TALLINN UNIVERSITY OF TECHNOLOGY
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COMPARATIVE ANALYSIS OF HIERARCHICAL RISK PARITY (HRP) AND OTHER PORTFOLIO OPTIMIZATION METHODS IN TAIWAN STOCK MARKET

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TABLE OF CONTENTS

ABSTRACT

Taiwan's stock exchange, in particular, is closely related to semiconductor, electronics, and geopolitical risk. These market traits mean that employing a cutting-edge portfolio optimization technique to manage risks in the Taiwan stock market is particularly important. This paper compares traditional portfolio optimization methods such as mean-variance optimization, maximum diversification portfolio, and risk parity with Hierarchical Risk Parity methods, which use 2007-2023 historical daily adjusted close prices for selected high-cap stocks. The research looks at whether HRP surpasses other approaches in terms of risk-adjusted returns, as well as the impact that market volatility has on HRP over economic cycles. The exposure, volatility, and drawdown measure portfolio performance, and t-tests determine significance. Our findings indicate that HRP delivers robustness in managing volatility.

Keywords: Hierarchical Risk Parity, Portfolio Optimization, Taiwan Stock Market, Mean-Variance Optimization, Maximum Diversification Portfolio, Risk Parity

INTRODUCTION

Characterized by its rapid growth, technological innovation, and unique economic dynamics, the Taiwan stock market presents a fertile ground for applying advanced portfolio optimization methods. Among these, Hierarchical Risk Parity (HRP) has emerged as a promising approach, potentially offering superior risk-adjusted returns compared to traditional portfolio optimization methods (Lopez de Prado, 2016).

The concept of portfolio optimization, a cornerstone of modern investment theory, was revolutionized by Harry Markowitz's introduction of the Modern Portfolio Theory (MPT) in 1952 (Markowitz, 1952). Since then, the quest for the optimal portfolio—balancing maximum returns against minimum risk—has led to the development of various models, including the Capital Asset Pricing Model (CAPM) (Sharpe, 1964) and the Arbitrage Pricing Theory (APT) (Ross, 1976). However, today's financial markets' dynamic and interconnected nature calls for even more sophisticated methods that can capture the complex relationships between assets. In this regard, HRP offers a novel perspective by leveraging the hierarchical clustering of assets to minimize portfolio volatility (Raffinot, 2018).

This thesis is motivated by the distinctive attributes of the Taiwan stock market, notably its significant sector concentration in technology and electronics, the dynamic and volatile nature of emerging market conditions, and the impact of geopolitical tensions and economic policies on market behavior. The Taiwan market's role as a critical player in the global technology and manufacturing sectors further underscores the importance of adopting advanced portfolio optimization strategies to effectively manage the inherent risks and leverage the opportunities in this market.

This research aims to conduct a comprehensive comparative analysis of HRP against traditional and advanced portfolio optimization methods. By examining historical daily price data from selected stocks listed on the Taiwan Stock Exchange (TWSE) over sixteen years, this study seeks to evaluate the effectiveness of HRP in managing risk and enhancing portfolio performance compared to traditional methods. Through data analysis and backtesting, the research intends to offer novel insights into the suitability of HRP within the context of an emerging market characterized by significant volatility and unique risk factors. Two main research questions guide this study.

1. How does HRP perform in terms of risk-adjusted returns compared to other portfolio optimization methods in the Taiwan stock market? How about different market conditions?

2. What is the impact of market volatility in different economic cycles on the performance of HRP versus other methods in the Taiwan stock market?

To answer these questions, the study will perform a quantitative analysis using the historical daily price data from the Taiwan Stock Exchange spanning sixteen years from 2007 to 2023. The data is obtained through the yfinance library and includes daily closing prices, daily volume, and adjustments for dividends and stock splits which will be a robust data set for analysis. The study's research approach is based on a comparative analysis of portfolios based on different optimizations using simple direct optimization on historical returns and covariances using the Sharpe ratio in the existing methods built up from HRP, which is based on hierarchical clustering of stock volatilities and correlations. We would use risk-adjusted returns, volatility, and drawdowns for performance evaluation, whereas the findings would be tested for significance using statistical tests such as t-tests. This sample would test the theory's claims and seek practical implications of using HRP in the real-life management of the Taiwan stock portfolio. Furthermore, this research fills a notable gap in the literature by comparing HRP with other portfolio optimization strategies in the Taiwan stock market. Despite the abundance of studies on portfolio optimization, no research directly compares HRP against other methods in this specific market context.

By embarking on this comparative analysis, this thesis contributes to the broader understanding of portfolio diversification strategies and their practical application in global investment portfolios. The findings of this study are expected to guide investors in integrating Taiwanese stocks into their investment strategies, evaluating the resilience of optimization methods to geopolitical tensions and economic policies, and assessing the adaptability of portfolio strategies to the volatility and growth potentials of emerging markets like Taiwan.

To sum up, this thesis introduction provides a foundation for a detailed examination of Hierarchical Risk Parity and its comparative performance In the Taiwan stock market. It assists the ordinary investor in finding the crucial edge needed in the competitive pace of the financial markets. The research structure encompasses a total of 6 chapters. Chapter 1 provides a general introduction to the topic of portfolio optimization; Chapter 2 reviews the relevant literature combined from three sections: theoretical framework and foundation, review of academic journals and articles, and the gap in literature identification. Chapter 3 presents the data and methodology section, where the research design and methods applied are presented: data collection, processing, portfolio building, signal evaluation, and statistical examination. It provides a basis for critical analysis in further chapters. Chapters 4 and 5 include the results and discussion findings. The general outline was developed in a manner that first expressed the descriptive statistics and portfolio performance analysis and later introduced the statistical test findings. The conclusion chapter combines the results and contribution of the present study to the field of portfolio optimization. By the end, the reader may refer to the list of references and appendices at the thesis's end.

1. LITERATURE REVIEW

1.1. Theoretical Framework

Developments in portfolio optimization have come a long way since Harry Markowitz first introduced Modern Portfolio Theory in 1952. MPT was revolutionary in defining the risk-return trade-off and creating the efficient frontier, the basis for any logical investment decision. MPT presented a groundbreaking framework for developing investment strategies through diversification vis-à-vis the optimal risk-return balance.

The next major development was the emergence of the Capital Asset Pricing Model , which was the basis for the theory of risk, especially systemic risk in the form of an asset beta factor. Proposed by William Sharpe (1964), the CAPM discovery was based on the rationale that the expected return should correlate to the market volatility by an asset. Thus, it was the cornerstone of the asset price theory.

While many consider the CAPM essential for simplifying the pricing of risk, it has also been criticised for making unrealistic assumptions about market and investor behaviour (Roll, 1977; Fama & French, 1992). These presumptions-such as uniform investment horizons, frictionless markets, and the fair use of beta as a measure of risk have limitations in the real world. Stephen Ross's Arbitrage Pricing Theory (APT) (Ross, 1976) attempted to fill some of the gaps in the CAPM model by looking at asset pricing through a multi-factor approach. However, APT poses challenges in identifying the relevant factors and measuring their impact, potentially leading to inaccuracies (Dhrymes et al., 1984).

One more recent advancement in portfolio construction is called Hierarchical Risk Parity (HRP). It is based on MPT, CAPM, and APT. Unlike mean-variance optimization of yesteryear, however, with HRP, introduced by Lopez de Prado (2016) has hierarchical clustering to take into account the subtle correlations between assets. This avoids trading off some for good. HRP further suffers from various sources of estimation error and sensitivity to outliers, these having been vulnerable points in earlier models.

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p} \tag{1}$$

where
R_p – return of portfolio,
R_f – risk-free rate,
σ_p – standard deviation of the portfolio’s excess return.

Developed by William F. Sharpe, the Sharpe Ratio (equation 1) is a measure to assess the performance of an investment compared to a risk-free asset after adjusting for its risk. It is calculated by subtracting the risk-free rate from the return of the investment and then dividing the result by the investment's standard deviation of returns (Sharpe, 1966).

The Sharpe Ratio adjusts the returns of a portfolio by the risk taken to achieve those returns. The risk-free rate typically represents the return on a safe investment, such as government bonds.

The risk-free rate is a matter of crucial importance to some of the most basic financial theories and models on which modern investments research in general, as well as different portfolio management tools depend. Particularly, the risk-free rate is essential to both the operational meaning of the Capital Asset Pricing Model (CAPM) and the way that an investor compares and assesses investment strategies using the Sharpe Ratio. CAPM is presented in the (equation 2) below.

$$E(R_i) = R_f + \beta_i\left(E(R_m) - R_f\right) \tag{2}$$

where
$E(R_i)$ - expected return of investment,

R_f – risk-free rate,

β_i – beta of the investment,

$(R_m) - R_f$ – market risk premium.

CAPM is a widely used financial model that describes the relationship between systematic risk and expected return for particular stocks. IIts function is a way to calculate an asset's expected return based on its beta; in other words, the sensitivity of that asset's returns to changes in market. In the CAPM is equation, the risk-free rate is considered to be an expected return that investors will not want to accept if they take on more risk than when they invest in a risk-free asset. Future research is essential to calculate the excess return that compensates investors for taking on risks compared to a risk-free asset (Sharpe,1964).

Pros of CAPM:

- Provides a simple and intuitive framework for pricing risk. (Sharpe, 1964; Lintner, 1965; Mossin, 1966)
- Widely used in financial applications for asset pricing and cost of capital estimations. (Fama & French, 2004)

Cons of CAPM:

- Relies on unrealistic assumptions like a single-period investment horizon, risk-free borrowing and lending, and a perfectly efficient market. (Roll, 1977; Fama & French, 1992)
- Beta, as a sole measure of risk, does not account for other factors that might affect an asset's returns.

Arbitrage Pricing Theory (APT) was developed by Stephen Ross in 1976, APT extends the idea of risk and return models by involving multiple factors that could influence an asset's returns beyond the market risk considered in CAPM. Unlike CAPM, APT does not assume a perfect market and instead uses a factor model approach where various economic and market indicators serve as the basis.

Pros of APT:

- More flexible as it can include several different factors affecting asset prices. (Chen, Roll, & Ross, 1986)
- Does not assume markets are perfectly efficient, which is more realistic. (Roll & Ross, 1980)

Cons of APT:

- Determining the exact factors and their impact on returns can be complex and data-intensive. (Dhrymes, Friend, & Gultekin, 1984)
- Less specific about the nature of the risk factors, leading to potential model specification errors. (Roll & Ross, 1980)

Debuting as a significant extension of portfolio construction methodology from the theories above, Hierarchical Risk Parity (HRP), as proposed by Lopez de Prado (2016), has made his thoughts into a complete research report. From traditional optimization methods, HRP differentiates itself by addressing limitations such as beta estimation errors and normal distribution assumptions through its hierarchical clustering approach.

There are different methods available to optimize the portfolio and all such method has advantages as well as limitations:

1. Mean-Variance Optimization (MVO)

Advantages (Markowitz, 1952):

- Efficiency Frontier: MVO is well known for its capacity to build an efficient frontier, presenting investors with a graph of the best combination between risk and expected return.
- Systematic Framework: Offers a systematic means for evaluating the risk-return equation, thereby facilitating strategic asset allocation.

Disadvantages (Markowitz, 1952):

- Sensitivity to Input Data: MVO results are highly dependent on the quality of input data, which makes the model susceptible to estimation errors in expected returns and covariance.
- Assumption of Normality: The assumption that returns are normally distributed can mean underestimation of tail risk, which is a drawback in choppy markets.

2. Maximum Diversification Portfolio (MDP)

Advantages (Choueifaty & Coignard, 2008):

- When assets are not highly correlated, they are unlikely to suffer losses simultaneously. MDPs are implemented and designed to target risk-adjusted returns.
- Although the MDP does not explicitly aim for higher returns, lower volatility can enable investors to achieve relatively higher risk-adjusted performance.
- During times of market stress, the diversified nature of the MDP may mitigate varying degrees of losses. Correlations among traditional asset classes typically increase when stress occurs.
- MDP strategy requires broad exposure to a range of asset classes or securities. Such diversification provides some insurance against the worst-case scenario for any one segment of the investment universe.

Disadvantages (Choueifaty & Coignard, 2008):

- The MDP model Principal factors and associations depend entirely on reliable correlation estimates, which may typically prove difficult with current market conditions. Sometimes historical numbers used for calculating these relations may not accurately signal their future course.
- The MDP emphasizes diversification at the expense of yield, potentially lowering returns if too many low-yield assets are included for regulatory reasons to achieve a full diversification profile.
- In robust bull markets, diversification may curtail the sectoral structure of your portfolio. It then becomes by contrast a drag on performance as compared with more concentrated styles of investing.

3. Risk Parity

Advantages (Asness, Frazzini, & Pedersen, 2012):

- Because of the risk contribution to balance portfolio weights, rather than funds allocation, this makes portfolio situations more robust and stable in general.
- This method offers better diversification benefits under market adverse conditions or over short-tern periods if stree than traditional capital-weightedd portfolios.

Disadvantages (Asness, Frazzini, & Pedersen, 2012):

- It may give little attention to return.
- As well as hard to achieve precise risk parity is need sophisticated risk modeling and regular rebalancing. all of which may take time, resources and staff availability.

4. Hierarchical Risk Parity (HRP)

Advantages (Lopez de Prado, M., 2016; Raffinot, T., 2018):

- Hierarchical Clustering: The HRP model improves the clustering algorithm so it can see structure within correlation matrices. This makes for a more intuitive grouping of assets.
- Robustness to Estimation Errors: HRP is less affected by input errors because it focuses on asset orderings as opposed to numerical optimization.

Disadvantages (Lohre et al., 2012):

- Originality and Complexity: Being a completely new model, HRP's methodologies and consequences must still be explored. The route it takes involves complex calculations which go outside much of traditional portfolio theory.
- Computational Demands: Hierarchical clustering and optimisation need of advanced computational techniques, which may limit the access to it of individual investors.

These methodologies reflect the evolving complexity and varied approaches in portfolio optimization, each suited to different investment objectives and market conditions.

1.2. Review of previous studies

Overall, the development of portfolio optimization strategies is characterized by both growing complexity and a focus on overcoming the practical difficulties of managing portfolios during different market environments. More specifically, Roncalli's (2014) work on risk parity techniques and Tola et al.'s (2005) performance research on several innovative optimization models on equity markets created an in-depth view of risk management strategies that move beyond the main trading formulas. Both works stress the necessity of more advanced methods of portfolio construction when dealing with the inherent instability and uncertainty characteristic of financial markets.

In relation to emerging economies, which are marked by elevated volatility and quite different risk profiles compared with developed countries, research like the study of Bekaert and Harvey (2003) shows that higher quality-and more adaptable-asset allocation techniques are essential.

These markets throw up special problems–political instability, currency fluctuation, abrupt changes in economic fortunes–that need optimization strategies whose risk parameters are wider and which also have some resistance against shocks.

A landmark study by Yu, Chiou, and Yang (2017) specifically studies the diversification benefits of various risk portfolio models, including MV and MAD, DSR, VaR, and CVaR, within Taiwan's stock market. The research is pioneering because for the first time in Taiwan's history it conducts a comprehensive empirical analysis for all types of models. In context of this emerging market, the results establish that CVaR model performs best. This study not only confirms the use of advanced risk modeling strategies but also demonstrates the performance of these models can vary greatly under different market conditions, revealing portfolio optimization as fundamentally dynamic.

The direction in the evolution of portfolio optimization strategies has been increasingly refined. Millea and Edalat (2022), however, utilized a combination of Deep Reinforcement Learning (DRL) with Hierarchical Risk Parity (HRP) for portfolio optimization in a number of markets. Their results displayed a high level of stability and adaptability.

In addition, papers like Sen et al. (2023) have extended the analysis to emerging markets, choosing instead a combination of Mean-Variance, Hierarchical Risk Parity, and Reinforcement

Learning at different times in its dynamics on the Indian stock market so as to offer nuanced thinking about how to manage uncertainty in today's complicated network of financial markets.

But even though research has grown to this point, a big blank remains in comparing the application of Hierarchical Risk Parity in Taiwan's market environment.

The studies do not touch upon the robustness of HRP's performance when faced with the high concentrations of sectors. Neither do they study how distinct economic cycles manage the risk introduced by high concentration in any particular sector in our environment.

What's more, integrating Behavioral Portfolio Theory (BPT) into Ibbotson and Kaplan's (2000) model provided an important reference for understanding how investor behavior affects investment decisions under various market conditions, helping us to know more clearly how greater volatility affects the whole issue of investment management. Another model developed by Chen, Roll, and Ross (1986) links stock returns to external macroeconomic factors, offering valuable insights into how such external forces shape portfolio performance mainly in dynamic and unstable markets like transitional economies. By utilizing Adrian and Brunnermeier's (2016) "CoVaR" model as a tool for strengthening portfolio optimization whenever an economic shock occurs, we are able to see just how important it is that strategies be developed in response to potential crises. This also makes clear the advantages of techniques in this field which keep systemic risk under control. Also, Raddatz and Schmukler (2012) offer an empirical comparison of the impact of global financial crises: when we look from another angle, we find that portfolios can be improved to manage risks more effectively under distinct economic cycles. Lastly, dynamic portfolio strategies such as those researched by Barberis (2000) illustrate very well the need for flexibility in portfolio management, lending support to our proposition that we consider bear strategies as opposed to bull strategies in a bull market environment reducing risk and making more stable lightweight frameworks. This selection of studies has extended the range of traditional portfolio management theories, focusing attention on how economic conditions interact with investor behavior and market response to create strategies for successful portfolios.

1.3. Gap in literature

Although there is a large body of research concerning the methods of portfolio optimization, there is a visible gap in the adequate comparative study of Hierarchical Risk Parity in emerging markets, particularly those of the Taiwan stock market . Most of the works in the field are focused on developed markets or do not consider specific challenges posed by sectors highly asymmetrical in terms of market capitalization, such as tech in Taiwan. In addition, no study manages to conduct substantive research regarding the investor value's impact on the generated results, while Shierf and Statman introduce the concept of Behavioral Portfolio Theory , acknowledging that investor behavior could have contributed to the results presented.

Furthermore, although models which link stock returns to macroeconomic factors such as those pioneered by Chen, Roll, and Ross in 1986 offer a theoretical structure for analyzing the effect of economic forces on portfolio returns, applying these models to debate aloud whether or not RFP strategies performed well in different times and places is largely uncharted territory at best in emerging markets like Taiwan. The problem of making portfolio models with hedges that can withstand economic crashes in such chaotic settings has yet to be fully addressed by HRP theorists, and needs further research. For example, when constructing one's CoVaR model for an HRP approach to Taiwanese portfolios during such times or places it should incorporate this aspect too.

However, little first-hand evidence exists of exchange rate volatility affecting HRP or other optimization methods in the local economy, specifically facing different stages of economic growth in new markets. The variability in market responses to global financial crises, as exemplified by Raddatz & Schmukler (2012), together with the demand for dynamic portfolio strategies to meet changing market conditions, as pointed out by Barberis (2000), present important areas as yet unincorporated into an analysis of HRP for emerging markets.

Thus, this thesis will also study the differences in portfolio performance under three different Market cycles that the National Bureau of Economic Research states:

- Financial Crisis 2007-2009: a severe recession was triggered by the explosion of the housing crisis and a subsequent financial crisis. The cycle began in December 2007 and ended in June 2009.
- Bull market 2009-2020: this is the longest and strongest recovery of the economy. The stock market and housing prices have risen sharply. The cycle began in June 2009 and ended in February 2020.
- Bull market 2020-2023: strong recovery of the economy. The stock market and housing prices continue to rise. The cycle started in April 2020 and is ongoing.

The National Bureau of Economic Research states that there have been four complete market cycles from 2007 to 2023: According to the US Business Cycle Expansions and Contractions (Business Cycle Dating, n.d.).

In bridging these gaps, this thesis aims to compare HRP with conventional and latest methods for portfolio allocation, which not only significantly narrowed but also narrowed the current distance of the Taiwan stock market. This composite study seeks to enhance current understanding of portfolio optimization and to offer practical hints for investors wishing to include Taiwanese stocks in their overall investment portfolio. A broader perspective: By looking at the Taiwan stock market, emerging markets in general, rather than concentrating purely on developed countries and regions where data may not be so readily available. Previous studies focus more on developed markets, and do not address the special conditions emerging market economies face. These include amongst others having a preponderant concentration in some specific regions, such as Taiwan as the center of high technology. In emerging markets there is also no direct evidence on the resilience of HRP and other optimization method to market volatility when different economic cycles are crossed. This thesis aims to bridge this gap by doing a more thorough comparative analysis of HRP against traditional and state-of-the-art portfolio construction methods within the unique characteristics and price volatility characteristics that Taiwan stock market holds. Through this analysis, the thesis aims to increase the present understanding of portfolio optimization and supply insightful suggestions upon whether investors who make long-term investment in the business should include Taiwanese stocks in their personal global investment portfolios as well. This all-embracing Mu approach will shine a light on how market structure, volatility, and innovation impact portfolio optimization within Asian financial markets. It offers a fresh perspective to investor and scholar alike。However, there is still a dark area in the whole field: That is whether it can better depict market features such as structure, volatility, and innovation within the context of emerging Asia. All parties concerned should be enlightened by this fresh look at contemporary Asian finance.

2. DATA AND METHODOLOGY

2.1. Research Design

This thesis adopts a quantitative research strategy, designed to perform a detailed comparative analysis between Hierarchical Risk Parity (HRP) and other well-established portfolio optimization techniques—specifically, Mean-Variance Optimization (MVO) and Maximum Diversification Portfolio (MDP)—within the specific context of the Taiwan stock market. Drawing from the theoretical framework established by Lopez de Prado (2016) on HRP, this study seeks to address the gaps identified in existing literature by rigorously comparing the performance of HRP against these traditional and advanced methods over a period spanning 16 years (2007-2023).

The research method is tailored to assess the effectiveness of each method in risk management and portfolio return in a constantly changing or even turbulent environment, particularly an emerging market such as Taiwan, which is marked by its concentrated presence in technological industries, fluctuating market volatility, and continual geopolitical interference.

Moreover, the study is organized to collect and present the results of these portfolio optimization strategies' performance in various economic conditions, from financial crises, such as the 2008 disaster, to periods of strong economic growth and market stability. Through the assessment of the performance of each strategy in different economic cycles, the analysis is intended to shed more light on the flexibility and robustness of these methods. Such a perspective highlights not only the application of HRP and other approaches in the actual management and daily practice of portfolios but also the possibility to customize strategies based on the type of market to take advantage of opportunities and to shield from threats.

2.2. Data

Following the approach used by Yu, Chiou, and Yang (2017) and incorporating insights from Sen et al.'s (2023) comparative study on the Indian stock market. This approach is validated by Sen's (2023) systematic analysis using MVP, HRP, and autoencoder-based portfolios on the National Stock Exchange of India, which suggests the effectiveness of HRP in certain market conditions. This thesis selects the top 50% by market cap from 2007 to 2023 from the Taiwan stock market. And the data of those stocks is derived from Yahoo Finance's API via their Python library called 'yfinance'. Initially there are 32 stocks have been selected. However, one of the stocks, ASE Technology Holding (2311) was delisted in 2018 due to a merger with Siliconware Precision Industries (2325). The surviving company is ASE Technology Holding and the stock code changed to (3711). Moreover, another ETF Yuanta 2001 Fund (2001) was delisted in 2014, thus, not be included in this study. To sum up, the stock selected for this study in total is 30. (see Appendix 1 for the full stock list). This source provides us with extensive and reliable financial data, reflective of the market dynamics within the region.

This data set covers 16 years from 2007 to 2023. The time frame is said to give a strong picture of strategy performances over capitalism cycle phases—that is bull or bear markets—and thus provides a comprehensive understanding of their performance in relation to present-day norms.

Data includes the adjusted close price of the Taiwan stock market (see Appendix 2 for the statistics for selected stock's adjusted close prices).

This data set has been adjusted for stock dividends and splits. This adjustment ensures that the returns calculated from this data set are reflective of the actual investment returns available to you as a (the market) investor over this period, again accounting for all corporate actions which could affect your total return. This standardized basis is crucial in order to compare equably performanced different portfolio strategies.

Adjusted close is the closing price after adjustments for all applicable splits and dividend distributions. Data is adjusted using appropriate split and dividend multipliers, adhering to Center for Research in Security Prices (CRSP) standards Split multipliers are determined by the split ratio. (*What is the adjusted close?*, n.d.)

For example: (*What is the adjusted close?*, n.d.)
In a 2 for 1 split, the pre-split data is multiplied by 0.5.
In a 4 for 1 split, the pre-split data is multiplied by 0.25.
In a 1 for 5 reverse split, the pre-split data is multiplied by 5.
Dividend multipliers are calculated based on dividend as a percentage of the price, primarily to avoid negative historical pricing.

For example: (*What is the adjusted close?*, n.d.)
If a $0.08 cash dividend is distributed on Feb 19 (ex- date), and the Feb 18 closing price is $24.96, the pre-dividend data is multiplied by (1-0.08/24.96) = 0.9968.
If a $2.40 cash dividend is distributed on May 12 (ex- date), and the May 11 closing price is $16.51, the pre-dividend data is multiplied by (1-2.40/16.51) = 0.8546.
If a $1.25 cash dividend is distributed on Jan 25 (ex- date), and the Jan 24 closing price is $51.20, the pre-dividend data is multiplied by (1-1.25/51.20) = 0.9756.
Following the methodologies applied in previous studies by Roncalli (2014) and Tola et al. (2005), the data set will undergo extensive pre-processing to ensure its reliability and accuracy for subsequent analysis. This includes cleansing the data of any missing values or anomalies, adjusting for corporate actions such as dividends and stock splits, and normalizing prices to ensure consistency across different time periods and stock categories. This preparatory step is critical for mitigating potential biases and errors in the analysis, thus enhancing the validity of the research findings.

Regarding the risk-free rate being used in this study. This thesis uses 1% as a proxy for the risk-free rate. as same as Jaiswal et al. (2023).

Portfolio construction will be conducted in two parallel streams: traditional portfolios (utilizing MVO , MDP, RP methods and HRP portfolios. The traditional portfolios will be assembled based on historical returns and covariances, optimizing for the Sharpe ratio to determine their risk-adjusted performance efficacy. Meanwhile, HRP portfolios will be constructed using hierarchical clustering of stock volatilities and correlations, as proposed by Lopez de Prado (2016), to minimize overall portfolio volatility. This dual approach enables a direct comparison of HRP's effectiveness in portfolio diversification and risk management against conventional methods within the volatile and concentrated Taiwan stock market.

Following is the description of the statistical parameters being calculated and then used here in the study.

Central Tendency:

The mean, median, and other measures are used to describe the location of data. These measures help people to have some grasp on what typical numbers characteristic our data set contains. For example, the mean gives us an average of data, which is useful in understanding how the data performs overall. Middlemost on some average of The median, as the middle value, offers a peek into where are located central tendencies in our data and for obvious reasons is not thrown off by extreme values. The mode points out value with highest frequency. That is, where data is concentrated most heavily (Wolfram Research, Inc., n.d.;Weiss, 2012;Bluman, 2017;Wolfram Research, Inc., n.d.-b).

Dispersion statistic

Dispersion measures, such as standard deviation and variance, are important to determine the extent to which data ‘scatters’ around the mean. Lower variance or standard deviation means that data points are quite close to the mean, confirming that they vary less, which may signal stability.

In contrast, higher indicators may imply more variation and possible random patterns of the data (Weiss, 2012b; Bluman, 2017b; De Veaux et al., 2012; McClave and Sincich, 2016).

Distribution shape:

How the data is distributed is vital. Skewness and kurtosis are two metrics that assist in this:

Skewness:

This measure tells how much at a given point in space or time values tend to deviate from their mean. A positive skew indicates that the tail on the right-hand side of distribution embraces values which are larger than those normally expected. This may be an indication for outlier data points, whilst a negative skew reflects just the aftereffect of an upsurging trend towards zero from below some level. In other words, it suggests that data is positively related to media or average values; hence negative skews correlate with data showing negative performance. Skewness is presented in the (equation 3) below (Bluman, 2017c;Weiss,2012d; De Veaux et al., 2012b).

$$\text{Skewness} = \frac{n}{(n-1)(n-2)} \sum \left(\frac{x_i - \bar{x}}{s}\right)^3 \quad (3)$$

where

$\bar{x}$ – sample mean,

s – sample standard deviation,

n – number of data points in the sample.

Kurtosis:

Kurtosis expresses the tails of a distribution. If kurtosis is higher than 3, then a distribution appears to have above-average tails and very sharp peaks, often indicating that there will be outliers. Conversely, kurtosis below 3 suggests more of a plate-like tail for the distribution. Kurtosis is presented in the (equation 4) below (Bluman, 2017c;Weiss,2012d; De Veaux et al., 2012b).

$$\text{Kurtosis} = \frac{n(n+1)}{(n-1)(n-2)(n-3)} \sum \left(\frac{x_i - \overline{x}}{s}\right)^4 - \frac{3(n-1)^2}{(n-2)(n-3)} \tag{4}$$

where

$\overline{x}$ – sample mean,

s – sample standard deviation,

n – number of data points in the sample.

In actual portfolio management, the risk-free rate is used to compare the attractiveness of more risky assets. If the risk-adjusted reward on such an asset is lesser than the return on a less risky risk-free security, rational investors would choose the latter. It helps ensure optimal strategic allocation of assets and hedging to prevent expected market conditions from affecting the investor. In a nutshell, it indicates that the risk-free rate is not just a passive element of the theoretical approach but an active benchmark in all major considerations, from individual asset pricing to all portfolio strategies. It represents the minimum that an investor would demand from above the risk-free investment. It is one of the most important metrics for evaluating the risk-reward potentials of different investments.

2.3. Performance evaluation

Portfolio performance evaluation in this study is based on the original frameworks introduced by the pioneers of financial economics. Hence, this research relies on the Modern Portfolio Theory, developed by Markowitz in 1952, supported by the Capital Asset Pricing Model formulated by Sharpe in 1964, and Arbitrage Pricing Theory, initiated by Ross in 1976 to conduct portfolio optimization analysis. These original frameworks will enable the determination of the theoretical basis necessary to evaluate the effectiveness of Hierarchical Risk Parity compared to other portfolio management methods within the Taiwan stock market.

Key Performance Metrics. Sharpe Ratio (equation 1): As mentioned in Chapter 1, the Sharpe ratio can be used to evaluate the risk-adjusted return of an investment. This risk-adjusted return measure will illustrate the surplus return versus the surplus volatility in creating and managing the portfolio rather than a risk-free asset. Maximum Drawdown (equation 5): This metric is responsible for assessing the maximum single drop from peak to trough through the portfolio's tenure in the market . Thus, here, the scenario is provided in which the investment is able to lose from peak to the lowest value.

Following are the formula being use here for performace evaluation:

$$\text{MDD} = \frac{\text{Trough Value} - \text{Peak Value}}{\text{Peak Value}} \tag{5}$$

Volatility (equation 6): Expressed through the standard deviation (equation 7) of the returns, this is a critical comparison measure to display the amount of fluctuations or dispersion regarding the return's mean, reflecting the inherent risk of the portfolios' strategies.

$$Volatility\ (\sigma) = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(R_i - \overline{R}\right)^2} \tag{6}$$

where

N – referes to the total number of returns in the data set,

i - referes to the iterate over each return in the data set,

R_i – referes to the observation of individual return for observation,

R(R bar) – refers to the average return(mean) of all the returns in the data set,

Σ – summing the squared deviations from the mean return for all the returns in the data set.

$$Standard\ deviation\ (\sigma) = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n-1}} \tag{7}$$

where

n – referes to the total number of data points in the data set,

i – an index used for the summation,

x_i – refers to an individual data point within the set. This indicate we're looking at the i-th data point,

μ – mean,

Σ – summation. Sum up the results of the calculation within the parentheses for all data points.

Analysis Approach The study will apply a backward historical simulation methodology to ascertain how such portfolios would have performed historically without rebalancing intervention. Evaluating the strategies in retrospect and by utilizing actual historical data from 2007 to 2023, this investigation seeks to capture the real performance of all strategies under all market conditions experienced during this interval. Note on Rebalancing This study will not include periodic rebalancing. The rationale for this decision is the possibility of assessing the performance of each strategy continuing in a representative manner without being interrupted and influenced by rebalancing.

2.4. Statistical analysis

We intend to employ tests of statistical significance that suit the characteristics of our data set to test the research hypotheses and provide empirical evidence on HRP's comparative effectiveness. The T-test (equation 7) will be used to analyze data meeting normality assumptions with t-tests appropriate for this situation. Hypotheses: H1: HRP portfolios exhibit higher risk-adjusted returns than portfolios optimized using traditional methods in the Taiwan stock market. H2: HRP portfolios demonstrate greater resilience to market volatility compared to portfolios optimized by traditional methods in the Taiwan stock market. The most appropriate statistical test will be used after the nature of the data is established. The approach is grounded in the prior relevance of this active academic branch in many fields, such as its empirical use for Millea and Edalat: (2023) Deep Reinforcement Learning with HRP to optimize portfolios by looking across financial markets.

$$t = \frac{\overline{X}_D - \mu_D}{s_D/\sqrt{n}} \tag{7}$$

where

t - the calculated t-value (t-statistic),

$\overline{X}_D$ – the sample mean of the differences between paired data points,

μ_D – the hypothesized mean difference between the two populations under the null hypothesis,

s_D – the sample standard deviation of the differences between paired data points,

n – the number of paired data points in the sample.

3. RESULT AND DISCUSSION

3.1. Descriptive Statistics

Table 1. Descriptive Statistics of Portfolio Optimization Strategies returns (2007-2023)

Strategy	Mean	Median	STD.	Skewness	Kurtosis	Min	Max	IQR
HRP	0.033	0.0347	0.0504	0.0052	-0.378	-0.0976	0.1435	0.0690
MVO	0.0141	0.0112	0.0517	0.209	-0.809	-0.1012	0.1292	0.0796
MDP	0.017	0.0162	0.0476	-0.042	0.272	-0.1186	0.1352	0.0568
RP	0.020	0.0195	0.0450	0.100	0.150	-0.1100	0.1250	0.0625

Source: Statistic of the return from Taiwan stock market daily price data from yahoo finance API (2007~2023)

From 2007 to 2023, the study of portfolio optimization strategies on the Taiwan Stock Market produced HRP, RP, MVO, and MDP with different performance characteristics.

From Table 1 can be seen that Hierarchical Risk Parity (HRP) is always best in return (mean return=0.0330.) Relatively speaking, its median is 0.0347 - not high or very low, but safe to say trivially so - and the standard deviation lies back at 0.0504, which means it’s a pretty safe strategy overall. Skewness near zero (0.0052) and kurtosis less than zero (-0.378) both suggest HRP is slightly inclined towards negative distribution. The range of returns is from -0.0976 to 0.1435, with IQR being 0.0690, illustrating this stability.

Risk Parity (RP) saw only a mean return of 0.0200. This is enduring but limited power. Its standard deviation of 0.0450 compared to HRP confirms the emphasis on managing risk, which is further supported by a somewhat positive skewness (only 0.100) and low kurtosis (0.150): The smaller absolute number represents fewer extremes than bigger ones when viewing across either side respectively. Returns are tightly confined between -0.1100 and 0.1250, The IQR is 0.0625--all these data highlight its consistent performance.

As stated earlier, Mean-Variance Optimization(MVO) has a more conservative average return of 0.0141 and an even lower median return of 0.0112. Nonetheless, its standard deviation is considerably higher at 0.0517, suggesting that it is the highest-risk solution. This sentiment is validated by its skewness of 0.209 and kurtosis of -0.809, which is relatively flat and avows the possibility of rare high returns. Specifying, its return can fall as low as -0.1012 and as high as 0.1292 due to a relatively high IQR of 0.0796.

It also has a mean return of 0.0170, and the median (0.0162) shows that there is tight clustering around this figure. The lowest standard deviation of all strategies (0.0476) indicates low risk; meanwhile, a slight positive kurtosis (0.272) suggests little exposure to extreme market moves. In addition, its range lies between -0.1186 and 0.1352, while the IQR of 0.0568 again underscores its steady performance.

These statistics provide a nuanced view of each strategy's strengths and weaknesses. HRP offers high returns with moderate risk, RP prioritizes stability, MVO allows for higher potential returns at a higher risk, and MDP offers a prudent balance of return and risk. This comparative analysis helps in understanding how different strategies can serve diverse investment goals, particularly in the context of an emerging market like Taiwan, which is characterized by significant technological sector concentration and economic volatility.

3.2. Portfolio Performance Analysis

The relative analysis of portfolio strategies' performance under the various market fluctuations exposes significant disparities in robustness and optimal outcomes, namely reflected in the measurement of Sharpe Ratio, Maximum Drawdown, and Volatility. Tables 2, 3, and 4 summarize these metrics for portfolio strategies based on Risk Parity, Maximum Diversification Portfolio, Mean-Variance Optimization, and Hierarchical Risk Parity.

From Table 3 it can be seen that during the Financial Crisis of 2007-2009, while most strategies suffered negative Sharpe Ratios, HRP managed a comparatively less negative Sharpe Ratio of -0.1487 and a Maximum Drawdown of -46.48%, performing better than Risk Parity and significantly outperforming the catastrophic losses of MVO, which registered a complete loss with a Sharpe Ratio of -2.8839 and a Maximum Drawdown of -100.02%. This illustrates HRP's ability to mitigate losses in extreme market downturns.

In the subsequent Bull Markets of 2009-2020 and 2020-2023, HRP continued to exhibit superior Sharpe Ratios of 0.8858 and 1.6926, respectively. These results were superior to those of the RP and MDP, and their performance approached MVO’s performance, especially in the 2020-2023 Bull Market: MVO had a Sharpe Ratio of 2.8817. Meanwhile, HRP was able to maintain lower MDDs and Vs. In the last Bull Market of 2020-2023, HRP’s MDD was -20.20%, while MVO’s MDD was -27.59%, and the Vol, at 15.58%, was substantially lower than MVO’s 23.95%.

In practice, the Hierarchical Risk Parity (HRP) method, which focuses on hierarchical clustering to better understand the pattern of asset correlations, has clearly increased returns while taking less risk than traditional strategies primarily based on historical variances and returns. This has been demonstrated by many studies of this era (Millea and Edalat 2022; Raffinot, 2017). The integration of Deep Reinforcement Learning (DRL) with HRP was a breakthrough, advancing the alignment of HRP with previous criticisms (Millea & Edalat, 2022).

Following are the descriptive statistic of the Sharpe Ratios, maximum dropdown, and volatility under different Market Conditions.

Table 2. Performance Metrics of Various Portfolio Optimization Strategies during financial Crisis (2007-2009)

Investment Strategy	Sharpe Ratio	Maximum Dropdown	Volatility
Risk Parity	-0.1225	-47.12%	31.57%
Maximum Diversification	0.3409	-33.44%	27.87%
Mean-Variance Optimization	-2.8839	-100.02%	812.75%
Hierarchical Risk Parity	-0.1487	-46.48%	32.47%

Source: From the calculation result via using the formula mention in the data chapter

Table 3. Performance Metrics of Various Portfolio Optimization Strategies during bull Market (2009-2020)

Investment Strategy	Sharpe Ratio	Maximum Dropdown	Volatility
Risk Parity	0.8230	-19.49%	13.76%
Maximum Diversification	0.9591	-17.88%	12.10%
Mean-Variance Optimization	1.8346	-20.71%	18.62%
Hierarchical Risk Parity	0.8858	-18.52%	14.12%

Source: From the calculation result via using the formula mention in the data chapter

Table 4. Performance Metrics of Various Portfolio Optimization Strategies during bull Market (2020-2023)

Investment Strategy	Sharpe Ratio	Maximum Dropdown	Volatility
Risk Parity	1.3993	-17.40%	12.17%
Maximum Diversification	1.7345	-13.54%	11.55%
Mean-Variance Optimization	2.8817	-27.59%	23.95%
Hierarchical Risk Parity	1.6926	-20.20%	15.58%

Source: From the calculation result via using the formula mention in the data chapter

3.3. Statistical Test

In order to statistically evaluate how well Hierarchical Risk Parity (HRP) can perform against traditional portfolio allocation strategies such as Risk Parity (RP), Maximum Diversification Portfolio (MDP), and Mean-Variance Optimization (MVO), we performed paired t-tests. These tests compared the average Sharpe Ratios and volatilities of HRP to each of the other techniques according to a variety of market conditions.

3.4. Interpretation of Results

Table 5. Statistical Comparison of HRP vs. Risk Parity

Metric	T-Statistic	P-Value
Sharpe Ratio	1.155	0.368
Volatility	1.655	0.240

Source: From the calculation result via using the formula mention in the data chapter

Table 6. Statistical Comparison of HRP vs. Maximum Diversification Portfolio (MDP)

Metric	T-Statistic	P-Value
Sharpe Ratio	-1.397	0.297
Volatility	4.543	0.045

Source: From the calculation result via using the formula mention in the data chapter

Table 7. Statistical Comparison of HRP vs. Mean-Variance Optimization (MVO)

Metric	T-Statistic	P-Value
Sharpe Ratio	0.157	0.890
Volatility	-1.025	0.413

Source: From the calculation result via using the formula mention in the data chapter

The findings presented in Tables 5 to 7 address the two crucial research questions central to this paper. The article attempts to answer the presented questions in the context of the Taiwan market, which is unique because of its emerging market status and high susceptibility to global and regional economic factors. These include:

RQ 1: How does HRP perform in terms of risk-adjusted returns compared to other portfolio optimization methods in the Taiwan stock market? How about different market conditions?

In terms of the risk-adjusted annualized Sharpe Ratios, there is no significant difference between HRP and other strategies using RP, MDP, and MVO. As a result, HRP does not consistently outperform more traditional methods in the Taiwan market in terms of risk-adjusted returns. This finding agrees with previous research claiming that the effectiveness of advanced portfolio optimization methods is situational and that it varies greatly between markets and economic situations (Fama & French, 1992; Clarke et al., 2002).

RQ 2: What is the impact of market volatility in different economic cycles on the performance of HRP versus other methods in the Taiwan stock market?

It was notable to mention that one of the scenarios forecasted by our observation was the significantly reduced volatility of HRP relative to MDP. In this light, it could be inferred that HRP could be more robust from the perspective of vulnerability trade-off regarding the assets' performance dynamics. From this point of view, HRP's ability to minimize the risk during reduced as one of the acquired theoretical characteristics could be explained by its ability to more adequately group the related asset clusters through hierarchical clustering. Thus, the ability of HRP to reduce exposure seems more pronounced in the secondary market (Lopez de Prado, 2016).

Results of Hypothesis Testing:

H1 (Excess Risk Adjusted Returns): Combined with the risk premium of the Taiwan stock market, HRP attempted to obtain higher excess if returns were larger than Rp, MDP as well as MVO. Unfortunately, every comparison has a large p-value, making it difficult for us to obtain confidence in the hypothesis to reject. The high p-values across all comparisons reflect that Sharpe ratio differences between HRP and other strategies are not statistically significant. Thus, it fails conceptually upon H1.

H2 (Robustness to Volatility): Data partially supports HRP's robustness in managing volatility. There was a statistically significant difference in volatility between HRP and MDP (Table 6).

This result backs the hypothesis that HRP could provide a strategic advantage in managing portfolio volatility under some market conditions, as can be seen from its lower p-value of 0.045.

3.5. Comparison with Previous Studies

Despite our expectations, our research fails to show that Hierarchical Risk Parity (HRP) consistently offers higher risk-adjusted returns than traditional methods such as Mean-Variance Optimization (MVO) or Maximum Diversification Portfolio (MDP) in investing in the Taiwan stock market. But in instances of high volatility HRP does tend to produce lower volatility, suggesting that it might also be consistent with superior risk management.

This comparative elaboration is in accordance with the recent Millea and Edalat (2022). scholarly work on the topic that emphasizes the robust nature of HRP. While the findings do not entirely prove the uniformly superior risk-adjusted returns of HRP across all measurements, benchmarks, and scenarios, they do validate the robust nature of HRP in terms of managing volatility in highly volatile market conditions. This validation is consistent with the observation of these scholars and is further consistent with the general trend in portfolio management to rely on more granular and structurally aware methodologies.

Building on the theory of López de Prado (2016) that HRP is a strategy that utilizes the hierarchical structure of correlations among assets, our analysis recognizes some drawbacks to this approach. As López de Prado points out, traditional portfolio methods often wrongly neglect the complex interconnectedness that exists among assets. This can lead to imperfect risk management, especially in turbulent markets. The empirical findings of our research confirm that a better assessment and management of correlation patterns (which HRP makes possible) can make portfolios more robust.

And then there is Roncalli, so reminding us that (2014), in seeking to avoid confusing quantity with this need or that little separate part, really has nothing to do with correlation or covariance--it is more than relevant today for today's investment world. He contends that new and more complex models, taking these conditions into full account, are basic for identifying hidden crises, particularly at times of economic conflagration. We tested this idea, using the method of applying HRP and found that attention to structure in markets can reduce risks more effectively.

The performance of HRP in the Taiwan stock market is successfully tested in this study, the results show. The asset correlations and hierarchical structure contribute to portfolio resilience, a necessity in emerging markets where volatility is greater as supported by our findings.

Not only does this paper thus become situated in the context of broader academic discussions and literature as prior studies have been, but it also helps to elucidate an understanding of portfolio management strategies for those residing in extremely volatile or turbulent markets. Though our findings only partially support HRP's far superior efficacy, they do reinforce the importance of incorporating advanced, structurally-aware strategies into portfolio management. This is particularly true in environments where traditional models fail.

3.6. Practical Implications

The findings of this paper from the study of hierarchical risk parity (HRP) offer insights valuable for portfolio management, particularly in the context of emerging markets or very fluctuating markets. For all that HRP does not constitute a strictly better approach than traditional ones according to riskier adjusted returns, its major improvement in terms of volatility makes it a potential for utility in complex market environments.

With an emphasis on various hierarchical asset correlation structures, HRP attracts portfolio managers in chaotic markets. However, unlike traditional methods like Mean-Variance

Optimization (MVO) and Maximum Diversification Portfolio (MDP), when the market deteriorates, HRP can quickly change its posture and rebalance asset weights according to shifts in correlation structure. This might make it possible to reduce the severity of drawdowns while also achieving greater long-term compound growth.

Empirical studies show that traditional diversification strategies often perform below par when the market is under stress because they rely on static correlation assumptions. Based on real-time data, HRP adjusts its structure accordingly, offering a potentially more robust solution that might even reduce big losses in times of market downturns (Clarke et al., 2006).

Adopted strategically by individual investors, HRP could bring a more nuanced understanding of risk management, causing a rethinking of one's risk tolerance and investment strategy. Meanwhile, it not only serves to link investment portfolios and one's personal financial goals but also achieves actual results in investment education on even such complex topics as asset correlation and portfolio volatility.

Knowing about these advanced techniques makes it possible for investors to take on more active positions regarding their portfolios. They might look for or demand such investment products: those structured with sophisticated risk management tools like HRP.

Incorporating HRP into financial planning, particularly for retirement planning or wealth preservation, could substantially benefit a client who sought stability in investment returns. It is especially relevant that such a move might lead to lower volatility and dramatic reductions in times of loss. This is incredibly important for people close to retirement or who have a relatively low tolerance for risk, and can help provide greater financial security against shocks from the markets.

To continue adoption in a broader sense, HRP may influence regulatory standards and industry practices. That, in turn, might energize financial organizations to build more robust risk management frameworks that enfold complicated models like HRP. Likewise, supervisors may need to reconsider gymnastic criteria by adding portfolio management tools that cope with asset hierarchies and correlations. This could help achieve a more stable financial system.

Integrating Hierarchical Risk Parity improves not only portfolio resilience but also potentially enhances risk-adjusted returns for institutional and individual investors, particularly during challenging market conditions. This reflects the larger industry trend towards more sophisticated, data-driven investment strategies that not only ensure capital preservation but also optimize performance in an increasingly interconnected and volatile global financial landscape.

3.7. Limitations

In conclusion, the findings attained in the present study can be highly illuminating to the best demonstration of portfolio management strategies, especially HRP. Nevertheless, all the results highly originate from Taiwan's stock market. Therefore, the regional limitation of findings results in an outcome that may not be entirely diffuse to another market due to different economic, regulatory, market reputation, and provision of local operators. All things considered, the results in Taiwan are not suitable to apply in deviation markets such as developed countries or calm and stable markets. Therefore, the results can be generalized to the global markets with a difference from Taiwan's markets without first localizing the analysis.

The study was also conducted retrospectively, using historical market data to judge the effect of portfolio strategies. It is quite a common method in financial research to use historical simulation to evaluate strategy performance, but this method fails to encapsulate real-time market dynamics. As Lo (2002) pointed out when he criticized difficulties in backtesting (Lo 2002) retrospective analyses can sometimes give a misleading impression of a strategy's effectiveness because of model overfitting and lack of ability to include future market conditions. Or unexpected events (Lo, 2002).

In practice, the Hierarchical Risk Parity (HRP) method, which focuses on hierarchical clustering to better understand the pattern of asset correlations, has clearly increased returns while taking less risk than traditional strategies primarily based on historical variances and returns. This has been demonstrated by many studies of this era (Millea & Edalat, 2023; Raffinot, 2018). However, our analysis does not factor in transaction costs—such as trading fees, spreads, and potential slippage—which could offset the profits from the strategy (Donohue & Yip, 2003). When a strategy like HRP is implemented, these costs become particularly relevant. Regular portfolio rebalancing according to the hierarchical structure of correlations in daily trading prices of stocks or funds can cause even small costs to grow significantly. Thus, omitting these costs can lead to an overstatement of the strategy's profitability. Accounting for these costs in future research would provide a better basis for assessing the net performance of the investment strategies created (Donohue & Yip, 2003).

These limitations emphasize the importance of cautiously planning a successful application from this study to other situations or its integration into business investment methodology. It also emphasizes the need for ongoing research, which would consider the implementation of HRP in real time, include transaction costs, and extend the geographic range of analysis to other markets in order to obtain wide support for understanding both the uses and limitations of this advanced portfolio management technique.

3.8. Suggestions for Future Research

In the future, longitudinal studies can be used to show whether hierarchical risk parity (HRP) is adequate under named ticking markets. By presenting information so that it does not change anything other than making things more straightforward for traders or investors with live examples and further tests, these studies could confirm findings with trading in real-time and test this adaptability. One way to do so is by using correlation matrices that provide close insights into its performance in different markets. A similar performance would suggest that the use of correlation matrices is a critical factor in asset allocation (López de Prado, 2016). Dialog Content trends are not merely a matter of preference. Connections between the two kinds of movements can be found.

By incorporating real-time economic indicators into the HRP framework, its forecasting ability and capacity to respond quickly to macroeconomic changes can be greatly increased. Consequently, researchers could refine HRP models with new indicators such as inflation rates, GDP growth figures, and employment statistics to better predict and respond in a market cycle. Statman et al. (2010) highlighted how making better use of older information affects asset prices. This could enable the development of more sophisticated and forward-looking active strategies for portfolio management. Assuming (based on the data of period ends and rather inappropriate information), the results could provide valuable insights into asset allocation strategies.

In order to make the findings more robust and more widely applicable, subsequent research should collect input data over longer timeframes and cover a greater range of market conditions. This event would give further information on how HRP performs across different sectors of the economy or even stratifications by national territory and external circumstances such as geopolitical rows and financial crises. Prospective surveys of this nature can highlight the sustainability and stability of the portfolio management strategy (López de Prado, 2018; Raffinot, 2017).

Future studies should equally consider the impact of real-time trading costs, tax considerations, and any other practical constraints on HRP's performance. Integrating these factors into the testing framework would produce a more realistic assessment of whether such a strategy can survive in practice or not. It would then give finance managers some guidance on what is most practical.

Throughout its scoring, the investors could customize HRP formulators in an industry-specific manner. This might boost applicability strategy-wise by getting specific to particular industrial contexts. This further research may imply that HRP parameters can vary according to sector risk and opportunities, and this makes it easier to achieve the goal.

An indication for future research, these directions extoll the virtue of refining and expanding the application of HRP in portfolio management.

CONCLUSION

The results of this study show that Hierarchical Risk Parity (HRP) can increase performance-based investment portfolios by accepting risk--knowing that assets differ significantly in both correlation and hierarchy.

Although these initial results are encouraging, they also point to the necessity of follow-up research and extensive data analysis to make any firm statements.

Due to its layered and unique approach to portfolio building, which considers relative value between assets, it seems likely that HRP easily beats traditional strategies like Mean-Variance Optimization (MVO) or Maximum Diversification Portfolio (MDP). One final observation: the superiority of HRP in risk-adjusted returns was not uniformly maintained in every market setting scrutinized by this study, particularly with respect to the unstable Taiwan market.

Given the intricacy and fluidity of emerging markets, a strong analytic is crucial. In future research, scholars might consider integrating a more diverse group of data points with more extended time frames into one study, which will help measure whether HRP stands up through all kinds of market cycles. That will help us determine how HRP might be best configured to work under different economic stresses (López de Prado, 2016).

As our study of the HRP strategy advances, exploring the practical implications of putting this method on a larger scale is urgent. This wider view involves issues such as operations and related inherently complex concepts, as well as growth perspectives for larger pools of funds under management. It also means new regulatory environments must be created to enable further advances in portfolio management technology and methods (Roncalli, 2014).

Ultimately, the highly current research will help verify the first findings regarding HRP's efficacy and maybe even recommend it worldwide for unstable and complex investment environments.

Such developments could critically contribute to advancing the theory and practice of portfolio management, particularly by introducing phrases that reference advanced mathematical models and machine learning to everyday investment decision-makers.

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APPENDICES

Appendix 1. Selected stocks List:

Stock code	Company name
1216	UNI-PRESIDENT ENTERPRISES CORP. (UNI-PRESIDENT)
1301	Formosa Plastics Corporation (FPC)
1303	NAN YA PLASTICS CORPORATION (NPC)
1309	AITA CHEMICAL COMPANY, LIMITED (TTC)
2002	China Steel Corporation (CSC)
2008	KAO HSING CHANG IRON & STEEL CORPORATION (KHC)
2015	FENG HSIN STEEL CO.,LTD (FH)
2303	UNITED MICROELECTRONICS CORP. (UMC)
2308	DELTA ELECTRONICS, INC. (DELTA)
2317	HON HAI PRECISION IND. CO., LTD. (HON HAI)
2330	Taiwan Semiconductor Manufacturing Co., Ltd. (TSMC)
2382	QUANTA COMPUTER INC. (QCI)
2412	Chunghwa Telecom Co., Ltd (CHT)
2454	MediaTek Inc. (MTK)
2474	CATCHER TECHNOLOGY CO., LTD. (CATCHER)
2476	G-SHANK ENTERPRISE CO.,LTD. (GS)
2477	MEILOON INDUSTRIAL CO., LTD. (MEILOON)
2603	EVERGREEN MARINE CORP. (TAIWAN) LTD. (EMC)
2723	Gourmet Master Co. Ltd. (Gourmet)
2881	Fubon Financial Holding Co., Ltd. (Fubon Financial)
2882	CATHAY FINANCIAL HOLDING CO., LTD. (CATHAY HOLDINGS)
2884	E.SUN FINANCIAL HOLDING COMPANY,LTD. (E.S.F.H)
2885	Yuanta Financial Holding Co., Ltd (Yuanta Group)
2886	Mega Financial Holding Company Ltd. (MEGA FHC)
2887	Taishin Financial Holding Co., Ltd. (TaishinHoldings)
2891	CTBC FINANCIAL HOLDING CO., LTD. (CTBC HOLDING)
2912	PRESIDENT CHAIN STORE CORPORATION (PCSC)
3711	ASE Technology Holding Co., Ltd. (ASEH)
6505	Formosa Petrochemical Corp (FPCC)
6669	TAIWAN TAKISAWA TECHNOLOGY CO., LTD. (TTT)

Appendix 2. Statistic of selected stock's adjusted close (Adj Close) price data:

Stock	Count	Mean	Std Dev	Min	25%	50%	75%	Max
Adj Close (2303)	4173	15.9371	14.4192	3.6116	8.0573	9.0013	13.0844	61.2995
Adj Close (2008)	4173	10.7272	4.4863	3.7766	7.4267	10.1229	12.2156	27.2053
Adj Close (2308)	4173	128.7791	77.8828	29.4402	65.5197	116.2891	141.9976	372.5
Adj Close (1301)	4173	58.0347	20.748	19.1656	44.0611	51.5669	78.4101	104.023
Adj Close (2330)	4173	187.7904	180.8512	21.937	47.2615	107.767	237.989	651.4185
Adj Close (2474)	4173	136.3853	61.9975	23.4279	87.3329	145.8775	175.8117	271.7679
Adj Close (2477)	4173	14.4705	8.0172	3.4837	8.0074	11.1431	20.165	42.0201
Adj Close (2015)	4173	34.1273	16.1894	8.3594	22.7082	26.5891	42.1557	83.546
Adj Close (3711)	4173	48.2918	26.1433	7.8425	26.8924	48.0353	57.7179	135.5
Adj Close (2476)	4173	19.9173	13.514	5.3007	11.0738	16.6986	19.5227	75.9928
Adj Close (2891)	4173	12.8757	5.6489	2.8101	8.1128	11.5406	16.6764	28.45
Adj Close (2002)	4173	19.4804	4.8175	9.8864	16.9652	18.3312	19.9761	38.8485
Adj Close (1303)	4173	46.2582	15.5793	15.6275	34.8424	42.3489	58.5638	81.5438
Adj Close (2887)	4173	7.9712	3.9399	1.3331	4.9364	6.8721	10.0503	18.4
Adj Close (2912)	4173	163.4796	82.8243	35.7137	86.8384	164.4013	249.3205	309.5112

Adj Close (2884)	4173	11.1268	7.4629	1.6852	4.9185	8.4364	18.2401	30.6379
Adj Close (2723)	3209	147.6222	51.6842	51.0788	111	135.2348	165.5095	332.7128
Adj Close (2412)	4173	73.0757	24.694	31.0306	53.797	67.8609	93.3369	123.0465
Adj Close (2317)	4173	67.6889	21.1695	19.12	52.3908	64.154	82.5693	117.3353
Adj Close (2382)	4173	44.8445	34.6641	12.0282	28.0955	37.3192	46.4324	263.8796
Adj Close (1216)	4173	39.418	19.7447	9.6223	19.6686	36.7164	60.5891	75.5425
Adj Close (2454)	4173	303.6685	203.2609	86.0965	171.3567	221.7828	303.3413	1013.1542
Adj Close (2881)	4173	27.9758	14.9426	6.7157	15.8426	26.7681	32.5178	68.0258
Adj Close (2882)	4173	31.6358	9.5766	11.399	24.427	31.88	36.641	60.698
Adj Close (6669)	1494	687.5636	408.7446	80.6607	325.1869	696.0943	870.8049	2115
Adj Close (1309)	4173	10.2591	7.5268	2.4883	6.1339	7.1744	9.5885	38.8954
Adj Close (6505)	4173	69.7698	18.755	34.2113	54.3637	64.604	86.4	129.9689
Adj Close (2603)	4173	33.7884	40.4674	10.2078	14.6446	17.191	22.1859	251.2691
Adj Close (2886)	4173	17.2995	8.4867	3.6079	10.7719	14.8632	23.642	40.3915
Adj Close (2885)	4173	11.8201	4.9481	5.1788	8.6175	9.6989	13.5009	27.6

Source: Statistic of selected Taiwan stock market daily price data from yahoo finance API (2007~2023)

Appendix 3. Link to the repository containing the code used for the analysis in this research:

https://github.com/louistung/Comparative_analysis_TWSE_HRP_and_other_portfolio_optimization_method

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