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COMPARATIVE ANALYSIS OF EXCHANGE-TRADED FUNDS AND INDEX FUNDS

Bachelor's thesis

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ABSTRACT

The aim of this thesis is to comparatively analyze index funds and exchange-traded funds (ETFs), which are often regarded as substitutes because they share an identical objective and have analogous aspects. This thesis examines whether they differ in terms of their performance, volatility, expenses and risk-adjusted return. The research has a geographical focus on the United States (U.S.) market, hence the sample comprises a total of 16 different passive funds that solely track U.S. market capitalization-weighted indices. The research covers the post-financial crisis period from 2009 to 2018. The quantitative analysis encompasses evaluation of annualized return, tracking error, beta, standard deviation, expense ratio, Sharpe ratio and Treynor ratio. Empirical results show that the greatest difference between ETFs and index funds occurs in returns and expense ratios. ETFs had slightly higher returns and lower expense ratios than their respective index funds. Furthermore, the statistical significance was assessed and the results indicate that index funds and ETFs are not statistically significantly different in terms of their performance, volatility, expenses and risk-adjusted return.

Keywords: index funds, exchange-traded funds, passive management, investing

INTRODUCTION

Passive management is an investment strategy that has become extensively popular among investors as passive funds have proliferated over the past decades. Passive funds, such as index funds and exchange-traded funds (ETFs), attempt to track and replicate a specific market index by investing in the same constituent securities that comprise the underlying index. Passive funds have seen substantial growth primarily because they can offer various benefits to investors such as broad diversification, cost efficiency and tax efficiency. (Philips 2011).

Index funds and ETFs are often considered to be substitutes as they have an identical purpose and analogous aspects. Despite bearing a strong resemblance to each other, they also have a few characteristics that differentiate them. The aim of this thesis is to comparatively analyze index funds and ETFs, and examine if they significantly differ in terms of their performance, volatility, expenses and risk-adjusted return.

Two research questions that align with the aim are established and addressed in this thesis. Those questions are: “what are the underlying differences of index funds and ETFs” and “whether they significantly differ in terms of performance, volatility, expenses and risk-adjusted return”. The pertinent research methods used are literature review, quantitative analysis and empirical research. Furthermore, quantitative analysis encompasses annualized returns, tracking error, standard deviation, beta, expense ratio, Sharpe ratio and Treynor ratio. Subsequently, null and alternative hypotheses are defined and t-test is conducted to assess the statistical significance.

The sample comprises 16 different index funds and ETFs. This research has a geographical focus on the U.S. (United States) market, therefore the passive funds included in the sample track a variety of U.S. market capitalization-weighted indices, such as Nasdaq 100, Nasdaq Composite, S&P MidCap 400, S&P 500, S&P SmallCap 600, Russell 2000 and CRSP U.S. Total Market Index. The research covers a period of 10 years, from 2009 to 2018. The sample is limited to 16 index funds and ETFs due to the scarcity of U.S. passive funds that had inception before 2009.

This thesis is structured into three main parts. The thesis begins with the theoretical background and gives a comprehensive overview of passive management, index funds, ETFs and their differences. In addition, active management is elucidated because previous empirical studies have indicated its impact on the growth of passive funds. This is followed by the data and methodology part, where the relevant information pertaining to the research is presented. The methodology section introduces the quantitative methods for evaluating the performance, volatility, expenses and risk-adjusted return. Moreover, the hypothesis testing is succinctly explained and the hypotheses of this research are defined. The last part of the thesis presents the empirical results and addresses the limitations of the research.

1. THEORETICAL BACKGROUND

1.1. Passive management

Passive management refers primarily to an investment strategy that seeks to match the performance of a specific stock market index. Essentially, this investment strategy does not aim to outperform the market but tries to accurately track and replicate the returns of an underlying index. Passive funds, such as index funds or ETFs, attempt to replicate the performance of a stock market index by assembling a portfolio that invests in the same constituent securities that comprise the underlying index with weights proportionate to their market value, or alternatively by a sampling of the securities. (Philips 2011)

The rationale for passive management originates from the investment theory of efficient market hypothesis by Eugene Fama. According to Fama (1970), security prices fully reflect all available information at any point in time, such market in which the prices fully reflect available information is referred to as efficient. The main implication of the efficient market hypothesis is that it is impossible for an investor to outperform the market on a consistent basis.

Since the inception of the first public index mutual fund in 1976, passive management has become increasingly popular among investors as the growth of passive funds has been exponential, particularly in the U.S. after the financial crisis of 2007-2009 (Rowley *et al.* 2018, 2). In 2018, actively managed U.S. equity funds had cash outflows of 174 billion dollars, in contrast, passive U.S. equity funds had inflows of 207 billion dollars. At the end of 2018, actively managed U.S. equity funds had a market share of 51.3%, while passive equity funds had a market share of 48.7%. (McDevitt, Schramm 2019). Actively managed funds have existed significantly longer than passive funds and for this reason they have presently a larger market share. However, the market share of actively managed funds has progressively declined over the past decade, and the growth of passive funds unceasingly dominates over the growth of actively

managed funds. For perspective, Figure 1 presents the market share as a percentage of both U.S. active and passive equity funds from the year 2009 to 2018.

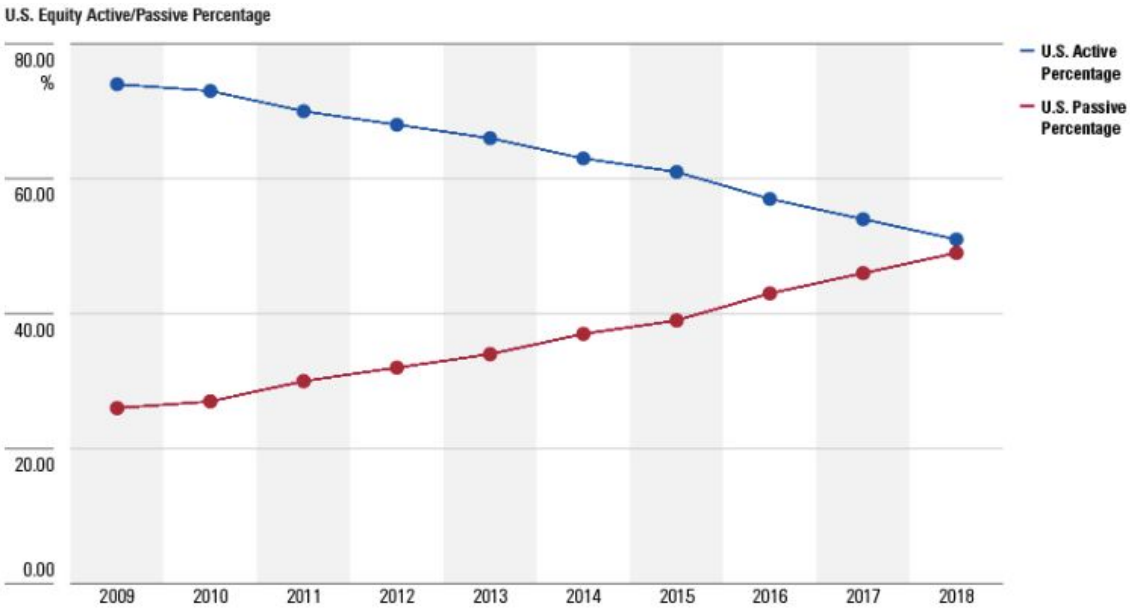

Figure 1. U.S. Equity Active-Passive Market Share.
Source: McDevitt, Schramm (2019); Morningstar Direct.

Passive funds have proliferated over the past decades because they provide a simplified and efficient investment with the potential to increase investors wealth across a broad range of asset classes and sub-asset classes. Main beneficial aspects of passive management are broad diversification, cost efficiency and tax efficiency, which have all contributed to the substantial growth of passive funds. (Philips 2011)

Passive versus Active management

The two main investment strategies, that can be used with the purpose to generate returns, are active portfolio management and passive portfolio management. Active management involves the use of specialized means, such as market timing ability or security selection, with the investment objective to outperform the market and generate above-average returns. The underlying reason for an investor to embark on active management instead of passive management is the possibility to generate returns that exceed the stock market index. Active management is based on the idea of inefficient markets, and that it is possible to outperform the

market and identify mispriced securities. Consequently, investors who use active management do not believe in the efficient market hypothesis. (Ofili 2014)

There is a perennial subject of debate about active versus passive management and their respective merits and demerits. The essence of the debate revolves around whether actively managed funds can outperform the market and therefore is a better investment option compared to passive funds. Some of the most prevailing studies by the proponents of passive management are by Malkiel (1995) and Gruber (1996), who both assert that in the long term actively managed funds do not outperform the market or generate higher returns than passive funds. However, according to Rompotis (2009), the findings on this topic are relatively ambiguous, because there are studies that show both outperformance and underperformance of actively managed funds with respect to the market indices and passive funds. The underperformance of actively managed funds is mainly the result of the increased expenses incurred by active management. This was also examined by Sharpe (1991), who states that after costs, the return on the average actively managed dollar will be less than the return on the average passively managed dollar. Furthermore, the inability of numerous actively managed funds to generate superior returns and outperform the market on a consistent basis, in conjunction with the beneficial aspects of passive management, have both contributed to the significant growth of passive funds (Rompotis 2009).

1.2. Index funds

A variety of prominent studies, such as modern portfolio theory framework developed by Markowitz (1952) and the unmanaged investment company idea proposed by Renshaw and Feldstein (1960), subsequently led the way to the launch of the first index funds in the early 1970s. American financial services company Wells Fargo established the first index portfolio in 1971, however it was initially only for institutional investors. This was followed by the inception of the first public index fund for the individual investor in 1976 by the American investment management company Vanguard Group and its renowned founder John C. Bogle. (Chovancová 2005, 28)

An index fund is a subset of mutual funds with an investment objective of tracking the returns of the underlying index. The portfolio of index funds encompasses all of the constituent securities included in the underlying index, both in terms of structure and value. Index funds are often distinguished based on the indices that they are tracking. Several index funds track broad-based market indices, such as S&P 500, Russell's indices or Wilshire 5000. Some index funds also mimic the dynamics of a particular sector or industry, such as Dow Jones U.S Financial Services Index. Additionally, there are index funds that track international indices, for example, the MSCI Europe Index. (Ibid.)

One of the characteristics of index funds is the low-cost aspect. Index funds derive their low-cost structure primarily from low turnover and low management fees (Philips 2011, 8). As index funds do not have active management to attempt to outperform the market, they do not have additional costs in order to find and capitalize on opportunities for outperformance. For instance, costs associated with research and trading can reduce potential returns realized by the investor. Consequently, by avoiding these costs, index funds can offer a broad market exposure at a lower cost in comparison to actively managed funds. (Rowley *et al.* 2018). Another characteristic of index funds is tax efficiency. From an after-tax perspective, index funds rarely realize and distribute capital gains because of the low turnover and the way they are managed (Philips 2011, 16).

In general, index funds are highly diversified because they have holdings that encompass a wide range of securities, except for index funds that track narrow market segments. A myriad of index funds have holdings of a large variety of securities, which in return reduces the risk associated with specific securities and eliminates the component of return volatility. Moreover, for index funds it is crucial to have holdings of a wide range of constituent index securities in order to precisely track a particular market index, whether by implementing replication or sampling strategy. (Ibid.)

Index funds can provide exposure to a broad market, or a particular segment of the market through varying degrees of index replication (Rowley *et al.* 2018, 14). Index portfolio requires investment in all the constituent securities and in the exact proportions as the underlying index, which is often defined as the full replication strategy. For instance, index funds that track

broad-based market indices are predominantly employing a full replication strategy. Alternative approaches to the full replication strategy are stratified sampling and optimization, which are considered to be non-replication strategies. Optimization and stratified sampling strategies involve matching the index through investments in a subset of index securities, while simultaneously ensuring that the portfolio has an analogous risk and return characteristics as the stock market index. (Frino, Gallagher 2001)

1.3. Exchange-traded funds

The first U.S. ETF, named SPDR S&P 500 (stock ticker symbol SPY), was introduced to the market in 1993 by State Street Global Advisors. Nowadays it is the largest ETF by assets under management and also the most actively traded security in the world. The earliest adopters of ETFs were institutional investors looking to access diversified and broad equity market exposure, and to hedge risk and volatility. The multitude of benefits of ETFs broadened its appeal to include all types of investors. The ETFs market grew substantially over time and new ETFs became available to the market. Today, the market counts more than 5,000 ETFs and it is possible to invest in various asset classes, such as equities, bonds, commodities, and emerging markets. (Bioy 2013). Moreover, there are special types of ETFs available such as leveraged, inverse, growth and value ETFs.

ETFs combine the essential aspects of conventional mutual funds with those of stock. ETF is a fund that can be continuously traded on a stock exchange during trading hours. Essentially, ETF is a passive investment that generates market returns rather than tries to seek outperformance. The primary investment objective of a passive ETF is identical to the index fund, which is to track and replicate the returns of its underlying index. Therefore, ETF can provide risk-return profile analogous to the underlying index. (Khan *et al.* 2015, 40)

Since their inception in 1993, ETFs have seen substantial growth. The most vital drivers of ETFs success, according to Bollapragada *et al.* (2013), are cost efficiency, tax efficiency, transparency, intraday trading, broad-market exposure and liquidity. ETFs are designed to provide exposure to

broad-based indices at a lower cost. Similarly to index funds, it is achieved by being passively managed and not engaging in extensive research and trading. A particular strength of ETFs is tax efficiency. ETFs have low turnover, which minimizes the number of taxable events realized in the fund, which will result in lower capital gains being passed to an investor. Another aspect of ETFs is transparency, which means that the information about the ETFs constituent securities and other data is updated on a daily basis and is easily accessible to investors. (Ibid.)

Among other characteristics of ETFs, according to Kostovetsky (2003), is the possibility to buy or sell ETFs at any time of the trading day similarly to stocks. ETFs offer the ability to buy on margin and sell short, additionally, investors can place stop and limit orders on them. However, trading ETFs requires a brokerage account, which in return can lead to brokerage transaction fees and bid-ask spreads. (Ibid.)

Similarly to open-end funds, ETFs allow the creation and redemption of shares in the fund (Ben-David *et al.* 2017, 4). The ETFs ability to create and redeem shares on a consistent basis ensures the underlying depth of liquidity. ETFs are perceived to be a more liquid alternative to mutual funds because they can be traded at market prices throughout the trading day. (Vanguard 2016)

ETFs can be classified into physical and synthetic based on their replication strategy. Physical ETF tries to accurately track and replicate the returns of the underlying index by holding all or a representative sample of the constituent index securities with weights to closely mimic those in the index. Synthetic ETF is a newer innovation of ETFs, which has gained significantly more popularity in Europe than in the U.S. The objective of synthetic ETF is to track and deliver the performance of an index by entering into derivative contracts, for instance swap contracts. (Ben-David *et al.* 2017)

An alternative to passive ETFs emerged in 2008 when U.S. Securities and Exchange Commission permitted the creation of actively managed ETFs, which did not have to track an index (Sharifzadeh, Hojat 2011, 6). The performance of both passive and active ETFs was assessed in the study by Schizas (2014), who showed that passive ETFs tend to perform better in comparison to active ETFs.

1.4. Differences of index funds and exchange-traded funds

Index funds and ETFs are analogous by nature and often regarded as substitutes. That is predominantly because they are both passive funds and index-based, referring to the fact that they both attempt to track and replicate a specific stock market index by investing in the same constituent securities of the underlying index and in the similar proportions. Therefore, they compete for the same investors in the market. (Sharifzadeh, Hojat 2011, 10). Additionally, both can offer similar benefits to investors such as tax efficiency, cost efficiency, broad diversification. (Philips 2011)

Agapova (2009) analyzed the substitutability of index funds and ETFs. The findings of this study show that index funds and ETFs are substitutes, although not perfect substitutes. The study further implies that the coexistence can be explained by the clientele effect that classifies them into different market niches. (Ibid.)

Although index funds and ETFs are analogous, they have a few underlying differences. Index funds and ETFs diverge in their trading features. ETFs can offer various trading features that are unavailable in index funds, which is attributable to the fact that ETFs have a resemblance to stocks. For instance, features such as stop and limit orders, margin trading, short selling and intraday trading, which can also offer higher liquidity to investors. (Bennyhoff 2008). Furthermore, index funds might be a more favorable choice by investors with lower liquidity and lower trading needs. From another perspective, investors with higher liquidity and higher trading needs might favor ETFs over index funds. (Agapova 2009, 7)

ETFs offer intraday pricing and execution, which is not possible with index funds that are priced at Net Asset Value (NAV) at the end of the trading day. On the contrary, ETFs are priced by market throughout the day and the market price of an ETF can also differ slightly from NAV. (Bennyhoff 2008). According to the study by Engle and Sarkar (2006), the creation-redemption process allows ETFs to trade on a stock exchange throughout the day at a price determined by supply and demand rather than at the NAV. In addition, arbitrage opportunities can be profitably exploited, whenever the price of ETF deviates from the NAV. Arbitrage activity ensures that the

ETF price trades closely to the value of the underlying securities. Thus, the quoted market prices for an ETF will closely match the value of the underlying index (Vanguard 2016, 31)

According to Kostovetsky (2003), the key areas of quantitative differences between ETFs and index funds are transaction fees, management fees and tax efficiency. The first factor, that distinguishes index funds from ETFs, is that ETFs have transaction fees because they trade similarly to stocks. In contrast to index funds, trading ETFs entails transaction fees associated with brokerage accounts and bid-ask spreads. Additionally, index funds and ETFs slightly differ in terms of management fees. Although both can offer cost efficiency, ETFs can often offer even lower expense ratio because the accounting is done at the shareholder level. Another difference relies on the tax efficiency. In comparison to index funds, ETFs are even more tax efficient because they almost never distribute capital gains. (Ibid.)

2. DATA AND METHODOLOGY

2.1. Data

The sample comprises a total of 16 different U.S. market index funds and ETFs. For a better perspective, index funds and ETFs are further classified based on their underlying index. The research has a geographical focus on the U.S. market, hence the sample includes solely index funds and ETFs that track the U.S. stock market indices such as Nasdaq 100, Nasdaq Composite, S&P MidCap 400, S&P 500, S&P SmallCap 600, Russell 2000 and CRSP U.S. Total Market Index. All those aforementioned indices are U.S. market capitalization-weighted indices. The relevant data pertaining to the index funds and ETFs is obtained from the Thomson Reuters Eikon database. The research covers a period of 10 years, ranging from 2009 to 2018. Consequently, the sample is limited to 16 passive funds due to the scarcity of index funds and ETFs that had inception before 2009. The information about the title, stock ticker symbol, stock market index and the inception date of the respective index fund and ETF is presented in Table 1.

Table 1. U.S. Index Funds and ETFs.

Title	Ticker symbol	Market Index	Inception
Invesco QQQ Trust Series 1 ETF	QQQ	Nasdaq 100	10.03.1999
USAA Nasdaq-100 Index Fund	USNQX	Nasdaq 100	27.10.2000
Fidelity Nasdaq Composite Index ETF	ONEQ	Nasdaq Composite	25.09.2003
Fidelity Nasdaq Composite Index Fund	FNCMX	Nasdaq Composite	25.09.2003
SPDR S&P MidCap 400 ETF	MDY	S&P MidCap 400	28.04.1995
Principal MidCap S&P 400 Index Fund	PMFMX	S&P MidCap 400	06.12.2000
SPDR S&P 500 ETF	SPY	S&P 500	22.01.1993
Vanguard 500 Index Fund	VFINX	S&P 500	31.08.1976
iShares Core S&P 500 ETF	IVV	S&P 500	19.05.2000
Schwab S&P 500 Index Fund	SWPPX	S&P 500	19.05.1997
SPDR S&P 600 Small Cap ETF	SLY	S&P SmallCap 600	08.11.2005

Principal S&P SmallCap 600 Index Fund	PSSIX	S&P SmallCap 600	01.03.2001
iShares Russell 2000 ETF	IWM	Russell 2000	22.05.2000
iShares Russell 2000 Small-Cap Index Fund	MASKX	Russell 2000	09.04.1997
Vanguard Total Stock Market ETF	VTI	CRSP U.S. Total Market	24.05.2001
Vanguard Total Stock Market Index Fund	VTSMX	CRSP U.S. Total Market	27.04.1992

Source: Thomson Reuters Eikon database, compiled by author.

2.2. Methodology

2.2.1. Performance measures

Annualized return shows how much an investment earns over a specific period of time. It represents the yearly movement in the value of an investment, including the effect of compounding (Charles Schwab 2009). Annualized return is a measure of performance. The higher is the annualized return, the better is the performance.

Annualized return formula: (1)

$$AR = [(1 + r_1)\, x \,\ldots\, x\, (1 + r_n)^{\frac{1}{N}} - 1]$$

where

AR – annualized return

r_1 – actual rate of return for the first year

r_n – actual rate of return for the n year

N – number of years

Tracking error denotes the difference between the returns of an index tracking fund and the returns of the corresponding benchmark index. Tracking error is an indicator of how accurately the fund tracks the underlying index. It is determined by the standard deviation of return difference between the fund and the index. The primary factors that affect the tracking error of funds are fund cash flows, transaction fees, treatment of dividends by the index, volatility of the benchmark, corporate activity and index composition changes (Frino, Gallagher 2001). These factors might prevent a passive fund from perfectly replicating the performance of the underlying

index. Tracking error can be an efficient performance comparison for passive funds since they replicate the returns of the market index. (Rompotis 2009, 5). Low tracking error, around zero, is more favourable for passive funds as it indicates that the fund is accurately tracking the index. On the contrary, the greater the tracking error, the more the fund is deviating from following the index.

Tracking error formula: (2)

$$TE = \sqrt{\frac{\sum_{i=1}^{n} (Rp - Rb)^2}{N - 1}}$$

where

TE – tracking error
Rp – return of a fund, portfolio
Rb – return of a benchmark, index
N – number of return periods

2.2.2. Risk and volatility measures

Standard deviation is a statistical measure of volatility in finance, which measures the asset's total risk and reflects dispersion of asset returns (Haslem 2003, 248). Standard deviation determines the dispersion of a set of data values and is calculated as the square root of the variance by determining the variation between each data point relative to the mean. Therefore, the higher the deviation within the data set, the higher is the standard deviation. High standard deviation indicates high volatility, whereas low standard deviation indicates the opposite. In general, a passive fund is expected to have a lower standard deviation. (Hargrave 2019).

Standard deviation formula: (3)

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{N - 1}}$$

where

σ – standard deviation
x_i – each of the value of the data

$\bar{x}$ – mean of x_i
N – number of data points

Beta is a measure of systematic risk and volatility. Beta measures the systematic risk of asset returns relative to the changes in the benchmark index (Haslem 2003, 248). It is calculated by dividing covariance of the returns of security and market by the variance of the market over a specific period. If the beta is below one it indicates that the security is less volatile than the market or the price movements of security have a lower correlation with the market. A beta greater than one indicates the opposite. According to Rompotis (2011), beta can be an indicator of the adopted strategy in the case of funds. A beta of one suggests a full replication strategy. On the contrary, when the beta significantly differs from one, then it suggests that the fund implements another strategy.

Beta formula: (4)

$$\beta = \frac{Cov\ (Rp, Rb)}{Var\ (Rb)}$$

where
β – beta
Cov (Rp, Rb) – covariance of the security and the market
Var (Rb) – variance of the market

2.2.3. Risk-adjusted return measures

Sharpe ratio is a measure of risk-adjusted return. The ratio was developed by William F. Sharpe (1966) and is determined by subtracting risk-free rate from the return and dividing it by standard deviation. Sharpe (1994) defines it as the measure of the expected return per unit of risk. The greater the value of Sharpe ratio, the greater is the return generated per unit of risk. Thus, the greater the value of Sharpe ratio the more excess return investors can expect for the extra risk they are exposed. High Sharpe ratio depicts better risk-adjusted performance, whereas a low Sharpe ratio signifies low risk-adjusted performance. Sharpe ratio can be also negative, which is attributable to return being lower than the risk-free rate.

Sharpe ratio formula: (5)

$$S = \frac{Rp - Rf}{\sigma}$$

where

S – Sharpe ratio
Rp – return of a fund, portfolio
Rf – risk free rate
Σ – standard deviation

Another risk-adjusted measure is the Treynor ratio, which was developed by Jack L. Treynor (1965) and is the ratio of excess return in relation to its beta. The ratio measures excess return per unit of systematic risk. Treynor ratio and Sharpe ratio are analogous, except that the Treynor ratio uses beta in the denominator instead of standard deviation. Thus, the Treynor ratio utilizes systematic risk instead of total risk. The greater the value of the Treynor ratio, the better is the risk-adjusted performance.

Treynor ratio formula: (6)

$$T = \frac{Rp - Rf}{\beta}$$

where

T – Treynor ratio
Rp – return of a fund, portfolio
Rf – risk free rate
B – beta

2.2.4. Expense ratio

All funds incur expenses for managing and operating the fund and therefore they charge a percentage of fund's assets to cover the expenses. Those expenses include management fees and other operating expenses. The expense ratio measures management fees and operating expenses as a percentage of total managed assets (Kostovetsky 2003, 83). Additionally, Sharpe (1966) states that the expense ratio does not encompass all expenses and some fees are omitted from the ratio, such as brokerage fees. Hence, the expense ratio does not depict all the differences in expenses among funds.

In general, the expense ratio is a critical factor to consider when choosing a fund to invest, because it can have a tremendous impact on the returns. The lower the expense ratio, the more beneficial it is for the investor. Higher management fees and other operating expenses will reduce potential returns. According to Sharpe (1966) the smaller a fund's expense ratio, the better are the results obtained by investors. This is in line with the study by Carhart (1997), who implies that the expense ratio and transaction fees are negatively related to the performance of a fund.

Both index funds and passive ETFs can offer low expense ratio which is attributable to the fact that they are both passively managed. For perspective, the expense ratio of actively managed funds can be as high as 2%, but for the index fund and ETFs it is usually below 0.5% per year. In general, ETFs have been able to offer even lower expense ratio than the index funds, which is primarily because ETFs are not in charge of shareholder accounting. (Kostovetsky 2003, 83).

Total expense ratio formula: (7)

$$TER = \frac{Total\ fund\ costs}{Total\ fund\ assets}$$

where
TER – total expense ratio

2.3. Hypothesis testing

Hypothesis testing is the use of statistics to determine if the hypothesis is true, whether to accept or reject the hypothesis. The two types of hypotheses are the null hypothesis and alternative hypothesis. The null hypothesis, denoted as H_0, is a general statement, which is assumed to be true unless the data provide convincing evidence that it is false. The alternative hypothesis, denoted as H_a, is the contradictory statement to the null hypothesis, which is only accepted when sufficient evidence exists to establish its truth. (McClave *et al.* 2018).

The hypotheses in this research are tested with a t-test, conducted as a two-tailed test and two-sample test assuming unequal variances. The t-test is performed with the purpose of examining whether the index funds and ETFs are significantly different from each other in terms of performance, volatility, risk-adjusted return and expenses. The t-test determines the statistical significance of returns, tracking errors, standard deviations, Sharpe ratios, Treynor ratios and expense ratios.

The hypotheses for examining the performance of index funds and ETFs, more specifically the statistical significance of returns and tracking errors:

$H_{0,1}$ There is no significant difference between the returns of index funds and ETFs.

$H_{a,1}$ There is a significant difference between the returns of index funds and ETFs.

$H_{0,2}$ There is no significant difference between the tracking error of index funds and ETFs.

$H_{a,2}$ There is a significant difference between the tracking error of index funds and ETFs.

The hypotheses for testing the statistical significance of volatility in terms of standard deviations:

$H_{0,3}$ There is no significant difference between the standard deviations of index funds and ETFs.

$H_{a,3}$ There is a significant difference between the standard deviations of index funds and ETFs.

The hypotheses for examining the risk-adjusted returns, more specifically the statistical significance of Sharpe ratios and Treynor ratios:

$H_{0,4}$ There is no significant difference between the Sharpe ratios of index funds and ETFs.

$H_{a,4}$ There is a significant difference between the Sharpe ratios of index funds and ETFs.

$H_{0,5}$ There is no significant difference between the Treynor ratios of index funds and ETFs.

$H_{a,5}$ There is a significant difference between the Treynor ratios of index funds and ETFs.

Finally, the hypotheses for testing the statistical significance of the expense ratios:

$H_{0,6}$ There is no significant difference between the expense ratios of index funds and ETFs.

$H_{a,6}$ There is a significant difference between the expense ratios of index funds and ETFs.

3. EMPIRICAL RESULTS

3.1. Performance

The performance of index funds and ETFs is assessed in terms of annualized return and tracking error. The results pertaining to the annualized return and tracking error are presented in Table 2. The higher the annualized return, the more beneficial it is for the investor. The best performing passive fund in terms of 10-year annualized return is Invesco QQQ Trust 1 ETF with a value of 20.62%, which is closely followed by the respective index fund that tracks also Nasdaq 100 index. On the contrary, the lowest 10-year annualized return is 15.26%, which is attributed to iShares Russell 2000 Small-Cap index fund. The largest discrepancy occurs between S&P SmallCap 600 index tracking passive funds, where the 10-year annualized return of SPDR S&P 6000 SmallCap ETF exceeds respective index fund. On all occasions, ETFs outperformed their respective index funds in terms of 10-year annualized return. In addition, the 10-year ETFs annualized return mean is 30 basis points greater than index funds annualized return mean. Therefore, it can be inferred that ETFs slightly outperform their respective index funds.

Tracking error denotes the difference between the returns of an index tracking fund and the returns of the underlying index. Thus, lower tracking error is more favorable as it indicates that the passive fund is accurately tracking the underlying index. All the S&P 500 index tracking passive funds and iShares Russell 2000 ETF have the lowest 10-year tracking error of 0.01%. On the contrary, Nasdaq 100 tracking passive funds have the highest tracking error values, which indicates that they deviate the most from accurately tracking the underlying index. This might be affected by factors such as fund cash flows, transaction fees, treatment of dividends, volatility of the benchmark, corporate activity and index composition changes (Frino, Gallagher 2001). From the perspective of difference, the greatest difference exists between S&P SmallCap 600 tracking passive funds, as they differ 40 basis points from each other. Furthermore, the 10-year tracking error mean of index funds is 4 basis points lower than the mean of ETF.

Table 2. Annualized return and tracking error 10-year results.

Title	Annualized return	Tracking error
Invesco QQQ Trust Series 1 ETF	20.62%	1.36%
USAA Nasdaq-100 Index Fund	20.11%	1.37%
Fidelity Nasdaq Composite Index ETF	18.70%	1.03%
Fidelity Nasdaq Composite Index Fund	18.68%	1.04%
SPDR S&P MidCap 400 ETF	15.92%	0.73%
Principal MidCap S&P 400 Index Fund	15.37%	0.73%
SPDR S&P 500 ETF	15.78%	0.01%
Vanguard 500 Index Fund	15.76%	0.01%
iShares Core S&P 500 ETF	15.84%	0.01%
Schwab S&P 500 Index Fund	15.80%	0.02%
SPDR S&P 600 Small Cap ETF	17.73%	0.44%
Principal S&P SmallCap 600 Index Fund	16.67%	0.04%
iShares Russell 2000 ETF	15.35%	0.01%
iShares Russell 2000 Small-Cap Index Fund	15.26%	0.05%
Vanguard Total Stock Market ETF	16.05%	0.03%
Vanguard Total Stock Market Index Fund	15.92%	0.03%
Mean of ETFs	17.00%	0.45%
Mean of Index Funds	16.70%	0.41%

Source: Thomson Reuters Eikon database, compiled based on author's calculations.

Annualized return t-test results are presented in Table 3. The t-test results show that there is no statistical significance between ETFs and index funds in terms of annualized return. Both null and alternative hypotheses were defined in 2.3., based on the annualized return t-test results it can be inferred that the null hypothesis, denoted as $H_{0,1}$, is accepted and the alternative hypothesis, denoted as $H_{a,1}$, is rejected.

Table 3. Annualized return t-test results.

	Mean	Variance	Observation	df	t Stat	P-value
ETFs	17.00	3.45	8			
Index Funds	16.70	3.10	8	14	0.33	0.74

Source: compiled based on author's calculations.

The tracking error t-test results are presented in Table 4. The results indicate that there is no statistical significance between index funds and ETFs in terms of tracking error. Therefore, the null hypothesis $H_{0,2}$ is accepted and the alternative hypothesis $H_{a,2}$ is rejected. Furthermore, the conclusion can be drawn that the U.S. market indices tracking index funds and ETFs are not statistically significantly different in terms of performance, as both t-test results of tracking error and annualized return show that there is no statistical significance between index funds and ETFs.

Table 4. Tracking error t-test results.

	Mean	Variance	Observation	df	t Stat	P-value
ETFs	0.45	0.29	8			
Index Funds	0.41	0.31	8	14	0.15	0.88

Source: compiled based on author's calculations.

3.2. Risk and volatility

Risk and volatility are examined in terms of beta and standard deviation. The corresponding results are presented in Table 5. Beta is a measure of systematic risk and a beta of 1 is the most preferred for passive funds. The 10-year beta of 1 occurs with 5 passive fund pairs. According to the study by Rompotis (2011), earlier discussed in 2.2.2., a beta could be an indicator of the adopted strategy in the case of passive funds. For instance, a beta of 1 can indicate that the fund is employing a full replication strategy. S&P MidCap 400 tracking passive funds have the lowest beta of 0.95, which indicates that the funds are less volatile than the market. On the contrary, Nasdaq 100 tracking passive funds have the highest value of 1.14. All the betas of the ETFs and their respective index funds are equal, hence the beta means of both index funds and ETFs are also identical.

Standard deviation is a measure of volatility. The higher the value of standard deviation, the higher is the volatility of a passive fund. The highest 10-year standard deviation of 17.69% have

both Russell 2000 index tracking passive funds. Contrarily, SPDR S&P 500 ETF has the lowest 10-year standard deviation of 12.65%. It is evident that all the passive funds that track the S&P 500 index have the lowest standard deviation values and they only vary a few basis points from each other. After assessing the difference of passive funds, it can be deduced that the greatest difference is 36 basis points, which occurs between the SPDR S&P 600 Small Cap ETF and its respective index fund. Overall, the 10-year standard deviation mean of ETFs is greater only by 3 basis points than the mean of index funds.

Table 5. Beta and standard deviation 10-year results

Title	Beta	Standard deviation
Invesco QQQ Trust Series 1 ETF	1.14	15.24%
USAA Nasdaq-100 Index Fund	1.14	15.29%
Fidelity Nasdaq Composite Index ETF	1.11	15.08%
Fidelity Nasdaq Composite Index Fund	1.11	15.11%
SPDR S&P MidCap 400 ETF	0.95	15.36%
Principal MidCap S&P 400 Index Fund	0.95	15.36%
SPDR S&P 500 ETF	1.00	12.65%
Vanguard 500 Index Fund	1.00	12.69%
iShares Core S&P 500 ETF	1.00	12.67%
Schwab S&P 500 Index Fund	1.00	12.66%
SPDR S&P 600 Small Cap ETF	1.00	17.59%
Principal S&P SmallCap 600 Index Fund	1.00	17.23%
iShares Russell 2000 ETF	1.00	17.69%
iShares Russell 2000 Small-Cap Index Fund	1.00	17.69%
Vanguard Total Stock Market ETF	1.00	13.19%
Vanguard Total Stock Market Index Fund	1.00	13.18%
Mean of ETFs	1.03	14.93%
Mean of Index Funds	1.03	14.90%

Source: Thomson Reuters Eikon database, compiled based on author's calculations.

All the beta values of the passive fund pairs are identical, hence it is not rational to conduct the t-test to assess the statistical significance. The t-test is conducted only for standard deviation and

the results are presented in Table 6. The t-test results of standard deviation imply that there is no statistical significance between index funds and ETFs, therefore the null hypothesis $H_{0,3}$ is accepted and alternative hypothesis $H_{a,3}$ is rejected.

Table 6. Standard deviation t-test results.

	Mean	Variance	Observation	df	t Stat	P-value
ETFs	14.93	4.05	8			
Index Funds	14.90	3.78	8	14	0.03	0.97

Source: compiled based on author's calculations.

3.3. Risk-adjusted return

The risk-adjusted returns of index funds and ETFs are assessed in terms of Sharpe ratio and Treynor ratio. The results of both ratios are presented in Table 7. Essentially, the greater the value of the Sharpe ratio, the greater is the return generated per unit of risk. The highest 10-year Sharpe ratio value of 0.35 has Invesco QQQ Trust Series 1 ETF, which is closely followed by the respective index fund that has lower value by only 1 basis points. Contrarily, Russell 2000 index tracking passive funds have the lowest 10-year Sharpe ratio value of 0.23. There is a minor discrepancy of Sharpe ratios between ETFs and index funds as some passive funds only differ by 1 basis points from each other. Moreover, the 10-year mean of the Sharpe ratio of index funds and ETFs is identical.

Treynor ratio is analogous to Sharpe ratio. However, the excess return is in relation to beta instead of standard deviation. The higher is the Treynor ratio value, the better is the risk-adjusted performance. The highest 10-year Treynor ratio value is 1.34, and similarly to Sharpe ratio, it is attributed to the ETF that tracks Nasdaq 100 index. On the contrary, the lowest value of the Treynor ratio belongs to iShares Russell 2000 Small-Cap index fund, which is closely followed by the respective ETF. Moreover, the greatest difference of 10-year Treynor ratio exists between the S&P 600 tracking passive funds, where ETF exceeds the respective index fund by 6 basis

points. ETFs exceed index funds 10-year Treynor ratio on 5 occasions, other times they were identical. Overall, the 10-year Treynor ratio means differ imperceptibly, because the mean of ETFs is only 2 basis point greater.

Table 7. Sharpe ratio and Treynor ratio 10-year results.

Title	Sharpe ratio	Treynor ratio
Invesco QQQ Trust Series 1 ETF	0.35	1.34
USAA Nasdaq-100 Index Fund	0.34	1.30
Fidelity Nasdaq Composite Index ETF	0.32	1.26
Fidelity Nasdaq Composite Index Fund	0.32	1.26
SPDR S&P MidCap 400 ETF	0.27	1.26
Principal MidCap S&P 400 Index Fund	0.26	1.22
SPDR S&P 500 ETF	0.33	1.19
Vanguard 500 Index Fund	0.33	1.18
iShares Core S&P 500 ETF	0.33	1.19
Schwab S&P 500 Index Fund	0.33	1.19
SPDR S&P 600 Small Cap ETF	0.26	1.31
Principal S&P SmallCap 600 Index Fund	0.25	1.25
iShares Russell 2000 ETF	0.23	1.16
iShares Russell 2000 Small-Cap Index Fund	0.23	1.15
Vanguard Total Stock Market ETF	0.32	1.20
Vanguard Total Stock Market Index Fund	0.32	1.20
Mean of ETFs	0.30	1.24
Mean of Index Funds	0.30	1.22

Source: Thomson Reuters Eikon database, compiled based on author's calculations.

Sharpe ratio t-test results are presented in the following Table 8. The t-test results show that there is no statistical significance between index funds and ETFs in terms of Sharpe ratio. Thus, the null hypothesis $H_{0,4}$ is accepted and the alternative hypothesis $H_{a,4}$ is rejected.

Table 8. Sharpe ratio t-test results.

	Mean	Variance	Observation	df	t Stat	P-value
ETFs	0.30	0	8			
Index Funds	0.30	0	8	14	0.18	0.86

Source: compiled based on author's calculations.

Table 9 presents the t-test results of the Treynor ratio. The t-test results of Treynor ratio show that there is no statistical significance between the index funds and ETFs. The null hypothesis $H_{0,5}$ is accepted and the alternative hypothesis $H_{a,5}$ is rejected. Moreover, both Sharpe ratio and Treynor ratio t-test results indicate that they are not statistically significantly different, therefore it can be concluded that the index funds and ETFs are not statistically significant different in terms of risk-adjusted return.

Table 9. Treynor ratio t-test results.

	Mean	Variance	Observation	df	t Stat	P-value
ETFs	1.24	0	8			
Index Funds	1.22	0	8	13	0.70	0.49

Source: compiled based on author's calculations.

3.4. Expense ratio

One of the most beneficial aspects of passive funds is cost efficiency, which is evident in the low expense ratio they offer to the investors. The total expense ratios are evaluated as of 2018 and the results are presented in Table 10. Lower expense ratio is more desirable to the investors because the expense ratio can have a significant impact on the potential returns. The lowest total expense ratio of 0.03% is offered by Vanguard Total Stock Market ETF and also by Schwab S&P 500 index fund. On the contrary, Principal MidCap S&P 400 index fund has the highest total expense ratio of 0.73%. The greatest difference accounts to 49 basis points between S&P MidCap 400 index tracking passive funds. Furthermore, ETFs have lower expense ratio than index funds on 7 occasions and the total expense ratio mean of ETFs is lower by 12 basis points. These results are in line with the study by Kostovetsky (2003), as discussed in 2.2.4, who asserted that in general ETFs can offer slightly lower expense ratio than index funds.

Table 10. Total Expense ratio results

Title	Total Expense Ratio
Invesco QQQ Trust Series 1 ETF	0.20%
USAA Nasdaq-100 Index Fund	0.51%
Fidelity Nasdaq Composite Index ETF	0.21%
Fidelity Nasdaq Composite Index Fund	0.29%
SPDR S&P MidCap 400 ETF	0.24%
Principal MidCap S&P 400 Index Fund	0.73%
SPDR S&P 500 ETF	0.09%
Vanguard 500 Index Fund	0.14%
iShares Core S&P 500 ETF	0.04%
Schwab S&P 500 Index Fund	0.03%
SPDR S&P 600 Small Cap ETF	0.15%
Principal S&P SmallCap 600 Index Fund	0.22%
iShares Russell 2000 ETF	0.20%
iShares Russell 2000 Small-Cap Index Fund	0.12%
Vanguard Total Stock Market ETF	0.03%
Vanguard Total Stock Market Index Fund	0.14%
Mean of ETFs	0.15%
Mean of Index funds	0.27%

Source: Thomson Reuters Eikon database, compiled based on author's calculations.

Total expense ratio t-test results are presented in Table 11. The t-test results indicate that ETFs and index funds are not statistically significantly different in terms of expense ratio. Therefore, the null hypothesis $H_{0,6}$ is accepted and the alternative hypothesis $H_{a,6}$ is rejected.

Table 11. Total Expense ratio t-test results.

	Mean	Variance	Observation	df	t Stat	P-value
ETFs	0.15	0.01	8			
Index Funds	0.27	0.06	8	9	-1.45	0.18

Source: compiled based on author's calculations.

3.5. Limitations of the research

The research had a few limitations regarding to the sample. The main limitation was the scarcity of specific U.S. market index funds and ETFs, hence the sample size was limited to 16 passive funds. As the research covers the period from 2009 to 2018, several passive funds were excluded from the sample because their inception was after 2009. For instance, the Russell 1000 and Russell 3000 tracking passive funds that had inception in 2010 were excluded, because they could not be assessed on the 10-year basis. In the longer perspective the research can be made with the inclusion of more passive funds. In addition, other prominent U.S. market indices were excluded, such as Dow Jones Industrial average and Wilshire 5000, because the respective passive fund was unavailable or the inception of the passive fund was after 2009.

CONCLUSION

The aim of this thesis was to comparatively analyze index funds and ETFs, and to examine whether they significantly differ in terms of their performance, volatility, expenses and risk-adjusted return. The sample comprised 16 different index funds and ETFs, which were further classified based on the U.S. market capitalization-weighted indices that they track, such as Nasdaq 100, Nasdaq Composite, S&P MidCap 400, S&P 500, S&P SmallCap 600, Russell 2000 and CRSP U.S. Total Market Index. The quantitative analysis encompassed evaluation of annualized return, tracking error, beta, standard deviation, expense ratio, Sharpe ratio, and Treynor ratio. The research covered a period of 10 years, from 2009 to 2018.

Index funds and ETFs share an identical objective, which is to track and replicate a specific stock market index by investing in the same constituent securities of the underlying index. Philips (2011) implied that they offer similar benefits to investors such as cost efficiency, tax efficiency, and broad diversification. Moreover, Sharifzadeh and Hojat (2011) asserted that index funds and ETFs compete for the same investors in the market. According to the study by Agapova (2009), index funds and ETFs are substitutes, however not perfect substitutes.

Although index funds and ETFs are analogous by nature, they also have few underlying differences. According to Bennyhoff (2008), they primarily diverge on their trading features as ETFs can offer trading features because of the resemblance to stocks. Additionally, ETFs have intraday price, whereas index funds are priced at NAV at the end of the trading day. Moreover, Kostovetsky (2003) asserted that the key areas of quantitative differences between ETFs and index funds are transaction fees, management fees and tax efficiency. ETFs can offer even lower management fees and better tax efficiency than index funds, however trading ETFs entails transaction fees. Furthermore, ETFs can offer greater liquidity to investors than index funds. According to Agapova (2009), index funds might be a more favorable choice by investors with lower liquidity and trading needs, whereas ETFs are suitable for investors with higher liquidity and trading needs.

The performance of passive funds was assessed in terms of annualized return and tracking error. The empirical results show that ETFs had 10-year annualized return mean of 17%, thus ETFs outperform index funds by 30 basis points. However, 10-year tracking error mean of index funds was 4 basis points lower than the mean of ETFs, which indicates that index funds track their underlying indices more accurately. The risk and volatility were assessed in terms of beta and standard deviation. The results of the 10-year beta of ETFs were identical to their respective index funds. The ETFs 10-year standard deviation was 14.93%, which was 3 basis points higher than the index funds mean. The risk-adjusted performance was assessed in terms of Treynor ratio and Sharpe ratio. The 10-year Sharpe ratio means were identical for both index funds and ETFs. The 10-year Treynor ratio mean of ETFs was 1.24, which indicates 2 basis points better risk-adjusted performance than the index funds. Lastly, the expense ratio mean of ETFs was 0.15%, which was 12 basis points lower than the index funds. It can be concluded, that the ETFs and index funds greatest difference occurs in returns and expense ratios. ETFs had slightly higher returns and lower expense ratios than index funds.

In addition, the statistical significance was assessed with a t-test for annualized return, tracking error, standard deviation, expense ratio, Sharpe ratio and Treynor ratio. All the results indicated that index funds and ETFs are not statistically significantly different. A conclusion can be drawn that index funds and ETFs do not significantly differ in terms of their performance, volatility, expenses and risk-adjusted return.

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