DOCTORAL THESIS

Aspects of Higgs Boson Physics: Vacuum Stability, Yukawa Couplings and SO(10) Unification

Ruiwen Ouyang

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Declaration:
Hereby I declare that this doctoral thesis, my original investigation and achievement, submitted for the doctoral degree at Tallinn University of Technology, has not been submitted for any academic degree elsewhere.

Ruiwen Ouyang ______________________
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Higgsi bosoni füüsika aspektid: vaakumi stabiilsus, Yukawa interaktsioonid ja SO(10) ühendteooriad

RUIWEN OUYANG

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List of Publications

The present Ph.D. thesis is based on the following publications that are referred to in the text by Roman numbers.

I Hiroyuki Ishida, Shinya Matsuzaki, and Ruiwen Ouyang. Unified interpretation of scalegenesis in conformally extended standard models: a dynamical origin of Higgs portal. *Chin. Phys. C*, 44(11):111002, 2020

II Emidio Gabrielli, Luca Marzola, Kristjan Müürsepp, and Ruiwen Ouyang. Vacuum stability with radiative Yukawa couplings. *JHEP*, 01:142, 2022

III Abdelhak Djouadi, Ruiwen Ouyang, and Martti Raidal. Yukawa coupling unification in non-supersymmetric SO(10) models with an intermediate scale. *Phys. Lett. B*, 824:136788, 2022

IV Abdelhak Djouadi, Renato Fonseca, Ruiwen Ouyang, and Martti Raidal. Non-supersymmetric SO(10) models with Gauge and Yukawa coupling unification. *Eur. Phys. J. C*, 83(6):529, 2023

Author's Contributions to the Publications

I The author played a significant role in initiating the collaboration, generating ideas, analyzing results, and writing portions of the manuscript as one of the main contributors

II The author contributed to the project as one of the participants by writing sections of the code, executing numerical computations for the renormalization group equations, analyzing the results, and performing consistency checks.

III The author served as the primary and corresponding author, taking responsibility for writing the complete code, executing numerical computations, analyzing the results, preparing figures, and drafting the manuscript.

IV The author was the primary and corresponding author, who independently completed all theoretical and numerical computations, and generated data through self-written algorithms to produce all figures and tables. The author then summarized analytical and numerical results, discussed their physical significance, and drafted the manuscript.

Abbreviations

2HDM	Two-Higgs-Doublet Model
BSM	Beyond Standard Model
CC	Cosmological Constant
CSI	Classical Scale Invariance
CSIMs	Classical Scale Invariant Models
EFT	Effective Field Theory
EWSB	Electroweak Symmetry Breaking
FCNC	Flavor-Changing Neutral Current
GUT	Grand Unified Theory
IR	Infrared
LRSMs	Left-Right Symmetric Models
MSSM	Minimal Supersymmetric Standard Model
QCD	Quantum Chromodynamics
QED	Quantum Electrodynamics
QFT	Quantum Field Theory
QG	Quantum Gravity
RG	Renormalization Group
SM	Standard Model
SMEFT	Standard Model Effective Field Theory
SSB	Spontaneous Symmetry Breaking
SUGRA	Supergravity
SUSY	Supersymmetry
UV	Ultraviolet
VEV	Vacuum Expectation Value
YM	Yang-Mills

1 Introduction

The Standard Model (SM) of particle physics is a theory that describes the fundamental building blocks of matter and three of the interactions that govern their motions: the electromagnetic [5, 6, 7, 8, 9, 10, 11], weak [12, 13, 14] and strong [15, 16, 17, 18, 19, 20] interactions. It is a widely accepted and successful theory that has been extensively tested through a variety of experiments and confirmed by numerous observations including the discovery of the Higgs boson a decade ago [21, 22, 23].

The guiding philosophy for particle physics is the idea that all matter is made up of elementary point-like particles, the smallest units of matter that cannot be further divided, which are classified into two categories: fermions that make up matter such as quarks and leptons, and bosons that mediate forces such as photons and gluons. As Wigner showed in 1939 [24], particles are defined to be physical fields that are uniquely classified by the mass m and spin J where m is a non-negative real number and spin is a non-negative half-integer, and should be embedded into the irreducible representations of the Poincaré group. The physical multi-particle states are then constructed by defining the creation and annihilation operators as functions of fields that satisfy certain commutation or anti-commutation relations. This method is now formalized within a robust framework known as the quantum field theory (QFT) [25, 26, 27, 28, 29], which has been extensively tested through experiments with remarkable precision.

A QFT is usually built from the action $S[\Phi]$ which is a classical functional of the local fields $\Phi(x)$, so that any physical operator $\mathscr{O}(\Phi)$ can be calculated from the path integrals [30, 31, 32]

$$\langle \mathscr{O}(\Phi) \rangle \sim \int \mathscr{D}\Phi \, e^{iS[\Phi]} \mathscr{O}(\Phi) \,, \tag{1}$$

where the symmetry principle [33] is usually applied to determine the action of a QFT as well as the properties of the fields.

Along the path of building the SM as a QFT to describe the fundamental interactions, symmetry always plays a central role. Indeed, one of the key questions that are central to particle physics in the middle of the 20th century is to identify all the fundamental symmetries existing in nature and to understand how they are realized in the unified framework. Such an approach has achieved great success in the development of SM, for example, the first discovery of gauge symmetry led to the development of quantum electrodynamics (QED) [5, 6, 7, 8, 9, 10, 11], while in the 1950s the approximate internal symmetries among mesons or baryons known as the eightfold way [34, 35] has driven the discovery of quark model and the underlying quantum chromodynamics (QCD) [15, 16, 17, 18, 19, 20]. It was not until the late 1960s did people finally confirmed the existence of all three gauge symmetries corresponding to the three fundamental interactions in nature, namely the electromagnetic and weak interactions described by the Glashow-Weinberg-Salam model [36, 37, 38], and the strong interaction described by QCD, which then became the foundation of the Standard Model.

However, the golden epoch of discovery in theoretical particle physics has not lasted for a long time, as people soon realized that the SM is not complete. For example, it fails to describe the gravity or predict the existence of the masses of neutrinos, and even worse, it is not "natural" even at the energy scale of a few TeV [39] in the sense that a physical parameter of the SM, the Higgs mass, receives a large quantum correction scaling as $m_H^2 \sim E^2$ rather than a logarithmic correction. Therefore, it seems that the SM can precisely describe the phenomena at energies close to the Electroweak scale, but is not sufficient at energies far from that. This simply implies that it is an effective description of physics that is only valid up to an energy scale denoted as a cut-off scale Λ.

Indeed, such effective descriptions, known as the Effective Field Theories (EFTs), have proven to be very useful in different aspects of physics, e.g. in particle physics (the Fermi theory [12, 13, 14]), mathematical physics (the Seiberg-Witten theory [40, 41]), condensed matter physics (the BCS theory [42]), nuclear physics (the chiral perturbation theory [43]), and cosmology (general relativity [44]), etc. One common feature that EFTs share is that they always have an intrinsic cut-off scale Λ, above which the EFTs break down and need to be modified. At energy scales much below the cut-off, it is always possible to integrate out some degrees of freedom and thereby obtain an EFT valid at even lower energy. In general, based on the technique of renormalization group (RG) [45, 46, 47, 48, 49, 50, 51, 52, 53], we can divide the effective Lagrangian of a d-dimensional EFT into a renormalizable part and a tower of non-renormalizable operators suppressed by the cut-off scale Λ as

$$\mathscr{L}_{\text{eff}} = \mathscr{L}_{\text{ren}} + \sum_{n=d}^{\infty} \frac{c_n \mathscr{O}_n}{\Lambda^{n-d}} \,. \tag{2}$$

where $\mathscr{L}_{\text{ren}}$ is the renormalizable part of a QFT in d-dimensional spacetime, $\mathscr{O}_n$ are the gauge-invariant non-renormalizable operators with scaling dimension n suppressed by the cut-off scale Λ, and c_n are their Wilson coefficients.

Hence, a natural question arises asking what is the cut-off scale of the SM, as the SM itself only contributes the renormalizable part of an underlying EFT, and those non-renormalizable operators **must** exist at an ultraviolet (UV) scale, for example, when gravity is switched on at the energy of the Planck scale (about 10^{19} GeV) the non-renormalizable interactions between gravitational field (the metric $g_{\mu\nu}$) and matter fields are induced. This question remains controversial since the birth of SM yet a consensus has hardly been achieved: on one hand, an unnatural fine-tuning between the bare Higgs mass and the radiative corrections of the Higgs mass existing in the infrared (IR) scale seems to suggest that the SM should have a cut-off scale close to the TeV scale [39], while on the other hand, the SM seems still valid above the TeV scale because all dedicated and non-conclusive searches in the experiment of LHC at CERN have found no significant signal deviating the predictions of SM [21], indicating that the cut-off scale should be much higher. If the SM cut-off scale is indeed only slightly above the TeV scale, phenomena of new physics can be expected to appear within our reach, which thus motivated a lot of efforts to construct new theories beyond the Standard Model (BSM) predicting the new physics scale Λ and studying their interesting phenomenology.

The methodology applied in many BSM research, which is similar to developing the SM from the Fermi theory in the past, is to complete the SM by proposing physical assumptions or by trial and error with a new EFT valid at a new physics scale (usually not necessary to be the UV scale), and then to check whether such a new theory can still be consistent with the experiments by realizing the SM as a low energy EFT. This idea also motivates the study of the SMEFT [54], in which all beyond the Standard Model (BSM) effects are described by the non-renormalizable operators in the effective Lagrangian at the electroweak scale obtained from integrating out heavy degrees of freedom, with suppression by the cutoff scale Λ which is usually related to the mass of heavy fields.

From a top-down perspective, in order to consistently realize the SM as an EFT, it is typically assumed that a UV-complete theory exists at the UV scale (e.g., the Planck scale $M_P \sim 10^{19}$ GeV) that is renormalizable. As we descend to lower energy scales, certain information about the UV theory may be lost, leaving us with an effective description. This effective description of the fundamental theory can be represented as an EFT, which can be divided into two components, as illustrated in eq. (2). As observers residing in the IR regime, we are primarily concerned with the relevant or marginal operators, which are encoded in the renormalizable part (e.g., operators with dimension $d \leq 4$). Consequently, when the cutoff scale Λ greatly exceeds the IR scale of interest, as a nice approximation we can neglect non-renormalizable operators, such as the tower of states induced by gravity, to significantly simplify the considered EFT. Thus, the renormalizable part of the EFT discussed in this context is a subset of a general EFT, which could be UV-completed into a fundamental theory incorporating gravity. The exploration of these renormalizable parts within well-motivated EFTs constitutes a key motivation for this thesis.

Following this rationale, a natural question arises: How can one construct an EFT that remains valid at high energy scales while reducing to the SM at lower energy scales? A straightforward bottom-up approach involves starting from the SM and subsequently introducing additional elements to account for phenomena beyond its original scope, such as massive neutrinos, dark matter, dark energy, etc. From the EFT perspective, this method is equivalent to adding new degrees of freedom above the scale where SM breaks down in the process called "integrating in" in contrast to the concept of "integrating out", resulting in the definition of a new EFT that is valid from SM cut-off to some new cut-off. Usually, this process is guided by the symmetry principle [55], in which the new degrees of freedom will be embedded into the representations of some symmetries, such as global symmetry, extra gauge symmetry, supersymmetry, conformal symmetry, etc. The only problem is, this process cannot be repeated indefinitely [56]: any EFT must have a cut-off (dubbed the QG cut-off) above which it cannot be amended to give a consistent QFT weakly coupled to Einstein gravity. In other words, it is not possible to "integrate in" new light degrees of freedom while preserving the QFT description. This occurs, for example, if an infinite tower of new light states appears, they cannot simply be "integrated in": quantum gravitational effects become important and the EFT completely breaks down [33]. However, since this story pertains to the far UV scale, it is better for us to set it aside and focus instead on the EFT that functions on much lower energy scales.

A more important issue is, there seem to be enormous possibilities for constructing phenomenologically acceptable models just by integrating in new degrees of freedom because the inverse of the RG transformation is not unique. There might be many different consistent EFTs at the UV giving rise to the same dynamics at the IR. In practice, even counting the total number of consistent BSM models is challenging. What is possible is that this number is actually much larger than all of the known models we have constructed so far for BSM. If this is true, we would need a systematic way to analyze a category of similar EFTs classified by similar dynamics or phenomena, instead of analyzing only one model each time. On the contrary, if this number is very small, it would indicate that a consistent completion of SM is highly non-trivial, and thus very strong constraints should be imposed in BSM research. In either case, the conditions that determine the constraints of BSM models may lead to some universal observable effects that can be tested in future experiments.

This thesis makes progress in understanding the Standard Model as a low-energy effective theory, while also exploring the implications of certain theoretical constraints on different models. For instance, it investigates the phenomenological predictions of a class of classical scale invariant models [1], the requirement for an absolutely stable vacuum at all scales [2], as well as the constraints of gauge and Yukawa coupling unification [3, 4]. The goal of all of these research is to find out the possible constraints of BSM models and study the possible observable effects of these constraints. In the end, we hope that it advances the realization of the SM as a low-energy effective theory, and also provides some useful insights on the principles of physics behind all of these BSM models.

In particular, the authors of publication I [1] explored a universal interpretation of a specific class of conformal extensions of SM. Within this study, models are categorized into a universality class at the IR scale predicting similar phenomenology in relation to Higgs physics. The author showed how the two primary scale generation mechanisms in the classical scale-invariant model, the perturbative and the non-perturbative type, both generate the Higgs portal coupling with the negative sign. These models contain at least two fundamental scalar fields (H_1, H_2) and a dilaton χ, forming an extended Higgs sector beyond the Standard Model. The Electroweak scale is then generated via the conformal anomaly if the mixing between two Higgses is small and the mass of the second scalar field H_2 greatly exceeds the masses of the dilaton χ as well as the first scalar field H_1, which makes it possible to integrate out the heavy scalar field H_2. From the EFT perspective, integrating out H_2 yields a vertex operator as $\mathscr{O} \sim \lambda_H \chi^2 |H_1|^2$, where the Higgs portal coupling λ_H is negative, given by $\lambda_H \sim -c_1^2/c_2 < 0$ where $c_{1,2}$ are originally the positive quartic coupling constants. After the dilaton acquires a vacuum expectation value $\langle\chi\rangle = \eta$ which breaks the conformal symmetry, this negative Higgs portal coupling becomes the SM Higgs mass term with a negative sign. Phenomenologically, the presence of such a light dilaton χ coupling to the Higgs boson via a negative portal coupling defines a low-energy testable discriminator for a specific universality class of models. This universality class offers a unified interpretation for various models arising from distinct UV completions with conformal symmetry breaking by the light dilaton. Thus, this universality class could possibly lead to model-independent predictions, and hence, improve the understanding of the longstanding gauge hierarchy problem if it could be found.

In his publication II [2], the author studied a possible extension of the SM where the Yukawa couplings are generated radiatively from a hidden sector at one-loop level. Possible tree-level Yukawa couplings are forbidden because of a new underlying symmetry (such as the Z_2) assumed to be spontaneously broken by the vacuum expectation value of a new scalar field above the electroweak scale. In this case, the SM Yukawa couplings are realized as effective operators by integrating out the messenger scalar fields and dark fermions at higher energies. The theoretical setup of the model was provided and the calculation for deriving effective Yukawa couplings was presented in detail. In particular, the stability of the electroweak vacuum was examined for two different scenarios. In the first scenario, all interactions are taken to be common and well-perturbative benchmark values, but then to match with the observed effective top Yukawa coupling the unitarity bound can be violated at energies close to or below the matching scale so a Lee-Wick extension of the model is needed. After scanning parameters within the allowed regions, it was shown that the stability of the Electroweak vacuum can be achieved regardless of the current experimental uncertainties affecting the Higgs boson or the top quark mass, as was illustrated in Figure. 2. In the second scenario, instead of fixing those couplings with benchmark values, the unitarity bound was used to determine the maximal values of the trilinear coupling Λ_S. The Higgs quartic coupling is then evolved up to the UV cutoff of this scenario which is set by the first Landau pole of its' perturbative RGEs. The electroweak vacuum is again stable in this scenario within the considered perturbative regions as was illustrated in Figure. 3 of the paper. Therefore, in both cases, the stability of the Electroweak vacuum will be guaranteed because the main contribution to the running of Higgs quartic couplings is absent as long as the tree-level top-Higgs Yukawa coupling is forbidden.

In publication III [3], the author undertook an analysis of the constraint of gauge and Yukawa coupling unification within non-supersymmetric (non-SUSY) SO(10) models. This study marked the first successful realization of Yukawa coupling unification in non-SUSY SO(10) models by introducing an intermediate symmetry breaking scale and an additional

Higgs doublet at the Electroweak scale. Through the evolution of the renormalization group from UV to IR scales, the author showed how the constraint from Yukawa unification impacts the viable parameter space of two Higgs doublet models (2HDMs) at the IR scale. In particular, the realization of unification of Yukawa couplings involved three detailed steps. Firstly, gauge coupling unification with an intermediate scale was enforced by matching the renormalization group equations of gauge couplings at the two-loop level at the symmetry breaking scales, where the 1-loop threshold corrections are included by randomly sampling the masses of the heavy particles that emerge at the corresponding scales. Secondly, with both the intermediate scale and unification scale determined, the running of the Yukawa couplings for the top quark, bottom quark, and tau lepton was studied at the two-loops level in two possible breaking chains, where appropriate matching conditions for the Yukawa couplings are imposed. Thirdly, a parameter scan was conducted within the Yukawa sector of the SO(10) model, leading to the identification of specific parameter sets that enabled the unification of Yukawa couplings for the third generation while maintaining a viable fermion spectrum at the electroweak scale. The results showed that the Yukawa coupling unification can be fulfilled for certain parameters, as summarized in Table 1 of the paper. Notably, the ratio of the vacuum expectation values of the two Higgs doublet fields was found to be large, with $\tan\beta \approx 60$. This implies that the requirement of achieving Yukawa coupling unification for the third generation, which represents a constraint of the model at the UV scale, has implications for the possible parameters of the EFT at the IR scale. Consequently, our model predicts the existence of additional Higgs particles with weak scale masses and $\tan\beta \approx 60$, which could be the subject of search and potential observation at the Large Hadron Collider or future high-energy colliders.

In publication IV [4], the author conducted a comprehensive investigation of non-supersymmetric SO(10) Grand Unification Theories (GUTs) with an intermediate symmetry breaking scale. The focus of this study was not only on achieving gauge coupling unification but also enforcing Yukawa coupling unification through appropriate threshold corrections and matching conditions, and at the same time incorporating several important phenomenological constraints such as the proton decays and the absence of Flavor-Changing Neutral Currents (FCNCs) at tree-level. The model presented in this publication differs from the previous one in several aspects. Firstly, the scalar representation $\mathbf{10_H}$ is complexified, and an additional global $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry is introduced to address the strong CP problem and axions. Furthermore, the gauge coupling unification is achieved for four distinct intermediate-scale breaking chains of SO(10). These chains correspond to intermediate gauge groups such as $\mathrm{SU}(4)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ (Pati-Salam) and $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ (minimal left-right symmetry), both with or without a D-parity.

A significant contribution of the publication IV is the derivation of approximately analytical solutions for achieving gauge coupling unification at the two-loop level in the non-supersymmetric SO(10) model with a single intermediate symmetry breaking scale. These analytical results extend the previous findings derived for supersymmetric SO(10) models to the non-supersymmetric case with an intermediate symmetry breaking scale. Subsequently, the phenomenological constraint arising from proton decay was applied to examine the viability of various intermediate symmetry breaking patterns. By sampling the threshold corrections from the masses of heavy particles, the range of the intermediate scale and unification scale was determined for each symmetry breaking pattern, which was summarized in the Figure 1 of the paper. The analysis concluded that only the Pati-Salam and minimal left-right symmetry breaking chains survive the proton decay constraints when large threshold corrections are included. Specifically focusing on the Pati-Salam intermediate breaking pattern, the author further imposed the unification of Yukawa couplings for third-generation fermions at the gauge unification scale, once again at the two-loop level. In the considered context, Yukawa coupling unification implies a relationship between the fermion couplings to the 10- and 126-dimensional scalar representations of the SO(10) group. One such possible relation, which is attainable in an E_6 model where the previous two scalar fields are part of a single multiplet, was investigated. Taking into account phenomenological constraints such as the absence of flavor-changing neutral currents at tree-level, constraints on the parameters of the low-energy EFT, specifically the Two-Higgs-Doublet Model (2HDM), were derived, in particular on the ratio of the two Higgs doublets vacuum expectation values $\tan\beta$.

This thesis is structured as follows: in Chapter 2 the current theoretical description of the SM is briefly reviewed. In Chapter 3 some of the most important problems that exist within the SM are examined, in particular the hierarchy problems. In Chapter 4 an overview of some of the most promising approaches that attempt to complete the SM while offering potential solutions to specific hierarchy problems is provided. The summary is given in Chapter 5. Finally, in Appendix, the author's publications are appended on which this thesis is based.

2 Standard Model of Particle Physics

The Standard Model of Particle Physics describes with amazing parsimony how the basic building blocks of matter interact via the fundamental forces of nature over vastly different scales: from the Hubble radius of 10^{30} cm all the way down to the scales of the order of 10^{-16} cm [57]. It is the relativistic quantum field theory that describes strong interactions [15, 16, 17, 18, 19, 20], and the weak and electromagnetic interactions of Glashow, Weinberg, and Salam [36, 37, 38]. The latter two are unified in the theory of electroweak interaction. Gravity, the fourth fundamental interaction, is negligible at the energy scales where the Standard Model is considered to be valid.

A possible definition [58] of the Standard Model (SM) is that it is the most general renormalizable quantum field theory [59, 60, 61] with the gauge group

$$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y\,, \tag{3}$$

and three generations of fermions plus a scalar which transforms under the representations of the gauge group as

$$(\mathbf{3},\mathbf{2})_{1/6} + (\bar{\mathbf{3}},\mathbf{1})_{-2/3} + (\bar{\mathbf{3}},\mathbf{1})_{1/3} + (\mathbf{1},\mathbf{2})_{-1/2} + (\mathbf{1},\mathbf{1})_{1} \quad \text{and} \quad (\mathbf{1},\mathbf{2})_{1/2}\,, \tag{4}$$

respectively, where the boldface numbers are the dimensions of representations of $\mathrm{SU}(3)_C$ and $\mathrm{SU}(2)_L$, and the subscripts denote the $\mathrm{U}(1)_Y$ hypercharge.

In this chapter, we will provide a detailed introduction to the ingredients necessary for understanding the SM, review its construction, and highlight its key features.

2.1 Quantum Field Theory

Quantum field theory (QFT) is a theory describing the motions and interactions of (quantum) fields [26], which are considered to be the most fundamental degrees of freedom generalized from quantizing the classical fields such as the electromagnetic and gravitational fields observed in everyday life. It provides a more unified view of the fundamental degrees of freedom in the relativistic and short-distance limit, rather than the old dualistic interpretation in terms of both particles and waves in quantum mechanics: in QFT, particles are understood as bundles of energy and momentum of the fields, while the wave function is a functional of these fields rather than a function of particle coordinates [55].

The quantum field theory is described by the Lagrangian formalism (or more generally the action formalism), where the symmetry of nature can be manifest when the action is invariant under transfomrations of fields or spacetime, implying the existence of Lie algebras of suitable quantum operators [55]. The (quantum) fields, under such formalism, should satisfy certain commutation (or anti-commutation) relations, and be defined as a representation of the symmetry groups. Therefore, from a top-down perspective, one of the essential steps for studying the fundamental degrees of freedom is to identify the symmetry groups (or their Algebras).

Fortunately, it is not difficult to classify the possible algebras and formulating the corresponding quantum field theory in mathematics. A most straightforward example is the spacetime symmetry: if we are living in a universe with 3+1 macroscopic spacetime dimensions, a Poincaré symmetry is present and the **physical** fields are classified by the Wigner's definition [24, 26] which says physical multi-particle states transform under unitary irreducible representations of the Poincaré group uniquely classfied by the mass m and spin J. In addition to the spacetime symmetry, another example is the gauge symmetries, where fields transform under the representations given in eq. (4).

With the well-defined physical fields, the quantum field theory can be formulated [26], to the latest knowledge, by the use of path integrals [30, 31, 32]. Path integrals can be thought of as a generalization of classical trajectories to quantum scattering amplitudes. More precisely, path integrals involve integration over all possible classical field configurations of the phase given by the classical action evaluated in those field configurations, as shown in eq. (1). This can be calculated most conveniently by the generating functional $Z[J]$ which defines all the correlation functions and, therefore, the entire theory. In this formulation, we can also directly prove the existence of a localized symmetry, namely gauge invariance, which results from the freedom of redefining the field configurations [29].

One of the key elements in the triumph of quantum field theory was the development of renormalizable theory, in which the infinities encountered in the calculation of physical observables can be absorbed into the "redefinition" of fields, masses, or charges via the process called renormalization.

However, given a set of fields (or operators) that are assumed to be valued over some algebra, their classical action can generally contain as many symmetric operators as possible up to any scaling dimensions without assuming the renormalization. Besides, there are non-perturbative operators taking the form of total derivatives of the Lagrangian. These ambiguities make it difficult to fully determine a unique form of the Lagrangian from a top-down perspective based on the first principles. This brings in the usage of effective field theories [62] , where the low energy dynamics are well-approximated by the EFTs below their intrinsic cutoff scale Λ_{EFT}.

The essential point in using an EFT is that we are not allowed to make any assumption of simplicity about the Lagrangian, in particular, the renormalizability [55]. Therefore, an EFT can be in general non-renormalizable by definition, and we adapt ourselves to the definition that we have used in eq. (2):

$$\mathscr{L}_{\text{eff}} = \mathscr{L}_{\text{ren}} + \sum_{n=d}^{\infty} \frac{c_n \mathscr{O}_n}{\Lambda^{n-d}} \,, \tag{5}$$

where the $\mathscr{L}_{\text{ren}}$ is the renormalizable part of a QFT in d spacetime dimension, $\mathscr{O}_n$ are the gauge-invariant non-renormalizable operators with scaling dimension n suppressed by the cut-off scale Λ, and c_n are their Wilson coefficients that run as functions $c_n(\mu)$ of the renormalization group scale μ.

The biggest advantage of EFTs is that they are powerful tools in theoretical physics that allow us to describe the low-energy behavior of complex systems without knowing the underlying microscopic theory. When using an EFT, we are not allowed to assume that the Lagrangian is simple or that it is renormalizable, because the renormalizability is not a fundamental requirement of a QFT, according to the folk theorem [55]. Therefore, as long as we write down the most general Lagrangian consistent with the symmetries of the theory [62], we are actually writing down the most general theory we could possibly write down. This approach allows us to make predictions about observable phenomena at low energies without being concerned about the properties of high-energy physics. Therefore, we naturally introduce a UV cut-off Λ_{UV} associated with our QFT in order to regularize the QFT to ignore everything happening above the cut-off scale.

In the following chapter, we will develop a particular type of QFT, namely the Yang-Mills gauge field theory, to understand the SM. It's important to note that although the SM turned out to be renormalizable, the non-renormalizable part of a fundamental theory is still unknown. Treating the SM as an EFT (the so-called SMEFT) and seeking to understand its non-renormalizable part is an interesting problem in phenomenological studies beyond the SM, which will be discussed at the end of the chapter.

2.2 Yang-Mills Theory

The Standard Model successfully describes three of the fundamental interactions, the Electromagnetic interaction, the weak interaction, and the strong interaction, in terms of the Yang-Mills Theory [63] based on the Lie groups of $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. The Yang-Mills theory is a generalization of the quantum field theory of the massless spin-1 particle, which is the quantum electrodynamics (QED) based on the U(1) gauge symmetry. In QED, the massless spin-1 particle, the photon, is embedded into a vector field $A_\mu(x)$, which has four degrees of freedom but two of them are removed by the gauge invariance under the local transformation for an arbitrary $\alpha(x)$

$$A_\mu(x) \to A_\mu(x) + \frac{1}{e}\partial_\mu \alpha(x)\,. \tag{6}$$

One can prove that the Lagrangian with interacting spin-1 and spin-1/2 fields which is invariant under this symmetry is

$$\mathscr{L}_{\mathrm{QED}} = -\frac{1}{4}F_{\mu\nu}^2 + \bar{\psi}(iD_\mu\gamma^\mu - m)\psi\,, \tag{7}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the vector field tensor, γ^μ is the γ-matrices satisfying the spinor algebra, m is the mass of fermion ψ, and $D_\mu = \partial_\mu - iQeA_\mu$ is the covariant derivative with the coupling constant e and the charge of fermion Q. In fact, the covariant derivative contains the unique renormalizable interaction which is gauge invariant. The Yang-Mills theory is a unique generalization of the QED to the case of local non-Abelian symmetry based on SU(N) Lie group. So the renormalizable gauge-invariant interactions among the massless spin-1 particles can also be characterized by elevating the ordinary derivatives to the covariant derivatives defined by

$$D_\mu = \partial_\mu - igA_\mu^a T^a\,, \tag{8}$$

where g is a real number defined as the coupling constant, T^a are the generators of the Lie group SU(N) in the fundamental representation, and A_μ^a is a set of vector fields of the gauge bosons which transform infinitesimally as

$$A_\mu^a(x) \to A_\mu^a(x) + \frac{1}{g}\partial_\mu \alpha^a(x) - f^{abc}\alpha^b(x)A_\mu^c(x)\,, \tag{9}$$

where f^{abc} are the structure constants of SU(N). The natural field tensor in the non-Abelian case is thus defined as

$$\mathbf{F}_{\mu\nu} \equiv \frac{i}{g}\left[D_\mu,\, D_\nu\right] = (\partial_\mu A_\nu - \partial_\nu A_\mu) - ig\left[A_\mu,\, A_\nu\right]\,. \tag{10}$$

Or, in terms of components $\mathbf{F}_{\mu\nu} = F_{\mu\nu}^a T^a$, we can write

$$F_{\mu\nu}^a = \partial_\mu A_\nu - \partial_\nu A_\mu + gf^{abc}A_\mu^b A_\nu^c\,. \tag{11}$$

With these concepts, the locally $\mathrm{SU}(N)$ invariant Lagrangian of interacting gauge bosons and N_f massive fermions can be written as

$$\mathscr{L}_{\mathrm{YM}} = -\frac{1}{4}(F_{\mu\nu}^a)^2 + \sum_{i,j=1}^{N_f} \bar{\psi}_i(\delta_{ij}i\partial\!\!\!/ + gA\!\!\!/^a T_{ij}^a - m\delta_{ij})\psi_j\,. \tag{12}$$

This is the Lagrangian of the Yang-Mills theory, where the contraction has been used for $\not{k} = k_\mu \gamma^\mu$, and the coupling constant g now represent the strength of the interaction between the gauge bosons and the fermions.

It should be noted that the Yang-Mills theory requires the gauge bosons associated with the gauge symmetry to be massless, as any inclusion of the mass term of the spin-1 vector field will explicitly break the gauge invariance. However, as we know that the weak interaction is constrained within a finite subatomic length scale which implies that the gauge bosons associated with the weak interaction must be massive. On the other hand, if the fermions are massive, the Yang-Mills theory must be a vector-like theory where the gauge bosons equally couple to the left-handed and right-handed massive fermions, just like the quantum chromodynamics (QCD). Therefore, without further mechanism, the Yang-Mills theory itself fails to describe the weak interactions among the massive quarks and leptons. However, as we will see in the next subsection, the introduction of the Higgs mechanism cures this problem and thus generalize the Yang-Mills theory to be applicable to a wider range of topics.

2.3 Spontaneous Symmetry Breaking

Spontaneous symmetry breaking (SSB) is a vital concept in quantum field theory, playing a crucial role in understanding various phenomena in particle physics and condensed matter physics. In general, SSB occurs when the Lagrangian or Hamiltonian of a physical system is invariant under a symmetry transformation, but the lowest-energy state (the ground state $|\Omega\rangle$) of the theory is not.

$$[S,H] = 0\,, \quad S|\Omega\rangle \neq 0\,. \tag{13}$$

SSB provides explanations for various phenomena, including simple phase transitions like conventional superconductivity, super-fluid $^4\mathrm{He}$, and spontaneous magnetization in ferromagnetic materials. In particle physics, the electroweak symmetry breaking explains the masses of particles in the Standard Model. Cosmological phase transitions, such as the electroweak phase transition and QCD phase transition, are driven by the spontaneous breaking of the electroweak and chiral symmetries as well as color confinement.

The concept of SSB has been around for a long time without being recognized dating back to 1907 when Pierre Curie studied crystal symmetries. However, it was not until 1957 that the original concept of SSB was proposed in condensed matter physics by John Bardeen, Leon Cooper, and Robert Schrieffer (BCS) in 1957 [42]. This theory explains superconductivity as the spontaneous breaking of an approximate two-dimensional rotation symmetry, which was later recognized as the $\mathrm{U}(1)_{\mathrm{EM}}$ electromagnetic gauge invariance. As a consequence, products of any even number of electron fields acquire non-vanishing expectation values in a superconductor, resulting in the unique properties of superconductors such as zero electrical resistance and the Meissner effect.

The discovery of spontaneous symmetry breaking in a medium quickly led to a revolution in elementary particle physics. Soon after BCS, Yoichiro Nambu applied the SSB to an approximate global symmetry in relativistic quantum field theory where the vacuum is the ground state. In his subsequent paper with Giovanni Jona-Lasinio [64], he predicted the existence of pions, a particle with a very small mass, associated with the spontaneous broken approximate chiral symmetry. This paper became the first illustrative quantum field theory with SSB.

Nambu believed that spontaneous symmetry breaking is highly relevant to the problem of fermion masses [65]. Whenever a massless fermion exhibits chiral symmetry, it can be spontaneously broken to give the fermion mass. As a result, mass becomes a dynamic quantity that can be explained theoretically.

An immediate consequence of the SSB is the Goldstone's theorem [66, 67], which states that the spontaneous breaking of a continuous global symmetry implies the existence of a massless particle known as the Nambu-Goldstone boson [68, 67]. If the continuous global symmetry is not exact, i.e. if it is explicitly broken, then the SSB does not result in the existence of massless Goldstone bosons, but rather massive pseudo-Nambu-Goldstone bosons. The pions are typical pseudo-Nambu-Goldstone bosons resulting from the spontaneous breaking of QCD chiral symmetry $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ by the condensation of fermion bilinear $\langle \bar{q}q \rangle$ in the vacuum, as well as explicitly breaking by the tiny quark mass terms.

After a decade following the discovery of SSB, an exception to Goldstone's theorem was eventually found, which applies to gauge symmetry, particularly to Yang-Mills theory based on non-Abelian gauge symmetries. Brout, Englert, Guralnik, Hagen, Higgs, and Kibble demonstrated [69, 70, 71, 72, 73] that when local gauge symmetry is spontaneously broken, neither the vector bosons associated with the gauge symmetry nor the Nambu–Goldstone bosons produced by the symmetry breaking have zero mass. This mechanism was later named the Higgs mechanism for various reasons, and will be reviewed in detail in the next subsection.

To sum up, we conclude that the SSB in general has the following characteristics as per Nambu [65]:

1. Degeneracy of the ground state.

2. Existence of the NG modes when the symmetry is continuous and the system is infinite (the thermodynamic limit).

3. Possibility of hierarchical SSB - This means that an SSB can trigger another in a hierarchical way.

Lastly, it is worth mentioning that in many cases, symmetry can be spontaneously broken in a non-trivial way without introducing a fundamental scalar that acquires a vacuum expectation value. A renormalizable Yang-Mills theory based on the SU(N) local symmetry with N massless fermions, whose Lagrangian can be obtained by setting $m = 0$ in eq. (12), has two tensor products capable of developing vacuum expectation values at the leading order. The first one is the fermion bilinear $\langle \bar{q}q \rangle$, known as the quark condensate, which breaks the global chiral symmetry of the vacuum. This is the scenario discussed in the Nambu-Jona-Lasinio model[64], where SSB of chiral symmetry by the quark condensates leads to the existence of light pions as pseudo-Nambu-Goldstone bosons. The formation of quark condensates in QCD is not yet fully understood [1], but it is believed to be related to one of the deepest problems of QCD which is the confinement problem. The second

[1]Although the mechanism responsible for the formation of quark condensates in QCD remains poorly understood theoretically, their existence is well-established through lattice calculations and is widely accepted as a basic fact. This is because, in the massless limit of QCD, the global symmetry $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_\mathrm{R} \times \mathrm{U}(1)_V$ has a 't Hooft anomaly, which means that this global symmetry is exact and cannot be gauged [39]. The exactness of this symmetry is enforced by the anomaly matching condition, which requires that any effective description of the strongly-interacting fermions below the phase transition scale must incorporate the same anomalies as the underlying QCD theory above the phase transition scale. In the language of the renormalization group, this implies that a certain RG invariant, namely the 't Hooft anomaly, must be preserved when transforming a theory from the UV scale to an effective description at the IR scale.

one is the field tensor bilinear, such as $\langle \tilde{F}_{\mu\nu} F^{\mu\nu} \rangle$, that breaks the $\mathrm{U}(1)_A$ global symmetry. The formation of such condensation can be ascribed to the instanton effect, provided that fermionic zero modes exist within non-trivial gauge field configurations.

2.4 Quantum Anomalies

In general, if a symmetry of the system holds only at the classical level but gets violated by quantum corrections, we refer to it as an anomalous symmetry [74, 75, 76], indicating that it is not an exact symmetry. In four dimensions, local anomalies are linked to triangle Feynman diagrams featuring three external boson lines.

Anomalies can in principle happen for any symmetries, including the spacetime, gauge, global, or conformal, etc. But if the gauge invariance is not preserved at the quantum level, then the theory is inconsistent.

The most well-known anomalies are the global anomalies associated with the U(1) global symmetries, such as the baryon number and the lepton number. Indeed, the anomalies are often related to the instantons, which are self-dual solutions to the classical equation of motion in the non-Abelian gauge theory. The chiral anomaly is developed when instantons interact with the quantum theory.

As symmetries play a crucial role in the behavior of quantum systems, as suggested by Weinberg, for an EFT to be valid in describing the same dynamics as an underlying theory, it must have the same amount of symmetries, as well as carry all the information about how broken symmetries are non-linearly realized.

Therefore, it is important to calculate all the anomalies in each EFT so that we know whether a gauge or mixed anomalies exist in certain EFT. When running from one EFT to another EFT, we also need to rely on the t' Hooft anomaly matching in order to determine whether the two EFT indeed have similar symmetric structures.

2.5 The Higgs Mechanism and the Electroweak Theory

As was discussed in the earlier subsection, the Higgs mechanism is the unique exception to Goldstone's theorem, in which both the vector bosons associated with the gauge symmetry and the Nambu-Goldstone bosons produced by the symmetry breaking become massive when the gauge symmetry is spontaneously broken. To spontaneously break the gauge symmetry from the initial symmetry group $\mathscr{G}$ to the residual symmetry group $\mathscr{H}$, we need to introduce an order parameter of SSB, which can be identified as a new scalar field ϕ carrying the specific charges of $\mathscr{G}$ and being a singlet of the residual symmetry $\mathscr{H}$. This particular scalar field ϕ only plays the role of SSB by acquiring a vev through the shape of its potential, and interacts with the other fields in a gauge-invariant way.

Taking the SM as an example, the Higgs mechanism is the spontaneous breaking of $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge symmetry to the $\mathrm{SU}(3)_C \times \mathrm{U}(1)_{\mathrm{EM}}$ gauge symmetry that the vacuum possesses to preserve the fact that both the photons and gluons remain massless. This can be achieved by introducing a complex scalar field ϕ, which was called the Higgs field and transforms as a doublet under the $\mathrm{SU}(2)_L$ gauge symmetry and can be parameterized as

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} . \tag{14}$$

The Lagrangian involving the gauge-invariant kinetic terms should be written as

$$\mathscr{L}_\phi = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi), \tag{15}$$

where the covariant derivative is

$$D_\mu \phi = \partial_\mu \phi - ig_2 W_\mu \frac{\sigma^a}{2}\phi - ig_Y Y_H B_\mu \phi\,, \tag{16}$$

in which the hypercharge of the Higgs multiplet is defined to be $\frac{1}{2}$ to be consistent with its neutral electrical charge.

The gauge-invariant potential including the self-interaction for the Higgs boson is

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2. \tag{17}$$

As a requirement for triggering the SSB of the SM gauge symmetry, the potential is written in a seemingly artificial way to guarantee the existence of a non-zero vacuum expectation value. The vacuum state that corresponds to the non-trivial minimum of the above potential, should satisfy the relation $\frac{\partial V(\phi)}{\partial \phi} = 0$, which yields the solution

$$\phi^\dagger \phi = \frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2}\,, \tag{18}$$

with $v = \sqrt{\frac{\mu^2}{\lambda}}$. Therefore, the negative sign in the Higgs quadratic term in eq. (17) actually indicates the ground state has an expectation value chosen to be the neutral component of the Higgs field ϕ_3

$$\langle \phi \rangle_0 \equiv \langle 0|\phi|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{19}$$

It is simplest to study this theory in unitary gauge, in which the Higgs field can be expressed as the excitation above its' vev

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}. \tag{20}$$

Therefore, we can expand the kinetic terms in terms of the new field $h(x)$ and get

$$|D_\mu \phi|^2 \supseteq \frac{g^2 v^2}{8}\left[\left(W_\mu^1\right)^2 + \left(W_\mu^2\right)^2 + \left(\frac{g_Y}{g_2}B_\mu - W_\mu^3\right)^2\right] + \frac{1}{2}(\partial_\mu h)^2 + \ldots. \tag{21}$$

The above equation clearly shows that, when the gauge symmetry is spontaneously broken, the interaction between the Higgs field and the gauge field will develop mass terms of the gauge bosons which is driven by the existence of nonzero vacuum expectation value $v \neq 0$ of the Higgs field.

However, there is a mixing mass term between the B_μ field and the W_μ^3 field. To find out the mass eigenstates, we can diagonalize the mass matrices by performing a rotation between B_μ and the W_μ^3 field, thus we have

$$\begin{aligned} Z_\mu &\equiv \cos\theta_w W_\mu^3 - \sin\theta_w B_\mu\,, \\ A_\mu &\equiv \sin\theta_w W_\mu^3 + \cos\theta_w B_\mu\,, \end{aligned} \tag{22}$$

with the weak mixing angle defined as

$$\tan\theta_w = \frac{g_Y}{g_2}\,. \tag{23}$$

To express the charged degrees of freedom for the W gauge boson, we define the charged W boson field as

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right), \tag{24}$$

associated with the new $\mathrm{SU}(2)_L$ generators $\tau^\pm = \frac{1}{\sqrt{2}}\left(\tau^1 \pm i\tau^2\right)$.

After rotating to the mass eigenstates, we found the kinetic terms for the gauge bosons in terms of new field variables to be

$$\mathscr{L}_K = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{4}Z_{\mu\nu}^2 - \frac{1}{4}W_{\mu\nu}^+W^{-\mu\nu} + \frac{1}{2}m_Z^2(Z_\mu)^2 + m_W^2 W_\mu^+ W^{-\mu}, \tag{25}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the photon field tensor, $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$ is the Z boson field tensor, and $W_{\mu\nu}^\pm = \partial_\mu W_\nu^\pm - \partial_\nu W_\mu^\pm$ is the physical charged W boson field tensor. The masses for each field are all proportional to the vev of the Higgs field v, which are correspondingly

$$m_A = 0, \quad m_Z = \frac{g_2 v}{2\cos\theta_w}, \quad m_{W^\pm} = \frac{g_2 v}{2}. \tag{26}$$

This is consistent with the fact that there is only one massless fermion in nature, which is the photon field in QED. The associated gauge coupling of the photon field after rotations, which is the the electromagnetic coupling, is

$$e = g_2 \sin\theta_w = g_Y \cos\theta_w. \tag{27}$$

As we have clearly seen above, the spontaneous breaking of the gauge symmetry gave masses to the gauge bosons by acquiring a nonzero vev of the Higgs field.

2.6 The Standard Model Lagrangian

We have seen how to describe the strong interaction and the electroweak interaction in nature by applying the spontaneous symmetry breaking in the Yang-Mills theory, it is therefore the proper time for us to combine the electroweak interaction and the strong interaction together to form a unified description for three of the fundamental interactions, which is known as the Standard Model of particle physics.

The Standard Model Lagrangian can be divided into four parts

$$\mathscr{L}_{\mathrm{SM}} = \mathscr{L}_G + \mathscr{L}_F + \mathscr{L}_Y + \mathscr{L}_H, \tag{28}$$

for which we will explain in detail separately.

Gauge sector

The Standard Model contains three gauge interactions, the strong interaction, the weak interaction, and the electromagnetic interaction by the direct product of the (non-)Abelian gauge symmetry groups $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. The gauge sector of the Standard Model thus comprises a Yang-Mills theory with three gauge fields, one for each of the three gauge groups, which is an outcome of the Yang-Mills theory being the unique renormalizable theory possessing non-Abelian gauge symmetry.

$$\mathscr{L}_G = -\frac{1}{4}\sum_{A=1}^{8}(G_{\mu\nu}^A)^2 - \frac{1}{4}\sum_{a=1}^{3}(W_{\mu\nu}^a)^2 - \frac{1}{4}(B_{\mu\nu})^2, \tag{29}$$

where $G^A_{\mu\nu}$, $W^a_{\mu\nu}$, and $B_{\mu\nu}$ are the field tensors for the eight gluons of $\mathrm{SU}(3)_C$, three W-bosons of $\mathrm{SU}(2)_L$, and a B-boson of $\mathrm{U}(1)_Y$, correspondingly. The gauge fields are all initially massless in the symmetric phase to preserve the gauge invariance. After the spontaneous breaking of the SM electroweak gauge symmetry $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ down to $\mathrm{U}(1)_{\mathrm{EM}}$, the gauge fields W^a_μ and Z_μ become massive because of the Higgs mechanism. Though the gauge field B_μ and the third component of the $\mathrm{SU}(2)_L$ gauge field W^3_μ are mixed together in the flavor eigenstates, as we have seen in the last subsection it can be diagonalized in the mass eigenstates to form the neutral Z boson and the photon field observed in nature.

Fermion sector

The interactions between the Fermions and the gauge bosons in the SM can also be written in a similar way as the second term in Eq. (12). The Standard Model contains three generations of fermions with the same quantum numbers except for their masses, and in each generation, we have two quarks with different hypercharges known as the up-type or down-type quarks, one lepton and one neutrino.

One of the important observations of particle physics is that the weak interactions of left-handed and right-handed fermions are different. This phenomenon indicates that the fundamental weak interaction separates fermion with different chirality, and thus the SM must have a chiral structure. This was later been established by Glashow, Weinberg, and Salam [66, 36, 37] in the electroweak theory that only the left-handed fermions transform as doublets under the weak SU(2) isospin, while the right-handed fermions transform as singlets. Therefore, the chiral fermions of the SM for each generation should be embedded into the following representations under the $\mathrm{SU}(2)_L$ gauge symmetry

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad Q_{R,u} = u_R\,, \quad Q_{R,d} = d_R \quad L_R = e_R\,. \tag{30}$$

The Lagrangian of the fermion sector including the kinetic terms and their interaction with the gauge bosons is thus

$$\mathscr{L}_F = \sum_{i=1}^{3} \bar{L}^i_L i\not{D} L^i_L + \bar{Q}^i_L i\not{D} Q^i_L + \bar{e}^i_R i\not{D} e^i_R + \bar{u}^i_R i\not{D} u^i_R + \bar{d}^i_R i\not{D} d^i_R\,, \tag{31}$$

where the index $i = 1,2,3$ runs through all three generations of quarks and leptons, and the covariant derivative is

$$D_\mu = \partial_\mu - ig_3\theta_S G^A_\mu T^A - ig_2\theta_W W^a_\mu \tau^a - ig_Y Y B_\mu\,, \tag{32}$$

where $\theta_S = 0,1$ for singlets or triplets of $\mathrm{SU}(3)_C$ and $\theta_W = 0,1$ for singlets or doublets of $\mathrm{SU}(2)_L$, $\tau^a = \sigma^a/2$ is the canonically normalized $\mathrm{SU}(2)$ generators, $T^a = \lambda^a/2$ is the canonically normalized $\mathrm{SU}(3)_C$ generators with λ to be the Gell-Mann matrices, and g_3, g_2, g_Y are the corresponding gauge coupling constants of the strong $\mathrm{SU}(3)_C$, weak $\mathrm{SU}(2)_L$, and $\mathrm{U}(1)_Y$ hypercharge interactions. The hypercharge Y, along with the third component of the $\mathrm{SU}(2)_L$ weak isospin T_3, gives the electrical charge after the SSB of the SM gauge symmetry

$$Q = T_3 + Y\,. \tag{33}$$

Specifically, we can denote the color, weak isospin, and hypercharge assignments of all SM fermion representations in the form $(\mathrm{SU}(2)_L, \mathrm{SU}(3)_c)_{\mathrm{U}(1)_Y}$ as,

$$L^i_L \sim (\mathbf{2},\mathbf{1})_{y_1}\,, e^i_R \sim (\mathbf{1},\mathbf{1})_{y_2}\,, Q^i_L \sim (\mathbf{2},\mathbf{3})_{y_3}\,, u^i_R \sim (\mathbf{1},\mathbf{3})_{y_4}\,, d^i_R \sim (\mathbf{1},\mathbf{3})_{y_5}\,. \tag{34}$$

Classically, the partial Lagrangian $\mathscr{L}_G + \mathscr{L}_F$ displays large global symmetries. For example, one can perform a global transformation on the lepton doublet $L_i \to L_i' = U_{ij} L_j$ between three different families, where U_{ij} is a 3×3 unitary matrix and leaves the partial Lagrangian invariant. It turns out that such invariance holds for all five types of chiral representations of fermions defined in eq. (30), so we have the global family symmetry $\mathrm{U}(3) \times \mathrm{U}(3) \times \mathrm{U}(3) \times \mathrm{U}(3) \times \mathrm{U}(3)$ in eq. (31). Much of this enormous global symmetry is explicitly broken by the Yukawa sectors $\mathscr{L}_Y$ defined in the following.

Yukawa sector

The Yukawa sector contains the interactions between fermion pairs and spinless particles, whose existence is actually indicated by the renormalizability of the SM. Even though most of them are very small, they explicitly break the global symmetries of the massless Yang-Mills sector. As a consequence, this generates masses of fermions after the spontaneous breaking of the electroweak symmetry.

$$\mathscr{L}_Y = -Y_{ij}^{\ell} \bar{L}_L^i \Phi e_R^j - Y_{ij}^d \bar{Q}_L^i \Phi d_R^j - Y_{ij}^u \bar{Q}_L^i \tilde{\Phi} u_R^j + h.c. \tag{35}$$

We will see shortly that such a scalar field Φ triggers the Higgs mechanism to be responsible for the spontaneous breaking of the electroweak $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ symmetry down to the electromagnetic $\mathrm{U}(1)_{\mathrm{EM}}$ symmetry. Note that the $\mathrm{U}(1)_Y$ invariance indicates that the scalar field must take the form of $\tilde{\Phi} = i\tau_2 \Phi^*$ when being coupled to the up quark pairs $\bar{Q}_L^i u_R^j$.

At the classical level, the hypercharge assignments in the SM are arbitrary, as noted in equation (34). However, at the quantum level, they are constrained by the Adler-Bell-Jackiw anomaly. In order to ensure the vanishing of $\mathrm{U}(1)_Y$ gauge anomalies, the hypercharge assignments of the SM chiral fermions must satisfy the following equation [57]:

$$2y_3 + y_4 + y_5 = 0\,,\ y_1 + y_3 = 0\,,\ 2y_1^3 + y_2^3 + 3(2y_3^3 + y_4^3 + y_5^3) = 0\,. \tag{36}$$

Additionally, the SM may have another type of anomaly, namely the mixed gravitational anomaly generated by the triangle diagram with one hypercharge gauge boson and two gravitons. Combining this with the above equation, we obtain the following condition for anomaly cancellation [57]:

$$18y_3(2y_3 - y_5)(4y_3 + y_5) = 0\,, \tag{37}$$

which only supports two solutions: $y_5 = 2y_3$ and $y_3 = 0$. However, the existence of Yukawa couplings in the form of equation (35) forbids the second possibility, since $\mathrm{U}(1)_Y$ invariance implies that

$$y_h = y_1 + y_2 = -(y_3 + y_4) = (y_3 + y_5)\,. \tag{38}$$

Thus, the Yukawa couplings determine all the hypercharge assignments to be (in units of Y_h),

$$y_1 = -1\,, y_2 = +2\,, y_3 = +\frac{1}{3}\,, y_4 = -\frac{4}{3}\,, y_5 = +\frac{2}{3}\,. \tag{39}$$

Therefore, the absence of all kinds of anomalies from the SM requires hypercharge to be quantized. It is noteworthy that if additional right-handed neutrinos are included, their hypercharge must be zero, $y_{\nu_R} = 0$, in order not to spoil the hypercharge assignments of the SM fermions. This implies that either the right-handed neutrino is absent, or it is

Majorana (in which case it is its own antiparticle and cannot have any quantum numbers, including hypercharge), or it does not have hypercharge for some other reason.

It is important to point out that the global symmetries that remained in the Yukawa sector can be used to simplify the Yukawa couplings. Without loss of generality, we can write any matrix as the product of unitary matrix times a real diagonal matrix times another unitary matrix, yielding the lepton Yukawa matrix

$$Y^\ell = U_\ell M^\ell V_\ell^\dagger \, , \tag{40}$$

where U_ℓ and V_ℓ are unitary matrices in the family space, and M^ℓ is a real diagonal matrix. Now without affecting the rest parts of the Lagrangian, we can absorb the unitary matrices into the redefinition of the lepton fields as

$$\bar{L}_L \to \bar{L}_L \, U_\ell^\dagger \, , \; e_R \to V_\ell \, e_R \, . \tag{41}$$

And then the leptonic part of the Yukawa couplings will become flavor-diagonal

$$\mathscr{L}_{Y,\ell} = -m_i^\ell \bar{L}_L^i \Phi e_R^i + h.c. \, , \tag{42}$$

where m_i^ℓ is the diagonal element of the lepton mass matrix M^ℓ. Therefore, the leptonic Yukawa couplings explicitly break the global family symmetry $U(3) \times U(3)$ down to the three $U(1)$ phase transformations

$$\bar{L}_L^i \to e^{-i\alpha_i} \, \bar{L}_L^i \, , \; e_R \to e^{i\alpha_i} \, e_R \, . \tag{43}$$

The associated global charges of these three transformations are called the lepton numbers L_i ($i = e, \mu, \tau$) for each family.

Now we can repeat the same simplification to the quark Yukawa sector by introducing

$$Y^u = U_u M^u V_u^\dagger \, , \; Y^d = U_d M^d V_d^\dagger \, , \tag{44}$$

where $U_{u,d}$ and $V_{u,d}$ are unitary family matrices, and $M^{u,d}$ are both real diagonal matrices. What we will find out immediately from eq. (35) is that $V_{u,d}$ can be both absorbed by the redefinition of right-handed quark fields as

$$u_R \to V_u \, u_R \, , \; d_R \to V_d \, d_R \, , \tag{45}$$

while $U_{u,d}$ cannot be both absorbed into the redefinition of left-handed quark fields as we only have one left-handed chiral representation for the quark fields, which is $\bar{Q}_L$. The best way to redefine the left-handed quark fields is to first define the following matrix

$$\mathscr{V} = U_u^\dagger U_d \, , \tag{46}$$

and then decompose it by Iwasawa decomposition [57] as

$$\mathscr{V} = \mathscr{P} \mathscr{U} \mathscr{P}'^T \, , \tag{47}$$

where $\mathscr{P}$ and $\mathscr{P}'$ are diagonal phase matrices generated by elements of the Cartan subalgebra, and $\mathscr{U}$ contains the remaining parameters.

Now expand the quark Yukawa couplings sector in eq. (35) in terms of eqs. (44)-(47), we obtain

$$\mathscr{L}_{Y,q} = -\bar{Q}_L U_d M^d \Phi V_d^\dagger d_R - \bar{Q}_L U_d (\mathscr{P}' \mathscr{U}^\dagger \mathscr{P}^T) M^u \tilde{\Phi} V_u^\dagger u_R + h.c. \, , \tag{48}$$

which can be rewritten by field redefinitions as

$$\bar{Q}_L \to \bar{Q}_L U_d^\dagger \mathscr{P}'^T \ , \ d_R \to \mathscr{P}' V_d d_R \ , \ u_R \to \mathscr{P} V_u u_R \, . \tag{49}$$

As a result, the quark Yukawa sector in eq. (35) becomes [57]:

$$\mathscr{L}_{Y,q} = -m_i^d \bar{Q}_L^i \Phi d_R^i - m_j^u \bar{Q}_L^i \mathscr{U}_{ij}^\dagger \tilde{\Phi} u_R^j + h.c. \, , \tag{50}$$

where m_i^d and m_j^u are the diagonal elements of the quark mass matrices M^d and M^u, respectively. The matrix $\mathscr{U}$ is called the Cabibbo-Kobayashi-Maskawa (CKM) matrix [77, 78]. In the quark sector, the only remnant of the previous global symmetries $\mathrm{U}(3)^3$ is a common $\mathrm{U}(1)$ phase transformation for all the quarks,

$$\bar{Q}_L^i \to e^{-i\alpha} \, \bar{Q}_L^i \ , \ d_R^i \to e^{i\alpha} \, d_R^i \ , \ u_R^i \to e^{i\alpha} \, u_R^i \, , \tag{51}$$

which is nothing but the quark number, or $1/3$ of the baryon number B. It is also straightforward to check that both the baryon number and the lepton number are both anomalous, while the difference between the baryon number and the lepton number ($B-L$) is conserved at the quantum level.

Lastly, it is worth mentioning that if there had been only two families, such decomposition (the Euler decomposition) would have left us with only one rotation angle and no phase. Therefore, we conclude that, with three families, in total the Yukawa sector of the SM depends on thirteen (real) parameters: nine masses, three mixing angles and one phase.

Higgs sector

We now turn to the final part of the SM Lagrangian, which is the Higgs sector that describes the Higgs doublet and its interactions with the gauge fields and with itself. The Lagrangian of the Higgs sector is simply composed of the covariant term of the Higgs field and the scalar potential in the Higgs mechanism we introduced in the last subsection

$$\mathscr{L}_H = (D_\mu \Phi)^\dagger (D^\mu \Phi) + m^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2, \tag{52}$$

where the Higgs field Φ is a doublet under the $\mathrm{SU}(2)_L$ gauge symmetry and a singlet under $\mathrm{SU}(3)_C \times \mathrm{U}(1)_Y$ with $\frac{1}{2}$ hypercharge. The potential of the Higgs field is arranged in a special way to generate a vev for the Higgs field at $v = m/\sqrt{\lambda} \approx 246$ GeV, which spontaneously breaks the electroweak symmetry and generate the mass terms for the electroweak gauge bosons as the Higgs mechanism shows.

The expansion of the Higgs kinetic terms $(D_\mu \Phi)^\dagger (D^\mu \Phi)$ actually contains more interactions besides the masses of gauge bosons, such as the mass and self-interaction of the Higgs boson. By parameterizing the Higgs field in unitary gauge as

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} , \tag{53}$$

the Higgs boson is thus identified as the field $h(x)$ as an excitation from the vacuum, which is a neutral scalar boson under the $\mathrm{U}(1)_{\mathrm{EM}}$ gauge symmetry.

Now we can expand the Lagrangian of the Higgs sector eq. (52) by eq. (53)

$$\begin{aligned} \mathscr{L}_H &= \left| D_\mu \left(\frac{v+h}{\sqrt{2}} \right) \right|^2 + \frac{1}{2} m^2 (v+h)^2 - \frac{1}{4} \lambda (v+h)^4 \\ &= \left(\frac{1}{2} m^2 v^2 - \frac{1}{4} \lambda v^4 \right) + \left| D_\mu \left(\frac{v+h}{\sqrt{2}} \right) \right|^2 + \left(\frac{1}{2} m^2 - \frac{3}{2} \lambda v^2 \right) h^2 - \lambda v h^3 - \frac{1}{4} \lambda h^4 . \end{aligned} \tag{54}$$

This is the Lagrangian of the Higgs sector in the unitary gauge after the SSB of the electroweak symmetry. We can simplify this expression further by applying the minimization condition of the potential that is $v = m/\sqrt{\lambda}$ and replace the bare parameters m and λ in the potential with the physical observables assigned with specific physical meanings.

The first term in eq. (54), which is a constant term in the Lagrangian, is the vacuum energy corresponding to the potential energy when $h = 0$ that read

$$V(h=0) = -\frac{1}{2}m^2v^2 + \frac{1}{4}\lambda v^4 = -\frac{m^4}{4\lambda}\,. \tag{55}$$

The result that the vacuum has a negative nonzero vacuum energy, indicates that the vacuum is not normalized properly in the asymmetric phase. In fact, this is just an artifact of the SSB because our starting potential in eq. (52) assumes a zero vacuum energy when $\Phi = 0$, which is, however, not the global minimum of the potential. Therefore, strictly speaking we should reformulate the original Higgs potential by adding this tiny constant to normalize the vacuum, but we usually neglect this term as there are no global effects to any observables except for the vacuum energy of the SM.

The second term in eq. (54) includes the kinetic term and the interactions with the gauge bosons $W_\mu^\pm$ and Z_μ. After rotating the gauge bosons W_μ^3 and Z_μ by the weak mixing angle θ_w defined in eq. (23), the covariant derivative of the Higgs boson in terms of mass eigenstates is

$$D_\mu = \partial_\mu - ig_2 W_\mu^+ \tau^+ - ig_2 W_\mu^- \tau^- - i\frac{g_2}{\cos\theta_w} Z_\mu \tau^3\,, \tag{56}$$

and thus, the Lagrangian is expanded to

$$\begin{aligned}\mathscr{L}_H \supset \left| D_\mu \left(\frac{v+h}{\sqrt{2}}\right)\right|^2 &= \frac{g_2^2 v^2}{4}\, W_\mu^+ W^{-\mu} + \frac{g_2^2 v^2}{8\cos\theta_w^2}\, Z_\mu Z^\mu + \frac{1}{2}(\partial_\mu h)^2 \\ &+ \left(\frac{g_2^2 v}{2}\, W_\mu^+ W^{-\mu} + \frac{g_2^2 v}{4\cos^2\theta_w}\, Z_\mu Z^\mu\right) h + \left(\frac{g_2^2}{2}\, W_\mu^+ W^{-\mu} + \frac{g_2^2}{4\cos^2\theta_w}\, Z_\mu Z^\mu\right) h^2 .\end{aligned} \tag{57}$$

The above expression clearly shows that the interactions between the gauge bosons and the Higgs field induce the mass terms of the gauge fields when the Higgs field acquires the vev. We can thus define the mass terms of the gauge boson as

$$m_Z = \frac{g_2 v}{2\cos\theta_w}\,, \quad m_{W^\pm} = \frac{g_2 v}{2}\,, \tag{58}$$

and the expansion of the covariant derivative of the Higgs field is thus expressed by the physical observables as

$$\mathscr{L}_H \supset \left| D_\mu \left(\frac{v+h}{\sqrt{2}}\right)\right|^2 = \frac{1}{2}(\partial_\mu h)^2 + \left(m_W^2\, W_\mu^+ W^{-\mu} + \frac{m_Z^2}{2}\, Z_\mu Z^\mu\right)\left(1 + 2\frac{h}{v} + \frac{h^2}{v^2}\right)\,, \tag{59}$$

where the last term depicts the interaction between the gauge bosons and the Higgs boson.

The third term of eq. (54) is actually the mass term of the Higgs boson m_h

$$\mathscr{L}_H \supset -\left(-\frac{1}{2}m^2 + \frac{3}{2}\lambda v^2\right) h^2 = -\frac{1}{2}m_h^2 h^2\,, \quad \text{with} \quad m_h^2 = 2\lambda v^2 = 2m^2\,. \tag{60}$$

Unlike the original negative quadratic coupling $-m^2\Phi^2$ in the Higgs potential, the m_h is positive definite implying that the physical Higgs boson field h has the correct sign which can be interpreted as the physical mass term. We now know that this corresponds to about 125 GeV by the discovery of the Higgs boson to be introduced in the next section.

Finally, the last two terms in eq. (54) are the self-interactions of the Higgs boson. They can be rewritten in terms of physical observables as

$$\mathscr{L}_H \supset -\lambda v h^3 - \frac{1}{4}\lambda h^4 = -\frac{g_2 m_h^2}{m_W} h^3 - \frac{g_2^2 m_h^2}{32 m_W^2} h^4 \,. \tag{61}$$

In summary, after the SSB of the electroweak symmetry, the expansion of the Lagrangian of the Higgs sector in the unitary gauge (eq. (54)) is

$$\begin{aligned} \mathscr{L}_H &= \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{2} m_h^2 h^2 - \frac{g_2 m_h^2}{m_W} h^3 - \frac{g_2^2 m_h^2}{32 m_W^2} h^4 \\ &\quad + \left(m_W^2 \, W_\mu^+ W^{-\mu} + \frac{m_Z^2}{2} \, Z_\mu Z^\mu \right) \left(1 + 2\frac{h}{v} + \frac{h^2}{v^2} \right) . \end{aligned} \tag{62}$$

2.7 Parameters of the Standard Model

Now we have gained a comprehensive understanding of the Standard Model, and it has become clear that every operators in the SM Lagrangian in eq. (28) has a coefficent fully determined by a set of independent fundamental parameters [79], such as the coupling constants. But without determining the specific values of these coefficients from experiments, it is just a mathematical model that cannot give any quantitative description. The way of fixing all these coefficients is to map them into the experimental obersevables from theoretical calculation, such as scattering amplitude, anomalous magnetic moments, etc, up to any order of accuracy in perturbation theory, and then compare the calulation results with the measured values of the observables. In this way, we can check how accurately the Standard Model, as a robust mathematical model, can describe the phenomena observed in the experiments. The Standard Model's predictive power, which enables accurate predictions of particle physics experiments with remarkable precision, has cemented its status as one of the most successful mathematical models in the history of science.

It is important to note that there are no explicit coupling constants in the partial Lagrangian $\mathscr{L}_G + \mathscr{L}_F$, or more explicitly, in the kinetic terms of fields [79]. Instead, we have chosen to absorb them into the definition of the gauge field A_μ and the spinor fields ψ, so the coupling constants are implicitly present in the covariant derivatives, defined in eq. (32), which describe the interactions between the gauge bosons and fermions. As we have seen, there are three gauge couplings associated with the three different groups: g_Y, g_2, and g_3 respectively. What's more, the Yukawa sector $\mathscr{L}_Y$ contains 13 parameters of the Yukawa couplings (12 real and 1 phase), while the Higgs sector $\mathscr{L}_H$ includes one Higgs self-coupling λ and one dimensionful parameter m associated with the Higgs mass. Thus, the classical SM depends on 18 parameters, not including the masses and mixing of neutrinos. In addition to being invariant under the gauge group $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, the Standard Model is also invariant (at the classical level) under four global phase transformations, the three lepton numbers $\mathrm{U}(1)_{L_e} \times \mathrm{U}(1)_{L_\mu} \times \mathrm{U}(1)_{L_\tau}$ and the baryon number $\mathrm{U}(1)_B$.

The above counting is actually altered by quantum effects [57]: quantum anomalies break a linear combination of baryon and lepton numbers. The chiral anomaly also introduces another parameter that describes the QCD vacuum. These anomalies, however, do not alter the renormalizability of the theory since they affect only ungauged symmetries.

On the other hand, the quantum effects shift the exact values of these parameters from their bare values because of the runnings of these parameters with respect to the energy scales. The reason behind the running of the couplings can be summarized in the following way: starting from a UV theory at a cut-off $\Lambda_{\rm UV}$, we expect that all the higher-energy excitations should contribute to a general interaction, so we must sum over all the one-particle irreducible diagrams containing the exchange of imaginary particles in order to get an effective coupling constant (vertex). But the exchanging of heavy modes are energy-dependent, so in a low energy theory the observed values of couplings must be suppressed because less quantum-exchange were counted. Such a physical picture can be summarized in a simplified formulation known as the renormalization groups [45, 46, 47, 48, 49, 50, 51, 52, 53], which transforms the EFT description of a theory from the UV scale to the IR scale. The importance of RG must be emphasized here because it is one of the most important concepts in modern quantum field theory, and is one of the deepest underlying principles of physics that we know today.

Nevertheless, all the couplings must be renormalized at a fixed energy scale, where it becomes meaningful to give the exact values of these 18 SM parameters, otherwise, they are merely functions of the energy scale. For instance, at the Electroweak scale which is commonly chosen as the Z boson mass $M_Z = 91.2$ GeV, we can calculate the precise values for all observables, including the particle masses, decay widths, scattering cross sections, etc. And then these parameters can be fixed by comparing the theoretical predictions with the experimental data through a standard procedure known as the mapping [54]. In this way, we are able to obtain all of the 18 parameters at the same Electroweak scale, which have now become standard inputs for the Standard Model and are listed in the handbook of Particle Data Group (PDG) [21].

Suppose we have now determined all the Standard Model parameters (as we have) and obtain a perfect mathematical model that describes the physics of our world. It is then logical to ask the following two questions:

1. What is the origin of these parameters in the Standard Model?

2. What are the conditions under which the Standard Model can be applied?

These two questions are very fundamental regardless of the specific values of the SM parameters. They lie at the core of our pursuit to comprehend the deepest origins of the Standard Model as a physical theory and to explore the fundamental laws of physics. Countless research efforts in particle physics have been dedicated to addressing these questions.

Regarding the first question, we can imagine a fundamental theory where all SM parameters can be in principle calculated from only one or a few “fundamental parameters” defined in the moduli space. In such a scenario, all SM parameters, such as the gauge and Yukawa coupling constants, can be derived as functions of these moduli. If this idea were to be realized, it would offer hope for resolving one of the biggest challenges in physics and shed light on many other profound questions across various fields of study. Such an achievement could potentially propel the progress of human civilization and benefit all of humankind.

On the other hand, the second question is comparatively more manageable and may indeed have practical solutions. We will delve into further details regarding this question in the subsequent chapters of this thesis.

2.8 Vacuum of the Standard Model

In our previous discussion about the Standard Model, the parameter counting remains incomplete due to the omission of a general type of terms in gauge theories called the topological or θ-terms [58]:

$$\mathscr{L} \supset \theta \mathrm{tr} F \wedge F \sim \theta \varepsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu} F^a_{\rho\sigma} \sim \theta \mathrm{tr} F \tilde{F} \,. \tag{63}$$

At first glance, this would seem to introduce three new parameters, one for each gauge group. However, it's important to note that these terms act as total derivatives when expressed in terms of A_μ through Chern-Simons current. Consequently, they remain invisible in perturbation theory. In the Abelian case, the θ-term in $\mathrm{U}(1)$ gauge theories remains unobservable, as there are no instantons present to contribute to its effects, either through perturbative or non-perturbative means. However, in the non-Abelian case, the existence of instantons leads to a nonzero Chern-Simons invariant of the form:

$$\int F \wedge F \,, \tag{64}$$

which goes to zero in the presence of charged massless fermions [58]. Therefore, the only non-zero possibility must arise from the QCD instanton configurations, giving rise to a physical QCD θ-term. A non-zero value of θ_{QCD} breaks the CP symmetry of QCD, resulting in a non-zero electric dipole moment of the neutron. However, experimental searches have thus far produced only an extremely small upper bound, approximately $\theta_{QCD} < 10^{-10}$.

In any case, we have now arrived at our final result of 19 parameters. However, it is essential to recognize that the status of these parameters varies significantly. In the following sub-chapter, we will delve into a detailed examination of these operators using the EFT language.

2.9 The Standard Model as an Effective Field Theory

At the beginning of the chapter, we demonstrated the advantages of adapting to an EFT description. Additionally, we have recognized the intricate nature of the Standard Model as a delicate mathematical model containing a set of parameters: 17+1 dimensionless parameters (including the QCD θ-term) and 1 dimensionful parameter m. Consequently, the key question that arises is how we can rewrite the Standard Model using the language of EFT, enabling a clear depiction of the explicit dependence on the dimensionless parameters and the SM cutoff scale Λ.

When treating the Standard Model as an EFT, our key assumption is that a finite cutoff $\Lambda \gg M_Z$ is present and that the SM is the effective theory valid below this cutoff. The rationale behind this assumption will be elaborated upon in the subsequent chapter. Presently, we entertain two possibilities for this cutoff: it can either correspond to the scale at which the EFT framework becomes insufficient, or it can be the scale at which the SM is replaced by a different, more fundamental, ultraviolet QFT.

Let us recall that any EFT, as defined in eq. (5), inherently incorporates a characteristic cutoff scale Λ along with a set of dimensionless Wilson coefficients. Whenever a n-dimensionful parameter is required, it must be supplied by a power of the cutoff scale as Λ^{4-n}. It turns out that this is very useful in the classification of operators within the EFT. Specifically, at low energies ($E < \Lambda$), terms inversely proportional to Λ become small, rendering them irrelevant operators in the IR that are non-renormalizable. This situation corresponds to the condition where the scaling dimension n of the irrelevant operator is greater than the spacetime dimension d, expressed as $n-d>0$. Conversely, terms proportional to positive powers of Λ require $n<d$ and are deemed relevant in the IR, indicating their super-renormalizability. The marginal operators are those with $n=d$, which can be either relevant or irrelevant depending on their anomalous dimensions.

We can now apply this classification to the SM and derive the Standard Model Effective Field Theory (SMEFT, for review see e.g. [54]) in order to clarify the dependence on the SM cutoff scale Λ. It is not necessary to consider the full SM Lagrangian for this purpose since, apart from the Higgs mass term associated with the dimensionful parameter m, all other operators in the SM are dimension-4 operators characterized by dimensionless parameters. So obviously, the Higgs mass term is clearly a relevant operator, while the remaining terms in the SM are considered marginal. With this understanding, we are now ready to write down the Lagrangian of SMEFT. We first generalize the SM Lagrangian with an SM cutoff scale Λ by incorporating all possible irrelevant operators into the SM Lagrangian in eq. (28) as

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \sum_{n=5}^{\infty} \frac{c_n \mathscr{O}_n}{\Lambda^{n-4}} \,. \tag{65}$$

We further rearrange all terms in SMEFT Lagrangian according to their scaling dimensions:

$$\begin{aligned} \mathscr{L}_{\text{SMEFT}} &= \mathscr{L}_2 + \mathscr{L}_4 + \mathscr{L}_5 + \mathscr{L}_6 + \dots \\ &= c_2 \Lambda^2 |\Phi|^2 + |D_\mu \Phi|^2 - \lambda |\Phi|^4 + \mathscr{L}_4' + \mathscr{L}_5 + \mathscr{L}_6 + \dots \,, \end{aligned} \tag{66}$$

where we have expanded $\mathscr{L}_{\text{SM}}$ and $\mathscr{L}_H$ by eq. (28) and eq. (52), and the $\mathscr{L}_4'$ term is the renormalizable SM Lagrangian without the Higgs part: $\mathscr{L}_4' = \mathscr{L}_G + \mathscr{L}_F + \mathscr{L}_Y$. We also express the dimensionful Higgs mass parameter as $m^2 = c_2 \Lambda^2$, with the expectation that $|c_2| \ll 1$ is necessary for the Higgs to be a dynamical field below the cutoff. The reason for this is because if $c_2 = \mathscr{O}(1)$, the mass of the scalar field Φ will be of the order of the cutoff scale Λ, and then at energies much below it we can always integrate out this heavy degree of freedom resulting in the absence of a dynamical Higgs field.

Hence, without loss of generality, the SMEFT transmutes the SM dimensionful parameter m into a dimensionless Wilson coefficient which is extremely small: $|c_2| \ll 1$, with the price of introducing the SM cutoff scale Λ. The implication is straightforward: a very particular UV completion must exist that enables the tuning of this coefficient to a very small value in a natural manner. However, at the moment it is insufficient to argue for the existence of such tuning without considering the inclusion of quantum corrections. We will revisit this point in greater detail in the subsequent chapter.

Before closing this chapter, it is crucial to address a fundamental question regardless of the origin of parameters in the SM. The question at hand is: whether it is reasonable to have extremely small parameters (dimensionless or dimensionful) in an EFT? Suppose we have a fundamental theory above the EFT cutoff scale that allows for the precise calculation of the exact Wilson coefficients in the low-energy EFT, can we obtain an accurate result where the theoretical uncertainties are less than $\mathscr{O}(10^{-10})$ or $\mathscr{O}(10^{-38})$? To accomplish this, we would need to compute all quantum corrections in order to define the measure for the fine-tuning [58].

Roughly speaking, if the quantum corrections are small (or logarithmic-divergent), no fine-tuning is required, and a small number remains small from the UV to the IR scale. In this sense, the small number is considered natural (also known as technical naturalness [39]) and predictable from a UV theory, as is typically the case for gauge and Yukawa couplings. However, if the corrections are large (or power-divergent), a fine-tuning is present, indicating the need for a large bare parameter to cancel out the large corrections. In such cases, an extremely small computational outcome will not trivially emerge in an EFT due to the loss of accuracy after an RG transformation. We have repeatedly observed examples of this in nature in various physics theories, and even our most precise measurements have not reached an accuracy of $\mathscr{O}(10^{-38})$. Hence, in an EFT, the presence of a small number that receives large quantum corrections seems to contradict our fundamental physics intuitions.

As we are already aware of the existence of a few such small numbers, such as the cosmological constant, Higgs mass term, and QCD θ-term, a more significant question arises: Do these unnatural small numbers indicate the existence of an underlying theory at the fundamental level, where all these small numbers can be precisely calculated? Alternatively, can we formulate a more fundamental theory at the scale Λ where $c_i \ll 1$ can be comprehended?

3 Problems of the Standard Model

Despite being a successful mathematical model that has withstood the test of time, the Standard Model is incomplete. Although it has undergone extensive testing, there are still theoretical problems that remain unsolved, and unknown phenomena that are left unexplained in the SM. Most importantly, it absents the explanation for at least four major phenomena observed in our everyday lives:

1. The gravitational field and interaction.

2. The accelerating expansion of the universe.

3. The masses and mixing of neutrinos.

4. Hierarchical assignments for three generations of matter fields and three gauge couplings, in addition to the existence of non-trivial topological vacuum structure.

The presence of these problems prevents the SM from serving as a fundamental theory of physics that provides a quantum mechanical description of all particles and interactions in nature. Regardless of any biases one may hold when selecting scenarios that provide aesthetically natural or logically reasonable explanations for well-known unsolved problems in physics, a fundamental theory of physics must describe these phenomena in a unified framework based on a set of principles. Therefore, the pursuit of a unified theory that explains these four phenomena is important, and the inability of the Standard Model to do so represents one of its biggest challenge. The physics Beyond the Standard Model (BSM), is a subject that study these problems and how Standard Model can be completed into a more fundamental theory at high energy scales in a consistent manner.

The SM faces other numerous challenges as well, although some of these issues admit well-established natural solutions. For example, the inflation scenario addresses the horizon and flatness problems of the universe, while the dark matter scenario explains the rotation curve problem of galaxies, and the axion scenario addresses the strong CP problem. However, most of the theoretically or phenomenologically intriguing problems of the SM, such as the hierarchy problems, proton decays, baryogenesis, and the B physics anomaly, are still the subject of debate. To address some of these problems, we may have to rely on certain criteria and biases to consider certain EFT models, even if these EFTs cannot solve all the issues simultaneously. Furthermore, as we attempt to generalize these well-motivated models to incorporate other problems, it is common for new questions to arise one after another.

There are so many new possibilities and so many challenges existing now, it is just difficult to perform an analytical study of all different BSM models. But there is something in common in the solution of these questions, and can be understood in a model-independent way. In this chapter, we will, therefore, discuss a few critical questions in SM mentioned above without specifying a model.

3.1 Gravity

Gravity cannot be described in a quantum mechanical way, as is widely recognized over the past. This is not a problem specific to the Standard Model, but a general limitation of quantum field theory in curved spacetime. The reason is simple, near the quantum gravity scale gravitons and higher excitations such as higher spin states and KK modes, start to contribute [80], leading to a tower of states that strongly interact with each other. The exchange of these UV degrees of freedom significantly changes the whole picture: for example, the effective coupling for graviton scattering goes like [33]

$$g_{eff}(E)^2 \sim \frac{E^2}{M_P^2}\,, \tag{67}$$

which is different than the logarithmic scaling behavior of gauge couplings in traditional gauge field theory where the gauge coupling $g(E)^{-2} \sim \ln(E/\Lambda)$ becomes strong for $E \sim \Lambda \gg M_P$. As a result, near the Planck scale the effective operators with higher scaling dimensions start contributing to the graviton scattering process, and it turns out that we encounter infinitely many vertex operators with UV-divergent amplitudes that are typically non-renormalizable. This loss of control over couplings and UV degrees of freedom seems to suggest a breakdown of a quantum field theory description of gravity.

However, as long as we are dealing with energies much lower than the Planck scale (i.e., $E \ll M_P$), the changes in the couplings are small. In this sense, the low energy theory can be considered as a nice classical effective field theory in which gravity can be decoupled below the cut-off scale Λ_{QG}, where the spacetime is flat and the conventional QFT provides a good approximation. The SM is such an EFT where the effects of gravity are negligible at the scale much below the Planck scale, where we are safe to consider the gravity as a perturbation around the flat spacetime, known as the QFT in curved spacetime. The crucial problem is that we must be very careful on the EFT breakdown scale, where the non-trivial gravitational interaction comes into play and ruins the whole conventional EFT approach.

But if the EFT does not work, what should a theory of gravity look like? Given the importance of unitarity as a fundamental principle of physics [79], it is natural to expect that a quantum mechanical description of gravity should exist and be formulated in a way that preserves unitarity (for example, in S-matrix [81]). However, only a handful of gravity theories have been able to achieve this requirement in a self-consistent way. Therefore, gravity must set a non-trivial constraint on gravitationally-interacting EFTs by conserving the unitarity in a non-trivial way, even if gravity ruins the renormalizability of the EFT in the presence of UV degrees of freedom. The detailed construction of models involving gravity [82] is beyond the scope of this thesis, we will not go any further in this topic.

3.2 The Accelerating Expansion of the Universe

The latest observations of the universe have shown that it is expanding at an accelerating rate. To account for this phenomenon, a constant term called the cosmological constant must be added to Einstein's equation that describes the energy density of the vacuum. We may shortly summarize the effect of this constant term from the EFT perspective [58]. For now, let's set aside most of the deep conceptual problems in quantum gravity and treat the metric as a gauge field expanded around the flat space as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. This allows us to derive the leading order coupling between $h_{\mu\nu}$ and SM matter fields, which is analogous to the coupling of gauge fields to matter via $gA^a_\mu j^\mu_a$ [58]:

$$\mathscr{L}_{\rm eff} \supset \kappa h_{\mu\nu} T^{\mu\nu}\,, \tag{68}$$

where $\kappa \equiv 1/M_P$ is the dimensionful coupling of gravitational field, and $T_{\mu\nu}$ is the energy-momentum tensor. Note that we have applied the rescaling $h \to \kappa h$ for convenience.

Since a nonzero cosmological constant implies a nonzero energy-momentum tensor in the vacuum proportional to the vacuum expectation value of the energy density as $\langle\rho\rangle \equiv \lambda$, we can express this tensor as $T^{\mu\nu} = -\eta^{\mu\nu}\lambda$, to be substituted into equation (68) to obtain a non-zero tadpole term for the flutation field $h_{\mu\nu}$:

$$\mathscr{L}_{\rm eff} \supset -\kappa h_{\mu\nu} \eta^{\mu\nu} \lambda\,. \tag{69}$$

This is the effective operator corresponding to the contribution of a nonzero vacuum energy density (cosmological constant) in the EFT language.

It is argued that the observed cosmological constant is a fine-tuned quantity in the EFT context [58]. In order to do so, we need to calculate the loop corrections to the bare parameter λ. The way of doing it is to introduce boson and fermion fields which contribute to the effective vertex λ of the metric field $h_{\mu\nu}$, and then integrate out all particles as heavy degrees of freedom. Let's rename the original cosmological constant term in the tree-level action as the bare coupling λ_0, the renormalized vacuum energy density can thus be expressed as:

$$\lambda = \lambda_0 + \frac{c_\lambda}{16\pi^2}\Lambda_{EFT}^4\,, \tag{70}$$

where Λ_{EFT} is the cut-off of the EFT. The coefficient c_λ is proportional to the sum of bosonic and fermionic degrees of freedom and does not include a small coupling constant.

For a single real scalar field with mass m in the Euclidean signature, this means the loop corrections are explicitly:

$$\delta\lambda = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\ln\left(k^2+m^2\right) = c_0\Lambda_{EFT}^4 + c_1 m^2\Lambda_{EFT}^2 + \ldots\,, \tag{71}$$

where the cutoff of the integration is chosen to be the EFT scale Λ_{EFT}.

By comparing the observed value of the vacuum energy, which is estimated to be approximately $\lambda \simeq (2.2\,\mathrm{meV})^4$ [58], with the right-hand side of eq. (70) where the cutoff is expected to be of the order of the Planck scale $\Lambda_{EFT} = M_P \simeq 10^{18}$ GeV, one finds a huge discrepancy of roughly 10^{120} orders of magnitude: $\lambda \sim 10^{-120}M_P^4$. This discrepancy represents the worst case of fine-tuning within the EFT framework, known as the cosmological constant problem (CC problem).

Moreover, the second term in eq. (71) are significant sub-leading order corrections to the vacuum energy proportional to $m^2\Lambda_{EFT}^2$, which is again huge even when only the light SM particles are included. In addition, the complex vacuum structure of the SM leads to further effects on the vacuum energy density due to the presence of the vacuum expectation value of the Higgs field. All these subtle issues suggest that a non-trivial completion of the SM is needed to explain the observed vacuum energy density in a natural way.

3.3 The Neutrinos

The existence of neutrino oscillation clearly proves that neutrinos must have tiny masses and mixings. So a realistic EFT in the UV must include the mechanism of masses and mixings in a certain way. Once we write down a UV model, by the standard matching and running in the EFT framework introduced in chapter 2.9, we can obtain certain operators in SMEFT incorporating all the UV effects corresponding to the neutrino masses. Therefore, without writing down any specific model, it is clear that such neutrino masses must come from the dimension-5 Weinberg operator [83], which is also the unique (up to the flavor structure) dimension-5 gauge-invariant operator allowed in SMEFT:

$$\mathscr{L}_5 = \frac{c_5}{\Lambda}\left(\bar{L}^i\tilde{H}\right)\left(\tilde{H}L^j\right)^\dagger\,. \tag{72}$$

After the electroweak symmetry breaking, the Higgs acquires a vev at v_{EW} so the low-energy effect of the above operator is to give mass to the neutrino as:

$$\mathscr{L}_5 \sim \frac{c_5 v_{EW}^2}{\Lambda}\left(\bar{\nu}_L^i\right)^C \nu_L^j\,. \tag{73}$$

Instead of the Dirac mass term, this term corresponds to the Majorana mass term for the left-handed neutrinos. Note that because the SMEFT only contains exactly the same degrees of freedom as the SM, a right-handed neutrino cannot be introduced, and hence, a Dirac mass term for the neutrinos is strictly forbidden. This term explicitly breaks the lepton number symmetry, but it is fine to have it here, because an EFT in principle does not need to respect any global symmetry.

Given that the current experimental bound on the left-handed neutrino masses are at the order of 0.1 eV, from an EFT perspective if the coefficient is $c_5 = \mathscr{O}(1)$, this immediately implies that a cutoff scale Λ for this operator is[2]

$$m_{\nu_L} \sim \frac{v_{EW}^2}{\Lambda} \quad \Rightarrow \quad \Lambda \sim 3 \times 10^{14}\,\mathrm{GeV}\,. \tag{74}$$

This defines a new scale as the cutoff scale of SMEFT, referred to as the neutrino scale. As any EFT is invalid beyond its cutoff scale, we know that this is where the SMEFT must be broken, and instead, a new UV EFT must be used. Fortunately, it is not difficult to write down such a UV-EFT where the Weinberg operator is generated by the renormalizable operators via the so-called seesaw mechanism [84, 85, 86].

There are many different ways of realizing the seesaw mechanisms, which can be roughly classified into three different categories according to the new degrees of freedom introduced in the high-scale EFT:

1. Type-I: the chiral partner of ν_L (the right-handed neutrino ν_R) is introduced at UV.
2. Type-II: a $\mathrm{SU}(2)_L$ triplet scalar is introduced at UV.
3. Type-III: a $\mathrm{SU}(2)_L$ triplet fermion is introduced at UV.

In addition, similar to the type-I seesaw, we may have an inverse seesaw model [87], or a linear seesaw model [88], where the tree-level Majorana mass term for ν_R can be absent with the price of introducing extra fermions.

The type I seesaw model seems to be the most economical choice, as the right-handed neutrino exists in many different UV models and naturally leads to the Left-Right symmetric models. But we could not exclude the other possibilities, as well as the most complicated scenario where we need a combination of all the three scenarios.

Let us focus on the simplest and probably the most natural seesaw model at the moment, the type-I seesaw model. It is remarkable that with the addition of just a single massive fermion, the right-handed neutrino ν_R, we can generate the Weinberg operator as the low-energy EFT where the observed mass of neutrinos becomes naturally small. Recall that the SM is anomaly-free, to retain the anomaly-free structure in a UV-EFT, the right-handed neutrino must be a singlet under the SM gauge group, and must not carry any hypercharge as was demonstrated in eq. (39). And in the case of neutral fermion, there is no gauge symmetry to forbid the existence of Majorana mass terms for the fermion. Therefore, rewrite the neutrino as the Weyl spinors, the most general mass term for right-handed neutrinos (of one generation) is simply [58]:

$$\mathscr{L}_{m_{\nu_R}} \supset -\beta \bar{L}\tilde{H}\nu_R - \frac{1}{2}M_R \bar{\nu}_R \nu_R\,. \tag{75}$$

The first term is a Dirac mass mimicking a mixing between the left-handed and right-handed neutrinos, and the second term is a Majorana mass term for the right-handed

[2] If instead an unnatural small coefficient $c_5 \ll \mathscr{O}(1)$ is introduced, the neutrino scale can be lower, but then we will leave the smallness of neutrino masses unexplained.

neutrino. The Majorana mass term for the left-handed neutrino, as we learned in eq. (74), is extremely small and should stay small even after running and matching to another UV EFT. Now integrating out the heavy right-handed neutrino, we obtain precisely the Weinberg operator with

$$c_5 \sim \beta^2 \quad \text{and} \quad \Lambda \sim M_R\,, \tag{76}$$

which gives the observed neutrino mass $m_{\nu_L} = \beta^2 v_{EW}^2/M_R$.

Hence, any high-energy EFT with the type-I seesaw mechanism must have an effective neutrino mass term taking the form of eq. (75). But similar to the case of other fermions, it seems that we can have a more fundamental mechanism to generate such an effective mass term in a natural manner and explain the origin of the neutrino scale M_R. For example, we can consider that the effective Majorana mass indeed originates from a more fundamental UV theory via the spontaneous symmetry breaking and the Yukawa coupling. This idea naturally leads to the Left-Right symmetric model, where the Majorana mass of right-handed neutrinos may be generated by couplings with an additional scalar triplet field Δ_R charged under a $\mathrm{SU}(2)_R$ gauge symmetry.

For simplicity, we have only discussed the neutrinos for one generation. For the three generations case, there are mixings in different flavor eigenstates, in analogy to the mixing in the quark sector. The existence of mixings has been confirmed in the neutrino oscillation experiments, and all the mixing angles have been measured. It is again non-trivial to explain the origin of these mixing angles and phases unless we have a more fundamental theory for neutrinos.

Lastly, the effective neutrino mass term in eq. (75), if present in a UV theory, contributes to the self-energy of the Higgs field and results in a large quantum correction proportional to M_R^2. If $M_R^2 \gg m_H^2$, then a large fine-tuning is required in order to make the physical Higgs mass term small. We shall discuss more details about it in the next sub-chapter.

3.4 The Hierarchy Problems

One of the most important problems to be solved for physics at the electroweak scale, is the gauge hierarchy problem, which is the question of why the electroweak scale is so much smaller than the Planck scale. From the EFT perspective, if the Standard Model can be renormalized to the Planck scale, we can set the SM cutoff at the Planck scale $\Lambda = M_P$, and then the difference between the electroweak scale and the Planck scale becomes m_H^2/M_P^2. We have also seen in the previous chapter that we can naturally write the Higgs mass term as $m_H^2 = c_2\Lambda^2 = c_2 M_P^2$, so the gauge hierarchy problem is equivalent to the question of why the coefficient c_2 is so small.

To seriously discuss why it is unacceptable to have an extremely small coefficient c_2, we must compute the quantum corrections to the coefficient, just as we did for the cosmological constant. However, different than the rest of the dimensionless parameters which run logarithmically as energy increases, the Higgs mass term runs quadratically [58]:

$$\delta m_H^2(\mu) = m_H^2(\Lambda) + \frac{c_H}{16\pi^2}\Lambda^2 + \mathscr{O}(\Lambda^0)\,, \tag{77}$$

where roughly speaking $c_H = \lambda_H + y_t^2 + g_2^2 + \ldots$ when suppressing all $\mathscr{O}(1)$ coefficients and disregarding all logarithmic coefficients. Therefore, $c_H \sim \mathscr{O}(1)$ if the cutoff scale is $\Lambda = 1\,\mathrm{TeV}$. If we set the cutoff scale to be the Planck scale, things are getting worse as we need a very large $m_H^2(\Lambda)$ to cancel the large quantum corrections. Similarly to the case of the cosmological constant, there exists a fine-tuning between the bare mass term

$m_H^2(M_P)$ and the loop corrections $c_H M_P^2 \sim (10^{19}\,\mathrm{GeV})^2$ canceling each other to obtain a small physical Higgs mass term at the electroweak scale $m_H^2(v_{EW}) \simeq (125\,\mathrm{GeV})^2$.

It was argued that a small renormalized value cannot be small, unless there is a symmetry-based explanation for this [39]. This is the well-known Naturalness principle, which states that if a bare parameter is set to zero, the radiative corrections should not lead to a renormalized non-zero value. The Higgs mass term clearly violated this with a quadratic divergent term as the cutoff scale increases. For example, one proposal is to explain via softly broken supersymmetry (SUSY) since SUSY implies non-renormalization theorems [89] that prevent the superpotential to be renormalized and hence no perturbative corrections arise. However, none of the explanations so far have been quite successful in providing a natural explanation for this puzzle while keep being compatible with experimental results.

On the other hand, there are other hierarchies within the Standard model. We have seen two of them in our previous discussion: the first one is the strong CP problem, and the second one is the hierarchies in the Yukawa couplings which basically ask why are the masses of some particles orders of magnitudes different from each other. The latter one relates to the origin of Yukawa couplings.

4 Beyond the Standard Model (BSM) Models

As mentioned in the previous chapter, not all of the problems within the SM can be simultaneously solved within a unified framework. Therefore, we must first consider toy models to better understand the essential issues related to the specific problem. The hope is that someday we will be able to combine all the crucial ideas proposed in these toy models into a complete theory within a unified framework that can solve all the problems of the SM simultaneously. This approach is known as the bottom-up approach, or EFT approach, where the models are usually based on extending the Standard Model, referred to as Beyond the Standard Model (BSM) models. While it may be challenging, theorists have been working on this method for decades.

In order for any BSM model to be compatible with all experimental observations, it must be constructed in a consistent manner, ensuring that the Standard Model Effective Field Theory (SMEFT) is realized as a low-energy EFT. This standard process is commonly referred to as the matching in the EFT framework. The objective is to match a specific UV-EFT to the SMEFT precisely at a high-energy scale Λ_{UV}. And it entails integrating out all the heavy degrees of freedom, thereby yielding all the Wilson coefficients which can be constrained by the experimental observations through the running of mappings that occur within the SMEFT [54]. Since all the renormalization group (RG) flows in the SMEFT are well-defined, our primary focus lies in the full effective action at a specific high-energy scale Λ_{UV}. This high-energy effective action encompasses the model-dependent aspects of BSM model building.

Before delving into the details of the EFT approach, we first raise a few fundamental questions that EFT theorists concern about. Suppose we have constructed a specific BSM model with its effective action remaining valid at a high-energy scale Λ_{UV}, to accurately describe the observed low-energy physics. In such a scenario, we would expect that starting from this EFT, we can derive every parameter of the Standard Model through the standard matching procedure, thereby establishing a fully determined renormalized EFT at the IR scale. However, this raises a more significant question: How can we precisely calculate all these small parameters (e.g. $\theta_{QCD} \lesssim 10^{-10}$) from a UV-EFT? Consequently, we are faced with three key challenges:

1. How can we explain the precise (small) values of the 17 dimensionless parameters of the SM, including gauge couplings, Yukawa couplings, and Higgs self-coupling?
2. How can we explain the fine-tuning parameters in the SM, including the Higgs mass term and the QCD θ-term?
3. How can we explain the cosmological constant with the observed value in EFT?

Additionally, if the theory predicts additional parameters for degrees of freedom not present in the SM, such as right-handed neutrinos, another challenge arises

4. If additional degrees of freedom are generated at a high-energy scale, how can we generate their parameters, such as masses and interactions, without exacerbating the hierarchy problem?

These challenges are all important and relate to the origins of parameters in the model, which should be addressed to develop a more complete understanding of the unified theory and its underlying principles.

But what will these underlying principles look like? Over the past few decades, theorists have gained considerable experience constructing BSM models and have come to recognize that two fundamental principles have been highly effective in making successful predictions and in addressing the above questions [33]. The first principle, known as **UV/IR decoupling**, asserts that low-energy physics can be effectively described independently of high-energy physics within the EFT framework. The second principle, **Naturalness** [90], assumes that coupling constants in a theory are of order one in the appropriate mass scale. Therefore, if any parameter is unusually small or large[3], a good explanation, such as an underlying symmetry, is require [33].

In this chapter, we will focus on examining toy models capable of understanding the origin of other parameters in the SM except for the CC. This approach is motivated by the fact that, given a well-defined UV model, we can separate it into two parts: one that generates the observed patterns for SM parameters and another that generates the CC and remaining UV degrees of freedom. With this strategy in mind, we will discuss several BSM models based on the principles of UV/IR decoupling and naturalness, including the Two-Higgs-Doublet Models (2HDMs) [91], Classical Scale Invariant Models (CSIMs) [92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 1], Left-Right Symmetric Models (LRSMs) [105], and Grand Unified Theories (GUTs) [105], all of which provide explanations for the origins of various SM parameters. We will also emphasize that most BSM models require a non-trivial flavor structure to avoid experimental bounds on flavor violation. Therefore, constructing a mechanism to explain the origin of this flavor structure or constraining parameters in the Yukawa sector, and understanding their phenomenological consequences, are critical in BSM model building [90].

4.1 Two Higgs Doublet Models

In a non-supersymmetric $\mathrm{SU}(N_c)$ YM theory described by equation (12), the IR dynamics are determined by the two numbers N_c and N_f. When a scalar sector is introduced to the YM theory, spontaneous symmetry breaking occurs via the Higgs mechanism. This symmetry-breaking pattern can be achieved in many ways with a general scalar sector, meaning that from an EFT perspective, there is no a-priori reason for the number of the scalar field N_s not to be a free parameter. This means that, from the naturalness point of view, the choice $N_s = 1$ made by the SM seems to be very special.

An experimental observation that highlights this issue is the precise measurement of the ρ-parameter $\rho = m_W^2/(m_Z^2 \cos^2\theta_W)$, which is found to be very close to 1 [21]. In a general $\mathrm{SU}(2)L \times U(1)Y$ gauge theory with N_s scalar multiplets Φ_i, having weak isospin I_i, weak hypercharge Y_i, and vacuum expectation value of the neutral components v_i, the ρ-parameter can be given at tree level by the expression [91]

$$\rho = \frac{\sum_{i=1}^{N_s} \left[I_i(I_i+1) - Y_i^2\right] v_i^2}{\sum_{i=1}^{N_s} 2Y_i^2 v_i^2} \tag{78}$$

In the simplest case with non-zero weak isospin, the above equation gives $\rho = 1$ when there are N_s $\mathrm{SU}(2)$ doublets with $Y_i = \pm\frac{1}{2}$, as they all give $I_i(I_i+1) = 3Y_i^2$. The SM Higgs doublet corresponds to the $\mathrm{SU}(2)$ doublet with $Y_h = +\frac{1}{2}$.

As the simplest extension of the Standard Model, we may consider extending the SM with a second Higgs doublet field that also acquires a vacuum expectation value and participates in breaking the electroweak symmetry at the electroweak scale. This gives rise to the Two Higgs Doublet Models (2HDMs), which enlarges the scalar potential (the Higgs sector) of the SM to include two Higgs doublet (Φ_1 and Φ_2), each of which is a doublet

[3] In a broader sense, the question of the smallness of parameters, even including the dimensions of spacetime, the rank of SM gauge groups, and the representations and numbers of fermions in the SM, can all be seen as naturalness problems, which may or may not have solutions yet [33].

of the $\mathrm{SU}(2)_L$ gauge group with hypercharges $Y_i = \pm\frac{1}{2}$, resulting in five physical scalar states: two CP even neutral scalar states, one CP odd pseudoscalar, and two charged Higgs states [91]. As a result, the scalar potential in general contains many more terms than the SM, including parameters, which thus allows for a richer phenomenology such as the possibility of having CP violation and flavor-changing neutral currents (FCNCs) mediated by scalar fields, as well as stabilizing the EW vacuum.

To study their scalar potential in detail, we note that the scalar sector of 2HDMs is weakly coupled, making it possible to classify the vacuum phase structure of 2HDMs in traditional perturbation theory. The most general scalar potential contains CP-violating or charge-violating minima, but several simplifying assumptions can be made to reduce the free parameters in the scalar potential in order to coincide with phenomenology. For example, one such assumption is the conservation of CP symmetry in the Higgs sector, which allows for the distinguishability of scalar and pseudoscalar states. Under these assumptions, the renormalizable scalar potential for two Higgs doublets, Φ_1 and Φ_2, both with hypercharge $+\frac{1}{2}$, is given by [91]

$$\begin{aligned} V_H &= m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - m_{12}^2\left(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1\right) + \lambda_1\left(\Phi_1^\dagger\Phi_1\right)^2 + \lambda_2\left(\Phi_2^\dagger\Phi_2\right)^2 \\ &\quad + \lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \lambda_5\left[\left(\Phi_1^\dagger\Phi_2\right)^2 + \left(\Phi_2^\dagger\Phi_1\right)^2\right] \end{aligned} \tag{79}$$

This simplified scalar potential contains 3 dimensionful couplings and 5 dimensionless scalar self-couplings, for a total of 8 real parameters, which is much richer than the SM Higgs sector. For a region of parameter space, the minimization of this potential gives [91]

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_1\end{pmatrix} \quad , \quad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_2\end{pmatrix}, \tag{80}$$

which generate masses to the W and Z bosons, thus implying the relation $\sqrt{v_1^2 + v_2^2} = v_{SM} \simeq 246$ GeV. We further define the ratio of these two vevs to be $\tan\beta = v_2/v_1$.

Next we proceed to generalize the SM Yukawa sector to include two Higgs doublets [90]. The introduction of a general Yukawa sector consisting of all interactions between fermion and scalar representations (the type III 2HDM) can lead to tree-level flavor changing neutral currents (FCNC), which are tightly constrained by experimental data. To avoid this, the 2HDMs are typically constrained by imposing global symmetries in different ways. The Paschos-Glashow-Weinberg theorem [106, 107] formalized this by stating that to prevent tree-level FCNC, all fermions of a given charge and helicity must transform under the same $\mathrm{SU}(2)$ irreducible representation. In the SM, this requires all fermions to couple to a single Higgs multiplet. However, in the 2HDM, discrete or continuous symmetries must be introduced to achieve this, leading to two possibilities: the type I 2HDM, where all fermions couple to just one of the Higgs doublets (typically Φ_2), and the type II 2HDM, where fermions with different weak isospins couple to different Higgs doublets. As an example, the Lagrangian of the Yukawa sector in type-II 2HDM for two scalar doublets (H_u and H_d) with different hypercharges $Y_{H_d} = -Y_{H_u} = +\frac{1}{2}$ is given by

$$-\mathscr{L}_Y^{\text{2HDM}} = Y_{ij}^\ell \bar{L}_L^i H_d\, e_R^j + Y_{ij}^d \bar{Q}_L^i H_d\, d_R^j + Y_{ij}^u \bar{Q}_L^i H_u\, u_R^j + \text{h.c.}, \tag{81}$$

with Q_L^i/L_L^i the quark/lepton left-handed doublets and $f_R^i\,(f = u, d, e)$ the right-handed singlets.

Finally, we emphasize that the 2HDM should also be regarded as a low-energy model from an EFT perspective. It serves as the low-energy EFT of many well-motivated UV models, such as the original axion model (Weinberg-Wilczek model [108, 109]), the Minimal

Supersymmetric Standard Model (MSSM) [110], CSIMs, LRSMs, GUTs, etc, which are potential candidates to solving the naturalness problems. These UV models usually impose continuous symmetries to produce the same Yukawa couplings as a type II 2HDM in the low-energy limit. For example, the Weinberg-Wilczek model requires an additional Higgs doublet to enforce the additional $U(1)_{PQ}$ global symmetry under which SM quarks are charged. Similarly, the MSSM requires two Higgses to ensure gauge invariance, because only with two scalars we can have two different chiral multiplets with different chiralities.

4.2 Classically Scale Invariant Models and the Higgs-portal Scenarios

In the EFT approach introduced in chapter 2.1, we start from a symmetry and write down the most general action consistent with that symmetry [62]. The more symmetry we have, the more constrained the action will be. Similarly, when generalizing the scalar sector of the SM with N_s scalar multiplets, we must constrain the general scalar and Yukawa sectors with symmetries based on our understanding of the fundamental principles of physics, otherwise the scalar interactions will be too complicated which could lead to dangerous phenomena. The question is, besides the conventional spacetime and gauge symmetry, what other symmetries can we use? One possibility is the global symmetry as in the previous 2HDM case, which appears as an approximate symmetry in the low-energy limit and is known as accidental symmetry.

Observing the SM, we notice that there is one special classical symmetry, which is an approximate global symmetry preserved by the part of SM with the 18 dimensionless parameters, but is explicitly broken by the Higgs mass term which is dimensionful. This symmetry is the classical scale invariance (CSI) [92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 1], which redefines the coordinates as a scale factor as $x \to e^{\varepsilon} x$, and can be seen as a generalization of the spacetime Poincare symmetry to the conformal symmetry.

The classical scale invariance is anomalous, which is described by the conformal anomaly that is proportional to the beta function of the full theory. As the tree level Higgs mass term explicitly breaks the CSI, a theory admitting CSI should not contain a tree-level Higgs mass term, hence, it would not develop any quadratic divergence and only logarithmic corrections would contribute to the running of Higgs mass. Therefore, this approach has the potential to solve the fine-tuning problem of Higgs mass [111]. Without a tree-level Higgs mass term, the Electroweak Symmetry Breaking (EWSB), including the observed Higgs mass, is then entirely generated by the conformal anomaly effect.

To see how Higgs mass is generated by the conformal anomaly, we start with a small bare coupling[4] λ of a scalar field ϕ in a gauge theory with CSI at the UV scale Λ_{UV}. By renormalization, the coupling receives contributions from both itself (λ^2) and the gauge coupling (α^2), leading to a one-loop effective potential that takes the form [112]

$$V = \frac{\phi^4}{4!} + (A\lambda^2 + B\alpha^2)\phi^4 \left(\ln \frac{\phi^2}{v^2} - c' \right), \tag{82}$$

where A, B, and c' are dimensionless parameters fixed by the renormalization. It is straightforward to note here that only logarithmic corrections are induced by the conformal anomaly, as mentioned above.

Minimization of the above potential gives $\lambda \sim \alpha^2$, which reduces one dimensionless parameter in the potential. As a result, the one-loop effective potential contains only two free parameters: the gauge coupling α and a dimensionful vev v, which is dynamically

[4] Note that the smallness of λ is motivated by the smallness of SM Higgs quartic coupling $\lambda = m_h^2/2v^2 \simeq 1/8 \ll 1$ at the tree-level.

generated by quantum correction [112]:

$$V \sim \alpha^2 \phi^4 \left(\ln \frac{\phi^2}{v^2} - \frac{1}{2} \right) . \tag{83}$$

This potential is very similar to the SM Higgs potential, with one dimensionless coupling λ and one dimensionful coupling m. By renormalizing this potential from the UV cutoff scale Λ_{UV} to the symmetry breaking scale v to one-loop order, we can determine that [113]

$$v = \Lambda_{UV} \exp \left[\left(\frac{\beta_0}{2\pi} \right)^{-1} \left(\frac{1}{\alpha(v)} - \frac{1}{\alpha(\Lambda_{UV})} \right) \right] , \tag{84}$$

with β_0 the one-loop beta coefficient. This result is the famous mechanism of dimensional transmutation.

Depending on the dynamics of the gauge theory, the above equation implies two possible ways of dynamically generating scale to be discussed in detail in Appendix I [1]:

1. $\beta_0 > 0$, the gauge coupling is asymptotic free: $\alpha(\Lambda_{UV}) = 0$, implying that the symmetry breaking scale $v \sim \Lambda_{UV} \exp \left[-\frac{2\pi}{\beta_0 \alpha(\Lambda_{UV})} \right] \ll \Lambda_{UV}$ is generated nonperturbatively, similar to the case of QCD, known as the **nonperturbative type**.

2. $\beta_0 < 0$, the gauge coupling develops a Landau pole at UV: $\alpha(\Lambda_{UV}) = \infty$, implying that the symmetry breaking scale $v \sim \Lambda_{UV} \exp \left[-\frac{2\pi}{|\beta_0| \alpha(v)} \right]$, as discussed in Coleman-Weinberg mechanism [112], known as the **perturbative type**.

In both cases, the conformal anomaly of gauge couplings generates a symmetry-breaking scale v, which is exponentially suppressed from the UV cutoff.

It is widely known that the above picture can be generalized by the Gildener-Weinberg mechanism [114] in a general case with N_s weakly interacting scalars S_i ($i = 1, 2, \ldots, N_s$). This can be applied to the SM, where in addition to Higgs, extra scalar fields are needed and the EWSB is realized by the Gildener-Weinberg mechanism. The realization of the Gildener-Weinberg mechanism in a class of CSIMs and their phenomenology are still actively being researched in the current BSM research. Moreover, these CSIMs can be generalized beyond the classical level to cancel all the conformal anomalies at the UV cutoff scale, ensuring a quantum scale invariance in a non-trivial UV-completed theory. This concept is known as asymptotic safety, and we will discuss it in more detail in Appendix I [1].

Finally, we would like to emphasize that triggering the dynamical breaking of Electroweak symmetry by radiative corrections requires the presence of one or more SM-singlet scalar fields S_i that interact with the visible sector through a biquadratic coupling to the Higgs doublet H, given by $\lambda_p |H|^2 S_i^2$, to preserve the CSI. This means that if there is a hidden sector of other heavy scalar degrees of freedom, the Higgs field naturally acts as a mediator between the SM and the hidden sector. This scenario is known as the **Higgs portal scenario**, with the coupling between the hidden sector and the Higgs called the portal coupling λ_p. In general, the hidden sector can be anything, and the SM-singlet scalar S_i can be charged under some hidden symmetries (gauge or global), as long as the portal coupling is tuned to be small enough to avoid low-energy observable effects from the hidden sector. In some cases, the Higgs portal scenarios do not necessarily admit classical scale invariance, but may instead be constrained by discrete global symmetries. This is yet another viable extension of the SM with respect to the Higgs sector.

4.3 Left-Right Symmetric Models

In the previous chapter, we explored the simplest and most elegant explanation for the smallness of left-handed neutrino masses, which is achieved through the type-I seesaw mechanism. From the perspective of EFT, it is natural to regard the effective mass term of the right-handed neutrinos as the cutoff scale as $\Lambda_{EFT} = M_R$, so that eq. (75) can be interpreted as an effective description valid only at energies below the cutoff scale M_R. In order to comprehend the origin of this seesaw term, we can study the possible forms of the potential above the scale M_R which should reduce to eq. (75) as a low-energy depiction after integrating out the higher-energy degrees of freedom.

There exist numerous possibilities to achieve this, and conventionally, it is accomplished through models where the majorana masses of right-handed neutrinos are generated by Yukawa couplings. In such cases, the introduction of a heavy SM-singlet scalar field becomes necessary, with a mass of approximately M_R, to mediate these Yukawa couplings. To generate the mass term for this heavy scalar field, it is therefore natural to assume the presence of an additional gauge group that undergoes spontaneous symmetry breaking at the scale of M_R, where the heavy scalar acquires a mass of the order of M_R due to the Higgs mechanism.

A simple and natural choice for an additional gauge symmetry is $\mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$, commonly referred to as Left-Right Symmetric Models (LRSMs or LR models) [115, 116, 117]. In these models, a typical Yukawa potential is given by:

$$-\mathscr{L}_Y \supset Y_Q \bar{Q}_L \Phi Q_R + Y_L \bar{L}_L \Phi L_R + Y_R L_R^T i\sigma_2 \Delta_R L_R + \mathrm{h.c.}\,, \tag{85}$$

It is clear that to match with the effective mass term in the seesaw mechanism, the right-handed triplet field Δ_R has to get a vev at the scale M_R and break the $\mathrm{SU}(2)_R$ symmetry spontaneously. The bidoublet field Φ is then responsible for generating the EWSB at the scale v_{EW}.

For simplicity, we neglect terms coupling to $\tilde{\Phi} \equiv \sigma_2 \Phi^* \sigma_2$. Here, Q and L represent quark and lepton fields with chiral subscripts, respectively, and σ_2 denotes the second Pauli matrix. The minimum couplings introduce two scalar representations: a Higgs bidoublet Φ transforming as $(\mathbf{2},\mathbf{2})$ and a right-handed triplet field Δ_R transforming as $(\mathbf{1},\mathbf{3})$ under the gauge symmetry group $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. Notably, in order to match with the effective mass term in the seesaw mechanism, the right-handed triplet field Δ_R must acquire a vev at the scale M_R, thereby spontaneously breaking the $\mathrm{SU}(2)R$ symmetry. On the other hand, the bidoublet field Φ is responsible for generating the EWSB at the scale v_{EW}.

The introduction of additional scalar fields also poses a significant challenge in studying the vacuum structure. At the renormalizable level, numerous gauge invariant operators emerge within the scalar sector, which makes the analysis extremely difficult.

However, for the simplest scenario where only two scalar fields, Φ and Δ_R, are introduced, it has been demonstrated that their vevs must satisfy a constraint that stems from a non-trivial alignment. This alignment is crucial in achieving the desired symmetry breaking pattern, and is expressed as follows:

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \kappa_u e^{i\theta_u} & 0 \\ 0 & \kappa_d e^{i\theta_d} \end{pmatrix}, \quad \langle \Delta_R \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ \kappa_R e^{i\theta_R} & 0 \end{pmatrix}. \tag{86}$$

By aligning the vevs in this manner, it ensures that the symmetry of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ can be broken down to the electromagnetic $\mathrm{U}(1)_{\mathrm{EM}}$ symmetry.

The Left-Right symmetric models serve as one of the simplest approaches to implement the type-I seesaw mechanism. However, a pertinent question arises: with numerous models available to apply the seesaw mechanism and explain the origins of neutrino masses, how can we justify which one is better suited to describe nature without direct experimental evidence? In addressing this question, Naturalness once again offers a reliable criterion: a theory is considered more natural when it explains more phenomena using the same set of parameters. While we have primarily focused on the matter of neutrino masses within LRSMs thus far, it is important to recognize that LRSMs, in general, provide extensive explanatory power for both theoretical and phenomenological issues. Below, we provide a concise summary of the advantages of LRSMs, highlighting their inherent capacity to account for the smallness of neutrino masses:

1. The presence of spontaneously broken $\mathrm{SU}(2)_R$ symmetry at a right-handed scale offers a dynamic explanation for the observed parity violation in weak interactions.

2. LRSMs provide a clear and meaningful interpretation of hypercharges [118, 119].

3. LRSMs incorporate baryon number violating interactions that can account for the generation of baryonic matter (baryogenesis) in the universe.

4. LRSMs open up the possibility of unifying gauge couplings within the context of SO(10) Grand Unified Theories with intermediate scales.

However, similar to many BSM models, the LRSMs are unable to address the gauge hierarchy problem. One apparent approach to tackle this issue within LRSMs is through supersymmetrizing the model [120, 121, 122]. Nevertheless, introducing supersymmetry results in strong constraints on Supersymmetric Left-Right models, particularly when attempting to obtain a vacuum for broken gauge symmetries [123, 124, 125]. Furthermore, realizing the natural unification of gauge couplings becomes notably challenging in such scenarios [126, 127, 128, 129]. Thus, on one hand, if one strongly insists on the unification of gauge couplings alongside the implementation of the seesaw mechanism, it appears that non-supersymmetric Left-Right models are simpler than their supersymmetric counterparts. On the other hand, finding an alternative solution to the gauge hierarchy problem becomes a challenging endeavor if we do not impose supersymmetry. There remains a possibility [130] of resolving this potential conflict, but it extends beyond the scope and focus of the current thesis. Therefore, in the upcoming subchapter, we temporarily set the hierarchy problem aside and shift our attention toward exploring simple phenomenological models that exhibit the unification of gauge couplings while simultaneously accommodating the seesaw mechanism.

4.4 The SO(10) Grand Unified Theories

In the previous chapters, we have explored various extensions of the Standard Model, focusing on the Higgs, fermion, and gauge sectors separately. Now, we are ready to take a further step by unifying these sectors into a single, elegant, realistic framework. This desire for unification led to the development of Grand Unified Theories (GUTs), which aim to bring together all the fundamental interactions, except for gravity, into a coherent theory. Surprisingly, this idea has proven to be even more successful than anticipated, providing profound insights into the fundamental aspects of physics.

In fact, the existence of Grand Unified Theories is implicitly suggested in the Standard Model. By considering the gauge couplings of the Standard Model at the electroweak scale and running them to a higher energy scale by the renormalization group equations (RGEs) of the SMEFT, we can track their behavior at higher energy scales. Remarkably, these RGEs reveal that the three SM gauge couplings converge to approximately the same value at a specific energy scale called the GUT scale, denoted as $M_U \sim 10^{16}$ GeV. At this scale, the gauge couplings become identical, enabling different gauge couplings to be interchangeable while preserving the system's invariance. This implies that, to a certain level of precision, we can formulate a theory at the GUT scale with a single gauge field that equally couples to the other species involved. In short, the concept behind Grand Unified Theories is essentially centered around such a gauge theory based on a single, comprehensive gauge group denoted as $\mathscr{G}$. Notably, the three gauge symmetries of the Standard Model, $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, must be a subgroup of this larger gauge group, $\mathscr{G}$.

The first choice of such bigger gauge group is the $\mathrm{SU}(5)$ group [131, 132], proposed as the first kind of known grand unified theories. However, there are two problems in this simple model. The first problem is that it must have a EW-scale supersymmetry for achieving the unification of gauge couplings, which was not seen in the LHC experiment. The other problem is that it only contains all the 15 SM chiral fermions as the fundamental spinor representation of $\mathrm{SU}(5)$ is $\bar{\mathbf{5}}+\mathbf{10}$, which makes it difficult to include the seesaw mechanism to explain the smallness of neutrino masses without invoking the right-handed neutrinos.

One such example that encompasses all three SM gauge couplings is the $\mathrm{SO}(10)$ group [115, 120]. Alternatives include the $\mathrm{SU}(5)$, E_6, E_7, and E_8 groups, but we are not going to discuss any of them here. So what we would like to do now is to write down an explicit renormalizable action that contains all the $\mathrm{SO}(10)$-invariant operators. To do that, it is necessary to assign specific representations to the various fields involved. The fundamental representation for fermionic fields in $\mathrm{SO}(10)$ is the 16-dimensional spinor representation, denoted as $\mathbf{16_F}$. On the other hand, it is worth noting that in the Standard Model, a single generation of fermions consists of exactly 15 chiral fermions. This includes six colored chiral quarks multiplied by two chiralities, one charged lepton, and a left-handed neutrino. So remarkably, these 15 SM fermions can all be embedded into the 16-dimensional spinor representation $\mathbf{16_F}$, together with an additional SM-singlet introduced as the right-handed neutrino.

Indeed, it is relatively straightforward to construct an EFT that incorporates all interactions between the spinor representation $\mathbf{16_F}$ and the gauge bosons associated with the $\mathrm{SO}(10)$ gauge symmetry, where the only work needed to be done is to list all the $\mathrm{SO}(10)$-invariant operators, so we will not do it again here. The same analogy can also be applied to the scalar and Yukawa sector of $\mathrm{SO}(10)$ GUTs. All the details about specific $\mathrm{SO}(10)$ model building can be found in the author's publication III and IV [3, 4].

Once we have a well-established GUT model, where the details of interactions and symmetry breaking patterns are explicitly given, it opens up avenues to address various fundamental puzzles and phenomena. The first remarkable feature is that it can achieve the unification of fundamental couplings, which is a crucial constraint for a UV EFT. This unification provides insights into the underlying symmetry structure and greatly simplifies the work at high energy. Moreover, it naturally incorporates the seesaw mechanism, which naturally explains the smallness of neutrino masses, when the $\mathrm{SO}(10)$ symmetry is broken to a Left-Right Symmetric Model via an intermediate step. Such kinds of GUTs can also be supersymmetrized, addressing the gauge hierarchy problem. But because a low-scale SUSY has not been seen in the LHC, a complete answer to the gauge hierarchy problem is still missing, and motivates us to study the non-SUSY GUTs where low-scale SUSY is not needed. In addition to these theoretical aspects, GUT models have diverse phenomenological implications like, solving the baryogenesis of the universe, and providing the dark matter candidates such as the axions.

Given that the GUT serves as a framework that is more elegant and simpler in the sense of unification, it greatly simplifies our work and enables the incorporation of numerous innovative techniques. For instance, as mentioned earlier, the supersymmetric Left-Right models do not naturally compatible with the unification of gauge couplings. To address this issue in a natural manner, one potential solution is the introduction of a new type of parameter, an extra spatial dimension [133, 134]. Furthermore, studies have demonstrated that the seesaw mechanism naturally emerges in scenarios involving large extra dimensions, providing an explanation for the suppressed neutrino masses [135].

In conclusion, the GUT is highly regarded due to its remarkable ability to provide a unified and elegant framework that addresses crucial questions simultaneously. Its existence in nature is indeed indirectly indicated by the Standard Model. However, as the GUT scale surpasses the Electroweak scale by numerous orders of magnitude, experimental testing becomes challenging. We eagerly await further investigations, particularly in the fields of cosmology and astroparticle physics, to propose more testable predictions and shed light on this promising theory.

5 Summary

The Standard Model of particle physics has proven to be a highly successful mathematical framework for describing the interactions and properties of fundamental particles at the Electroweak scale. However, countless of evidence indicate that it is an incomplete theory, leaving numerous fundamental questions unanswered and phenomena unexplained. For instance, it fails to account for the origins of neutrino masses, the hierarchy between the electroweak scale and the Planck scale, the observed vacuum energy density, and it lacks a description of gravitational interactions which is one of the most fundamental forces in nature. Thus, the Standard Model is widely regarded as an Effective Field Theory (EFT) that effectively captures low-energy phenomena while representing an approximation of a more fundamental theory.

In Chapter 2, we introduce the concept of EFTs and present the Standard Model as an EFT. By employing the language and framework of EFTs, our aim is to organize the thesis in a coherent and self-consistent manner. The EFT approach enables the systematic unification of various Beyond Standard Model (BSM) models and facilitates discussions on their universal properties in a model-independent way.

In Chapter 3, we delve into several of the most fundamental questions in particle physics and provide a comprehensive review of the current understanding regarding their answers. Consequently, it becomes apparent that a BSM framework is necessary to tackle these outstanding questions and offer potential solutions.

In Chapter 4, we focus on several specific BSM models as potential frameworks to address the fundamental questions raised earlier. These models are discussed to illustrate how answers to these questions can be realized in specific ways while maintaining a level of generality. Specifically, we explore the Two Higgs Doublet Models (2HDM) as the simplest extension of the Higgs sector, the Classical Scale Invariant Models (CSIMs) as a dynamical explanation for the origin of scales, the Left-Right Symmetric Models (LRSMs) as a natural realization of the type-I seesaw mechanism, and the SO(10) Grand Unified Theories (GUTs) as a potential framework for unifying all interactions excluding gravity and incorporating all these mechanisms.

Although there have been significant developments, it is still uncertain whether a consistent Effective Field Theory can be found at high energy scales that can simultaneously address the issues related to neutrinos, unification, and the gauge hierarchy problem, and that can be UV-completed consistently including the gravity. However, we maintain an optimistic outlook, as recent developments in new techniques and approaches hold the promise of constructing realistic models that are phenomenologically viable and capable of providing coherent explanations for all these fundamental problems.

Further details on these specific models and their corresponding phenomenological implications can be found in the author's publications I-IV [1, 2, 3, 4].

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Abstract

Aspects of Higgs boson physics: vacuum stability, Yukawa couplings and SO(10) unification

Despite the remarkable success of the Standard Model throughout the past decades, it is acknowledged to be incomplete due to the absence of crucial fundamental elements such as gravity and massive neutrinos. Consequently, there is a pressing need for endeavors that beyond the Standard Model (BSM) to achieve its completion at high energy scales. The quest for a more fundamental theory capable of elucidating theoretical puzzles and perplexing phenomena necessitates the exploration of works in BSM model buildings.

One of the essential stages in constructing BSM models involves identifying the degrees of freedom that exist at high energy scales and "integrating in" them within the Quantum Field Theory framework. This step can be most naturally accomplished through the utilization of Effective Field Theories (EFTs). Consequently, this thesis formulates both the Standard Model and the BSM models within the EFT framework to systematically investigate a comprehensive range of questions. By adopting this coherent approach, we can study the Standard Model and its extensions in a unified manner.

We have introduced the seesaw mechanism to offer a natural explanation for the tiny masses of neutrinos, which is present in various types of UV completions. We also discussed other existing hierarchy problems within the Standard Model, some of which may find resolution in certain toy models. We provided comprehensive insights into a selection of extensively studied BSM models, including the Two Higgs Doublet Models (2HDMs), the Classical Scale Invariant Models (CSIMs), the Left-Right Symmetric Models (LRSMs), and the SO(10) Grand Unified Theories (GUTs). In particular, the Grand Unified Theories offer a unified framework to address significant questions and implement many mechanisms.

In the appendix, the author conducted a detailed analysis of the phenomenological aspects of various BSM models. In particular, the author introduced a classification of the conformal extension of SM in his publication I [1], which is categorized into a universality class at the IR scale, and the low energy testable discriminator is an existence of dilaton coupling to the Higgs boson via a negative portal coupling that is necessary for a dynamically generated electroweak scale. In his publication II [2], the author studied a possible model that realizes the SM Yukawa couplings being radiatively generated as effective operators from a hidden sector at one-loop level. It was also shown that the fermiophobic nature of the Higgs boson, imposed by the symmetries, can ensure the stability of the EW vacuum, regardless of the precise value of the top quark mass. In his publication III and IV [3, 4], the author analyzed the constraint of gauge and Yukawa coupling unification in the context of non-supersymmetric SO(10) models. By the evolution of RG from UV to IR scales, we showed how such UV constraints affect the possible parameter spaces of an EFT at the IR scale. The goal of all of these researches is to find out the possible constraints of BSM models, and to find out the possible observable effects of these constraints. In the end, we hope it advances the realization of the SM as a low-energy effective theory, and also provides some useful insights on the principles of physics behind all of these BSM models.

Kokkuvõte

Higgsi bosoni füüsika aspektid: vaakumi stabiilsus, Yukawa interaktsioonid ja SO(10) ühendteooriad

Kuigi standardmudel on osutunud äärmiselt edukaks osakestefüüsika teooriaks on üldiselt aksepteeritud, et ta on poolik, kuna ta ei suuda seletada nähtusi nagu gravitatsioon või massivsete neutriinode füüsika. Järelikut peab olemas olema füüsika kõrgematel energiatel kui standardmudel seletab. Standardmudeli-ülese füüsika leidmine on kaasaegse osakestefüüsika üks eesmärke.

Üks standardmudeli järgse füüsika eesmärke on leida uue füüsika vabadusastmed, mis integreeritakse välja kui kirjeldatakse madala energia standardmudelit. Seda on võimalik teha kasutades efektiivse väljateooria metoodikat. Selles dissertatsioonis kasutame seda meetodit nii standardmudeli kui ka uue füüsika kirjeldamiseks. Kasutades sama meetodikat on võimalik nii standardmudelit kui ka uut füüsikat kirjeldada samadest printsiipidest lähtuvalt.

Käesolevas dissertatsioonis kasutame kiigemehhanismi kergete neutriinode masside seletamiseks. Samuti tõime sisse hierarhia probleemi lahendused, mis esinevad selles töös uuritavates mudelites. Töö eesmärgiks on seletada erinevaid lähenemisi uue füüsika mudelita valikuks, kaasaarvatud kahe Higgsi dubleti mudelid, klassikaliselt skaalainvariantsete mudelid, parem-vasak sümmeetrilised mudelid ja SO(10) sümmeetrial põhinevad ühendteooriad. Erilist tähelepanu pöörame ühendteooriate mudelitele, mis võimaldavad adresserida kõiki nimetatud probleeme ühes füüsikateoorias.

Käesoleva dissertatsiooni lisades on autor esitanud detailse analüüsi uue füüsika mudelite kohta. Artiklis I on autor esitanud klassifikatsiooni konformsete standardmudeli arenduste kohta, mille eripärad on dilatoni negatiivsed interaktsioonid standardmudeli Higgsi bosoniga selleks, et indutseerida standardmudeli energiaskaala. Artiklis II on autor uurinud võimalust, et kõik standardmudeli Yukawa interaktsioonid on indutseeritud kvantparanduste tasemel tumeda sektori interaktsioonidest. Selle töö tulemusel näidati, et selliste omadustega Higgsi boson on vaba vaakumi stabiilsuse probleemidest sõltumatult top-kvargi massist. Artiklites III ja IV uuris autor kalibratsiooniinteraktsioonide ja Yukwa interaktsioonide ühinemist mittesupersümmeetriliste SO(10) mudelite kontekstis. Uuriti nimetatud interaktsioonide jooksmist madalalt skaalalt kõrgete skaaladeni efektiivsete mudelite kontekstis. Selle töö tulemusena leidsime uusi piiranguid nii madala energia mudelite parameetritele kui ka uusi juhiseid kõrge energa mudelite kirjeldamiseks. Kokkuvõtvalt, käesolev uurimstöö viis meid lähemale standardmudeli mõistmisele suurte ühendteooriate kontekstis.

Appendix 1

I

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Unified interpretation of scalegenesis in conformally extended standard models: a dynamical origin of Higgs portal*

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Abstract: We present a universal interpretation of a class of conformal extended standard models that include Higgs portal interactions as realized in low-energy effective theories. The scale generation mechanism in this class (scalegenesis) arises along the (nearly) conformal/flat direction for breaking scale symmetry, where the electroweak symmetry-breaking structure arises similarly as in the standard model. A dynamical origin for the Higgs portal coupling can provide the discriminator for the low-energy "universality class," to be probed in forthcoming collider experiments.

Keywords: beyond standard model, hidden QCD, conformal symmetry

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1 Introduction

The standard model (SM) of particle physics has been completed by the Higgs discovery [1, 2]. It is, however, still unsatisfactory that the dynamics of electroweak symmetry breaking (EWSB) are not yet understood in detail: in the SM, the sign of the Higgs mass is necessarily assumed to be negative to realize EWSB; so in that sense, the SM only provides an incomplete answer to the origin of EWSB as well as the origin of mass. This issue would be related to the gauge hierarchy problem or fine tuning problem involving the physics bridging the EW and Planck scales through the unique dimensional parameter. Motivated by this longstanding problem, people have to date extensively worked on a possible new dynamics and/or mechanism that could be dormant behind the Higgs sector.

Scale symmetry could be one of the clues to penetrate this issue and is currently significant in the Higgs physics domain, as a possible solution to the gauge hierarchy problem, à la Bardeen [3]: Quadratic divergent corrections to the Higgs mass term, a critical part of the hierarchy problem, are assumed not to provide a physical scaling; hence, they should be removed, such that the Higgs mass term only undergoes logarithmic corrections that are proportional to the bare Higgs mass or SM particle masses coupled to the Higgs. In that case, no massive cancellation or instability of the radiative power corrections associated with the Planck scale is required for the Higgs mass term since no gauge hierarchy problem is present. If the Higgs mass parameter can be turned off at some scale in the renormalization evolution, possibly at the Planck scale, the Higgs mass will not develop up to the logarithmic corrections. This can be done by assuming the realization of scale symmetry at the Planck scale, and the physical Higgs mass then may arise by entering only the logarithmic corrections as the quantum scale anomaly effect.

Nature might have in fact supported the presence of an approximate scale (or conformal) invariance and an *orientation* nearly along a conformal theory: the observed SM-like Higgs is thought to be lying in a nearly conformal direction in the EW broken phase at the vacuum expectation value (VEV) of $v \simeq 246$ GeV, and it acquires mass due to the small quartic coupling breaking

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the scale symmetry (at the tree-level). $\lambda_H = (m_h^2/2v^2) \simeq 1/8 \ll 1$. Thus, a *flat (conformal) direction* can be seen in the SM by taking the limit $\lambda_H \to 0$, where the Higgs potential in the EW broken phase becomes completely flat.

Generically, flat (conformal) directions are observed as stationary hyper-surfaces spanned by aligned scalar VEVs: $v_i = n_i v$, where i runs the number of scalars and v is the average magnitude of the VEVs spontaneously breaking the scale symmetry [4]. Along the flat (conformal) direction, one finds the flat curvature, hence expects the presence of a massless scalar associated with the scale (conformal) symmetry broken by the VEVs, i.e., a "dilaton." The conformal limit in the SM ($\lambda_H \to 0$) can be understood as the simplest case for a generic flat direction argument [5], where the zero determinant of the quartic coupling matrix, reflected as the constraint on the n_i vector, is given as a necessary condition for a flat direction, which is trivially realized by $\lambda_H = 0$ in the SM. In that sense, the SM can indeed be termed a nearly (classically) conformal theory (the nearly conformal SM), and the 125 GeV Higgs can be regarded as a light "pseudo-dilaton" associated with the approximate conformal direction (with a small curvature).

This observation, which is still perturbatively operative even when including quantum corrections that will, however, directly be linked to the instability of the EW vacuum in the SM when λ_H is reduced, even approaches, or goes negative because of the sizable top loop correction [6-35]. Thus, the nearly conformal SM has to be remedied by new dynamics that remains in a conformal direction that includes the SM for low-energy description, maintaining the approximate scale invariance (without power corrections to the Higgs mass term) at the quantum level up to the Planck scale.

The realization of scale invariance at the quantum level (quantum scale-invariance) has been developed from Bardeen's initial proposal, as described above, that is currently not simply an ad hoc assumption, but rather involves two nontrivial dynamical issues: One is to dynamically achieve the initial renormalization condition at the Planck scale ($\Lambda_{\rm pl}$) as the Higgs mass parameter $m_H(\Lambda_{\rm pl}) = 0$, while the other is to eliminate the threshold corrections to m_H from the running of other couplings to realize the nontrivial ultraviolet (UV) fixed points. The former can be realized by a dynamical cancellation at $\Lambda_{\rm pl}$ over the Planckian contribution, as has been argued in [36-40]. The latter is subject to some UV completion for the conformal SM embedded in a nontrivial-UV safety theory (called asymptotic safety), which has recently been explored extensively [41-58]. With these dynamical conditions at hand, no corrections to the Higgs mass term can be generated at any loop order, which indeed gives rise to the quantum-scale invariant SM at the infrared EW scale; thus, no fine-tuning or unnatural Higgs mass parameter will arise.

The key point here is to note that this kind of conformally extended SM (embedded in the asymptotic safety) necessarily includes one SM-singlet scalar, S, which couples to the Higgs doublet via biquadratic forms with a real scalar [59], an extra $U(1)$-charged scalar [60], or a generic complex scalar with or without CP violation [61-63], such as $|H|^2 S^2$, known as the *Higgs portal scenario*. Then, the renormalization group evolution of λ_H necessarily receives positive contributions from such portal couplings and allows the λ_H bounded from zero to attain a stable EW vacuum. In this case, the EW scale can be generated dynamically from the scale-symmetry breaking at the quantum level in the following two ways:

- *perturbative-type* – Coleman-Weinberg (CW) mechanism [4, 64] for weakly coupled massless scalars ([59-63] and also see, e.g., related references that have cited those papers);
- *nonperturbative-type* – dimensional transmutation of a nonperturbatively created scale by a strongly coupled hidden sector [65-73].

Any scenario of this class can therefore be called a *Higgs-portal scalegenesis*.

A common feature that all perturbative-type Higgs-portal scalegenesis mechanisms share is the presence of a flat direction in the tree-level scalar potential. It is necessarily present if the determinant of the scalar quartic-coupling matrix vanishes [5]. It ensures that all scalars acquire their VEVs simultaneously by quantum corrections, i.e. the CW mechanism, and thus, an inevitably light dilaton-like scalar state emerges as a pseudo-Nambu-Goldstone (NG) boson associated with the anomalous scale symmetry [64] (also called the scalon in the original Gildener-Weinberg approach [4]).

Even in the nonperturbative-type Higgs-portal scenarios, similar flat directions can be observed in terms of low-energy effective scalar potential, where the Higgs portal interactions are established between the SM-like Higgs and composite scalars generated by the underlying strong dynamics. Nevertheless, the scale-symmetry breaking that is nonperturbatively generated appears built-in at the tree-level in low-energy effective scalar theory.

Note that Higgs portal scalegenesis posseses universal experimental evidences: the predicted dilaton, arising as a singlet scalar fluctuation mode from the portal field S, will couple to SM particles due to mixing with the SM-like Higgs boson h through the Higgs portal interaction $|H|^2|S|^2 = v_S v h s + \cdots$ with the vacuum expectation values of S and H, v_S and v. The size of the dilaton coupling to the SM particles as well as the 125 GeV Higgs couplings to them are then universally controlled by the mixing angle θ, respectively, in which the latter has been constrained severely by the Higgs coupling measurement experiments as $|\sin\theta| \lesssim 0.3$ [74]. When the dilaton mass is of the order of the EW scale, or higher, it is mainly produced by a gluon fusion process and decays to the EW-dibosons WW and ZZ at hadron collider, like the LHC. The possible excess events, which are $\propto \sin^2\theta\times$ SM-like Higgs events at the invariant mass around the EW scale in those diboson channels will thus be a generic prediction of the Higgs portal scenario.

In addition to the diboson signatures, the Higgs potential structure, including the higher order terms in the h field, such as the cubic h^3 term, will be modified by the SM prediction that is parametrized by functions of θ, with the ratio of v_s/v, and will be subject to the light dilaton resonance coupled to the diHiggs in the trilinear Higgs amplitude through the h-s conversion process, such as $h^{(*)} \to s^{(*)} \to hh$. Note also that the dilaton resonance is generically narrow due to the small coupling strength to SM particles set by the phenomenologically small $\sin\theta$. Thus, the diHiggs signatures will also be a characteristic indicator of this scenario, as has been discussed by means of a specific Higgs portal scalegenesis [73].

Those signals can be predicted no matter what kind of method is applied to realize the EWSB via Higgs portal interactions with a sufficiently light and narrow scalar (at some decoupling limit for heavier particles, if any); hence, they are universal predictions expected in an energy range that is within the reach of collider experiments [73]. Without having been analyzed, it is obvious that other models regarded as the Higgs portal scalegenesis [4, 5, 59-72] can generically predict similar collider signatures.

Thus, this kind of conformal extension of the SM, namely, the Higgs-portal scalegenesis, is thought to form a *universality class*, in the universal low-energy effective theory and related phenomenological sense.

Even in such a Higgs-portal scalegenesis, actually, the main focus on the realization of the EWSB is simply going to be moved from the origin of the Higgs mass itself to the origin of the portal coupling because the latter has to be "negative." Even working in the CW mechanism [64], one requires the portal coupling to be "negative" by hand; otherwise, none of the models can realize the EWSB (see, e.g., [75], [76, 77][1]) and references therein): The CW mechanism cannot simply be applied to generate the EW scale since the required parameters will not be compatible with the observed values.

This implies the requirement for a dynamical origin for the generation of both the scale and Higgs portal couplings, including the negative sign to propose the origin of mass for scenario completion. Furthermore, it should offer a definite phenomenological consequence distinguishable in the sense of a unified category for the Higgs-portal scalegenesis.

In this paper, we demonstrate a universal interpretation of models that lead to the Higgs-portal scalegenesis as a low-energy effective theory, which arises along a conformal/flat direction, with an EWSB structure similar to that encoded by the SM. This constructs the universality class in the (nearly) conformal/flat direction including the SM, without loss of generality as will be seen below. We then present a discriminator for the universality class, which must be closely related to the very origin of the negative Higgs mass term/origin of mass: *A dynamical origin of the Higgs portal*.

2 A dynamical origin of Higgs portal: a generic low-energy description

First, we demonstrate that a conventional scale-invariant Higgs portal scenario emerges in a decoupling limit for the scale-invariant realization of two-Higgs-doublet models with a light dilaton introduced. In addition, we observe that in this class of models, a softly broken $Z_2/U(1)_A$ for the Higgs sector plays a crucial role to realize the negative Higgs-portal coupling between the SM-like Higgs and the light dilaton.

Having in mind a scale-invariant realization of a two-Higgs-doublet model with a light dilaton (χ), one finds potential terms such as

$$V \ni \chi^2\left[c_0|H_1|^2 + c_1(H_1^\dagger H_2 + \text{H.c.}) + c_2|H_2|^2\right], \tag{1}$$

where $c_{0,1,2}$ are arbitrary dimensionless coefficients and $H_{1,2}$ are the Higgs doublets. Manifestly, to perceive a symmetry structure of interest, one may introduce a two-by-two Higgs matrix form, $\Sigma = (H_1, H_2^c)$ (with H_2^c being the charge conjugated field of H_2), to rewrite the terms as

$$V \ni \chi^2\left[\left(\frac{c_0+c_2}{2}\right)\text{tr}\left[\Sigma^\dagger\Sigma\right] + c_1(\det\Sigma + \text{H.c.}) + \left(\frac{c_0-c_2}{2}\right)\text{tr}\left[\Sigma^\dagger\Sigma\sigma^3\right]\right], \tag{2}$$

1) It has been discussed in that without a bare Higgs portal coupling, a mixing effect between the hypercharge gauge and a newly introduced gauge ($B-L$) bosons can radiatively generate the portal coupling between the $B-L$ Higgs (regarded as a dilaton in that case) and the SM-like Higgs at the two-loop level. However, because of the higher loop-induced coupling, its size is quite small ($\sim O(10^{-3})$), which is required to realize the $B-L$ breaking at TeV scale, hence the mixing strength with the SM-like Higgs gets small enough as well, so that the predicted light dilaton couplings to diEW and diHiggs bosons will be negligibly smaller than other models having the sizable (negative) Higgs portal coupling (by hand) at tree-level. To this respect, we may exclude this kind of radiative generation scenarios from the universality class that we presently work on.

where σ^3 is the third Pauli matrix. It is facile to see that the potential is structured on a global chiral $U(2)_L \times U(2)_R$ symmetry for the two Higgs flavors, where the $SU(2)_R$ component is in part explicitly broken down (to the subgroup corresponding to the third component of $SU(2)$) by the third term and the $U(1)_A$ component (that is usually called a soft-Z_2 breaking term in the context of two-Higgs-doublet models) is broken by the second c_1 term. The same chiral two-Higgs sector structure (without the scale invariance) has been discussed in [78, 79].

At this point, the dimensionless couplings $c_{0,1,2}$ are simply assumed to be real and positive for them to have a conformal/flat direction. In that case the conformal/flat direction for both the scale and EW breaking VEVs can be achieved, where the direction for the EW scale is somewhat deformed due to the mass mixing by c_1 as

$$\tilde{v}_2 \equiv v_2 + (c_1/c_2)v_1 = 0. \tag{3}$$

Note that this deformation is nothing but a base transformation: $v_{1,2} \to \tilde{v}_1 (= v_1), \tilde{v}_2$ and can generally and smoothly be connected to the SM limit with v_1 only.

Now, assume the maximal isospin breaking for the two-Higgs doublets, where $c_0/c_2 \to 0$, and the soft-enough $U(1)_A/Z_2$ is broken, by taking $c_1/c_2 \ll 1$. Then, one may integrate the heavy Higgs doublet H_2 to get the solution for the equation of motion, $H_2 \approx -(c_1/c_2)H_1$ [1]. Plugging this solution back into the potential, one finds

$$V \approx -\left(\frac{c_1^2}{c_2}\right)\chi^2 |H_1|^2, \tag{4}$$

which is nothing but a desired Higgs portal model, where the portal coupling $\lambda_{H\chi} = -c_1^2/c_2$ has been dynamically induced *including the minus sign* without any assumptions and is reflected by the attractive interaction of the scalar-exchange induced potential in the quantum mechanical sense. One should also realize that the small portal coupling can actually be rephrased by the small size of the soft-$Z_2/U(1)_A$ breaking for the underlying two-Higgs doublet model. Note also that the conformal/flat direction oriented in the original two-Higgs doublet model is smoothly reduced back to that in the Higgs portal model, as it should be.

This generation mechanism is nothing less than the bosonic seesaw [80-90], which one can readily check if the scalar mass matrix assumes the seesaw form, namely, its determinant is negative under the aforesaid assumption. Note also that the original conformal/flat direction $\tilde{v}_2 \equiv v_2 + (c_1/c_2)v_1 = 0$ can also be rephrased in terms of the bosonic seesaw relation: when the mixing is reduced (i.e. $c_1/c_2 \ll 1$), the heavy Higgs partner arises via the bosonic seesaw as approximately $\tilde{H}_2 \simeq H_2 + (c_1/c_2)H_1$, so the conformal/flat direction has been realized due to the presence of an approximate inert H_2. Thus, the bosonic seesaw provides the essential source for the Higgs-portal scalegenesis to predict the universal low-energy new-physics signatures such as significant deviations for Higgs cubic-coupling measurements compared with the SM prediction, and for the light dilaton signatures in di-Higgs, diEW bosons, as aforementioned.

3 The very origin of the Higgs portal: a UV completion

One can further observe that a hidden strong gauge dynamics – often called hidden QCD (hQCD) or hypercolor [85-87, 90] – provides the dynamical origin for the softly-broken Z_2 or $U(1)_A$ symmetry and alignment to the flat direction that are supplied as ad hoc assumptions in the framework of the scale-invariant realization of the two-Higgs doublet model, as executed immediately above. Indeed, a class of the hQCD as explored in [85-87, 90] can dynamically generate a composite dilaton (arising generically as an admixture of fluctuating modes for the hQCD fermion bilinear, like conventional sigma mesons in QCD and gluon condensates such as glueballs. Even in a naive scale-up version of QCD with the small number of flavors as applied in the literature [86, 87, 90], it has recently been argued [91] that there might exist an infrared conformality, supporting the QCD dilaton to be light enough, compared to the dynamical intrinsic scale. Even if it is not the case, the hQCD flavor structure can straightforwardly be extended from the three flavor to many flavors, say, eight's [92, 93], with keeping the bosonic seesaw mechanism, so that a manifest light composite dilaton can be generated by the nearly conformal dynamics, as has recently been discussed [94]).

Consider an hQCD with three colors and three flavors, as a minimal model to realize the bosonic seesaw as discussed in [85-87], where the hQCD fermions form the $SU(3)$-flavor triplets, $F_{L,R} = (\Psi_i, \psi)^T_{L,R}$, having vectorlike charges with respect to the SM gauges like $\Psi_{i(i=1,2)} \sim (N, 1, 2, 1/2)$ and $\psi \sim (N, 1, 1, 0)$for the hQCD color group $SU(N=3)$ and $SU(3)_c \times SU(2)_W \times U(1)_Y$. Thus, this hQCD possesses the (approximate) "chiral" $U(3)_{F_L} \times U(3)_{F_R}$ symmetry as well as classical-scale invariance, of which the former is explicitly broken by the vectorlike SM gauges. Besides, we shall introduce the following terms, which are SM gauge-invariant but explicitly break the chiral symmetry: $\mathcal{L}_{y_H} = -y_H \bar{F}_L \cdot \begin{pmatrix} 0_{2\times 2} & H \\ H^\dagger & 0 \end{pmatrix} \cdot F_R + \text{H.c.}$. Note that in addition to this y_H-Yukawa term,

1) The H_2 mass term takes a χ field-dependent form like $m^2_{H_2}(\chi) = c_2\chi^2$, with $c_2 > 0$ and $O(1)$ as assumed in the text. This $mH_2(\chi)$ becomes the H_2 mass after the χ develops the VEV $\langle\chi\rangle$, which is by construction lager than the EW scale, or the lightest Higgs mass identified as the 125 GeV Higgs's. Thereby, one can safely integrate out the heavy H_2 by taking into account its a priori heaviness.

the $U(1)_{F_A}$ symmetry is explicitly broken also by the anomaly coupled to hQCD gluons that can, however, be transferred to this y_H-Yukawa term by the $U(1)_{F_A}$ rotation, so that it fully controls the size of the $U(1)_{F_A}$ symmetry breaking.

The remaining (approximate) chiral $SU(3)_{F_L}\times SU(3)_{F_R}(\times U(1)_{F_V})$ symmetry is broken down by the chiral condensate invariant under the SM gauge symmetry, $\langle\bar{F}F\rangle = \langle\bar{\Psi}_i\Psi_i\rangle = \langle\bar{\psi}\psi\rangle \neq 0$, down to the diagonal subgroup $SU(3)_{F_V}(\times U(1)_{F_V})$ at the scale $\Lambda_{\rm hQCD}$, similar to the ordinary QCD. This spontaneous chiral breaking thus leads to the low-energy spectrum with the eight NG bosons.

The low-energy description for $\mathcal{L}_{y_H}$, below the scale $\Lambda_{\rm hQCD}$, can be as follows:

$$\chi^2\left[c_1(H_1^\dagger\Theta+{\rm H.c.})+c_2|\Theta|^2\right] = \chi^2\left\{c_1({\rm det}\Sigma+{\rm H.c.})+c_2{\rm tr}\left[\Sigma^\dagger\Sigma\left(\frac{1-\sigma^3}{2}\right)\right]\right\}, \tag{5}$$

where $\Sigma=(H,\Theta^c)$ with $\Theta\sim\bar{\psi}_R\Psi_L$ is a composite Higgs doublet (Note that when one works on hQCD theory with hQCD fermions in higher dimensional representations, like a real or a pseudo-real representation, the seesaw partner Θ would be a composite Nambu-Goldstone Higgs-doublet, as employed in [89][1)]); $c_1\simeq y_H$ up to some renormalization effect scales down to $\Lambda_{\rm hQCD}$; and c_2 has been generated by the chiral condensate $\langle\bar{F}F\rangle$ scaled by the VEV of the composite hQCD dilaton χ. This is nothing but the form of a scale-invariant two-Higgs doublet model as discussed above, so the bosonic seesaw should work, to bring the theory back to the Higgs portal model as the low-energy description. It is important to note also that the approximate inertness of the second Higgs doublet that is necessary for the conformal/flat direction is now manifest because of the robust Vafa-Witten theorem [95].

This ensures the zero VEV for the non-vectorlike condensates such as $\Theta\sim\bar{\psi}_R\Psi_L$, in this vectorlike hQCD and the positiveness of the c_2 (i.e. the positive mass square of the Θ), as long as the chiral manifold describing the low-energy hQCD is stable.

4 Discriminating the universality class

One may identify $U(1)_{F_A}$ in the hQCD as $U(1)_A$ for the previous two-Higgs sector. Then one can say that the ad hoc assumption (the soft-$Z_2/U(1)_A$ broken by taking $c_1/c_2\ll 1$ and maximal isospin breaking for the Higgs sector: $c_0=0$) is naturally realized by the hQCD in which the bosonic seesaw mechanism is built and where the smallness of c_1 can be understood by the presence of light hQCD pions. Although the y_H gives a tachyonic mass to the lightest neutral hQCD pion, one can resolve it by introducing extra singlet pseudoscalar as discussed in [86, 87, 90] without conflicting any discussions in the present paper. Thus, the origin of the EWSB derived from the negative portal coupling is tied to the explicit-hidden chiral symmetry-breaking (and/or $U(1)_A$ breaking) in the hQCD sector.

The small y_H coupling can lead to custodial symmetry breaking, and oblique corrections, such as the T-parameter constraint, must be discussed due to the corrections from the EW-charged hQCD pions. Such EW charged pions also significantly contribute to the 125 GeV Higgs decaying to diphotons in addition to overall suppression by the mixing angle and the light dilaton that is universally present in Higgs-portal scalegenesis. We have confirmed that parameter spaces are sufficiently allowed under those constraints and this will be reported in detail in another publication. For instance, when we take $\Lambda_{\rm hQCD}=1(2)$ TeV, the EW-charged hQCD pion mass is bound to be $\gtrsim 450(700)$ GeV for the Higgs-dilaton mixing strength $\sin^2\theta=0.1$, and $\gtrsim 400(600)$ GeV for $\sin^2\theta=0.05$, along with the soft-$Z_2/U(1)_A$ breaking coupling $y_H\lesssim 0.1$, which yields the Higgs portal coupling $\lambda_{H\chi}\lesssim 0.1$, and the hQCD dilaton having the mass around 300 GeV as in [73]. Such light pions can be produced at the LHC by EW interactions (vector boson fusions) via the chiral anomaly in the hQCD for the predicted production cross sections to be quite small (roughly at most $\sim 10^{-1}$ fb at 13 TeV) due to the loop factor suppression that is compared with the currently stringent upper bound $\sim 10^2$ fb at the corresponding mass range [96], and it may be hard to detect directly, even in the high-luminosity epoch (for similar EW-charged pion signals, e.g., see [97]). Note even in that case that the presently proposed hQCD can be probed by correlated deviations for the 125 GeV Higgs to decay to diphotons by hQCD pion loops and the diboson channels including diHiggs and di-EW

1) In that case, one would have $c_2=0$ at the $\Lambda_{\rm hQCD}$ scale. Going down to lower scales, however, EW radiative corrections would generate the Θ mass on the order of $O(g_W/(4\pi)\Lambda_{\rm hQCD})$, where $\Lambda_{\rm hQCD}=O(1)$ TeV, as will be seen from the phenomenological bounds later. Hence, this Θ mass scale should be of $O(100)$ GeV, less than the EW scale and smaller than a composite dilaton (χ) mass. Therefore, one cannot integrate out the Θ, instead, the dilaton χ will be integrated out such that the theory will be away from the conformal direction. In other words, this hQCD model does not blelong to the universality class in which the Higgs portal between the SM-like Higgs H_1 and a SM-singlet dilaton χ is necessarily present at the low-energy theory. This is the case for a minimal setup only with the HC theory and the y_H-like Yukawa term as well as the SM gauge interactions. Going beyond the minimal setup could make the theory come back on the track of the conformal direction.

bosons, as discussed in [73], that are definitely characteristic of the universality class of the Higgs-portal scale-genesis.

Thus, the light hQCD pions will be definite discriminators for the universality class of the Higgs portal scale-genesis. If both a light dilaton and hQCD pions (the masses of which are expected to be around/below TeV scale) are detected at forthcoming collider experiments it would be the hQCD that indicates the very origin of the Higgs portal coupling, and hence the very origin of the Higgs mass term. In addition, the y_H term which breaks chiral symmetry explicitly potentially induces a significant deviation of the T parameter [98, 99]. Thus, the EW precision data also provide some hints to explore the models of this universality class.

5 Conclusion

In conclusion, the universality class that is presently proposed can be depicted as in Fig. 1. The universality class and its disentangled dynamical origin would provide a novel guideline along the conformal extension of the SM and possibly lead to resolving the longstanding inquiry into the gauge hierarchy (fine tuning) problem. This would also provide a clear understanding of the hidden new physics in the search for the dynamical origin of the Higgs sector and hence the origin of mass that can be tested in upcoming collider experiments.

More detailed studies regarding distinct collider signatures for the two-Higgs-doublet model type and hQCD type are worth performing and will be pursued elsewhere. In addition, the thermal histories as well as possible gravitational wave signals for this universality class could be discriminated, which is a worthy future research direction.

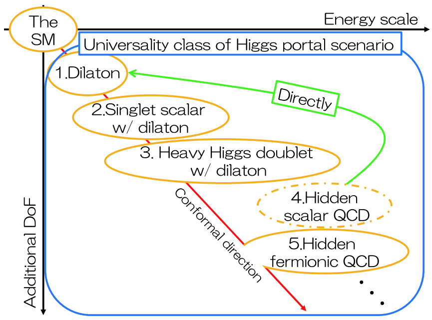

Fig. 1. (color online) A schematic picture of the universality class for Higgs portal scenarios. All the extensions correspond to 1). [4, 64], 2). [59-63], 3). [78, 79, 84], 4). [65, 67, 70-73], and 5). [66, 82, 83, 85-90], respectively. All categorized models predict the same low-energy phenomenology related to the Higgs physics, as do those noted in the main text. A dynamical origin for this universality class can be encoded in a type of fermionic hQCD (#5 in the figure) with distinct light pion signatures as well as the universal Higgs-related ones.

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Appendix 2

II

Emidio Gabrielli, Luca Marzola, Kristjan Müürsepp, and Ruiwen Ouyang. Vacuum stability with radiative Yukawa couplings. *JHEP*, 01:142, 2022

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Vacuum stability with radiative Yukawa couplings

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Abstract: We explore the electroweak vacuum stability in the framework of a recently proposed paradigm for the origin of Yukawa couplings. These arise as low energy effective couplings radiatively generated by portal interactions with a hidden, or dark, sector at the one-loop level. Possible tree-level Yukawa couplings are forbidden by a new underlying symmetry, assumed to be spontaneously broken by the vacuum expectation value of a new scalar field above the electroweak scale. As a consequence, the top Yukawa interaction ceases to behave as a local operator at energies above the new sector scale and, therefore, cannot contribute to the running of the quartic Higgs coupling at higher energies. By studying two complementary scenarios, we explicitly show that the framework can achieve the stability of the electroweak vacuum without particular tuning of parameters. The proposed mechanism requires the existence of a dark sector and new portal messenger scalar interactions that, connecting the Standard Model to the dark sector fields, could be tested at the LHC and future collider experiments.

Keywords: Beyond Standard Model, Higgs Physics

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Contents

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1 Introduction

The discovery of the Higgs boson by the ATLAS and CMS collaborations in proton-proton collisions data with centre-of-mass energies of 7 TeV and 8 TeV [1, 2] has marked a milestone in our understanding of the electroweak symmetry breaking (EWSB). To-date, the tested properties of this particle are all in good agreement with the Standard Model (SM) predictions [3, 4]. In particular, the recent observations of the Higgs boson decay modes into bottom quark [5–7] and tau-lepton pairs [8, 9] are consistent with the SM Yukawa coupling strength and, therefore, support the existence of the corresponding interactions in Nature. Likewise, the detection of the Higgs boson production in association with top anti-top quark pairs [10, 11] matches the SM expectations and thus provides a direct confirmation of the existence of top quark Yukawa coupling. Given the Higgs boson vacuum expectation value (VEV) inferred from weak interactions, $v = 246\,\mathrm{GeV}$, these results inevitably strengthen our confidence in the SM and in the Yukawa coupling origin of elementary fermion masses.

The precise measurement of the Higgs boson mass, $m_H = 125.25 \pm 0.17$ [12], has also disclosed the expected value of the quartic Higgs boson coupling at the electroweak (EW) scale, which is about $\lambda_H(m_t) \sim 0.126$. This inference allows us to speculate on the stability of the EW vacuum by computing the relevant quantum corrections to the Higgs scalar potential [13–18]. For large field values, $H \gg v$, the renormalization group (RG) improved effective potential is well approximated by the tree-level form with a running coupling

$$V_{\rm eff}^{\rm tree}(H) \simeq \frac{\lambda_H(\mu_H)}{4} H^4 \,, \tag{1.1}$$

where the scale $\mu_H \sim H$ is of the order of the Higgs field value. Therefore, the problem of vacuum stability can be studied by simply analyzing the RG evolution of λ_H and, at least in principle, solved by requiring that $\lambda_H(\mu) > 0$ up to energies close to the Planck scale.

Impressive efforts have been dedicated to the computation of the higher-order corrections to the RG flow controlling the evolution of the Higgs quartic coupling [19–25]. The resulting relation that at low energy connects $\lambda_H(\mu)$ to the Fermi constant G_F is

$$\lambda_H(\mu) = \frac{G_F m_H^2}{\sqrt{2}} + \Delta\lambda_H(\mu) \,, \tag{1.2}$$

where $\Delta\lambda_H(\mu)$ contains finite threshold corrections that arise beyond the tree-level. These corrections are quite large and are the main source of uncertainty in the determination of the value of λ_H on the considered energy span. Recently, the determination of the next-to-next-to-leading-order (NNLO) corrections to $\Delta\lambda_H(\mu)$, including the complete two-loop Yukawa-QCD contributions [24, 25], has allowed a reduction of the error in the determination of the Higgs mass of about $\pm 0.7\,$GeV [24].

Given the current experimental values of the SM parameters that enter the RG evolution of the Higgs quartic coupling, these analyses reveal that λ_H becomes negative well below the Planck scale and that the EW vacuum is thus metastable. In particular, taking into account the theoretical and experimental uncertainties, ensuring the absolute stability of the EW vacuum up to the Planck scale requires $m_H > (129.4 \pm 1.8)\,$GeV. Equivalently, the SM vacuum stability is excluded at 2σ for $m_H < 126\,$GeV [24]. The result still critically depends on the value of the top quark mass, which is the dominant source of uncertainty in the determination of the Higgs mass, and affects the RG equations (RGE) of the Higgs quartic coupling through the negative contribution induced by the related Yukawa coupling.

A possible way to ensure the stability of SM vacuum is to assume that all Yukawa couplings, including that of the top quark, are effective low energy parameters. Importantly, these are to be radiatively generated in absence of direct couplings of the Higgs boson to any fermion field, that is, by requiring a fundamentally *fermiophobic* Higgs boson. In fact, if the SM Yukawa couplings were to be generated through new fundamental interactions of the Higgs boson with fermion fields, the vacuum instability problem could still occur due to the corresponding — and potentially sizeable — new fermion-loop contributions. An explicit example is provided by models based on the universal seesaw mechanism [26–31], which generate the SM Yukawa couplings through fundamental interaction of the Higgs boson with new vector-like fermions that might still drive the scalar potential to negative values.

Provided that new physics (NP) is below the SM instability scale, the fermiophobic Higgs condition can instead guarantee the stability of the EW vacuum. The underlying idea is straightforward: radiatively generated Yukawa operators cease to be local operators above the NP scale where they are generated. Then, due to the fermiophobic nature of the Higgs boson, the RGE of λ_H at higher energies receives only the positive contributions of the *bosonic* degrees of freedom which thus enforce the vacuum stability. Clearly, in order to ensure that the Higgs quartic coupling remain positive throughout its complete RG evolution, the NP scale where the SM Yukawa operators are effectively generated must be below the SM instability scale. The latter therefore provides a theoretical upper bound on the NP scale required for the validity of the proposed solution.

The fermiophobic Higgs condition can be naturally implemented by extending the theory to any local or global symmetry that forbids all SM Yukawa operators. For instance, the mechanism is straightforwardly embedded in the scenarios of refs. [32–34] which were originally designed to solve the flavor hierarchy problem. In more detail, the tree-level Yukawa operators are forbidden by a new symmetry S: a discrete symmetry in ref. [32] and a local $SU(2)_R$ extension of the SM gauge group in refs. [33, 34]. In either case, the new symmetry is spontaneously broken by the vacuum expectation value v_S of a dedicated scalar field which, thereby, allows for the emergence of the SM Yukawa operators.

The framework predicts the existence of massive vector-like dark fermion fields — heavy SM gauge-singlet replicas of the SM fermions — and a set of scalar messenger fields that mediate the interactions between the SM and the dark sector. The messenger fields carry the same quantum numbers of squarks and sleptons of known supersymmetric models, in addition to a new $U(1)_D$ gauge charge under which the dark sector fields are also charged. The chiral symmetry breaking necessary for the Yukawa coupling generation is provided by the dark-fermion masses and communicated, at the 1-loop level, to the SM fields by the messengers. After the spontaneous breaking of the symmetry S, the emerging Yukawa couplings are then necessarily proportional to the involved dark-fermion mass. A non-perturbative dynamics in the dark-sector, related to the $U(1)_D$ gauge symmetry, is responsible for the exponential spread of the dark fermion masses and, therefore, for the observed hierarchy of the SM Yukawa couplings [35]. The same framework also allows for the radiative origin of flavor mixing, modelled in the Cabibbo-Kobayashi-Maskawa matrix [34].

Adopting the simplest model delineated by the framework [32], in the present paper we explore the EW vacuum stability in light of the fermiophobic Higgs mechanism. In particular, we compute the 1-loop contributions to the β-function of the Higgs quartic coupling induced by the new degrees of freedom and analyse the conditions required for the positivity of this parameter on the whole of its RG evolution.

The paper is organized as follows. In the next section we discuss the theoretical framework at the basis of the construction, detailing the relevant interactions of the new fields. In section 3 we review how the SM Yukawa coupling are generated from the interactions of messengers and dark fermions, whereas in section 4 we compute the 1-loop β-functions of the model. In section 5 we study the vacuum stability of the theory by analyzing the RG evolution of the λ_H coupling from the EW scale up to the ultraviolet (UV) cutoff of the theory. We conclude with section 6, where we summarize our findings.

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Field	**Spin**	$\mathbb{Z}_2$ charge	$\mathrm{U}(\mathbf{1})_{\mathbf{D}}$ charge	$\mathrm{U}(\mathbf{1})_{\mathbf{Y}}$ charge	$\mathrm{SU}(\mathbf{2})_{\mathbf{L}}$ repr.	$\mathrm{SU}(\mathbf{3})_{\mathbf{c}}$ repr.
			Extended SM sector:			
q_L^i	1/2	1	0	1/6	2	3
U_R^i	1/2	1	0	2/3	1	3
D_R^i	1/2	1	0	-1/3	1	3
L_L^i	1/2	1	0	-1/2	2	1
E_R^i	1/2	1	0	-1	1	1
ν_R^i	1/2	1	0	0	1	1
$\hat{H}$	0	-1	0	1/2	2	1
H_S	0	-1	0	0	1	1
			Mediator sector:			
$\hat{S}_L^{U_i}$	0	1	$-e_D^{U_i}$	1/6	2	3
$\hat{S}_L^{D_i}$	0	1	$-e_D^{D_i}$	1/6	2	3
$S_R^{U_i}$	0	1	$-e_D^{U_i}$	2/3	1	3
$S_R^{D_i}$	0	1	$-e_D^{D_i}$	-1/3	1	3
$\hat{S}_L^{N_i}$	0	1	$-e_D^{N_i}$	-1/2	2	1
$\hat{S}_L^{E_i}$	0	1	$-e_D^{E_i}$	-1/2	2	1
$S_R^{N_i}$	0	1	$-e_D^{N_i}$	0	1	1
$S_R^{E_i}$	0	1	$-e_D^{E_i}$	-1	1	1
			Dark sector:			
Q^{U_i}	1/2	1	$e_D^{U_i}$	0	1	1
Q^{D_i}	1/2	1	$e_D^{D_i}$	0	1	1
Q^{E_i}	1/2	1	$e_D^{E_i}$	0	1	1
Q^{N_i}	1/2	1	$e_D^{N_i}$	0	1	1

Table 1. Particle content of the model and gauge assignments. The index $i = 1, 2, 3$ runs over the SM generations. The electric charge of each field is given by $Q = I_3 + Y$, where Y is the hypercharge and I_3 is the eigenvalue of the third weak isospin generator.

2 Theoretical framework

We summarize here the main features of the model at the basis of the present work, using the original formulation of ref. [32] for the sake of simplicity. Because the discussion of the vacuum stability issue does not significantly depend on the nature of the symmetry used to forbid the existence of tree-level Higgs Yukawa couplings, our results can be applied also to the framework based on the Left-Right (LR) gauge symmetry presented in ref. [33].

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Concretely, we enlarge the SM gauge group by a $\mathbb{Z}_2$ discrete symmetry under which the fields transform as specified in table 1. The full Lagrangian is then given by

$$\mathcal{L} = \mathcal{L}_{\rm SM}(Y_f = 0) + \mathcal{L}_{\rm MS}(q) + \mathcal{L}_{\rm MS}(L) + \mathcal{L}_{\rm Dark} - V_{H_S} - V_{\rm MS}\,, \tag{2.1}$$

where $\mathcal{L}_{\rm SM}(Y_f = 0)$ represent the SM Lagrangian without the usual Yukawa interactions, which necessarily vanish for the considered $\mathbb{Z}_2$ assignments. The remaining terms host portal interactions and the dark sector fields, in particular $\mathcal{L}_{\rm MS}(q)$ and $\mathcal{L}_{\rm MS}(L)$ contain the Lagrangian for the messenger sector, including the portal interactions with quarks and leptons, respectively. The term $\mathcal{L}_{\rm Dark}$, instead, contains a set of massive Dirac fermions, the dark fermions, singlet under the SM gauge group but charged under a vectorial $\mathrm{U}(1)_D$ dark gauge theory. Next, $V(H_S)$ collects the terms of the scalar potential that involve the scalar field H_S, responsible for the spontaneous breaking of the new Z_2 symmetry. Explicitly

$$V_{H_S} = \lambda_{H_S}\frac{H_S^4}{4} - \mu_S^2\frac{H_S^2}{2} + \frac{1}{2}\lambda_{HH_S}H_S^2\hat{H}^\dagger\hat{H}\,, \tag{2.2}$$

where $\hat{H}$ stands for the SM Higgs doublet. As both $\hat{H}$ and H_S develop non-vanishing VEVs, the last term in eq. (2.2) results in a tree-level mass mixing between the two scalars. In fact, focusing for the moment on the two Higgs fields only, the minimization conditions for the corresponding scalar potential set

$$\mu_S^2 = v^2\lambda_{HH_S} + v_S^2\lambda_{H_S} \tag{2.3}$$
$$\mu_H^2 = v^2\lambda_H + v_S^2\lambda_{HH_S} \tag{2.4}$$

and the matrix of squared masses of the CP-even bosons thus is:

$$\mathrm{M}^2 = \begin{pmatrix} 2\,v^2\,\lambda_H & 2v\,v_S\,\lambda_{HH_S} \\ 2v\,v_S\,\lambda_{HH_S} & 2\,v_S^2\,\lambda_{H_S} \end{pmatrix}\,. \tag{2.5}$$

The mass eigenstates are determined upon a rotation of the original scalar fields by an angle of

$$\tan 2\theta = \frac{2\,v\,v_S\,\lambda_{HH_S}}{v^2\,\lambda_H - v_S^2\,\lambda_{H_S}}\,. \tag{2.6}$$

Because in the rest of the paper we consider scenarios where $v_S \gg v$, we expand the above relation in powers of v/v_S, obtaining at the first order that

$$\tan 2\theta \approx -2\frac{\lambda_{HH_S}}{\lambda_{H_S}}\frac{v}{v_S}\,. \tag{2.7}$$

As we can see, the effects of mass mixing in the considered limit are suppressed by the large hierarchy between the EW and NP scales and thus can be safely neglected. Still, because the λ_{HH_S} coupling receives important radiative contributions form the interactions contained in the $\mathcal{L}_{\rm MS}(q)$ and $\mathcal{L}_{\rm MS}(L)$ terms of eq. (2.1), we retain the full RG evolution of this coupling in our analyses.

Finally, the last contribution in eq. (2.1), $V_{\rm MS}$, contains the full scalar potential for the messenger fields and is separately discussed in the following.

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The portal interactions in $\mathcal{L}_{MS}(q)$, responsible for the radiative generation of Yukawa couplings, are shaped by the SM quantum numbers and transmit the chiral symmetry breaking sourced by the dark fermion masses to quarks and leptons [32].

In more detail, for the quark sector we have

$$\mathcal{L}_{MS}(q) = \mathcal{L}^0_{MS}(q) + \mathcal{L}^I_{MS}(q)\,, \tag{2.8}$$

where $\mathcal{L}^0_{MS}(q)$ contains the kinetic terms, mass parameters and gauge interactions of the messenger fields and $\mathcal{L}^I_{MS}(q)$ specifies the portal interactions with the SM quarks:

$$\begin{aligned}\mathcal{L}^I_{MS}(q) = {} & g^U_L \sum_{i=1}^{3} \left[\bar{q}^i_L Q^{U_i}_R\right] \hat{S}^{U_i}_L + g^D_L \sum_{i=1}^{3} \left[\bar{q}^i_L Q^{D_i}_R\right] \hat{S}^{D_i}_L \\ & + g^U_R \sum_{i=1}^{3} \left[\bar{U}^i_R Q^{U_i}_L\right] S^{U_i}_R + g^D_R \sum_{i=1}^{3} \left[\bar{D}^i_R Q^{D_i}_L\right] S^{D_i}_R \\ & + \lambda^U_S \sum_{i=1}^{3} \tilde{H}^\dagger \hat{S}^{U_i}_L S^{U_i\dagger}_R H_S + \lambda^D_S \sum_{i=1}^{3} \hat{H}^\dagger \hat{S}^{D_i}_L S^{D_i\dagger}_R H_S \; + \; H.c.\,.\end{aligned} \tag{2.9}$$

In the equation above, we have left all the color and $\mathrm{SU}(2)_L$ contractions understood and we have indicated the chiral projections of the dark fermion fields Q^{D_i} and Q^{U_i} with a subscript L, R. All sums run over $i = 1, 2, 3$, corresponding to the SM fermion generations. The $\mathrm{SU}(2)_L$ doublets $q^i_L = \left(U^i_L\, D^i_L\right)^T$ represent the SM up (U) and down (D) quark fields, $\hat{S}^{U_i,D_i}_L = \left(S^{U_i,D_i}_{L_1}\, S^{U_i,D_i}_{L_2}\right)^T$, and $\hat{H} = \left(H^+\, H^0\right)^T$ is the SM Higgs doublet. As usual, $H^0 = (v+H)/\sqrt{2}$ and the conjugate doublet is $\tilde{H} = i\sigma_2 \hat{H}^\star$. The fields carrying an R subscript are $\mathrm{SU}(2)_L$ singlets, including the complex scalar fields S^{U_i,D_i}_R. The constants $g^{U,D}_L$ and $g^{U,D}_R$ in eq. (2.9) are flavor-universal parameters that we require to lie in the perturbative regime throughout the following analysis.

The Lagrangian $\mathcal{L}_{MS}(L)$ that connects leptons to the corresponding dark fermions possesses a similar structure:

$$\begin{aligned}\mathcal{L}^I_{MS}(L) = {} & g^N_L \sum_{i=1}^{3} \left[\bar{L}^i_L Q^{N_i}_R\right] \hat{S}^{N_i}_L + g^E_L \sum_{i=1}^{3} \left[\bar{L}^i_L Q^{L_i}_R\right] \hat{S}^{E_i}_L \\ & + g^N_R \sum_{i=1}^{3} \left[\bar{\nu}^i_R Q^{N_i}_L\right] S^{N_i}_R + g^E_R \sum_{i=1}^{3} \left[\bar{E}^i_R Q^{L_i}_L\right] S^{E_i}_R \\ & + \lambda^N_S \sum_{i=1}^{3} \tilde{H}^\dagger \hat{S}^{N_i}_L S^{N_i\dagger}_R H_S + \lambda^E_S \sum_{i=1}^{3} \hat{H}^\dagger \hat{S}^{L_i}_L S^{E_i\dagger}_R H_S \; + \; H.c.\,.\end{aligned} \tag{2.10}$$

Here $L^i_L = \left(\nu^i_L\, E^i_L\right)^T$ represent the SM lepton doublets, with E^i and ν_i being the charged lepton and neutrino fields, respectively. Similar to the case of quarks, we have $\hat{S}^{N_i,E_i}_L = \left(S^{N_i,E_i}_{L_1}\, S^{N_i,E_i}_{L_2}\right)^T$. The particle content of the SM has also been extended with right-handed neutrinos so that these particles can acquire mass in the same way as the remaining SM fermions, that is via effective Yukawa couplings. The framework is therefore compatible with the presence of three light *Dirac* neutrinos.

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The Lagrangian terms in eqs. (2.9) and (2.10) contain the minimal set of interactions needed to produce the SM Yukawa couplings radiatively. Due to the fact that the dark fermions Q^{U_i,D_i,N_i,E_i} are SM gauge singlets, the quantum numbers of the messenger scalar fields necessarily coincide with those of squarks and slepton of supersymmetric models.

According to the proposals in refs. [32, 33], the assumption of flavor (generation) universality for the $g_{L,R}^{U,D,E,N}$, $\lambda_S^{U,D,E,N}$ couplings appearing in eqs. (2.9) and (2.10) attributes a potential flavor dependence of the Yukawa couplings in the quark or lepton sector solely to the involved dark-fermion masses. In fact, flavour universality is also preserved by the one-loop corrections to the mentioned couplings since only the dark-fermion masses break the universality. As a result, the spread of SM Yukawa couplings is directly related to the dark fermion mass spectrum, regardless of its origin. For instance, the use of non-perturbative dynamics in the dark sector easily allows exponentially spread dark fermions masses. The mechanism can therefore generate hierarchical SM Yukawa couplings that naturally fit the observed values, thereby solving the SM flavor puzzle [32–34]. Since in the present paper we are mainly concerned with the stability of the EW vacuum, which is not sensitive to the details of the flavour structure, we restrict ourselves to the generation of flavor-diagonal Yukawa couplings.

After the spontaneous symmetry breaking operated by the SM Higgs doublet and by the H_S field, the last term in eqs. (2.9) and (2.10) result in the following trilinear couplings

$$\begin{aligned}\mathcal{L}_3 \supset\ & \lambda_S^U v \sum_{i=1}^3 \hat{S}_L^{U_i} S_R^{U_i\dagger} H_S + \lambda_S^D v \sum_{i=1}^3 \hat{S}_L^{D_i} S_R^{D_i\dagger} H_S + \lambda_S^U v_S \sum_{i=1}^3 \tilde{H}^\dagger \hat{S}_L^{U_i} S_R^{U_i\dagger} + \lambda_S^D v_S \sum_{i=1}^3 \hat{H}^\dagger \hat{S}_L^{D_i} S_R^{D_i\dagger} \\ & + \lambda_S^N v \sum_{i=1}^3 \hat{S}_L^{N_i} S_R^{N_i\dagger} H_S + \lambda_S^E v \sum_{i=1}^3 \hat{S}_L^{E_i} S_R^{E_i\dagger} H_S + \lambda_S^N v_S \sum_{i=1}^3 \tilde{H}^\dagger \hat{S}_L^{N_i} S_R^{N_i\dagger} + \lambda_S^E v_S \sum_{i=1}^3 \hat{H}^\dagger \hat{S}_L^{E_i} S_R^{E_i\dagger} \\ & + H.c., \end{aligned} \tag{2.11}$$

where v_S is the VEV of the H_S field. As we show in the next section, these trilinear couplings are crucial for the radiative generation of the SM Yukawa couplings.

To conclude the section, we report below the most general expression for the minimal form of quartic potential V_{MS} of messenger scalar fields allowed by the symmetries of the theory. The expression takes into account the following aspects:

i) the hypothesis of flavor universality of the $g_{L,R}^i$, λ_S^i couplings, $i = U,D,E,N$, in eqs. (2.9) and (2.10).

ii) the fact that the messengers and dark-fermions are both charged under $\mathrm{U}(1)_D$ gauge interactions, with different $\mathrm{U}(1)_D$ charges.

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The above conditions result in a radiatively generated potential given by

$$\begin{aligned}
V_{\rm MS} = {}& \lambda^{U}_{LL}\sum_{i=1}^{3}\left[\left(\hat{S}^{U_i}_{L}\right)^{\dagger}\hat{S}^{U_i}_{L}\right]^{2} + \lambda^{D}_{LL}\sum_{i=1}^{3}\left[\left(\hat{S}^{D_i}_{L}\right)^{\dagger}\hat{S}^{D_i}_{L}\right]^{2} \\
&+\lambda^{U}_{RR}\sum_{i=1}^{3}\left[\left(S^{U_i}_{R}\right)^{\dagger}S^{U_i}_{R}\right]^{2} + \lambda^{D}_{RR}\sum_{i=1}^{3}\left[\left(S^{D_i}_{R}\right)^{\dagger}S^{D_i}_{R}\right]^{2} \\
&+\lambda^{U}_{LR}\sum_{i=1}^{3}\left(\hat{S}^{U_i}_{L}\right)^{\dagger}\hat{S}^{U_i}_{L}\left(S^{U_i}_{R}\right)^{\dagger}S^{U_i}_{R} + \lambda^{D}_{LR}\sum_{i=1}^{3}\left(\hat{S}^{D_i}_{L}\right)^{\dagger}\hat{S}^{D_i}_{L}\left(S^{D_i}_{R}\right)^{\dagger}S^{D_i}_{R} \\
&+\lambda^{N}_{LL}\sum_{i=1}^{3}\left[\left(\hat{S}^{N_i}_{L}\right)^{\dagger}\hat{S}^{N_i}_{L}\right]^{2} + \lambda^{E}_{LL}\sum_{i=1}^{3}\left[\left(\hat{S}^{E_i}_{L}\right)^{\dagger}\hat{S}^{E_i}_{L}\right]^{2} \\
&+\lambda^{N}_{RR}\sum_{i=1}^{3}\left[\left(S^{N_i}_{R}\right)^{\dagger}S^{N_i}_{R}\right]^{2} + \lambda^{E}_{RR}\sum_{i=1}^{3}\left[\left(S^{E_i}_{R}\right)^{\dagger}S^{E_i}_{R}\right]^{2} \\
&+\lambda^{N}_{LR}\sum_{i=1}^{3}\left(\hat{S}^{N_i}_{L}\right)^{\dagger}\hat{S}^{N_i}_{L}\left(S^{N_i}_{R}\right)^{\dagger}S^{N_i}_{R} + \lambda^{E}_{LR}\sum_{i=1}^{3}\left(\hat{S}^{E_i}_{L}\right)^{\dagger}\hat{S}^{E_i}_{L}\left(S^{E_i}_{R}\right)^{\dagger}S^{E_i}_{R}\,. \qquad (2.12)
\end{aligned}$$

The number of independent parameters included in the expression above is the minimal compatible with the symmetries of the theory and flavor universality. The number of couplings could be further reduced only by assuming extra symmetries. For instance, the LR gauge symmetry considered in ref. [33] forces $\lambda^{U,D,E,N}_{LL} = \lambda^{U,D,E,N}_{RR}$ and $g^{U,D,E,N}_{L} = g^{U,D,E,N}_{R}$ at the high energy scale where the LR symmetry is first broken. In our case, these relations are not preserved by radiative corrections because the $L-$ and R-type messenger fields have different $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ quantum numbers.

In the following analysis we also disregard all quartic couplings of the form $\hat{H}^\dagger\hat{H}\hat{S}^{I\dagger}_{L}\hat{S}^{I}_{L}$ or $\hat{H}^\dagger\hat{H}S^{I\dagger}_{R}S^{I}_{R}$, involving two messenger fields of the same type (I runs over the SM fields) and the SM Higgs doublet $\hat{H}$ or the singlet H_S. Even if vanishing at a scale, these couplings are inevitably re-generated by radiative corrections already at the one-loop level. However, with the interactions of the Higgs boson and H_S field included in eq. (2.10), the β-functions of the couplings that multiply the $\hat{H}^\dagger\hat{H}S^{I\dagger}_{X}S^{I}_{X}$ operators, $X = L, R$, receive a first contribution proportional to the square of the EW gauge couplings at the one-loop level. Operators involving H_S, instead, begin to run only at higher orders. By setting these parameters to small and positive values at a given scale, the slow running then ensures that their effect on the RGEs of the model — and in particular on the evolution of the Higgs quartic coupling — is always negligible. Consequently, we expect that a more careful assessment of the RGE evolution in the present model would only marginally change the results of our vacuum stability analysis.

3 Radiative generation of Yukawa couplings

We begin our investigation by identifying the couplings and mass scales relevant to the problem of vacuum stability, using the simplified framework introduced in the previous section as a benchmark.

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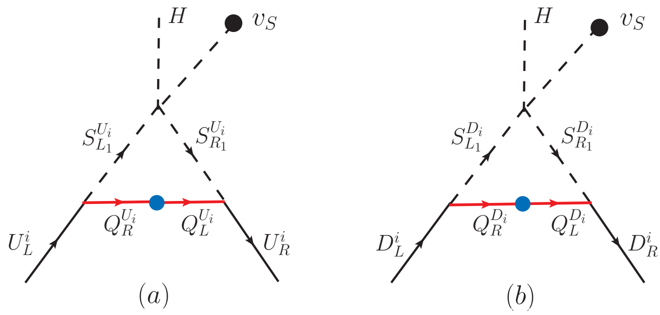

Figure 1. The diagrams responsible for the radiative generation of the Yukawa couplings of up-type quarks (a) and down-type quarks (b). The black circle on the external scalar line implies that the field H_S is set to its vacuum expectation value v_S. The blue dot on the dark fermion line (in red), instead, represents a mass insertion.

The SM Yukawa couplings arise at the one-loop level through diagrams analogous to that of figure 1, which explicitly show the case of quarks.

The expressions for the resulting SM Yukawa couplings can be obtained by matching the fundamental 5-dimensional amplitudes to the local SM Yukawa operators. In particular, for the quark sector we have [32]

$$Y_i^q = \frac{g_L^q g_R^q}{16\pi^2}\left(\frac{M_{Q_i}\Lambda_S}{m^2}\right) f_1(x_i,\xi) \tag{3.1}$$

where the index i stands for the quark flavor (for both up and down types) and $f_1(x,\xi)$ is a loop function given by

$$f_1(x,\xi) = \frac{1}{2}\left[C_0\left(\frac{x}{1-\xi}\right)\frac{1}{1-\xi} + C_0\left(\frac{x}{1+\xi}\right)\frac{1}{1+\xi}\right], \tag{3.2}$$

with

$$C_0(x) = \frac{1 - x\,(1-\log x)}{(1-x)^2}\,. \tag{3.3}$$

In the above equations we have used $x_i = M_{Q_i}^2/m^2$, where m^2 is the average mass of the colored scalar messengers and M_{Q_i} is the mass of the dark fermion associated to the quark q^i. We have also defined

$$\begin{aligned} \Lambda_S &= \lambda_S v_S\,, \\ \xi &= \frac{\Lambda_S v}{m^2}\,, \end{aligned} \tag{3.4}$$

which indicate the scale of new physics and the strength of the mixing in the colored messenger mass sector, respectively.

The request that messenger fields be unstable, or analogously that the dark fermions be stable particles, forces the latter to be lighter than the former. The condition translates into the following bound

$$M_{Q^i}^2 < m^2(1-\xi)\,, \tag{3.5}$$

where we have neglected the contributions of quark masses, subdominant with respect to those of dark-fermions.

In analogy to the above results, the SM Yukawa couplings of leptons are

$$Y_i^{E,N} = \frac{g_L^{E,N} g_R^{E,N}}{16\pi^2} \left(\frac{M_{Q_i} \Lambda_S}{\bar{m}^2} \right) f_1(x, \bar{\xi}) \tag{3.6}$$

where now M_{Q_i} is the mass of the dark fermion associated to the lepton of generation i and $\bar{m}$ and $\bar{\xi}$ are the common mass and the mixing angle in the leptonic sector of messengers, respectively.

Without loss of generality, henceforth we set the masses of all the messenger fields to a common scale, imposing $m = \bar{m}$, and likewise require that $\xi = \bar{\xi}$. In fact, these simplifications do not preclude the framework from reproducing arbitrary Yukawa hierarchies, which can be matched by considering suitable dark fermion mass spectra and rescaling of the $g^q_{L,R}$, $g^E_{L,R}$ and $g^N_{L,R}$ couplings. Notice that since avoiding tachyons and color- or charge-breaking minima in the messengers sector requires $\xi < 1$, we can bound the scale of new physics through

$$\Lambda_S < \frac{m^2}{v} \,. \tag{3.7}$$

In order to further reduce the parameter space, we require that $g^q_L = g^q_R \equiv g^q_{LR}$, $g_L^{N,E} = g_R^{N,E} \equiv g^L_{LR}$ at the energy scale $\mu_{\rm mes} \sim m$ of the order of the common messenger mass. Since the messengers are the heaviest fields running inside the relevant loop contributions, we match the fundamental amplitude of figure 1 to the SM Yukawa operator at the same scale $\mu_{\rm mes}$.

A closer inspection of eq. (3.1) reveals that the radiative suppression factor is to be compensated by large couplings $g^q_{L,R} \simeq \mathcal{O}(1)$ to reproduce the observed value of $Y_t \sim 1$. Alternatively, it is possible to take $\Lambda_S/m \gg 1$ (but $\Lambda_S/m < m/v$ to avoid tachyons) and match the value of top Yukawa coupling for perturbative values of $g^q_{L,R}$. Indeed, large values of the trilinear coupling Λ_S are allowed at high energy because the associated operator has dimension three. However, they can break the perturbative unitarity of the S matrix at low energies [36]. In particular, Λ_S appears in the interaction vertex $\Lambda_S H S_i^\dagger S_i$ between generic messenger fields denoted by the index i. Large values of the parameter can therefore cause the elastic scatterings cross section for the $S_i S_i \to S_i S_i$, mediated by the SM Higgs boson, to grow beyond the unitarity limit. This might signal the formation of bound states of messenger fields, which would allow to recover the unitarity of the theory in a non-perturbative way.

The issue of perturbative unitarity in the radiative generation of Y_t becomes evident once the value of the dark fermion mass associated to the top quark is chosen so as to maximize the radiative contributions to Y_t. Setting the parameter to the same order of the lightest messenger mass, $M_{Q_t} \sim m$, implies $x_t = 1 - \xi$ and gives

$$Y_t = \frac{(g^q_{LR})^2}{16\pi^2} \left(\frac{m}{v} \right) F_Y(\xi) \,. \tag{3.8}$$

The function $F_Y(\xi)$ is

$$F_Y(\xi) = \frac{2\xi + (1-\xi)^2 \log\left(\frac{1-\xi}{1+\xi}\right)}{8\xi\sqrt{1-\xi}}\,, \tag{3.9}$$

and admits the limit $F_Y(\xi) \simeq \frac{\xi}{2} + \mathcal{O}(\xi^2)$ in the small mixing case, $\xi \ll 1$. We can then express the Yukawa coupling as a function of the Λ_S scale

$$Y_t \sim \frac{(g^q_{LR})^2}{32\pi^2}\frac{\Lambda_S}{m} + \mathcal{O}(\xi^2)\,, \tag{3.10}$$

and the value of Y_t can be matched by adjusting the product $(g^q_{LR})^2\Lambda_S$. For the purpose of assessing the vacuum stability, we can then use the relation in eq. (3.8) to infer the initial condition for g^q_{LR}, given at the messengers scale as a function of m and ξ. Equivalently, in the small mixing regime, we can use eq. (3.10) to determine the value of Λ_S/m as a function of g^q_{LR}.

The same strategy also allows to determine the g^L_{LR} couplings, although perturbative unitarity can be easily respected in the leptonic sector because $Y_\tau \ll 1$. Requiring that the mass of the dark-fermion associated to the tau lepton be of the same order of the lightest messenger mass, we then have

$$Y_\tau = \frac{(g^L_{LR})^2}{32\pi^2}\left(\frac{m}{v}\right)F_Y(\xi)\,, \tag{3.11}$$

where, as anticipated, we have used the same average messenger mass and mixing as in the quark sector. For the measured values of SM Yukawa couplings, we do not expect the simplification to induce qualitative changes in the RG evolution of the SM Higgs boson quartic coupling because the hierarchy between quark and lepton messengers spans at most two orders of magnitude.

In the following, after detailing the relevant β-functions, we analyze the EW vacuum stability in two complementary scenarios delineated by the above considerations:

I) We consider perturbative values of the couplings $g^{L,q}_{LR} \lesssim 1$ and a set of values for the common messenger mass m. The scale Λ_S is then adjusted so as to reproduce the observed value of Y_t through eq. (3.8) and we extend the vacuum stability analysis up to the Planck scale $\mu \sim M_{\rm Pl}$. Due to the large ratio $\Lambda_S/m \gg 4\pi$, we assume that the non-perturbative phenomena needed to recover unitarity at low energy in messenger sector do not affect the running of λ_H at the large scales relevant for the vacuum stability. This is justified by the fact that operators of dimension 3, potentially responsible for breaking unitarity at low energy, are super-renormalizable in the UV. We also speculate on a possible UV completion which allows to have a large ratio $\Lambda_S/m \gg 1$ compatible with perturbative couplings $g^q_{LR} \ll 1$ at low energy.

II) We use $\Lambda_S/m \lesssim 4\pi$, within the limit of perturbative unitarity. The initial values of the couplings g^q_{LR} and g^L_{LR} are then extracted from Y_t and Y_τ in eq. (3.8). Because g^q_{LR} is necessarily borderline with the perturbative limit, we analyze the running of λ_H only up to the scale where the first Landau pole is reached.

In both scenarios, we regard the common messenger mass, m, and the trilinear Higgs-messengers coupling, Λ_S, as input parameters. For the sake of simplicity, we set the quartic coupling λ_{H_S} in a way that the mass of the H_S field matches the common messenger mass scale. Beside the quantities that directly determine the Higgs boson quartic coupling, we track the RG evolution of the remaining couplings to ensure the absence of color breaking[1] and assess their perturbativity.

3.1 Further phenomenological implications

We conclude the section with a brief review of the phenomenological implications of the framework, based on the works of refs. [34, 37–44].

In this scenario, the generation of the SM Yukawa interactions requires the existence of heavy scalar messenger fields and light dark-fermions, both charged under an unbroken $U(1)_D$ gauge interaction in the dark sector. Importantly, the model then clearly allows for the direct production of a pair of colored scalar messenger fields at collider experiments, via gluon-gluon fusion or quark-antiquark annihilation (the latter proceeding through the exchange of dark fermions in the t- or u-channel). Each messenger field eventually decays into the corresponding quark and dark-fermion, resulting thereby in a jet accompanied by missing energy. This signature is quite similar to the squark production of supersymmetric models with a stable neutralino, which plays here the role of a dark fermion. Although a dedicated collider analysis is still missing, we expect that the sensitivity of the experiment to the cross sections will be reduced by the mass of the messenger fields with respect to the corresponding supersymmetric case. In particular, the LHC can only probe the direct production of messenger fields with masses up to a few TeV.

The framework also foresees the existence of a light sector containing the massless dark photon, $\bar{\gamma}$, associated to the dark $U(1)_D$ gauge symmetry. This $U(1)_D$ guarantees the stability of dark fermions, required by DM phenomenology, and protects the theory from inducing large tree-level flavour-changing neutral current (FCNC) transitions. Although the dark photon does not couple to ordinary matter at the tree-level, effective couplings are generated by higher dimensional operators involving quarks and leptons in the loop. Then, another distinguishing feature of this scenario is the predicted decay of the Higgs boson into photon and dark photon $H \to \gamma\bar{\gamma}$ [38, 40], which gives rise to a monochromatic photon plus (neutrino-like) missing energy signature at the LHC [38, 40] or future e^+e^- colliders [39]. This process is induced at the one-loop level by the exchange of messenger fields in the loop. For the non decoupling properties of the Higgs boson, we expect that sizeable ratios of the percent level could be achieved even for very large masses of the messenger fields. In regard of this, the ATLAS [45, 46] and CMS [47–49] collaborations have recently begun their investigation of this signature producing quite stringent upper bounds on the process.

Another signature of the model are the FCNC processes induced by the decay of a SM fermion (f) into a lighter one (f') of same charge plus a dark-photon, $f \to f' + \bar{\gamma}$ [42],

[1]The emergence of color breaking minima is prevented by requiring that quartic couplings involving messenger fields remain positive at all scales. This also prevents the appearance of mass mixing terms involving messenger fields and the SM Higgs boson, which would be otherwise generated through the $\hat{H}^\dagger \hat{H} S_X^{I\dagger} S_X^I$ operators, $X = L, R$, neglected in this analysis.

active in both the quark and lepton sectors. In particular, implications for the dark photon production via the rare charged Kaon decay $K^+ \to \pi^+\pi^0 + \bar{\gamma}$ (induced by $s \to d\bar{\gamma}$ transitions) have been analyzed in ref. [44]. The large branching ratios expected for this decay could be of interest for experiments dedicated to rare K^+ decays like the NA62 at CERN.

Finally, the production of light dark-fermions in invisible decays of neutral hadrons has been investigated in ref. [43], finding that the expected branching ratios of the K_L and B^0 mesons are comparable to the current experimental limits.

4 RGEs for the full model

We present here the β-functions of the parameters that we track in our analysis of vacuum stability. Due to the approximations adopted and the precision used in the computation of the effective Yukawa couplings, it is sufficient to compute the corresponding RGEs at the 1-loop order.

In studying our benchmark model, we have assumed a common $\mathrm{U}(1)_D$ charge for all the dark fermions and mediator fields expecting that the generalization to non-universal charges will not change the conclusion of the analysis. This assumption also guarantees the flavor (family) universality of the 1-loop RGEs.

The convention we use for the β-function is:

$$\beta\left(X\right) \equiv \mu \frac{dX}{d\mu} \equiv \frac{1}{\left(4\pi\right)^2}\beta^{(1)}(X)\,.$$

In our analysis, we run the SM RGEs from the top quark mass scale to the matching scale $\mu = \mu_{\mathrm{mes}} \simeq m$, where the parameters of our benchmark model are initialized.[2] We then continue the RG evolution of the quantities under investigation by using the β-functions obtained for the benchmark model, up to a scale $\mu = \Lambda_{\mathrm{UV}}$ corresponding to the UV cutoff of the theory. As for this, in absence of a UV completion for gravitational interactions, it is customary [50] to assume as a UV cutoff the lowest between the Planck mass, M_{Pl}, and the scale M_{LP} at which the first Landau pole appears in the evolution of a coupling:

$$\Lambda_{\mathrm{UV}} = \min\left[M_{\mathrm{Pl}}, M_{\mathrm{LP}}\right]\,. \tag{4.1}$$

In particular, in the second scenario we consider, Landau poles might appear in the RG flow of $g^q_{L,R}(\mu)$ well below the Planck scale, $M_{\mathrm{LP}} < M_{\mathrm{Pl}}$, due to the large initial values of these couplings imposed by the matching with the SM top Yukawa coupling. The above criterion was introduced in ref. [50] to investigate the stability of the SM vacuum under the assumption that quantum gravity does not introduce additional particle threshold above the Planck scale, as expected for instance in asymptotic safety scenarios [51].

For the sake of convenience, we also report the SM 1-loop β-function for the Higgs boson quartic coupling used for the RG evolution of the parameter in the range $m_t < \mu < \mu_{\mathrm{mes}}$,

[2] Dark Matter phenomenology forces the dark fermions to be lighter than the messengers. Therefore, the scale m corresponds to the largest scale associated to the degrees of freedom that circulate in the loop diagrams responsible for the Yukawa couplings generation.

with $\mu_{\text{mes}} \sim \mathcal{O}(m)$:

$$\beta^{(1)}(\lambda_H) = 24\lambda_H^2 - 6Y_t^4 + 12\lambda_H Y_t^2 - \frac{9}{5}g'^2\lambda_H - 9g^2\lambda_H + \frac{27}{200}g'^4 + \frac{9}{20}g^2g'^2 + \frac{9}{8}g^4 \,. \tag{4.2}$$

4.1 Quartic couplings

We provide below the 1-loop β-functions for the quartic couplings of the model as defined in section 3, valid for $\mu_{\text{mes}} < \mu < \Lambda_{\text{UV}}$, with the scale Λ_{UV} as defined in eq. (4.1):

$$\beta^{(1)}(\lambda_H) = 24\lambda_H^2 + \frac{1}{2}\lambda_{HH_S}^2 - \frac{9}{5}g'^2\lambda_H - 9g^2\lambda_H + \frac{27}{200}g'^4 + \frac{9}{20}g^2g'^2 + \frac{9}{8}g^4 \,, \tag{4.3}$$

$$\beta^{(1)}(\lambda_{H_S}) = 18\lambda_{H_S}^2 + 2\lambda_{HH_S}^2 \,, \tag{4.4}$$

$$\begin{aligned}\beta^{(1)}(\lambda_{HH_S}) = {}& 4N_F\Big(3(\lambda_S^U)^2 + 3(\lambda_S^D)^2 + (\lambda_S^E)^2 + (\lambda_S^N)^2\Big) \\ & + \lambda_{HH_S}\left(12\lambda_H + 6\lambda_{H_S} + 4\lambda_{HH_S} - \frac{9}{10}g'^2 - \frac{9}{2}g^2\right),\end{aligned} \tag{4.5}$$

$$\beta^{(1)}(\lambda_S^q) = \lambda_S^q\left(2\lambda_{HH_s} + 2\lambda_{LR}^q - C_S^q g'^2 - \frac{9}{2}g^2 - 8g_3^2 - 6g_D^2 + |g_L^q|^2 + |g_R^q|^2\right), \tag{4.6}$$

$$\beta^{(1)}(\lambda_S^L) = \lambda_S^L\left(2\lambda_{HH_s} + 2\lambda_{LR}^L - C_S^L g'^2 - \frac{9}{2}g^2 - 6g_D^2 + |g_L^L|^2 + |g_R^L|^2\right), \tag{4.7}$$

$$\begin{aligned}\beta^{(1)}(\lambda_{LL}^q) = {}& \lambda_{LL}^q\left(40\lambda_{LL}^q - \frac{1}{5}g'^2 - 9g^2 - 16g_3^2 - 12g_D^2\right) + 3(\lambda_{LR}^q)^2 \\ & + \frac{1}{600}g'^4 + \frac{1}{20}g^2g'^2 + \frac{9}{8}g^4 + g^2g_3^2 + 3g^2g_D^2 + \frac{1}{15}g_3^2g'^2 + \frac{13}{6}g_3^4 \\ & + 4g_3^2g_D^2 + \frac{1}{5}g_D^2g'^2 + 6g_D^4 + 4\lambda_{LL}^q|g_L^q|^2 - 2|g_L^q|^4 \,,\end{aligned} \tag{4.8}$$

$$\begin{aligned}\beta^{(1)}(\lambda_{LL}^L) = {}& \lambda_{LL}^L\left(24\lambda_{LL}^L - \frac{9}{5}g'^2 - 9g^2 - 12g_D^2\right) + (\lambda_{LR}^L)^2 + \frac{27}{200}g'^4 + \frac{9}{20}g^2g'^2 + \frac{9}{8}g^4 \\ & + 3g^2g_D^2 + \frac{9}{5}g_D^2g'^2 + 6g_D^4 + 4\lambda_{LL}^L|g_L^L|^2 - 2|g_L^L|^4 \,,\end{aligned} \tag{4.9}$$

$$\begin{aligned}\beta^{(1)}(\lambda_{RR}^q) = {}& \lambda_{RR}^q\left(28\lambda_{RR}^q - (C_Y^q)^2\frac{4}{5}g'^2 - 16g_3^2 - 12g_D^2\right) + 6(\lambda_{LR}^q)^2 + \frac{2}{75}(C_Y^q)^4g'^4 \\ & + \frac{4}{15}(C_Y^q)^2g_3^2g'^2 + \frac{13}{6}g_3^4 + 4g_3^2g_D^2 + \frac{4}{5}(C_Y^q)^2g_D^2g'^2 + 6g_D^4 + 4\lambda_{RR}^q|g_R^q|^2 - 2|g_R^q|^4 \,,\end{aligned} \tag{4.10}$$

$$\begin{aligned}\beta^{(1)}(\lambda_{RR}^L) = {}& \lambda_{RR}^L\left(20\lambda_{RR}^L - \frac{36}{5}(C_Y^L)^2g'^2 - 12g_D^2\right) + \frac{54}{25}(C_Y^L)^4g'^4 + 2(\lambda_{LR}^L)^2 \\ & + \frac{36}{5}(C_Y^L)^2g_D^2g'^2 + 6g_D^4 + 4\lambda_{RR}^L|g_R^L|^2 - 2|g_R^L|^4 \,,\end{aligned} \tag{4.11}$$

$$\begin{aligned}\beta^{(1)}(\lambda_{LR}^q) = {}& 2(\lambda_S^q)^2 + \lambda_{LR}^q\left(28\lambda_{LL}^q + 16\lambda_{RR}^q + 4\lambda_{LR}^q - C_{LR}^q g'^2 - \frac{9}{2}g^2 - 16g_3^2 - 12g_D^2\right) \\ & + (C_Y^q)^2\frac{1}{75}g'^4 + C_Y^q\frac{4}{15}g_3^2g'^2 + \frac{13}{3}g_3^4 + 8g_3^2g_D^2 + C_Y^q\frac{4}{5}g_D^2g'^2 + 12g_D^4 \\ & + 2\lambda_{LR}^q|g_L^q|^2 + 2\lambda_{LR}^q|g_R^q|^2 \,,\end{aligned} \tag{4.12}$$

$$\begin{aligned}\beta^{(1)}(\lambda_{LR}^L) = {}& 2(\lambda_S^L)^2 + \lambda_{LR}^L\left(12\lambda_{LL}^L + 8\lambda_{RR}^L + 4\lambda_{LR}^L - C_{LR}^L g'^2 - \frac{9}{2}g^2 - 12g_D^2\right) \\ & + (C_Y^L)^2\frac{27}{25}g'^4 + C_Y^L\frac{36}{5}g_D^2g'^2 + 12g_D^4 + 2\lambda_{LR}^L|g_L^L|^2 + 2\lambda_{LR}^L|g_R^L|^2 \,,\end{aligned} \tag{4.13}$$

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where the superscript $q = U,D$, $L=E,N$, the couplings g', g, g_3, and g_D correspond to the gauge groups $\mathrm{U}(1)_Y$, $\mathrm{SU}(2)_L$, $\mathrm{SU}(3)_c$ and $\mathrm{U}(1)_D$ respectively, and $N_F = 3$ is the number of SM generations or families. The constants coefficients that differentiate between the β-functions are $C_S^U = 13/10$, $C_S^D = 7/10$, $C_S^E = 27/10$, $C_S^N = 9/10$, $C_Y^U = 2$, $C_Y^D = -1$, $C_Y^E = 1$, $C_Y^N = 0$, $C_{LR}^U = 17/10$, $C_{LR}^D = 1/2$, $C_{LR}^E = 9/2$, $C_{LR}^N = 9/10$.

Notice that the large negative contribution of the top quark Yukawa coupling to $\beta^{(1)}(\lambda_H)$ (corresponding to $-6Y_t^2$ in eq. (4.2)), which is the main cause of vacuum instability in the SM, vanishes above the messenger scale. The new contribution to the RGE of λ_H is given by the positive term proportional to $\lambda_{HH_S}^2$.

4.2 Dark Yukawa couplings

The β-function for the flavor universal couplings $g_{L,R}^X$ of the model, $X=U,D,E,N$, defined in section 3 and valid for $\mu_{\text{mes}} < \mu < \Lambda_{\text{UV}}$, are:

$$\beta^{(1)}(g_L^U) = g_L^U \left(\frac{9}{2} |g_L^U|^2 + \frac{1}{2} |g_L^D|^2 - \frac{1}{20} g'^2 - \frac{9}{4} g^2 - 4g_3^2 - 3g_D^2 \right) \tag{4.14}$$

$$\beta^{(1)}(g_R^U) = g_R^U \left(3 |g_R^U|^2 - \frac{4}{5} g'^2 - 4g_3^2 - 3g_D^2 \right) \tag{4.15}$$

$$\beta^{(1)}(g_L^D) = g_L^D \left(\frac{9}{2} |g_L^D|^2 + \frac{1}{2} |g_L^U|^2 - \frac{1}{20} g'^2 - \frac{9}{4} g^2 - 4g_3^2 - 3g_D^2 \right) \tag{4.16}$$

$$\beta^{(1)}(g_R^D) = g_R^D \left(3 |g_R^D|^2 - \frac{1}{5} g'^2 - 4g_3^2 - 3g_D^2 \right) \tag{4.17}$$

$$\beta^{(1)}(g_L^N) = g_L^N \left(\frac{5}{2} |g_L^N|^2 + \frac{1}{2} |g_L^E|^2 - \frac{9}{20} g'^2 - \frac{9}{4} g^2 - 3g_D^2 \right) \tag{4.18}$$

$$\beta^{(1)}(g_R^N) = g_R^N \left(2 |g_R^N|^2 - 3g_D^2 \right) \tag{4.19}$$

$$\beta^{(1)}(g_L^E) = g_L^E \left(\frac{1}{2} |g_L^N|^2 + \frac{5}{2} |g_L^E|^2 - \frac{9}{20} g'^2 - \frac{9}{4} g^2 - 3g_D^2 \right) \tag{4.20}$$

$$\beta^{(1)}(g_R^E) = g_R^E \left(2 |g_R^E|^2 - \frac{9}{5} g'^2 - 3g_D^2 \right) . \tag{4.21}$$

4.3 Scalar mass parameters

Finally, we report the β-functions for the mass parameters of the considered scalar fields.

$$\beta^{(1)}(\mu_S^2) = 6\lambda_{H_S}\mu_S^2 + 4\lambda_{HH_S}\mu_H^2 \tag{4.22}$$

$$\beta^{(1)}(\mu_H^2) = \mu_H^2 \left(-\frac{9}{10} g'^2 - \frac{9}{2} g^2 + \frac{\lambda_{HH_S}\mu_S^2}{\mu_H^2} + 12\lambda_H \right) \tag{4.23}$$

$$\beta^{(1)}(m_{S_L^q}^2) = 6\lambda_{LR}^q m_{S_R^q}^2 + m_{S_L^q}^2 \left(-\frac{1}{10} g'^2 - \frac{9}{2} g^2 - 8g_3^2 - 6g_D^2 + 28\lambda_{LL}^q + 2 |g_L^q|^2 \right) \tag{4.24}$$

$$\beta^{(1)}(m_{S_L^L}^2) = 2\lambda_{LR}^L m_{S_R^L}^2 + m_{S_L^L}^2 \left(-\frac{9}{10} g'^2 - \frac{9}{2} g^2 - 6g_D^2 + 12\lambda_{LL}^L + 2 |g_L^L|^2 \right) \tag{4.25}$$

$$\beta^{(1)}(m_{S_R^q}^2) = 12\lambda_{LR}^q m_{S_L^q}^2 + m_{S_R^q}^2 \left(-\frac{2}{5} (C_Y^q)^2 g'^2 - 8g_3^2 - 6g_D^2 + 16\lambda_{RR}^q + 2 |g_R^q|^2 \right) \tag{4.26}$$

$$\beta^{(1)}(m_{S_R^L}^2) = 4\lambda_{LR}^L m_{S_R^L}^2 + m_{S_R^L}^2 \left(-\frac{18}{5} (C_Y^L)^2 g'^2 - 6g_D^2 + 8\lambda_{RR}^L + 2 |g_R^L|^2 \right) , \tag{4.27}$$

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where again $q = U,D$ and $L = E,N$, and $m^2_{S^{q,L}_{L,R}}$ denote the mass parameters of the messenger fields. Although quantum correction inevitably generate a splitting in the messenger mass spectrum, the effect can be safely disregarded in the assessment of vacuum stability.

5 Vacuum stability analysis

We can proceed with the analysis of the SM vacuum stability, dividing the study into the two aforementioned scenarios that cover complementary cases. In more detail, we consider:

- **Scenario S1.** After setting all the dark Yukawa couplings to a common perturbative value $g_{LR} \equiv g^q_L = g^L_L = g^q_R = g^L_R$ at the matching scale $\mu_{\rm mes} \sim \mathcal{O}(m)$, we compute the trilinear coupling $\Lambda_S = \lambda_S v_S$ needed to match the top Yukawa coupling through eq. (3.8). Since the required value is in tension with the unitarity bound for the ratio Λ_S/m, we separately discuss a possible UV complete theory where the bound is avoided. In this scenario, the vacuum stability analysis is extended up to the Planck scale $\Lambda_{\rm UV} = M_{\rm Pl}$ due to the absence of Landau poles at lower scales.

- **Scenario S2.** We assume a trilinear coupling $\Lambda_S = 4\pi m$, corresponding to the maximum value allowed by perturbative unitarity at low energy, and a large value of the mixing parameter $\xi = 0.95$. We then initialize the couplings $g^q_{LR} \equiv g^q_L = g^q_R$ and $g^L_{LR} \equiv g^L_L = g^L_R$ by matching the Yukawa couplings of top quark and tau lepton. In this case, the resulting value of g^q_{LR} is necessarily borderline with perturbation theory and inevitably causes the emergence of a Landau pole at a scale $M_{\rm LP} \ll M_{\rm Pl}$. Therefore, in the present scenario we aim to ensure that vacuum stability is at least achieved for energies as large as $\Lambda_{\rm UV} = M_{\rm LP}$, remarking that a complete assessment valid at arbitrarily large energies requires a dedicated study of the non-perturbative regime of the theory.

In both the cases, to assess the stability of the EW vacuum we analyze the running of the Higgs boson quartic coupling, using eq. (1.1) which approximates well the full RG-improved potential. The vacuum stability is then ensured if $\lambda_H(\mu_H) > 0$ for all values of the scale μ from the EW scale up to the UV cutoff $\Lambda_{\rm UV}$. Concerning the quartic couplings in the messenger sector, since they do not play a direct role in the vacuum stability analysis, we set them to a common perturbative value at the matching scale $\mu_{\rm mes}$. The coupling λ_{HH_S} in eq. (2.2) is set instead to vanish at the matching scale to minimize its contribution to the running of λ_H. Concretely, we choose:

$$\begin{aligned} \lambda^X_S(\mu_{\rm mes}) = \lambda^X_{LL}(\mu_{\rm mes}) = \lambda^X_{RR}(\mu_{\rm mes}) = \lambda^X_{LR}(\mu_{\rm mes}) &= 0.1\,, \\ \lambda_{HH_S}(\mu_{\rm mes}) = 0\,, \quad \lambda_{H_S}(\mu_{\rm mes}) &= 0.1\,, \end{aligned} \tag{5.1}$$

where the superscript $X = U,D,E,N$.

In order to avoid color and charge breaking minima, throughout the following analyses we require that all the mass parameters and quartic couplings of mediators be positive up to the scale $\Lambda_{\rm UV}$. In regard of this, the corresponding β-functions force the initial values of these parameters to be sizeable, although still well perturbative, in order to overcome the large negative contribution due to the $\mathrm{SU}(3)_c$ gauge group.

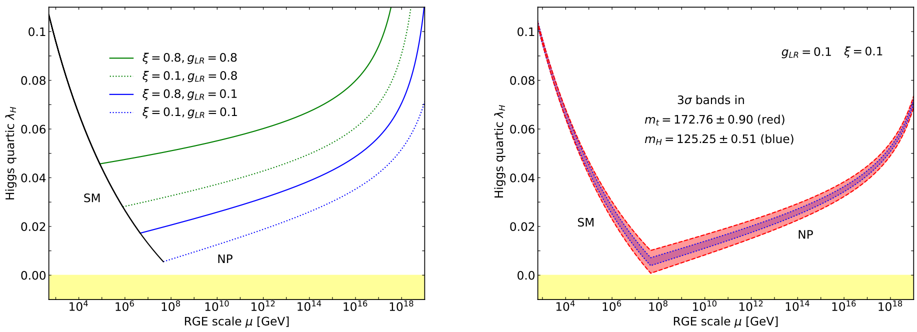

Figure 2. Evolution of the effective Higgs boson quartic coupling λ_H with the renormalization scale $\mu \sim H$, eqs. (4.2) and (4.3). In the first panel, the colored lines represent the RG evolutions obtained for the indicated benchmark setups. In each case, the Higgs boson quartic coupling evolves according to the SM trajectory, shown in black, until it reaches the considered colored curve, then proceeds along the latter. The value of the renormalization scale at the point where the two curves meet corresponds to the matching scale. The second panel shows the effect of the current experimental uncertainties on one of the setups analyzed in the previous panel. In both the panels, the yellow band indicates the region excluded by vacuum stability.

5.1 Scenario S1

Following the standard approach, we approximate the effective SM Higgs potential with its tree-level form improved by the running coupling, $V_{\text{eff}}(H) = \lambda_H(\mu)H^4/4$, and identify $\mu \sim H$.

In figure 2 we present the results obtained by running the SM β-functions from the EW scale to the scale μ_{mes}, performing the matching and then running the parameters with the β-functions in section 4.

The first panel shows the evolution of the Higgs boson quartic coupling with the RGE scale, from $\mu = 10^3$ GeV up to the UV cutoff identified in this scenario with the Planck scale. The yellow band signals the region of the parameter space ($\lambda_H(\mu) < 0$) where the EW vacuum is *not* stable.

The different curves are obtained by considering the indicated combinations of the benchmark values used for the mixing parameter, $\xi = 0.1, 0.8$, and the initial values of all dark Yukawa couplings, $g_{LR} = 0.1, 0.8$. Once g_{LR} and ξ are selected, the value of the common messenger mass scale m is set by the top Yukawa coupling in eq. (3.8) through the matching conditions. As m is also used as the matching scale, the full system of RGEs is solved iteratively until sufficient precision is obtained.

In each of the analyzed cases, the matching scale corresponds to the value of the RG scale at which the running departs from the SM evolution, indicated by the black line. Correspondingly, at this point the top Yukawa coupling ceases to contribute to the λ_H beta-function. The kinks in the curves are an artefact of the approximation used for the threshold conditions. For every curve, the ratio Λ_S/m can be computed by inverting eq. (3.4): $\Lambda_S/m = \xi m/v$. For the values reported in the figure, in order of increasing matching scale $\mu_{\text{mes}} \sim m$, we obtain $\Lambda_S/m = 2.6 \times 10^2, 3.3 \times 10^2, 1.5 \times 10^5, 1.9 \times 10^5$.

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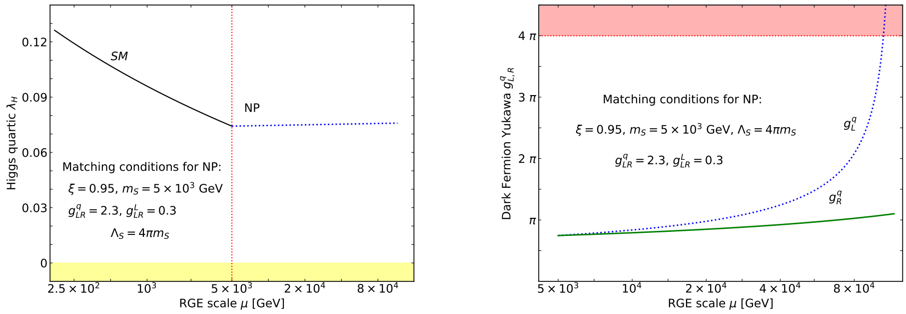

Figure 3. Left panel: evolution of the Higgs boson quartic coupling with the RG scale. The trilinear coupling Λ_S, entering the expressions obtained for the SM Yukawa operators, is set at the maximum value allowed by unitarity. The red dashed line indicates the matching scale, which separates the SM evolution of the parameter from its continuation determined by new physics. Right panel: evolution of the dark Yukawa coupling with the RG scale. Because of the large initial value selected by the considered value of Λ_S, the evolution of parameters is stopped by the emergence of a first Landau pole at $\mu \simeq 10^5$ GeV, which defines the UV cutoff for the scenario.

In the second panel of figure 2, we show, instead, the effect of the current experimental uncertainties affecting the top quark and Higgs boson mass for the previously analyzed case that reaches closer to the instability zone. As we can see from these results, given the present measurements of these quantities, the stability of vacuum is always guaranteed in the present scenario.

5.2 Scenario S2

The results obtained under the assumptions that specify the scenario S2 are shown in figure 3.

In more detail, the first panel shows again the RG evolution of the Higgs boson quartic coupling with the RG scale. In the plot, the trilinear coupling Λ_S is set at the maximum value allowed by unitarity, which results in the initial conditions $g^q_{LR} \simeq 2.3$, $g^L_{LR} \simeq 0.3$ of the dark Yukawa couplings at the matching scale. The latter is denoted by the red dashed line and separates the SM evolution of λ_H (black solid line) from its new physics continuation, rendered by the dotted segment in blue. Once again, the yellow band indicates the region of the parameter space where the vacuum is unstable. The RG evolution of the Higgs boson quartic coupling has been computed only up to $\mu \sim 10^5$ GeV, where the first Landau pole appears in the RG flow of dark Yukawa couplings related to quarks. This is illustrated in the second panel of figure 3, which shows that the g^q_L coupling is led to non-perturbative values (red band) already at a scale two order of magnitudes larger than the considered matching scale. The difference in the behaviors of g^q_L and g^q_R is solely due to the extra SU(2) contributions in the β-function of the former.

In Scenario S2, the EW vacuum stability is thus achieved at least in the energy range where the theory maintains perturbativity. The assessment of the vacuum structure of the theory for larger energies, however, requires non-perturbative methods that go beyond the scope of the present paper.

5.3 Large trilinear couplings and unitarity bounds

In this section we discuss the unitarity bound violated by the large trilinear coupling $\Lambda_s/m \gg 4\pi$ of scenario S1 and speculate on a UV scenario where the constraint is relaxed.

As is well known, trilinear scalar interactions are UV-safe because the corresponding dimension 3 operator guarantees that processes mediated by these interactions do not violate unitarity in the limit of high energy, regardless of the value of the trilinear scalar coupling. On the other hand, at a set energy scale, the same interactions spoil unitarity when the trilinear coupling is much larger than any of the masses associated to the fields entering the trilinear vertex, as shown for instance in [36].

For the case of scenario S1, the problematic trilinear coupling is due to the interaction vertex between the messenger fields and the Higgs boson H, arising after the H_S field acquires a VEV: $\mathcal{L} \supset \Lambda_S \sum_i S_i S_i^\dagger H$ — see eq. (2.11). We can then consider the scattering process

$$S_i S_i \to S_j S_j \tag{5.2}$$

with $i \neq j$ allowed by the interaction in eq. (2.11). The only diagram contributing to the amplitude has a SM Higgs boson propagator in the s-channel. The amplitude $\mathcal{M}$ is given by

$$\mathcal{M}(s) = i \frac{\Lambda_S^2}{s - m_H^2}\,, \tag{5.3}$$

where $s = (p_1 + p_2)^2$ is the square of the center of mass energy and p_1 and p_2 the four-momenta of the initial state messengers S_i.

For center of mass energies comparable with the messengers mass threshold, $s \simeq 4m^2$, the amplitude tends to

$$\mathcal{M}(s \to 4m^2) \to i \frac{\Lambda_S^2}{4m^2}\,, \tag{5.4}$$

where we assumed $m \gg m_H$ and neglected the contribution of the Higgs mass in the denominator. As we can see, the amplitude of the process grows arbitrarily for $\Lambda_S/m \gg 1$, breaking the S-matrix unitarity at any fixed value of the scattering energy s. In particular, one can also show that for $\Lambda_S > 4\pi m$ perturbative unitarity is broken [36]. In fact, Λ_S/m effectively works as a dimensionless coupling and consequently the ratio cannot be arbitrarily large if perturbation theory is to work. Still, for any fixed value of the trilinear coupling, the cross section at large energies $s \gg m^2$ scales as $1/s$ and thus the unitarity problem appears only at a set energy scale. Similar conclusions apply to the case of elastic scattering, where additional t- and u-channels diagrams contribute to the amplitude.

In order to recover perturbative unitarity, we explore an extension of the framework that adds a Lee-Wick (LW) higher derivative term [52, 53] for the SM Higgs boson in the Lagrangian. The resulting kinetic term, which contains a fourth derivative of the field, can be rewritten as a sum of the usual scalar propagator plus the propagator of an unstable massive particle with negative norm: the LW ghost. The instability of the LW ghost formally allows to recover the unitarity of the S matrix upon restricting the asymptotic (stable) states of Hilbert space to positive norm states [52–54]. The LW extension of the SM has been previously proposed in the context of the hierarchy problem related to the Higgs boson mass [55–57].

Extending our model to include a LW higher derivative term for the Higgs field, the amplitude in eq. (5.3) is modified by the propagation of the associated LW ghost, of mass M_H, as follows:[3]

$$\mathcal{M}^{\mathrm{LW}}(s) = i\Lambda_S^2 \left(\frac{1}{s - m_H^2} - \frac{1}{s - M_H^2} \right) . \tag{5.5}$$

Assuming now that the messenger mass is larger than the LW ghost scale, $m \gg M_H$, the amplitude at the threshold in eq. (5.4) becomes

$$\mathcal{M}^{\mathrm{LW}}(s \to 4m^2) \to -i\frac{\Lambda_S^2 M_H^2}{16m^4} + \mathcal{O}(M_H^2/m^2) . \tag{5.6}$$

Then, in the LW modified theory, perturbative unitarity of the process at hand is always guaranteed if $\frac{\Lambda_S^2 M_H^2}{m^4} \lesssim 1$, implying

$$m_H \ll M_H < \frac{m^2}{\Lambda_S} . \tag{5.7}$$

Interestingly, for the typical values used during the analysis of scenario S1, the characteristic scale of LW ghost is of order $\mathcal{O}(1-10)$ TeV, consistently with the lower bounds on the scenario from the LHC [58]. In the present context, this indicates the expected cutoff that softens the SM Higgs hierarchy problem in the proposed LW extension.

As for the problem of EW vacuum stability, the presence of a higher derivative kinetic term for the Higgs boson modifies the RGEs of section 4. In particular, the increased dependence of the propagator on inverse powers of the momentum makes all the one-loop diagrams presenting at least one Higgs boson circulating in the loop finite. Therefore, the corresponding contributions to the β-functions vanish above the mass scale associated to the LW ghost and the expressions in section 4 are modified as follows:

$$\beta^{(1)}(\lambda_H) = \frac{1}{2}\lambda_{HH_S}^2 - \frac{9}{5}g'^2\lambda_H - 9g^2\lambda_H + \frac{27}{200}g'^4 + \frac{9}{20}g^2 g'^2 + \frac{9}{8}g^4 , \tag{5.8}$$

$$\beta^{(1)}(\lambda_{H_S}) = 18\lambda_{H_S}^2 , \tag{5.9}$$

$$\begin{aligned}\beta^{(1)}(\lambda_{HH_S}) = {} & 4N_F\Big(3(\lambda_S^U)^2 + 3(\lambda_S^D)^2 + (\lambda_S^E)^2 + (\lambda_S^N)^2\Big) \\ & + \lambda_{HH_S}\left(6\lambda_{H_S} - \frac{9}{10}g'^2 - \frac{9}{2}g^2\right) ,\end{aligned} \tag{5.10}$$

$$\beta^{(1)}(\lambda_S^q) = \lambda_S^q\left(2\lambda_{LR}^q - C_S^q g'^2 - \frac{9}{2}g^2 - 8g_3^2 - 6g_D^2 + |g_L^q|^2 + |g_R^q|^2\right) , \tag{5.11}$$

$$\beta^{(1)}(\lambda_S^L) = \lambda_S^L\left(2\lambda_{LR}^L - C_S^L g'^2 - \frac{9}{2}g^2 - 6g_D^2 + |g_L^L|^2 + |g_R^L|^2\right) , \tag{5.12}$$

[3] The imaginary contribution in the propagator is neglected since the latter is computed off-shell.

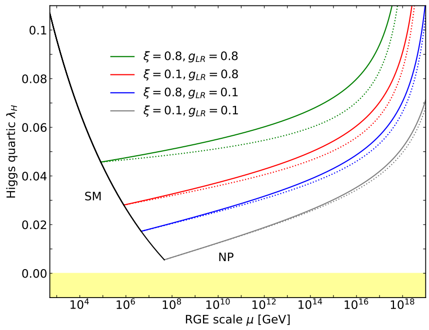

Figure 4. Evolution of the effective Higgs boson quartic coupling λ_H with the renormalization scale $\mu \sim H$ in the framework of Scenario S1 (solid lines) and in its LW extension (dotted lines) for the choice of parameters indicated in the legend.

with the superscript $q = U,D,\ L{=}E,N$, $C_S^U = 13/10$, $C_S^D = 7/10$, $C_S^E = 27/10$ and $C_S^N = 9/10$. As before, the couplings g', g, g_3, and g_D correspond to the gauge groups $\mathrm{U}(1)_Y$, $\mathrm{SU}(2)_L$, $\mathrm{SU}(3)_c$ and $\mathrm{U}(1)_D$ respectively, and $N_F = 3$ is the number of SM generations or families.

In order to assess the EW vacuum stability in the proposed LW extension, we have repeated the analysis of Scenario S1 using the above β-functions in place of the corresponding expressions presented in section 4. In figure 4 we compare the RG evolution of the Higgs boson quartic coupling obtained in the LW extension (dotted lines) to the results previously obtained (solid lines), shown also in the first panel of figure 2. In this example we have set the LW ghost mass scale to $M_H = 1\,\text{TeV}$, saturating the lower bound due to eq. (5.7) to maximize the difference in the RG evolutions. As we can see, the Higgs boson quartic coupling remains positive on the whole range of values considered for the renormalization scale and its RG evolution qualitatively remains the same. Therefore, we conclude that the stability of EW vacuum can be guaranteed also in the proposed LW extension of the model.

6 Conclusions

We have analysed the stability of the EW vacuum in the context of a previously proposed framework [32–34] for the radiative generation of the SM Yukawa interactions.

In the simpler version [32] adopted in this paper, the framework uses a new discrete symmetry to first forbid the usual SM dimension 4 Yukawa operators. Then, non-perturbative effects related to a new $\mathrm{U}(1)_\mathrm{D}$ gauge interaction yield a strongly hierarchical mass spectrum for a set of fermions charged under the symmetry. These dark fermions are in a one-to-one correspondence with the SM (Dirac) fermions and are responsible for sourcing the chiral symmetry breaking necessary to produce the SM Yukawa operators. The dark fermions and the SM particles are connected by a mediator sector, which hosts a set of scalar fields in a one-to-one correspondence with the SM Weyl fermions. Because the dark fermions are only charged under the $\mathrm{U}(1)_\mathrm{D}$ gauge group, the messengers necessarily carry the same quantum numbers as squarks and sleptons of supersymmetric theories. The

SM Yukawa couplings are thus generated at the one-loop level through processes allowed by the interactions of mediators, such as the one shown in figure 1, after the spontaneous breaking of the discrete symmetry.

It is a peculiarity of the framework that the SM Higgs boson is naturally prevented from interacting directly with both the SM or the dark fermions, i.e. that it is *fermiophobic*. As a consequence, at energies higher than the mediator and dark fermion mass scales, fermions cannot contribute to the running of the Higgs boson Lagrangian parameters, in particular to its quartic coupling.

Because the top quark Yukawa coupling provides the main contribution towards the metastability of the EW vacuum in the SM, in the present paper we set out to analyze the same problem of stability in light of the possible fermiophobic nature of the Higgs boson.

After detailing the model in section 2 and reviewing the radiative generation of Yukawa couplings in section 3, we show the RGEs for the full model in section 4. In order to study the stability of the EW vacuum, we delineate two complementary scenarios that exemplify well the reach of the considered framework. The common strategy is to solve the SM RGEs up to a matching scale, identified with the messenger scale, and then evolve the parameters according to the RGEs of the full model.

In the first scenario, we study different cases were all the interactions between the SM fermions, the dark fermions and messenger fields are set to common and well perturbative benchmark values. Matching the top quark Yukawa coupling then requires the trilinear coupling appearing in the relevant amplitude, eq. (3.10), to violate the unitarity bound of the S-matrix at energies close or below the matching scale. Postponing this issue momentarily, the results in figure 2 show that stability of EW vacuum can be achieved in the considered framework regardless of the current experimental uncertainties affecting the Higgs boson or the top quark mass. We argue that the unitarity of the S-matrix can be recovered in a LW extension of the framework that adds a LW ghost partner for the Higgs boson along the lines of the SM extension proposed in refs. [52, 53]. In this case, the model predicts a LW mass scale below 10 TeV, as required to solve the naturalness problem affecting the Higgs boson mass scale.

In the second scenario, instead, we take the maximal value of the trilinear coupling allowed by unitarity and initialize the relevant new physics interaction by matching the SM Yukawa couplings. As is evident from eq. (3.10), the top quark case requires coupling with values that are borderline with perturbation theory. The stability analysis is then performed up to the scale of the first Landau pole emerging in the RGEs of the full model, identified here as an effective UV cutoff. The results shown in figure 3 show that the EW vacuum stability is ensured also in this case.

Because the issue of vacuum stability does not significantly depend on the symmetry used to forbid the SM Yukawa couplings, the results obtained can be straightforwardly extended to the LR model presented in ref. [33].

In conclusion, our analyses shows that the fermiophobic nature of the Higgs boson, imposed by the symmetries, can ensure the stability of the EW vacuum, regardless of the precise value of the top quark mass. The framework remarkably predicts the existence of weakly coupled dark sector fields, as well as of new messenger scalar interactions, that can be explored in the next generation of experiments at the LHC and future colliders.

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Appendix 3

III

Abdelhak Djouadi, Ruiwen Ouyang, and Martti Raidal. Yukawa coupling unification in non-supersymmetric SO(10) models with an intermediate scale. *Phys. Lett. B*, 824:136788, 2022

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Yukawa coupling unification in non-supersymmetric SO(10) models with an intermediate scale

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ABSTRACT

We discuss the possibility of unifying in a simple and economical manner the Yukawa couplings of third generation fermions in a non-supersymmetric SO(10) model with an intermediate symmetry breaking, focusing on two possible patterns with intermediate Pati-Salam and minimal left-right groups. For this purpose, we assume a minimal Yukawa sector at high energy, starting with two Higgs bi-doublets at the intermediate scale which then simply reduce to a two Higgs doublet model at the electroweak scale. We first enforce gauge coupling unification at the two-loop level by including the threshold corrections in the renormalization group running which are generated by the heavy fields that appear at the intermediate symmetry breaking scale. We then study the running of the Yukawa couplings of the top quark, bottom quark and tau lepton at two-loops in these two breaking schemes, when the appropriate matching conditions are imposed. We find that the unification of the third family Yukawa couplings can be achieved while retaining a viable spectrum, provided that the ratio of the vacuum expectation values of the two Higgs doublet fields is large, $\tan\beta \approx 60$.

1. Introduction

The paradigm of grand unification [1,2] is at the heart of particle physics as it exploits the power of symmetries to unify in a most elegant way the electromagnetic, weak and strong interactions of the Standard Model (SM) into a single force [3]. Grand unified theories (GUTs) provide natural solutions to theoretical questions such as charge quantization and anomaly cancellation, in addition to the explanation of the existence of three separate gauge symmetry groups. GUTs can also successfully address most, if not all, of the important issues that call for beyond the SM physics. This is particularly the case for the problems of neutrino masses and mixing, the baryon asymmetry in the universe and the nature of the dark matter. Hence, leaving aside the issue of naturalness and the large hierarchy between the weak and Planck scales that induces quadratic "divergences" to the observed Higgs boson mass (for which one can, for instance, adopt an anthropic point of view just as in the case of the cosmological constant), non-supersymmetric GUTs can be viewed as the royal path to physics beyond the SM.

Unification in the context of SO(10) [4] is particularly interesting as this symmetry group possesses a representation of dimension 16 in which, for each generation, one can accommodate the 15 chiral fermions of the SM and an additional Majorana neutrino. If the mass of this new state is very large, somewhere at a scale of 10^{10} GeV, the see-saw mechanism [5] could explain the present pattern in the neutrino sector, the baryon asymmetry could be achieved through leptogenesis [6] and a suitable axion [7] could account for dark matter; see Refs. [8–10] for reviews. This intermediate scale can naturally be present in SO(10) as the group is of rank five, *i.e.*, larger than the rank of the SM group by one unit, so that the symmetry breaking may occur in three steps, one at the GUT scale M_U, one at this intermediate scale M_I and a last one at the electroweak scale. This solves one of the main drawbacks of non-supersymmetric GUTs, namely, the failure of the gauge couplings to unify at the high energy scale. Indeed, threshold effects [11] are generated by the contributions of the scalar multiplets that break the intermediate symmetry down to the SM group at the energy M_I, and these modify the renormalization group evolution of the three coupling constants such that they finally intersect at the scale M_U [12,13]. Hence, gauge coupling unification can also be realized without the need of supersymmetry, which was one of its main attractive points [14].

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Another argument in favor of supersymmetry was the possibility of also unifying the Yukawa couplings of third generation fermions [15]. Indeed, in the minimal supersymmetric extension of the SM, the MSSM, two-Higgs doublets fields are required in order to generate separately masses for the isospin up- and down-type fermions and, in constrained scenarios with universal "soft" SUSY–breaking parameters, these Yukawa couplings can be unified at the GUT scale. This occurs at large values of the ratio of the vacuum expectation values of the two Higgs fields, $\tan\beta = v_u/v_d$, which induces the proper hierarchy for the starting top and bottom quark masses, $\tan\beta \approx m_t/m_b \approx 60$.

In this letter, we show that the unification of the Yukawa couplings of third generation fermions can also be achieved in a rather simple and most economical way in a non-supersymmetric SO(10) scenario taking as examples two of the most interesting and widely discussed intermediate breaking patterns: the Pati-Salam [2] and the minimal left-right symmetric [16] groups. As a matter of fact, and in contrast to most earlier studies, only two Higgs bi-doublet fields will be necessary to describe the Yukawa interactions of standard fermions above the scale at which the intermediate breaking occurs, and this spectrum then reduces to two Higgs doublets only below this intermediate scale and down to the electroweak scale. Hence, one would have an effective two Higgs doublet model (2HDM) of type II [17] at low energies, just as in the MSSM, with vacuum expectation values such that the parameter $\tan\beta$ is large as to obtain the correct hierarchy for the top and bottom quark masses. Using this minimal scalar sector, it is possible to make that the renormalization group running of third generation Yukawa couplings in these two breaking schemes, with suitable matching conditions at the intermediate scale for which gauge coupling unification occurs, leads to Yukawa coupling unification at the GUT scale. This can be achieved while reproducing the third family fermion and electroweak gauge boson masses and preserving some important features such as ensuring the stability of the electroweak vacuum up to the intermediate scale and keeping the Yukawa couplings perturbative at all scales.

The paper is organized as follows. In the next section, we introduce our theoretical framework and discuss the breaking of SO(10) with intermediate steps. In section 3, we discuss the known issue of gauge couplings unification in SO(10) when threshold corrections are added at an intermediate scale but with a new ingredient, namely, the presence of an additional Higgs doublet field at low energies. In section 4, we study the running of the third generation Yukawa couplings and show that they can reach a common value at the same scale that allows for gauge coupling unification, while keeping a viable low energy spectrum. Our conclusions are given in section 5.

2. Theoretical framework

The SO(10) group has many attractive features [8–10] and most of them follow from the fact that it possesses a fundamental representation of dimension-16 in which, for each generation, the 15 SM chiral fermions as well as one right-handed neutrino can be embedded. In this case, the Yukawa couplings of the scalar bosons to pairs of these fermions belong to the direct product of $\mathbf{16}\otimes\mathbf{16}$, which can be decomposed into

$$\mathbf{16_F}\otimes\mathbf{16_F} = \mathbf{10}+\mathbf{120}+\mathbf{126}\,. \tag{1}$$

Thus, the most general Yukawa interaction which is SO(10) invariant is given by

$$-\mathcal{L}_Y = \mathbf{16_F}(Y_{10}\mathbf{10_H} + Y_{126}\overline{\mathbf{126}}_\mathbf{H} + Y_{120}\mathbf{120_H})\mathbf{16_F}\,. \tag{2}$$

The special case with only the first two Yukawa terms with the $\mathbf{10_H}$ and $\overline{\mathbf{126}}_\mathbf{H}$ representations, leading to the so-called minimal SO(10) model, has been thoroughly discussed, see *e.g.* Refs. [9,18]. The extended SO(10) model including the $\mathbf{120_H}$ representation has been also explored [19,20]. The first model usually requires an extra U(1) symmetry to complexify the $\mathbf{10_H}$ representation to achieve the required splitting in the fermionic spectrum, otherwise the ratio m_t/m_b would be fixed to unity at the GUT scale [9]. In turn, in the latter scenario, it has been shown that a realistic fermion spectrum can be achieved with or without introducing such an extra U(1) symmetry [20].

In this work, we will restrict to the minimal and most studied SO(10) scenario in which only the $\mathbf{10_H}$ and $\overline{\mathbf{126}}_\mathbf{H}$ representations are kept, but without an extra U(1) symmetry to complexify the $\mathbf{10_H}$ representation. This will constrain the parameter space, making the model more predictive, while allowing the possibility of neutrino mass generation via a seesaw mechanism and being consistent with present data [9,18].

The breaking of SO(10) to the SM gauge group $\mathcal{G}_{\rm SM} \equiv \mathcal{G}_{321} = \mathrm{SU(3)_C}\times\mathrm{SU(2)_L}\times\mathrm{U(1)_Y}$ can be triggered in several ways, but we will be only interested in two patterns that involve one intermediate gauge group at a high scale M_I: the Pati-Salam (PS) group [2] $\mathcal{G}_{422} = \mathrm{SU(4)_C}\times\mathrm{SU(2)_L}\times\mathrm{SU(2)_R}$ and the minimal left-right (LR) symmetry group [16] $\mathcal{G}_{3221} = \mathrm{SU(3)_C}\times\mathrm{SU(2)_L}\times\mathrm{SU(2)_R}\times\mathrm{U(1)_{B-L}}$. To achieve the desired symmetry-breaking in these two scenarios, one would necessarily need to introduce scalar multiplets that acquire vacuum expectation values (vevs) at the corresponding high scales.

For the Pati-Salam scenario, the breaking chain from SO(10) to the SM gauge group is *e.g.* achieved by the $(\mathbf{15},\mathbf{1},\mathbf{1})$ component of the scalar representation $\mathbf{210_H}$ which acquires a vev at the GUT scale M_U, and by the $\overline{\mathbf{126}}_\mathbf{H}$ that acquires a vev at the intermediate scale M_I. In turn, in the minimal left-right scenario, the symmetry should be broken first by the $\mathbf{45_H}$ which acquires a vev at the GUT scale and then by the $\overline{\mathbf{126}}_\mathbf{H}$ that acquires it at the intermediate scale [20]. One thus has

$$\mathrm{PS:}\quad \mathrm{SO(10)}|_{M_U} \xrightarrow{\langle\mathbf{210_H}\rangle} \mathcal{G}_{422}|_{M_I} \xrightarrow{\langle\overline{\mathbf{126}}_\mathbf{H}\rangle} \mathcal{G}_{321}|_{M_Z} \xrightarrow{\langle\mathbf{10_H}\rangle} \mathcal{G}_{31}\,; \tag{3}$$

$$\mathrm{LR:}\quad \mathrm{SO(10)}|_{M_U} \xrightarrow{\langle\mathbf{45_H}\rangle} \mathcal{G}_{3221}|_{M_I} \xrightarrow{\langle\overline{\mathbf{126}}_\mathbf{H}\rangle} \mathcal{G}_{321}|_{M_Z} \xrightarrow{\langle\mathbf{10_H}\rangle} \mathcal{G}_{31}\,. \tag{4}$$

According to the extended survival hypothesis [21], all the scalar fields that do not participate in the symmetry breaking patterns above by acquiring vevs will have masses of the order of the high scales M_U and M_I. In these two breaking chains, the scalar content that acquires vevs at the intermediate scale M_I or at the electroweak scale M_Z consists of, respectively, the $\overline{\mathbf{126}}_\mathbf{H}$ and $\mathbf{10_H}$ representations. More specifically, of the SO(10) scalar representations that can be decomposed under the intermediate gauge groups, only certain scalar fields from $\mathbf{10_H}$ and $\overline{\mathbf{126}}_\mathbf{H}$ have masses below the GUT scale and will contribute to the renormalization group equations (RGEs) between the two scales M_I and M_U. These are, in the PS scenario, $(\mathbf{1},\mathbf{2},\mathbf{2})$ (Φ_{10}) from $\mathbf{10_H}$ and $(\mathbf{15},\mathbf{2},\mathbf{2})\oplus(\mathbf{10},\mathbf{1},\mathbf{3})$ $(\Sigma_{126}\oplus\Delta_R)$ from $\overline{\mathbf{126}}_\mathbf{H}$ and, in the minimal LR scenario, $(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{0})$ from $\mathbf{10_H}$ and the $(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{0})\oplus(\mathbf{1},\mathbf{1},\mathbf{3},\mathbf{2})$ from $\overline{\mathbf{126}}_\mathbf{H}$. We will thus only consider this restricted set of scalar fields between the GUT and intermediate scales.[1]

At low energies, among the two Higgs bi-doublets that we had at a high energy scale, only two Higgs doublets survive and develop vevs at the electroweak scale. Thus, in our study, we will have in fact a model with two Higgs doublet fields H_u and H_d

[1] This in contrast to most studies which are done in this context as, generally, a very complicated scalar sector of the SO(10) group is needed to fit the low energy spectrum, in particular the fermion (including the light and sometimes even the heavy neutrino sector) masses and mixings, by adjusting the numerous input parameters that are available.

that couple separately to isospin $+\frac{1}{2}$ and $-\frac{1}{2}$ fermions and acquire vevs v_u and v_d

$$\langle H_u\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_u\end{pmatrix} \quad , \quad \langle H_d\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_d\end{pmatrix}, \tag{5}$$

to give masses to the W, Z bosons implying the relation $\sqrt{v_u^2+v_d^2} = v_{\rm SM} \simeq 246\,\text{GeV}$; we then define the ratio of these two vevs to be $\tan\beta = v_u/v_d$. The most general renormalizable scalar potential of this two Higgs doublet model can be found in Ref. [17] to which we refer for all details. The Yukawa interactions of the fermions are those of a type-II 2HDM with a Lagrangian given by

$$-\mathcal{L}_Y^{\rm 2HDM} = Y_u \bar{Q}_L H_u\, u_R + Y_d \bar{Q}_L H_d\, d_R + Y_e \bar{L}_L H_d\, e_R + \text{h.c.}\,, \tag{6}$$

with Q_L/L_L the quark/lepton left-handed doublets and f_R the right-handed singlets. In our study, only the third generation of fermions will be considered and the small Yukawa couplings of the first two generations will be neglected.

At the intermediate scale M_I, the minimal Yukawa interaction Lagrangian is obtained when only two Higgs bi-doublets couple to fermions. One of them should be from the $\overline{\mathbf{126}}_{\mathbf{H}}$ which also has a triplet field that breaks the left-right symmetry. The other can be chosen to be the $\mathbf{10_H}$. Starting from eq. (2), the Yukawa Lagrangian for fermions at the intermediate scale M_I can be written in the considered two schemes as

$$-\mathcal{L}_Y^{PS} = \bar{F}_L(Y_{10}^{PS}\Phi_{10} + Y_{126}^{PS}\Sigma_{126})F_R + F_R^T Y_R^{PS} C\overline{\Delta_R} F_R + \text{h.c.}\,, \tag{7}$$

$$\begin{aligned}-\mathcal{L}_Y^{LR} &= \bar{Q}_L(Y_{10,q}^{LR}\Phi_{10} + Y_{126,q}^{LR}\Sigma_{126})Q_R \\ &+ \bar{L}_L(Y_{10,l}^{LR}\Phi_{10} + Y_{126,l}^{LR}\Sigma_{126})L_R \\ &+ \frac{1}{2}L_R^T Y_R^{LR} i\sigma_2\Delta_R L_R + \text{h.c.}\,,\end{aligned} \tag{8}$$

where $F_{L,R}$ are generic left or right-handed quark/lepton fields and σ_2 a Pauli matrix. In both cases, we have assumed that terms like $\bar{F}_L^T\tilde{\phi}F_R$ with $\phi = \Phi$ or Σ and $\tilde{\phi} = \sigma_2^T\phi^*\sigma_2$ are forbidden by suitably chosen $\rm U(1)_Y$ charges [22]. Below the intermediate scale, the PS and LR models include, besides the triplet field Δ_R that gives masses to the heavy neutrino species, four Higgs doublets: two doublets ϕ_1 and ϕ_3 with opposite hypercharge from the $(\mathbf{1},\mathbf{2},\mathbf{2})$ representation and the doublets ϕ_2 and ϕ_4 again with opposite hypercharge from $(\mathbf{15},\mathbf{2},\mathbf{2})$. The fields ϕ_1 and ϕ_2 couple to up-type quarks and heavy neutrinos, while ϕ_3 and ϕ_4 couple to down-type quarks and the light leptons.

While the triplet fields acquire a very large vev, $\langle\Delta_R\rangle = v_R \sim \mathcal{O}(M_I)$, the bi-doublet fields acquire vevs of the order of the electroweak scale which implies that $\sum_{i=1}^4 v_i^2 = v_{\rm SM}^2$. This ensures that the right-handed gauge bosons are very heavy, $M_{W_R}, M_{Z_R} \approx g v_R$, while the $\rm SU(2)_L$ W and Z bosons have weak scale masses, $M_W, M_Z \approx g v_{\rm SM}$. In fact, only two linear combinations of the four scalar doublet fields $\phi_1 \cdots \phi_4$ will have weak scale masses, while the two other field combinations will have masses close to the very high scale. One has thus to tune the scalar potentials of the two scenarios to achieve this situation and discussions about the constraints to which it leads can be found in Refs. [23,24] for instance. The two fields with weak scale masses will be ultimately identified with the doublets H_u and H_d of our low energy 2HDM. At the intermediate scale M_I, these fields should match the Φ_{10} and Σ_{126} fields, the interactions of which have been given in eqs. (7), (8) as will be discussed shortly.

3. Gauge coupling unification

Assuming the 2HDM structure at low energies and the two breaking patterns of SO(10) down to the SM group with the intermediate scale M_I discussed previously, namely PS and LR, we study the renormalization group running of the three SM gauge couplings $\alpha_i = g_i^2/(4\pi)$. We closely follow Ref. [13] in which the standard case with only one electroweak Higgs doublet was studied. The analytical expressions for the gauge coupling RGEs at the two loop level, including the relevant β functions can be found, *e.g.*, in Ref. [26] where the dependence of the number of Higgs doublets is explicitly given. Naively, the more intermediate scale scalar particles are included in the running of the couplings, the lower would be the resulting unification scale.

At the intermediate scale, threshold effects [11] due to all the particles that have masses in the vicinity of M_I, and in particular all the scalar fields that develop vevs at this scale, will be active. These higher order corrections will modify the matching conditions of the gauge couplings at the scale of symmetry breaking, depending on the particle content. For a symmetry breaking from a group $\mathcal{G}$ to a subgroup $\mathcal{H}$ at the scale μ, the matching conditions with the threshold corrections take the form

$$\alpha_{i,\mathcal{H}}^{-1}(\mu) = \alpha_{i,\mathcal{G}}^{-1}(\mu) - \lambda_{i,\mathcal{H}}^{\mathcal{G}}/(12\pi)\,, \tag{9}$$

where $i = 1, 2, 3, \ldots$ refers to the particular gauge coupling α_i and $\lambda_{i,\mathcal{H}}^{\mathcal{G}}$ are usually weighted by the parameters $\eta_i = \ln(M_i/\mu)$ with M_i being the masses of the heavy particles integrated out at the low energy. The complete expressions for the one-loop threshold corrections $\lambda_{i,\mathcal{H}}^{\mathcal{G}}$ are given in Ref. [13] (see Tables IV and VI of their Appendices B and C, respectively). As a result, the intermediate and the unification scales M_I and M_U could be shifted by an order of magnitude or more even when only small threshold corrections are included. In the following, we show an explicit example of gauge coupling unification in the PS and LR breaking chains when these thresholds are included.

We start with the following initial conditions for the SM gauge couplings calculated in the $\overline{\rm MS}$ renormalization scheme with two-loop accuracy and first evaluated at the electroweak scale that we take to be the Z boson, mass $M_Z = 91.2$ GeV [27],

$$[g_Y(M_Z), g_2(M_Z), g_3(M_Z)] = [0.3574, 0.6517, 1.2182]\,, \tag{10}$$

where g_Y should be normalized with the usual GUT condition leading to $\alpha_1/\alpha_Y = 5/3$. Using the two-loop RGEs in the case in which two Higgs doublets are present at low energies and including the relevant threshold corrections following Ref. [13], we determine the point at which the couplings intersect when appropriately adjusting the intermediate scale M_I. In the two symmetry breaking chains that we consider, the tree-level matching conditions that determine the gauge couplings of the intermediate scale models from the low energy ones read

$$\begin{aligned}&\text{PS}: \alpha_4^{-1}(M_I) = \alpha_3^{-1}(M_I)\,,\ \alpha_{2L}^{-1}(M_I) = \alpha_2^{-1}(M_I)\,,\\ &\quad \alpha_{2R}^{-1}(M_I) = \frac{5}{3}\alpha_Y^{-1}(M_I) - \frac{2}{3}\alpha_3^{-1}(M_I)\,,\\ &\text{LR}: \alpha_3^{-1}(M_I) = \alpha_3^{-1}(M_I)\,,\ \alpha_{2L}^{-1}(M_I) = \alpha_2^{-1}(M_I)\,,\\ &\quad \alpha_{B-L}^{-1}(M_I) = \kappa\alpha_{2R}^{-1}(M_I) = \kappa\left(\frac{2\kappa+3}{5}\right)^{-1}\alpha_Y^{-1}(M_I)\,,\end{aligned} \tag{11}$$

where in LR we assume $\alpha_{B-L}^{-1}(M_I) = \kappa\alpha_{2R}^{-1}(M_I)$ as we are matching three couplings to four; this normalization factor κ of $\mathcal{O}(1)$ is to be solved with the scales M_I and M_U.

Note that in eq. (10), we have ignored, for simplicity, the experimental errors on the couplings constants (as well as the theoretical uncertainties) and kept only the central values. These errors, the largest of which being the one that affects the strong coupling constant α_3 which is at the percent level, will generate an uncertainty on the derived GUT an intermediate scales of the order of

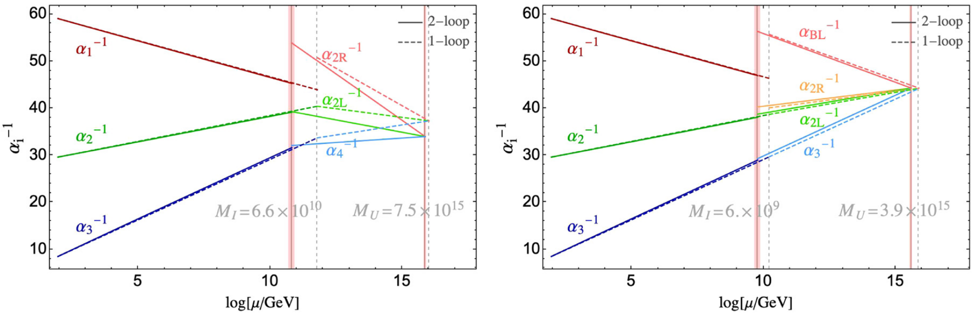

Fig. 1. The evolution of the inverse of the gauge coupling constants $\alpha_i = g_i^2/(4\pi)$ as a function of the energy scale μ in the 2HDM+$\mathcal{G}_{422}$ Pati-Salam model (left) and 2HDM+$\mathcal{G}_{3221}$ minimal LR model (right) at the one-loop (dashed lines) and two-loop (solid lines) orders. The GUT and intermediate scales M_U and M_I are indicated and the threshold effects are included. The red bands reveal the effects of the experimental uncertainties in the measurement of the couplings.

a few percent only and, hence, do not affect our discussion in a significant way as will be shown shortly.

In our analysis, the RGEs of the gauge couplings are solved up to two loop order with the help of the program SARAH [29], and the inclusion of the thresholds corrections is performed by randomly sampling parameters $\eta_i = \ln(M_i/\mu)$ within the range of $\eta_i \in [-1, 1]$. We then impose the tree-level matching conditions of the gauge couplings at the intermediate scale as in eq. (11) when the one-loop threshold corrections are included as in eq. (9), and determine the values of the two scales M_I and M_U for each sampling parameter set. More precisely, we take at least 10,000 points for the parameters η_i within the range of $\eta_i \in [-1, 1]$ and determine the sets of all scales that allow for gauge coupling unification.

In the two intermediate SO(10) scenarios that we consider, gauge coupling unification with the inclusion of threshold corrections can, for instance, be achieved for the following values of the unification and intermediate scales

$$\text{PS}: \; M_U = 7.5 \times 10^{15}\ \text{GeV and } M_I = 6.6 \times 10^{10}\ \text{GeV},$$
$$\text{LR}: \; M_U = 3.9 \times 10^{15}\ \text{GeV and } M_I = 6.0 \times 10^{9}\ \text{GeV}. \quad (12)$$

The evolution of the inverse of the coupling constants α_i^{-1} from the scale M_U down to M_I and then down to M_Z is shown in Fig. 1 as a function of the energy scale in the two breaking patterns PS (left panel) and LR (right panel) when the two-loop (solid lines) and one-loop (dashed lines) RGEs are used and the threshold effects are included at the intermediate scale. While the three couplings are clearly different at the scale M_I of the order of a few times 10^{10} GeV (as required to reproduce neutrino phenomenology), the slope is significantly modified at this energy by the additional contributions so that the couplings meet at a scale M_U of the order of a few times 10^{15} GeV (which is high enough to prevent fast proton decay). Both the two-loop corrections and the threshold corrections have a noticeable impact and make the intermediate scale lower. The small impact of the experimental errors on the couplings is illustrated by the narrow red bands at the scales M_I and M_U.

4. Yukawa coupling unification

We now turn to the Yukawa sector of the theory. As already mentioned, we will ignore the very small Yukawa couplings of the first and second generation fermions[2] and consider only those of the top quark, the bottom quark and the tau lepton, neglecting all possible mixings. Below the intermediate scale M_I, the Yukawa interactions of these fermions are those of a type-II 2HDM with a Lagrangian given by eq. (6). It leads to the following relations between the fermion masses and the Yukawa couplings

$$m_t = \frac{1}{\sqrt{2}} Y_t v_u\,, \quad m_b = \frac{1}{\sqrt{2}} Y_b v_d\,, \quad m_\tau = \frac{1}{\sqrt{2}} Y_\tau v_d\,. \quad (13)$$

In the region between the intermediate scale and the GUT scale, we assume the Yukawa structure of eqs. (7) and (8) for the PS and minimal LR breaking patterns, respectively. With a real $\mathbf{10_H}$ representation with its vevs denoted by $v_{10}^u = v_{10}^{d*} = v_{10}$ and by adopting a phase convention in which v_{10} is real (this can be done via, *e.g.*, an SU(2) rotation) [20], and denoting by $v_{126}^{u,d}$ the vevs of the Σ_{126} field, the fermion masses for the two considered breaking chains will be given by

$$m_t = \frac{v_{10} Y_{10}^{PS} + v_{126}^u Y_{126}^{PS}}{\sqrt{2}}, \; m_b = \frac{v_{10} Y_{10}^{PS} + v_{126}^d Y_{126}^{PS}}{\sqrt{2}},$$
$$m_\tau = \frac{v_{10} Y_{10}^{PS} - 3 v_{126}^d Y_{126}^{PS}}{\sqrt{2}},$$
$$m_t = \frac{v_{10} Y_{10,q}^{LR} + v_{126}^u Y_{126,q}^{LR}}{\sqrt{2}}, \; m_b = \frac{v_{10} Y_{10,q}^{LR} + v_{126}^d Y_{126,q}^{LR}}{\sqrt{2}},$$
$$m_\tau = \frac{v_{10} Y_{10,l}^{LR} + v_{126}^d Y_{126,l}^{LR}}{\sqrt{2}}. \quad (14)$$

Finally, above the GUT scale M_U, the third generation Yukawa couplings are unified as in eq. (2) and are given by

$$m_t = v_{10} Y_{10} + v_{126}^u Y_{126}\,, \; m_b = v_{10} Y_{10} + v_{126}^d Y_{126}\,,$$
$$m_\tau = v_{10} Y_{10} - 3 v_{126}^d Y_{126}, \quad (15)$$

(with the additional masses for neutrinos $M_{\nu_D} = v_{10} Y_{10} - 3 v_{126}^u Y_{126}$ and $M_{\nu_R} = v_R Y_{126}$). The normalization factors can be absorbed into the redefinition of the Yukawa couplings at the GUT scale. The factors of 3 and the relative signs between the various terms are due to the Clebsh-Gordan coefficients coming from the vev of the traceless adjoint 15 of SU(4) in $(\mathbf{2}, \mathbf{2}, \mathbf{15})$.

As the evolution of the couplings near the scale M_I should be affected by threshold corrections, one should expect a significant discontinuity of the Yukawa couplings when the contributions of the numerous scalar and vector fields are included in the RGEs. Nevertheless, as these Yukawa couplings are directly related to the masses of the fermions, one can simply assume that the physical fermion masses are continuous at the scale M_I [25] when these

[2] As in the supersymmetric case, fermions with masses below a few GeV cannot be realistically described in our approach as it will be plagued by strong interaction uncertainties when running the RGEs down to the fermion mass scale.

threshold corrections are included. This means that the masses calculated in the low-energy 2HDM should coincide with those obtained from the intermediate left-right or Pati–Salam models or the unified SO(10) model, up to their running. One can then consider this relation as the matching conditions for the Yukawa couplings at the intermediate and the GUT scales. For example, at the scale M_I, equating eqs. (13)–(14) for the PS and LR breaking chains leads to[3]

$$
\begin{aligned}
Y_t(M_I) &= Y_{10}^{PS}(M_I)\frac{v_{10}}{v_u} + Y_{126}^{PS}(M_I)\frac{v_{126}^u}{v_u} \\
&\text{or } Y_{10,q}^{LR}(M_I)\frac{v_{10}}{v_u} + Y_{126,q}^{LR}(M_I)\frac{v_{126}^u}{v_u}, \\
Y_b(M_I) &= Y_{10}^{PS}(M_I)\frac{v_{10}}{v_d} + Y_{126}^{PS}(M_I)\frac{v_{126}^d}{v_d} \\
&\text{or } Y_{10,q}^{LR}(M_I)\frac{v_{10}}{v_d} + Y_{126,q}^{PS}(M_I)\frac{v_{126}^d}{v_d}, \\
Y_\tau(M_I) &= Y_{10}^{PS}(M_I)\frac{v_{10}}{v_d} - 3Y_{126}^{PS}(M_I)\frac{v_{126}^d}{v_d} \\
&\text{or } Y_{10,l}^{LR}(M_I)\frac{v_{10}}{v_d} + Y_{126,l}^{LR}(M_I)\frac{v_{126}^d}{v_d}.
\end{aligned}
\tag{16}
$$

As for the matching conditions at the GUT scale, one has to carefully take the Clebsh-Gordan factors into account for the Yukawa interactions when the field representations are embedded into the SO(10) group [22,30]. One can then enforce Yukawa coupling unification by requiring the matching conditions at the GUT scale to be

$$Y_f(M_U) \equiv Y_{10}^{PS}(M_U) = \frac{1}{4}Y_{126}^{PS}(M_U), \tag{17}$$

$$
\begin{aligned}
Y_f(M_U) &\equiv Y_{10,q}^{LR}(M_U) = \frac{1}{4}Y_{126,q}^{LR}(M_U) = Y_{10,l}^{LR}(M_U) \\
&= -\frac{1}{12}Y_{126,l}^{LR}(M_U),
\end{aligned}
\tag{18}
$$

where the unified Yukawa coupling $Y_f(M_U)$ is taken to be a free parameter of SO(10).

One has then to fit these parameters with the actual observables, namely the top, bottom and tau masses using the relations in eq. (13) at the low energy scale, chosen again to be $M_Z = 91.2$ GeV. We use the following input $\overline{\rm MS}$ running fermion masses in the SM [27,28] (we again ignore the related experimental uncertainties for now),

$$[m_t(M_Z), m_b(M_Z), m_\tau(M_Z)] = [168.3, 2.87, 1.73]\ \text{GeV}, \tag{19}$$

and we then turn them into the corresponding input masses in the 2HDM by using the appropriate RGEs in the evolution from the scale of the fermion masses to M_Z.

In the PS model, the colored-quarks and leptons are charged under the same local SU(4) symmetry so that all fermions can be unified into the same representation $F_{L,R}$. When these fermions couple to the Higgs fields $\mathbf{10_H}$, one cannot distinguish the bottom quark from the tau lepton and one should have $m_b = m_\tau$ if the vev v_{126}^d is small, $v_{126}^d/v_d \ll 1$, as can be seen from eq. (14). If this mass equality is still valid slightly below the intermediate scale, we should then have $Y_b(M_I) = Y_\tau(M_I)$ in our low energy 2HDM by virtue of eqs. (16) and (13). The scale at which the bottom and tau Yukawa couplings are equal, that we denote by $M_{b\tau}$, is simply determined (within some accuracy) by the point at which the curves for their RG running from the weak scale M_Z upwards intersect, which critically depends on the value of the parameter $\tan\beta$. In order to use the matching conditions given by the equations above, the scale for b-τ unification should be identical to the intermediate PS breaking scale, $M_{b\tau} = M_I$, and this can be achieved by selecting the appropriate value of $\tan\beta$.

The RGEs for the Yukawa couplings from M_U to the intermediate scale M_I and from M_I to the electroweak scale M_Z up to the two-loop level have been given in Ref. [31] in the standard case with one electroweak Higgs doublet only. In our case, we also include the additional contributions of the extra Higgs doublet at the low scale. We solve the system using again the program SARAH [29].

In the PS model, the running of the third generation Yukawa couplings from the low to the high energy scales are shown in the left panel of Fig. 2, for the specific case where the input value $\tan\beta = 58$ is chosen. One can see that, indeed, the curves for Y_b and Y_τ intersect at an energy scale $M_{b\tau} \simeq 7 \times 10^{10}$ GeV, which is very close to the intermediate scale for which the gauge couplings unify in the PS scheme.

The right panel of the figure shows the dependence of the bottom-tau unification scale $M_{b\tau}$ on the ratio of vevs $\tan\beta$ and, as can be seen, intermediate scale values between $M_I = 10^9$ GeV and $M_I = 10^{11}$ GeV would imply high values of $\tan\beta$, in the range $\tan\beta \approx 50 - 60$. Note that the value of $\tan\beta$ cannot be arbitrarily high, $\tan\beta \lesssim 70$ in the specific cases we are discussing here, in order to avoid that the Yukawa couplings run to non-perturbative values at these scales.

In the minimal LR model, the discussion above does not hold and the bottom and tau Yukawa couplings do not unify at the intermediate scale. Nevertheless, one should have close if not equal values for the Yukawa terms $Y_{10,q}^{LR}$ and $Y_{10,l}^{LR}$ such that they can run to a common value at the scale M_U where, according to eq. (18), one has $Y_{10,q}^{LR} = Y_{10,l}^{LR}$.

We come now to the unification of all Yukawa couplings. With the four Yukawa couplings, the randomly chosen one $Y_f(M_U)$ and the three weak scale ones Y_t, Y_b, Y_τ, the scales M_U and M_I to be determined from gauge coupling unification, five parameters are needed to entirely describe our Yukawa sector: the 2HDM vevs v_u, v_d at M_Z and the vevs v_{126}^u, v_{126}^d and v_{10} of the bi-doublets at the scale M_I. Nevertheless, we have many constraints at hand as, besides the matching relations given in eqs. (16-18), one needs to reproduce the experimental values of the standard particle masses.

Indeed, at both the scales M_Z and M_I, the correct W, Z masses should be reproduced, giving $\sqrt{v_u^2 + v_d^2} = v_{\rm SM} = 246\,\text{GeV} = v_{10}^2 + v_{126}^2 + v_{126d}^2$ [24]. One needs also to reproduce the heavy fermion masses at the weak scale M_Z, eq. (19), using the relations of eq. (13). We will assume that there is an uncertainty of the order of 2% in reproducing all these particle masses. This uncertainty, which is sufficiently small for our purpose (and allows us to have some solutions for the coupled RGE's), is introduced not only because of the experimental errors (e.g. on α_3 and the top and bottom masses) but also the theoretical ones from various sources such as the higher order effects in the RGEs, the higher order threshold corrections, the possible running of the vev's, etc.

For completeness, we make sure in addition that the electroweak vacuum remains stable up to the intermediate scale $M_I \approx 10^{10}$ GeV for the chosen top quark and SM-like Higgs boson masses. To do so, we use the necessary and sufficient conditions of Refs. [17,32] on the 2HDM quartic scalar couplings to ensure the scalar potential to be bounded from below, with the input values for the relevant weak scale parameters given in Ref. [33]. As an additional and final constraint, we force the three Yukawa

[3] For this exploratory work, we simply follow Ref. [25] and ignore the small running of the vevs. This issue, together with other refinements, will be postponed to a forthcoming publication.

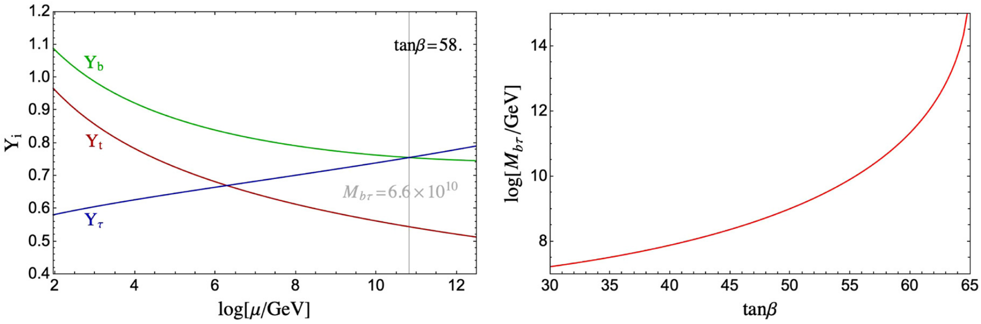

Fig. 2. An example of the running of third generation fermion Yukawa couplings from the weak to the high scales for the value $\tan\beta = 58$ for which the bottom and tau couplings unify at a scale $M_{b\tau} = 6.6 \times 10^{10}$ (left) and the dependence of this unification scale $M_{b\tau}$ on the value of $\tan\beta$ (right).

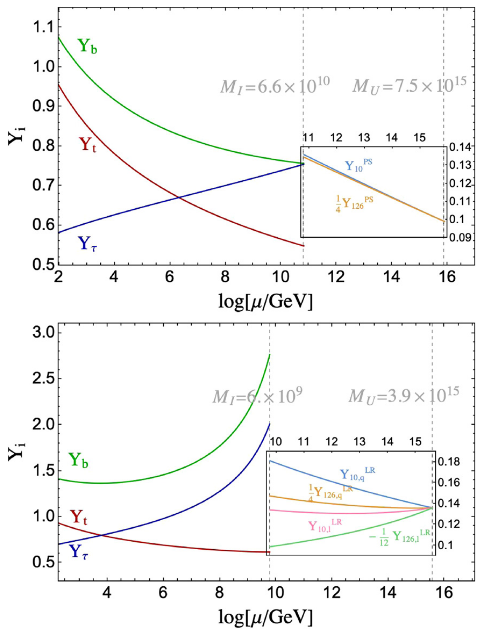

Fig. 3. The renormalization group running of the Yukawa couplings at two-loop in the 2HDM+$\mathcal{G}_{422}$ model (top) and 2HDM+$\mathcal{G}_{3221}$ model (bottom) including matching conditions and threshold effects at an intermediate scale M_I for a parameter set that is compatible with the observed top, bottom, tau as well as gauge boson masses.

couplings to remain perturbative at all scales by imposing the conditions $Y_i^2(\mu)/(4\pi) \le 1$.

We then scan this constrained parameter space in order to find a viable solution to the system of equations. Our main results are displayed in Fig. 3 and in Table 1.

Fig. 3 shows the evolution of the three Yukawa couplings as a function of the energy scale in the PS (upper panel) and LR (lower panel) scenarios and it can be seen that all of them reach a common value, $Y_f(M_U) \approx 0.1$, at the same GUT scale M_U that leads to gauge coupling unification eq. (12). At the intermediate scale M_I that is also required by gauge coupling unification, eq. (12), one notices the discontinuity for the Yukawa couplings which is due to the matching conditions.

Finally, in Table 1, we show examples of points in the 2HDM+PS and 2HDM+LR model parameter spaces that satisfy all the criteria discussed above and list the sets of values for the three fermion couplings and all the relevant vevs which lead to Yukawa coupling unification, with the GUT and intermediate scales that allow for gauge coupling unification, eqs. (12), and with all constraints implemented.

One can see that in both the PS and LR breaking schemes, one obtains approximately the same unified Yukawa coupling $Y_f(M_U) = \mathcal{O}(0.1)$. One can also see that the relations $\sum_i v_i^2 = v_{\rm SM}^2$ are fulfilled at the relevant scales and that all Yukawa couplings are such that their squares are smaller than 4π even at M_I.

In both cases, the obtained values of the input 2HDM parameter $\tan\beta = v_u/v_d$ at the electroweak scale are large but still reasonable, approximately $\tan\beta = 58$ and $\tan\beta = 70$ for the PS and LR models respectively, as they ensure that the bottom quark Yukawa coupling remains perturbative at all energy scales before M_U, and gives the correct hierarchy of quark masses at the electroweak scale, $\tan\beta \approx m_t/m_b$.

Hence, Yukawa coupling unification can also be achieved in a simple manner in a non-supersymmetric SO(10) scenario. One can arrange to achieve it for lower values of $\tan\beta$ than above, at a minimal cost and without affecting the simplicity of the approach, by complexifying the $\mathbf{10_H}$ representation. One still makes use of two Higgs bi-doublets above the scale M_I but there are four non-zero vevs instead of three as $v_{10}^u \neq v_{10}^d$. This additional input can be adjusted to have more adequate solutions to the system of Yukawa coupling RGEs. This possibility, as well as other interesting extensions of the simple scheme proposed here, will be addressed in a forthcoming publication.

5. Conclusions

We have analyzed the possibility of unifying the Yukawa couplings of third generation fermions in the context of a non-supersymmetric SO(10) scenario with intermediate breaking, focusing on the Pati-Salam and minimal left-right breaking chains. The framework that we adopt is rather simple as the relevant scalar sector of the theory consists of only two Higgs bi-doublets at the intermediate breaking scale, $M_I = \mathcal{O}(10^{10})$ GeV, reducing to a two Higgs doublet model of type II at the electroweak scale.

We first discussed gauge coupling unification which can indeed be achieved at a GUT scale close to $M_U \approx 10^{16}$ GeV, by including the threshold effects of the scalar multiplets that appear at the intermediate scale. This is somehow expected as the contribution of the additional electroweak Higgs doublet (and all scalar fields in general) does not significantly modify the running of the gauge coupling constants.

We have then studied the renormalization group running of the Yukawa couplings of the top and bottom quarks and the tau lepton in the Pati-Salam and minimal left-right SO(10) scenarios, with the proper matching conditions at the unification, intermediate

Table 1
A set of the third generation fermion Yukawa couplings at the scales M_Z, M_I and M_U, and the relevant vevs at the weak and intermediate scales, which fit all observables within 2% accuracy at the two-loop level and lead to both gauge coupling and Yukawa coupling unification in SO(10) with PS and LR intermediate breaking.

Scale	M_Z			M_I			M_U	M_Z		M_I		
	Y_t	Y_b	Y_τ	Y_t	Y_b	Y_τ	Y_f	v_d	v_u	v_{10}	v^u_{126}	v^d_{126}
PS	0.97	1.09	0.58	0.55	0.76	0.75	0.10	4.21	246.2	23.4	244.7	0.004
LR	0.97	1.44	0.68	0.62	2.76	2.01	0.14	3.50	246.2	53.2	241.0	0.079

and electroweak scales. We have performed a scan of the parameter space of the two models, imposing that the phenomenology at low energy and, in particular the third generation fermion and the electroweak gauge boson masses, is correctly reproduced within 2% accuracy. We find that the unification of the Yukawa couplings of third generation can be indeed realized in regions of the parameter space in which the ratio of the two electroweak Higgs doublet vevs is large, $\tan\beta \approx 60$.

Hence, similarly to the well known and widely studied supersymmetric case, not only gauge but also Yukawa coupling unification can be achieved in SO(10) while using a rather simple Higgs sector and retaining a viable particle spectrum at the weak scale.

An interesting feature of this possibility is that while most of the ingredients of the conventional SO(10) model are expected to be at a too high scale, $\mathcal{O}(10^{10})$ GeV, to be probed effectively in collider experiments, our scenario requires a second Higgs doublet at low energies. The model thus predicts additional Higgs particles with weak scale masses which could be searched for and eventually be observed at the Large Hadron Collider or at the next generation of high-energy colliders.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix 4

IV

Abdelhak Djouadi, Renato Fonseca, Ruiwen Ouyang, and Martti Raidal. Non-supersymmetric SO(10) models with Gauge and Yukawa coupling unification. *Eur. Phys. J. C*, 83(6):529, 2023

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Non-supersymmetric SO(10) models with Gauge and Yukawa coupling unification

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Abstract We study a non-supersymmetric SO(10) Grand Unification Theory with a very high energy intermediate symmetry breaking scale in which not only gauge but also Yukawa coupling unification are enforced via suitable threshold corrections and matching conditions. For gauge unification, we focus on a few symmetry breaking patterns with the intermediate gauge groups $\mathrm{SU(4)_C \times SU(2)_L \times SU(2)_R}$ (Pati–Salam) and $\mathrm{SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}}$ (minimal left-right symmetry) assuming an additional global U(1) Peccei–Quinn symmetry, and having the Standard Model supplemented by a second Higgs doublet field at the electroweak scale. We derive the conditions as well as the approximate analytical solutions for the unification of the gauge coupling constants at the two-loop level and discuss the constraints from proton decay on the resulting high scale. Specializing to the case of the Pati–Salam intermediate breaking pattern, we then impose also the unification of the Yukawa couplings of third generation fermions at the high scale, again at the two-loop level. In the considered context, Yukawa unification implies a relation between the fermion couplings to the 10- and 126-dimensional scalar representations of the SO(10) group. We consider one such possible relation which is obtainable in an $\mathrm{E_6}$ model where the previous two scalar fields are part of a single multiplet. Taking into account some phenomenological features such as the absence of flavor changing neutral currents at tree-level, we derive constraints on the parameters of the low energy model, in particular on the ratio of the two Higgs doublets vacuum expectation values $\tan\beta$.

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1 Introduction

A Grand Unified Theory (GUT) which describes the four fundamental forces that are present in Nature has always been the Holy Grail of particle physics. Leaving aside the gravitational force which has a rather special status, it has been shown already in the 1970s [1,2] that the concept of gauge symmetries makes it possible to combine in a very elegant manner the electromagnetic, weak and strong interactions of the Standard Model (SM) into a single force at a very high energy scale [3]. This would have been the case of SU(5), the simplest and most economical gauge symmetry group that contains the $\mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y}$ group of the SM as a subgroup. Alas, when the three SM gauge couplings are evolved with the energy scale, starting from their experimentally measured values and including the SM particle content, they shortly fail to meet at a single point, the presumed GUT scale M_U [4–7].

One solution to this problem was to invoke Supersymmetry (SUSY) [8–13], a theory that predicts the existence of a partner to each SM particle and has an extended Higgs sector consisting of two complex scalar fields to break the electroweak symmetry down to the electromagnetic U(1) group [14–16]. The new particle content modifies the slopes of the renormalisation group evolution (RGE) of the gauge couplings such that they meet at a GUT scale that is high enough, $M_U \approx 2 \times 10^{16}$ GeV, to prevent a too fast decay of the proton [4–7]. Another virtue of SUSY, which made it extremely popular in the past four decades, is that it solves the problem of the large hierarchy between the weak and Planck scales that induces quadratic "divergences" to the observed Higgs boson mass. However, in order to resolve both the unification and hierarchy problems, SUSY needs to be broken at an energy not too far from the electroweak scale and, hence, should involve superpartners with masses of a few hundred

GeV to order a TeV at most. Unfortunately, such a low SUSY-breaking scale has been excluded for most superparticles (in particular the strongly interacting ones that are copiously produced) by dedicated and non-conclusive searches at the CERN LHC [17]. Thus, the theory lost some of its appeal as it appears now to be less "natural".

In principle, the existence of extra particles with the appropriate masses and quantum numbers to give the necessary contributions to the RGEs is all what is needed to achieve unification of the three gauge couplings. However, postulating the existence of extra fields for this reason alone might be considered a somewhat contrived solution to the problem. A more appealing possibility is to consider symmetry groups larger than SU(5) which break down to the SM gauge group via a chain that involves intermediate symmetries. In this case, the new scalar multiplets that break these intermediate symmetries (and some of the associated new gauge bosons) will generate additional contributions at the intermediate scale M_I, which will modify the RG evolution of the gauge couplings. Taking into account these threshold corrections, it is then possible to unify the three couplings at a scale M_U [18,19].

Such a unification with an intermediate step can be realized in the context of SO(10) [20,21]; see Refs. [22–25]. This group is particularly interesting as it has a representation of dimension 16 which can accommodate the 15 SM chiral fermions of each generation, as well as an additional Majorana neutrino. If the mass of this neutrino is very large, of the order of 10^{12-14} GeV, other very pressing problems in particle physics can also be addressed. This is, for instance, the case of the complicated pattern of the SM neutrino masses and mixings which can be explained by the see-saw mechanism. This is also the case of the baryon asymmetry in the Universe which could be achieved through a leptogenesis triggered by the additional Majorana neutrino. Hence, SO(10) with an intermediate scale of $\mathcal{O}(10^{12-14})$ GeV, could explain the most acute problems of the SM that call for new physics beyond it, leaving aside the hierarchy problem and introducing a suitable axion that could account for the particle that forms the dark matter in the Universe; see Refs. [26–28] for reviews.

Another issue for which low-energy SUSY theories gained popularity, is the unification of the Yukawa couplings of third generation fermions [29–32]. This additional step in the unification paradigm is accomplished in the minimal supersymmetric extension of the SM (MSSM), thanks to the presence of the two-Higgs doublets fields that are required by the extended symmetry. In constrained scenarios, such as the minimal supergravity model with universal "soft" SUSY-breaking parameters [33–35], the top, bottom and tau Yukawa couplings can be unified at the same scale M_U that allows for gauge coupling unification. Indeed, for large values of the ratio of the vacuum expectation values of the two Higgs fields, $\tan\beta$, one can generate the required hierarchy for the top and bottom quark masses, $\tan\beta \approx m_t/m_b \approx 60$, and the RG evolution that allows the couplings to also meet at M_U.

In a recent letter, we have contemplated the possibility of Yukawa coupling unification in the context of a non-supersymmetric SO(10) model as well [36]. Focusing on the most widely studied scenarios with intermediate symmetry breaking, namely the Pati–Salam scenario with the intermediate group $SU(4)_C \times SU(2)_L \times SU(2)_R$ [2] and the minimal left-right symmetry group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [37–39], we have shown that in a two-doublet Higgs model (2HDM) extension present at the electroweak scale, exactly like in the MSSM, one can first obtain the correct hierarchy for the masses m_t and m_b by again taking a ratio $\tan\beta$ that is sufficiently high. In both schemes, it is then possible to arrange such that the RG running of third generation Yukawa couplings, with suitable matching conditions at the same intermediate scale M_I for which gauge coupling unification occurs, leads to Yukawa coupling unification at the same GUT scale M_U. This can be achieved while preserving important phenomenological features such as reproducing third family fermion and weak gauge boson masses, ensuring the stability of the electroweak vacuum up to the high scales and keeping the Yukawa couplings perturbative.

In this paper, we perform a more exhaustive analysis of the possibility of simultaneous gauge and Yukawa coupling unification, extending the earlier analysis [36] in several directions. Firstly, the present discussion is more thorough and general, as our results are valid for any breaking chain of non-SUSY SO(10) models with only one intermediate scale and we consider the interplay between gauge coupling unification, proton decay, the perturbativity of the Yukawa couplings and, more importantly, the absence of flavor changing neutral currents at tree-level. Secondly, for gauge coupling unification, we present some approximations which highly simplify the analytical discussions of the RGEs and we discuss unification in models in which one adds a global U(1) Peccei–Quinn symmetry [40] that would allow the resulting axion to address the dark matter problem; this will have important repercussions on the breaking pattern, the RGE running of the couplings as well as on the fermionic mass pattern. A third difference when compared to Ref. [36] is that in the present work, we study the case in which the condition for Yukawa couplings unification at the high scale is inspired by the existence of an even larger E_6 gauge symmetry.

The rest of the paper is organized as follows. In the next section, we introduce the non-SUSY SO(10) model, discuss its various intermediate breaking schemes and the weak scale 2HDM structure. In Sect. 3, we enforce gauge coupling unification using threshold effects and discuss some approximations. In Sect. 4, we analyze the issue of simultaneously unifying the gauge and third generation fermion Yukawa couplings. A short conclusion is made in Sect. 5 and some analytical complementary material is given in an Appendix.

2 Non-SUSY SO(10) with an intermediate scale

In this section, we will summarize how unification of the three gauge interactions of the SM can be achieved in a non-supersymmetric SO(10) GUT with a spontaneous symmetry breaking pattern that involves an intermediate gauge group at a very high scale which breaks down to the SM gauge group. A very interesting aspect of the SO(10) model is that all fermions can be embedded into a single representation of the symmetry group.

Indeed, the SO(10) group possesses a fundamental 16-dimensional representation $\mathbf{16_F}$ in which, for each generation, the 15 SM chiral fermions[1] as well as one right-handed neutrino can be embedded. In this case, the allowed Yukawa couplings of the scalar bosons to pairs of these fermions belong to the direct product representation $\mathbf{16_F} \times \mathbf{16_F}$, which can be decomposed into

$$\mathbf{16_F} \times \mathbf{16_F} = \mathbf{10} + \overline{\mathbf{126}} + \mathbf{120}. \tag{1}$$

Thus, the most general Yukawa interactions are given by the expression

$$-\mathcal{L}_{\text{Yukawa}} = \mathbf{16_F}(Y_{10}\mathbf{10_H} + Y_{126}\overline{\mathbf{126}}_\mathbf{H} + Y_{120}\mathbf{120_H})\mathbf{16_F}, \tag{2}$$

where $\mathbf{10_H}$, $\overline{\mathbf{126}}_\mathbf{H}$, and $\mathbf{120_H}$ denote the scalar representations of SO(10) group. However, among the large number of scalar field components in these representations, we will assume all those that do not participate in the symmetry breaking mechanism by acquiring vacuum expectations values (vevs) will have masses of the order of the SO(10) symmetry-breaking scale. This is known as the extended survival hypothesis [41–44], by which one can safely decouple most of the redundant ingredients in the SO(10) scalar representations at the GUT scale and be left only with the light Higgs boson spectrum of the low-energy effective theory which is present at the electroweak scale. The hypothesis helps to drastically reduce the number of scalar fields that couple to fermions and, hence, to simplify the structure of the Yukawa sector of the model.

As was discussed in many instances, see for instance Ref. [27], the Yukawa sector of the SO(10) model must consist of a $\overline{\mathbf{126}}_\mathbf{H}$ representation, to trigger the see-saw mechanism via the breaking of the left-right symmetry at an intermediate scale M_I. One additional scalar representation, either the $\mathbf{10_H}$ or the $\mathbf{120_H}$, is needed to break the SM gauge symmetry. Because the main difference between these two representations is that the $\mathbf{120_H}$ decomposes into four scalar doublets under the SM group, while the $\mathbf{10_H}$ representation decomposes only in two scalar doublets, based on minimality one should consider the Yukawa sector of SO(10) with only the $\mathbf{10_H}$ and the $\overline{\mathbf{126}}_\mathbf{H}$ representations, leading to the so-called minimal SO(10) models. Note that more scalar representations, which do not affect fermion masses, are needed to achieve the correct symmetry breaking pattern down to the Standard Model group, as will be discussed in the breaking patterns below.

Given that the 10-dimensional representation of SO(10) is real, the field $\mathbf{10_H}$ could in principle be real. However, it was shown [27] that a scalar sector composed of a real $\mathbf{10_H}$ and a complex $\overline{\mathbf{126}}_\mathbf{H}$ leads to an unrealistic mass spectrum for the second and third generation fermions.[2]

The simplest possible extension is to complexify the original real $\mathbf{10_H}$, which leads to the following minimal SO(10) model with complex scalar fields

$$-\mathcal{L}_Y = \mathbf{16_F}(Y_{10}\mathbf{10_H} + Y_{10^*}\mathbf{10_H^*} + Y_{126}\overline{\mathbf{126}}_\mathbf{H})\mathbf{16_F}. \tag{3}$$

The price to pay is that we need to introduce a new Yukawa coupling Y_{10^*} which makes the theory less predictive. To avoid the extra independent Yukawa coupling associated with the $\mathbf{10_H^*}$, we will assume in this paper an additional global U(1) Peccei–Quinn symmetry [40] with the following charge assignment for some real parameter α

$$\mathbf{16_F} \to e^{i\alpha}\mathbf{16_F}, \quad \mathbf{10_H} \to e^{-2i\alpha}\mathbf{10_H}, \quad \overline{\mathbf{126}}_\mathbf{H} \to e^{-2i\alpha}\overline{\mathbf{126}}_\mathbf{H}. \tag{4}$$

It reduces Eq. (3) to

$$-\mathcal{L}_Y = \mathbf{16_F}(Y_{10}\mathbf{10_H} + Y_{126}\overline{\mathbf{126}}_\mathbf{H})\mathbf{16_F}. \tag{5}$$

There are two additional motivations for adding such a global symmetry to our model. A first one is that, when the $\mathrm{U(1)_{PQ}}$ symmetry is broken by assigning a vev to a SO(10) scalar, it influences the symmetry breaking pattern and the renormalization group running of the gauge couplings, as we will see shortly. Another and more phenomenological motivation is that it implies the existence of an axion which can solve the strong CP problem [40] and, at the same time, provide a good candidate for the dark matter in the Universe.

The breaking of SO(10) to the SM gauge group, which we will denote for shortness by

$$\mathrm{G_{SM}} \equiv \mathrm{G_{321}} = \mathrm{SU(3)_C} \times \mathrm{SU(2)_L} \times \mathrm{U(1)_Y}, \tag{6}$$

can be triggered in several ways. As mentioned in the introduction, in the case of non-SUSY SO(10) models, the exis-

[1] For a single generation of the SM fermions, one has two chiralities time six colored quarks and one charged lepton, plus a left-handed neutrino; this makes 15 degrees of freedom in total.

[2] If the $\mathbf{10_H}$ field is real and there is more than one generation of fermions, the mass ratio of isospin up and down-type quarks is predicted to be of order 1 [27], in contradiction with the observed quark masses: $m_t/m_b \gg 1$. This indicates that a more complicated scenario should be considered.

tence of intermediate scales M_I play an important role in the unification of the gauge couplings at some scale M_U. More precisely, as the evolution of the U(1) coupling needs to be strongly modified to meet with the other two couplings at a unique scale that should be high enough, large contributions from additional gauge bosons are required. These contributions can be provided by the particles that are present at the intermediate breaking step, i.e. at the scale M_I. We will stick to the breaking chains with only one intermediate-step involving a left-right (LR) symmetry – with a $SU(2)_R$ group – to invoke the see-saw mechanism for neutrinos. The embedding of the $SU(2)_R$ symmetry above the intermediate scale M_I strongly affects the gauge coupling evolution.

In our analysis, we will interested in the following breaking patterns:

$$422: \quad SO(10)|_{M_U} \xrightarrow{\langle \mathbf{210_H} \rangle} \mathcal{G}_{422}|_{M_I} \xrightarrow{\langle \overline{\mathbf{126}}_\mathbf{H} \rangle} \mathcal{G}_{321}|_{M_Z} \xrightarrow{\langle \mathbf{10_H} \rangle} \mathcal{G}_{31}; \quad (7)$$

$$422D: \quad SO(10)|_{M_U} \xrightarrow{\langle \mathbf{54_H} \rangle} \mathcal{G}_{422} \times \mathcal{D}|_{M_I} \xrightarrow{\langle \overline{\mathbf{126}}_\mathbf{H} \rangle} \mathcal{G}_{321}|_{M_Z} \xrightarrow{\langle \mathbf{10_H} \rangle} \mathcal{G}_{31}; \quad (8)$$

$$3221: \quad SO(10)|_{M_U} \xrightarrow{\langle \mathbf{45_H} \rangle} \mathcal{G}_{3221}|_{M_I} \xrightarrow{\langle \overline{\mathbf{126}}_\mathbf{H} \rangle} \mathcal{G}_{321}|_{M_Z} \xrightarrow{\langle \mathbf{10_H} \rangle} \mathcal{G}_{31}; \quad (9)$$

$$3221D: \quad SO(10)|_{M_U} \xrightarrow{\langle \mathbf{210_H} \rangle} \mathcal{G}_{3221} \times \mathcal{D}|_{M_I} \xrightarrow{\langle \overline{\mathbf{126}}_\mathbf{H} \rangle} \mathcal{G}_{321}|_{M_Z} \xrightarrow{\langle \mathbf{10_H} \rangle} \mathcal{G}_{31}, \quad (10)$$

where $\mathcal{D}$ refers to a left-right discrete symmetry, called $\mathcal{D}$ parity, transforming spinors of opposite chirality [45–48]. We have used the following abbreviations for the gauge groups

$$\begin{aligned} \mathcal{G}_{422} &\equiv SU(4)_C \times SU(2)_L \times SU(2)_R, \\ \mathcal{G}_{3221} &\equiv SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}, \end{aligned} \quad (11)$$

with the former being the Pati–Salam (PS) group and the latter being the minimal left-right (LR) gauge group.

To achieve the desired symmetry breaking in these scenarios, one would necessarily need to introduce scalar multiplets that acquire vevs at the corresponding high scales. In the breaking chains above, the scalar content that acquires vevs at the intermediate scale M_I or at the electroweak scale M_Z consists of, respectively, the $\overline{\mathbf{126}}_\mathbf{H}$ and $\mathbf{10_H}$ representations; while at the GUT scale M_U, the relevant representations that break the SO(10) symmetry are $\mathbf{210_H}$, $\mathbf{54_H}$ and $\mathbf{45_H}$; the latter will not enter our discussion here.

Despite the large number of scalars, under the extended survival hypothesis, most of them have a mass of the order of M_U, and only certain scalar components from the $\mathbf{10_H}$ and $\overline{\mathbf{126}}_\mathbf{H}$ representations acquire masses below the GUT scale; they are the only ones to contribute to the running of the various couplings between the two scales M_I and M_U.

Table 1 List of scalar multiplets containing light fields, for each intermediate symmetry. They are the only ones which are not integrated out below the SO(10) symmetry breaking scale mass M_U

Intermediate symmetry	Scalar multiplets
422	$\Phi_{10} \oplus \Sigma_{126} \oplus \Delta_R \oplus \Delta_{45R}$
422D	$\Phi_{10} \oplus \Sigma_{126} \oplus \Delta_L \oplus \Delta_R \oplus \Delta_{45L} \oplus \Delta_{45R}$
3221	$\Phi_{10} \oplus \Sigma_{126} \oplus \Delta_R$
3221D	$\Phi_{10} \oplus \Sigma_{126} \oplus \Delta_L \oplus \Delta_R \oplus \Delta_{45L} \oplus \Delta_{45R}$

In the different scenarios, these are: $(\mathbf{1},\mathbf{2},\mathbf{2})$ or $(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{0})$ (Φ_{10}) from $\mathbf{10_H}$ and $(\mathbf{15},\mathbf{2},\mathbf{2}) \oplus (\mathbf{10},\mathbf{1},\mathbf{3})$ or $(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{0}) \oplus (\mathbf{1},\mathbf{1},\mathbf{3},\mathbf{2})$ $(\Sigma_{126} \oplus \Delta_R)$ from $\overline{\mathbf{126}}_\mathbf{H}$ for the gauge group $\mathcal{G}_{422}$ or $\mathcal{G}_{3221}$, correspondingly.

On the other hand, the global $U(1)_{PQ}$ symmetry in these chains can be simultaneously broken at a distinct scale by assigning a PQ charge to an SO(10) scalar [49–55]. For the Pati–Salam model, one of the options could be that the $\mathbf{45_H}$ scalar representation from SO(10) acquires a vev, in addition to the vev of $\overline{\mathbf{126}}_\mathbf{H}$ that allows to break the linear combination of PQ, $B-L$ and T_{3R} at the intermediate scale [27,56,57]. This allows for the breaking of the Peccei–Quinn symmetry and the Pati–Salam symmetry with the minimal ingredients from SO(10) and, at the same time, avoids the unnecessary fine-tuning of introducing an SO(10) singlet [25]. For the latter, the only price we need to pay is to have an extra scalar field $(\mathbf{1},\mathbf{1},\mathbf{3})$ or $(\mathbf{1},\mathbf{1},\mathbf{3},\mathbf{0})$ (Δ_{45R}) from the $\mathbf{45_H}$ that contributes to the running of gauge couplings between the intermediate and the GUT scales.

For the breaking chain involving an intermediate $\mathcal{D}$ parity, namely 422D or 3221D in our case, similar arguments can be invoked, except that we have to also add the representation $(\mathbf{10},\mathbf{3},\mathbf{1})$ or $(\mathbf{1},\mathbf{3},\mathbf{1},-\mathbf{2})$ (Δ_L) from $\overline{\mathbf{126}}_\mathbf{H}$ representation as well as the $(\mathbf{1},\mathbf{3},\mathbf{1})$ or $(\mathbf{1},\mathbf{3},\mathbf{1},\mathbf{0})$ (Δ_{45L}) from $\mathbf{45_H}$ representation in order to preserve the $\mathcal{D}$ parity. Lastly, for the special case of the 3221 chain in Eq. (9) and because the $\mathbf{45_H}$ is assigned a vev directly at the GUT scale to break down both the SO(10) and the Pati–Salam symmetry to their $\mathcal{G}_{3221}$ subgroup, the PQ symmetry is expected to be also broken at the GUT scale together with the SO(10) symmetry, and thus no scalars from the $\mathbf{45_H}$ scalar representation survive at the intermediate scale. Table 1 gives, for each breaking chain, the surviving scalars at the intermediate scale.

As a result, in the different breaking patterns, the number of Higgs multiplets that contribute to the renormalization group running of the gauge couplings above the intermediate scale is different. Broadly speaking, the larger the number of Electroweak scalars that contribute to the RGEs is, the faster the couplings will evolve, and the lower the intermediate or the GUT scale will be. As we will shortly see, this will have very important consequences in the scenarios that we are adopting in our analysis, which has an extended Higgs sector at the electroweak scale.

Above the intermediate scale M_I, only the two Higgs bi-doublet fields, decomposing from the $\mathbf{10_H}$ and $\overline{\mathbf{126}}_\mathbf{H}$ representations, will couple to the fermions. Starting from Eq. (5), the Yukawa Lagrangian for fermions at the intermediate scale M_I can be written in each of considered schemes as

$$\begin{aligned}
-\mathcal{L}_Y^{422} &= \bar{F}_L(Y_{10}\Phi_{10} + Y_{126}\Sigma_{126})F_R \\
&\quad + Y_R F_R^T C\overline{\Delta_R} F_R + \text{h.c.}, \\
-\mathcal{L}_Y^{422D} &= \bar{F}_L(Y_{10}\Phi_{10} + Y_{126}\Sigma_{126})F_R + Y_L F_L^T C\overline{\Delta_L} F_L \\
&\quad + Y_R F_R^T C\overline{\Delta_R} F_R + \text{h.c.}, \\
-\mathcal{L}_Y^{3221} &= \bar{Q}_L(Y_{10,q}\Phi_{10} + Y_{126,q}\Sigma_{126})Q_R + \bar{L}_L(Y_{10,l}\Phi_{10} \\
&\quad + Y_{126,l}\Sigma_{126})L_R \\
&\quad + Y_R L_R^T i\sigma_2 \Delta_R L_R + \text{h.c.}, \\
-\mathcal{L}_Y^{3221D} &= \bar{Q}_L(Y_{10,q}\Phi_{10} + Y_{126,q}\Sigma_{126})Q_R \\
&\quad + \bar{L}_L(Y_{10,l}\Phi_{10} \\
&\quad + Y_{126,l}\Sigma_{126})L_R \\
&\quad + Y_L L_L^T i\sigma_2 \Delta_L L_L + Y_R L_R^T i\sigma_2 \Delta_R L_R + \text{h.c.},
\end{aligned} \tag{12}$$

where $F_{L,R}$ are generic left or right-handed SU(4) fermion fields, Q, L are quark/lepton fields, and σ_2 one of the Pauli matrices. In both cases, we have assumed that terms like $\bar{F}_L^T\tilde{\phi}F_R$ with $\phi = \Phi$ or Σ and $\tilde{\phi} = \sigma_2^T\phi^*\sigma_2$ are forbidden by suitably chosen $U(1)_Y$ charges [58].

At the intermediate scale, the corresponding left-right symmetry is broken by the right-handed triplet. As we assume that both the $\mathbf{10_H}$ and the $\overline{\mathbf{126}}_\mathbf{H}$ representations are complex, their vevs should be aligned in the following way to break the intermediate $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry down to the electromagnetic $U(1)_{EM}$ symmetry

$$\begin{aligned}
\langle\Phi_{10}\rangle &= \frac{1}{\sqrt{2}}\begin{pmatrix}\kappa_{10}^u e^{i\theta_{10}^u} & 0 \\ 0 & \kappa_{10}^d e^{i\theta_{10}^d}\end{pmatrix}, \\
\langle\Sigma_{126}\rangle &= \frac{1}{\sqrt{2}}\begin{pmatrix}\kappa_{126}^u e^{i\theta_{126}^u} & 0 \\ 0 & \kappa_{126}^d e^{i\theta_{126}^d}\end{pmatrix}, \\
\langle\Delta_L\rangle &= \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 0 \\ \kappa_L e^{i\theta_L} & 0\end{pmatrix}, \\
\langle\Delta_R\rangle &= \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 0 \\ \kappa_R e^{i\theta_R} & 0\end{pmatrix},
\end{aligned} \tag{13}$$

where for later convenience we use the following notation to denote the $\mathbf{10_H}$ and $\overline{\mathbf{126}}_\mathbf{H}$ vevs

$$v_a^b = \kappa_a e^{i\theta_a^b}\ (a = 10, 126;\ b = u, d). \tag{14}$$

In our vevs assignment, the left-handed triplet should also acquire a tiny but nonzero vev,[3] $v_L \sim 0$, while the right-handed triplet should have an intermediate-scale vev, $v_R \sim M_I$.

Below the intermediate scale, the low-energy models include, besides the triplet field Δ_R that gives masses to the heavy right-handed neutrino species, four Higgs doublet fields $\phi_{1,\dots,4}$: the two doublets ϕ_1 and ϕ_3 from the bi-doublet Φ_{10} and which have opposite hypercharge $Y_\phi = \pm 1$ and the doublets ϕ_2 and ϕ_4 again with opposite hypercharge from the bi-doublet Σ_{126}. The fields ϕ_1 and ϕ_2 will couple to up-type quarks and the heavy right-handed neutrinos, while the fields ϕ_3 and ϕ_4 will couple to down-type quarks and the light leptons. While the triplet fields acquire a very large vev, $\langle\Delta_R\rangle = v_R \sim \mathcal{O}(M_I)$, the bi-doublet fields acquire vevs of the order of the electroweak scale. This should imply the relation $\sum_{i=1}^4 v_i^2 = v_{SM}^2 \simeq (246\,\text{GeV})^2$ between vevs, when their running is neglected. This ensures that the right-handed gauge bosons are very heavy, $M_{W_R}, M_{Z_R} \approx g v_R$, while the $SU(2)_L$ W and Z bosons have weak scale masses, $M_W, M_Z \approx g v_{SM}$.

In fact, one should arrange such that only two linear combinations of the four scalar doublet fields $\phi_1, \dots \phi_4$ acquire masses of the order of the electroweak scale, while the masses of the two other field combinations should be close to the very high scale M_I. The two fields with weak scale masses will be ultimately identified with the doublets H_u and H_d of the low energy 2HDM that we adopt here. At the intermediate scale M_I, these fields should match the Φ_{10} and Σ_{126} fields, the interactions of which have been given in Eq. (12), and will be discussed in details in Sect. 4.

To achieve this peculiar configuration, one has to tune the parameters of the scalar potential of the model and a discussion of this issue, together with the constraints to which these parameters should obey, has been made in e.g. Refs. [59, 60] and we refer to them for the relevant details.

Hence, for each decomposing bi-doublet, only one Higgs doublet remains light and the rest of the scalar multiplets acquire intermediate scale masses. In this respect, at low energies, we will have in fact a model with two Higgs doublet fields H_u and H_d that couple separately to isospin $+\frac{1}{2}$ and $-\frac{1}{2}$ fermions and acquire vevs v_u and v_d

$$\langle H_u\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0 \\ v_u\end{pmatrix}, \quad \langle H_d\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0 \\ v_d\end{pmatrix}, \tag{15}$$

to generate masses to the W and Z bosons, thus implying the relation $\sqrt{v_u^2 + v_d^2} = v_{SM} \simeq 246$ GeV. We further define the ratio of these two vevs to be $\tan\beta = v_u/v_d$. The most general renormalizable scalar potential of this two Higgs doublet

[3] The vev of the left-handed triplet must be tiny but non-zero in order to comply with the phenomenology of the light neutrinos which have masses of the order of 1 eV or less.

model may be written [61]

$$V_H = m_{11}^2 H_d^\dagger H_d + m_{22}^2 H_u^\dagger H_u - \left(m_{12}^2 H_d H_u + \text{h.c.}\right) \\ +\lambda_1 \left(H_d^\dagger H_d\right)^2 + \lambda_2 \left(H_u^\dagger H_u\right)^2 \\ +\lambda_3 \left(H_d^\dagger H_d\right)\left(H_u^\dagger H_u\right) \\ +\lambda_4 \left(H_d H_u\right)\left(H_u^\dagger H_d^\dagger\right) \\ +\left[\lambda_5 \left(H_d H_u\right)^2 + \lambda_6 \left(H_d^\dagger H_d\right)\left(H_d H_u\right) \right. \\ \left. +\lambda_7 \left(H_u^\dagger H_u\right)\left(H_d H_u\right) + \text{h.c.}\right]. \quad (16)$$

We will later discuss in details the above scalar sector, in particular when it comes to the perturbativity of the various couplings and the stability of the corresponding vacua. These impose severe constraints on the model as we will see.

The Yukawa interactions of the fermions are those of a Type-II 2HDM [61] with a Lagrangian given by

$$-\mathcal{L}_Y^{2\text{HDM}} = Y_u \bar{Q}_L H_u\, u_R + Y_d \bar{Q}_L H_d\, d_R \\ +Y_e \bar{L}_L H_d\, e_R + \text{h.c.}, \quad (17)$$

with Q_L/L_L the quark/lepton left-handed doublets and f_R the right-handed singlets. In our discussion, only the third generation fermions will be considered and the small Yukawa couplings of the first two generations will be neglected. The relations between the masses and Yukawa couplings are then simply given by

$$m_t = \frac{1}{\sqrt{2}} Y_t v_u, \quad m_b = \frac{1}{\sqrt{2}} Y_b v_d, \quad m_\tau = \frac{1}{\sqrt{2}} Y_\tau v_d. \quad (18)$$

Having introduced these essential elements, we can now discuss the unification of the gauge and Yukawa couplings.

3 Gauge couplings unification with thresholds

3.1 Approximate solutions of the RGEs

In this section, we present some analytical expressions for the renormalization group evolution of the three SM gauge couplings, which can be used to derive the unification scale M_U and the universal coupling constant α_U at this scale for any breaking pattern of the non-SUSY SO(10) GUTs with an intermediate scale M_I. The RGEs with an energy scale μ of the couplings $\alpha_i = g_i^2/4\pi$, where g_i are the coupling constants of the SU(3), SU(2) and U(1) groups for respectively $i = 3, 2, 1$, are given by the following differential equations

$$\frac{\mathrm{d}\alpha_i^{-1}(\mu)}{\mathrm{d}\ln\mu} = -\frac{a_i}{2\pi} - \sum_j \frac{b_{ij}}{8\pi^2 \alpha_j^{-1}(\mu)}. \quad (19)$$

Including the Yukawa interactions, the solutions take the following approximate form in terms of a reference scale μ_0

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(\mu_0) - \frac{a_i}{2\pi} \ln\frac{\mu}{\mu_0} \\ -\frac{1}{4\pi} \sum_j \frac{b_{ij}}{a_j} \ln\frac{\alpha_j(\mu)}{\alpha_j(\mu_0)} + \Delta_Y^i. \quad (20)$$

The one- and two-loop β coefficients (as they are usually called; not to be confused with the ratio of vevs $\tan\beta$), a_i and b_{ij}, are given explicitly in Appendix A1 for the symmetry groups and representations that we are considering. Δ_Y^i stands for the Yukawa couplings contributions that enter at two-loops but, as they only have a very small impact on the running of the gauge couplings compared to the other two-loop contributions, we will neglect them in our computation. The detailed calculation including the two-loop Yukawa contributions to the gauge couplings can be found in Ref. [62] for instance.

In addition, at the intermediate symmetry breaking scales, threshold effects [18,19] due to all the particles that have masses in the vicinity of these scales and, in particular, all the scalar fields that develop vevs at these scales, will be active. These higher order corrections will modify the matching conditions of the gauge couplings at the symmetry breaking scale, depending on the particle content. For a general symmetry breaking from a group $\mathcal{G}$ to a subgroup $\mathcal{H}$ at the scale μ, the matching conditions with the threshold corrections included take the form

$$\alpha_{i,\mathcal{G}}^{-1}(\mu) = \alpha_{i,\mathcal{H}}^{-1}(\mu) + \frac{\lambda_{i,\mathcal{H}}^{\mathcal{G}}}{12\pi}, \quad (21)$$

where $\lambda_{i,\mathcal{H}}^{\mathcal{G}}$ are weighted by the parameters $\eta_i = \ln(M_i/\mu)$ with M_i being the masses of the heavy particles integrated out at the low energy scale. The complete expressions for the one-loop threshold corrections $\lambda_{i,\mathcal{H}}^{\mathcal{G}}$ at the relevant scale are given in Ref. [25] for the models that we are considering here.

At this stage, combining Eqs. (20) and (21), the gauge couplings at a low scale $\alpha_{i,\mathcal{H}}^{-1}(\mu_0)$ can be evolved to an arbitrary high scale μ where the gauge couplings are embedded into a higher symmetric group $\mathcal{G}$ as

$$\alpha_{i,\mathcal{G}}^{-1}(\mu) = \alpha_{i,\mathcal{H}}^{-1}(\mu_0) - \frac{a_i^{\mathcal{H}}}{2\pi} \ln\frac{\mu}{\mu_0} \\ -\frac{1}{4\pi} \sum_j \frac{b_{ij}^{\mathcal{H}}}{a_j^{\mathcal{H}}} \ln\frac{\alpha_{j,\mathcal{H}}(\mu)}{\alpha_{j,\mathcal{H}}(\mu_0)} + \frac{\lambda_{i,\mathcal{H}}^{\mathcal{G}}}{12\pi}, \quad (22)$$

where the two-loop corrections can be approximated with the following relation

$$-\frac{1}{4\pi} \sum_j \frac{b_{ij}^{\mathcal{G}}}{a_j^{\mathcal{G}}} \ln\frac{\alpha_{j,\mathcal{G}}(\mu)}{\alpha_{j,\mathcal{G}}(\mu_0)} \approx -\frac{\alpha_U}{8\pi^2} \theta_i^{\mathcal{G}} \ln\frac{\mu}{\mu_0}, \quad (23)$$

where α_U is the universal gauge coupling at the GUT scale and the coefficient

$$\theta_i^{\mathcal{G}} \equiv \sum_j b_{ij}^{\mathcal{G}} \frac{\ln(1+a_j^{\mathcal{G}}\alpha_U t)}{a_j^{\mathcal{G}}\alpha_U t} \quad \text{and} \quad t = \frac{1}{2\pi}\ln\frac{\mu}{\mu_0} \tag{24}$$

are defined with the same way as in Ref. [62]. The exact forms of the coefficients $\theta_i^{\mathcal{G}}$ are given in Appendix A2 for all the considered symmetry groups.

The unification of the gauge couplings at the scale M_U sets the boundary conditions for the RGEs, which are valid for any breaking pattern of SO(10) with an intermediate gauge group $\mathcal{G}_I$

$$\alpha_U^{-1} = \alpha_{i,\mathcal{G}_I}^{-1}(M_U) + \frac{\lambda_{i,\mathcal{G}_I}^{\mathrm{SO}(10)}}{12\pi}. \tag{25}$$

At the intermediate scale M_I, depending on the symmetry breaking chain, the gauge couplings are related with the ones at low-energy by proper normalization of the generators. As an example, in the particular symmetry breaking chains that we consider, one has

$$\begin{aligned} 422/422D: \ & \alpha_{4,\mathcal{G}_{422}}^{-1}(M_I) = \alpha_{3,\mathcal{G}_{321}}^{-1}(M_I), \\ & \alpha_{2_L,\mathcal{G}_{422}}^{-1}(M_I) = \alpha_{2,\mathcal{G}_{321}}^{-1}(M_I), \\ & \alpha_{2_R,\mathcal{G}_{422}}^{-1}(M_I) = \frac{5}{3}\alpha_{1,\mathcal{G}_{321}}^{-1}(M_I) - \frac{2}{3}\alpha_{3,\mathcal{G}_{321}}^{-1}(M_I), \end{aligned} \tag{26}$$

in the 422/422D cases and, in the case of the 3221 and 3221D breaking chains,

$$\begin{aligned} 3221: \ & \alpha_{3,\mathcal{G}_{3221}}^{-1}(M_I) = \alpha_{3,\mathcal{G}_{321}}^{-1}(M_I), \\ & \alpha_{2_L,\mathcal{G}_{3221}}^{-1}(M_I) = \alpha_{2,\mathcal{G}_{321}}^{-1}(M_I), \\ & \alpha_{B-L,\mathcal{G}_{3221}}^{-1}(M_I) = \kappa\alpha_{2_R,\mathcal{G}_{3221}}^{-1}(M_I) \\ & = \left(\frac{2\kappa+3}{5\kappa}\right)^{-1}\alpha_{1,\mathcal{G}_{321}}^{-1}(M_I), \\ 3221D: \ & \alpha_{3,\mathcal{G}_{3221}}^{-1}(M_I) = \alpha_3^{-1}(M_I), \ \alpha_{2_L,\mathcal{G}_{3221}}^{-1}(M_I) \\ & = \alpha_{2_R,\mathcal{G}_{3221}}^{-1}(M_I) = \alpha_{2,\mathcal{G}_{321}}^{-1}(M_I), \\ & \alpha_{B-L,\mathcal{G}_{3221}}^{-1}(M_I) = \frac{5}{2}\alpha_{1,\mathcal{G}_{321}}^{-1}(M_I) - \frac{3}{2}\alpha_{2,\mathcal{G}_{321}}^{-1}(M_I). \end{aligned} \tag{27}$$

In the 422D chain we also require $\alpha_{2_L,\mathcal{G}_{422}}^{-1}(M_I) = \alpha_{2_R,\mathcal{G}_{422}}^{-1}(M_I)$ to preserve the $\mathcal{D}$ parity, and in the 3221 chain, we assume $\alpha_{B-L,\mathcal{G}_{3221}}^{-1}(M_I) = \kappa\alpha_{2_R,\mathcal{G}_{3221}}^{-1}(M_I)$ as we are matching three couplings to four. This normalization factor κ of $\mathcal{O}(1)$ is to be solved for together with the scales M_I and M_U.

For the purposes of achieving unification, it is enough to consider the differences between the various gauge couplings, $\alpha_{i,\mathcal{G}}^{-1} - \alpha_{j,\mathcal{G}}^{-1}$, whose running depends only on the parameters

$$\Delta_{ij}^{\mathcal{G}} = \frac{a_i^{\mathcal{G}} - a_j^{\mathcal{G}}}{2\pi} + \frac{\theta_i^{\mathcal{G}} - \theta_j^{\mathcal{G}}}{8\pi^2}\alpha_U. \tag{28}$$

In fact, it turns out that for each intermediate symmetry $\mathcal{G}_I$, it is enough to consider only one combination of the various $\Delta_{ij}^{\mathcal{G}}$, which we will call $C_{\mathcal{G}_I}$. For the cases $\mathcal{G}_I = \mathcal{G}_{422}$ and $\mathcal{G}_I = \mathcal{G}_{3221}$ they read

$$C_{\mathcal{G}_{422}} = \frac{3}{5}\frac{\Delta_{42_R}^{\mathcal{G}_{422}}}{\Delta_{42_L}^{\mathcal{G}_{422}}} \quad \text{and} \quad C_{\mathcal{G}_{3221}} = \frac{3\Delta_{32_R}^{\mathcal{G}_{3221}} + 2\Delta_{3B-L}^{\mathcal{G}_{3221}}}{5\Delta_{32_L}^{\mathcal{G}_{3221}}}. \tag{29}$$

With the boundary conditions defined in Eq. (25) and the matching conditions at M_I for different breaking chains (e.g. Eqs. (26)–(27)), the RGEs of SO(10) in Eq. (22) can be transformed into the following general equations, where the intermediate scale M_I, the unification scale M_U, and the universal SO(10) coupling α_U, are related to the initial conditions of the gauge couplings in the SM

$$\ln\left(\frac{M_I}{M_Z}\right) = \frac{(\alpha_{1_{\mathrm{EW}}}^{-1} - \alpha_{3_{\mathrm{EW}}}^{-1}) - C_{\mathcal{G}_I}(\alpha_{2_{\mathrm{EW}}}^{-1} - \alpha_{3_{\mathrm{EW}}}^{-1}) + D_{\mathcal{G}_I}}{C_{\mathcal{G}_I}\Delta_{32}^{\mathcal{G}_{321}} - \Delta_{31}^{\mathcal{G}_{321}}}, \tag{30}$$

$$\ln\left(\frac{M_U}{M_I}\right) = -\frac{\alpha_{2_{\mathrm{EW}}}^{-1} - \alpha_{3_{\mathrm{EW}}}^{-1}}{\Delta_{3_I 2L_I}^{\mathcal{G}_I}} - \frac{\Delta_{32}^{\mathcal{G}_{321}}}{\Delta_{3_I 2L_I}^{\mathcal{G}_I}}\ln\left(\frac{M_I}{M_Z}\right) - \frac{D'_{\mathcal{G}_I}}{\Delta_{3_I 2L_I}^{\mathcal{G}_I}}, \tag{31}$$

$$\begin{aligned} \alpha_U^{-1} \simeq \alpha_{3_{\mathrm{EW}}}^{-1} - & \frac{1}{C_{\mathcal{G}_I}\Delta_{32}^{\mathcal{G}_{321}} - \Delta_{31}^{\mathcal{G}_{321}}} \\ & \times\left[\left(\frac{a_3^{\mathcal{G}_{321}}}{2\pi} - \frac{\Delta_{32}^{\mathcal{G}_{321}}}{\Delta_{3_I 2L_I}^{\mathcal{G}_I}}\frac{a_{3_I}^{\mathcal{G}_I}}{2\pi} + \mathcal{O}\left(\frac{\alpha_U\theta_i^{\mathcal{G}}}{8\pi^2}\right)\right)\right. \\ & \times(\alpha_{1_{\mathrm{EW}}}^{-1} - \alpha_{3_{\mathrm{EW}}}^{-1}) \\ & - \left(C_{\mathcal{G}_I}\frac{a_3^{\mathcal{G}_{321}}}{2\pi} - \frac{\Delta_{31}^{\mathcal{G}_{321}}}{\Delta_{3_I 2L_I}^{\mathcal{G}_I}}\frac{a_{3_I}^{\mathcal{G}_I}}{2\pi} + \mathcal{O}\left(\frac{\alpha_U\theta_i^{\mathcal{G}}}{8\pi^2}\right)\right) \\ & \left.\times(\alpha_{2_{\mathrm{EW}}}^{-1} - \alpha_{3_{\mathrm{EW}}}^{-1})\right]. \end{aligned} \tag{32}$$

The four constant terms $C_{\mathcal{G}_I}$, $\Delta_{31}^{\mathcal{G}_{321}}$, $\Delta_{32}^{\mathcal{G}_{321}}$ and $\Delta_{3_I 2L_I}^{\mathcal{G}_I}$ (in this last term, 3_I and $2L_I$ refer to the corresponding gauge couplings in the intermediate gauge group $\mathcal{G}_I$, containing the SM $SU(3)_C$ and $SU(2)_L$ components; for instance, if $\mathcal{G}_I = \mathcal{G}_{422}$, the factor refers to $\Delta_{42_L}^{\mathcal{G}_{422}}$) are all determined by the β coefficients of the low-energy models $\mathcal{G}_{321}$ and the intermediate-scale model $\mathcal{G}_I$ from Eqs. (28) and (29).

We have calculated these factors using the work of Refs. [63,64] for each breaking chains we consider and we list them in the Appendix A3; they can easily be calculated from the quantum number of the light fields in the different breaking chains of SO(10). The shorthand notation $(\alpha_{1_{\mathrm{EW}}}^{-1}, \alpha_{2_{\mathrm{EW}}}^{-1}, \alpha_{3_{\mathrm{EW}}}^{-1})$ was used to denote the gauge couplings at the electroweak scale ($\alpha^{-1}_{1,\mathcal{G}_{321}}(M_Z), \alpha^{-1}_{2,\mathcal{G}_{321}}(M_Z), \alpha^{-1}_{3,\mathcal{G}_{321}}(M_Z)$). The factors $D_{\mathcal{G}_I}$ and $D'_{\mathcal{G}_I}$ include all threshold corrections for each breaking chain and are given by

$$
\begin{aligned}
D_{\mathcal{G}_{422}} &= D^{\mathcal{G}_{422}}_{13,\mathcal{G}_{321}} + \frac{3}{5} D^{\mathrm{SO}(10)}_{2_R4,\mathcal{G}_{422}} - C_{\mathcal{G}_{422}} D'_{\mathcal{G}_{422}},\\
D'_{\mathcal{G}_{422}} &= D^{\mathcal{G}_{422}}_{23,\mathcal{G}_{321}} + D^{\mathrm{SO}(10)}_{2_L4,\mathcal{G}_{422}},\\
D_{\mathcal{G}_{3221}} &= D^{\mathcal{G}_{3221}}_{13,\mathcal{G}_{321}} + \frac{3}{5} D^{\mathrm{SO}(10)}_{2_R3,\mathcal{G}_{3221}}\\
&\quad + \frac{2}{5} D^{\mathrm{SO}(10)}_{B\text{–}L3,\mathcal{G}_{3221}} - C_{\mathcal{G}_{3221}} D'_{\mathcal{G}_{3221}},\\
D'_{\mathcal{G}_{3221}} &= D^{\mathcal{G}_{3221}}_{23,\mathcal{G}_{321}} + D^{\mathrm{SO}(10)}_{2_L3,\mathcal{G}_{3221}},
\end{aligned}
\tag{33}
$$

where the parameter $D^{\mathcal{G}}_{ij,\mathcal{H}}$ depicts the difference between the threshold corrections of the gauge couplings $\alpha^{\mathcal{H}}_i$ and $\alpha^{\mathcal{H}}_j$ defined as

$$
D^{\mathcal{G}}_{ij,\mathcal{H}} = \frac{1}{12\pi}\left(\lambda^{\mathcal{G}}_{i,\mathcal{H}} - \lambda^{\mathcal{G}}_{j,\mathcal{H}}\right). \tag{34}
$$

3.2 Uncertainties of the calculation at the two-loop order

The initial conditions on the SM gauge couplings (α_1^{-1}, α_2^{-1}, α_3^{-1}), evaluated in the $\overline{\mathrm{MS}}$ renormalization scheme with two-loop accuracy, are the coupling values at the electroweak scale that we take to be the Z boson mass $M_Z = 91.2$ GeV, namely [65],

$$
\left(\alpha^{-1}_{1_{\mathrm{EW}}},\ \alpha^{-1}_{2_{\mathrm{EW}}},\ \alpha^{-1}_{3_{\mathrm{EW}}}\right) = \left(59.0272,\ 29.5879,\ 8.4678\right),
\tag{35}
$$

where the hypercharge coupling α_Y has been normalized with the usual GUT condition leading to $\alpha_1/\alpha_Y = 5/3$. In the equation above, we have neglected for convenience the experimental errors on the inverse couplings constants (as well as the estimated theoretical uncertainties) and kept only the central values. These errors, in particular the one that affects the strong coupling α_3 will lead to an uncertainty on the obtained scales M_U and M_I of the order of a few percent at most and will therefore not affect our discussion in a significant way.

With the above initial conditions, the solutions to Eqs. (30) and (31) can be derived order by order. At one-loop order, the two-loop coefficients can be ignored, which is equivalent to setting α_U to zero in Eqs. (28) and (29). Neglecting also the one-loop threshold corrections $D_{\mathcal{G}_I}$ and $D'_{\mathcal{G}_I}$, the solutions of Eqs. (30) and (31) in this case, denoted as $\ln(M_I/M_Z)_1$ and $\ln(M_U/M_I)_1$, are determined by the one-loop values of the four constants ($C_{\mathcal{G}_I}$, $\Delta^{\mathcal{G}_{321}}_{31}$, $\Delta^{\mathcal{G}_{321}}_{32}$ and $\Delta^{\mathcal{G}_I}_{3_I 2L_I}$) (see Appendix A3 for details). The universal coupling at one-loop order $\alpha_U^{\text{1-loop}}$ can also be obtained in a similar way by substituting back the one-loop values of these four constants in the right-handed side of Eq. (32).

We summarize the results for the three one-loop quantities $\ln(M_I/M_Z)_1$, $\ln(M_U/M_I)_1$ and $\alpha_U^{\text{1-loop}}$ in the first panel of Table 2 for some considered breaking chains when the threshold corrections (as well as the Yukawa couplings) are neglected.

At two-loop order, Eqs. (30) and (31) can be seen as implicit functions of the independent variables α_U, $\ln(M_I/M_Z)$ and $\ln(M_U/M_I)$. Denoting the right-handed sides of these equations as $F(\alpha_U, \ln(M_I/M_Z), \ln(M_U/M_I))$ and $G(\alpha_U, \ln(M_I/M_Z), \ln(M_U/M_I))$ correspondingly, Eqs. (30) and (31) can be rewritten as

$$
F\left(\alpha_U, \ln\left(\frac{M_I}{M_Z}\right), \ln\left(\frac{M_U}{M_I}\right)\right) - \ln\left(\frac{M_I}{M_Z}\right) = 0, \tag{36}
$$

$$
G\left(\alpha_U, \ln\left(\frac{M_I}{M_Z}\right), \ln\left(\frac{M_U}{M_I}\right)\right) - \ln\left(\frac{M_U}{M_I}\right) = 0. \tag{37}
$$

Because the one-loop solutions $\ln(M_I/M_Z)_1$ and $\ln(M_U/M_I)_1$ when $\alpha_U = 0$ are exact solutions to the above Eqs. (36) and (37), the small required corrections can be found by performing the following variations to the one-loop solutions

$$
\begin{aligned}
&\left.\frac{\partial F}{\partial \alpha_U}\right|_{\alpha_U=0} \delta\alpha_U + \left[\left.\frac{\partial F}{\partial \ln\left(\frac{M_I}{M_Z}\right)}\right|_{\alpha_U=0} - 1\right] \delta \ln\left(\frac{M_I}{M_Z}\right)\\
&\quad + \left.\frac{\partial F}{\partial \ln\left(\frac{M_U}{M_I}\right)}\right|_{\alpha_U=0} \delta \ln\left(\frac{M_U}{M_I}\right) = 0,
\end{aligned}
\tag{38}
$$

$$
\begin{aligned}
&\left.\frac{\partial G}{\partial \alpha_U}\right|_{\alpha_U=0} \delta\alpha_U + \left.\frac{\partial G}{\partial \ln\left(\frac{M_I}{M_Z}\right)}\right|_{\alpha_U=0} \delta \ln\left(\frac{M_I}{M_Z}\right)\\
&\quad + \left[\left.\frac{\partial G}{\partial \ln\left(\frac{M_U}{M_I}\right)}\right|_{\alpha_U=0} - 1\right] \delta \ln\left(\frac{M_U}{M_I}\right) = 0.
\end{aligned}
\tag{39}
$$

A careful investigation of the above differential forms reveal that indeed all the other derivatives vanish when $\alpha_U = 0$ except for $\partial F/\partial\alpha_U$ and $\partial G/\partial\alpha_U$ due to the fact that the derivatives of the two-loop factor $\Delta^{\mathcal{G}}_{ij}$ satisfies the relation $\partial\Delta^{\mathcal{G}}_{ij}/\partial t|_{\alpha_U=0} = 0$. Therefore, the two-loop solutions $\ln(M_I/M_Z)_2$ and $\ln(M_U/M_I)_2$ can be approximated by

$$
\begin{aligned}
\ln\left(\frac{M_I}{M_Z}\right)_2 &= \ln\left(\frac{M_I}{M_Z}\right)_1 + \delta\ln\left(\frac{M_I}{M_Z}\right) \approx \ln\left(\frac{M_I}{M_Z}\right)_1\\
&\quad + \left.\frac{\partial F}{\partial\alpha_U}\right|_{\alpha_U=0} \delta\alpha_U,
\end{aligned}
\tag{40}
$$

$$
\begin{aligned}
\ln\left(\frac{M_U}{M_I}\right)_2 &= \ln\left(\frac{M_U}{M_I}\right)_1 + \delta\ln\left(\frac{M_U}{M_I}\right) \approx \ln\left(\frac{M_U}{M_I}\right)_1\\
&\quad + \left.\frac{\partial G}{\partial\alpha_U}\right|_{\alpha_U=0} \delta\alpha_U,
\end{aligned}
\tag{41}
$$

Table 2 A summary table of our approximate analytical estimates of the intermediate scales M_I, the unification scales M_U, and the values of the universal SO(10) coupling constant α_U for different intermediate breaking groups $\mathcal{G}_I$ and low-energy models $\mathcal{G}_{\rm SM}$, where the Yukawa contributions and the threshold corrections are neglected

$\mathcal{G}_{321}$	$\mathcal{G}_I$	$\log\left(\frac{M_{I1}}{\rm GeV}\right)$	$\log\left(\frac{M_{U1}}{\rm GeV}\right)$	$\alpha_U^{\text{1-loop}}$	$\log\left(\frac{M_{I2}}{\rm GeV}\right)$	$\log\left(\frac{M_{U2}}{\rm GeV}\right)$	$\alpha_U^{\text{2-loop}}$
SM	$\mathcal{G}_{422}$	11.102	16.314	0.0275	9.627	16.718	0.0313
SM	$\mathcal{G}_{3221}$	9.807	16.165	0.0223	9.942	15.929	0.0262
2HDM	$\mathcal{G}_{422}$	11.429	15.988	0.0273	10.133	16.346	0.0304
2HDM	$\mathcal{G}_{3221}$	10.234	15.896	0.0226	10.398	15.652	0.0230

where $\delta\alpha_U = \alpha_U^{\text{1-loop}}$ should be substituted in the above equation. The universal grand unified coupling at the two-loop level $\alpha_U^{\text{2-loop}}$ can also be solved numerically from Eq. (32) by substituting the one-loop value of $\ln(M_I/M_Z)_1$ and $\ln(M_U/M_I)_1$ into the parameters $C_{\mathcal{G}_I}$, $\Delta_{31}^{\mathcal{G}_{321}}$, $\Delta_{32}^{\mathcal{G}_{321}}$, and $\Delta_{3_I 2L_I}^{\mathcal{G}_I}$.[4]

In summary, neglecting all the threshold corrections, as the coupling constant α_U is rather small, it is a good approximation to expand the coefficients $\Delta_{ij}^{\mathcal{G}_I}$ in terms of this coupling to find, first the one-loop solutions. The two-loop solutions are then obtained by inserting the one-loop solutions into Eqs. (40) and (41).

We summarize our results for the one-loop and two-loop predictions for $\ln(M_I/M_Z)$, $\ln(M_U/M_I)$, and α_U separately in Table 2. This approximation is in a good agreement with the numerical results to be discussed in Sect. 3.4. One can observe from Table 2 that the one-loop solutions agree with the numerical results given in Ref. [25] at the 4σ confidence level, while the two-loop solutions agree with the numerical results in Table 3 from Sect. 3.4 at the 2σ confidence level.

In practice, one can solve these equations iteratively, as is done for instance in Ref. [62], to obtain more accurate predictions of the scales M_I and M_U. However, as we are assuming the approximation in Eq. (23) for a unification of gauge couplings, without integrating out higher derivatives, the approximation by the first derivatives in Eqs. (40) and (41) already includes uncertainties of the order of one-percent, which is also comparable with the contributions from the Yukawa couplings that we neglect in our computation. Besides the uncertainties from our approximations and leaving aside the Yukawa contributions, the largest uncertainty actually comes from the threshold corrections $D_{\mathcal{G}_I}$ and $D'_{\mathcal{G}_I}$, which are shown in several analyses to be able to modify the predictions of the unification scales by more than an order of magnitude; see for instance Refs. [25,36,66].

Finally, we should note that in principle, analytical expressions cannot be derived when a multi-step symmetry breaking with more than one intermediate scale is present, unless additional constraints on the intermediate scale are imposed. Our analytical results generalize the formulae derived in Ref. [62] for SUSY-SO(10) GUTs to the non-SUSY case and to the case with one intermediate symmetry breaking.[5]

3.3 Impact of proton decay

Before moving to the numerical results, let us first have a brief discussion[6] on the constraints that come from proton decay on our SO(10) GUTs with intermediate breaking, and more precisely on the values of the unification scale M_U and unification coupling α_U. The most-constraining decay channel on the proton lifetime is the one in which one has a pion and a positron in the final state [69,70]. In this particular mode, the proton lifetime in years can be roughly estimated to be [25]:

$$\tau(p\to e^+\pi^0) \simeq (7.47\times 10^{35}\ \text{year}) \left(\frac{M_U}{10^{16}\ \text{GeV}}\right)^4 \left(\frac{0.03}{\alpha_U}\right)^2 . \tag{42}$$

The strongest current experimental constraint, including other decay channels, for proton decay come from the Super-Kamiokande experiment [71–75] which sets the bounds on the proton lifetime

$$\tau(p\to e^+\pi^0) > 1.67\times 10^{34}\ \text{year} \tag{43}$$

at the 90% confidence level, which yields the following bound

[4] At the two-loop level, the most important contributions to α_U are due to $(C_{\mathcal{G}_I}, \Delta_{31}^{\mathcal{G}_{321}}, \Delta_{32}^{\mathcal{G}_{321}}$, and $\Delta_{3_I 2L_I}^{\mathcal{G}_I})$ corrected by the two-loop β coefficients (see explicitly Appendix A3), so that one can safely neglect the terms proportional to $(\alpha_U\theta_i^{\mathcal{G}}/8\pi^2)$ in Eq. (32), which can be considered as higher-order corrections [62].

[5] This formalism can be generalized to the supersymmetric case discussed for example in Ref. [67] where a general analytical method is applied for a SUSY SU(5) GUT. For SUSY SO(10) GUTs like the ones discussed in Ref. [68], one can identify the intermediate scale to be the SUSY-breaking scale and use the formalism presented here to derive the unification scale M_U.

[6] For a detailed account, see the recent and more general discussion given in Ref. [66].

Table 3 A summary table of the numerical results of the intermediate scale, the unification scale, and the universal gauge coupling at the two-loop level, neglecting all the threshold corrections as well as the estimated proton lifetimes obtained for each considered breaking chain with two Higgs doublets at the electroweak scale. The ratio of vevs is fixed to $\tan\beta = 65$ as the results do not change significantly for lower values of $\tan\beta$

Breaking chain	$\log\left(\frac{M_{Ic}}{\text{GeV}}\right)^{\text{2-loop}}$	$\log\left(\frac{M_{Uc}}{\text{GeV}}\right)^{\text{2-loop}}$	$\alpha_U^{\text{2-loop}}$	$\tau(p \to e^+\pi^0)$/year
422	10.03	16.19	0.032	3.82×10^{36}
3221	10.66	15.45	0.023	7.84×10^{33}
422D	13.65	14.66	0.026	4.22×10^{30}
3221D	11.82	14.63	0.024	3.89×10^{30}

$$\ln\left(\frac{M_U}{M_Z}\right) + \frac{1}{2}\ln\left(\alpha_U^{-1}\right) > 33.1, \tag{44}$$

where the unification scale $\ln\left(\frac{M_U}{M_Z}\right)$ and coupling α_U^{-1} can be obtained from Eqs. (30)–(32) with values that are summarized in Table 2 given in the previous subsection.

In the general case, the analytical expressions for the β coefficients can be found in Refs. [63,64], where the dependence on the number of fermion families and Higgs doublets is explicitly given. We can thus express all β coefficients as a function of the number of scalars running from the electroweak scale to the intermediate scale. One can generally state that the more colorless scalars contribute to the gauge couplings, the lower the unification scale would be and, thus, the shorter the proton lifetime would be.

For the low-energy model $\mathcal{G}_{321}$ studied in our paper, namely the 2HDM, and without including the threshold corrections as is shown for example in Table 2, the only two breaking chains that survive the constraint from proton decay are the 422 and the 3221 breaking chains, with the latter one sitting right on the edge of the proton decay bounds that could be spoiled easily by slightly going beyond our approximation. Including the threshold corrections could raise the unification scale by an order of magnitude to avoid a too fast proton decay. The shift of scales M_I and M_U when including the threshold corrections numerically is discussed in the next subsection.

3.4 Numerical results

In this subsection, we will give more precise results that we obtain numerically by deriving and solving the RGEs for each considered breaking chain up to two-loop order, using the Mathematica package SARAH [76]. In our present case, from the electroweak scale M_Z to the intermediate scale M_I, the low-energy model $\mathcal{G}_{321}$ is not the SM but is assumed to be the 2HDM whose two-loop RGEs are also given in Appendix B. Note also that in our numerical treatment, the contributions of the Yukawa couplings, determined from the fermion masses at the electroweak scale and the parameter $\tan\beta$ of the 2HDM, have been also included.

We also include the one-loop threshold corrections numerically at the scales M_I and M_U, by randomly sampling the parameters $\eta_i = \ln(M_i/\mu)$ of Eq. (21) within the range of values $\eta_i \in [-1, 1]$. The systems of two-loop RGEs would then be solved together with the given one-loop threshold corrections to determine the values of the two scales M_I and M_U for each sampling parameter set, by requiring all the gauge couplings to match at the grand unified scale M_U including the threshold corrections when appropriately adjusting the intermediate scale M_I. We took at least 10,000 points for the parameters η_i within the selected range of η_i values and determined the sets of all scales (M_I, M_U) that allow for gauge coupling unification for each breaking chain.

The results are given by the four panels of Fig. 1 which shows for the four considered breaking patterns, the scatter plots for the set of scales (M_I, M_U) with the randomly sampled threshold corrections, when the ratio of the 2HDM vevs is chosen to be $\tan\beta = 65$. The intermediate and the GUT scales when all the threshold corrections are taken to be zero ($\eta_i = 0$) are defined as the central values (M_{Ic}, M_{Uc}) that are specified in each plot. As we have already noticed in Ref. [36], both the two-loop corrections and the threshold corrections have a significant impact.

In particular, our results in Fig. 1 show the effect of the additional Higgs doublet contributions to the gauge coupling running for our considered breaking chains (again the β coefficients are given in Appendix A1), when comparing for instance to the work of Ref. [25] (especially to their Figure 3), where only the SM particle content is used in the running at low energy. The extra contributions in the 2HDM to the running of the gauge couplings, even though not very large, results in a unification scale M_U that is significantly smaller. In fact, for some of the breaking scenarios that we consider, in particular the 422D and 3221D chains, the resulting M_U values could easily fall into the values excluded by proton-decay bounds even when large threshold corrections are included.

As an example, the evolution of the inverse of the gauge coupling constants squared α_i^{-1} for the selected 2HDM ratio of vevs $\tan\beta = 65$ when all threshold corrections η_i are

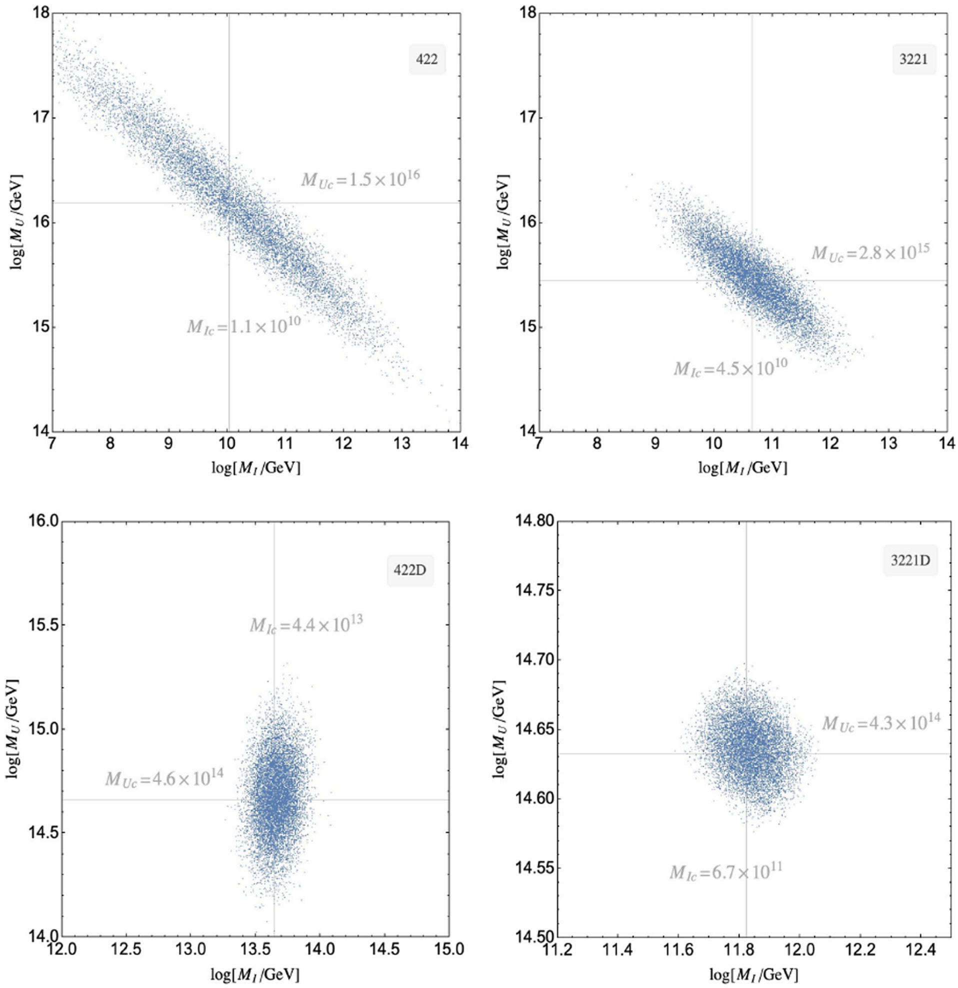

Fig. 1 The scatter plots for the set of (logarithms of the) scales (M_I, M_U) of the four breaking patterns considered, with randomly sampled threshold corrections for $\eta_i \in [-1, 1]$ when the ratio of vevs of the two Higgs fields is chosen to be $\tan\beta = 65$. Note that these results are not sensitive to $\tan\beta$. The central values (M_{Ic}, M_{Uc}) that we indicate represent the intermediate and the GUT scales with all threshold corrections taken to be zero, $\eta_i = 0$

taken to be zero from the scale M_U down to the scale M_I and then down to the weak scale M_Z is shown in Fig. 2 as a function of the (logarithm of the) energy scale μ. We have used the program SARAH in which we have implemented the full two-loop RGEs for the considered breaking patterns 422 (upper left), 422D (bottom left), 3221 (upper right) and 3221D (bottom right). While the three couplings are clearly different at the scale M_I, of the order of a few times 10^{10-13} GeV, the slope are significantly modified at this energy by the additional contributions so that the couplings meet at a scale M_U of the order 10^{14-16} GeV. The small impact of the experimental errors on the couplings is illustrated by the narrow vertical red bands that are drawn at the scales M_I and M_U.

Finally, we also summarize in Table 3 our numerical results for our four considered breaking patterns, when the threshold corrections are not included. The relevant intermediate and unification scales at the two-loop level M_{Ic} and M_{Uc} as well as the unification coupling α_U, are to be compared with those given in Table 2; in addition, we display the estimated proton lifetime in each scenario.

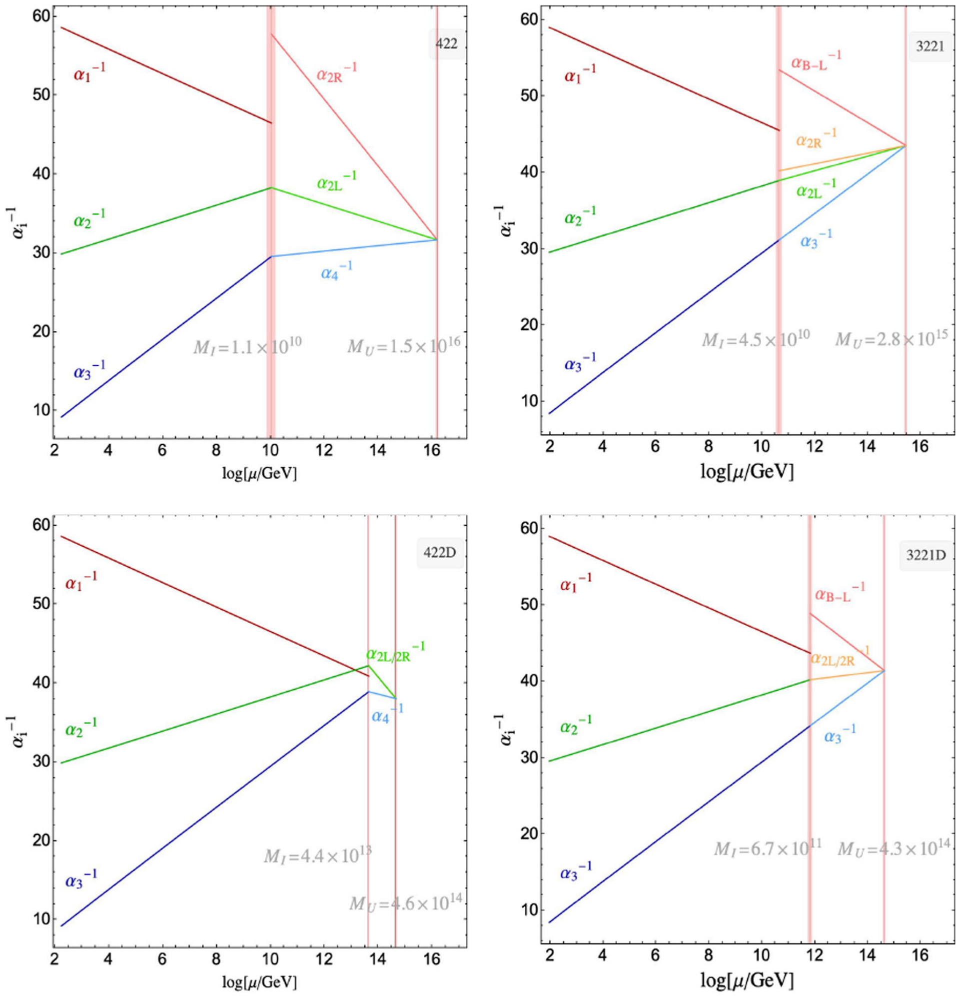

Fig. 2 The evolution of the inverse of the gauge coupling constants squared α_i^{-1} as a function of the (logarithm of the) energy scale μ for the value $\tan\beta = 65$, when all threshold corrections η_i are taken to be zero from the electroweak scale to the GUT scale in the 2HDM+422 (upper left), 2HDM+3221 (upper right), 2HDM+422D (bottom left) and 2HDM+3221D (bottom right) models. The red vertical bands reveal the uncertainty on the measurement of gauge couplings at the electroweak scale

From this table, one can see that when the threshold corrections are switched off, only the breaking chain 422 with the Pati–Salam symmetry as an intermediate step and a 2HDM at the low energy scale, survives the proton decay bound from Kamiokande, namely $\tau(p \to e^+\pi^0) > 1.67 \times 10^{34}$ year. In addition, even though the 3221 chain seems to lie at the edge of the dangerous region excluded by proton decay, any small amount of threshold corrections at a given symmetry breaking scale could easily rescue it, by raising the unification scale by an order of magnitude, as can be seen from Fig. 1. The same situation occurs in the 422D breaking chain, but large threshold corrections ($\eta_i \simeq 1$) would be needed to prevent fast decay of proton in this case. Finally, for the 3221D breaking chain resulting to a 2HDM at the low energy scale, we find that the bound from proton decay is violated unless extremely large (an potentially unrealistic) threshold corrections ($\eta_i \gg 1$) are taken into account.

Before we close this section, let us make a brief comment on the fact that gauge coupling unification in non-SUSY SO(10) models with only one intermediate scale suffers from the severe constraints from proton decay, if no large threshold corrections are imposed, except for the 422 breaking chain. As we have seen above, the two-loop RGEs of these SO(10) models have approximate analytical solutions which are completely determined by the β coefficients for any breaking chain. In other words, there are no free parameters in determining the symmetry breaking scales except for the threshold corrections (e.g. in the D factors given in Eq. (33)) and the value of the parameter $\tan\beta$ of the low energy 2HDM. The latter parameter generally affects only marginally gauge coupling unification, but it will be strongly constrained when the Yukawa interactions of the fermions are included as we will see in the next section. This will be particularly the case when one invokes the requirement of the perturbativity of the Yukawa couplings (the absence of Landau poles) and by the consistency of the values for the fermion masses that can be obtained at the intermediate scale.

Thus, the surviving parameter space for non-SUSY minimal SO(10) models with an intermediate scale is rather small, thus rendering the model quite predictive. We move now to the unification of third generation Yukawa couplings. In this case, we will ignore the models 422D and 3221D with a PQ symmetry as they lead to a low unification scale and, hence, are in conflict with the limits from proton decay.

4 Yukawa coupling unification

4.1 Yukawa unification in non-SUSY SO(10)

Following the paradigm of the unification of the gauge coupling constants, one is tempted to push the idea further and to consider also the possibility of unifying the fermionic Yukawa couplings in the framework of the same GUT symmetry group. In this context, one is forced to ignore the rather small Yukawa couplings of the first- and second-generation fermions as the masses of these particles are below the few GeV scale which allows them to be realistically described without being affected by the strong interaction uncertainties that are encountered at the corresponding mass scale. In our work, we will thus consider only the Yukawa couplings of third-generation fermions, the top quark, the bottom quark and the tau lepton, with the additional simplification of neglecting all possible mixings. The three fermions will be assumed to have a common Yukawa coupling at the GUT scale M_U within the natural context of SO(10) unification where the fermions are embedded into a single irreducible representation 16_F of the symmetry group.

Below the intermediate scale M_I and down to the electroweak scale, the Yukawa interactions of these fermions are those of a Type-II 2HDM with a Lagrangian given by Eq. (17), which leads to the masses given in Eq. (18) in terms of the two vevs v_u and v_d defined at the electroweak scale. The choice of the 2HDM as the low-energy scale directly follows from the requirement that the top and bottom Yukawa couplings should be comparable and this cannot be achieved in the context of the SM with its single Higgs doublet field. In turn, in extended Higgs sectors, the large ratio between the top and bottom quark masses could be due to a large ratio of the vevs of the Higgs multiplets that give rise to the masses of the up- and down-type fermions. The simplest of such an extension[7] is a 2HDM of Type II. More specifically, one would have for the parameter $\tan\beta$ which is defined as the ratio of the two vevs v_u and v_d of the fields H_u and H_d that break the electroweak symmetry

$$\tan\beta = v_u/v_d \sim m_t/m_b \approx \mathcal{O}(60). \quad (45)$$

Note that in the equation above, m_t and m_b are the mass parameters of the top and bottom quarks evaluated at the weak scale M_Z, and not the physical masses.

In the context of the SO(10) unification group that we are considering here, with either a $\mathbf{10_H}$ or a $\overline{\mathbf{126}}_\mathbf{H}$ scalar representation coupling to fermions, the third generation masses must depend on a single parameter for consistency reasons. However, instead of just one SO(10) scalar representation, we consider the possibility that both a complex $\mathbf{10_H}$ and a $\overline{\mathbf{126}}_\mathbf{H}$ scalar interact with fermions; see Eq. (5). Fermion masses can therefore receive non-negligible contributions from two Yukawa couplings. Thus, a discussion on Yukawa unification implies that Y_{10} and Y_{126} are somehow related, which in turn implies some connection between the two scalars in our model. One tantalizing possibility is that both the $\mathbf{10_H}$ and the $\overline{\mathbf{126}}_\mathbf{H}$ are part of a single irreducible representation of an even larger gauge group. A natural candidate is the exceptional group E_6 for the following reasons:

- The smallest non-trivial representation of E_6, the one of dimension **27**, decomposes as $\mathbf{16}+\mathbf{10}+\mathbf{1}$ and, therefore, contains the SM fermions plus vector-like ones.
- A scalar representation $\mathbf{351'_H}$ can couple to the bilinear product of fermions in the representation $\mathbf{27}\times\mathbf{27}$ and, furthermore, it decomposes as $\mathbf{10_H}+\overline{\mathbf{126}}_\mathbf{H}+\cdots$ under the SO(10) group. Note that $\mathbf{351'}$ is a complex representation and, therefore, $\mathbf{10_H}$ must be associated to a complex field.
- The E_6-symmetric Yukawa interaction $Y\times\mathbf{27_F}\cdot\mathbf{27_F}\cdot\mathbf{351'}_H$ can be written as a sum of terms which, individually, are symmetric only under SO(10): $c_{10}Y\times\mathbf{16_F}\cdot\mathbf{16_F}\cdot\mathbf{10_H}+c_{126}Y\times\mathbf{16_F}\cdot\mathbf{16_F}\cdot\overline{\mathbf{126}}_\mathbf{H}+\cdots$ for specific Clebsch–Gordon factors $c_{10}, c_{126}, \ldots$. These last numbers are therefore a prediction of an E_6-symmetric theory, hence the enlarged symmetry enforces a particular

[7] The idea of Yukawa coupling unification emerged and was developed in the late 1980s in the context of supersymmetric theories, to predict the mass of the not yet discovered top quark and to understand the origin of the top-bottom mass hierarchy; see e.g. Ref. [77]. For consistency reasons, the minimal supersymmetric standard model or MSSM required two Higgs doublets fields of Type-II.

ratio Y_{10}/Y_{126} since

$$\frac{Y_{10}}{Y_{126}} = \frac{c_{10}Y}{c_{126}Y} = \frac{c_{10}}{c_{126}}. \tag{46}$$

- An exactly E_6-symmetric theory does not involve the coupling $\mathbf{16_F} \cdot \mathbf{16_F} \cdot \mathbf{10}^*_H$ and, hence, there is not such an interaction at leading order. Its absence can be understood by the fact that E_6 contains an extra U(1) subgroup which commutes with SO(10), under which the fields are changed precisely in the manner described in Eq. (4).

The crucial ratio of Yukawa couplings discussed above turns out to be

$$\left(\frac{Y_{10}}{Y_{126}}\right)_{E6} = \sqrt{\frac{3}{5}}, \tag{47}$$

with the understanding that at the GUT scale, the SO(10) contractions $\mathbf{16_F} \cdot \mathbf{16_F} \cdot \mathbf{10}_H$ and $\mathbf{16_F} \cdot \mathbf{16_F} \cdot \overline{\mathbf{126}}_\mathbf{H}$ normalized in such a way that the two SM doublets (one in $\mathbf{10}_H$ and the other in $\overline{\mathbf{126}}_\mathbf{H}$) contribute to the top quark mass with the same Clebsch–Gordon factor. If we were to write the $\mathcal{G}_{321}$-invariant Yukawa interactions involving the four Higgs doublets $H_{u/d,10/126}$ contained in $\mathbf{10_H}$ and $\overline{\mathbf{126}}_\mathbf{H}$, they would have the form

$$\begin{aligned}&\bar{Q}_L\left(Y_u^{10}H^*_{u,10} + Y_u^{126}H^*_{u,126}\right)u_R\\ &+\bar{Q}_L\left(Y_d^{10}H_{d,10} + Y_d^{126}H_{d,126}\right)d_R\\ &+\bar{L}_L\left(Y_e^{10}H_{d,10} + Y_e^{126}H_{d,126}\right)e_R + \text{h.c.}\end{aligned} \tag{48}$$

At the unification scale, the matching relations are as follows:

$$Y_u^{10} = Y_d^{10} = Y_e^{10} = Y_{10};\ Y_u^{126} = Y_d^{126} = -\frac{1}{3}Y_e^{126} = Y_{126}. \tag{49}$$

Combining these two expressions, we obtain the fermion mass formulas

$$\begin{aligned}&m_t = v_{10}^u Y_{10} + v_{126}^u Y_{126},\ m_b = v_{10}^d Y_{10} + v_{126}^d Y_{126},\\ &m_\tau = v_{10}^d Y_{10} - 3v_{126}^d Y_{126}.\end{aligned} \tag{50}$$

In addition, we have the Dirac neutrino mass which is given by

$$m_{\nu_D} = v_{10}^u Y_{10} - 3v_{126}^u Y_{126}. \tag{51}$$

Note however that we do not consider a direct breaking of the SO(10) symmetry to the SM group $\mathcal{G}_{321}$; the purpose of the previous equations is simply to clarify the normalization of the SO(10)-invariant Yukawa couplings that we are considering in Eq. (5). (Furthermore, we consider only two light Higgs doublets, which are necessarily a combination of the four doublets in Eq. (48).)

The number indicated in Eq. (47) is quite peculiar since the ratio of Clebsch–Gordon factors is often a rational number (Ref. [78] contains a large list of examples, none of which involves an irrational number). We also cannot avoid commenting on the fact that $\sqrt{3/5}$ is also used to canonically normalize the SM hypercharge; nevertheless, as far as we can tell, this equality is just a coincidence.

The ratio of Eq. (47) was derived with the `Subgroup Coefficients` function of `GroupMath` [79] but it can also readily be derived from the available literature. Note in particular that eqs. (77) and (78) and Table 6 of Ref. [80] directly imply that $Y \times \mathbf{27_F} \cdot \mathbf{27_F} \cdot \langle \mathbf{351'}_H \rangle$ contains the terms $\left[1/\left(2\sqrt{10}\right) Y v_{10}^u - 1/\left(2\sqrt{6}\right) Y v_{126}^u\right] t t^c$.[8]

In the following, we will consider the consequences of the above relation. However, it is beyond the scope of the present work to present a fully realistic E_6 model for Yukawa unification as this would entail several challenges.[9]

Let us also mention that in our earlier work [36], we have considered some of the implications of requiring the simple relation $Y_{10} = Y_{126}$ at the scale where the gauge couplings unify. While we do not have a mechanism that would prescribe this relation, we will nevertheless consider here in more detail some of its consequences. It is worth keeping in mind that $\sqrt{3/5} \approx 0.77$ is not far off from 1, hence the two Yukawa unification conditions, $Y_{10} = CY_{126}$ with $C = \sqrt{3/5}$ or 1, should not lead to dramatically different results.

4.2 Matching conditions in the 422 breaking chain

Having introduced the Yukawa unification conditions in our SO(10) model from a top-down perspective, we then seek the relations of the low-energy Yukawa couplings in different breaking chains of SO(10). We first note that the field content needed to enforce the $\mathcal{D}$ parity symmetry yields a unification scale that is unacceptably low, making the proton lifetime too short in the 422D and the 3221D breaking chains, as can be seen from Table 3 and the relevant discussion in the Sect. 3.4. We thus ignore these two possibilities in our next discussion. In fact, we will also not discuss the 3221 breaking chain of this particular SO(10) model; some of the elements have been presented in Ref. [36] and others will be postponed to a

[8] The individual values of the two Clebsch–Gordon factors, including their relative sign, are convention-depend and therefore unphysical; the absolute value of their ratio is not.

[9] For example, a realistic scalar sector would necessarily have a large number of scalars transforming as $(\mathbf{2}, \pm\mathbf{1/2})$ or $(\mathbf{1}, \mathbf{0})$ under the electroweak group, whose vevs affect fermion masses. Providing masses to all the scalars is another challenge. Also, on top of the $\mathbf{16_F}$ (three generations of it), one would have vector-like fermions transforming as $\mathbf{10_F}$ and $\mathbf{1_F}$ which can mix with the spinor representation, complicating the identification of what are the SM fermions. We thank Vasja Susič for his helpful comments on this and other E_6-related topics.

forthcoming paper. Thus, for illustration, we will discuss in the following only the evolution of the Yukawa couplings in the 422 breaking chain.

One should first recall that enforcing Yukawa unification can be seen as finding a solution for the system of RGEs of Yukawa couplings satisfying the boundary conditions and the initial conditions obtained from the experimental observables. However, even in a model as simple as a 2HDM, the general RGEs of the Yukawa couplings, which can be read from Appendix B, do not admit an approximate solution like the ones for the gauge couplings discussed in Sect. 3. Therefore, we will leave the discussion of the numerical evaluation of all the Yukawa couplings between different energy scales to the end of the present section, and we first concentrate here on the boundary conditions.

The boundary conditions for Yukawa couplings at the GUT scale M_U, where the full SO(10) is restored, relate the couplings in a very specific way which is dictated by appropriate Clebsch–Gordon coefficients originated from the decomposition of the tensor product for the fermion bilinear and the scalar field representation. Below the SO(10) scale, given that the gauge symmetry group is smaller and less constraining, there can be more than two Yukawa couplings, as shown in Eq. (12). For the 422 breaking chain, in which the Yukawa couplings can be identified as Y_{10}^{422}, Y_{126}^{422} and Y_R^{422}, the matching conditions can be read from Ref. [48] which gives:

$$Y_{10}(M_U) = \frac{1}{\sqrt{2}} Y_{10}^{422}(M_U), \quad Y_{126}(M_U) = \frac{1}{4\sqrt{2}} Y_{126}^{422}(M_U) = \frac{1}{4} Y_R^{422}(M_U). \tag{52}$$

Note that the numerical factors shown here are not intrinsically physical since they depend on how one contracts the $SU(4)_C \times SU(2)_L \times SU(2)_R$ group indices which, incidentally, are not shown in Eq. (12). Obviously, whatever convention is adopted, it must be followed consistently. In the present case, this means that the factors of $\sqrt{2}$ and 4 shown above must drop out when matching the 321 and 422 Yukawa couplings at M_I.

As was mentioned in Sect. 4.1, the Yukawa unification in non-SUSY SO(10) can be defined as $Y_{10} = C Y_{126}$, with C the ratio of CG coefficients decomposing the scalar representation of higher symmetry into SO(10) multiplets $\mathbf{10_H}$ and $\overline{\mathbf{126}}_\mathbf{H}$.[10] Motivated by E_6 in Eq. (47), we take this factor to be $\sqrt{3/5}$, which, after combining the GUT-scale matching condition in Eq. (52), implies that the 422-Yukawa couplings at M_U must fell on the line $\ell(M_U)$ in the two-dimensional parameter space $(Y_{10}^{422}(M_U), Y_{126}^{422}(M_U))$ defined by

$$\frac{Y_{10}^{422}(M_U)}{Y_{126}^{422}(M_U)} = \frac{1}{4}\sqrt{\frac{3}{5}}. \tag{53}$$

Comparing Eqs. (49) and (52), we obtain from Eq. (50) the following fermion masses in the 422-symmetric phases

$$m_t = \frac{v_{10}^u}{\sqrt{2}} Y_{10}^{422} + \frac{v_{126}^u}{4\sqrt{2}} Y_{126}^{422}, \; m_b = \frac{v_{10}^d}{\sqrt{2}} Y_{10}^{422} + \frac{v_{126}^d}{4\sqrt{2}} Y_{126}^{422},$$
$$m_\tau = \frac{v_{10}^d}{\sqrt{2}} Y_{10}^{422} - \frac{3 v_{126}^d}{4\sqrt{2}} Y_{126}^{422}, \tag{54}$$

in addition to the Dirac/Majorana neutrino masses written as

$$m_{\nu_D} = \frac{v_{10}^u}{\sqrt{2}} Y_{10}^{422} - \frac{3 v_{126}^u}{4\sqrt{2}} Y_{126}^{422}, \; m_{\nu_R} = \frac{1}{4} v_R Y_R^{422}. \tag{55}$$

We can now match the intermediate-scale fermion mass matrices in Eq. (54) to the low-energy ones in Eq. (18), as the consistency between both theories implies that the masses predicted from the low-energy effective theory and the high-energy theory should be the same at the symmetry breaking scale. It follows that for the breaking chains 422, the matching conditions of the Yukawa couplings at the intermediate scale read

$$Y_t v_u = v_{10}^u Y_{10}^{422} + \frac{1}{4} v_{126}^u Y_{126}^{422},$$
$$Y_b v_d = v_{10}^d Y_{10}^{422} + \frac{1}{4} v_{126}^d Y_{126}^{422},$$
$$Y_\tau v_d = v_{10}^d Y_{10}^{422} - \frac{3}{4} v_{126}^d Y_{126}^{422}. \tag{56}$$

The above matching conditions contain six free parameters: the four vevs of the Higgs bi-doublets and two intermediate-scale Yukawa couplings. With the three electroweak-scale Yukawa couplings $Y_{t,b,\tau}(M_Z)$ in the 2HDM obtained from the experimental inputs, we actually have enough degrees of freedom to be able to fix these free parameters by the Yukawa couplings RGEs, as has been done in the literature, see Refs. [36,53–55]. We will discuss such a numerical fitting procedure in detail in the next subsection. However, by imposing the constraints from the scalar potential, such as forbidding dangerous flavor changing neutral currents (FCNCs), we find that the allowed parameter spaces can be largely reduced as will be discussed shortly.

Finally, we emphasize that Y_R is not a free parameter which contributes to the running of other Yukawa couplings. This is because, for every possible set of $(Y_{10}^{422}(M_I), Y_{126}^{422}(M_I))$, there is a uniquely determined $Y_R^{422}(M_I)$ defined by the GUT-scale matching condition in Eq. (52) as their values at M_I and at M_U are related by their RGEs

$$Y_{126}^{422}(M_U) = \sqrt{2} Y_R^{422}(M_U). \tag{57}$$

Therefore, in practice, we scan for all the possible values of $Y_R(M_I)$ to satisfy the above relation (within a certain accu-

[10] The special case where $C = 1$ has been studied in [36] for a simplified SO(10) model with a real $\mathbf{10_H}$ representation without implying any further unification of the scalar representation.

racy), together with $(Y_{10}^{422}(M_I), Y_{126}^{422}(M_I))$ to determine the initial conditions at M_I for solving the RGEs from the intermediate scale to the GUT scale.

4.3 The evolution of Yukawa couplings

In this subsection, we give the details of the numerical fitting procedure for the parameter space allowing to address the possibility of Yukawa coupling unification, following Ref. [36], where two breaking patterns of a non-SUSY SO(10) model with a real $\mathbf{10_H}$ representation were discussed. The analysis is restricted to the 422 case, and is based on numerically solving the RGEs and the matching conditions in Eq. (56) simultaneously.

In our numerical evaluation of the RGEs, the Yukawa couplings at the electroweak scale chosen to be the Z boson mass $M_Z = 91.2$ GeV, have to be fitted with the physical observables which are the top, bottom and tau masses using the relations in Eq. (18). The following input values of the $\overline{\text{MS}}$ running fermion masses in the SM [65,81,82] (we again ignore here the related experimental uncertainties) will be used

$$[m_t(M_Z), m_b(M_Z), m_\tau(M_Z)] = [168.3, 2.87, 1.73]\ \text{GeV}. \tag{58}$$

We convert these inputs into the corresponding masses in the 2HDM by using the appropriate RGEs in the evolution from the scale of the fermion masses to the scale M_Z. With the value of $\tan\beta$ and the fermion masses at the electroweak scale M_Z, one can obtain the Yukawa couplings $Y_{t,b,\tau}(M_Z)$ in the 2HDM, which are then evaluated from M_Z to M_I by the Mathematica program SARAH [76] similar to what we did in the case of the evolution of the gauge couplings in Sect. 3.4.

As was discussed in Sect. 2, after complexifying the $\mathbf{10_H}$ field by introducing an extra $\text{U}(1)_{\text{PQ}}$ symmetry, we can separate the up- and down-type Higgs component of the bi-doublet field Φ_{10} in our intermediate-scale left-right symmetric model. As a result, we will have a few more free parameters, which are the vevs v_{10}^u and v_{10}^d instead of a single vev v_{10} in Ref. [36], and also the relative phases between them, for fitting all the experimental inputs. Counting on the freedom of modifying the scales M_I and M_U by appropriate threshold corrections when enforcing gauge coupling unification, it turns out that within some corners of the huge possible parameter space, we will always be capable of finding solutions for Yukawa coupling unification, unless there are additional constraints from the scalar potential. One such example is the constraints from FCNCs when matching the intermediate-scale left-right model to the low-energy 2HDM, which will be discussed shortly after this subsection.

In the 2HDM, when electroweak symmetry breaking is achieved, the $\text{SU}(2)_L$ gauge bosons W_L will acquire masses from the vevs of both Higgs doublets. This implies a relation between $\tan\beta$ and the SM vev given by $v_u^2 + v_d^2 = v_{\text{SM}}^2 \approx (246\,\text{GeV})^2$ at the scale M_Z. Similarly, in the intermediate left-right model, the electroweak symmetry was broken by the vevs of bi-doublets which then gives the following relation

$$\left(v_{10}^u\right)^2 + \left(v_{10}^d\right)^2 + \left(v_{126}^u\right)^2 + \left(v_{126}^d\right)^2 = v_u^2 + v_d^2 = v_{\text{SM}}^2. \tag{59}$$

In the absence of knowledge of technical details about the intermediate-scale scalar potential, this is the only constraint that we would have for constraining the parameter space.

With the above equation, we can eliminate one free vev. Furthermore, with the matching conditions and the Yukawa coupling conditions defined in Eqs. (46) and (52), we can eliminate one free 422-Yukawa coupling. Note that all the other Yukawa couplings and vevs in the 2HDM can be computed from the sole parameter $\tan\beta$ by the masses of the top and bottom quarks and the tau lepton that are experimentally given. As a result, we have $\tan\beta$, one 422-Yukawa coupling and three vevs, in total five free parameters, when solving the three Eq. (56).[11] We can thus numerically scan for some definite values of the two free parameters, $\tan\beta$ and Y_U which is the free Yukawa coupling at the GUT scale, to get the numerical solutions of these matching conditions for obtaining Yukawa unification.

Therefore, differently from the case discussed in Ref. [36] where in addition to the different matching condition at M_U, the parameter space is very constrained because of the fact that the field $\mathbf{10_H}$ is real. We conclude that the model with a complexified $\mathbf{10_H}$ field is more general and has a much larger parameter space, thus allowing for Yukawa unification that is not restricted to high values of $\tan\beta$ anymore as found in Ref. [36].

Because of the largely allowed parameter spaces, we show in Fig. 3 only two particular examples of the sets of parameters needed to achieve Yukawa coupling unification: one for $\tan\beta = 30$ (in the top panels) and the other for $\tan\beta = 65$ (in the bottom panels) when $Y_{126}^{422}(M_U) = 1$, where the GUT-scale matching conditions motivated from E_6 in Eqs. (52) and (53) have been applied to numerically solve the RGEs of Yukawa couplings from M_Z to M_U. In Table 4, we explicitly list the important free parameters for Yukawa unification.

[11] Rigorously speaking, we should take into account the effects from the runnings of vevs for v_u and v_d when solving the matching conditions in Eq. (56) at the intermediate scale, which makes about 10% deviation from their electroweak-scale values after running to M_I by their RGEs.

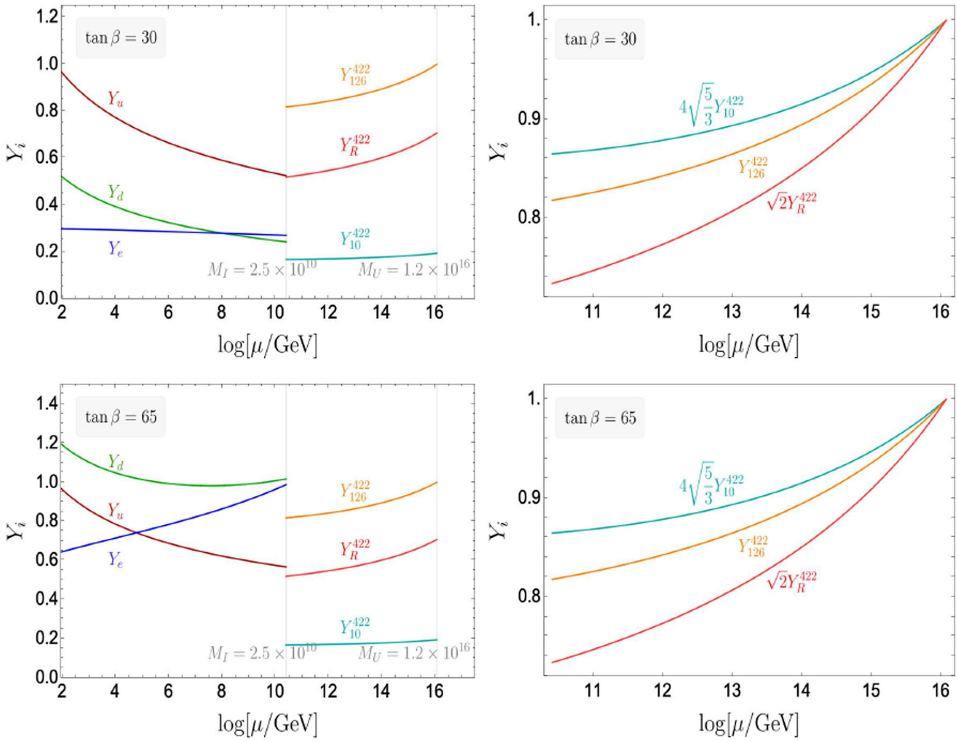

Fig. 3 The runnings of Yukawa couplings in the 422 breaking chains of our non-SUSY SO(10) model including the threshold corrections of gauge couplings, where the E6 factor in Eq. (47) has been used to define the Yukawa unification at the GUT scale. Because of the large parameter spaces allowed, we only show here two particular examples when $\tan\beta = 30$ (in the top) and $\tan\beta = 65$ (in the bottom)

Table 4 The set of third generation fermion Yukawa couplings at the scales M_Z, M_I and M_U, and the relevant vevs at the electroweak and intermediate mass scales at the two-loop level that lead to both gauge coupling and Yukawa coupling unification in our non-SUSY SO(10) model with intermediate 422 breaking

Scale	M_Z			M_I			M_U		M_I			
$\tan\beta$	Y_t	Y_b	Y_τ	Y_{10}^{422}	Y_{126}^{422}	Y_R^{422}	Y_{10}^{422}	Y_{126}^{422}	v_{10}^u	v_{10}^d	v_{126}^u	v_{126}^d
30	0.97	0.35	0.20	0.17	0.82	0.52	0.19	1.0	204.9	12.7	105.1	−0.33
65	0.97	1.19	0.64	0.17	0.82	0.52	0.19	1.0	194.3	16.0	118.2	0.09

4.4 Matching conditions with constraints from FCNCs

At the intermediate scale, the two bi-doublets (Φ_{10} and Σ_{126}) first split into four intermediate-scale Higgs doublets (denoted as $H_{u/d,10}$ and $H_{u/d,126}$ in Eq. (48)), and then two linear combinations of them become light forming the two Higgs doublets (H_u and H_d) of the low-energy 2HDM, while the other two linear combinations acquire masses at the intermediate-scale [59]. Without presenting the technical details about the splitting of bi-doublets in the scalar sector and to simplify our model, we adopt a simple parameterization to forbid the FCNCs for the four intermediate-scale Higgs doublets in Eq. (48) in the mass eigenstates assuming no complex phases involved as

$$\begin{pmatrix} H_u \\ H_u^{\text{heavy}} \end{pmatrix} = \begin{pmatrix} \cos\theta_U & \sin\theta_U \\ -\sin\theta_U & \cos\theta_U \end{pmatrix} \begin{pmatrix} H_{u,10} \\ H_{u,126} \end{pmatrix}$$
$$\begin{pmatrix} H_d \\ H_d^{\text{heavy}} \end{pmatrix} = \begin{pmatrix} \cos\theta_D & \sin\theta_D \\ -\sin\theta_D & \cos\theta_D \end{pmatrix} \begin{pmatrix} H_{d,10} \\ H_{d,126} \end{pmatrix}, \quad (60)$$

where the H_u and H_d are the admixtures of two scalar doublets coupling only to the isospin up/down fermionic sector,[12] which will be identified as the two Higgs fields in low-energy 2HDM, and $H_u^{\rm heavy}$ and $H_d^{\rm heavy}$ are the doublets acquiring intermediate-scale masses via the interactions like $\mathrm{tr}(\Phi^2\Delta_R^2)$. Indeed, this assumption implies that the up/down-type Higgs doublet consists of the up/down components of the two Higgs bidoublets (Φ_{10} and Σ_{126}) at M_I, which can be seen as the definition of the mixing angle $\theta_{U/D}$

$$\cos\theta_U = \frac{v_{10}^u}{v_u},\ \sin\theta_U = \frac{v_{126}^u}{v_u},\ \cos\theta_D = \frac{v_{10}^d}{v_d},\ \sin\theta_D = \frac{v_{126}^d}{v_d}. \tag{61}$$

The above parameterization includes the constraints from FCNCs when matching the intermediate-scale left-right model to the low-energy 2HDM, so equivalently, we can express the matching conditions for Yukawa couplings at M_I derived in Eq. (56) by the mixing angles $\theta_{U/D}$ as

$$Y_t(M_I) = \cos\theta_U Y_{10}^{422}(M_I) + \frac{1}{4}\sin\theta_U Y_{126}^{422}(M_I), \tag{62}$$

$$Y_b(M_I) = \cos\theta_D Y_{10}^{422}(M_I) + \frac{1}{4}\sin\theta_D Y_{126}^{422}(M_I), \tag{63}$$

$$Y_\tau(M_I) = \cos\theta_D Y_{10}^{422}(M_I) - \frac{3}{4}\sin\theta_D Y_{126}^{422}(M_I). \tag{64}$$

Because above the intermediate scale the bottom quark will couple exactly the same way to the Higgs bi-doublets as the tau lepton does, we can eliminate one free parameter θ_D from the last two matching conditions for $Y_b(M_I)$ and $Y_\tau(M_I)$, and get a relation for the Yukawa couplings at M_I:

$$\left(Y_{10}^{422}(M_I)\right)^2 = \frac{\left(Y_{126}^{422}(M_I)\right)^2 (3Y_b(M_I) + Y_\tau(M_I))^2}{16\left[\left(Y_{126}^{422}(M_I)\right)^2 - (Y_b(M_I) - Y_\tau(M_I))^2\right]}. \tag{65}$$

This equation defines a curve $\gamma(M_I)$ in the parameter space $(Y_{10}^{422}(M_I),\ Y_{126}^{422}(M_I))$ as a function of $\tan\beta$. Note that Eq. (65) also implies the lower bound for $Y_{10}^{422}(M_I)$ by

$$Y_{10}^{422}(M_I) > \frac{3Y_b(M_I) + Y_\tau(M_I)}{4}, \tag{66}$$

while requiring $Y_{10}^{422}(M_I) < \sqrt{4\pi}$ implies the lower bound of $Y_{126}^{422}(M_I)$ by

$$Y_{126}^{422}(M_I) > \frac{Y_b(M_I) - Y_\tau(M_I)}{\sqrt{1 - \frac{(3Y_b(M_I)+Y_\tau(M_I))^2}{64\pi}}}. \tag{67}$$

For a straightforward comparison, we show in Fig. 4 several curves $\gamma(M_I)$ depicted by Eq. (65) for certain values of $\tan\beta$ and the intermediate scale M_I taken from 10^8 to 10^{11} GeV, where the minimum of $Y_{10}^{422}(M_I)$ and $Y_{126}^{422}(M_I)$ are given by Eqs. (66) and (67) correspondingly.

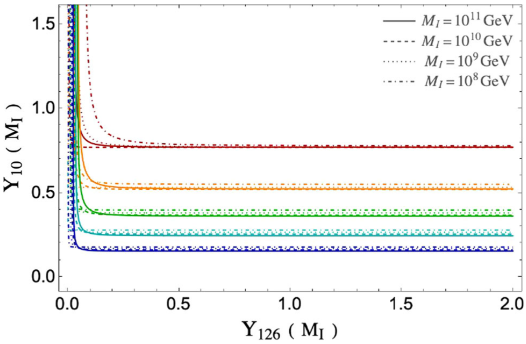

Fig. 4 The curves $\gamma(M_I)$ defined in Eq. (65) when $\tan\beta$ is taken to be 60 (red), 50 (orange), 40 (green), 30 (cyan), 20 (blue), where the parameter M_I is chosen to be 10^{11} GeV (solid), 10^{10} GeV (dashed), 10^9 GeV (dotted), 10^8 GeV (dot-dashed)

If we assume all the vevs are positive, i.e. $0 < \theta_D < \pi/2$, from Eqs. (60)–(61) we can separate the intermediate-scale Yukawa couplings $Y_{10}^{422}(M_I)$ and $Y_{10}^{422}(M_I)$ as

$$Y_{10}^{422}(M_I)\cos\theta_D = \frac{1}{4}\left(3Y_b(M_I) + Y_\tau(M_I)\right),$$
$$Y_{126}^{422}(M_I)\sin\theta_D = Y_b(M_I) - Y_\tau(M_I). \tag{68}$$

It suggests that if the coupling $Y_{126}^{422}(M_I)$ had the same positive sign as $Y_{10}^{422}(M_I)$ in order to be able to be unified at M_U, then at M_I we must have $Y_b(M_I) - Y_\tau(M_I) > 0$. Indeed, it is only an artifact by assuming the positivity of vevs, as the latter relation is nothing but

$$\sqrt{2}(m_b - m_\tau) = (Y_b - Y_\tau)v_d = 4Y_{126}v_{126}^d, \tag{69}$$

from subtracting the SO(10) mass matrices in Eq. (50) if we are matching the SO(10) directly to the 2HDM.

As was discussed in Sect. 3, the intermediate scale M_I and the unification scale M_U are totally fixed by the input parameters $\tan\beta$ and the threshold corrections, irrelevant of the Yukawa couplings. However, once the intermediate-scale Yukawa couplings are switched on, what we deduce from Eq. (68) is that an upper bound on M_I exists such that $Y_b(M_I) > Y_\tau(M_I)$. As both Y_b and Y_τ are functions of $\tan\beta$ only, we can thus derive a scale $M_{b\tau}$ as a function of $\tan\beta$, determined (within some accuracy) by the point at which the curves for their RG running from the weak scale M_Z upwards intersect so that $Y_b(M_{b\tau}) = Y_\tau(M_{b\tau})$. Then the assumption of Eq. (68) translate to

$$M_I \le M_{b\tau} \quad \text{(assuming the positivity of vevs).} \tag{70}$$

This is exemplified in Fig. 5 where the scale $M_{b\tau}$ leading to the unification of the bottom quark and tau lepton couplings in 2HDM are shown as a function of the input value of $\tan\beta$

[12] More general combinations of the 4 scalars of the type

$$H_u = \alpha_1^u H_{u,10} + \alpha_2^u H_{u,126} + \beta_1^u H_{d,10}^* + \beta_2^u H_{d,126}^*,$$
$$H_d = \alpha_1^d H_{d,10} + \alpha_2^d H_{d,126} + \beta_1^d H_{u,10}^* + \beta_2^d H_{u,126}^*,$$

are highly constrained by FCNC [58,59].

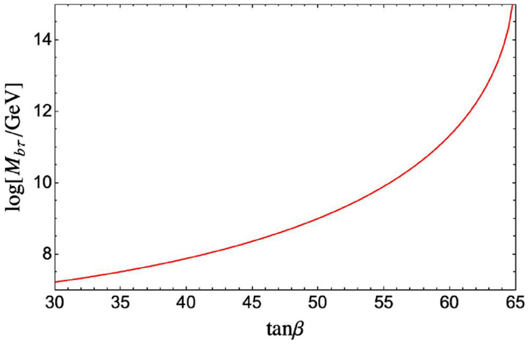

Fig. 5 The unification scale for the bottom quark and tau lepton ($M_{b\tau}$) in the 2HDM which is determined by the point at which the curves for their RG running from the weak scale M_Z upwards intersect

at the low scale. This scale increases with increasing values of tan β and, in order to have reasonably high values of $M_I > 10^{10}$ GeV, needs rather large tan β values,[13] tan $\beta > 55$.

The parameter θ_U for determining the top Yukawa coupling in the 2HDM, on the other hand, cannot be eliminated without further assumptions. Thus, in practice, we treat it as a free parameter that should fit the mass of the top quark. Again by assuming the positivity of vevs, from Eq. (62) we can estimate that the top Yukawa coupling in the 2HDM should lie in the region of

$$\text{Min}\left[Y_{10}^{422}(M_I),\ \frac{Y_{126}^{422}(M_I)}{4}\right] \leq Y_t(M_I) \leq \sqrt{\left(Y_{10}^{422}(M_I)\right)^2 + \left(\frac{Y_{126}^{422}(M_I)}{4}\right)^2}, \tag{71}$$

which, when combined with Eqs. (66) and (67), gives

$$Y_t(M_I) > \text{Min}\left[\frac{3Y_b(M_I)+Y_\tau(M_I)}{4}\ \frac{Y_b(M_I)-Y_\tau(M_I)}{4\sqrt{1-\frac{(3Y_b(M_I)+Y_\tau(M_I))^2}{64\pi}}}\right] = \frac{Y_b(M_I)-Y_\tau(M_I)}{4\sqrt{1-\frac{(3Y_b(M_I)+Y_\tau(M_I))^2}{64\pi}}}. \tag{72}$$

This criterion thus helps us check easily whether a parameter θ_U exists for fitting the mass of the top quark.

[13] Note that we cannot have much higher values of tan β, i.e. tan $\beta < 65$ in general, to avoid the bottom quark Yukawa couplings running into a non-perturbative regime at high energy scales.

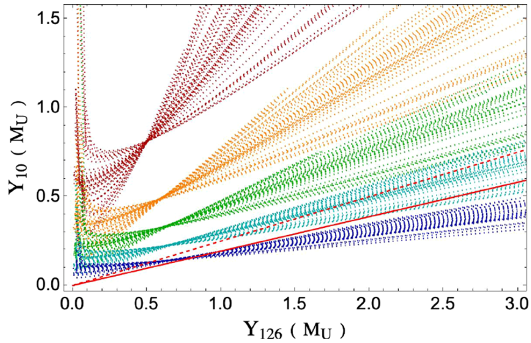

Fig. 6 The curves $\gamma(M_U)$ obtained from numerically evaluating the curves $\gamma(M_I)$ from M_I to M_U by the RGEs of 422 breaking chain, where the scales M_I and M_U are determined by enforcing the gauge unification with randomly taking threshold corrections for $\eta_i = \ln(M_i/\mu) \in [-1, 1]$. The uncertainties of random threshold correction thus generate the uncertainties of these curves $\gamma(M_U)$, which are plotted for the parameter tan β corresponding to 60 (dark red), 50 (orange), 40 (green), 30 (cyan), and 20 (blue). The red line $\ell(M_U)$ corresponds to the condition of Yukawa unification motivated by E_6 unification given in Eq. (53). The intersections of $\ell(M_U)$ and $\gamma(M_U)$ thus define the solutions for Yukawa unification motivated from E_6

4.5 Numerical results for Yukawa unification

In principle, with the RGEs obtained for the 422 breaking chain of our SO(10) model, we can run all the 422-Yukawa couplings on the curves $\gamma(M_I)$ from M_I to M_U to get a new curve $\gamma(M_U)$. The intersections of the curve $\gamma(M_U)$ with the line $\ell(M_U)$ defined in Eq. (53) at the GUT scale M_U thus define the solutions admitting the Yukawa unification in the 422 breaking chain.

When evaluating the curve $\gamma(M_I)$ to the GUT scale, the exact values of M_I and M_U will be determined by numerical solving the RGEs to ensure the unification of gauge couplings as done in Sect. 3.4. The randomly-taken threshold corrections would thus bring some uncertainties in determining the exact values of the two scales M_I and M_U which eventually affect the curves $\gamma(M_I)$ and $\gamma(M_U)$. However, as can be seen from the analytical results in Fig. 4, the curves $\gamma(M_I)$ almost remain intact when varying the scales M_I, suggesting that the threshold corrections of gauge couplings only make a tiny difference in determining the curves $\gamma(M_I)$ and similarly to $\gamma(M_U)$, contrary to what happens in gauge coupling unification, i.e. Fig. 1. We can thus safely choose some random-sampling threshold corrections when visualizing the curves $\gamma(M_U)$ as shown in Fig. 6 below, where for each tan β we explicitly show the uncertainty regions allowed by varying the parameters of threshold corrections from $\eta_i = \ln(M_i/\mu) \in [-1, 1]$.

When considering a different scenario of Yukawa unification for the 422 breaking chains of the non-SUSY SO(10)

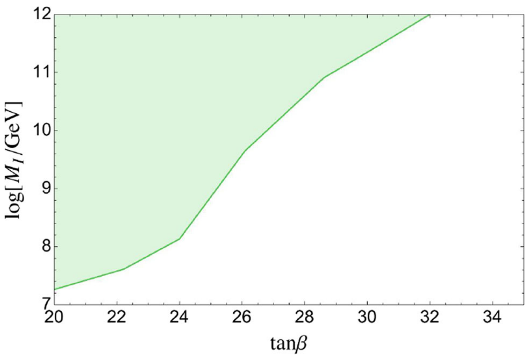

Fig. 7 Values of the intermediate scale M_I and $\tan\beta$ consistent with gauge and Yukawa unification (the green region), using the E_6 factor in our non-SUSY SO(10) model

models, one can merely change the slope of the line $\ell(M_U)$ by defining the ratios of Y_{10} and Y_{126} in Eq. (46), while the curves $\gamma(M_U)$ remain the same. To be compared with the E_6 case, we also show the condition of Yukawa unification for the ratio $C = c_{10}/c_{126} = 1$, which is presented by the dashed red line in Fig. 6.

As a consistency check, we must combine all the conditions that we derive to constrain the parameter space, including the proton decay bound of Eq. (44), the GUT-scale matching condition in Eqs. (52) and (53), the lower bounds of the Yukawa couplings in Eqs. (66) and (67), and the perturbative bound when requiring that all Yukawa couplings must be smaller than $\sqrt{4\pi}$ at all energy scales.

One then immediately finds that these constraints also influence the intermediate scale M_I, especially through Eq. (70). Thus, the unification of third generation Yukawa couplings also has a repercussion on gauge coupling unification. This refines the naive statement that we initially made in Sect. 3, namely that the contributions of the Yukawa couplings hardly affect the RGEs of the gauge couplings and, hence, their unification.

Including all the constraints, and enforcing the unification of the Yukawa couplings with the E_6 ratio, one can visualize the numerical solutions in the 422 breaking chain of our non-SUSY SO(10) model in Fig. 7. It shows, in green, the parameter region in which both gauge and Yukawa coupling unification can be achieved in the plane $[\tan\beta, \log(M_I/\text{GeV})]$ as $\tan\beta$ is the most important parameter in determining unification in the two cases. As can be seen, for each $\tan\beta$ value, there can be multiple solutions depending on the exact threshold corrections resulting in the different intermediate scale M_I and at a later stage, the unification scale M_U. Thus, after considering the constraints of FCNCs, in addition to all the other constraints, the parameter $\tan\beta$ is again very constrained in this 422 intermediate breaking model and only relatively lower values (compared to those discussed in the earlier analysis of Ref. [36]), $\tan\beta \lesssim 30$ for $M_I \lesssim 10^{12}$ GeV, are favored[14]

As a preliminary conclusion, the constraints from FCNCs largely reduce the allowed parameter spaces for Yukawa coupling unification motivated by E_6 symmetry. This only favors lower values of $\tan\beta$ in the 422 breaking chain for instance. Thus, the 422 breaking chain of our non-SUSY minimal SO(10) model is very constrained, with the only parameter which can be varied being the value of the input $\tan\beta$ of the low-energy 2HDM. This renders the model quite predictive. The other nice feature is that Yukawa unification, with a common coupling at the high scale being naturally of order unity, implies that a condition at the high scale has an impact on the low energy parameters such as $\tan\beta$.

5 Conclusions

The unification of fundamental forces plays an extremely important role in particle physics. A wide range of studies have dealt with the unification of the three gauge couplings of the SM either by sticking to the minimal SU(5) gauge group and extending the SM particle spectrum, as is the case in Supersymmetric theories, or keeping the SM particle content and extending the unifying gauge symmetry group. In this last option, the SO(10) group has been the most widely studied as it is the simplest one beyond the minimal SU(5) group. It possibly leads to a left-right symmetry group and it has a fundamental representation of dimension 16 which could contain all SM fermions plus an additional Majorana neutrino. If the mass of the latter particle is high enough, $\mathcal{O}(10^{12}-10^{14})$ GeV, one could explain the pattern of masses and mixing of the SM light neutrino species and address the problem of the baryon asymmetry in the universe by invoking a leptogenesis triggered by this additional heavy neutrino. Unification is achieved by considering that this large mass of the Majorana neutrinos is in fact due to the intermediate scale of the breaking of SO(10) into the SM group via an intermediate step, corresponding, for instance, to the Pati–Salam or the minimal left-right symmetry groups. This is achieved by including the threshold effects of the additional Higgs and gauge bosons at this intermediate scale M_I, which then modify the renormalization group evolution of the coupling constants and make them intersect at a single point, the unification scale M_U.

[14] We should note that in this green region in which both gauge and Yukawa unification occur, the vevs of the Higgs bi-doublets, for example, v^d_{126}, are complex and are negative according to Eq. 70 and Fig. 5. If we require the vevs to be positive, unification occurs only for lower values of M_I and higher values of $\tan\beta$. The corresponding region will not intersect with the green region.

It is very tempting to extend the unification paradigm to the case of the Yukawa couplings of fermions, in particular those of the third generation which are heavy enough to allow for a perturbative treatment at the low energy scale. This has been attempted in an earlier analysis in Ref. [36] in both the Pati–Salam and the minimal left-right intermediate schemes, which showed that, ignoring constraints from flavor changing neutral currents, one can achieve the Yukawa unification in the context of a low energy two-Higgs doublet model in which the ratio of the two vevs is very high, $\tan\beta \approx 60$, and reproduce the hierarchy of the fermion masses of the third generation from the running of Yukawa couplings.

In this paper, we generalized our previous analysis made in Ref. [36] to the case with a complex 10_H field, where a $U(1)_{PQ}$ global symmetry was introduced to forbid the Yukawa couplings with the field $\mathbf{10}^*_\mathbf{H}$, in order to relax the parameter space in the previous over-constrained model which also changes the RGEs of the gauge couplings. We then derived the analytical approximate solutions of the RGEs of gauge couplings enforced by unification at the two-loop level. The procedure in our chosen non-SUSY SO(10) model with an intermediate scale can also be applied for any breaking patterns of SO(10). The uncertainties of our approximation were also discussed, including the constraints from proton decay experiments. All our approximate analytical results have been compared with the numerical results given in Tables 2 and 3, and a good agreement was found.

We have then discussed the possibility of unifying the Yukawa couplings of third generation heavy fermions at the high scale which, in the present context, implies a relation between the fermion couplings to the scalar representations 10 and 126. Specializing to the Pati–Salam intermediate SO(10) breaking chain, we have considered the particular case where the coupling is obtainable in an E_6 model where the previous two scalars are part of a single multiplet and which leads to the relation $Y_{10} = \sqrt{3/5}\, Y_{126}$. We concluded that Yukawa unification is a very strong constraint which, when imposing the absence of flavor changing neutral currents at tree–level induced by the two light Higgs doublet fields, is achieved only for $\tan\beta$ values that are not too large. Our non-SUSY SO(10) model is thus very predictive and can be testified by future electroweak-scale experiments.

Our present exploratory analysis raises rather interesting questions which require further attention and studies of the subject. In particular, there are still some phenomenological issues to be discussed within this model, such as the problem of the stability of the electroweak vacuum and the origin of neutrino masses. Because our low-energy effective theory is based on a 2HDM scenario, we must constrain our scalar potential to enforce a stable vacuum that is bounded from below as, for instance, discussed in Refs. [83–88]. We expect the discussions held in these references to also apply in our case as we are dealing with the same Type-II 2HDM scenarios. On the other hand, if the neutrinos are to acquire masses from a Type-I/II see-saw mechanism, the scale of the right-handed neutrino mass, which is assumed to be of the order of the intermediate scale M_I, cannot be too small as to avoid unnatural fine-tuning in the determination of the light neutrino masses. These neutrino masses thus contribute to setting another constraint on the intermediate scale. All these aspects and others need further attention and we plan to address them in future work.

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Appendix A: Lists of useful coefficients

A.1 β coefficients for different gauge groups and representations

The one-loop and two-loop β coefficients a_i and b_{ij}, can be calculated from Refs. [63,64] in the general case. We list the values of the β coefficients for some particular gauge groups $\mathcal{G}_I$ with the considered scalar representations that are relevant for our discussions and which are given in Table 5.

Table 5 The coefficients a_i and b_i of the β functions of the RGEs of the gauge couplings α_i for the different breaking schemes that we are considering

$\mathcal{G}_I$	a_i	b_{ij}
$\mathcal{G}_{321}$(SM)	$\begin{pmatrix} -7 \\ -\frac{19}{6} \\ \frac{41}{10} \end{pmatrix}$	$\begin{pmatrix} -26 & \frac{9}{2} & \frac{11}{10} \\ 12 & \frac{35}{6} & \frac{9}{10} \\ \frac{44}{5} & \frac{27}{10} & \frac{199}{50} \end{pmatrix}$
$\mathcal{G}_{321}$(2HDM)	$\begin{pmatrix} -7 \\ -3 \\ \frac{21}{5} \end{pmatrix}$	$\begin{pmatrix} -26 & \frac{9}{2} & \frac{11}{10} \\ 12 & 8 & \frac{6}{5} \\ \frac{44}{5} & \frac{18}{5} & \frac{104}{25} \end{pmatrix}$
$\mathcal{G}_{422}$	$\begin{pmatrix} -\frac{7}{3} \\ 2 \\ \frac{28}{3} \end{pmatrix}$	$\begin{pmatrix} \frac{2435}{6} & \frac{105}{2} & \frac{249}{2} \\ \frac{525}{3} & 73 & 48 \\ \frac{1245}{2} & 48 & \frac{835}{3} \end{pmatrix}$
$\mathcal{G}_{422} \times \mathcal{D}$	$\begin{pmatrix} \frac{2}{3} \\ \frac{28}{3} \\ \frac{28}{3} \end{pmatrix}$	$\begin{pmatrix} \frac{3551}{6} & \frac{249}{2} & \frac{249}{2} \\ \frac{1245}{2} & \frac{835}{3} & 48 \\ \frac{1245}{2} & 48 & \frac{835}{3} \end{pmatrix}$
$\mathcal{G}_{3221}$	$\begin{pmatrix} -7 \\ -\frac{8}{3} \\ -2 \\ \frac{11}{2} \end{pmatrix}$	$\begin{pmatrix} -26 & \frac{9}{2} & \frac{9}{2} & \frac{1}{2} \\ 12 & \frac{37}{3} & 6 & \frac{3}{2} \\ 12 & 6 & 31 & \frac{27}{2} \\ 4 & \frac{9}{2} & \frac{81}{2} & \frac{61}{2} \end{pmatrix}$
$\mathcal{G}_{3221} \times \mathcal{D}$	$\begin{pmatrix} -7 \\ -\frac{4}{3} \\ -\frac{4}{3} \\ 7 \end{pmatrix}$	$\begin{pmatrix} -26 & \frac{9}{2} & \frac{9}{2} & \frac{1}{2} \\ 12 & \frac{149}{3} & 6 & \frac{27}{2} \\ 12 & 6 & \frac{149}{3} & \frac{27}{2} \\ 4 & \frac{81}{2} & \frac{81}{2} & \frac{115}{2} \end{pmatrix}$

A.2 The two-loop $\theta_i^{\mathcal{G}}$ coefficients in our approximations

At two loop level, the solutions of the two-loop RGEs of gauge couplings takes the general implicit form of Eq. (20):

$$\alpha_{i,\mathcal{G}}^{-1}(\mu) = \alpha_{i,\mathcal{G}}^{-1}(\mu_0) - \frac{a_i^{\mathcal{G}}}{2\pi} \ln \frac{\mu}{\mu_0} + \gamma_i^{\mathcal{G}} + \Delta_{i,Y}^{\mathcal{G}}, \tag{A.1}$$

where the two-loop contributions are functions of gauge couplings $\alpha_{j,\mathcal{G}}^{-1}(\mu)$ read

$$\gamma_i^{\mathcal{G}} = -\frac{1}{4\pi} \sum_j \frac{b_{ij}^{\mathcal{G}}}{a_j^{\mathcal{G}}} \ln \frac{\alpha_{j,\mathcal{G}}(\mu)}{\alpha_{j,\mathcal{G}}(\mu_0)}. \tag{A.2}$$

This two-loop factor can be approximated by expanding the variables $\alpha_{j,\mathcal{G}}^{-1}(\mu)$ using the one-loop RGEs [62]:

$$\gamma_i^{\mathcal{G}} \approx -\frac{1}{4\pi} \sum_j \frac{b_{ij}^{\mathcal{G}}}{a_j^{\mathcal{G}}} \ln \frac{\alpha_{j,\mathcal{G}}^{-1}(\mu_0)}{\alpha_{j,\mathcal{G}}^{-1}(\mu_0) - a_j^{\mathcal{G}} t}$$
$$= -\frac{1}{4\pi} \sum_j \frac{b_{ij}^{\mathcal{G}}}{a_j^{\mathcal{G}}} \ln \left(1 + a_j^{\mathcal{G}} t\, \alpha_{j,\mathcal{G}}(\mu)\right), \tag{A.3}$$

where we define $t = \frac{1}{2\pi} \ln \frac{\mu}{\mu_0}$.

In Grand Unified Theories, all the gauge couplings intersect at the unification scale M_U for the value of α_U, so we can approximate the gauge couplings at an arbitrary high scale μ to be the universal gauge couplings α_U at the GUT scale. Now the two-loop factors $\gamma_i^{\mathcal{G}}$ become independent of the gauge couplings at the high scale so we can express them by the θ_i coefficients as in Refs. [62,67]:

$$\gamma_i^{\mathcal{G}} \approx -\frac{\alpha_U}{8\pi^2} \theta_i^{\mathcal{G}} \ln \frac{\mu}{\mu_0} \quad \text{and} \quad \theta_i^{\mathcal{G}} \equiv \sum_j b_{ij}^{\mathcal{G}} \frac{\ln(1 + a_j^{\mathcal{G}} \alpha_U t)}{a_j^{\mathcal{G}} \alpha_U t}. \tag{A.4}$$

In summary, the above equation shows the leading-order corrections of the two-loop β coefficients b_{ij} to the full two-loop RGEs, which provides the possibility to obtain analytical solutions for the original implicit differential equations. The coefficients θ_i are a combination of two-loop β coefficients $b_{ij}^{\mathcal{G}}$ scaling by the one-loop β coefficients $a_j^{\mathcal{G}}$ times universal coupling α_U and the logarithmic scales t. We therefore define the following combination to simplify the common factor appearing in the coefficient $\theta_i^{\mathcal{G}}$ between the scales M_a and M_b as:

$$\Theta_{ab} \equiv \frac{1}{2\pi} \alpha_U \ln \left(\frac{M_a}{M_b}\right), \tag{A.5}$$

where M_a is the high scale to be identified as either the GUT scale M_U or the intermediate scale M_I later, while M_b is the reference low scale to be identified as either the intermediate scale M_I or the Electroweak scale M_Z. These scaling factors will finally appear in the four constant terms $C_{\mathcal{G}_I}$, $\Delta_{31}^{\mathcal{G}_{321}}$, $\Delta_{32}^{\mathcal{G}_{321}}$ and $\Delta_{3_I 2L_I}^{\mathcal{G}_I}$ from definition Eq. (28), and they will be determined from solving Eqs. (30)–(32) in Sect. 3.2. We summarize the explicit form of the corresponding coefficients $\theta_i^{\mathcal{G}}$ for the symmetry groups and representations we considered in Table 6.

A.3 Some constant coefficients for the SO(10) breaking chains

We have shown in Sect. 3.1 that the two-loop RGEs with the boundary conditions defined as the gauge coupling unification in Eq. (25) and the matching conditions with an intermediate scale, e.g. Eqs. (26) and (27), will have the solutions in Eqs. (30)–(32). These solutions are only dependent on the four constant coefficients $C_{\mathcal{G}_I}$, $\Delta_{31}^{\mathcal{G}_{321}}$, $\Delta_{32}^{\mathcal{G}_{321}}$ and $\Delta_{3_I 2L_I}^{\mathcal{G}_I}$, where $\Delta_{ij}^{\mathcal{G}}$ gives the difference between the β coefficients of the gauge coupling $\alpha_{i,\mathcal{G}}^{-1}$ and those of $\alpha_{j,\mathcal{G}}^{-1}$:

$$\Delta_{ij}^{\mathcal{G}} = \frac{a_i^{\mathcal{G}} - a_j^{\mathcal{G}}}{2\pi} + \frac{\theta_i^{\mathcal{G}} - \theta_j^{\mathcal{G}}}{8\pi^2} \alpha_U. \tag{A.6}$$

As explained in the main text, for each intermediate symmetry group $\mathcal{G}_I$ it is enough to consider a particular combination of the $\Delta_{ij}^{\mathcal{G}}$, which we call $C_{\mathcal{G}_I}$. They can be proven to have the following forms for the typical breaking chains

Table 6 The coefficients θ_i for the symmetry groups and different breaking schemes that we are considering in our study

$\mathcal{G}_I$	$\theta_i^{\mathcal{G}}$
$\mathcal{G}_{321}$(SM)	$\begin{pmatrix} -\frac{44\ln(1-7\Theta_{IZ})}{35\Theta_{IZ}} - \frac{81\ln((6-19\Theta_{IZ})/6)}{95\Theta_{IZ}} + \frac{199\ln((10+41\Theta_{IZ})/10)}{205\Theta_{IZ}} \\ -\frac{12\ln(1-7\Theta_{IZ})}{7\Theta_{IZ}} - \frac{35\ln((6-19\Theta_{IZ})/6)}{19\Theta_{IZ}} + \frac{9\ln((10+41\Theta_{IZ})/10)}{41\Theta_{IZ}} \\ \frac{26\ln(1-7\Theta_{IZ})}{7\Theta_{IZ}} - \frac{27\ln((6-19\Theta_{IZ})/6)}{19\Theta_{IZ}} + \frac{11\ln((10+41\Theta_{IZ})/10)}{41\Theta_{IZ}} \end{pmatrix}$
$\mathcal{G}_{321}$(2HDM)	$\begin{pmatrix} -\frac{44\ln(1-7\Theta_{IZ})}{35\Theta_{IZ}} - \frac{6\ln(1-3\Theta_{IZ})}{5\Theta_{IZ}} + \frac{104\ln((5+21\Theta_{IZ})/5)}{105\Theta_{IZ}} \\ -\frac{12\ln(1-7\Theta_{IZ})}{7\Theta_{IZ}} - \frac{8\ln(1-3\Theta_{IZ})}{3\Theta_{IZ}} + \frac{2\ln((5+21\Theta_{IZ})/5)}{7\Theta_{IZ}} \\ \frac{26\ln(1-7\Theta_{IZ})}{7\Theta_{IZ}} - \frac{3\ln(1-3\Theta_{IZ})}{2\Theta_{IZ}} + \frac{11\ln(5+21\Theta_{IZ})/5)}{42\Theta_{IZ}} \end{pmatrix}$
$\mathcal{G}_{422}$	$\begin{pmatrix} -\frac{2435\ln(1-7\Theta_{UI}/3)}{14\Theta_{UI}} + \frac{105\ln(1+2\Theta_{UI})}{4\Theta_{UI}} + \frac{747\ln(1+28\Theta_{UI}/3)}{56\Theta_{UI}} \\ -\frac{75\ln(1-7\Theta_{UI}/3)}{\Theta_{UI}} + \frac{73\ln(1+2\Theta_{UI})}{2\Theta_{UI}} + \frac{36\ln(1+28\Theta_{UI}/3)}{7\Theta_{UI}} \\ -\frac{3735\ln(1-7\Theta_{UI}/3)}{14\Theta_{UI}} + \frac{24\ln(1+2\Theta_{UI})}{\Theta_{UI}} + \frac{835\ln(1+28\Theta_{UI}/3)}{28\Theta_{UI}} \end{pmatrix}$
$\mathcal{G}_{422} \times \mathcal{D}$	$\begin{pmatrix} \frac{3551\ln(1+2\Theta_{UI}/3)}{4\Theta_{UI}} + \frac{747\ln(1+28\Theta_{UI}/3)}{28\Theta_{UI}} \\ \frac{3735\ln(1+2\Theta_{UI}/3)}{4\Theta_{UI}} + \frac{979\ln(1+28\Theta_{UI}/3)}{28\Theta_{UI}} \\ \frac{3735\ln(1+2\Theta_{UI}/3)}{4\Theta_{UI}} + \frac{979\ln(1+28\Theta_{UI}/3)}{28\Theta_{UI}} \end{pmatrix}$
$\mathcal{G}_{3221}$	$\begin{pmatrix} -\frac{27\ln(1-8\Theta_{UI}/3)}{16\Theta_{UI}} - \frac{9\ln(1-2\Theta_{UI})}{4\Theta_{UI}} + \frac{26\ln(1-7\Theta_{UI})}{7\Theta_{UI}} + \frac{\ln(1+11\Theta_{UI}/2)}{11\Theta_{UI}} \\ -\frac{37\ln(1-8\Theta_{UI}/3)}{8\Theta_{UI}} - \frac{3\ln(1-2\Theta_{UI})}{\Theta_{UI}} - \frac{12\ln(1-7\Theta_{UI})}{7\Theta_{UI}} + \frac{3\ln(1+11\Theta_{UI}/2)}{11\Theta_{UI}} \\ -\frac{9\ln(1-8\Theta_{UI}/3)}{4\Theta_{UI}} - \frac{31\ln(1-2\Theta_{UI})}{2\Theta_{UI}} - \frac{12\ln(1-7\Theta_{UI})}{7\Theta_{UI}} + \frac{27\ln(1+11\Theta_{UI}/2)}{11\Theta_{UI}} \\ -\frac{27\ln(1-8\Theta_{UI}/3)}{16\Theta_{UI}} - \frac{81\ln(1-2\Theta_{UI})}{4\Theta_{UI}} - \frac{4\ln(1-7\Theta_{UI})}{7\Theta_{UI}} + \frac{61\ln(1+11\Theta_{UI}/2)}{11\Theta_{UI}} \end{pmatrix}$
$\mathcal{G}_{3221} \times \mathcal{D}$	$\begin{pmatrix} -\frac{27\ln(1-4\Theta_{UI}/3)}{4\Theta_{UI}} + \frac{26\ln(1-7\Theta_{UI})}{7\Theta_{UI}} + \frac{\ln(1+7\Theta_{UI})}{14\Theta_{UI}} \\ -\frac{167\ln(1-4\Theta_{UI}/3)}{4\Theta_{UI}} - \frac{12\ln(1-7\Theta_{UI})}{7\Theta_{UI}} + \frac{27\ln(1+7\Theta_{UI})}{14\Theta_{UI}} \\ -\frac{167\ln(1-4\Theta_{UI}/3)}{4\Theta_{UI}} - \frac{12\ln(1-7\Theta_{UI})}{7\Theta_{UI}} + \frac{27\ln(1+7\Theta_{UI})}{14\Theta_{UI}} \\ -\frac{243\ln(1-4\Theta_{UI}/3)}{4\Theta_{UI}} - \frac{4\ln(1-7\Theta_{UI})}{7\Theta_{UI}} + \frac{115\ln(1+7\Theta_{UI})}{14\Theta_{UI}} \end{pmatrix}$

Table 7 The four constant coefficients $C_{\mathcal{G}_I}$, $\Delta_{31}^{\mathcal{G}_{321}}$, $\Delta_{32}^{\mathcal{G}_{321}}$ and $\Delta_{3_I 2L_I}^{\mathcal{G}_I}$, and their corresponding derivatives appearing in the solutions of the RGEs of SO(10) in Eqs. (30)–(32) for our considered breaking chains. The numerical results presented here are those when α_U is taking to zero, which is relevant for calculating the two-loop solutions in Eqs. (40) and (41)

Breaking chains	$C_{\mathcal{G}_I}$	$\Delta_{31}^{\mathcal{G}_{321}}$	$\Delta_{32}^{\mathcal{G}_{321}}$	$\Delta_{3_I 2L_I}^{\mathcal{G}_I}$	$\frac{\partial C_{\mathcal{G}_I}}{\partial \alpha_U}$	$\frac{\partial \Delta_{31}^{\mathcal{G}_{321}}}{\partial \alpha_U}$	$\frac{\partial \Delta_{32}^{\mathcal{G}_{321}}}{\partial \alpha_U}$	$\frac{\partial \Delta_{3_I 2L_I}^{\mathcal{G}_I}}{\partial \alpha_U}$
$\mathcal{G}_{422} \to \mathcal{G}_{321}$(SM)	$\frac{21}{13}$	$-\frac{111}{20\pi}$	$-\frac{23}{12\pi}$	$-\frac{13}{6\pi}$	$\frac{266349}{6760\pi}$	$-\frac{897}{200\pi^2}$	$-\frac{587}{120\pi^2}$	$\frac{1721}{48\pi^2}$
$\mathcal{G}_{422} \to \mathcal{G}_{321}$(2HDM)	$\frac{21}{13}$	$-\frac{28}{5\pi}$	$-\frac{2}{\pi}$	$-\frac{13}{6\pi}$	$\frac{266349}{6760\pi}$	$-\frac{231}{50\pi^2}$	$-\frac{26}{5\pi^2}$	$\frac{1721}{48\pi^2}$
$\mathcal{G}_{3221} \to \mathcal{G}_{321}$(SM)	$\frac{24}{13}$	$-\frac{111}{20\pi}$	$-\frac{23}{12\pi}$	$-\frac{13}{6\pi}$	$-\frac{669}{3380\pi}$	$-\frac{897}{200\pi^2}$	$-\frac{587}{120\pi^2}$	$-\frac{145}{24\pi^2}$
$\mathcal{G}_{3221} \to \mathcal{G}_{321}$(2HDM)	$\frac{24}{13}$	$-\frac{28}{5\pi}$	$-\frac{2}{\pi}$	$-\frac{13}{6\pi}$	$-\frac{669}{3380\pi}$	$-\frac{231}{50\pi^2}$	$-\frac{26}{5\pi^2}$	$-\frac{145}{24\pi^2}$

$\mathcal{G}_I = \mathcal{G}_{422}$ and $\mathcal{G}_I = \mathcal{G}_{3221}$:

$$C_{\mathcal{G}_{422}} = 3\Delta_{42_R}^{\mathcal{G}_{422}}/(5\Delta_{42_L}^{\mathcal{G}_{422}}), \quad C_{\mathcal{G}_{3221}} = (3\Delta_{32_R}^{\mathcal{G}_{3221}} + 2\Delta_{3B-L}^{\mathcal{G}_{3221}})/(5\Delta_{32_L}^{\mathcal{G}_{3221}}), \tag{A.7}$$

which are basically a combination of the difference between the β coefficients of the gauge couplings of intermediate symmetry group $\mathcal{G}_I$. At one-loop level, we can neglect all the two loop coefficients $\theta_i^{\mathcal{G}}$ by setting $\alpha_U = 0$ in Eq. (A.6), so the coefficients $\Delta_{ij}^{\mathcal{G}}$ are merely constants. At two-loop order, because the coefficients $\Delta_{ij}^{\mathcal{G}}$ are a functions of the set of variables $(\ln(M_I/M_Z), \ln(M_U/M_I), \alpha_U)$, Eqs. (30)–(32) are implicit functions and were solved approximately using Eqs. (40) and (41). For this approximation, we need to calculate the derivatives $\left.\frac{\partial F}{\partial \alpha_U}\right|_{\alpha_U=0}$ and $\left.\frac{\partial G}{\partial \alpha_U}\right|_{\alpha_U=0}$, which is equivalent to finding $\left.\frac{\partial \Delta_{ij}^{\mathcal{G}}}{\partial \alpha_U}\right|_{\alpha_U=0}$. These derivatives are independent of the scale factor $t = \frac{1}{2\pi}\ln\frac{\mu}{\mu_0}$, so they are also constants when $\alpha_U = 0$. We summarize the numerical values of these coefficients for our considered breaking chains in the following Table 7.

Appendix B: Two-loop RGEs of the low-energy 2HDM

The RGEs for the three gauge couplings g_1, g_2, g_3 are:

$$16\pi^2 \frac{\mathrm{d}g_1}{\mathrm{d}t} = \frac{21}{5}g_1^3 + \frac{g_1^3}{800\pi^2}\left(208g_1^2 + 180g_2^2 + 440g_3^2 -85Y_t^2 - 25Y_b^2 - 75Y_\tau^2\right), \quad (A.8)$$

$$16\pi^2 \frac{\mathrm{d}g_2}{\mathrm{d}t} = -3g_2^3 + \frac{g_2^3}{160\pi^2}\left(12g_1^2 + 80g_2^2 + 120g_3^2 -15Y_t^2 - 15Y_b^2 - 5Y_\tau^2\right), \quad (A.9)$$

$$16\pi^2 \frac{\mathrm{d}g_3}{\mathrm{d}t} = -7g_3^3 - \frac{g_3^3}{160\pi^2}\left(-11g_1^2 - 45g_2^2 + 260g_3^2 +20Y_t^2 + 20Y_b^2\right), \quad (A.10)$$

while those for the third generation Yukawa couplings Y_t, Y_b, Y_τ, are:

$$\begin{aligned}16\pi^2 \frac{\mathrm{d}Y_t}{\mathrm{d}t} &= Y_t\left(3Y_t^2 - \frac{17g_1^2}{20} - \frac{9g_2^2}{4} - 8g_3^2\right) + \frac{1}{2}Y_t\left(Y_b^2 + 3Y_t^2\right)\\ &+ \frac{1}{16\pi^2}\Bigg[Y_b^2Y_t\left(-\frac{9Y_b^2}{4} - \frac{3Y_\tau^2}{4} - \frac{41g_1^2}{240} + \frac{33g_2^2}{16} + \frac{16g_3^2}{3} - 2\lambda_3 + 2\lambda_4\right)\\ &+ Y_t^3\left(-\frac{27}{4}Y_t^2 + \frac{223}{80}g_1^2 + \frac{135}{16}g_2^2 + 16g_3^2 - 12\lambda_2\right) - \frac{1}{4}\left(Y_b^2Y_t^3 + Y_b^4Y_t - 6Y_t^5\right)\\ &+ Y_t\bigg(-\frac{27Y_t^4}{4} - \frac{9}{4}Y_b^2Y_t^2 + \frac{1}{8}\left(17g_1^2 + 45g_2^2 + 160g_3^2\right)Y_t^2 + \frac{1267g_1^4}{600} - \frac{9}{20}g_2^2g_1^2 + \frac{19}{15}g_3^2g_1^2\\ &- \frac{21g_2^4}{4} - 108g_3^4 + 9g_2^2g_3^2 + 6\lambda_2^2 + \lambda_3^2 + \lambda_4^2 + 6\lambda_5^2 + \lambda_3\lambda_4\bigg)\Bigg], \quad (A.11)\end{aligned}$$

$$\begin{aligned}16\pi^2 \frac{\mathrm{d}Y_b}{\mathrm{d}t} &= Y_b\left(3Y_b^2 + Y_\tau^2 - \frac{1}{4}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2\right) + \frac{1}{2}Y_b\left(Y_t^2 + 3Y_b^2\right)\\ &+ \frac{1}{16\pi^2}\Bigg[Y_bY_t^2\left(-\frac{9Y_t^2}{4} - \frac{53g_1^2}{240} + \frac{33g_2^2}{16} + \frac{16g_3^2}{3} - 2\lambda_3 + 2\lambda_4\right)\\ &+ Y_b^3\left(-\frac{27Y_b^2}{4} - \frac{9Y_\tau^2}{4} + \frac{187g_1^2}{80} + \frac{135g_2^2}{16} + 16g_3^2 - 12\lambda_1\right) - \frac{1}{4}\left(Y_b^3Y_t^2 + Y_bY_t^4 - 6Y_b^5\right)\\ &+ Y_b\bigg(-\frac{27Y_b^4}{4} - \frac{9Y_\tau^4}{4} - \frac{9}{4}Y_b^2Y_t^2 + \frac{5}{8}\left(g_1^2 + 9g_2^2 + 32g_3^2\right)Y_b^2 + \frac{15}{8}\left(g_1^2 + g_2^2\right)Y_\tau^2 - \frac{113g_1^4}{600}\\ &- \frac{27}{20}g_2^2g_1^2 + \frac{31}{15}g_3^2g_1^2 - \frac{21g_2^4}{4} - 108g_3^4 + 9g_2^2g_3^2 + 6\lambda_1^2 + \lambda_3^2 + \lambda_4^2 + 6\lambda_5^2 + \lambda_3\lambda_4\bigg)\Bigg], \quad (A.12)\end{aligned}$$

$$\begin{aligned}16\pi^2 \frac{\mathrm{d}Y_\tau}{\mathrm{d}t} &= Y_\tau\left(3Y_b^2 + Y_\tau^2 - \frac{9}{4}g_1^2 - \frac{9}{4}g_2^2\right) + \frac{3}{2}Y_\tau^3\\ &+ \frac{1}{16\pi^2}\Bigg[Y_\tau^3\left(-\frac{27}{4}Y_b^2 - \frac{9}{4}Y_\tau^2 + \frac{387}{80}g_1^2 + \frac{135}{16}g_2^2 - 12\lambda_1\right) + \frac{3Y_\tau^5}{2}\\ &+ Y_\tau\bigg(-\frac{9Y_\tau^4}{4} - \frac{27Y_b^4}{4} - \frac{9}{4}Y_b^2Y_t^2 + \frac{5}{8}\left(g_1^2 + 9g_2^2 + 32g_3^2\right)Y_b^2 + \frac{15}{8}\left(g_1^2 + g_2^2\right)Y_\tau^2\\ &+ \frac{1449g_1^4}{200} + \frac{27}{20}g_2^2g_1^2 - \frac{21g_2^4}{4} + 6\lambda_1^2 + \lambda_3^2 + \lambda_4^2 + 6\lambda_5^2 + \lambda_3\lambda_4\bigg)\Bigg]. \quad (A.13)\end{aligned}$$

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- 2018, Enhanced DiHiggs Signal from Hidden Scalar QCD at Leading Order Scale Symmetry Limit, MSc, supervisor Dr. Andi Hektor, Tallinn University of Technology, National Institute of Chemical Physics and Biophysics, Estonia
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Papers

1 Abdelhak Djouadi, Renato Fonseca, Ruiwen Ouyang, and Martti Raidal. Non-supersymmetric SO(10) models with Gauge and Yukawa coupling unification. *Eur. Phys. J. C*, 83(6):529, 2023

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1. R. Ouyang. *The implication of Yukawa unification in SO(10) GUTs.*, The 30th International Conference on Supersymmetry and Unification of Fundamental Interactions (SUSY 2023): 17 July –21 July 2022, Southampton, UK

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3. Haridus

2019–2023	Tallinna Tehnikaülikool, Loodusteaduskond, Küberneetika instituut, doktorantuur
2018–2018	Tallinna Tehnikaülikool, Loodusteaduskond, Küberneetika instituut, magistriõpe, *cum laude*
2014–2018	Jilini Ülikool, Loodusteaduskond, Füüsika instituut, bakalaureuse õpe

4. Keelteoskus

hiina keel	emakeel
inglise keel	kõrgtase

5. Teenistuskäik

2018–2023	Keemilise ja Bioloogilise Füüsika Instituut, Nooremteadur

6. Vabatahtlik töö

2016–2016	vabatahtlik töö naabruskonnas

7. Computer skills

- Operatsioonisüsteemid: Microsoft Windows, MacOS
- Programmeerimiskeeled: Python, Wolfram Mathematica
- Teaduskeskkonnad: Susyno, SARAH

8. Autasud

- 2022, Dora Plus stipendium
- 2018, Teadusuuringute talendiotsingu programmi eriauhind
- 2018, Füüsika instituudi dekaani stipendium
- 2017, Teadusuuringute talendiotsingu programmi esikoht
- 2017, TAQ välisõpingute eristipendium

9. **Kaitstud lõputööd**

- 2018, Enhanced DiHiggs Signal from Hidden Scalar QCD at Leading Order Scale Symmetry Limit, MSc, juhendaja Dr. Andi Hektor, Tallinna Tehnikaülikool, Keemilise ja Bioloogilise Füüsika Instituut, Eesti
- 2018, Charged pions tagged with polarized photons probing strong CP violation in a chiral-imbalance medium, juhendaja Dr. Liang Yu, Jilini Ülikool, Füüsika instituut, Hiina

10. **Teadustöö põhisuunad**

- Kõrge energia füüsika, fenomenoloogia ja teooria
- Osakestefüüsika ja väljateooria
- Fenomenoloogia mudeliehitus
- Elementaarosakeste füüsika
- Kosmoloogia

11. **Teadustegevus**

Teaduspublikatsioonide nimestik on ära toodud ingliskeelse CV juures.

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