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MODELING VOLATILITY OF BALTIC STOCK MARKETS

Master thesis

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ABSTRACT

This study models and forecasts volatility of three Baltic stock market indexes (Riga, Tallinn, Vilnius) using GARCH, EGARCH and GJR volatility models. In order to specify, estimate and test the volatility models Box-Jenkins methodology is used. Forecasting performance of the models is evaluated with four forecast error measurements. Period from 2000 to 2012 is used for estimation of the models and period from 2013 to 2014 for testing out of sample. The aim of this study is to find the model for every stock market index that delivers the most accurate volatility forecast. The main results of this study is that GARCH(2,1) appeared to be the best performing model for volatility of stock market index of Riga, EGARCH(2,1) is the best model for volatility of stock market index of Tallinn and GARCH(2,2) works best for stock market index of Vilnius. It was also found that differences in error measurements are very small and largest differences in forecasts appear when large price shocks take place.

Keywords: Baltic stock market indexes, volatility forecasting, GARCH models, leverage effect, emerging markets.

INTRODUCTION

Typically, in financial markets, we are concerned with the variation of asset returns, which is defined as volatility in financial theory. As a proxy for market risk volatility has become one of the key variables in modern finance theory. It is used in such areas like derivatives pricing as a part of Black-Sholes model of valuation of option prices. In risk management area it is used in Value-at-Risk model to quantify the level of potential losses over a specific period of time. Worldwide adoption of Basel Accord lead to widespread application of this model in financial institutions around the globe. Modern portfolio theory assumes volatility is a measure of risk of particular security or portfolio. Portfolio managers have certain risk level over investment holding period, which is described by volatility. Investor may want to reduce his exposure to certain asset if volatility is going to rise, on the other hand option trader may employ volatility forecasts implementing trading strategy. Financial assets volatility appears in many cases to be time-varying, therefore in many cases previous period observation cannot be used as reliable source of information about volatility of the next period thus there is a need for forecasting method to estimate the risk accurately.

The larger number of studies has been devoted to volatility modelling during last decades and many different approaches have been proposed. Perhaps the easiest way to forecast volatility is simply calculation of standard deviation or dispersion over some historical period of time. Obviously, this method will deliver poor forecast of next period volatility, but it can be used to find a long term benchmark level. As an extension of this method, exponentially weighted moving average (EWMA) model has been developed. The model has become a part of popular risk management software, so it is used widely. Another way of making volatility forecast is to derive it from option prices, or to find so called implied volatility (IV). The method is also widely used in practice as it provides anticipation of market participants about future volatility, thereby it reflects collective opinion about future volatility. The evidence of successful application of IV has been reported by many studies. Although, a lot of different model exists, the most widespread volatility model is autoregressive conditionally heteroscedastic (ARCH) model, and in particular generalized autoregressive conditionally heteroscedastic (GARCH). This type of models allows user to capture such features of volatility like mean-reversion, clustering and asymmetric response to price shocks, so it has become very useful in many financial applications.

Since the introduction of ARCH model in 1982, the model turn to be so popular that whole family of ARCH-type models has been developed to capture different properties of financial series. It is well documented that many stock markets demonstrate asymmetric volatility, so there is a trend in empirical literature to use asymmetric-type models. Yet some emerging markets studies show that asymmetry is not always the case for those markets. Since Baltic States stock markets refer to emerging markets, the choice of volatility model turn to be a hard task. More recent studies showed that there is no asymmetry in volatility of Tallinn stock exchange index returns for the period from 2004 to 2008. Analysis of literature showed that volatility the problem of volatility modeling in this region has not been studied widely so far. This circumstance inspired author to study Baltic stock markets volatility.

The thesis is important because it provides comparison of performance of different ARCH-type volatility models on Baltic stock markets, thus helps to choose the appropriate model for forecasting the risk associated with investments in this market. This study results can be particularly useful for institutional risk management purposes where volatility forecast can be employed in Value-at-Risk model. Most of developed and some emerging stock markets are accompanied by well-functioning derivatives market, where the volatility expectation can be observed using by founding IV. The absence of derivative market in Baltic States limits variety of forecasting technics, making volatility forecasting issue more complex. This argument emphasizes contribution of this study to practice.

The aim of current study is to analyze volatility of daily returns on stock market indexes of Riga (OMXR), Tallinn (OMXT), and Vilnius (OMXV) for the period from 4.01.2000 to 30.12.2014, with three ARCH-type models: GARCH, EGARCH and GJR. Analyzed period covers almost whole time of existence of those markets. The time series is sufficiently long and consist of 3801 observations. The period from 4.01.2000 to 28.12.2012 is used for estimation of the models and period from 03.01.2013 to 30.12.2014 for testing out-of-sample. The data was obtained from the home page of Nasdaq Baltic stock exchanges. Author used statistical package EVievs 8 to make all necessary test in order to specify and estimate models, and evaluate forecasts delivered by those models.

The study is divided into five chapters. The first chapter attended to problem of volatility measuring and main properties of volatility. This chapter discusses various approaches employed to measure volatility of financial assets and different properties of volatility that has been confirmed by many studies. The second chapter provides overview of various volatility models, which have been analyzed in academic literature. The third chapter describes methodology applied in order to specify and estimate volatility models, and to evaluate forecasting performance. The fourth chapter describes data series used in this study and the fifth comments results of empirical part of current study.

The author would like to thanks his supervisor professor Karsten Staehr for valuable advices and recommendations.

1. VOLATILITY MEASURE AND PROPERTIES

Statistically, volatility is often measured by the sample variance or standard deviation as it shown in equation 1.1. Since the variance is simply the square of standard deviation, it makes no difference which measure to use when comparing volatility of two assets (Poon 2005, 1).

$$\sigma = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(r_t - \mu)^2} \tag{1.1}$$

where

σ - standard deviation,
r_t - return on time t,
μ - sample mean,
T - number of observations.

Standard deviation σ in equation 1.1 is unconditional measure of volatility, which imply that it is assumed to be constant over period T. Since volatility appears to be varying in time, the conditional standard deviation that changes during period T is more relevant reflection of risk (Poon 2005, 10).

It was noted by Figlewski (1997) that the statistical properties of the sample mean make it very inaccurate estimate of the true mean especially for small samples. Taking deviations around zero instead of the sample mean as in equation 1.1 typically increase volatility forecast accuracy. For this reason different variants of volatility measure emerged.

1.1. Various measures of volatility

There are various ways to measure the volatility, which caused by aspiration to receive more predictable time-series within the framework of particular model. Volatility estimation procedure depends on how much information is available at each sub-interval and the length of volatility reference period. Depending on availability of data, volatility can be measured as absolute or squared return, it can be also measured using daily highest and lowest prices in so called high-low method or using intraday returns. Before high-frequency data became widely available, many researchers used daily squared returns and absolute returns, calculated from market daily closing prices, to measure volatility. Due to rapid development of information technology industry in recent decade, high frequency data became easily obtainable, and thus bring more accurate measure of volatility.

In the times when first volatility studies came out only closing prices were obtainable, even nowadays many macroeconomic series are available only at the monthly interval, so the practice is to use absolute monthly value to proxy for macro volatility. This presumes taking absolute value of return to measure volatility (Poon 2005, 11).

Another way of volatility measurement is proxy squared returns to volatility estimator. Producing a series of daily squared returns trivially involves taking a column of observed returns and squaring each observation. The squared return at each point in time, then becomes the daily volatility estimate (Brooks 2008, 386).

Instead of absolute or squared returns a range estimator can be applied to proxy volatility. This method is also known as high-low or extreme-value based method. It is very convenient because nowadays high, low, opening and closing prices are easily obtainable. A range estimator can be calculated using the log of the ratio of the highest observed and the lowest observed price for trading day, which then becomes volatility estimate for the day. This approach is expressed in equation 1.2 (Brooks 2008, 386):

$$\sigma_t^2 = log\ \left(\frac{high_t}{low_t}\right) \tag{1.2}$$

where

σ_t^2 - conditional variance,
$high_t$ - day t highest price,
low_t - day t lowest price.

There is also another way to proxy volatility using daily extremes. Applying the Parkinson (1980) high-low measure to a price process that follows a geometric Brownian motion results in the following volatility estimator exhibited in equation 1.3 (Bollen et al. 2002):

$$\sigma_t^2 = \frac{\left(ln\, H_t - lnL_t\right)}{4ln2} \tag{1.3}$$

where

H_t - highest price on day t,

L_t - lowest price on day t.

The Garman and Klass (1980) estimator is an extension of Parkinson (1980), where information about opening and closing prices in incorporated in volatility estimation expressed in equation 1.4:

$$\sigma_t^2 = 0.5(ln\frac{H_t}{L_t})^2 - 0.39(ln\frac{p_t}{p_{t-1}})^2 \tag{1.4}$$

where

p_t - price of asset in time t.

As the high-low volatility measure is very sensitive to outliers, it will be useful to remove them from the series. The removal of outliers does not remove volatility persistence, described in the next chapter. In fact, that data trimming make long memory in volatility increase (Poon 2005, 13-18).

With the increase of availability of tick data in recent years, new approach called realized volatility came into use. Realized volatility involves calculating daily volatility with intraday squared returns. Estimating volatility with realized volatility involves calculations in equation 1.5 (McAleer et al. 2008):

$$RV_t = \sum_{j=0}^{T} r_{m,t}^2 \tag{1.5}$$

where

RV_t - realized volatility,

$r_{m,t}$ – intraday return of day t.

For a series, which has zero mean and does not contain jumps, realized volatility converges to the continuous time volatility (Poon 2005, 14).

1.2. Overview of various measures of volatility in literature

There is a number of ways to measure volatility, many different opinions exist about how volatility should be estimated. Ding, Granger and Engle (1993) suggested measuring volatility directly from absolute returns, Davidian and Carroll (1987) show absolute returns volatility specification is more robust against asymmetry and non-normality. However there is some empirical evidence that absolute returns based models produce better volatility forecast than models that are based on squared returns, the majority of volatility models are squared return models. One of the main arguments against using squared return is that it leads to low coefficient of determination (R^2) and undermine the inference on forecast accuracy (Poon 2005, 12).

Provided that there are no destabilizing large values, high-low volatility estimator is very efficient and, unlike the realized volatility, it is least affected by market microstructure (Poon 2005, 13).

Blair, Poon and Taylor (2001) reported an increase of R^2 by three to four times for the 1-day-ahead forecast when intraday 5-minutes squared returns instead of daily squared returns are used to proxy actual volatility. The main disadvantage of this approach is that, intraday data is mostly available only for period of recent decade, although there is a trend in academic literature to use realized volatility.

1.3. Stylized facts about volatility

Before turning to volatility models it worth paying attention to common properties of financial time-series. A number of so called stylized facts about the volatility of financial asset prices has been found and confirmed in the last decades.

1.3.1. Mean reversion

It is well documented fact that stock market prices are mean-reverting, first evidence of mean-reversion in stock market was documented by DeBondt and Thaler (1985), later by Fama and French (1988) and others. In terms of volatility, mean reversion implies that periods of high volatility will start to cease at some point thus turning to long-run average level, also when it approaches low levels it will then start to rise moving back to some historical average level.

1.3.2. Clustering

Financial time-series demonstrates clustering, the evidence was first reported by Mandelbrot (1963) and Fama (1965), who found that large price changes in both directions are often followed by large price changes and the same is valid for small price changes. Later this evidence was documented by other studies. Clustering effect can be seen from Figure 1, which displays daily returns on Dow Jones Industrial Index over period from 23 August 1988 to 22 August 2000. It can be seen that there are periods of relatively high and low volatility.

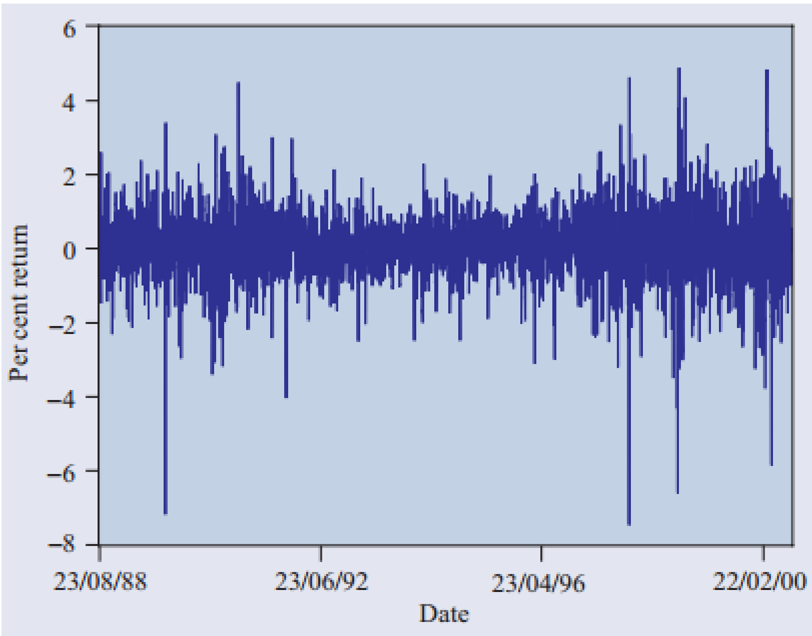

Figure 1. Returns on the Dow Jones Industrial Index, 23 August 1988 – 22 August 2000
Source: Engle and Patton, 2001, 241.

Clustering imply that volatility shocks today will influence the expectation of volatility many periods in the future. Volatility persistence can be measured as the time taken for volatility to move halfway back towards its unconditional mean following a deviation from it, which is described by equation 1.6 (Engle, Patton 2001, 239):

$$\tau = \frac{1}{2}\left|\sigma_t^2 - \sigma^2\right| \qquad (1.6)$$

where

τ - measure of volatility persistence,
σ^2 -unconditional variance.

It was shown by Engle and Patton (2001) that volatility persistence for Dow Jones Industrial Index time series mentioned above is 73 days. Existing of persistence in volatility series implies presence of autocorrelations. Unlike return series, the correlogram of squared return series indicates substantial dependence. Figure 2 reveals differences in autocorrelation functions of returns and squared returns.

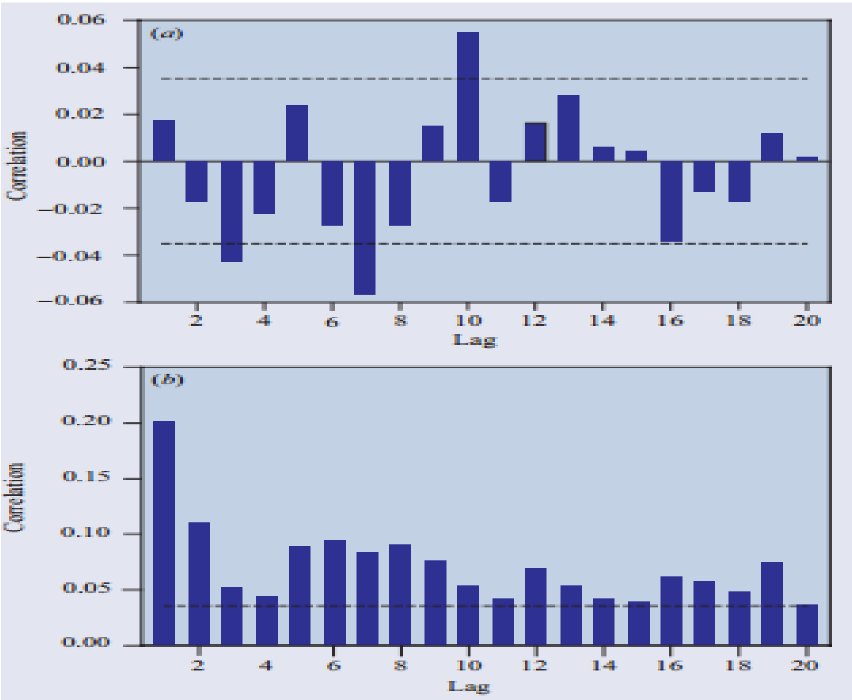

Figure 2. Correlograms of returns (*a*) and squared returns (*b*) of Dow Jones Industrial Index, 23 August 1988 – 22 August 2000

Source: Engle, Patton 2001, 241.

Although correlagram of returns rarely demonstrate correlation coefficients more than 0.04, correlation of squared returns turn to be higher on the whole scale of lags presented in figure 2, thus volatility is more predictable than returns. It must be pointed out that correlation is the highest at the first lag, meaning there is the greatest dependence on previous day.

1.3.3. Asymmetric response to price shocks

For stock market volatility it is likely to have greater response to negative shocks, than to positive, thus volatility is greater in bear market rather than in bull market. There is also a number of studies that provide evidence of asymmetry in stock market volatility. It was first documented by Black (1976) and later explained with leverage effect by Christie (1982). More recent studies proposed other explanations for asymmetry like positive feedback effect of volatility (Bekaert et al. 2000), short selling (Jayasuriya et al. 2005) and behavioral aspects (Hens et al. 2009). Latest studies show relation between volatility asymmetry and level of economic development of particular country, pointing out the behavior of non-professional investors as a reason of asymmetry (Talpsepp et al. 2009). Figure 3 is a good example of asymmetric relation between sign of return and volatility, it can be seen that in most cases at times of bear market volatility appears to be greater.

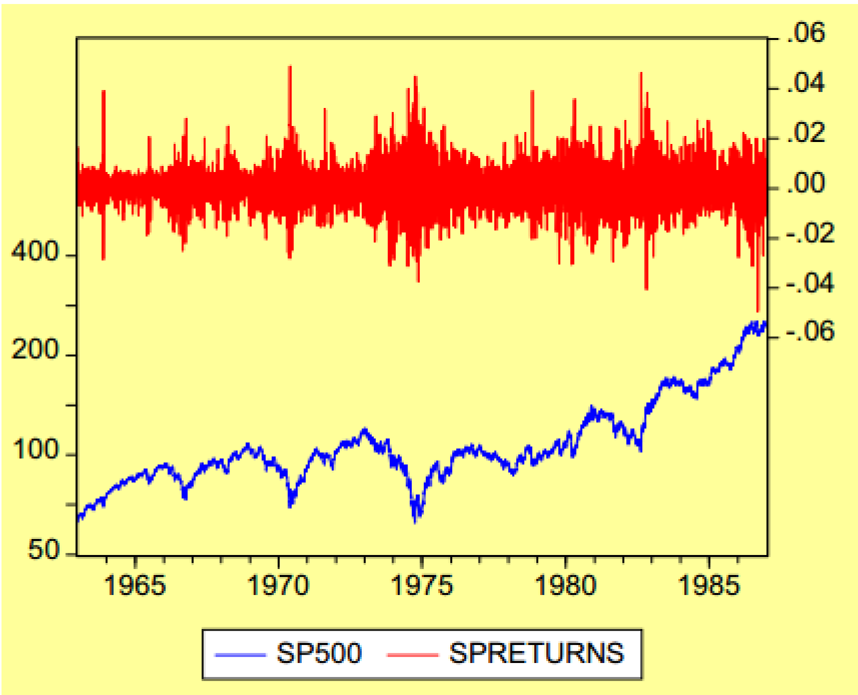

Figure 3. Standard and Poor's 500, daily, 1963 – 1987
Source: Engle 2004, 335.

Most of developed stock markets demonstrate asymmetric volatility. The evidence was documented by Ding, Granger and Engle (1993) in US stock market, by Bekaert and Wu (2000) in stock market of Japan, Jayasuria, Shambora and Rossiter (2005) in a set of developed markets. Brooks (2007) found a set of emerging markets where asymmetry was not found, moreover in some cases higher volatility was associated with positive shocks. Dzielinski, Rieger and Talpsepp (2010) found no asymmetry in OMXT using data for period 2004 – 2008, which seems to be in line with conclusion about the reasons of asymmetry (young market, almost non-existing analyst coverage of listed companies).

1.3.4. Departures from normal distribution

Another well-known fact is that distribution of asset returns has heavy tails. Every financial market experiences one or more daily price moves of four standard deviations or more each year. In any year, there is usually at least one market that has a daily move greater than 10 standard deviations (Jorion 2003, 361). Typical, kurtosis of financial asset series estimates range from 4 to 50 showing very extreme non-normality. (Engle, Patton 2001, 240)

To deliver precise forecast the model should capture the properties of financial series and be able to reflect them. Next chapter discuss various models for volatility forecasting.

2. VOLATILITY MODELS

As it was mentioned earlier, a number of different approaches to forecast volatility exists. The historical volatility (HIS) models will be considered first, after that IV and autoregressive models (AR) will be briefly discussed and in the end of the chapter the broadest class of volatility models – ARCH-type models will be described.

2.1. Historical volatility models

Compared with the other types of volatility models, the HIS models are the easiest to manipulate and construct. HIS may simply involve calculating the variance or standard deviation of returns over some historical period, and this then becomes the volatility forecast for all future periods (Brooks 2008, 383). In this case HIS can be easily found from equation 1.1 in the first chapter. However there are other types of HIS models.

All HIS models differ by the number of lag volatility terms included in the model and weights assigned to them. The simplest HIS model is the "random walk" model, where the difference between consecutive period volatility is modeled as random noise. The model can be expressed by equation 2.1:

$$\sigma_t = \sigma_{t-1} + \upsilon_t \tag{2.1}$$

where

υ_t - random noise.

Thus the best volatility forecast for the next period is the volatility of current period (Poon 2005, 32). In contrast with random walk, the historical average method makes a forecast based on the series of historical observations. The model is shown in equation 2.2:

$$\sigma_{t+1} = \frac{1}{t}(\sigma_t + \sigma_{t-1} + \ldots + \sigma_1) \tag{2.2}$$

Comparing with historical average, moving average discard older information. The model is shown in equation 2.3 (Poon 2005, 33):

$$\sigma_{t+1} = \frac{1}{\tau}(\sigma_t + \sigma_{t-1} + ... + \sigma_{t-\tau-1}) \tag{2.3}$$

where

τ - lag length to past information.

There is also another version of this method, which assign different weights to series of historical observations.

2.1.1. Exponentially weighted moving average

The EWMA is an extension of the historical average model, which allows more recent observations to have a stronger impact on the forecast of volatility than older data points. Under EWMA specification, the latest observation carries the largest weight, and weights associated with previous observations decline exponentially over time. The EWMA model is expressed in equation 2.4:

$$\sigma_t^2 = (1-\lambda)\sum_{j=0}^{\infty}\lambda^j\left(r_{t-j} - \bar{r}\right)^2 \tag{2.4}$$

where

$\bar{r}$ - mean return,
λ - relative weight or “decay factor”, $(0 < \lambda < 1)$.

The model has some clear advantages over simple historical models. Volatility is likely to be affected by recent events, which carry more weight, than events further in the past. As it was shown in previous section, volatility series is likely to have higher correlation on first lag, which means that assigning the largest weight to the last observation follows one of volatility properties. The influence of single given observation declines at an exponential rate as weights attached to recent events fall. In case of simple historical model, the influence of shock may be still included in the measurement sample, assigning equal weights to all observations in the sample, the forecast will remain at a high level even if the market is subsequently tranquil (Brooks 2008, 384-385).

On the other hand there are two limitations of EWMA model. First of them is that in case of finite sum of observable data, the weights from equation 2.4 will sum to less than one. In the case of small samples, this could make a large difference to the computed EWMA and thus a correction may be necessary. Second, many time-series models, like for example ARCH-type models, will have forecasts that tend towards the unconditional variance of the series as the prediction horizon increases. This is a good property for volatility forecasting models, since mean-reversion is one of the properties of volatility series. This implies that if the volatility is currently at high level relative to its historical average level, it will have tendency to fall back towards its average level, and the same is true about low level. EWMA does not have such feature (Brooks 2008, 386).

2.2. Implied volatility

As volatility is one of the variables that option price depend on, all option pricing models require volatility estimate over a period of lifetime of particular option contract. Volatility appears to be the only unobservable variable in Black-Sholes-Merton option pricing model (Black & Sholes 1973, Merton 1973), since strike price and time to maturity are determined in contract specification, current underlying asset price, risk-free rate and option price can be taken from market data. When all necessary data is obtained, it is possible to find forecast of volatility made by market participants. IV cannot be found directly from Black-Sholes-Merton, but using such numerical procedures like method of bisections or Newton- Raphson (Watsham, Parramore 2004) IV can be derived. (Brooks 2008, 384)

2.3. Autoregressive volatility models

AR volatility model is a simple example of the class of stochastic volatility specifications. The idea is that a time-series of observations on some volatility proxy are obtained. If the object of forecast is day volatility, various volatility measures discussed in the first chapter can be employed to construct time series of observations. When data series is obtained standard procedures of Box-Jenkins approach for estimating models, described in the third chapter, can then be applied. Given AR model can be then estimated using ordinary lest squares or maxim likelihood method (Brooks 2008, 386).

As an example of this specific type of models first order AR model can be considered. The model is expressed in equation 2.5:

$$\sigma_t^2 = \beta_0 + \sum_{j=1}^{p} \beta_j \sigma_{t-j} + \varepsilon_t \qquad \varepsilon_t \sim N(0,\sigma^2) \tag{2.5}$$

where

$\beta_j ; \beta_0$ - parameters,

ε_t - random error.

The main disadvantage of this model is that it assumes variance of errors to be constant, thus the data series is homoscedastic, which is not the case of the most of financial time series. As it was shown in the first chapter volatility appears to be varying over time, thus the model can deliver very inaccurate forecast of volatility (Ibid). This limitation has been overcome in ARCH model.

2.4. Autoregressive conditionally heteroskedastic models

As it was mentioned earlier, ARCH type models are in widespread use in finance. The model was first introduced by Engle (1982), where it was applied on variation of inflation rate of United Kingdom. Soon it was applied on financial markets by other researchers. Domowitz and Hakkio (1985) studied foreign exchange market, Engle, Lilien and Robins (1987) studied risk premium variation. The reason of popularity of ARCH model hides in the ability of a model to correspond to main properties of volatility of financial asset returns discussed in the beginning of chapter. Unlike HIS models, ARCH models do not use past volatility observations to forecast volatility, but formulate conditional variance from residuals of the model (Poon 2005, 36).

In contrast with classical linear regression model, ARCH model imply that the variance of errors is not constant. It is unlikely for financial time-series that the variance of the errors will be constant over time, therefore the model, which let the variance of the errors evolve, captures one of the main properties of financial series, which is heteroskedasticity. To understand the model, conditional variance of a random variable must be defined. Equation 2.6 shows that conditional variance of error can be denoted as series variance:

$$\sigma_t^2 = var(u_t | u_{t-1}, u_{t-2}, ...) = E[u_t^2 | u_{t-1}, u_{t-2}, ...] \qquad u_t \sim N(0, \sigma_t^2) \qquad (2.6)$$

where

u – random error or residual of the model.

Equation above states that conditional variance of zero mean normally distributed random error u_t is equal to the conditional expected value of the square u_t. Under ARCH model, autocorrelation in volatility is modeled by allowing the conditional variance σ_t^2 to depend on previous values of squared errors (Brooks 2008, 387). Equation 2.7 express general case of ARCH model:

$$\sigma_t^2 = a_0 + a_1 u_{t-1}^2 + a_2 u_{t-2}^2 + ... + a_q u_{t-q}^2 \qquad (2.7)$$

where

q – numebr of lags.

Condition mean equation in ARCH model is very flexible and can take many forms, in practice autoregressive moving average (ARMA) process is often used to model mean equation (Sauga 2013). Since variance cannot be negative by definition, conditional variance in equation above must be positive. The variables in the right side of equation are all squares of lagged errors, thus they cannot be negative. In order to ensure that these always result in positive conditional variance estimates, all of the coefficients in the conditional variance are required to be non-negative. If one or more coefficients takes negative value, then for a sufficiently large lagged squared innovation term attached to that coefficient, the fitted value from the model for the conditional variance could be negative. Therefore, all coefficients in the ARCH model must be non-negative (Brooks 2008, 388-389).

However ARCH model have some clear advantages comparing with other models, there is a number of difficulties associated with this model:

- How should be defined the number of lags of squared residuals? One of approaches could be likelihood test, however there is no best solution.
- The number of lags of the squared error that are required to capture the dependence in the conditional variance might be very large, which results in a large model.

- Non-negativity constraint might be violated. As the number of parameters in conditional variance equation increase, the likeliness that any of parameters will have negative value increases too (Brooks 2008, 391-392).

Those difficulties has been overcome in the extension of ARCH model or its generalized form.

2.4.1. GARCH model

The GARCH model was developed independently by Bollerslev (1986) and Taylor (1986) to generalize ARCH model. The model can be expressed in the form of equation 2.8 (Brooks 2008, 387):

$$\sigma_t^2 = a_0 + \sum_{i=1}^{q} a_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{2.8}$$

where

u_{t-i}^2 - unconditional variance of previous period,

σ_{t-j}^2 - conditional variance from previous period,

q - lag of unconditional variance,

p – lag of conditional variance.

It is possible to interpret the current fitted variance, as a function of a long-term average value (dependent on a_0), information about variance during the previous periods ($a_i u_{t-i}^2$) and the fitted variance from the model during the previous periods ($\beta_j \sigma_{t-j}^2$) (Brooks 2008, 394).

Since the time GARCH was introduced, a huge number of model extensions have been proposed. However GARCH captured some of main properties of financial time-series like volatility clustering, mean-reversion and leptokurtosis, the model still faced some difficulties. GARCH cannot deal with asymmetric response of volatility described in previous chapter, since it produces symmetric response of volatility disregarding the sign of shock. (Brooks 2008, 404) In next two sections two most popular asymmetric extensions of GARCH will be discussed.

2.4.2. EGARCH model

The model was proposed by Nelson (1991) to improve GARCH model response to negative shocks and illuminate non-negativity constraints discussed in the beginning of section. Examples of application of this model can be found in Pagan and Schwert (1990), where different GARCH-type models employed to forecast exchange rate volatility. Conditional variance of EGARCH can be expressed using equation 2.10 (Sauga 2013):

$$ln\left(\sigma_t^2\right) = a_0 + \sum_{j=1}^{q}\beta_j\, ln\left(\sigma_{t-j}^2\right) + \sum_{j=1}^{r}\gamma_j \frac{u_{t-j}}{\sqrt{\sigma_{t-j}^2}} + \sum_{j=1}^{p}a_j \frac{\left|u_{t-j}\right|}{\sigma_{t-j}} \tag{2.10}$$

where

γ - parameter of asymmetry.

In case of EGARCH, variance is modelled in logarithmic form, so even if all the parameters are negative, conditional variance will be positive, thus no non-negativity constraints needed. If modelled time-series indicates relation between negative sign of the return and volatility, the parameter γ will be negative, therefore allowing model to respond to leverage effect (Brooks 2008, 406). Since γ present volatility asymmetry in this model, it is possible to test data for asymmetry using this parameter. In this test null hypothesis means that γ = 0, and alternative hypothesis states that $\gamma \neq 0$ (Sauga 2013).

2.4.3. GJR model

Another GARCH-type models that takes into account asymmetric effect, was GJR model, introduced by Glosten et al. (1993). Evidence of application of GJR can be found in Franses and van Dijk (1996) or Brailsford and Faff (1996) where different GARCH-type models are compared in modelling stock market volatility. The conditional variance in this model can be given by equation 2.11 (Sauga 2013):

$$\sigma_t^2 = a_0 + \sum_{j=1}^{q}\beta_j\sigma_{t-j}^2 + \sum_{j=1}^{p}a_j u_{t-j}^2 + \sum_{j=1}^{r}\gamma_j u_{t-j}^2 I_{t-j}^{-} \tag{2.11}$$

where

I_t^- - dummy variable.

The dummy variable I_t^- is in charge for asymmetry in this model. When news cause negative shock in stock prices, u_t < 0 and I_t^- = 1, if there is a positive shock, u_t > 0 and I_t^- = 0. If negative shock causes increase in volatility, then γ > 0. Again, it is possible to test data for asymmetry with GJR. Null hypothesis states that γ = 0, and alternative hypothesis means that $\gamma \neq 0$. The non-negativity constraint remains, therefore, $a_0 > 0$, $a_j > 0$ and $\beta \geq 0$, but the model is still admissible, if and $a_j + \gamma \geq 0$. (Ibid)

2.5. Review of forecasting performance

Analysis of literature showed no consensus between researchers about what model performs best. Nevertheless, there is a huge body of volatility studies and major share of it stresses supremacy of GARCH-type model, it is easy to find supporters for every type of volatility models described above. It should be mentioned that volatility models comparison based on these studies is very difficult task, as analyzed models applied on time-series of various assets, various data frequencies and various error distribution functions are used. Some examples of these studies are listed below.

Unlike ARCH-type models, simpler methods like HIS and EWMA, do not separate volatility persistence from shocks and most of them do not incorporate mean reversion. These models tend to provide larger volatility forecast most of the time because there is no constraint on stationarity or convergence to the unconditional variance, and may result in larger forecast errors. (Poon 2005, 44) However such HIS models like random walk and historical average, seems to be too simple to provide reliable forecast, they work well for medium and long horizons. Models perform best with low frequency data, when longer than 6 month forecast horizons are considered. Although it is emphasized by many studies, that long period of data is required, to make good forecast. (Poon 2005, 35) HIS models have been shown to have a good forecasting performance comparing with other volatility models. Overview of 93 studies made by Poon and Granger (2003) showed there is more support to HIS models, than GARCH-type models, as 22 studies suggested HIS model is best performer and 17 suggested that GARCH performs best. However such distribution of opinions might be caused by the choice of author.

Bluhm and Yu (2000) compared different volatility models using German stock market returns, among which IV model, HIS model, EWMA, four GARCH-type models. They stated that it is difficult to decide, which method is better, however, they suggested to use IV for option pricing purposes only.

The GARCH-type models have more supporters than any other type of model. Akgiray (1989) found that GARCH model outperforms EWMA and random walk in stock market volatility forecasting, West and Cho (1995) found that GARCH provide more accurate forecast of exchange rate volatility then five others. In the recent decades the trend to study asset return volatility with GARCH models emerged in financial literature. This is caused by the ability of these models to copy financial series properties, especially the ability to reflect negative relation of volatility and shocks in stock market prices (Poon 2005, 43).

The evidence of superior performance of asymmetric GARCH-type models can be found in many studies. Pagan and Schwert (1990) compared GARCH, EGARCH, Markov switching regime and three non-parametric models for forecasting stock market volatility and found that EGARCH performed better than others. Mittnik and Paolella (2000) show that asymmetric GARCH outperformed regular GARCH model in modelling volatility of exchange rate of East Asian currencies against US dollar. Hansen and Lunde (2005) compared 330 ARCH family models in terms of their ability to describe conditional volatility of DM/USD exchange rate and IBM stock returns, they concluded that APARCH was the best model for stock and GARCH for exchange rate volatility. More recent study of Liu and Hung (2010) analyzed performance of distribution-type GARCH models and asymmetric GARCH models on time-series of S&P 100 returns, authors found that best performer was GJR followed by EGARCH. Yet, there is no consensus about what model should be applied in practice, more evidence of successful ARCH-type models has been found.

Next chapter discusses problem of specification, estimation and diagnostic of ARCH-type models.

3. METHODOLOGY

This chapter discusses methods for model building, estimating and evaluating forecasting performance. The first section is dedicated to model estimation technique, the second chapter covers complex solution for constructing and testing of the model, and the third discusses problem of forecast evaluation.

3.1. Maximum likelihood method

Before moving to Box-Jenkins methodology it is sensible to review method of estimation of ARCH-type models. ARCH-type models are not linear, thus ordinary least squares (OLS) cannot be employed. The main reason of why OLS cannot be used is that it minimizes the residual sum of squares (RSS), which is not the objective of GARCH modelling. In order to estimate GARCH-type models maximum likelihood method is used. Maximum likelihood method consist of three steps. On the first step model equations for mean and variance should be specified. As it was mentioned earlier, in order to specify mean equation for ARCH type models ARMA model can be applied. For example AR(1) can be used as a mean equation in GARCH (1,1) model, in this case GARCH model can be specified by equations 3.1 and 3.2 :

$$y_t = \mu + \varphi y_{t-1} + u_t \qquad u_t \sim N\left(0, \sigma_t^2\right) \tag{3.1}$$

where

ϕ - parameter of the model.

$$\sigma_t^2 = a_0 + a_1 u_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{3.2}$$

After equations are specified, the log-likelihood (LLF) function must be defined. Example of LLF for GARCH (1,1) is shown in equation 3.3:

$$L = \frac{T}{2} log(2\pi) \frac{1}{2} \sum_{t=1}^{T} log(\sigma_t^2) \frac{1}{2} \sum_{t=1}^{T} \frac{(y_t - \mu - \varphi y_{t-1})^2}{\sigma_t^2} \tag{3.3}$$

On the third step values of parameters of the function are sought to maximize the function (Brooks 2008, 395). When procedure of third step has been accomplished standard errors can be constructed to diagnostic checking of the model, which is a part of Box –Jenkins approach discussed above.

3.2. Box-Jenkins methodology

Initially developed for ARMA models Box-Jenkins methodology can be used in order to construct appropriate ARCH type model. Box and Jenkins (1976) were first to introduce complex solution for model estimation. The methodology can be divided into three steps: model identification, model estimation and diagnostic checking.

On the first step data dynamics is studied. This stage involves plotting the data to the graph, analyzing descriptive statistic, checking for stationarity and studying autocorrelation function to determine order of volatility model. When many specifications of the model need to be compared information criteria can be employed to find best specification (Brooks 2008, 230).

Second step involves estimation of the parameters of the model specified on step one. On this step statistical significance and signs of the parameters should be assessed. When appropriate specification has been found, it should be decided whether the model specified and estimated adequately (Brooks 2008, 231).

Third step involves analyzing the process generated by residuals of the model. If model managed to capture data dynamics of data, the residuals generate random stochastic process. In this sense residuals should be check for autocorrelation, heteroskedasticity and normal distribution (Ibid).

The tests implied by Box-Jenkins methodology are discussed further in this chapter.

3.2.1. Testing stationarity

Before specifying the model it is sensible to check data for stationarity. When time-series behave with trend it generates non-stationary process. One of the disadvantages of non-stationary series is the persistence of shocks. For stationary series, shocks will gradually die away, which means that if shock appears in time *t*, the impact of the shock will be smaller in time *t*+1 and even smaller in *t+2*. In case of non-stationary data, the influence of shock will be infinite and may lead to very inaccurate forecast (Brooks 2008, 319).

White noise is a pure example of stationary process, however it is not the only stationary process. White noise can be characterized by constant mean and variance, and zero autocovariance, thus each observation is uncorrelated with other values of series. Under stationarity is often thought weak stationarity, which unlike white noise assumes constant autocovariance (Sauga 2013).

Financial data series often represent trending process, which is non-stationary because its values are correlated. If one happened to deal with trending data, there are two different functions the trend can be described. One of them is deterministic trend, which can be described by equation 3.4:

$$y_t = a + \beta t + u_t \tag{3.4}$$

Deterministic trend process does not have constant mean, it exhibits random fluctuation around its trend. Figure 4 provide example of deterministic trend.

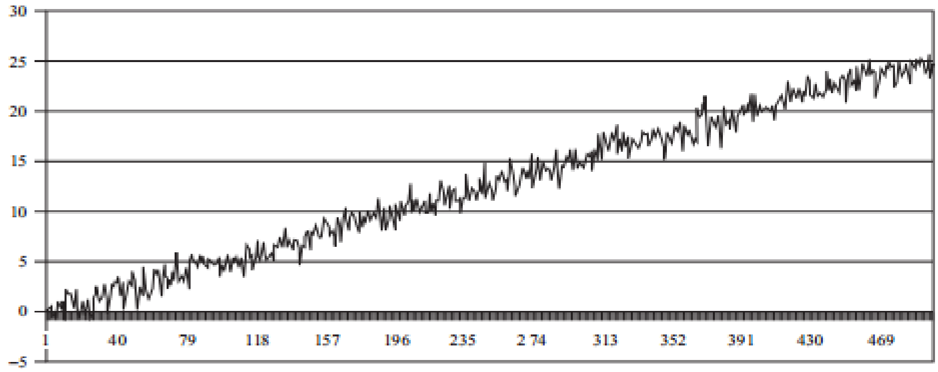

Figure 4. Time series plot of deterministic trend process
Source: Brooks 2008, 325.

The process is also called trend-stationary process, because it is stationary around linear trend (Brooks 2008, 322-323).

Another type of trending process is random walk model with drift or difference-stationary process, which can be described by equation 3.5 (Brooks 2008, 322):

$$y_t = \mu + y_{t-1} + u_t \tag{3.5}$$

In figure 5 random walk and random walk with drift are compared, positive drift in model described above lead series to up trending dynamic.

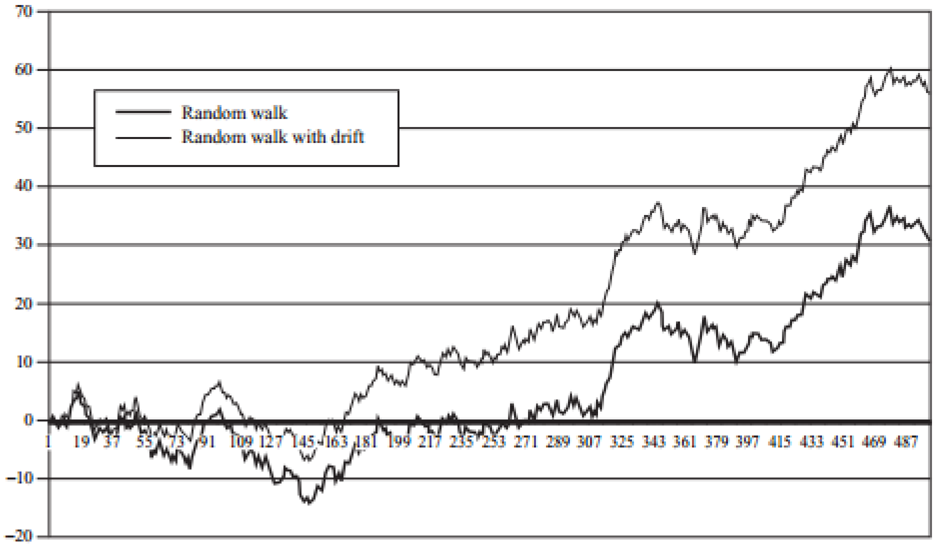

Figure 5. Time series plot of deterministic trend process
Source: Brooks 2008, 324.

The two models of trending behavior should be treated with different approaches in order to overcome non-stationarity. In first case, a model with linear trend should be estimated and residuals should be checked for stationarity. In second case, to achieve stationarity the data series should be differentiated (Sauga 2013).

One of possibilities to check data for stationarity is augement Dickey-Fuller test for unit root. In current study this test is used to test data for stationarity. Dickey-Fuller test can be expressed by equation 3.6:

$$y_t - y_{t-1} = (\rho - 1)y_{t-1} + u_t \qquad \delta = \rho - 1 \tag{3.6}$$

where

y_t - data point of time series,
ρ - parameter of the model.

If process is stationary δ <0, thus $|\rho| < 1$, otherwise non-stationary process, which is unit root process. According to Dickey-Fuller test, null hypothesis states that δ = 0, which means that process is not stationary. Alternative hypothesis in opposite states that δ <0, thus analyzed process is stationary (Sauga 2013).

In this study Dickey-Fuller test is used on the first step of Box-Jenkins methodology. When stationarity is confirmed, one can take further steps of analyzing data dynamic.

3.2.2. Testing normal distribution

When data is obtained it is reasonable to study its distribution. Distribution is usually defined by two measures: skewness and kurtosis. Skewness is a measure of symmetry in distribution of series around its mean value. Financial series usually has skewed distribution. Normal distribution in contrast, has zero coefficient of skewness as it is symmetric to its mean (Brooks 2008, 161). Figure 6 reveals difference between normal and skewed distribution.

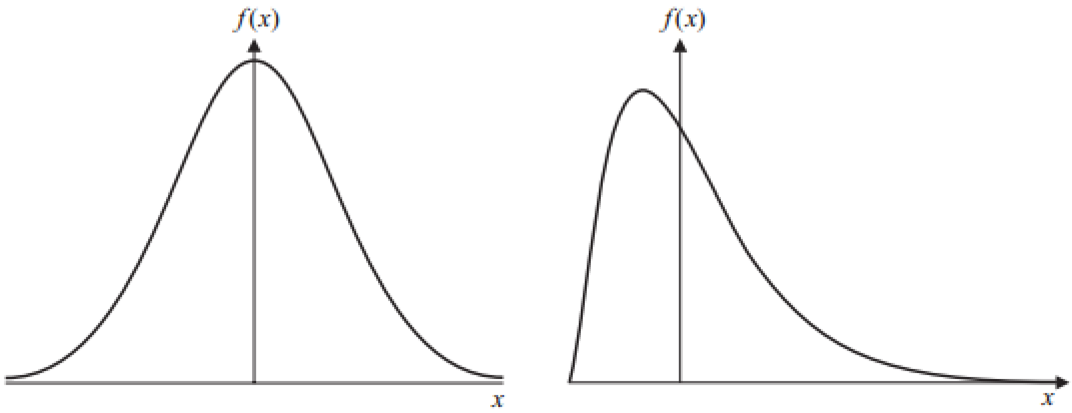

Figure 6. Normal (left side) and skewed distribution (right side)
Source: Brooks 2008, 162.

On the left side of the figure 6 values of series are distributed symmetrically around mean, in the left of figure 6 values are distributed asymmetrically as distribution is skewed to the right.

Kurtosis measures how fat the tails of distributions are. A leptokurtic distribution is the case, when distribution has fat tails and it is more peaked at the mean value than normal distribution. In most cases financial series is leptokurtic. Normal distribution has kurtosis of 3, which is known as mesokurtic distribution (Brooks 2008, 162). In Figure 7, provide example of difference between leptokurtic and mesokurtic distribution.

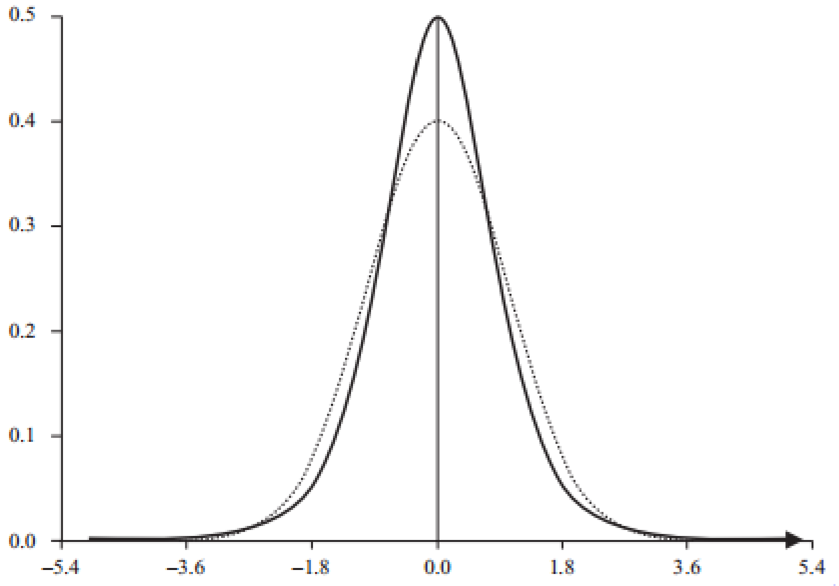

Figure 7. Mesokurtic (dashed line) and leptokurtic distribution (bold line)
Source: Brooks 2008, 162.

Figure 7 indicates that distribution marked with bold line (leptokurtic) is more peaked around mean comparing with distribution marked with dashed line (mesokurtic).

Bera and Jarque (1981) proposed test, which is based on idea that if series is normally distributed, it has coefficient of skewness and coefficient of excess kurtosis equal to zero. The Bera-Jarque test statistic is described by equation 3.7 (Brooks 2008, 163):

$$W = T \left[\frac{b_1^2}{6} + \frac{(b_2 - 3)^2}{24} \right] \tag{3.7}$$

where

T – is the size of sample,
b_1 - coefficient of skewness,
b_2 - coefficient of kurtosis.

b_1 and b_2 can be estimated using the residuals from the OLS regression. Under BJ test hypothesis are formulated as following: null hypothesis means series is normally distributed, and alternative hypothesis means series distribution is different from normal (Ibid). In current study author uses descriptive statistic to make inference about distribution of data and Bera-Jarque statistic to check residuals of models for normal distribution in the third step of Box-Jenkins methodology.

3.2.3. Testing autocorrelation

In order to find appropriate specification of model dependence between values of time series should be analyzed. Temporal dependence in the series is usually studied with correlogram, which displays coefficients of autocorrelation (AC) function and partial autocorrelation (PAC) function. Correlogram represents series autocorrelation with column graph allowing to estimate structure of autocorrelation visually. In order to estimate significance of AC and PAC Ljung-Box (1978) Q-statistic can be used. Calculation that stands behind this test can be described by equation 3.8:

$$Q = T(T+2)\sum_{k=1}^{m}\frac{\hat{\tau}_k^2}{T-k} \tag{3.8}$$

where
Q - Q-statistic,
T – sample size,
m – maximum lag length,
$\hat{\tau}_k$ - autocorrelation coefficient at lag k.

Under this method, joint hypothesis that all lags of correlation coefficients are simultaneously zero is tested and if any of autocorrelation coefficients is statistically significant, the hypothesis will be rejected. In this test null hypothesis states that all of coefficients of autocorrelation are zero, therefore alternative hypothesis should be accepted if any of coefficients is statistically significant (Brooks 2008, 209-210).

In current study this test is used on the first step of Box-Jenkins methodology, when model specifications are sought, and on the third step to check presence of autocorrelation in residuals of the models on the third step.

3.2.4. Determining model specification

Financial theory does not limit number of lags that should be specified in the model. Adding a large number of lags to the model can lead to insignificance of the parameters, while too small number of lags can be a reason of inappropriate specification, which cannot capture data process properly (Sauga 2013). In order to identify the appropriate form of the model correlorgam can be used, but this is subjective method, furthermore data may exhibit patterns, which makes it difficult to recognize process generated by data. This makes plots of autocorrelation function hard to interpret, and thus it is difficult to specify the model. Subjectivity associated with graphical analysis can be removed when information criteria is used (Brooks 2008, 232).

Information criteria uses two factors: a term, which is a function of the residual sum of squares (RSS), and some penalty for loss of degree of freedom from adding extra parameters. Therefore, adding new variable or additional lag to a model will have to competing effects on the information criteria: RSS will fall and value of penalty term will increase. The main idea of this method is to choose a number of parameters, which minimizes the value of information criteria, thus best model specified will have lowest value of information criteria. To receive lower value, the fall in RSS should be sufficient to more than outweigh the increase of the value of penalty term (Ibid).

There are several different criteria, the most popular are Akaike's (1974) information criterion (AIC), Schwartz's Bayesian (1978) information criterion (SBIC), and Hannan-Quinn criterion (HQIC). The adjusted R^2 can also be used as information criterion, but it typically selects only the largest models, for this reason it is not used here. Calculations of criterions listed above are expressed in equations 3.9 – 3.11 (Brooks 2008, 233):

$$AIC = ln\left(\hat{\sigma}^2\right) + \frac{2k}{T} \tag{3.9}$$

$$SBIC = ln\left(\hat{\sigma}^2\right) + \frac{k}{T} ln T \tag{3.10}$$

$$HQIC = ln\left(\hat{\sigma}^2\right) + \frac{2k}{T} ln\left(ln(T)\right) \tag{3.11}$$

where

$\hat{\sigma}^2$ - residual variance,
T - sample size,
k – number of parameters.

Since different criterion can deliver different result, and none of them is definitely superior, it was decided to use all three information criteria described above to avoid subjectivity on the first step of Box-Jenkins methodology. The model, which is suggested by more than one criterion will be chosen. In order to find best specification, combinations of volatility models with orders of p and q up to 2 are compared.

When specifications of volatility models are found and parameters of the models are estimated, residuals of the models should be tested for autocorrelation, heteroskedasticity and normal distribution. Since autocorrelation and normal distribution test were discussed in previous subsections, next subsection discusses heteroskedasticity test.

3.2.5. Testing heteroskedasticity

In order to test residuals for heteroskedasticity ARCH LM test can be used. If model fits data correctly then it should not present ARCH effect, thus residuals should be homoscedastic. The test implies estimation of regression model, which is described by equation 3.12:

$$u_t^2 = y_0 + \sum_{i=1}^{q} y_1 u_{t-1}^2 + \upsilon_t \tag{3.12}$$

Under null hypothesis all lags (q) of the squared residuals have coefficient values that are not significantly different from zero, thus there is residuals are homoskedastic. If the value of test statistic is greater than the critical value from the χ^2 distribution, then null hypothesis is rejected and alternative hypothesis should be accepted, meaning that residuals are heteroskedastic (Sauga 2013).

When all steps of Box-Jenkins methodology passed, ability of the models to forecast volatility of stock market indexes can be compared.

3.3. Forecast evaluation

The ability of a model to forecast future values of time-series usually involves comparing values forecasted by the model and actual values in some aggregated way. The measure of forecast error individually does not say a lot about model, but it allows researcher to compare different models, that were applied on the same time-series and forecast period. The model with lowest value of error measurement would be the most accurate (Brooks 2008, 252). There is no clearly best measurement of forecasting error, every function has its own advantages depending on feature of data series. Some of the most popular measurements of forecast accuracy will be given next.

Mean square error (MSE) is one of the common loss functions. MSE is defined by equation 3.13 (Brooks 2008, 253):

$$MSE = \frac{1}{T - (T_1 - 1)} \sum_{t=T_1}^{T} (y_{t+s} - f_{t,s})^2 \qquad (3.13)$$

where

T_1 - first out-of-sample forecast observation,
y_{t+s} - actual value,
$f_{t,s}$ - forecasted value.

MSE provides a quadratic loss function, thus it can be useful if large forecast errors are more disproportionately serious than smaller ones. However, in straight opposite situation, this characteristic can be thought as disadvantage. (Brooks 2008, 252) In practice squared root of MSE or RMSE is used more widely. It has the same units of measurement as the data it applied on and can be explained as average distance between actual and forecast value. (Sauga 2013) RMSE is defined by equation 3.14 (Brooks 2008, 253):

$$RMSE = \sqrt{\frac{1}{T - (T_1 - 1)} \sum_{t=T_1}^{T} (y_{t+s} - f_{t,s})^2} \qquad (3.14)$$

If series represent outliers, it may be better to use mean absolute error (MAE), which measure average absolute forecast error. It was mentioned by Dielman (1986) that if series present outliers, least absolute value should be used to determine model parameters. MAE is shown in equation 3.15 (Ibid):

$$MAE = \frac{1}{T - (T_1 - 1)} \sum_{t=T_1}^{T} \left| y_{t+s} - f_{t,s} \right| \tag{3.15}$$

Some authors, like for example Makridakis (1993), suggest to use mean absolute percentage error (MAPE), stating that it includes best characteristics among various accuracy criteria. MAPE is expressed in equation 3.16 (Ibid):

$$MAPE = \frac{100}{T - (T_1 - 1)} \sum_{t=T_1}^{T} \left| \frac{y_{t+s} - f_{t,s}}{y_{t+s}} \right| \tag{3.16}$$

Another popular criterion is Theil's U-statistic. This method of estimation implies comparing analyzed model with some benchmark model, like naive or random walk. Calculation of U-statistic is shown in equation 3.17:

$$U = \frac{\sqrt{\sum_{t=T_1}^{T} \left(\frac{y_{t+s} - f_{t,s}}{y_{t+s}} \right)^2}}{\sqrt{\sum_{t=T_1}^{T} \left(\frac{y_{t+s} - fb_{t,s}}{y_{t+s}} \right)^2}} \tag{3.17}$$

where

$fb_{t,s}$ - forecast obtained from benchmark model.

U-statistic greater than one means that analyzed model is worse than benchmark, in opposite case analyzed model is superior to the benchmark, and U-statistic of one indicates that the model and benchmark are equal in accuracy (Brooks 2008, 254).

Once the model has been estimated and checked for adequacy it can be employed to deliver forecasts. In order to generate forecasted values two types of forecasting can be used: dynamic and static. Dynamic forecast can be used to generate multi-step forecast and static is used one-step ahead forecast. Since dynamic forecast imply taking forecasted values to generate new ones, the forecasts quickly converge upon the long-term unconditional value of series. Static analysis, in contrast, roll the sample forwards one observation after each forecast in order to use actual values for lagged dependent variables (Brooks 2008, 256). In current study static forecasting is used, since it allows model to respond to shocks quickly and deliver more precise forecast. To evaluate forecasts delivered by models four error statistics are applied: RMSE, MAE, MAPE and Theil's U statistic.

Next two chapters review data and discuss results of the tests assumed by methodology applied in this study.

4. DATA

The daily closing prices of three Baltic stock market indexes were obtained from Nasdaq OMX web page. The time-series consist of 14 years of observations, which covers period from 04.01.2000 to 30.12.2014. In order to transform prices into returns log differences of prices were taken, as it shown in equation 4.1:

$$r_t = log \frac{p_t}{p_{t-1}} \tag{4.1}$$

Received values were multiplied by 100 to present index return in percentages of closing prices.

4.1. Analyzing data graphs

As it can be seen from figure 8, Baltic indexes grew rapidly for the period from 2000 to 2007. In the period of Global Financial Crisis markets fell sharply to the lowest historical level.

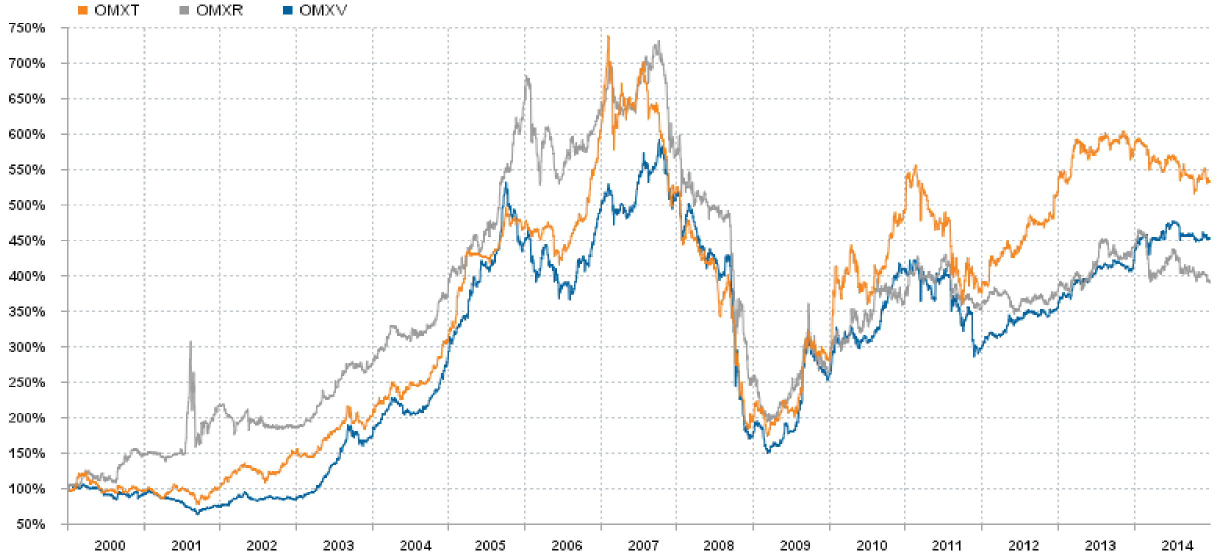

Figure 8. Baltic stock market indexes 04.01.2000 – 30.12.2014

Source: NASDAQ…

The period can be described by sharpest downturn of economic growth over the time of existence of Baltic stock markets. In 2009 markets approached turning point and recovered substantially during next five years. Volatility of Baltic stock market indexes can be represented graphically in figure 9, where daily returns of indexes are plotted on the graphs against time.

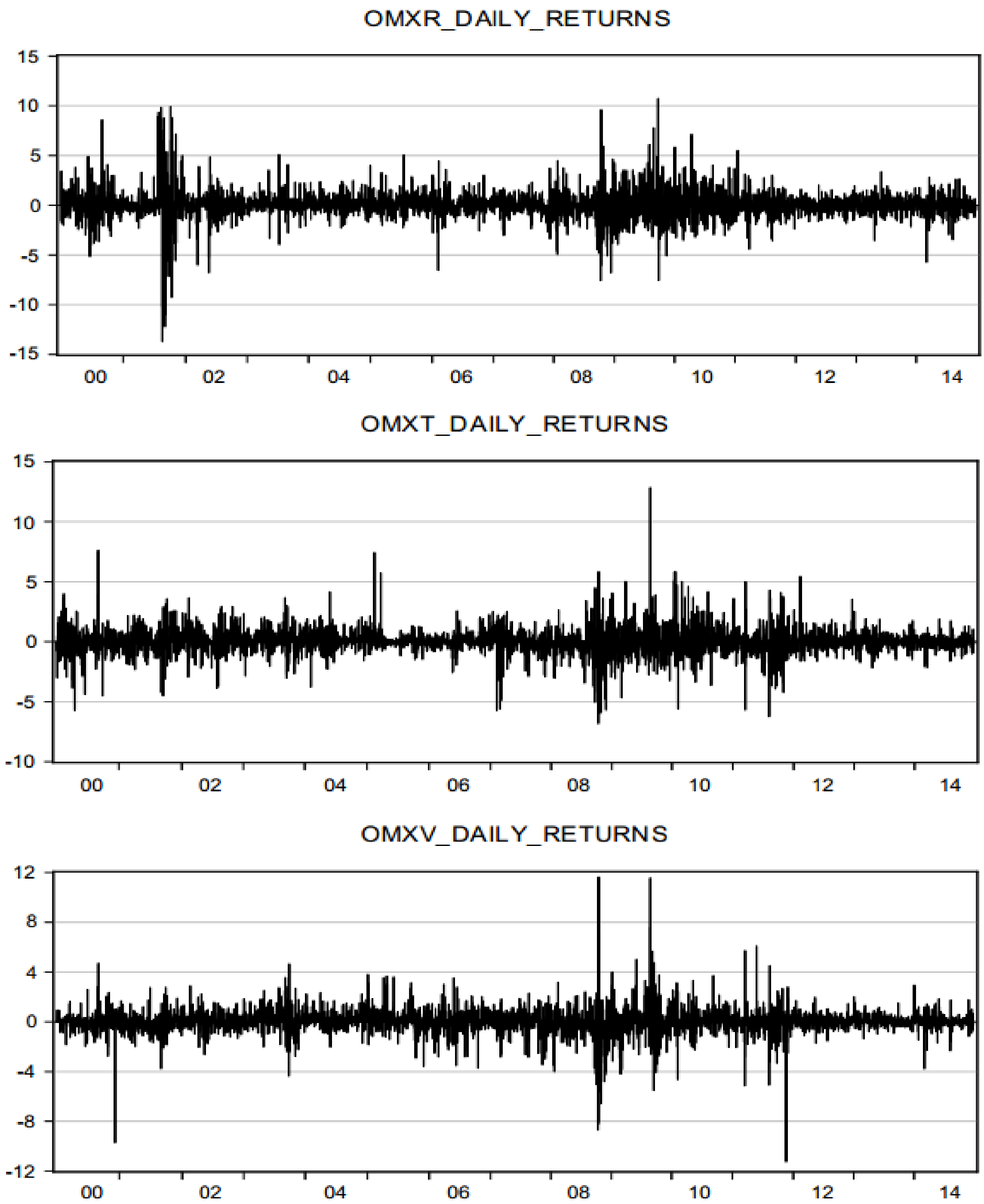

Figure 9. Returns of Baltic stock market indexes 04.01.2000 – 30.12.2014

Source: author’s figure from Eviews.

All graphs presented in figure 9 indicate high volatility in the second half of 2008 and 2009. Both periods can be described by sharp changes in the value of indexes, however as it was mentioned above those were changes in opposite directions, thus increase in volatility was caused by both negative and positive shocks.

However all Baltic markets experienced period of high volatility in 2008 and 2009, there are still substantial differences in volatilities whole span of time. OMXR and OMXT present more volatility clustering in 2010 when markets recovered than OMXV. It can be also seen that OMXR experienced high volatility in the middle of year 2001. This was caused by the action, during which government sold substantial amount of shares of Latvijas Gaze for the price three times higher than market price. (NASDAQ…) The event had a great impact to local stock market index and as a result of this action OMXV jumped up and bounces back, when the influence of shock eased. Nevertheless there are differences in previous periods, all markets remain relatively calm in terms of volatility during last three years.

4.2. Analyzing descriptive statistic

Table 1 shows that mean daily return for all Baltic indexes is very small and close to zero. Data present a bit different levels of standard deviations among markets, OMXR has largest and OMXT and OMXV have similar levels of standard deviation. Extremums of returns shown in table 1 indicate large fluctuations, largest negative return was observed in returns of OMXR, and largest positive in returns of OMXT. Minimum values of OMXR and OMXV can be described by approximately ten standard deviation of those seires, indicating higher risk comparing with OMXT.

Table 1. Descriptive statistic of daily data

Series	Min (%)	Max (%)	Mean (%)	SD (%)	Kurtosis	Skewness
OMXR	-13.68	10.72	0.05	1.46	17.34	-0.25
OMXT	-6.80	12.86	0.05	1.14	12.30	0.32
OMXV	-11.25	11.63	0.04	1.08	22.25	-0.17

Source: author’s table.

All indexes have excess kurtosis, which is much greater than three, meaning leptokurtic distribution, thus there are fat tails. Distributions of OMXR and OMXV are negatively skewed and OMXT has positive skewness, thus the tail on the left side of probability density functions of OMXR and OMXV is longer, and OMXT distribution has longer right tail. Therefore all of data series indicate substantial departures from normal distribution.

Graphs in figure 9 show, that none of data series generate deterministic trend process, since patterns in graphs look very different from the one shown in figure 4, thus model without trend should be used in unit root test for stationarity. The values of test statistic and probability values are given in table 2.

Table 2. Unit root test results

Series	Obs*R-square	P-value
OMXR	834.4479	0.00
OMXT	198.8276	0.00
OMXV	524.2070	0.00

Source: appendix 1.

P-value indicates very high level of significance for all data series, thus null hypothesis can be rejected and it can be stated that all data series analyzed in current study are stationary.

5. RESULTS

This chapter discuses results of procedures of specification, estimation and testing of volatility models, and also provides evidence of comparison of forecasting performance of those models. Since ARMA model is applied to specify mean equation for GARCH models, Box-Jenkins methodology is also used for ARMA modeling purposes. Chapter begins with discussion of results of estimation appropriate ARMA models and specification of GARCH models for every data series, next section discusses results of models diagnostics and last section reveals results of forecasting performance.

5.1. Estimating mean equation of volatility models

Returns of indexes analyzed in this study present different patterns of liner dependence. Correlograms in appendix 2 show, that the returns of OMXR have highest autocorrelation on the second lag, while returns of OMXT and OMXV have highest correlation on the first lag. In order to find best specification of ARMA model for conditional mean equation of volatility models, combinations of ARMA model with specification from AR(1) to ARMA(2,2) were compared with information criterions and checked for autocorrelation in model residuals. Models with lowest information criterion and less correlated residuals were chosen. Results presented in table 3.

Table 3. ARMA models specifications

Series	Order	AIC	SBIC	HQIC
OMXR	2,2	3,6687	3,6724	3,6700
OMXT	1,1	3,1888	3,1924	3,1900
OMXV	2,1	3,0716	3,0753	3,0730

Source: appendix 4.

As it can be seen from table 4 all of data series have different specification of ARMA model, thus different mean equations will be applied in GARCH modeling. Results of testing for autocorrelation in residuals presented in appendix 4 show that coefficients of AC and PAC function of OMXR and OMXV are very significant at all of lags considered in the test, thus null hypothesis about absence of correlation in residuals can be rejected, and it can be stated that there is autocorrelation in residuals, meaning that specification of mean equation is not correct. Coefficients of autocorrelation for residuals of ARMA model for OMXT are significant on the level of 1% at lags from 10 to 14, so, again, null hypothesis can be rejected, indicating that there is autocorrelation in residuals. However it should be mentioned that with ARMA model the major part of dependence in data series of OMXT was captured, since only a small number coefficients are statistically significant.

5.2. Testing heteroskedasticity

Before moving to volatility models, it is sensible to test residuals for heteroskedasticity to make sure that ARCH type models should be used here. Table 4 reveals results of test for heteroskedasticity in the residuals. The test statistic for all of data series is very significant, thus null hypothesis can be rejected and it can be stated that there is a heteroskedasticity in residuals of the models.

Table 4. ARCH test results

Series	Obs. R-square	P-value
OMXR	648.3534	0.00
OMXT	116.9457	0.00
OMXV	405.3217	0.00

Source: appendix 5.

Since ARCH test points to heteroskedasticity in residuals ARCH type models can be applied on the data series analyzed in this study. Next sections reviews results of estimation of volatility models.

5.3. Estimating volatility models

When mean equations for every data series was found author compared various specifications of volatility models with information criterions and best specifications were chosen from the models with all parameters significant on the level of 5%. The results of this procedure are presented in appendix 6. The chosen specification of volatility models is described in tables from 5 to 7 and commented in text below.

Table 5 shows that all analysed models presented in the table has significant parameters on the level of statistical significance of one percent.

Table 5. Parameters estimation results, OMXR

Parameter	GARCH(2,1)		EGARCH(2,2)		GJR(2,2)	
	value	p-value	value	p-value	value	p-value
a_0	0.0210	0.00	-0.0164	0.00	0.0819	0.00
a_1	0.2332	0.00	0.3664	0.00	0.0927	0.00
a_2	-0.1609	0.00	-0.3425	0.00	0.1167	0.00
β_1	0.9188	0.00	1.7959	0.00	-0.0629	0.00
β_2	-	-	-0.7977	0.00	0.7912	0.00
γ	-	-	0.0026	0.01	0.0696	0.00

Source: appendix 7.

Since EGARCH and GJR models have asymmetric parameter γ, volatility of OMXR can be checked for asymmetric response for negative price shocks. Asymmetric parameters of both models are very significant, thus null hypothesis about symmetric response of volatility to price shocks can be rejected on the level of significance of one percent. The value of asymmetric parameter in EGARCH is very small, so it has little impact on conditional variance, furthermore the parameter has positive sign, meaning that negative shocks reduce volatility. In opposite with previous model, asymmetric parameter of GJR has larger value and its sing indicates that negative shocks lead to increase in volatility.

Table 6 shows that all parameters of volatility models for OMXT are statistically significant on the level of five percent.

Table 6. Parameters estimation results, OMXT

Parameter	GARCH(2,1)		EGARCH(2,1)		GJR(1,2)	
	value	p-value	value	p-value	value	p-value
a_0	0.0104	0.00	-0.1457	0.00	0.0192	0.00
a_1	0.2400	0.00	0.3987	0.00	0.1745	0.00
a_2	-0.1335	0.00	-0.1937	0.00	-	-
β_1	0.8995	0.00	0.9809	0.00	0.3827	0.00
β_2	-	-	-	-	0.4414	0.00
γ	-	-	-0.0091	0.02	0.0263	0.02

Source: appendix 8.

It can be seen that parameter γ is less significant than others in both models which allows asymmetric response of volatility. In both models asymmetric parameter is significant and null hypothesis can be rejected on the level of significance of five percent. Unlike OMXR, volatility of OMXT increase when negative shocks take place, which is confirmed by the signs of parameters of both models, however the value of γ is again small, meaning very limited impact on conditional variance.

Table 7 indicates that all parameters of models for volatility of OMXV are significant on the level of one percent. Since asymmetric parameters of EGARCH and GJR are significant, null hypothesis about asymmetric response of volatility to price shocks can be rejected. Sings of parameters of both models mean that volatility of OMXV increase in response to negative shock.

Table 7. Parameters estimation results, OMXV

Parameter	GARCH(2,2)		EGARCH(1,2)		GJR(2,2)	
	value	p-value	value	p-value	value	p-value
a_0	0.0034	0.00	-0.1749	0.00	0.1050	0.00
a_1	0.2081	0.00	0.2505	0.00	0.1052	0.00
a_2	-0.1905	0.00	-	-	0.0516	0.00
β_1	1.4332	0.00	0.5307	0.00	-0.0428	0.00
β_2	-0.4521	0.00	0.4336	0.00	0.7383	0.00
γ	-	-	-0.0275	0.00	0.1610	0.00

Source: appendix 9.

The value of those parameters are highest for returns of OMXV, meaning there is greater asymmetry between volatility of index returns and negative shocks in returns.

Since parameters are estimated, the models now should be tested for adequacy and then forecasts generated by those models can be evaluated.

5.4. Diagnostics

If volatility models have been specified correctly there should not be correlation in the series of squared residuals of the model, which can be tested with Q-statistics.

5.4.1. Testing autocorrelation

Q-statistics of squared residuals of models for OMXR shown in appendix 10 indicate that on significance level of one percent null hypothesis should be accepted for residuals of models GARCH and GJR, since all values of probability are higher than 0.01, thus models are correctly specified. For residuals from EGARCH null hypothesis should be rejected, since values of test statistic are very significant at lags from 16 to 30, indicating that model miss necessary variable.

Test statistics of squared residuals for OMXT shown in appendix 11 indicate that for all models null hypothesis can be accepted up to lag 26 on significance level of five percent. It should be mentioned that p-values up to lag 26 are very high. From 27^{th} lag coefficients of autocorrelations become very significant, thus on the whole length null hypothesis should be rejected, indicating problems in model specification.

Q-statistics for squared residuals of OMXV presented in appendix 14 shows that null hypothesis can be accepted on significance level of five percent on the whole length of lags observed in this test. P-values are very high on the whole range of lags, thus no problems with specification of volatility models has been found for OMXV return series.

In order to make sure that model conditional variance of the series ARCH test can be applied on squared residuals of volatility models. Next subsection discusses ARCH test results.

5.4.2. Testing heteroskedasticity

Results of ARCH test for heteroskedasticity presented in the table 8. All values of test statistic have p-value above one percent, thus null hypothesis cannot be rejected on significance level of one percent. On the level of five percent null hypothesis can be rejected for residuals from model GJR applied on series of returns OMXR, indicating that condition variance was incorrectly specified.

Table 8. ARCH test results

Series	GARCH		EGARCH		GJR	
	Obs*R-square	p-value	Obs*R-square	p-value	Obs*R-square	p-value
OMXR	0.0449	0.83	0.6796	0.41	0.5366	0.02
OMXT	0.2028	0.65	0.0064	0.94	0.0098	0.92
OMXV	0.0249	0.87	0.5752	0.45	0.0010	0.98

Source: appendix 13-15.

Next subsection reveals results of Bera-Jarque test for normal distribution of residuals of volatility models.

5.4.3. Testing normal distribution

It can be seen from table 9, where Bera-Jarque test statistic is calculated for residuals, that test statistic is very significant, thus null hypothesis about normal distribution of the residuals from models can be rejected, meaning none of series of residuals is normally distributed.

Table 9. Residuals normality test results

Series	GARCH		EGARCH		GJR	
	test statistic	p-value	test statistic	p-value	test statistic	p-value
OMXR	7 460.444	0.00	7 654.519	0.00	5 241.373	0.00
OMXT	7 810.457	0.00	7 460.515	0.00	7 109.209	0.00
OMXV	41 889.50	0.00	40 111.79	0.00	43 907.81	0.00

Source: author’s table.

Departure from normal distribution means there are problems with models specification. However diagnostic of volatility models reveals some problem with specification, the models can still be used in forecasting. Next section discusses forecasting performance of volatility models.

5.5. Forecast evaluation

The common measures of forecasting performance of volatility models analyzed in this study are shown in table 10. The model that exhibits the lowest value of error measurements is considered to be the best. U-statistic for all models is lower than one indicating that forecasts delivered by the models is better than naive forecast.

Table 10. Volatility models forecast evaluation

Data	Model	RMSE	MAE	MAPE	U-statistic
OMXR	GARCH(2,1)	**0.8951**	**0.6581**	**101.4485**	0.9549
	EGARCH(2,2)	0.8956	0.6590	102.9004	0.9503
	GJR(2,2)	0.8953	0.6584	101.8902	0.9526
OMXT	GARCH(2,1)	0.5952	0.4372	115.2957	0.8575
	EGARCH(2,1)	**0.5947**	**0.4370**	**114.9290**	0.8596
	GJR(1,2)	0.5955	0.4374	115.8126	0.8553
OMXV	GARCH(2,2)	**0.5473**	**0.3605**	**116.2928**	0.8859
	EGARCH(1,2)	0.5478	0.3614	118.2238	0.8751
	GJR(2,2)	0.5499	0.3632	121.8809	0.8674

Note: The lowest value of error statistics marked in bold.
Source: author’s table.

Results shown in table 9 indicate that the best model for OMXR is GARCH(2,1) since it received the minimum values of error measurements. GJR(2,2) performed a bit worse than GARCH(2,1) and EGARCH(2,2) is the worst performer among three models applied on returns of OMXR. However it should be pointed out that the differences between models are extremely small in terms of RMSE and MAE. Appendix 16 exhibits graphs of variation forecast produced by the models plotted against time. It can be seen that EGARCH delivered the largest forecast of variance in response to negative shock in the beginning of first quarter of 2014. Comparing with other models influence of shock died more slowly in case of GJR, as forecast of variance remains near maxim for longer time. GARCH delivered largest forecast to both negative and positive shocks in the second quarter of 2013, and to positive shocks in third quarter of 2014.

Forecasts of EGARCH(2,1) appeared to be the most accurate for volatility of OMXT, as the model received the smallest values of error statistics. The model is followed by GARCH(2,1), which received slightly higher values of error measurements and GJR(1,2) is the worst performed. Again, differences in RMSE and MAE are extremely small. Graphs of OMXT variation forecast located in appendix 17 indicates that GARCH again tend to deliver larger forecasts of volatility when price shocks take place, comparing with to other models.

GARCH(2,2) is the performer for series of returns of OMXV, it is followed by EGARCH(1,2) with a bit higher values of error measurement and GJR(2,2) happened to be the worst. Differences between models are fairly small, however it can be seen that differences in error statistics of GARCH and EGARCH are smaller than ones of EGARCH and GJR. Appendix 18 displays differences of how the models responded to negative and positive shocks in the first quarter of 2014. When positive shock took place in first quarter of 2014 the GARCH model delivered the largest forecast, and in case of negative shocks later this quarter the GJR delivered the largest forecast. It can be seen that EGARCH delivered smallest forecasts when price shocks occurred.

As graphs in appendixes 16 – 18 showed that serious differences in forecasts come out when large price shocks happens, in the periods of calm market, models tend to deliver relatively very similar forecasts, which is also confirmed by small differences in error statistics.

CONCLUSIONS

The aim of this thesis was to perform analysis of volatility of three Baltic stock market indexes (Riga, Tallinn and Vilnius) with three ARCH-type models (GARCH, EGARCH and GJR) and find the model that delivers the most precise forecast. In order to perform the analysis daily closing price for the period from January 2000 to December 2014 were obtained and transformed into daily returns. Using those three data series nine volatility models were constructed, estimated, tested and forecasting performance of those models was compared using various loss functions. In order to specify, estimate and test the volatility models Box-Jenkins methodology was applied.

All stock market indexes analyzed in this study exhibit significant departure from normal distribution in terms of skewness and excess kurtosis, which is line with stylized facts about financial asset returns. Analysis of dependence in returns reveals significant autocorrelation in case of every data series, which author could not capture with ARMA models on the stage of choosing mean equation for volatility models. Particularly significant autocorrelation remained in residuals of OMXR and OMXV. Test for heteroskedasticity of residuals confirmed appropriateness of ARCH-type models application in modelling volatility of those three stock market indexes.

Testing for asymmetry in volatility provided evidence of leverage effect in all of Baltic stock market indexes, which is in line with Ding, Granger and Engle (1993), Bekaert and Wu (2000) and Jayasuria, Shambora & Rossiter (2005), who found evidence of leverage effect in developed markets, and does not confirm absence of leverage effect in volatility of OMXT documented by Dzielinski, Rieger and Talpsepp (2010). Moreover, one of the models indicated opposite asymmetry, the parameter of EGARCH indicated decrease in volatility of OMXR, when negative shocks appear. This is contrary to leverage effect documented by studies of developed markets, but somewhat similar to behavior that was found by Brooks (2007) for stock markets of Egypt, Nigeria and Zimbabwe, where higher volatility was associated with positive shocks.

On the stage of testing autocorrelation in residuals of volatility models problems with specification of the models was found for EGARCH model applied on series of returns on OMXR and for all volatility models applied on OMXT, although in this case autocorrelation arised on very high lags. Test for heteroskedasticity of residuals on level of statistical significance of five percent indicated problems with specification of GJR model applied on data series of OMXR. On the level of one percent, no heteroskedasticy was found. Test for normality of residuals showed that none of models have normally distributed residuals. Therefore, author conclude that he could not fully capture dependence in data series, thus it might have negative impact on forecasting performance. In order to overcome those disadvantages it may be helpful to divide analyzed period to smaller samples, which exclude periods of extremely high volatility. Another option can be data trimming, which involves excluding outliers from data series.

All of models analyzed in this study generated better then naive forecast. Evaluation of ability of the models to forecast volatility of Baltic stock markets showed very small differences in forecast error measurements among three models. GARCH(2,1) appeared to be the best model for returns of OMXR, followed by GJR(2,2) and EGARCH(2,2). For OMXT asymmetric model delivered more precise forecasts. Best performer is EGARCH(2,1), followed by GARCH(2,1) and GJR(1,2). In case of OMXV, GARCH(2,2) worked best, a bit worse performance showed EGARCH(1,2) and GJR(2,2) respectively. Graphical analysis of forecasts indicated that in periods of calm market models tend to deliver very similar forecasts, which was confirmed by small differences in error statistic. Significant differences between forecasts can be found on extremums, although it should be pointed out that all models behave differently on various data series. For OMXR the EGARCH produced the largest forecast of volatility in response to large negative shock, in case of OMXT the GARCH model delivered large forecasts disregarding the sing of shocks, and for OMXV the GJR generated forecast larger than two other models, when substantial negative shock took place.

The results of current study are contrary to results of many studies about stock market volatility, which provide evidence of supremacy of asymmetric GARCH models to standard GARCH model. In current study standard GARCH outperformed asymmetric models in two of three cases. Although EGARCH appeared to deliver slightly more accurate forecasts of volatility of OMXT than GARCH, the difference between two models was fairly small.

RESÜMEE

VOLATIILSUSE MODELLEERIMINE BALTI AKTSIATURGUDEL

Sergei Guštšak

Volatiilsus on üks põhimuutujatest paljudes finantsmudelites kaasaegses rahanduses. Seda kasutatakse tuletisväärtpaberite hinnastamises *Black-Sholes* mudelis optsiooni õiglase hinda leidmiseks. Volatiilsus on samuti osa *Value-at-Risk* mudelist, mida kasutavad finantsasutused ülemaailma võimaliku kahjumi rahaliseks mõtmiseks. Kaasaegne portfelliteooria kasutab volatiilsust väärtpaberi riskimõõdikuna. Fintsvara volatiilsus ei ole ajas konstantne, seega riski mõtmiseks on vaja meetodit, mis võimaldaks prognoosida volatiilsust võimalikult täpselt.

Volatiilsuse modeleerimise probleemi on väga palju uuritud, selleks kasutatakse erinevaid meetodeid ja mudelied. Praktikas kasutatakse ajaloolised mudelid, mille hulka kuulub terve rida erinevaid mudeleid, samuti kasutatakse nii nimetud eeldatavat volatiilsust, mida leiakse turul kaubeldavate optsionide hinnadest, kuid kõige levinum mudeli tüüp on autoregresiivne tingliku heteroskedastivsusega mudel või ARCH mudel. Viimastel aastakümnetel kasutatakse volatiilsuse modeleerimiseks peamiselt ARCH mudeli edasiarendusi, mis võimaldavad modeleerida selleseid nähtusi, nagu näiteks volatiilsuse kuhjumine, teravatipuline jaotus ning volatiilsuse asümmeetriline reaktsioon erineva märgiga hinnašokidele. Kirjanduse analüüs näitas, et Balti aktsiaturgude volatiilsuse modeleerimine ei ole veel laialt uuritud. Seoses viimasega autor otsustas uurida volatiilsuse modeleerimise probleemi Balti turgude näitel.

Käesoleva magistritöö teema on aktuaalne, kuna töös võrreldakse mittu ARCH tüüpi mudeleid, mille abil on võimalik prognoosida kohalikke aktsiaturgude volatiilsust. Töö tulemused võivad olla eriti kasulikud riskijuhtimise valdkonnas, kus saadud volatiilsuse prognoose võib kasutada *Value-at-Risk* mudelis.

Käesoleva töö eesmärgiks oli leida sellise ARCH tüüpi mudeli, mis annab kõige täpsema prognoosi Balti aktsiaturgudel. Selleks oli kogutud kolme börsiindeksi (Riia, Tallinn, Vilnius) päeva sulgemisehinnad perioodil 2000-2014 ja valitud kolm ARCH tüüpi mudelit (GARCH, EGARCH, GJR), mida tihiti kasutatakse volatiilsuse modeleerimiseks. Volatiilsuse mudelite ehitamiseks, hindamiseks ja testimiseks kasutati *Box-Jenksi* methoodikat. Mudeli prognoosimisvõime hindamiseks kasutati staatilist prognoosi, mille täpsust hinnati nelja prognoosivea järgi.

Empirilise testide käigus leiti, et Balti aktsiaturgude indeksite volatiilsuses esineb asümmeetriat negatiivsete hinnašokkide suhtes. Oli samuti leitud, et Riia aktsiaturu indeksi volatiilsus väheneb negatiivsete hinnašokkide mõjul. Taoline tulemus oli leitud ka teise autoriga arenevatel aktsiaturgudel. Volatiilsuse mudelite testimisel selgus, et EGARCH mudeli kuju ei sobi selleks, et kirjeldada Riia aktsiaturu indeksi volatiilsus täiesti ning teatud probleemid mudeli kujuga tekkisid ka komle volatiilsuse mudelitega rakendatud Tallinna aktsiaturu indeksi suhtes.

Mudelite prognoosimisvõime hindamise käigus leiti, et Riia aktsiaturu indeksi volatiilsuse prognoosimiseks sobib kolmest mudelist paremini GARCH(2,1), Tallinna aktsiaturu indeksi volatiilsuse prognoosimiseks sobib EGARCH(2,1) ja Vilniuse aktsiaturu indeksi volatiilsuse kõige täpsema prognoosi annab GARCH(2,2) mudel. Samas autor leidis, et prognoosiveade väärtuste erinevused kõige indeksite puhul olid väga väiksed, viidates sellel, et mudelid annavad väga sarnasi prognoose. Prognoositud volatiilsuse graafikute analüüs näitas, et aktsiaturu suhteliselt rahulikul ajal genereerivad mudelid enam-vähem sarnasi prognoose ning järskude hinnašokide puhul prognoosid võivad erineda märkimisväärselt.

Käesoleva töö tulemused lähevad mõnevõrra lahku paljude teaduslikute tööde tulemustega, kuna akadeemilises kirjanduses reegline rõhutatakse nn. asümeetriliste GARCH mudelite eelisi aktsiaturgude volatiilsusse modeleerimisel. Antud töö näitas, et kahel juhul kolmest standartne GARCH mudel andis parema prognoosi kui selle mudeli asümmeetriline edasiarendus.

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APPENDICES

Appendix 1. Dickey-Fuller test results

Null Hypothesis: OMXR_DAILY_RETURNS has a unit root
Exogenous: Constant
Lag Length: 3 (Automatic - based on SIC, maxlag=29)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-30.94719	0.0000
Test critical values:	1% level	-3.431886	
	5% level	-2.862104	
	10% level	-2.567114	

Null Hypothesis: OMXT_DAILY_RETURNS has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=29)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-53.27141	0.0001
Test critical values:	1% level	-3.431885	
	5% level	-2.862104	
	10% level	-2.567113	

Null Hypothesis: OMXV_DAILY_RETURNS has a unit root
Exogenous: Constant
Lag Length: 7 (Automatic - based on SIC, maxlag=29)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-17.34520	0.0000
Test critical values:	1% level	-3.431888	
	5% level	-2.862105	
	10% level	-2.567114	

Source: author's figure from Eviews.

Appendix 2. Correlograms of returns

OMXR

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.032	0.032	3.7884	0.052
		2	0.097	0.096	39.235	0.000
		3	0.024	0.018	41.433	0.000
		4	-0.067	-0.078	58.546	0.000
		5	-0.026	-0.027	61.194	0.000
		6	-0.041	-0.026	67.560	0.000
		7	-0.037	-0.027	72.874	0.000
		8	0.013	0.018	73.503	0.000
		9	0.039	0.044	79.380	0.000
		10	0.076	0.068	101.29	0.000
		11	0.059	0.042	114.78	0.000
		12	0.078	0.061	138.23	0.000

OMXT

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.145	0.145	79.704	0.000
		2	0.057	0.037	92.206	0.000
		3	0.034	0.021	96.487	0.000
		4	0.031	0.022	100.25	0.000
		5	0.038	0.029	105.76	0.000
		6	0.036	0.024	110.62	0.000
		7	0.045	0.034	118.37	0.000
		8	0.035	0.020	123.04	0.000
		9	0.065	0.053	139.23	0.000
		10	0.055	0.034	150.63	0.000
		11	0.042	0.022	157.42	0.000
		12	0.037	0.020	162.64	0.000

OMXV

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.133	0.133	67.167	0.000
		2	0.063	0.046	82.282	0.000
		3	0.052	0.039	92.535	0.000
		4	0.048	0.034	101.29	0.000
		5	0.004	-0.011	101.36	0.000
		6	0.071	0.067	120.39	0.000
		7	0.069	0.050	138.42	0.000
		8	0.088	0.068	167.83	0.000
		9	0.056	0.028	179.79	0.000
		10	0.058	0.033	192.49	0.000
		11	0.037	0.015	197.81	0.000
		12	0.058	0.039	210.86	0.000

Source: author's figure from Eviews.

Appendix 3. ARMA models estimation output

Dependent Variable: OMXR_DAILY_RETURNS
Method: Least Squares
Date: 05/17/15 Time: 00:52
Sample (adjusted): 1/06/2000 12/28/2012
Included observations: 3302 after adjustments
Convergence achieved after 8 iterations
MA Backcast: 1/04/2000 1/05/2000

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(2)	-0.336336	0.110759	-3.036645	0.0024
MA(2)	0.452060	0.105050	4.303263	0.0000

R-squared	0.014636	Mean dependent var	0.051024
Adjusted R-squared	0.014337	S.D. dependent var	1.525487
S.E. of regression	1.514512	Akaike info criterion	3.668670
Sum squared resid	7569.368	Schwarz criterion	3.672366
Log likelihood	-6054.974	Hannan-Quinn criter.	3.669993
Durbin-Watson stat	1.924934		

Dependent Variable: OMXT_DAILY_RETURNS
Method: Least Squares
Date: 05/17/15 Time: 00:55
Sample (adjusted): 1/05/2000 12/28/2012
Included observations: 3303 after adjustments
Convergence achieved after 12 iterations
MA Backcast: 1/04/2000

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.590835	0.078156	7.559682	0.0000
MA(1)	-0.462244	0.085938	-5.378833	0.0000

R-squared	0.023361	Mean dependent var	0.057160
Adjusted R-squared	0.023065	S.D. dependent var	1.205388
S.E. of regression	1.191406	Akaike info criterion	3.188750
Sum squared resid	4685.596	Schwarz criterion	3.192445
Log likelihood	-5264.221	Hannan-Quinn criter.	3.190073
Durbin-Watson stat	1.974494		

Inverted AR Roots	.59
Inverted MA Roots	.46

Appendix 3 extension

Dependent Variable: OMXV_DAILY_RETURNS
Method: Least Squares
Date: 05/17/15 Time: 00:51
Sample (adjusted): 1/06/2000 12/28/2012
Included observations: 3302 after adjustments
Convergence achieved after 5 iterations
MA Backcast: 1/05/2000

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(2)	0.063798	0.017516	3.642348	0.0003
MA(1)	0.130862	0.017402	7.520155	0.0000

R-squared	0.019772	Mean dependent var	0.043873
Adjusted R-squared	0.019475	S.D. dependent var	1.134751
S.E. of regression	1.123647	Akaike info criterion	3.071641
Sum squared resid	4166.520	Schwarz criterion	3.075337
Log likelihood	-5069.280	Hannan-Quinn criter.	3.072964
Durbin-Watson stat	2.001822		

Inverted AR Roots	.25	-.25
Inverted MA Roots	-.13	

Source: author’s figure from Eviews.

Appendix 4. Autocorrelation test results for residuals of ARMA model

Lag	OMXR				OMXT				OMXV			
	AC	PAC	Q-Stat	Prob	AC	PAC	Q-Stat	Prob	AC	PAC	Q-Stat	Prob
1	0.036	0.036	4.4030	-	0.011	0.011	0.3908	-	-0.002	-0.002	0.0127	-
2	-0.005	-0.006	4.4738	-	-0.025	-0.026	2.5322	-	-0.004	-0.004	0.0583	-
3	0.022	0.022	6.0211	0.01	-0.012	-0.012	3.0128	**0.08**	0.037	0.037	4.6238	**0.03**
4	-0.030	-0.032	9.0923	0.01	-0.001	-0.002	3.0197	**0.22**	0.036	0.037	9.0055	0.01
5	-0.022	-0.020	10.742	0.01	0.013	0.012	3.5538	**0.31**	-0.016	-0.015	9.8272	**0.02**
6	-0.054	-0.053	20.382	0.00	0.016	0.016	4.4363	**0.35**	0.061	0.060	22.272	0.00
7	-0.046	-0.041	27.412	0.00	0.028	0.028	6.9569	**0.22**	0.052	0.050	31.266	0.00
8	0.014	0.017	28.095	0.00	0.018	0.018	7.9951	**0.24**	0.072	0.073	48.257	0.00
9	0.044	0.044	34.440	0.00	0.047	0.048	15.178	**0.03**	0.035	0.034	52.314	0.00
10	0.069	0.065	50.248	0.00	0.037	0.038	19.708	0.01	0.044	0.038	58.682	0.00
11	0.050	0.041	58.505	0.00	0.026	0.028	21.986	0.01	0.019	0.014	59.878	0.00
12	0.068	0.062	73.952	0.00	0.026	0.028	24.269	0.01	0.041	0.034	65.552	0.00
13	0.069	0.062	89.751	0.00	0.013	0.014	24.832	0.01	0.071	0.065	82.307	0.00
14	0.057	0.059	100.38	0.00	0.016	0.016	25.660	0.01	0.040	0.030	87.548	0.00
15	-0.025	-0.019	102.50	0.00	-0.006	-0.008	25.770	**0.02**	0.019	0.009	88.734	0.00
16	-0.053	-0.039	111.93	0.00	0.018	0.015	26.825	**0.02**	0.063	0.046	101.76	0.00
17	-0.059	-0.047	123.64	0.00	-0.008	-0.013	27.015	**0.03**	0.006	-0.007	101.87	0.00
18	0.004	0.017	123.68	0.00	-0.001	-0.006	27.018	**0.04**	0.021	0.010	103.29	0.00
19	-0.032	-0.028	127.16	0.00	0.008	0.001	27.228	**0.06**	0.054	0.037	112.83	0.00
20	-0.010	-0.009	127.49	0.00	-0.019	-0.026	28.490	**0.06**	0.006	-0.012	112.97	0.00
21	-0.029	-0.046	130.23	0.00	0.003	-0.003	28.521	**0.07**	-0.019	-0.035	114.20	0.00
22	-0.005	-0.028	130.32	0.00	0.007	0.002	28.706	**0.09**	0.018	-0.006	115.24	0.00
23	-0.003	-0.032	130.34	0.00	-0.004	-0.009	28.754	**0.12**	0.023	0.005	116.94	0.00
24	0.002	-0.015	130.35	0.00	0.024	0.022	30.679	**0.10**	0.003	-0.009	116.97	0.00
25	-0.037	-0.047	135.03	0.00	0.011	0.009	31.108	**0.12**	-0.001	-0.017	116.98	0.00
26	-0.012	-0.016	135.54	0.00	0.002	0.003	31.118	**0.15**	0.016	-0.004	117.88	0.00
27	0.017	0.018	136.55	0.00	0.019	0.022	32.373	**0.15**	0.012	-0.000	118.36	0.00
28	-0.027	-0.020	139.03	0.00	-0.026	-0.026	34.639	**0.12**	0.008	0.001	118.55	0.00
29	0.008	0.026	139.21	0.00	0.015	0.019	35.401	**0.13**	0.015	0.005	119.29	0.00
30	-0.035	-0.023	143.23	0.00	0.012	0.012	35.902	**0.15**	0.008	0.001	119.51	0.00
31	0.018	0.033	144.28	0.00	0.043	0.043	41.990	**0.06**	0.028	0.023	122.15	0.00
32	-0.014	-0.015	144.95	0.00	0.022	0.022	43.603	**0.05**	0.006	-0.002	122.28	0.00
33	-0.020	-0.004	146.28	0.00	0.002	0.003	43.617	**0.07**	0.021	0.020	123.78	0.00
34	-0.031	-0.023	149.50	0.00	-0.004	-0.004	43.676	**0.08**	0.042	0.042	129.59	0.00
35	-0.023	-0.008	151.31	0.00	-0.020	-0.022	45.017	**0.08**	-0.025	-0.030	131.60	0.00
36	-0.009	-0.007	151.60	0.00	-0.009	-0.014	45.297	**0.09**	-0.032	-0.035	134.96	0.00

Note: p-values higher than 0.01 marked in bold.

Source: author's table base on calculation in Eviews.

Appendix 5. ARMA residuals ARCH test output

OMXR

Heteroskedasticity Test: ARCH

F-statistic	806.3335	Prob. F(1,3299)	0.0000
Obs*R-squared	648.3534	Prob. Chi-Square(1)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 05/17/15 Time: 01:44
Sample (adjusted): 1/07/2000 12/28/2012
Included observations: 3301 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.276898	0.142449	8.963901	0.0000
RESID^2(-1)	0.443180	0.015607	28.39601	0.0000

R-squared	0.196411	Mean dependent var	2.292961
Adjusted R-squared	0.196168	S.D. dependent var	8.835814
S.E. of regression	7.921899	Akaike info criterion	6.977745
Sum squared resid	207033.6	Schwarz criterion	6.981442
Log likelihood	-11514.77	Hannan-Quinn criter.	6.979068
F-statistic	806.3335	Durbin-Watson stat	2.100428
Prob(F-statistic)	0.000000		

OMXT

Heteroskedasticity Test: ARCH

F-statistic	121.1662	Prob. F(1,3300)	0.0000
Obs*R-squared	116.9457	Prob. Chi-Square(1)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 05/17/15 Time: 01:49
Sample (adjusted): 1/06/2000 12/28/2012
Included observations: 3302 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.150547	0.081727	14.07789	0.0000
RESID^2(-1)	0.188180	0.017096	11.00755	0.0000

R-squared	0.035417	Mean dependent var	1.417576
Adjusted R-squared	0.035124	S.D. dependent var	4.565534
S.E. of regression	4.484636	Akaike info criterion	5.839797
Sum squared resid	66369.48	Schwarz criterion	5.843494
Log likelihood	-9639.506	Hannan-Quinn criter.	5.841120
F-statistic	121.1662	Durbin-Watson stat	2.044385
Prob(F-statistic)	0.000000		

Appendix 5 extension

OMXV

Heteroskedasticity Test: ARCH

F-statistic	461.7766	Prob. F(1,3299)	0.0000
Obs*R-squared	405.3217	Prob. Chi-Square(1)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 05/17/15 Time: 01:43
Sample (adjusted): 1/07/2000 12/28/2012
Included observations: 3301 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.819913	0.096113	8.530749	0.0000
RESID^2(-1)	0.350410	0.016307	21.48899	0.0000

R-squared	0.122788	Mean dependent var	1.262199
Adjusted R-squared	0.122522	S.D. dependent var	5.758264
S.E. of regression	5.393985	Akaike info criterion	6.209052
Sum squared resid	95984.66	Schwarz criterion	6.212749
Log likelihood	-10246.04	Hannan-Quinn criter.	6.210375
F-statistic	461.7766	Durbin-Watson stat	2.117818
Prob(F-statistic)	0.000000		

Source: author’s figure from Eviews.

Appendix 6. Information criterions for volatility models

Model	OMXR			OMXT			OMXV		
	AIC	SBIC	HQIC	AIC	SBIC	HQIC	AIC	SBIC	HQIC
GARCH (1,0)	3.3703	3.3778	3.3729	3.0359	3.0433	3.0385	2.8499	2.8573	2.8526
GARCH (2,0)	3.2904	3.2996	3.2937	3.0079	3.0172	3.0112	2.8172	2.8264	2.8205
GARCH(1,1)	3.1437	3.1529	3.1470	2.8910	2.9002	2.8943	2.7588	2.7680	2.7621
GARCH(1,2)	3.1393	3.1504	3.1433	2.8879	2.8990	2.8919	2.7540	2.7651	2.7580
GARCH(2,1)	3.1321	3.1432	3.1361	2.8850	2.8961	2.8890	2.7429	2.7540	2.7469
GARCH(2,2)	3.1445	3.1575	3.1492	2.8853	2.8982	2.8899	2.7364	2.7493	2.7410
EGARCH(1,0)	3.4315	3.4408	3.4348	3.0646	3.0738	3.0679	2.8844	2.8936	2.8877
EGARCH(2,0)	3.3634	3.3745	3.3674	3.0285	3.0396	3.0325	2.8533	2.8644	2.8572
EGARCH(1,1)	3.1478	3.1589	3.1518	2.8801	2.8912	2.8840	2.7451	2.7561	2.7490
EGARCH(1,2)	3.1430	3.1559	3.1476	2.8739	2.8868	2.8785	2.7403	2.7532	2.7449
EGARCH(2,1)	3.1378	3.1508	3.1425	2.8716	2.8845	2.8762	2.7330	2.7459	2.7376
EGARCH(2,2)	3.1370	3.1518	3.1423	2.8720	2.8868	2.8773	2.7259	2.7407	2.7312
GJR(1,0)	3.3709	3.3801	3.3742	3.0314	3.0407	3.0347	2.8499	2.8573	2.8526
GJR(2,0)	3.2908	3.3019	3.2947	3.0049	3.0160	3.0089	2.8172	2.8264	2.8205
GJR(1,1)	3.1443	3.1554	3.1483	2.8912	2.9023	2.8952	2.7588	2.7680	2.7621
GJR(1,2)	3.1406	3.1536	3.1453	2.8880	2.9010	2.8927	2.7540	2.7651	2.7580
GJR(2,1)	3.1327	3.1456	3.1373	2.8853	2.8983	2.8899	2.7429	2.7540	2.7469
GJR(2,2)	3.1411	3.1559	3.1464	2.8857	2.9004	2.8910	2.7395	2.7543	2.7448

Note: Information criterions of models with insignificant parameters marked with grey, best combination of information criterions of models with all significant parameters marked with yellow.

Source: author’s calculations.

Appendix 7. Volatility models estimation outputs for OMXR

GARCH(2,1)

Dependent Variable: OMXR_DAILY_RETURNS
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 05/22/15 Time: 13:17
Sample (adjusted): 1/06/2000 12/28/2012
Included observations: 3302 after adjustments
Convergence achieved after 32 iterations
MA Backcast: 1/04/2000 1/05/2000
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-2)^2 + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
AR(2)	-0.439832	0.149599	-2.940073	0.0033
MA(2)	0.482474	0.146433	3.294848	0.0010
		Variance Equation		
C	0.021039	0.002030	10.36235	0.0000
RESID(-1)^2	0.233174	0.016671	13.98648	0.0000
RESID(-2)^2	-0.160886	0.016076	-10.00775	0.0000
GARCH(-1)	0.918837	0.005120	179.4723	0.0000

R-squared	0.008767	Mean dependent var	0.051024
Adjusted R-squared	0.008466	S.D. dependent var	1.525487
S.E. of regression	1.519016	Akaike info criterion	3.132093
Sum squared resid	7614.452	Schwarz criterion	3.143182
Log likelihood	-5165.086	Hannan-Quinn criter.	3.136062
Durbin-Watson stat	1.917350		

EGARCH(2,2)

Dependent Variable: OMXR_DAILY_RETURNS
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 05/22/15 Time: 13:22
Sample (adjusted): 1/06/2000 12/28/2012
Included observations: 3302 after adjustments
Convergence achieved after 23 iterations
MA Backcast: 1/04/2000 1/05/2000
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5)
*ABS(RESID(-2)/@SQRT(GARCH(-2))) + C(6)*RESID(-1)
/@SQRT(GARCH(-1)) + C(7)*LOG(GARCH(-1)) + C(8)*LOG(GARCH(
-2))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
AR(2)	-0.590638	0.121587	-4.857721	0.0000
MA(2)	0.632470	0.115336	5.483738	0.0000
		Variance Equation		
C(3)	-0.016442	0.002622	-6.269883	0.0000
C(4)	0.366383	0.015122	24.22824	0.0000
C(5)	-0.342456	0.014855	-23.05401	0.0000
C(6)	0.002596	0.000952	2.726328	0.0064
C(7)	1.795916	0.020143	89.15696	0.0000
C(8)	-0.797714	0.019859	-40.16824	0.0000

R-squared	0.008853	Mean dependent var	0.051024
Adjusted R-squared	0.008552	S.D. dependent var	1.525487
S.E. of regression	1.518950	Akaike info criterion	3.137036
Sum squared resid	7613.792	Schwarz criterion	3.151820
Log likelihood	-5171.246	Hannan-Quinn criter.	3.142328
Durbin-Watson stat	1.916758		

Appendix 7 extension

GJR(2,2)

Dependent Variable: OMXR_DAILY_RETURNS
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 05/22/15 Time: 13:25
Sample (adjusted): 1/06/2000 12/28/2012
Included observations: 3302 after adjustments
Convergence achieved after 51 iterations
MA Backcast: 1/04/2000 1/05/2000
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-1)^2*(RESID(-1)<0) + C(6)*RESID(-2)^2 + C(7)*GARCH(-1) + C(8)*GARCH(-2)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
AR(2)	-0.437739	0.162168	-2.699296	0.0069
MA(2)	0.482816	0.158135	3.053191	0.0023
		Variance Equation		
C	0.081901	0.006664	12.29016	0.0000
RESID(-1)^2	0.092659	0.008354	11.09168	0.0000
RESID(-1)^2*(RESID(-1)<0)	0.069576	0.014250	4.882651	0.0000
RESID(-2)^2	0.116654	0.008776	13.29244	0.0000
GARCH(-1)	-0.062925	0.013376	-4.704188	0.0000
GARCH(-2)	0.791233	0.014946	52.93987	0.0000

R-squared	0.009159	Mean dependent var	0.051024
Adjusted R-squared	0.008858	S.D. dependent var	1.525487
S.E. of regression	1.518716	Akaike info criterion	3.141131
Sum squared resid	7611.441	Schwarz criterion	3.155915
Log likelihood	-5178.007	Hannan-Quinn criter.	3.146423
Durbin-Watson stat	1.917582		

Source: author's figure from Eviews.

Appendix 8. Volatility models estimation outputs for OMXT

GARCH(2,1)

Dependent Variable: OMXT_DAILY_RETURNS
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 05/22/15 Time: 13:56
Sample (adjusted): 1/05/2000 12/28/2012
Included observations: 3303 after adjustments
Convergence achieved after 18 iterations
MA Backcast: 1/04/2000
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-2)^2 + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	0.398295	0.081297	4.899242	0.0000
MA(1)	-0.230401	0.089577	-2.572110	0.0101
		Variance Equation		
C	0.010416	0.001538	6.772858	0.0000
RESID(-1)^2	0.240047	0.019976	12.01707	0.0000
RESID(-2)^2	-0.133537	0.020306	-6.576117	0.0000
GARCH(-1)	0.899530	0.004873	184.5954	0.0000

R-squared	0.022035	Mean dependent var	0.057160
Adjusted R-squared	0.021739	S.D. dependent var	1.205388
S.E. of regression	1.192214	Akaike info criterion	2.885043
Sum squared resid	4691.954	Schwarz criterion	2.896128
Log likelihood	-4758.648	Hannan-Quinn criter.	2.889011
Durbin-Watson stat	2.049900		
Inverted AR Roots	.40		
Inverted MA Roots	.23		

EGARCH(2,1)

Dependent Variable: OMXT_DAILY_RETURNS
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 05/22/15 Time: 13:57
Sample (adjusted): 1/05/2000 12/28/2012
Included observations: 3303 after adjustments
Convergence achieved after 26 iterations
MA Backcast: 1/04/2000
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5)
*ABS(RESID(-2)/@SQRT(GARCH(-2))) + C(6)*RESID(-1)
/@SQRT(GARCH(-1)) + C(7)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	0.324531	0.082892	3.915083	0.0001
MA(1)	-0.157365	0.089967	-1.749150	0.0803
		Variance Equation		
C(3)	-0.145748	0.007662	-19.02155	0.0000
C(4)	0.398718	0.025224	15.80720	0.0000
C(5)	-0.193686	0.025709	-7.533903	0.0000
C(6)	-0.009091	0.004025	-2.258609	0.0239
C(7)	0.980902	0.002229	440.1006	0.0000

R-squared	0.021623	Mean dependent var	0.057160
Adjusted R-squared	0.021327	S.D. dependent var	1.205388
S.E. of regression	1.192465	Akaike info criterion	2.871608
Sum squared resid	4693.934	Schwarz criterion	2.884541
Log likelihood	-4735.460	Hannan-Quinn criter.	2.876237
Durbin-Watson stat	2.047975		
Inverted AR Roots	.32		
Inverted MA Roots	.16		

Appendix 8 extension

GJR(1,2)

Dependent Variable: OMXT_DAILY_RETURNS
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 05/22/15 Time: 14:00
Sample (adjusted): 1/05/2000 12/28/2012
Included observations: 3303 after adjustments
Convergence achieved after 18 iterations
MA Backcast: 1/04/2000
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-1)^2*(RESID(-1)<0) + C(6)*GARCH(-1) + C(7)*GARCH(-2)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	0.403905	0.079712	5.067047	0.0000
MA(1)	-0.232667	0.087505	-2.658897	0.0078
Variance Equation				
C	0.019194	0.002761	6.951132	0.0000
RESID(-1)^2	0.174486	0.010528	16.57300	0.0000
RESID(-1)^2*(RESID(-1)<0)	0.026272	0.011059	2.375694	0.0175
GARCH(-1)	0.382665	0.058681	6.521134	0.0000
GARCH(-2)	0.441402	0.053734	8.214494	0.0000

R-squared	0.021868	Mean dependent var	0.057160
Adjusted R-squared	0.021571	S.D. dependent var	1.205388
S.E. of regression	1.192316	Akaike info criterion	2.888030
Sum squared resid	4692.760	Schwarz criterion	2.900963
Log likelihood	-4762.581	Hannan-Quinn criter.	2.892659
Durbin-Watson stat	2.056515		

Inverted AR Roots	.40
Inverted MA Roots	.23

Source: author’s figure from Eviews.

Appendix 9. Volatility models estimation outputs for OMXV

GARCH(2,2)

Dependent Variable: OMXV_DAILY_RETURNS
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 05/22/15 Time: 14:16
Sample (adjusted): 1/06/2000 12/28/2012
Included observations: 3302 after adjustments
Convergence achieved after 57 iterations
MA Backcast: 1/05/2000
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-2)^2 + C(6)*GARCH(-1)
+ C(7)*GARCH(-2)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
AR(2)	0.089115	0.018597	4.792017	0.0000
MA(1)	0.120896	0.021589	5.599845	0.0000
		Variance Equation		
C	0.003442	0.000777	4.431141	0.0000
RESID(-1)^2	0.208121	0.019059	10.91968	0.0000
RESID(-2)^2	-0.190468	0.016920	-11.25723	0.0000
GARCH(-1)	1.432449	0.060952	23.50122	0.0000
GARCH(-2)	-0.452072	0.057316	-7.887356	0.0000

R-squared	0.019001	Mean dependent var	0.043873
Adjusted R-squared	0.018704	S.D. dependent var	1.134751
S.E. of regression	1.124088	Akaike info criterion	2.736403
Sum squared resid	4169.797	Schwarz criterion	2.749339
Log likelihood	-4510.801	Hannan-Quinn criter.	2.741033
Durbin-Watson stat	1.984550		

EGARCH(1,2)

Dependent Variable: OMXV_DAILY_RETURNS
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 05/22/15 Time: 14:18
Sample (adjusted): 1/06/2000 12/28/2012
Included observations: 3302 after adjustments
Convergence achieved after 137 iterations
MA Backcast: 1/05/2000
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5)
*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1)) + C(7)
*LOG(GARCH(-2))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
AR(2)	0.080032	0.015193	5.267612	0.0000
MA(1)	0.145958	0.019734	7.396259	0.0000
		Variance Equation		
C(3)	-0.174897	0.010484	-16.68171	0.0000
C(4)	0.250460	0.015829	15.82295	0.0000
C(5)	-0.027513	0.006285	-4.377589	0.0000
C(6)	0.530664	0.080464	6.595074	0.0000
C(7)	0.433580	0.079562	5.449562	0.0000

R-squared	0.019344	Mean dependent var	0.043873
Adjusted R-squared	0.019047	S.D. dependent var	1.134751
S.E. of regression	1.123892	Akaike info criterion	2.740288
Sum squared resid	4168.340	Schwarz criterion	2.753225
Log likelihood	-4517.216	Hannan-Quinn criter.	2.744919
Durbin-Watson stat	2.032596		

Appendix 9 extension

GJR(2,2)

Dependent Variable: OMXV_DAILY_RETURNS
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 05/22/15 Time: 14:20
Sample (adjusted): 1/06/2000 12/28/2012
Included observations: 3302 after adjustments
Convergence achieved after 71 iterations
MA Backcast: 1/05/2000
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-1)^2*(RESID(-1)<0) +
C(6)*RESID(-2)^2 + C(7)*GARCH(-1) + C(8)*GARCH(-2)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
AR(2)	0.102603	0.016759	6.122369	0.0000
MA(1)	0.151785	0.018498	8.205438	0.0000
Variance Equation				
C	0.104969	0.006618	15.86229	0.0000
RESID(-1)^2	0.105215	0.011714	8.982207	0.0000
RESID(-1)^2*(RESID(-1)<0)	0.161013	0.018716	8.602959	0.0000
RESID(-2)^2	0.051617	0.007374	6.999690	0.0000
GARCH(-1)	-0.042769	0.006743	-6.342626	0.0000
GARCH(-2)	0.738346	0.010701	68.99619	0.0000

R-squared	0.018050	Mean dependent var	0.043873
Adjusted R-squared	0.017752	S.D. dependent var	1.134751
S.E. of regression	1.124633	Akaike info criterion	2.739534
Sum squared resid	4173.839	Schwarz criterion	2.754319
Log likelihood	-4514.971	Hannan-Quinn criter.	2.744826
Durbin-Watson stat	2.044766		

Source: author’s figure from Eviews.

Appendix 10. Q-statistics for squared residuals of models for OMXR

Lag	GARCH(2,1)		EGARCH(2,2)		GJR(2,2)	
	q-stat	p-value	q-stat	p-value	q-stat	p-value
1	0.0450	**0.83**	0.6804	**0.41**	5.3724	**0.02**
2	0.0468	**0.98**	1.2092	**0.55**	5.8979	**0.05**
3	6.1438	**0.11**	1.4816	**0.69**	6.4188	**0.09**
4	7.3297	**0.12**	2.7592	**0.60**	7.0601	**0.13**
5	7.3356	**0.20**	2.8486	**0.72**	7.0614	**0.22**
6	7.5705	**0.27**	2.8740	**0.82**	7.9442	**0.24**
7	7.6560	**0.36**	2.8743	**0.90**	8.4710	**0.29**
8	7.7276	**0.46**	3.6904	**0.88**	8.5053	**0.39**
9	8.7399	**0.46**	4.4701	**0.88**	9.8971	**0.36**
10	9.3110	**0.50**	4.5359	**0.92**	11.934	**0.29**
11	9.3147	**0.59**	4.7381	**0.94**	11.989	**0.36**
12	10.160	**0.60**	5.4173	**0.94**	13.815	**0.31**
13	10.291	**0.67**	6.5173	**0.93**	14.077	**0.37**
14	10.552	**0.72**	9.9336	**0.78**	14.321	**0.43**
15	10.693	**0.77**	9.9710	**0.82**	14.328	**0.50**
16	23.948	**0.09**	33.677	0.01	23.431	**0.10**
17	24.606	**0.10**	34.063	0.01	24.189	**0.11**
18	27.290	**0.07**	36.619	0.01	25.247	**0.12**
19	32.278	**0.03**	45.535	0.00	30.133	**0.05**
20	32.529	**0.04**	46.551	0.00	30.599	**0.06**
21	33.247	**0.04**	46.996	0.00	31.597	**0.06**
22	33.520	**0.06**	47.226	0.00	31.771	**0.08**
23	33.530	**0.07**	47.321	0.00	32.672	**0.09**
24	33.833	**0.09**	47.360	0.00	32.971	**0.11**
25	34.047	**0.11**	47.373	0.00	33.194	**0.13**
26	34.705	**0.12**	47.584	0.01	33.708	**0.14**
27	34.706	**0.15**	47.611	0.01	33.748	**0.17**
28	34.706	**0.18**	48.033	0.01	33.794	**0.21**
29	34.850	**0.21**	48.086	0.01	33.969	**0.24**
30	35.570	**0.22**	49.500	0.01	34.407	**0.27**
31	36.063	**0.24**	49.610	**0.02**	35.198	**0.28**
32	36.095	**0.28**	49.655	**0.02**	35.221	**0.32**
33	36.205	**0.32**	49.726	**0.03**	35.413	**0.36**
34	36.273	**0.36**	49.938	**0.04**	35.425	**0.40**
35	36.564	**0.40**	50.149	**0.05**	36.342	**0.41**
36	38.862	**0.34**	51.704	**0.04**	42.083	**0.22**

Note: p-values higher than 0.01 marked in bold.

Source: author’s table base on calculation in Eviews.

Appendix 11. Q-statistics for squared residuals of models for OMXT

Lag	GARCH(2,1)		EGARCH(2,1)		GJR(1,2)	
	q-stat	p-value	q-stat	p-value	q-stat	p-value
1	0.2030	**0.65**	0.0064	**0.94**	0.0098	**0.92**
2	0.2217	**0.90**	0.7275	**0.70**	0.1807	**0.91**
3	0.3838	**0.94**	0.8098	**0.85**	0.7069	**0.87**
4	1.2373	**0.87**	1.1272	**0.89**	1.6475	**0.80**
5	1.2497	**0.94**	1.1599	**0.95**	1.7111	**0.89**
6	1.4670	**0.96**	1.2856	**0.97**	2.1356	**0.91**
7	2.1922	**0.95**	2.2016	**0.95**	3.1954	**0.87**
8	2.2811	**0.97**	2.3898	**0.97**	3.4171	**0.91**
9	2.8907	**0.97**	2.8672	**0.97**	4.1619	**0.90**
10	2.9329	**0.98**	2.9140	**0.98**	4.3499	**0.93**
11	4.1770	**0.96**	3.8796	**0.97**	5.8047	**0.89**
12	4.4976	**0.97**	4.0569	**0.98**	6.2407	**0.90**
13	4.7078	**0.98**	4.3238	**0.99**	6.3629	**0.93**
14	5.2975	**0.98**	4.9397	**0.99**	7.0340	**0.93**
15	5.3057	**0.99**	4.9433	**0.99**	7.0380	**0.96**
16	6.4685	**0.98**	5.7638	**0.99**	8.2788	**0.94**
17	6.4729	**0.99**	5.8085	**0.99**	8.2886	**0.96**
18	7.8726	**0.98**	6.7585	**0.99**	9.6478	**0.94**
19	7.8729	**0.99**	6.9231	**1.00**	9.6498	**0.96**
20	8.3718	**0.99**	7.4668	**1.00**	10.336	**0.96**
21	10.046	**0.98**	9.0052	**1.00**	12.207	**0.93**
22	10.355	**0.98**	9.0984	**1.00**	12.565	**0.95**
23	10.714	**0.99**	9.2964	**1.00**	12.883	**0.95**
24	10.796	**0.99**	9.4032	**1.00**	12.982	**0.97**
25	11.468	**0.99**	10.174	**1.00**	13.598	**0.97**
26	11.753	**0.99**	10.587	**1.00**	13.821	**0.98**
27	64.918	0.00	104.05	0.000	78.668	0.000
28	65.687	0.00	104.57	0.000	79.492	0.000
29	66.116	0.00	104.79	0.000	79.931	0.000
30	66.882	0.00	105.45	0.000	80.720	0.000
31	66.930	0.00	105.46	0.000	80.749	0.000
32	67.036	0.00	105.48	0.000	80.824	0.000
33	67.230	0.00	105.68	0.000	80.951	0.000
34	68.028	0.00	106.47	0.000	81.654	0.000
35	68.478	0.00	106.98	0.000	82.074	0.000
36	68.478	0.00	106.99	0.000	82.078	0.000

Note: p-values higher than 0.01 marked in bold.

Source: author's table base on calculation in Eviews.

Appendix 12. Q-statistics for squared residuals of models for OMXV

Lag	GARCH(2,2)		EGARCH(1,2)		GJR(2,2)	
	q-stat	p-value	q-stat	p-value	q-stat	p-value
1	0.0249	**0.88**	0.5759	**0.45**	0.0009	**0.98**
2	0.0635	**0.97**	0.7911	**0.67**	0.0591	**0.97**
3	0.0656	**1.00**	0.7963	**0.85**	0.2596	**0.97**
4	0.2085	**1.00**	0.9605	**0.92**	0.3975	**0.98**
5	0.4893	**0.99**	1.3859	**0.93**	0.9746	**0.97**
6	0.4915	**1.00**	1.4343	**0.96**	0.9770	**0.99**
7	0.5515	**1.00**	1.4578	**0.98**	1.1419	**0.99**
8	0.7205	**1.01**	1.8116	**0.99**	1.3558	**1.00**
9	0.8024	**1.00**	1.8196	**0.99**	1.3863	**1.00**
10	0.8872	**1.00**	2.0187	**1.00**	1.5697	**1.00**
11	0.9175	**1.00**	2.0211	**1.00**	1.5775	**1.00**
12	0.9657	**1.00**	2.2203	**1.00**	1.6811	**1.00**
13	0.9712	**1.00**	2.3851	**1.00**	1.8297	**1.00**
14	1.3415	**1.00**	2.9765	**1.00**	1.9974	**1.00**
15	2.8701	**1.00**	3.6700	**1.00**	2.7563	**1.00**
16	2.8958	**1.00**	3.6854	**1.00**	3.4791	**1.00**
17	3.1557	**1.00**	3.9883	**1.00**	3.7628	**1.00**
18	3.2721	**1.00**	4.1679	**1.00**	3.7679	**1.00**
19	3.3272	**1.00**	4.2395	**1.00**	3.8202	**1.00**
20	3.4106	**1.00**	4.2742	**1.00**	3.8202	**1.00**
21	3.5040	**1.00**	4.4249	**1.00**	3.9226	**1.00**
22	3.5208	**1.00**	4.4635	**1.00**	4.0357	**1.00**
23	3.8846	**1.00**	4.9685	**1.00**	4.3667	**1.00**
24	4.1001	**1.00**	5.2043	**1.00**	4.4062	**1.00**
25	4.2182	**1.00**	5.2529	**1.00**	4.4683	**1.00**
26	4.2980	**1.00**	5.2744	**1.00**	4.4802	**1.00**
27	4.5724	**1.00**	5.4079	**1.00**	4.5801	**1.00**
28	4.7054	**1.00**	5.4728	**1.00**	4.5932	**1.00**
29	4.7063	**1.00**	5.4824	**1.00**	4.5938	**1.00**
30	4.8000	**1.00**	5.6669	**1.00**	4.9189	**1.00**
31	4.9543	**1.00**	5.8312	**1.00**	5.0226	**1.00**
32	5.0087	**1.00**	5.8407	**1.00**	5.0520	**1.00**
33	5.2464	**1.00**	6.0207	**1.00**	5.1269	**1.00**
34	5.5214	**1.00**	6.2478	**1.00**	5.2604	**1.00**
35	5.6447	**1.00**	6.6356	**1.00**	5.6086	**1.00**
36	5.8010	**1.00**	6.9657	**1.00**	6.0653	**1.00**

Note: p-values higher than 0.01 marked in bold.

Source: author’s table base on calculation in Eviews.

Appendix 13. Residuals ARCH test output for OMXR

GARCH(2,1)

F-statistic	0.044875	Prob. F(1,3299)	0.8322
Obs*R-squared	0.044901	Prob. Chi-Square(1)	0.8322

EGARCH(2,2)

F-statistic	0.679371	Prob. F(1,3299)	0.4099
Obs*R-squared	0.679643	Prob. Chi-Square(1)	0.4097

GJR(2,2)

F-statistic	5.371680	Prob. F(1,3299)	0.0205
Obs*R-squared	5.366199	Prob. Chi-Square(1)	0.0205

Source: author's table base on calculation in Eviews.

Appendix 14. Residuals ARCH test output for OMXT

GARCH(2,1)

F-statistic	0.202657	Prob. F(1,3300)	0.6526
Obs*R-squared	0.202767	Prob. Chi-Square(1)	0.6525

EGARCH(2,1)

F-statistic	0.006432	Prob. F(1,3300)	0.9361
Obs*R-squared	0.006435	Prob. Chi-Square(1)	0.9361

GJR(1,2)

Heteroskedasticity Test: ARCH

F-statistic	0.009802	Prob. F(1,3300)	0.9211
Obs*R-squared	0.009808	Prob. Chi-Square(1)	0.9211

Source: author's table base on calculation in Eviews.

Appendix 15. Residuals ARCH test output for OMXV

GARCH(2,2)

F-statistic	0.024852	Prob. F(1,3299)	0.8747
Obs*R-squared	0.024867	Prob. Chi-Square(1)	0.8747

EGARCH(1,2)

Heteroskedasticity Test: ARCH

F-statistic	0.574942	Prob. F(1,3299)	0.4484
Obs*R-squared	0.575190	Prob. Chi-Square(1)	0.4482

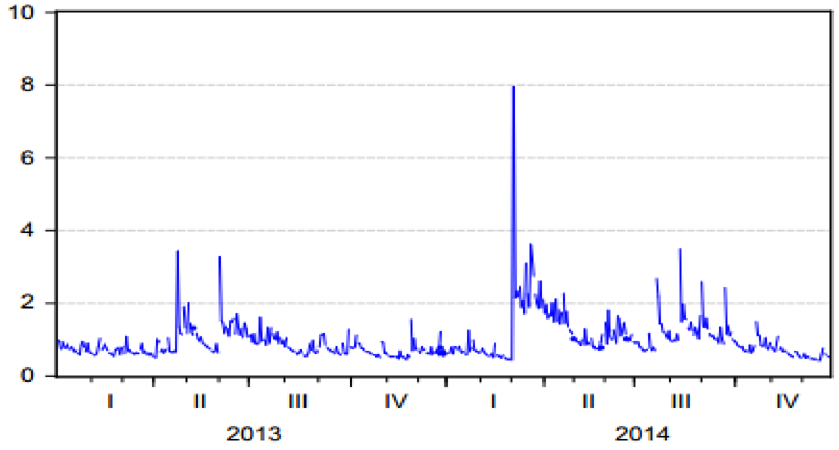
GJR(2,2)

F-statistic	0.000944	Prob. F(1,3299)	0.9755
Obs*R-squared	0.000945	Prob. Chi-Square(1)	0.9755

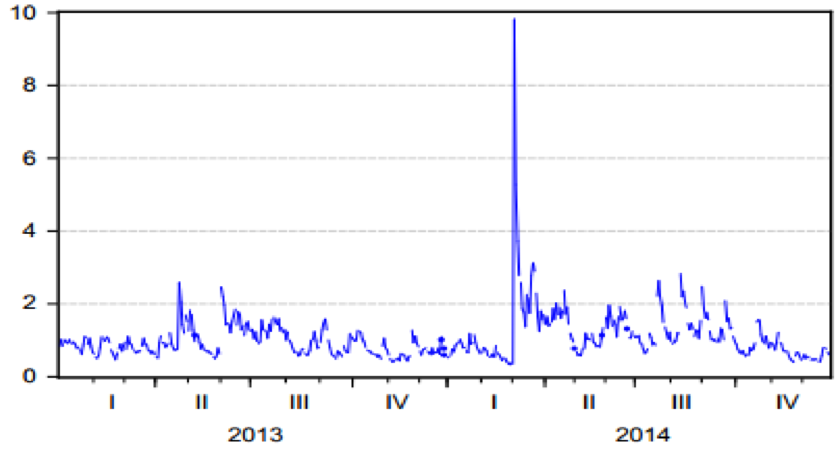
Source: author’s table base on calculation in Eviews.

Appendix 16. OMXR variance forecast

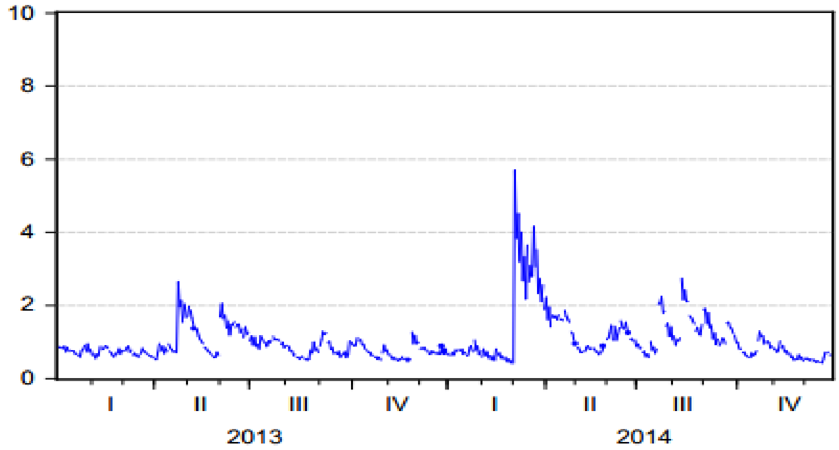
GARCH(2,1)



EGARCH(2,2)



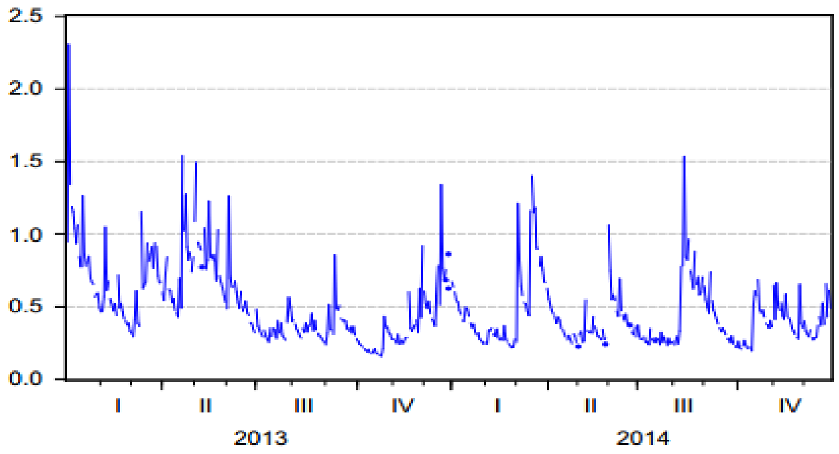
GJR(2,2)



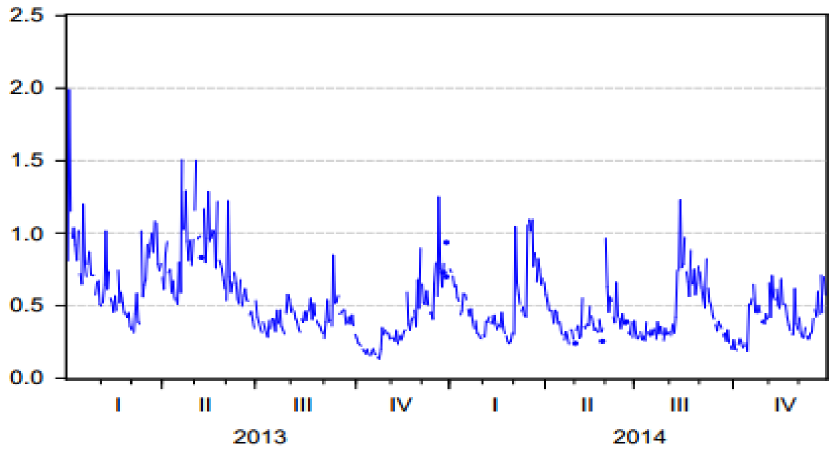
Source: author’s figures from Eviews.

Appendix 17. OMXT variance forecast

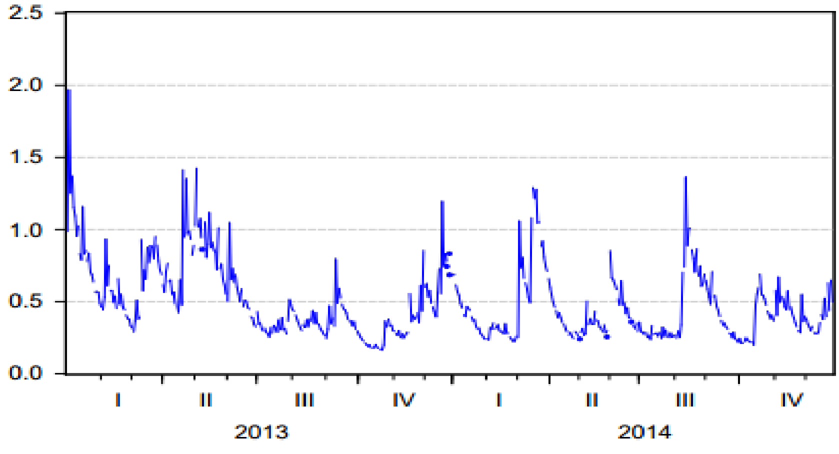
GARCH(2,1)



EGARCH(2,1)



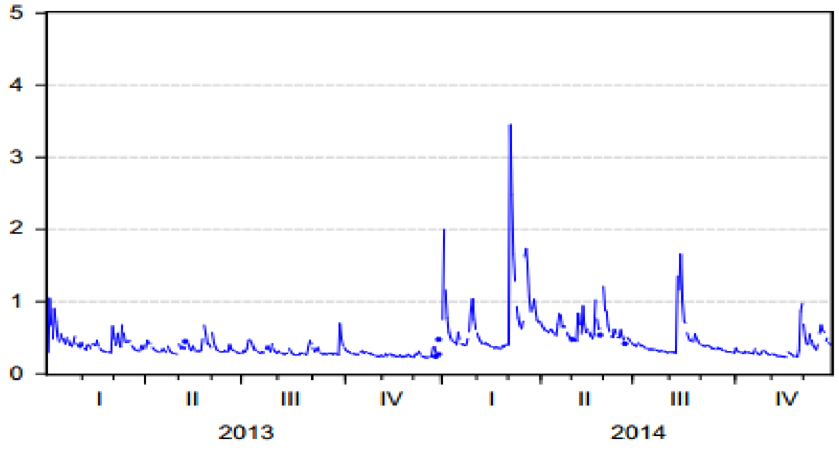
GJR(1,2)



Source: author's figures from Eviews.

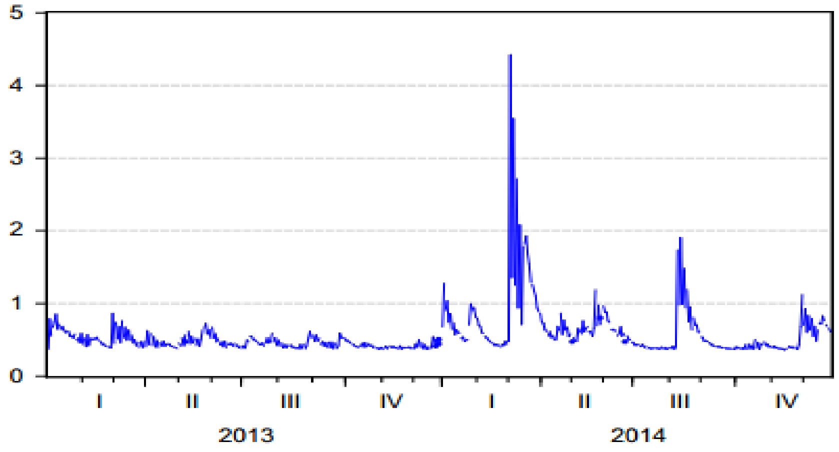
Appendix 18. OMXV variance forecast

GARCH(2,2)



EGARCH(2,1)

GJR(2,2)



Source: author’s figures from Eviews.