predict the initial stress distribution in a coating, the thickness of which differs from that of the experimental coating;

c) if the parameter c is known for a certain class of coatings, the value of the deformation parameter measured at coating of some simple substrate, e.g. strip-like substrate, will be enough for the express check of the residual stress.

The linear fractional distribution of the initial stress cannot be universal. Experimental investigations have shown that the formula (1.25) enables us effectively describe the initial stress distribution during the galvanic deposition in a bath [3,5,22,27,30,85] and by brushes [16].

2. DETERMINATION OF RESIDUAL STRESSES IN BARS, PLATES, CYLINDERS AND SPHERES WITH A THICK COATING

This chapter covers a short survey of the author's works which deal with the determination of residual stresses in thick coatings. Due to the limited size of the thesis a detailed treatment of a general algorithm composed for the case of multilayer coatings and substrate has been omitted and the computational formulas for determination of residual stresses and deformations for more frequently occurring special case of coating technology have been presented when substrate and coating are homogeneous, i.e. their elastic parameters are constant. For the sake of unification of presentation the results are given in the form, obtained by the differential approach.

2.1. Free plates with unilateral coating

In the article [3] using the integral approach the formula is derived for the residual stresses in unilateral coating of a rectangular bar or arbitrary contoured plate by deformation, measured on the free surface of a substrate, on the supposition that elastic parameters of coatings and substrate are constant and equal. A semiempirical formula for the approximation of experimental information is obtained from the linear fractional expression (1.25).

In the article [10] the same problem is solved in more general form when the elastic constants of the coating and the substrate are different. At the same time Volterra's first kind integral equation, typical of the integral approach, is analytically solved by the reduction to differential equation and numerically using the Newton-Cotes quadrature formulas.

In the report [85], using the differential approach on the ground of the hypothesis of theory of plates [61, 89], formulas are derived for the calculation of residual stresses in the unilateral coating of a rectangular bar or a plate by the deformation $\varepsilon(\zeta)$ or curvature $\varkappa(\zeta)$ of the free surface of a substrate on the supposition that the elastic parameters of the coating and the substrate change arbitrarily in the direction of thickness. From the formulas of the report [85] the following expressions can be obtained for the special case of constant elastic parameters:

$$\bar{\sigma}_2 = \begin{cases} (E'_1/2)[f(\zeta)/f_1(\zeta)]d\varepsilon(\zeta)/d\zeta, & (2.1)\\ (E'_1 h_1/6)[f(\zeta)/f_2(\zeta)]d\varkappa(\zeta)/d\zeta, & (2.2)\end{cases}$$

$$\sigma^*_2 = 2\nu[P_1(\eta)-3(1+\eta)P_2(\eta)], \qquad (2.3)$$

$$\sigma_1 = 2P_1(0)-6(1+\eta)P_2(0). \qquad (2.4)$$

In these formulas $P_j(\eta) = \int_\eta [f_j(\zeta)/f(\zeta)]\bar{\sigma}_2(\zeta)d\zeta$ $(j=1,\ 2)$.

2.2. Circular hollow cylinders with outer coating

The articles [83, 11, 13, 17] deal with the determination of residual stresses in the coated long circular cylinders. In the article [83] proceeding from the solution of thermoelastic problem of a hollow layered cylinder the algorithm has been obtained by the differential approach to the determination of residual stresses in the coated cylinders on the supposition that the elastic parameters of coating and substrate are constant. The analogous, at the same time considering the difference between the circumferential and longitudinal initial stress, algorithm obtained by the integral approach has been presented in the article [11].

The paper [13] includes the algorithm to determine the residual stresses in the thick coating of cylinders in the case of coating and substrate being non-homogeneous composed by the integral approach. The algorithm is based on the replacement of non-homogeneous coating and substrate with piecewise homogeneous layered cylinder and application of Lamé s formulas to the homogeneous layers, while functions, included in the formulas are expressed recurrently by the deformation measured on the free surface of the substrate using the continuity conditions of radial stresses and circumferential deformatioans on the interface of layers. This method avoids solving of linear systems of equations which occur in the algorithms [83, 42, 43] which are known.

Let us present the algorithm given in the articles [11, 13] for the special case when the coating and substrate are homogeneous and the circumferential and longitudinal initial stress equals ($\bar{\sigma}_{xx2} = \bar{\sigma}_{yy2} = \bar{\sigma}_2$):

$$\bar{\sigma}_2 = \begin{cases} -[E_1/2(1-\mu_2)][q(\xi)/p_0(\xi)\xi]d\varepsilon_\theta(\xi)/d\xi, & (2.5)\\ -[E_1/2(1-\mu_2)][q(\xi)/p_2(\xi)\xi]d\varepsilon_z(\xi)/d\xi, & (2.6)\end{cases}$$

$$(\sigma^*_{r2},\sigma^*_{\theta 2}) = -Q_4(\rho)\mp Q_6(\rho)/\rho^2, \qquad \sigma^*_{z2} = -2Q_8(\rho), \tag{2.7}$$

$$(\sigma_{r1},\sigma_{\theta 1}) = -2(1-\mu_2)(1\mp k_0^2/\rho^2)Q_{10}(1), \ \sigma_{z1} = -2(1-\mu_2)Q_{12}(1). \tag{2.8}$$

In the formulas (2.5)-(2.8)

$q(\xi) = a_0\xi^4 + a_1\xi^2 + a_2$,

$p_j(\xi) = b_j\xi^2 + b_{j+1}$ ($j = 0, 2, \ldots 12$),

$Q_j(\rho) = \int_{\rho}^{k_2} [p_j(\xi)\xi/q(\xi)]\bar{\sigma}_2(\xi)d\xi$ ($j=4, 6, \ldots 12$),

$a_0 = \vartheta[2(1-\mu_1)\vartheta+(\lambda-\vartheta)(1-k_0^2)]$,

$a_1 = [2(1-\mu_2+\vartheta-\mu_1\lambda)\vartheta+(\lambda-\vartheta)(1-k_0^2)](1-k_0^2)-4(1-\mu_1)\vartheta^2$,

$a_2 = \{[2(1-\mu_2)\lambda+\vartheta-\lambda](1-k_0^2)-\vartheta[2(1-\mu_1)\lambda+2(1-\mu_2)+\vartheta-\lambda]\}(1-k_0^2)+$
$+2(1-\mu_1)\vartheta^2$,

$b_0 = \mu_1(\vartheta-\lambda)(1-k_0^2)+2(1-\mu_1)\vartheta$, $b_1 = [2\lambda-\mu_1(\vartheta+\lambda)](1-k_0^2)-2(1-\mu_1)\vartheta$,

$b_2 = (\lambda-\vartheta)(1-k_0^2)+2(1-\mu_1)\vartheta$, $b_3 = [(1-2\mu_1)\lambda+\vartheta](1-k_0^2)-2(1-\mu_1)\vartheta$,

$b_4 = a_0$, $b_5 = [2(1-\mu_1)\vartheta\lambda+(\lambda-\vartheta)(1-\vartheta-k_0^2)](1-k_0^2)-2(1-\mu_1)\vartheta^2$,

$b_6 = \{[2(1-\mu_2)+\vartheta-\lambda](1-k_0^2)-2(1-\mu_1)\vartheta\}\vartheta$, $b_7 = a_2$, $b_8 = a_0$,

$b_9 = \{\vartheta[(1-2\mu_1)\lambda+\vartheta]+\mu_2(\lambda-\vartheta)(1-k_0^2)\}(1-k_0^2)-2(1-\mu_1)\vartheta^2$,

$b_{10} = \vartheta$, $b_{11} = (1-k_0^2)\lambda-\vartheta$, $b_{12} = (\lambda-\vartheta)(1-k_0^2)+2\vartheta$,

$b_{13} = (\lambda+\vartheta)(1-k_0^2)-2\vartheta$,

where

$\lambda = (1+\mu_2)/(1+\mu_1)$.

The formulas (2.5)-(2.7) allow us to determine the residual stresses in the external coating of a hollow cylinder by circumferential deformation $\varepsilon_\theta(\xi)$ or longitudinal deformation $\varepsilon_z(\xi)$ measured on the inner surface of the substrate. If the initial stress $\bar{\sigma}_2(\xi)$ is determined by the X-ray method or by the deformation of a thin-walled substrate, the residual stresses in the coating and the substrate may be calculated by the formulas (2.7) and (2.8).

We note that in the case of equal elastic constants of the coating and the substrate ($E_1=E_2=E$, $\mu_1=\mu_2=\mu$) the formulas which in the articles [50, 41] are derived specially for the X-ray method follow from the expressions (2.7) and (2.8).

In the case of a solid substrate we have to take $k_0=0$ in the formulas (2.5)-(2.8). The adequate expressions have also been obtained by the differential approach in the article [17], where they are used for the determination of residual stresses in fibres by longitudinal deformation of core measured during the removal of the outer layer (coating). In comparison with the methods recommended in the articles [36, 95], according to which the state of stress of coated fibres is considered uniaxial, the developed method makes it possible to determine all the three principal stresses of the residual stress state (see Appendix, Example A.1).

Taking the coated thin-walled tubular substrate $(h_1, h_2) \ll r_1$ into consideration the basic formulas (2.5)-(2.8) could be simplified:

$$\bar{\sigma}_2 = -E'_1(1+\nu\zeta)d\varepsilon(\zeta)/d\zeta, \tag{2.9}$$

$$\sigma_{r2} \approx 0, \quad \sigma^*_{\theta 2} = \sigma^*_{z2} = \sigma^*_2 = -\nu\int_{\eta}^{k} [\bar{\sigma}_2(\zeta)/(1+\nu\zeta)]d\zeta, \tag{2.10}$$

$$\sigma_{r1} \approx 0, \quad \sigma_{\theta 1} = \sigma_{z1} = \sigma_1 = \int_{0}^{k} [\bar{\sigma}_2(\zeta)/(1+\nu\zeta)]d\zeta, \tag{2.11}$$

where $\varepsilon(\zeta) = \varepsilon_\theta(\zeta) = \varepsilon_z(\zeta)$ is the deformation on the inner surface of the substrate.

The boundary effect at growth of a thin-walled tubular substrate with free edges is examined in the article [7]. Using the equation of radial displacements of the long cylindrical shell with uniform moment load on one edge, it is shown that with precision of 5% the boundary effect may be considered attenuated at length

$$z_{b\,*} = \left[\sqrt{r_1 h_1(1+k)} \;/\; \sqrt[4]{3(1-\mu^2)}\right] \ln\left[20\sqrt{6(1-\mu^2)} \;/\; (1-\mu)\right].$$

Hence, a thin-walled metallic tubular substrate (μ=0.3) may be considered as a long cylindrical shell if the length $l > 6.6\sqrt{r_1 h_1(1+k)}$.

2.3. Hollow spheres with outer coating

The works [84, 9] deal with the determination of residual stresses in coated spheres. In the article [84] using the solution of thermoelastic problem of multilayer hollow sphere the algorithm for the determination of residual stresses in spherical substrate with thick coating is composed for the case of layered coating and substrate. The same problem is solved by integral approach in the paper [9].

Let us consider the algorithm, presented in the articles [84, 9] for the special case of the homogeneous coating and substrate:

$$\bar{\sigma}_2 = -[E_1/9(1-\mu_1)(1-\mu_2)][(a_o\xi^3 - a_1)/\xi^2]d\varepsilon(\xi)/d\xi, \quad (2.12)$$

$$\sigma^*_{r2} = -2(a_o - a_1/\rho^3)S(\rho), \quad \sigma^*_{\theta 2} = -(2a_o + a_1/\rho^3)S(\rho), \quad (2.13)$$

$$\sigma_{r1} = -6(1-\mu_2)(1-k_o^3/\rho^3)S(1), \quad \sigma_{\theta 1} = -3(1-\mu_2)(2+k_o^3/\rho^3)S(1). \quad (2.14)$$

In these formulas

$$a_o = \theta[2(1-2\mu_1)+(1+\mu_1)k_o^3]+(1+\mu_2)(1-k_o^3),$$

$$a_1 = \theta[2(1-2\mu_1)+(1+\mu_1)k_o^3]-2(1-2\mu_2)(1-k_o^3),$$

$$S(\rho) = \int_{\rho}^{k_2} [\bar{\sigma}_2(\xi)\xi^2/(a_o\xi^3 - a_1)]d\xi.$$

It is possible to determine the residual stresses in the coating on the outer surface of a hollow sphere with the help of formulas (2.12) and (2.13) by the deformation $\varepsilon(\xi)$, measured on the inner surface of the substrate. If the initial stress $\bar{\sigma}_2(\xi)$ has been determined by the X-ray method or by the deformation of a thin-walled substrate, it is possible to calculate the residual stresses in the coating and substrate by the formulas (2.13) and (2.14).

In the case of a solid substrate in the formulas (2.13) and (2.14) $k_o = 0$ must be taken. Note that in the special case if $E_1 = E_2 = E$, $\mu_1 = \mu_2 = \mu$ and $k_o = 0$ from expressions (2.13) and (2.14) follow the formulas which in the article [41] were obtained especially for the X-ray method.

In the case of a coated thin-walled spherical substrate $(h_1, h_2) << r_1$ and basic formulas are simplified to the formulas (2.9)-(2.11) of a coated thin-walled tubular substrate.

3. METHODS FOR DETERMINATION OF INITIAL STRESS

As it was noted above the methods for the determination of initial stress are divided into deformation and force methods. The survey of these deformation methods the theory of which has been developed the author will be given below. The methods worked out by the author or with his participation are considered as well (Sec. 3.6 and 3.10).

3.1. Method of measuring the deformation parameters of a free strip or plate substrate with unilateral coating

In the case of deformation measuring method the initial stress is calculated by the formula (2.1), in the case of curvature measuring method - by the formula (2.2). On the assumption that the distribution of initial stress is linear fractional (1.25), the corresponding expressions for the approximation of experimental information are the following:

$$\varepsilon = (2/E_1')\bar{\sigma}_2(0)P_1(\zeta), \qquad (3.1)$$

$$\varkappa = (6/E_1'h_1)\bar{\sigma}_2(0)P_2(\zeta), \qquad (3.2)$$

where $P_j(\zeta) = \int_0^{\zeta} [f_j(\zeta)/f(\zeta)][(1+\nu\zeta)/(1+c\nu\zeta)]d\zeta \quad (j=1,\ 2).$

In general the values of the function $P_j(\zeta)$ are found numerically. In the case if the elastic constants of the coating and substrate are equal ($E_1=E_2=E$, $\mu_1=\mu_2=\mu$), the integral $P_j(\zeta)$ can be calculated analytically. For example, in the case of deformation measuring method an approximative equation is obtained:

$$\varepsilon = [2(1-\mu)\bar{\sigma}_2(0)/cE]\ln(1+c\zeta). \qquad (3.3)$$

The calculation of initial stress by the curvature of a plate substrate is also covered in the paper [4] where it is shown that in the article [91] the radii of gyration of the bimetallic and the homogeneous bar are erroneously identified. Using the differential approach the formula is obtained which follows from the expression (2.2) if the curvature is expressed by the deflection of free end of a cantilevered substrate. At the same time it is observed that in a special case while $\mu_1= \mu_2= \mu$, the formulas obtained in the articles [75, 82] follow from the corrected formula.

In a very thin coating (film) the residual stresses equal to the initial stress are determined on the assumption that the physico-chemical processes causing the initial deformation take place only after the formation of the coating with the finite thickness [45]. The expressions followed from the formulas (2.1) and (2.2) for the calculation of residual stresses in very thin coatings are:

$$\sigma_2 = \begin{cases} (E_1'/2k)\varepsilon, & (3.3) \\ (E_1'h_1/6k)\varkappa. & (3.4) \end{cases}$$

In comparison with the well-known Stoney's formula [57] the formula (3.4) takes the biaxial state of stress into consideration. The latter is shown as the factor $1/(1-\mu_1)$ in the formula (3.4) [82].

3.2. Method of measuring the deformation parameters of a plate substrate with slipping edges and unilateral coating

In the case of a plate substrate with slipping edges the linear displacements of substrate edges are possible only on the plane of the substrate, the edge moments (1.12) are balanced with reaction moments of redundant constraints and thus the coated substrate deforms without bending during the coating process, i.e. only due to the action of edge forces (1.12) [6]. As a thin-walled tubular substrate deforms also under the same conditions, in the case of unilateral growth of a plate substrate with slipping edges, the expression (2.9) between the initial stress and deformation measured on the free surface of the substrate is valid.

The initial stress can be determined by the curvature, arising after the release of the substrate (removal of redundant constraints). The corresponding formula in the article [6] could be expressed as follows:

$$\bar{\sigma}_2=(E_1'h_1/6)\{[f(\zeta)/f_2(\zeta)]d\varkappa(\zeta)/d\zeta+3\nu[f_2(\zeta)/(1+\nu\zeta)]\varkappa(\zeta)\}. \quad (3.5)$$

This formula differs from that in the monograph [90] since the deduction of which the edge moment is erroneously calculated proceeding from the distribution of initial stress. Determining the edge moment before releasing the substrate proceeding from the distribution of residual stresses on the basis of the methods of the work [90] the formula (3.5) can be obtained.

3.3. Method of measuring the deformation parameters of a plate substrate with fixed edges and unilateral coating

At the coating process of a plate substrate with fixed edges the edge forces and moments (1.12) are balanced by the reaction forces and moments owing to which no additional stresses will arise in coating and substrate. To determine a distribution of the initial stress, under the same conditions the coatings with different thickness are coated to a range of substrates. Then the deformation or curvature on the free surface of the substrate, generated by a release of the edges, is measured.

The general treatment of the curvature measuring method is given in the paper [15] where the elastic parameters of the substrate and coating are supposed to change arbitrarily through the thickness. Volterra s first kind integral equation is obtained for the determination of initial stress

$$\int_0^h \bar{\sigma}_2(z)[e(h)+z]dz = D(h)\varkappa(h), \quad (3.6)$$

where D(h) is the flexural rigidity of a coated substrate.

The solution of integral equation (3.6) is found by the reduction to differential equation. In the special case if elastic parameters of coating and substrate are constant the solution may be expressed in the following form

$$\bar{\sigma}_2 = \frac{E'_2 h_1}{6}\left[\frac{f(\eta)}{\nu f_2(\eta)}\frac{d\varkappa(\eta)}{d\eta} + \frac{4\omega(\eta)}{f_2(\eta)}\varkappa(\eta) + 2\int_0^\eta \frac{f(\zeta)}{f_2^2(\zeta)}\varkappa(\zeta)d\zeta\right], \quad (3.7)$$

where

$$\omega(\eta) = 1+3\eta+3\eta^2+\nu\eta^3.$$

Presuming the linear fractional distribution of the initial stress (1.25) a formula for the approximation of experimental information has been obtained

$$\varkappa = [6\bar{\sigma}_2(0)/E'_1 h_1 \nu^2 c^3 f(\eta)]\{c\nu[c\nu-2(1-c)+\nu(3c-2)\eta]\eta + (1-c)[2-c\nu+\nu(2+c\nu\eta)\eta]\ln(1+c\nu\eta)\}. \quad (3.8)$$

Note that the curvature measuring method of the strip substrate with fixed ends is for the first time described in the article [56], where the state of stress is supposed to be uniaxial, initial stress - constant and elastic parameters of coating and substrate - equal. In the article [39] the method is generalized for the case of different elastic constants of coating and substrate. The author has dealt with the case of coating of the plate substrate with fixed edges in the articles [82, 6].

The general treatment of the deformation measuring method is presented in the work [8] where the problem of determination of initial stresses is also reduced to the solution of Volterra s first kind integral equation. In the case of constant elastic parameters of the coating and substrate a formula for the calculation of initial stress follows from the solution obtained by reduction to differential equation:

$$\bar{\sigma}_2=E_2'\left[\frac{f(\eta)}{2\nu f_1(\eta)}\ \frac{d\varepsilon(\eta)}{d\eta}+2\frac{\omega(\eta)}{f_1(\eta)}\ \varepsilon(\eta)+3(1+\eta)\int_0^{\eta}\frac{f(\zeta)}{f_1^2(\zeta)}\ \varepsilon(\zeta)d\zeta\right]. \quad (3.9)$$

Presuming the linear fractional distribution of initial stress a formula for the approximation of experimental information is obtained

$$\varepsilon=[\bar{\sigma}_2(0)/cE_1' f(\eta)]\{\eta[2v(\eta)+3\eta\chi(\eta)]+[2(c-1)/c\nu][(1/c\nu)(c\nu v(\eta)-$$

$$-\ 3\chi(\eta))\ln(1+c\nu\eta)+3\eta\chi(\eta)]\}, \quad (3.10)$$

where

$v(\eta) = 1-3\nu\eta^2-2\nu\eta^3, \quad \chi(\eta) = 1+2\nu\eta+\nu\eta^2.$

3.4. Method of measuring the longitudinal deformation of a straight strip substrate with bilateral coating

According to this method for the determination of initial stress the longitudinal deformation of the straight strip substrate is measured during the simultaneous coating of the two sides, while the substrate is prestressed either by the force of gravity [92] or an elastic element (e.g. semiconductor gage) [34, 35]. The formula for the calculation of initial stress used in the articles [34,35] has been deduced to calcule an average stress in thin coatings.

In the paper [18] on the basis of the equilibrium equations, generalized Hooke s law and continuity conditions of deformations the system of equations is composed where the formula for the calculation of initial stress is obtained

$$\bar{\sigma}_2=\frac{1}{2}\left\{\frac{E_1'}{l_1}(1+2\nu\zeta)+\frac{C}{(1-\mu_1)bh_1}\left[\frac{l_o}{l_1}(1+2\nu\zeta)+\frac{\lambda+2\nu\zeta}{\lambda+2\theta\zeta}\right]\right\}\frac{d\Lambda(\zeta)}{d\zeta}, \quad (3.11)$$

where l_o is the length of region of the substrate without coating, l_1 - length of the coated region, b - width of the substrate, C - rigidity of the elastic element (if the pre-stressing is created by the force of gravity, then C = 0), $\Lambda(\zeta)$-axial displacement of the movable end of the substrate.

In the case of a very thin coating the initial stress may be calculated by an approximate formula

$$\bar{\sigma}_2=(E_1'/2)[1/l_1+C(l_0+l_1)/E_1bl_1h_1]d\Lambda(\zeta)/d\zeta \qquad (3.12)$$

obtained by limiting process $h \to 0$ from the formula (3.11).

As an example of the application the distribution of initial stresses calculated by formula (3.12) in cobalt coating of platinum substrate is given in Appendix (Example A.2).

3.5. Method of measuring the longitudinal deformation of a round wire substrate

In the works [83, 54] it is shown that the initial stresses in coatings may be determined by the longitudinal deformation of cylindrical substrate (see Sec.2.2). The paper [29] presents a more general method for the determination of initial stresses. Unlike the articles [83,54] it takes the pre-stressing of the substrate by an elastic element [44, 64] into consideration.

By using the solution of Lamé s problem for a long cylinder with uniform radial load, continuity conditions of deformation and equilibrium equation the basic formula of the method can be obtained:

$$\bar{\sigma}_2= -\frac{E_1}{2(1-\mu_2)l_1\psi_2(h)}\left\{\psi_1(h)+\frac{C}{\pi E_1}\left[l_1+\frac{l_2\psi_1(h)}{r_1^2-r_0^2}\right]\right\}\frac{d\Lambda(h)}{dh}. \qquad (3.13)$$

In this formula C is the rigidity of the elastic element,

$$\psi_1(h) = r_1^2- r_0^2+\theta(1+2(\mu_1-\mu_2)^2r_1^2/[2(1-\mu_2^2)r_1^2+$$
$$+ (1-\mu_2^2+\gamma)(2r_1+h)h]\}(2r_1+h)h,$$
$$\psi_2(h)=\{1-2(\mu_1-\mu_2)(1+\mu_2)r_1^2/[2(1-\mu_2^2)r_1^2+$$
$$+ (1-\mu_2^2+\gamma)(2r_1+h)h]\}(r_1+h),$$

where

$$\gamma = \theta(1-\mu_1^2)[(r_0^2+ r_1^2)/(r_1^2- r_0^2)-\mu_1/(1-\mu_1)]+\mu_2(1+\mu_2).$$

Some special cases of the method are thoroughly dealt with in the work [29]. We note that in the case of solid substrate in the formula (3.13) $r_o=0$ and in the case of pre-stressing by the force of gravity $C = 0$.

3.6. Method of measuring the deformation of a thin-walled tubular substrate

The deformation measuring method of a tubular substrate is orientated to the use of strain gages. The idea of the method worked out by the author is given in the article [83]. The determination of initial stresses by the deformation, measured on the inner surface of a thin-walled tubular substrate is dealt with in the articles [2, 5, 85, 13] while the most general treatment in the case of elastic parameters of coating and substrate which change arbitrarily through the thickness using the integral approach is presented in the report [85].

In the case of constant elastic parameters of coating and substrate, the initial stress is calculated by the formula (2.9). Integrating this equation as a differential equation with respect to the deformation $\varepsilon(\zeta)$, presuming that the initial stress retains the initial value $\bar{\sigma}_2(0)$ and introducing in the result the dimensionless parameter c in order to arise the approximation accuracy, we obtain the expression for the approximation of the experimental information [2]

$$\varepsilon = -[\bar{\sigma}_2(0)/cE_2']\ln(1+c\nu\zeta). \qquad (3.14)$$

Including this expression in differential equation (2.9), we get a linear fractional function (1.25) for describing the initial stress distribution.

3.7. Method of measuring the inner surface displacement of a thin-walled spherical substrate

Measuring the radial displacement $u(\zeta)$ on the inner surface during the coating of a thin-walled spherical substrate by principle of a liquid thermometer [49], the initial stresses may be calculated by the formula

$$\bar{\sigma}_2= -(E_1'/r_o)(1+\nu\zeta)du(\zeta)/d\zeta, \qquad (3.15)$$

which follows from the formula (2.12) on the assumption that $(h,\ h_1) \ll r_o$.

3.8. Method of measuring the circumferential deformation of a thin-walled ring substrate

The circumferential deformation measuring method of a thin-walled ring substrate was used in the article [73], while the coated substrate was treated as a bimetallic disc at calculating the residual stresses [77]. Taking into account that a thin-walled ring substrate can be better treated than a short cylindrical shell, the corresponding formula for initial stress is presented in the articles [20, 31]

$$\bar{\sigma}_2 = -E_1'\{(1+\nu\zeta)/[1+\Phi(\zeta)]\}d\varepsilon(\zeta)/d\zeta. \tag{3.16}$$

In the latter formula

$$\Phi(\zeta) = \sqrt{\frac{3(1+\mu)f_2^2(\zeta)}{(1-\mu)\quad f(\zeta)}}\ \frac{ch\lambda^o(\zeta)\sin\lambda^o(\zeta)-sh\lambda^o(\zeta)\cos\lambda^o(\zeta)}{sh\lambda^o(\zeta)ch\lambda^o(\zeta)+\sin\lambda^o(\zeta)\cos\lambda^o(\zeta)},$$

$$\lambda^o(\zeta) = \lambda(\zeta)/2,\ \lambda(\zeta) = b\sqrt[4]{[3(1-\mu^2)/h_1^2](1+\nu\zeta)^2/R^2(\zeta)f(\zeta)},$$

$$R(\zeta) = R_o + \nu h_1(1+\zeta)\zeta/2(1+\nu\zeta),$$

where b and R_o are the width of the ring substrate and the radius of the middle surface, respectively.

The formula (3.16) has been obtained solving the axisymmetric problem of a short cylindrical shell with surface and edge loads (1.11), (1.12) within the framework of the technical theory of shells [71] on the assumption that the elastic moduli of the coating and substrate are constant and Poisson s ratios are equal ($\mu_1=\mu_2=\mu$).

3.9. Method of measuring the angular deflection of a helical warped strip substrate with unilateral coating

The method of measuring the angular deflection of free end of strip substrate with the curvilinear axis and unilateral coating is used in stress measuring, above all for its great sensibility [38, 51, 55, 59, 77, 92].

The papers [22, 30] present the theory of the method, which, unlike the theory based on the bar substrate model, proceeds from the model of a cylindrical shell with curvilinear edges. The bending problem of a two-layer shell, loaded with edge moments (1.12) was solved by adding the states of stresses corresponding to the pure bending and edge effect [67, 68], on the assumption that the Poisson's ratios of coating and substrate are equal ($\mu_1=\mu_2=\mu$). As a result the following formula to calculate the initial stress by angular deflection $\varphi(\zeta)$ of the free end of the substrate was obtained:

$$\bar{\sigma}_2=(E_1h_1/12\pi n)[F(\zeta)f(\zeta)/R(\zeta)f_2(\zeta)]d\varphi(\zeta)/d\zeta. \qquad (3.17)$$

In this formula n is the number of substrate coils,

$$F(\zeta) = \frac{[F_1(\zeta)F_4(\zeta)\sin^2\alpha+F_2(\zeta)F_3(\zeta)]/(1-\mu^2)}{F_1(\zeta)F_3(\zeta)-\beta(\zeta)F_4(\zeta)\sin^2\alpha+(tg^2\alpha)[F_1^2(\zeta)+\beta(\zeta)F_2(\zeta)]/2},$$

where α is helix angle of a substrate coil,

$F_1(\zeta)=1-\beta(\zeta)(\sin^2\alpha+\mu\cos^2\alpha)$, $F_2(\zeta)=1-\beta(\zeta)(\sin^2\alpha+\mu\cos^2\alpha)^2$,
$F_3(\zeta)=1-\beta(\zeta)(1-\mu)\sin^2\alpha$, $F_4(\zeta)=\beta(\zeta)(1-\mu)(\sin^2\alpha+\mu\cos^2\alpha)$,
$\beta(\zeta)=[2/\lambda^*(\zeta)][ch\lambda^*(\zeta)-\cos\lambda^*(\zeta)]/[sh\lambda^*(\zeta)+\sin\lambda^*(\zeta)]$,
$\lambda^*(\zeta)=\lambda(\zeta)\cos\alpha$.

The function $F(\zeta)$ in the formula (3.17) may be considered as a correction factor for the formula obtained on the ground of the bar substrate model. It is shown that for the substrates used in practice (λ=10–13, α=12°–24°) the range of $F(\zeta)$ is $1.12\geq F(\zeta)\geq 1.04$. The formula (3.17) gives the initial stresses higher by 4–12% compared with the formulas used earlier [38, 39, 51, 55, 63, 74, 77, 92].

3.10. Method of measuring the deflection of an unclosed ring strip substrate with slipping edges and unilateral coating

The method for the determination of initial stresses in coatings grown by brush-plating is suitable on technological grounds [16]. For coating the outer surface, the substrate is fixed into the mandrel which makes free slipping of the edges as well as momentless deformation of the coated substrate possible. The coated substrate is released from the mandrel and the slit increment of the substrate $\delta(\zeta)$ is measured as bending deflection in order to determine the initial stresses. Since releasing the coated substrate from the mandrel is equivalent to the loading of the substrate edges by uniform moment (1.20) the determination of initial stresses reduces to solving the bending problem of a short two-layer unclosed cylindrical shell. The solution to this problem within the limits of technical theory of shells [68] has enabled us to obtain the Volterra s first kind integral equation, from which

$$\bar{\sigma}_2=(E_1h_1/12\pi R_o^2)F_o\{[f(\zeta)/f_2(\zeta)]d\delta(\zeta)/d\zeta+[3\nu f_2(\zeta)/(1+\nu\zeta)]\delta(\zeta)\}. \quad (3.18)$$

Here

$$F_o= (1-\mu^2\beta)/(1-\mu^2)(1-\mu\beta),$$

where

$$\beta = (2/\lambda)(ch\lambda-\cos\lambda)/(sh\lambda+\sin\lambda), \quad \lambda = b\sqrt[4]{3(1-\mu^2)/h_1^2R_o^2}.$$

4. THERMAL STRESSES IN COATED PARTS

As usually the coefficients of thermal expansion of the coating and substrate are different and the temperature of the coating process differs from the normal temperature (+20°C), the thermal stresses are generated in coated parts. The thermal stresses combined with the residual stresses determine the inherent stress state in coated parts.

At the experimental determination of residual stresses it may turn out that the coating temperature differs from that at which the deformation parameters are measured. In such cases one has to correct the measurement results taking the thermal strains into account. In addition the solutions of thermoelastic problems may be used to obtain the relations between the deformation parameters of the substrate and initial stress (see Sec.1.2).

Considering the circumstances above the author has dedicated his efforts to solving some thermoelastic problems. In this chapter a short review of these studies is given.

4.1. Free plates with unilateral coating

The articles [81, 12, 14] deal with the determination of thermal stresses in nonhomogeneous free rectangular bars and plates with arbitrary contour. The paper [14] includes the most general treatment where the thermoelastic problem of bars and plates with arbitrarily changing elastic parameters and temperature through thickness by the semiinverse method is solved. According to the solution a plate bends spherically, a beam - circularly. The solution includes, as a special case, a well-known result for a homogeneous plate and a beam [37].

In the case of constant elastic parameters and uniform temperature change ΔT, the curvature of coated substrate is given by the expression

$$\varkappa = 6\nu k(1+k)(\alpha_1-\alpha_2)\Delta T/f(k)h_1, \qquad (4.1)$$

where

$$f(k) = 4\nu k(1+k)^2 + (1-\nu k^2)^2. \qquad (4.2)$$

Thermal stresses in the coating and the substrate:

$$\sigma_2 = E_2'\{1+ \nu k[3k+ 4k^2- 6(1+k)\eta]\}(\alpha_1- \alpha_2)\Delta T/f(k), \qquad (4.3)$$

$$\sigma_1 = -E_2' k[4+3k+ \nu k^3+ 6(1+k)\eta](\alpha_1- \alpha_2)\Delta T/f(k). \qquad (4.4)$$

The maximum stresses in the coating and the substrate arise at the interface (η=0) [12].

The formulas (4.1) and (4.2) show that in the case of fixed total thickness $h_1+ h_2$, the thermosensibility of the bimetallic plate is maximum if $\nu k^2= 1$. This condition includes the Villarceau s condition as a special case.

Note that Timoshenko s well-known formula [60] results from the expression (4.1) in the special case $\mu_1=\mu_2=\mu$. Formulas (4.1),(4.3) and (4.4) for normal thermobimetal (νk^2=1) simplify [23]:

$$\varkappa = 3(\alpha_1-\alpha_2)\Delta T/2(1+k)h_1, \qquad (4.5)$$

$$\sigma_2 = E_2'(2k- 3\eta)(\alpha_1- \alpha_2)\Delta T/2(1+k), \qquad (4.6)$$

$$\sigma_1 = -E_1'(2+3\eta)(\alpha_1- \alpha_2)\Delta T/2(1+k). \qquad (4.7)$$

Researches made by the holographic method show (see Sec. 5.2) that the thermobimetallic strips with a relative thickness $(h_1+h_2)/b \le 1/20$ and width $b/l \ge 1/5$ bend spherically by uniform temperature change. As the relative thickness and width of the thermobimetallic springs used in the instrument engineering correspond to the ones of analyzed strips, at calculating it is not advisable to take them as strips as it is usually done [66], but as plates. At the same time, notice that the curvature formula of normal thermobimetal (4.5) does not include elastic constants and so it gives the same results for a strip and a plate. As regards to stress, according to the formulas (4.6) and (4.7) the stresses in the layers of a plate are $1/(1-\mu_2)$ and $1/(1-\mu_1)$ times greater than in the corresponding layers of a strip.

4.2. Coated circular cylinders

In the article [83] the axisymmetric thermoelastic problem of a multilayer (piecewise homogeneous) circular cylinder has been solved. According to the algorithm obtained from the system of linear equations the radial stresses on the interfaces of layers and the longitudinal deformation will be found and then the stresses in the layers will be calculated. Uniform temperature change ΔT causes the following stresses in the coated hollow cylinder [12]:

$$(\sigma_{r2},\sigma_{\theta 2}) = [p/(k_2^2-1)](1 \mp k_2^2/\rho^2),$$
$$\sigma_{z2} = E_2(\varepsilon_z-\alpha_2\Delta T)-2\mu_2 p/(k_2^2-1), \tag{4.8}$$

$$(\sigma_{r1},\sigma_{\theta 1}) = [p/(1-k_o^2)](1 \mp k_o^2/\rho^2),$$
$$\sigma_{z1} = E_1(\varepsilon_z-\alpha_1\Delta T)+2\mu_1 p/(1-k_o^2) \tag{4.9}$$

In these formulas

$$p = (c_{14}c_{20}- c_{10}c_{22})/(c_{10}c_{21}- c_{12}c_{20}),$$

$$\varepsilon_z= (c_{12}c_{22}- c_{14}c_{21})/(c_{10}c_{21}- c_{12}c_{20}),$$

where

$$c_{14} = [(1+ \mu_2)\alpha_2-(1+ \mu_1)\alpha_1]\Delta T/2,$$

$$c_{20} = [E_1(1-k_o^2)+ E_2(k_2^2-1)]/2, \qquad c_{10} = (\mu_1- \mu_2)/2,$$

$$c_{22} = -[E_1(1-k_o^2)\alpha_1+ E_2(k_2^2-1)\alpha_2]\Delta T/2, \quad c_{21} = \mu_1- \mu_2,$$

$$c_{12}= \frac{1+ \mu_1}{E_1}\left[\frac{1}{2} - \frac{1- \mu_1}{1-k_o^2}\right] - \frac{1+ \mu_2}{E_2}\left[\frac{1}{2} + \frac{1- \mu_2}{k_2^2-1}\right].$$

The formulas presented include the formulas of a coated solid cylinder as a special case (k_o=0).

In the case of a coated thin-walled tubular substrate the radial stresses are negligible, the circumferential and longitudinal stresses can be expressed as follows [12]:

$$\sigma_{\theta 2} \approx \sigma_{z2} = -E_2'(\alpha_2-\alpha_1)\Delta T/(1+\nu k),$$
$$\sigma_{\theta 1} \approx \sigma_{z1} = E_2'(\alpha_2-\alpha_1)\Delta T k/(1+\nu k). \tag{4.10}$$

4.3. Coated spheres

The thermoelastic problem of a central symmetric multilayer (piecewise homogeneous) hollow sphere is solved in the paper [84]. According to the gained algorithm the radial stresses on the interface of layers are found from the system of equations with three-diagonal matrix and then the stresses in layers are calculated.

Uniform temperature change causes the following stresses in a coated hollow sphere [12]:

$$\sigma_{r2} = [p/(k_2^3-1)](k_2^3-\rho^3)/\rho^3,$$
$$\sigma_{\theta 2} = -[p/2(k_2^3-1)](k_2^3+2\rho^3)/\rho^3, \tag{4.11}$$

$$\sigma_{r1} = [p/(1-k_o^3)](\rho^3-k_o^3)/\rho^3,$$
$$\sigma_{\theta 1} = [p/2(1-k_o^3)](k_o^3+2\rho^3)/\rho^3. \tag{4.12}$$

In these formulas

$$p = \frac{2E_1(\alpha_2-\alpha_1)\Delta T}{[2(1-2\mu_1)+(1+\mu_1)k_o^3]/(1-k_o^3)+[2(1-2\mu_2)+(1+\mu_2)k_2^3]/(k_2^3-1)\theta}.$$

For a coated solid sphere $k_o=0$ is taken. Radial stresses in a coated thin-walled spherical substrate are negligible, but circumferential stresses are equal to the corresponding ones of a thin-walled tubular substrate (4.10).

5. SOME RESULTS OF THE EXPERIMENTS

In this chapter a summary of the author s experimental research into modelling the continuous growth of coating in layers, verifying the deformation of a strip substrate and determination of initial stresses in galvanic steel coatings is presented.

5.1. Modelling of residual stresses by the photoelastic method

As noticed above (see Sec. 1.1), mechanical methods for the determination of residual stresses are based on the model of continuous growth in layers. To verify this model a unilateral coating growth of a rectangular bar in the case of constant initial stress was investigated by the photoelastic method [25, 33].

The width of the model, made of epoxide resin ЭД-5 was 9.5 mm, the height of the substrate 25.4 mm and the length 220 mm. The modelling part of the coating with the height of h_2= 25.3 mm was obtained by consecutively glueing eleven strips of 2.2x9.5 mm prestressed with the initial stress $\bar{\sigma}_2$= 2.7 MPa. For technological reasons momentless state was realized in the middle region of the model during its making. For that purpose the model in the test apparatus was loaded according to the scheme of the beam with two cantilevers so that during the growth the region of the model between supports was kept straight by reducing the deflection to zero.

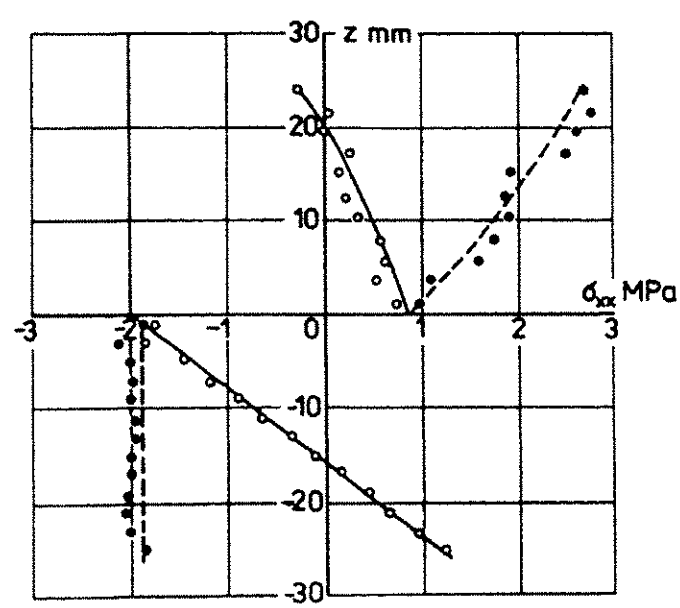

Fig.5.1. Distribution of the residual normal stresses σ_{xx} in the middle section of the model. Loaded model:---- theory, • photoelastic analysis. Free model: —— theory, • photoelastic analysis

The distribution of residual normal stresses in the middle section of the model (Fig.5.1) was determined before and after the release of the ends from the load. The calculated values were obtained by the formulas based on the model of continuous coating growth. The distribution of the residual shear stresses (Fig. 5.2) was determined on the end region of the free model, close to the interface between the substrate and the coating after cutting off the cantilevers. The theoretical values were obtained using the solution of a thermoelastic problem on a two-layer bar [40] as well by the finite elements method.

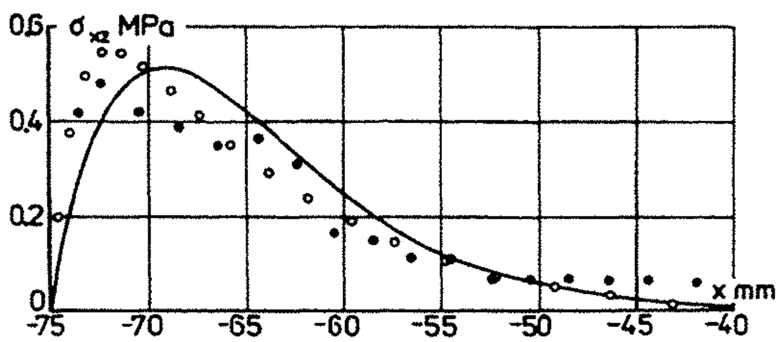

Fig.5.2. Distribution of the residual shear stresses σ_{xz} in the longitudinal section z=-1.1mm of the end region of the free model: ——— theory, • finite element analysis, ∘ photoelastic analysis

From Figs.5.1 and 5.2 it follows that the theoretical and experimental data satisfactorily coincide. The area of distribution of the edge effect is approximately equal to the height of the model. Thus, the results of the modelling confirm the validity of the model of continuous growth in layers and the well-known Saint-Venant s principle [78].

5.2. Deformation analysis of straight strip substrates

By the use of the method of measuring the deformation parameters unilaterally coated plate substrate, a rectangular plate is ordinarily used as a substrate. If the width of the substrate is big enough compared with the length, the substrate is considered as a plate which deforms spherically during the coating process [19]. A straight strip substrate (a narrow rectangular plate), the width of which is small compared with the length, but thickness is smaller than width is very often used as a substrate and the substrate cannot be taken as a bar. In the case of such a strip substrate the question arises about the character of its deformation, i.e. the problem is whether the substrate can be taken as a plate or is it necessary to form a new theory which takes the relative width and thickness into consideration.

In the article [19] within the scope of technical theory of plates it is shown that the deformation of the plate substrate in a growing or removal process is similar to the deformation of a bimetallic plate which is caused by uniform temperature change. Since the experimental study of a bimetal specimen deformation caused by a temperature change is technically easier, thermobimetal analogy was used to explain the effect of the width of the plate substrate on its deformation during the growing or removal of the coating.

In the article [19] there are given the results of the determination of curvature in longitudinal and lateral direction of the strips made of normal thermobimetal ТБ-1523 (thickness h = 0.6 mm, length l= 50 mm) [66]. To determine the curvature a specimen was fixed immovably on the short midline of one side. The angle of rotation in the middle of the opposite side, caused by a temperature change, was measured by an optical goniometer. The dependence of the curvatures $\varkappa$ calculated by the angle of rotation on the relative width b/l of the specimen is presented in Fig.5.3 which shows the data lie in the tolerance band of theoretical values. As a result the bending of the strip can be considered spherical, if its relative width $b/l \geq 0.05$.

The effect of the fixation length of the strip was also observed. It appeared the fixation length does not have any essential effect on the bending of a strip with the relative width $b/l \leq 0.3$.

As the determination of curvature of bimetal strip the angle of rotation of the edge assures the spherical bending of the strip only indirectly, the deformation of the strip was examined by the method of interference holography [21, 23] as well. The analysis of the interferograms of the deformations, caused by the uniform cooling of the strips, made of the bimetal TБ-1523 (thickness h = 0.6 mm, length l = 50 mm), has shown that the strips of interference were circular, i.e. the surface of the specimen is deformed into the surface of revolution. For the refinement of the surface the displacements measured on the longer axis of symmetry were approximated with a quadratic polynomial which in the case of small deflections corresponds quite precisely to the assumption that the specimen bends spherically.

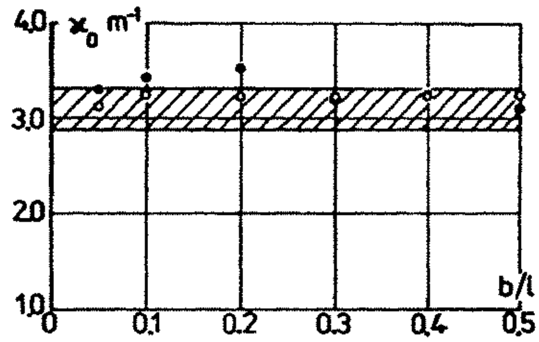

Fig.5.3. Curvature of a bimetal strip depending on the relative width: tolerance band of theoretical values (hatched), experimental values of longitudinal curvatures (∘) and lateral curvatures (•)

The polynomial constants - displacement w_o and angle of rotation φ_o at the origin of the coordinates (in the middle of the specimen) and the curvature $\varkappa$ in the middle region with length l_o were determined by the least-squares method. The results obtained by three specimens are given in Table 5.1, where the quadratic mean deviation of approximative function and experimental results δ are shown. As the latter is of the same order with measuring accuracy (79 nm), the approximation with quadratic polynomial may be considered optimal and surface of the specimen spherical.

Table 5.1. Deformation parameters of bimetal specimens

b (mm)	b/l	ΔT (°C)	l_o (mm)	$\varkappa\ 10^5$ (mm^{-1})	w_o (μm)	$\varphi_o\ 10^4$ (rad)	δ (nm)
7.8	0.16	-0.9	43.5	$\frac{4\ 16}{4\ 05}$	9.76	0.233	142
17.8	0.36	-1.0	36.6	$\frac{4.65}{4.50}$	7.72	0.081	37
25.0	0.50	-0.9	47.5	$\frac{4\ 08}{4\ 05}$	11.48	0.089	42

Note. Experimentally obtained curvature values are shown in the numerator, the values calculated by the formula (4.5) - in the denominator

Thus, proceeding from the results gained by the holographic method we can assure that coated strip substrate with the relative thickness $h/b \leq 0.08$ and the relative width $b/l \geq 0.16$ bend spherically during the growth or removal of the coating. Deformation study of the straight strip substrate during its unilateral covering with the colloid thin film [86] has confirmed this result.

5.3. Deformation analysis of curvilinear strip substrates

It is known that a uniform temperature change of the shell, nonhomogeneous in the direction of thickness, causes the same bending deformation as the uniform edge moment [65]. This analogy has allowed us to model the bending of a curvilinear strip substrate during the coating by bending of the curvilinear thermobimetal strip at uniform heating and thus experimentally check up the bending theory of the coated curvilinear strip substrate advanced in the article [22], according to which the edge moment equivalent to the uniform temperature change ΔT causes the angle of rotation

$$\varphi_T = 3\pi n R_o(\alpha_1-\alpha_2)\Delta T/F(k)(1-\mu)(h_1+h_2) \qquad (5.1)$$

of the free end of curvilinear strip made of normal thermobimetal. In the formula (5.1) F(k) is the parameter defined in Sec.3.9.

Table 5.2. Angle of rotation of free end of curvilinear thermobimetal strips at uniform heating

Specimen no.	Geometric parameters		Angle of rotation φ_T (rad)		Discrepancy (%)
	Width b (mm)	Angle of slope α (deg)	Theory	Experiment	
1	7.1	10.0	0.208	0.211	+1.4
2	8.9	14.7	0.217	0.222	+2.3
3	10.1	14.0	0.226	0.228	+0.9
4	10.1	18.9	0.228	0.230	+0.9
5	10.1	22.4	0.231	0.235	+1.7
6	11.0	13.5	0.227	0.228	+0.4
7	13.4	19.3	0.245	0.242	-1.2
8	15.5	19.8	0.255	0.248	-2.7

The specimens were made of thermobimetal TБ-1523 with the thickness of 0.78 mm. The number of coils of all specimens n=4 and radius of middle surface R_o=7,5 mm. The angle of rotation of the specimens free end while heating in the thermostat was measured by an optical goniometer. The comparison of theoretical and experimental values of the angle of rotation for ΔT = 70°C and $\alpha_1-\alpha_2$= $18\cdot10^{-6}$1/°C is presented in Table 5.2.

As shown in the table the discrepancy of theoretical and experimental data does not exceed 2.7%. Such result allows us to evaluate the bending theory of the coated curvilinear strip substrate [22] quite perfectly and at the same time recommend the formula (5.1) for calculating the curvilinear strip springs of bimetal thermometers which at present are calculated on the ground of the model of a curvilinear bar [66].

5.4. Study of residual stresses in galvanic steel coatings

Residual stresses in galvanic steel coatings used for restoration of machine parts have been studied by the deformation measuring method of a thin-walled tubular substrate [2, 5, 85]. The cathodes with the inner radius r_o=15 mm, the thickness h_1=1mm, the length of the coated region 130mm were used. The deformation was measured by a wire strain gage glued on the inner surface. The coating was carried out in the bath where the rotating anodes and automatically operating heating system were used for keeping uniform thickness and temperature of the electrolyte.

The coatings with the thickness of 0.47 – 0.52 mm were deposited from the electrolyte (g/l): iron chloride 500, sodium chloride 100, manganese chloride 5, free hydrochloride acid 0.8-1.0. The coating temperatures were 90, 92.5 and 95°C, current densities – 1.5 and 2 kA/m^2.

To determine the elastic constants of a coating the thin-walled tubular specimens with the inner diameter of 9mm, the thickness 0.5-0.7mm, the length of the coated region of 90 mm were used. The specimens were obtained by the deposition of a thin etching-safe coating and of the coating under investigation on a thin-walled tubular substrate and by following the etching of the substrate metal from the gage portion. The modulus of elasticity was determined by the tensile test. The Poisson s ratio was calculated by modulus of elasticity and the shear modulus from torsion test. In the limits of experimental errors for all coating conditions used the values E_2=200GPa, μ_2=0.3 were obtained.

In the first experiments the circumferential and longitudinal deformations were measured in order to explain the character of the state of stress in a coating. The analysis of the eight experiments has shown that these deformations are practically equal, i.e. the assumption according to which the biaxial state of stress with equal principal stresses arises in the superficial layer of the coating (see Sec. 1.1 and 2.2) is valid. Since the glueing of the strain gages measuring the longitudinal deformation comparing with circumferental one is easier, in remaining experiments an average longitudinal deformation of every cathode was measured by four gages.

The deformations measured were approximated by the logarithmic expression (3.14). The parameters c and $\bar{\sigma}_2(0)$ were determined by the use of the computer program which was based on the method of averages [76].

The analysis has shown that the variation extent 0.41 of the parameter c, obtained under the same conditions in the two experiments, is practically equal to the variation extent of the parameter c respective to different coating conditions. Considering this, the parameter c was taken as equal for all investigated coating regimes with the arithmetical mean c=1.8 gained from all the experiments.

The values of the parameter $\bar{\sigma}_2(0)$ are presented in Table 5.3. As we can see the initial value $\bar{\sigma}_2(0)$ of the initial stress, which in the case of the parameter c>1 is the greatest value of the initial stress in the coating, does not depend much on substrate metal. But this parameter is essentially influenced by the coating temperature and current density. As the temperature rises the parameter $\bar{\sigma}_2(0)$ decreases, but at an increase in current density it is also increased.

Table 5.3. Initial values of the initial stress of galvanic steel coatings

Cathode material	Coating regime		Initial value of initial stress $\bar{\sigma}_2(0)$ (MPa)
	Temperature (°C)	Current density (kA/m^2)	
Steel 15	90	1.5	268
	90	2.0	300
	92.5	2.0	301
	95	2.0	92
Steel 45	90	1.5	282
	90	2.0	345
	92.5	2.0	265
	95	2.0	103
Copper	90	1.5	286
	90	2.0	330
	92.5	2.0	260
	95	2.0	109

As an example in Appendix (Example A.3) the distributions of initial and residual stresses in a galvanic steel coating of the thin-walled tubular cathode are presented.

CONCLUSIONS

1.A general algorithm for the determination of residual stresses in coated parts is developed. The algorithm is universal and allows us to determine the residual stresses at coating growth or removal by deformation parameters, measured on the free surface of the substrate or on the moving surface of the coating. The algorithm may be used when the initial stress is determined by the method of surface physics or by measuring the deformation parameters of a thin--walled substrate.

2. The algorithms for determination of the residual stresses in multi-layer rectangular bars, plates, cylinders and spheres with a thick multi-layer coating are developed.

3. The theory of the methods for the determination of initial stresses is developed as follows:

a) in the deformation parameters measuring method of a unilaterally coated strip or plate substrate, both the coating and the substrate are considered non-homogeneous and the state of stress biaxial;

b) in the longitudinal deformation measuring method of a straight strip substrate with a bilateral coating, biaxial character of the state of stress and the possibility to prestress the substrate with an elastic element are considered;

c) for the inner surface displacements measuring method of the thin-walled spherical substrate (Mills method) the formula for the calculation of initial stress by radial displacement of the substrate inner surface is deduced;

d) the substrate is treated as a short cylindrical shell in the theory of circumferential deformation measuring method of a thin-walled ring substrates;

e) the substrate is treated as a cylindrical shell with curvilinear edges by the deduction of the formula for the method of measuring the angular deflection of a helical warped strip substrate.

4. For the determination of the initial stresses in galvanic coatings the deformation measuring method of a thin-walled tubular substrate is elaborated and for the

determination of the initial stress in tampon-galvanic coatings the deformation parameter measuring method of a thin-walled cut ring substrate is elaborated.

5. The methods for the determination of the thermal stresses and deformations in strip and plate substrates with unilateral coatings are perfected, assuming that the coating and the substrate are non-homogeneous and the state of stresses is biaxial. The algorithms for the determination of the thermal stresses and deformations in multi-layer cylinders and spheres with a multi-layer coating are composed.

6. The residual stress distribution in the unilateral coating of the straight bar substrate, determined on the ground of the model of a continuous layer growth of the coating, is verified by the photoelastic method.

7. The distribution of residual shear stresses in the end region of a unilaterally coated straight strip substrate is determined theoretically by thermobimetal analogy, numerically by using the finite-element method, and experimentally by the photoelastic method.

8. Using thermobimetal analogy it has been shown experimentally that at unilateral coating growth or removal a strip or plate substrate bends spherically.

9. Pure bending theory of the helical warped strip substrate with a unilateral coating is experimentally verified using thermobimetal analogy.

10. Residual stresses in thick galvanic steel coatings are investigated by the deformation measuring method of a thin-walled tubular substrate. It is shown that for the description of the initial stress distribution in a galvanic coating, the two-parametric linear fractional function is suitable.

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APPENDIX: EXAMPLES OF APPLICATION

Example A.1. Residual stresses in a boron fibre

In Fig.A.1 there is shown the distribution of residual stresses in boron fibre, which core radius $r_1 = 8.1\ \mu m$, outer radius $r_2 = 51\ \mu m$, $E_1 = 669$ GPa, $E_2 = 393$ GPa, $\mu_1 = \mu_2 = 0.21$. The stresses are calculated by the longitudinal deformation $\varepsilon_z(\xi)$ taken from the article [36] and previously smoothed by cubic splines. As it can be seen we obtain abased values for longitudinal stresses on the assumption of a linear stress state, at which the greatest difference is at the interface of the core and boron layer.

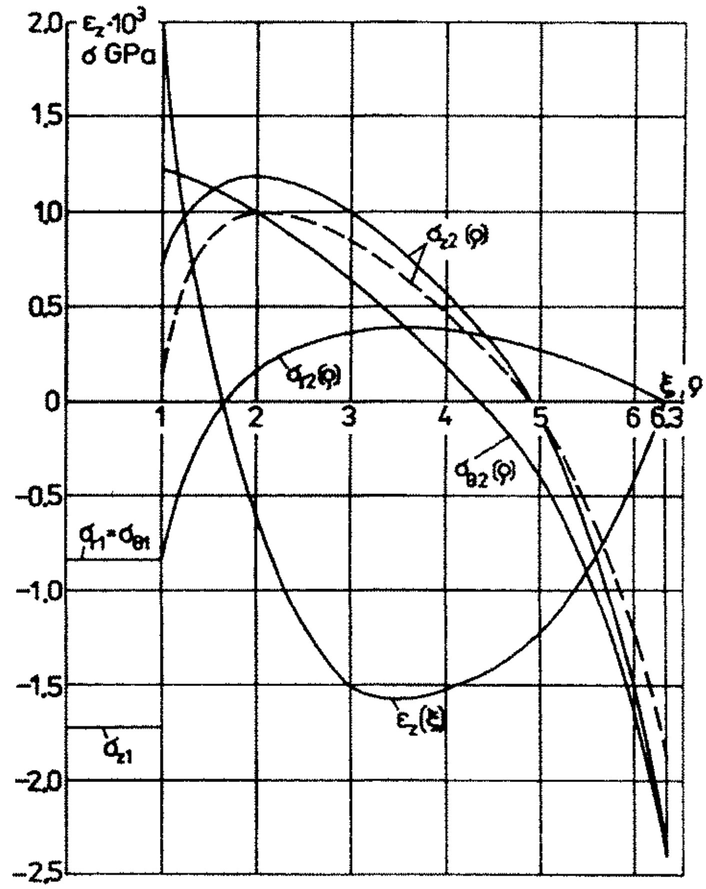

Fig.A.1. Longitudinal deformation ε_z measured during the etching of boron fibre and distribution of residual stresses σ_{r2}, $\sigma_{\theta 2}$, σ_{z2} determined by this deformation. Dash-line shows the distribution of stresses obtained on the assumption of a uniaxial stress state

Example A.2. Initial stresses in a cobalt coating

In Fig.A.2 the distribution of initial stresses in the bilateral cobalt coating (h_2=210 nm) of the platinum substrate (h_1= 25 μm, b =12.7 mm, l_1=44 mm, l_o=0, E_1=167 GPa, μ_1=0.39, C = 3.68 MN/m) is shown.

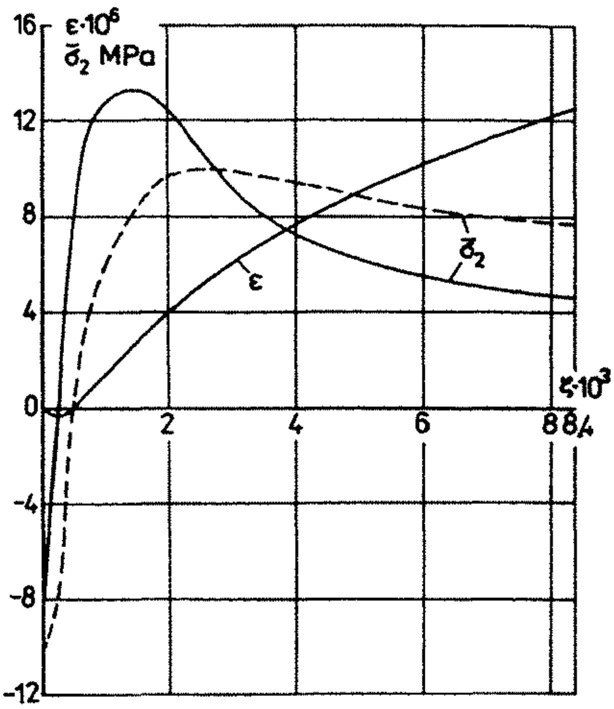

Fig.A.2. Deformation of the elastic element measured during the bilateral galvanic deposition of cobalt on the platinum strip substrate and determined according to this initial stress distribution. Dash-line shows the distribution of an average initial stress [35]

The initial stress distribution is obtained by the formula (3.12), by the deformation of the gage ε=Λ/l (l is the length of the gage), registered during the coating process of the substrate, prestressed with semiconductor strain gage [35], while the data have been previously smoothed with cubic splines. As it can be seen the distribution of average initial stresses differs essentially from that obtained by the formula (3.12).

Example A.3. Initial and residual stresses in a galvanic steel coating

Fig.A.3 shows the initial and residual stress distributions in the galvanic steel coating (h_2=0.49mm, E_2=200GPa, μ_2=0.3, α_2= 12$\cdot 10^{-6}$1/ °C) of thin-walled copper tubular cathode (r_o=15.01mm, h_1=0.98mm, E_1=110GPa, μ_1=0.3, α_1=17$\cdot 10^{-6}$1/°C).

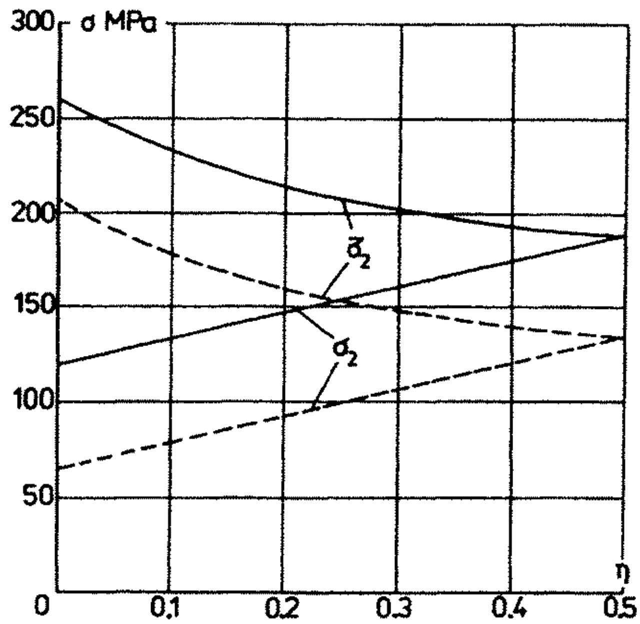

Fig.A.3. Distribution of initial $\bar{\sigma}_2$ and residual σ_2 stresses in the galvanic steel coating of thin-walled copper tubular cathode at the coating temperature of 92.5°C(——) and at normal temperature of 20°C(---)

The initial stresses are calculated by the formula (1.25) as follows:

$$\bar{\sigma}_2 = \bar{\sigma}_2(0)(1+\nu\eta)/(1+c\nu\eta). \qquad (A.1)$$

Additional stresses are found by the formula

$$\sigma_2^* = -[\bar{\sigma}_2(0)/c]\ln[(1+c\nu k)/(1+c\nu\eta)],$$

which follows from the formula (2.10) in the case of linear fractional initial stress distribution (A.1). To reduce the stress distribution to normal temperature the formula (4.10) was used.